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Robust Static Output Feedback H_2/H_∞ Control Synthesis with Pole Placement Constraints: An LMI Approach

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Abstract: This paper studies the robust static output feedback (SOF) problem considering pole placement constraints for linear systems with polytopic uncertainty as well as linear parameter varying (LPV) systems. New linear matrix inequality (LMI) approaches are proposed for the SOF controller design while the pole placement, H_2, and H_∞ constraints are guaranteed. In addition, the gain-scheduled SOF controller will be designed for LPV systems if system parameters are measured. The proposed methods can be applied to general linear systems without imposing any constraints on system matrices. The performance and effectiveness of the proposed methods are shown using two examples.

Keywords: Gain-scheduled controller, linear parameter dependent system, pole placement constraint, robust static output feedback.

1. INTRODUCTION

In control theory and practical applications, the robust static output feedback (SOF) controller design is one of the important and challenging problems. SOF controllers have simpler structures with a lower implementation cost in comparison with the other control methods [1]. In general, the design problems of such controllers are non-convex or the sufficient conditions cannot be derived by linear matrix inequality (LMI) optimization solutions [2–5]. However, many researchers have employed iterative and non-iterative solution algorithms to use LMI-based methods [6].

The SOF problem has been proposed in [7–9] through iterative LMI algorithms. A two-step optimization method has been developed to solve the H_∞ problem in [10]. Additionally, an SOF controller with a limited frequency range has been developed in [11] for an isolator model. In [12], by defining two change variables, a non-iterative method has been suggested for the SOF problem. However, the proposed methods in [7–12] are presented for linear time-invariant (LTI) systems. Using the change variables defined in [12], the parameter-dependent SOF control has been obtained in [13]. Moreover, sufficient LMI conditions in the SOF problem have been obtained in [6] for LTI systems with parametric uncertainties. In addition, the SOF controller has been given for LTI systems with polytopic uncertainty in [14–16]. However, the algorithms presented in [12,16] need to have the full row rank output matrix. Further, only the robust stability has been considered in [14, 15].

In addition to the robust stability, an SOF controller has been designed by imposing additional constraints on the transient closed-loop response which is related to the location of closed-loop poles [17]. The sufficient LMIs considering the pole placement constraints have been studied in [18,19], and [20]. Additionally, the problem of designing a robust linear parameter varying (LPV) state-feedback has been solved in [21] so that the pole placement constraints are satisfied.

This paper combines the robust SOF controller design with the closed-loop poles location concepts to derive the LMI approaches for both LTI systems with polytopic uncertainty and LPV systems. Accordingly, the closed-loop poles are located in the desired region of the complex plane. Furthermore, the robust stability is guaranteed without imposing any constraints on output matrix. In addition, the robust SOF is studied as a gain-scheduled problem for LPV systems to achieve better robustness.

The stability is the first requirement in control theory and practice. Sometimes, when noise, disturbance, and unmodeled dynamics are considered, the robust H_2 and H_∞ performances will be necessary [22]. Furthermore, a mixed H_2/H_∞ control can guarantee the robustness of the design as well as better performance on control and state signals [23]. An SOF H_∞ controller using an iterative algorithm in [7], SOF H_∞ controller with pole placement constraint using a non-smooth method in [24], SOF H_2

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controller with eigenvalue assignment in [25], and sub-optimal H_2/H_∞ controller by bilinear matrix inequality-based algorithm in [16] have been proposed for LTI systems. However, these methods can be applied for neither LPV systems nor LTI systems with polytopic uncertainty since the proposed linear inequalities become nonlinear due to time-varying/uncertain parameters. Moreover, for satisfying H_∞, H_2, and H_2/H_∞ performances for LTI systems with polytopic uncertainty, the SOF controller has been presented in [15, 16, 26], and [27], respectively. Furthermore, a line search method calculating the suboptimal gain-scheduled SOF controller has been introduced in [28] for LPV systems. In [29], the SOF control synthesis satisfying the H_2 performance has been developed for LPV systems using an iterative two-stage algorithm. Additionally, the problem of designing the robust H_∞ SOF controller for polynomial systems using an iterative sum of squares decomposition has been presented in [30]. A design method for the gain-scheduled SOF controller for an LPV system has been presented in [31]. However, the system output matrix in [31] is constant and the effects of disturbance and noise on the system output are not taken into account.

In this paper, some LMI-based algorithms based on H_2, H_∞, and H_2/H_∞ criteria have been proposed to design the SOF controller considering the pole placement constraints for general linear systems. These LMI approaches use a line search over some scalar variables. The methods can be applied even if the output system matrix is not full rank. Furthermore, for reducing the conservatism, the Lyapunov matrix is considered to be parameter-dependent for LTI systems with polytopic uncertainties. The SOF gain-scheduled controller is also designed for LPV systems to reduce the conservatism and achieve better performance. The gain-scheduled controllers use measured system parameters and interpolate them in real-time. However, if system parameters are not available, it will be shown that one fixed SOF can be designed. The proposed methods have several advantages over recent works: a) the controller is output feedback that can guarantee pole location constraints. However, the methods in [12, 14, 15] only satisfy the closed-loop stability. In addition, the full-state measurement has been assumed in [16, 19]. b) the techniques suggested in [15, 16, 26], and [27] cannot be used for LPV systems, neither would they consider the constraints on the closed-loop pole locations. c) in [27], a low pass filter, in series to the measured output of the system has been added to the open-loop model to derive SOF conditions. Without additional filters in [15] and [31], the problem synthesis has been extended to the SOF controller design where the output matrix should be full row rank. However, these studies on SOF synthesis do not take account the linear systems without imposing no limitations on system matrices. Furthermore, the proposed methods in [29] and [30] are iterative.

The rest of this paper is organized as follows: Section 2 presents the problem description and some preliminary lemmas. Section 3 provides a robust SOF method guaranteeing the stability and pole placement constraints. Two H_2 LMI-based methods are proposed in Section 4. In Section 5, the H_∞ and H_2/H_∞ algorithms are introduced with pole placement constraints. Finally, two numerical examples are given in Section 6.

The notation is fairly standard. In symmetric matrices, the symbol $(*)$ shows a term which can be derived by symmetry. I, $diag(\dots)$, $trace(\dots)$, and $\otimes$ indicate the identity matrix with the appropriate dimension, a block diagonal matrix, the trace of a matrix, and Kronecker product, respectively. The notation F_{pn} indicates that $F \in \mathcal{R}^{p\times n}$. $He(F) = F + F^T$ will also be used in this paper.

2. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider a continues-time system described by the following state-space equations:

$$\begin{aligned}
&\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) + E(\theta)w(t),\\
&z_2(t) = C_1(\theta)x(t) + D(\theta)u(t),\\
&z_\infty(t) = C_{\infty 1}(\theta)x(t) + D_\infty(\theta)u(t) + F_\infty(\theta)w(t),\\
&y(t) = C_2(\theta)x(t) + H(\theta)w(t), \qquad (1)
\end{aligned}$$

$$\Big\{\theta,\ A(\theta),\ B(\theta),\ E(\theta),\ C_1(\theta),\ D(\theta),\ C_{\infty 1}(\theta),\ D_\infty(\theta),\ F_\infty(\theta),\ C_2(\theta),\ H(\theta) = \sum_{i=1}^{\mu} \alpha_i \big[\theta_i, A_i, B_i, E_i, C_{1i}, D_i, C_{\infty 1i}, D_{\infty i}, F_{\infty i}, C_{2i}, H_i\big]\Big\}, \qquad (2)$$

where θ is the time-varying parameter vector, $x(t) \in \mathcal{R}^n$ is the state variable, $u(t) \in \mathcal{R}^m$ is the control input, $w(t) \in \mathcal{R}^f$ is the noise, disturbance, or un-modeled dynamics, $z_2(t) \in \mathcal{R}^{q_1}$ is the H_2 controlled output variable, $z_\infty(t) \in \mathcal{R}^{q_2}$ is the H_∞ output variable, and $y(t) \in \mathcal{R}^p$ is the measurement output. The system matrices and the parameters belong to the polytope set (see (2)) where μ is the number of parameter vertices. System (1) can be stated as an LTI system with polytopic uncertainty or an LPV system. Additionally, either $D(\theta)$ or $H(\theta)$ is zero in the open-loop system (1) because $z_2(t)$ in the closed-loop system should not have a direct dependency of $w(t)$ [27]. The goal of this paper is to design the control feedback

$$u(t) = K\,y(t) = K\left(C_2(\theta)x(t) + H(\theta)\,w(t)\right), \qquad (3)$$

such that the closed-loop system

$$\begin{aligned}
&\dot{x}(t) = A_{cl}(\theta)x(t) + B_{cl}(\theta)w(t),\\
&z_\infty(t) = C_{\infty cl}(\theta)x(t) + D_{\infty cl}(\theta)w(t),\\
&z_2(t) = C_{cl}(\theta)x(t), \qquad (4)
\end{aligned}$$

where

$$\begin{aligned}
&A_{cl}(\theta)=A(\theta)+B(\theta)KC_2(\theta),\\
&B_{cl}(\theta)=E(\theta)+B(\theta)KH(\theta),\\
&C_{cl}(\theta)=C_1(\theta)+D(\theta)KC_2(\theta),\\
&C_{\infty cl}(\theta)=C_{\infty 1}(\theta)+D_\infty(\theta)KC_2(\theta),\\
&D_{\infty cl}(\theta)=F_\infty(\theta)+D_\infty(\theta)KH(\theta)
\end{aligned}\tag{5}$$

satisfy the following conditions:

a) The closed-loop poles are located in the desired region of the complex plane.

b) The H_2 performance from $z_2(t)$ to $w(t)$ and H_∞ performance from $z_\infty(t)$ to $w(t)$ are simultaneously guaranteed or

$$\|T_\infty\|_\infty<\gamma,\quad \|T_2\|_2<\gamma.\tag{6}$$

Furthermore, the gain-scheduled controller will be proposed if the system is considered LPV. In the following, some preliminary lemmas have been brought.

Remark 1: In many previous studies, the matrices C_2 and H are based on some assumptions. The output matrix C_2 in [12] and [16] is assumed to be full-rank. The matrices C_2 and H are deemed to be constant and zero in [31], respectively. The matrix C_2 is an identity matrix and the matrix H is zero in [18, 19] and [20]. However, the methods proposed in this paper impose any limitations on these matrices.

Lemma 1 [12]**:** By assuming $C_2(\theta)$ is a full rank matrix, if there exist matrices $W=W^T>0$, M, and R such that

$$He(A(\theta)W+B(\theta)RC_2(\theta))<0,\tag{7a}$$
$$MC_2(\theta)=C_2(\theta)W,\tag{7b}$$

then the open-loop system (1) will be stable by considering $K=RM^{-1}$.

Lemma 2 [18]**:** The poles of the closed-loop system (4) are in the desired region, if there exists a symmetric matrix $W>0$ satisfying

$$\bar{\alpha}\otimes W+He\left(\bar{\beta}\otimes(A_{cl}(\theta)W)\right)<0,\tag{8}$$

where matrices $\overline{\alpha}$ and $\overline{\beta}$ define the desired region of the complex plane.

Lemma 3: If there exist matrices M, R, and $W=W^T>0$ such that

$$\bar{\alpha}\otimes W+He(\bar{\beta}\otimes(A(\theta)W+B(\theta)RC_2(\theta)))<0,\tag{9a}$$
$$MC_2(\theta)=C_2(\theta)W,\tag{9b}$$

then controller $K=RM^{-1}$ locates the poles of the closed-loop system (4) in the selected region of the complex plane. Furthermore, if $\overline{\alpha}=0$ and $\overline{\beta}=1$ are selected, the inequalities (9) conclude the inequalities (7), i.e., the problem is converted into the stability problem.

Proof: By substituting the change variable (7b) into (8), the inequalities (9) will be proved. □

Lemma 4 [32]**:** If there exist matrices $X=X^T>0$ and Z that satisfies

$$\begin{pmatrix} He(XA_{cl}(\theta)) & *\\ B_{cl}^T(\theta)X & -\gamma I\end{pmatrix}<0,$$
$$\begin{pmatrix} X & *\\ C_{cl}(\theta) & Z\end{pmatrix}>0,\ \ trace(Z)<\gamma,\tag{10}$$

or if there exist matrices $W=W^T>0$ and Z such that

$$\begin{pmatrix} He(A_{cl}(\theta)W) & *\\ C_{cl}(\theta)W & -\gamma I\end{pmatrix}<0,\tag{11a}$$
$$\begin{pmatrix} W & *\\ B_{cl}^T(\theta) & Z\end{pmatrix}>0,\ \ trace(Z)<\gamma,\tag{11b}$$

then $\|T_2\|_2<\gamma$ will be guaranteed.

Lemma 5: By considering $H(\theta)$ is zero and $C_2(\theta)$ is full rank, if there exist matrices M, R, and $W=W^T>0$ such that

$$\begin{pmatrix} He(A(\theta)W+B(\theta)RC_2(\theta)) & *\\ E^T(\theta) & -\gamma I\end{pmatrix}<0,\tag{12a}$$
$$\begin{pmatrix} W & *\\ C_1(\theta)W+D(\theta)RC_2(\theta) & Z\end{pmatrix}>0,\ \ trace(Z)<\gamma,\tag{12b}$$
$$MC_2(\theta)=C_2(\theta)W,\tag{12c}$$

or the following inequalities are guaranteed

$$\begin{pmatrix} He(A(\theta)W+B(\theta)RC_2(\theta)) & *\\ C_1(\theta)W+D(\theta)RC_2(\theta) & -\gamma I\end{pmatrix}<0,$$
$$\begin{pmatrix} W & *\\ E^T(\theta) & Z\end{pmatrix}>0,\ trace(Z)<\gamma,\tag{13a}$$
$$MC_2(\theta)=C_2(\theta)W,\tag{13b}$$

then controller $K=RM^{-1}$ satifies $\|T_2\|_2<\gamma$.

Proof: By defining $W=X^{-1}$ and pre- and post-multiplying (10) by $diag(W,I)$ and its transpose, the inequalities (12a) will be obtained from the change variable (12c). Furthermore, the inequality (13a) can be derived from the change variable (13b), the closed-loop system matrices (5), and the inequality (11). □

Lemma 6 [33]**:** $\|T_\infty\|_\infty$ is less than γ if there exists a symmetric matrix $W>0$ such that the following LMI is satisfied.

$$\begin{pmatrix} He(A_{cl}(\theta)W) & * & *\\ B_{cl}^T(\theta) & -\lambda I & *\\ C_{\infty cl}(\theta)W & D_{\infty cl}(\theta) & -\lambda I\end{pmatrix}<0.\tag{14}$$

Lemma 7 [26]**:** The following LMIs are equivalent.

$$\text{(i)}\ \begin{pmatrix} \bar{T} & *\\ \beta\bar{P}^T+\bar{U}\bar{A} & -\beta\left(\bar{U}+\bar{U}^T\right)\end{pmatrix}<0,$$
$$\text{(ii)}\ \bar{T}<0,\ \ \bar{T}+\bar{A}^T\bar{P}^T+\bar{P}\bar{A}<0,\tag{15}$$

where the matrices $\overline{T}$, $\overline{P}$, $\overline{U}$, and $\overline{A}$ have the appropriate dimension and parameter β is a positive scalar.

3. SYNTHESIS GUARANTEEING THE CLOSED-LOOP POLES LOCATION

In this section, the object is to introduce a sufficient LMI-based condition for SOF controller design considering pole placement constraints. These constraints define the location of the poles of the closed-loop system in the desired region of the complex plane. Further information about the determination of the desired region has been illustrated in [18]. By applying pole placement constraints, Theorem 1 shows the sufficient conditions for SOF controller design if the system (1) is LTI with polytopic uncertainty.

Theorem 1: Consider the open-loop system (1) with polytopic uncertainty (2). For known matrices $\overline{\alpha}$ and $\overline{\beta}$ that determine the desired region of the complex plane, and scalar β, if there exist matrices U, V, and symmetric matrices $Q_j > 0$, $i = 1, 2, \ldots, \mu$ satisfying

$$\begin{aligned} &\phi_{ii} < 0, \;\; i = 1,2,\ldots,\mu, \\ &\phi_{ij} + \phi_{ji} < 0, \;\; i < j = 1,2,\ldots,\mu, \end{aligned} \tag{16}$$

where

$$\begin{aligned} &\phi_{ij} = \begin{pmatrix} A_{11} & * \\ A_{21} & -\beta I \otimes \left(U + U^T\right) \end{pmatrix}, \\ &A_{11} = \bar{\alpha} \otimes Q_j + He\left(\bar{\beta} \otimes \left(A_i Q_j + B_i V F_{pn}^j\right)\right), \\ &A_{21} = (\bar{\beta} I_{m1}) \otimes (V^T B_i^T) + \bar{\beta} \otimes \left(C_{2i} Q_j - U F_{pn}^j\right), \end{aligned} \tag{17}$$

$$F_{pn}^j = \begin{cases} \left(C_2 C_2^T\right)^{-1} C_2, & C_2 \text{ is fixed and full rank}, \\ C_2, & C_2 \text{ is fixed and not full rank}, \\ C_{2j}, & C_2 \text{ is non-fixed}, \end{cases} \tag{18}$$

then the SOF controller

$$K = V U^{-1}, \tag{19}$$

locates the system poles (1) in the desired region of the complex plane.

Proof: The closed-loop poles will be in the desired region of the complex plane if the inequality (8) is guaranteed or

$$\bar{\alpha} \otimes Q(\theta) + He\left(\bar{\beta} \otimes A_{cl} Q(\theta)\right) < 0, \tag{20}$$

where $W = Q(\theta)$, $N = \mu$, and $A_{cl} = A(\theta) + B(\theta) K C_2(\theta)$. By utilizing the basic property of Kronecker product $(AC) \otimes (BD) = (A \otimes B)(C \otimes D)$, the inequality (20) can also be represented as $\overline{T} + \overline{A}^T \overline{P}^T + \overline{PA} < 0$ if

$$\begin{aligned} &\bar{T} = \bar{\alpha} \otimes Q(\theta) + He(\bar{\beta} \otimes (A(\theta) Q(\theta) + B(\theta) V F_{pn}(\theta))), \\ &\bar{P} = I \otimes (B(\theta) V), \\ &\bar{A} = \left(I \otimes U^{-1}\right) \left(\bar{\beta} \otimes (C_2(\theta) Q(\theta) - U F_{pn}(\theta))\right), \\ &\bar{U} = I \otimes U. \end{aligned} \tag{21}$$

From Lemma 7, if the following inequality is held, $\overline{T} + \overline{A}^T \overline{P}^T + \overline{PA} < 0$ is also satisfied.

$$\begin{aligned} &\begin{pmatrix} \zeta_{11} & * \\ \zeta_{21} & -\beta\left((I \otimes U) + (I \otimes U)^T\right) \end{pmatrix} < 0, \\ &\zeta_{11} = \bar{\alpha} \otimes Q(\theta) \\ &\qquad + He\left(\bar{\beta} \otimes (A(\theta) Q(\theta) + B(\theta) V F_{pn}(\theta))\right), \\ &\zeta_{21} = (\beta I \otimes B(\theta) V)^T + \bar{\beta} \otimes \left(C_2(\theta) Q(\theta) - U F_{pn}(\theta)\right). \end{aligned} \tag{22}$$

Now, by substituting the polytopic uncertainties (2) and defining $F_{pn}(\theta) = \int_{j=1}^{\mu} \alpha_j F_{pn}^j$ and $Q(\theta) = \int_{j=1}^{\mu} \alpha_j Q_j$, the inequality (22) can be rewritten as follows:

$$\sum_{i=1}^{\mu} \alpha_i^2 \phi_{ii} + \sum_{i=1}^{\mu} \sum_{i<j}^{\mu} \alpha_i \alpha_j (\phi_{ij} + \phi_{ji}) < 0, \tag{23}$$

where ϕ_{ij} is given in (17). The inequalities (23) will be satisfied if (16) is feasible. □

Remark 2: In this paper, the proposed techniques use a simple procedure to convert the inequality (22) into a set of inequalities. It should be noted that this technique is also used in [15, 26]. Furthermore, the suggested method in [34] can also be applied.

However, if the system is LPV and the inequality (8) is used, Theorem 1 cannot be applied because the Lyapunov matrix Q should be considered constant [32]. The sufficient conditions guaranteeing the pole placement constraints for an LPV system have been presented in the following corollary.

Corollary 1: Consider the closed-loop LPV system (4). By assuming known matrices $\overline{\alpha}$, $\overline{\beta}$, and scalar β, if there exist matrices V, U_i $\forall i = 1, 2, \ldots, \mu$, and symmetric fixed matrix $Q > 0$ such that the following LMIs are guaranteed,

$$\begin{aligned} &\varphi_{ii} < 0, \;\; i = 1,2,\ldots,\mu, \\ &\varphi_{ij} + \varphi_{ji} < 0, \;\; i < j = 1,2,\ldots,\mu, \end{aligned} \tag{24}$$

where

$$\begin{aligned} &\varphi_{ij} = \begin{pmatrix} A_{11} & * \\ A_{21} & -\beta I \otimes \left(U_i + U_i^T\right) \end{pmatrix}, \\ &A_{11} = \bar{\alpha} \otimes Q + He\left(\bar{\beta} \otimes \left(A_i Q + B_i V F_{pn}^j\right)\right), \\ &A_{21} = \left(\bar{\beta} I\right) \otimes \left(V^T B_i^T\right) + \bar{\beta} \otimes \left(C_{2i} Q - U_i F_{pn}^j\right), \end{aligned} \tag{25}$$

then the gain-scheduled SOF controller

$$K(\theta) = \sum_{i=1}^{\mu} V (\alpha_i U_i)^{-1}, \tag{26}$$

places the closed-loop poles in the desired region of the complex plane.

Proof: By defining $Q(\theta) = \sum_{j=1}^{\mu} \alpha_j Q$, $U(\theta) = \sum_{i=1}^{\mu} \alpha_i U_i$, $i = 1, 2, \ldots, \mu$, and the controller (26), this proof will be similar to the proof of Theorem 1. □

Remark 3: If $\overline{\alpha} = 0$ and $\overline{\beta} = 1$ are selected for Theorem 1 and Corollary 1, the pole placement problem is converted into a robust SOF problem. Additionally, for reducing the conservatism, Corollary 1 proposes an LPV controller in which the parameters α_i should be measured in real-time. However, if α_i cannot be measured or estimated, the gain-scheduled problem should be expressed as a fixed LTI controller. Therefore, $U_i = U$, $i = 1, 2, ..., \mu$ should be selected for Corollary 1.

Furthermore, if the output matrix C_2 has full row rank, Lemma 3 shows that the pole placement problem becomes an LMI-based approach. The following theorem illustrates that the proposed techniques in Theorem 1 and Corollary 1 can achieve the less, or at least the same conservatism results in comparison with the method presented by Lemma 3.

Theorem 2: If the conditions of Lemma 3 are held, then the proposed conditions of Theorem 1 and Corollary 1 will be satisfied too.

Proof: In this proof, the dependency of system matrices (1) on parameter θ has been ignored for simplicity; e.g., C_2 shows the matrix $C_2(\theta)$. From (9b), the following relation can be obtained.

$$C_2WC_2^T + C_2WC_2^T = MC_2C_2^T + C_2C_2^TM^T > 0. \quad (27)$$

From (27), if the inequality (9a) is held, then there exists a sufficiently small scalar $\beta > 0$ such that the following condition is held too.

$$\begin{aligned}&\bar{\alpha}\otimes W + He\left(\bar{\beta}\otimes(AW+BRC_2)\right)\\&\quad+\left(\beta I\otimes\left(RC_2C_2^T\right)^T B^T\right)^T\\&\quad\times\left(\beta I\otimes\left(MC_2C_2^T+C_2C_2^TM^T\right)\right)^{-1}\\&\quad\times\left(\beta I\otimes\left(RC_2C_2^T\right)^T B^T\right)<0.\end{aligned} \quad (28)$$

Further, the equality $\overline{\beta}\otimes C_2W = \overline{\beta}\otimes MC_2$ obtains from the change variable (9b). Therefore, the following equality can be written.

$$\begin{aligned}&\beta I\otimes\left(RC_2C_2^T\right)^T B^T\\&=\beta I\otimes\left(RC_2C_2^T\right)^T B^T+\bar{\beta}\otimes C_2W-\bar{\beta}\otimes MC_2.\end{aligned} \quad (29)$$

Then, the inequality (28) by using (29) can be rewritten as follows:

$$\begin{aligned}&\bar{\alpha}\otimes W + He\left(\bar{\beta}\otimes(AW+BRC_2)\right)\\&\quad+\left(\beta I\otimes\left(RC_2C_2^T\right)^T B^T+\bar{\beta}\otimes C_2W-\bar{\beta}\otimes MC_2\right)^T\\&\quad\times\left(\beta I\otimes\left(MC_2C_2^T+C_2C_2^TM^T\right)\right)^{-1}\\&\quad\times\left(\beta I\otimes\left(RC_2C_2^T\right)^T B^T+\bar{\beta}\otimes C_2W-\bar{\beta}\otimes MC_2\right)\\&<0,\end{aligned} \quad (30)$$

or

$$\begin{aligned}&\bar{\alpha}\otimes W + He\left(\bar{\beta}\otimes\left(AW+BRC_2C_2^T\left(C_2C_2^T\right)^{-1}C_2\right)\right)\\&\quad+\left(\beta I\otimes\left(RC_2C_2^T\right)^T B^T+\bar{\beta}\otimes C_2W\right.\\&\quad\left.-\bar{\beta}\otimes MC_2C_2^T\left(C_2C_2^T\right)^{-1}C_2\right)^T\\&\quad\times\left(\beta I\otimes\left(MC_2C_2^T+C_2C_2^TM^T\right)\right)^{-1}\\&\quad\times\left(\beta I\otimes\left(RC_2C_2^T\right)^T B^T+\bar{\beta}\otimes C_2W\right.\\&\quad\left.-\bar{\beta}\otimes MC_2C_2^T\left(C_2C_2^T\right)^{-1}C_2\right)\\&<0.\end{aligned} \quad (31)$$

Now, by defining the change variabl $V = RC_2C_2^T$, $U = MC_2C_2^T$, $Q = W$, and $F_{pn} = \left(C_2C_2^T\right)^{-1}C_2$, the inequality (31) will be

$$\begin{aligned}&\bar{\alpha}\otimes Q + He\left(\bar{\beta}\otimes(AQ+BVF_{pn})\right)\\&\quad+\left(\beta I\otimes V^TB^T+\bar{\beta}\otimes C_2Q-\bar{\beta}\otimes UF_{pn}\right)^T\\&\quad\times\left(\beta I\otimes\left(U+U^T\right)\right)^{-1}\\&\quad\times\left(\beta I\otimes V^TB^T+\bar{\beta}\otimes C_2Q-\bar{\beta}\otimes UF_{pn}\right)<0.\end{aligned} \quad (32)$$

Finally, by applying the schur complement of $\beta I\otimes\left(U+U^T\right)$ to (32), the inequality (22) obtains. In addition, if the inequality (22) is held, Theorem 1 using the controller (19) and symmetric matrices $Q = \sum_{j=1}^{\mu}\alpha_jQ_j > 0$ is proved. Furthermore, if inequality (22) is satisfied, Corollary 1 will be proved by using $U = \sum_{i=1}^{\mu}\alpha_iU_i$, $i = 1, 2, ..., \mu$, $Q = Q^T > 0$, and the controller (26). □

4. SOF H_2 CONTROL SYNTHESIS

In this part with the extension of Lemma 4, the LMI approaches are given for the SOF H_2 problem. The following theorem shows the SOF H_2 controller conditions if the system is LTI with polytopic uncertainty.

Theorem 3: Consider the LTI system (1) with the polytopic uncertainties (2) and matrix F_{pn}^j from (18). For known scalars ρ, ν, and β, if there exist matrices U, V, Z, and symmetric matrices $Q_i > 0$, $i = 1, 2, \ldots, \mu$ such that the following LMIs are satisfied,

$$\begin{aligned}&\delta_{ii}<0,\ \ i=1,2,...,\mu,\\&\delta_{ij}+\delta_{ji}<0,\ \ i<j=1,2,...,\mu,\end{aligned} \quad (33)$$

$$\begin{aligned}&\Theta_{ii}<0,\ \ i=1,2,...,\mu,\\&\Theta_{ij}+\Theta_{ji}<0,\ \ i<j=1,2,...,\mu,\\&trace(Z)<\gamma,\end{aligned} \quad (34)$$

or the following inequalities are feasible.

$$\psi_{ii}<0,\ \ i=1,2,...,\mu,$$

$$\psi_{ij}+\psi_{ji}<0,\ \ i<j=1,2,...,\mu, \tag{35}$$
$$\Psi_{ii}<0,\ \ i=1,2,...,\mu,$$
$$\Psi_{ij}+\Psi_{ji}<0,\ \ i<j=1,2,...,\mu,$$
$$trace(Z)<\gamma, \tag{36}$$

where

$$\delta_{ij}=\begin{pmatrix} He\left(A_iQ_j+B_iVF_{pn}^j\right) & * & * \\ E_i^T+H_j^TV^TB_i^T & -\gamma I & * \\ \beta\left(B_iV\right)^T+C_{2i}Q_j-UF_{pn}^j & H_i-UH_j & -\beta\left(U+U^T\right) \end{pmatrix}. \tag{37}$$

$$\Theta_{ij}=-\begin{pmatrix} Q_j & * & * \\ C_{1i}Q_j+D_iVF_{pn}^j & Z+He\left(\rho D_iVF_{pq_1}\right) & * \\ C_{2i}Q_j-UF_{pn}^j & -\left(D_iV\right)^T-\rho F_{pq_1} & \beta\left(U+U^T\right) \end{pmatrix}, \tag{38}$$

$$\psi_{ii}=\begin{pmatrix} He\left(A_iQ_j+B_iVF_{pn}^j\right) & * & * \\ C_{1i}Q_j+D_iVF_{pn}^j-\rho\left(B_iVF_{pq_1}\right)^T & -\gamma I-He\left(\rho D_iVF_{pq_1}\right) & * \\ \beta\left(B_iV\right)^T+C_{2i}Q_j-UF_{pn}^j & \beta\left(D_iV\right)^T+\rho UF_{pq_1} & -\beta\left(U+U^T\right) \end{pmatrix}, \tag{39}$$

$$\Psi_{ij}=\begin{pmatrix} -Q_j+\upsilon He\left(B_iVF_{pn}^j\right) & * & * \\ -E_i^T-H_j^TV^TB_i^T & -Z & * \\ \beta\left(B_iV\right)^T-\upsilon UF_{pn}^j & -H_i+UH_j & -\beta\left(U+U^T\right) \end{pmatrix}, \tag{40}$$

$$F_{pq_1}=\begin{cases} I, & p=q_1, \\ \begin{pmatrix} I_p & 0_{p\times(q_1-p)} \end{pmatrix}, & p<q_1, \\ \begin{pmatrix} I_{q_1} \\ 0_{(q_1-p)\times q_1} \end{pmatrix}, & p>q_1, \end{cases} \tag{41}$$

then the system (4) by using $K=VU^{-1}$ guarantees the H_2 performance γ as well as the asymptotic stability.

Proof: By considering $W=X^{-1}$ and pre- and post-multiplying (10) by $diag(W,I)$ and its transpose, the following conditions will be obtained.

$$\begin{pmatrix} He\left(A_{cl}(\theta)Q(\theta)\right) & * \\ B_{cl}^T(\theta) & -\gamma I \end{pmatrix}<0,$$
$$\begin{pmatrix} Q(\theta) & * \\ C_{cl}(\theta)Q(\theta) & Z \end{pmatrix}>0,\ \ trace(Z)<\gamma. \tag{42}$$

The inequality (42) can also be rewritten as

$$\begin{pmatrix} He\left(A_{cl}(\theta)Q(\theta)\right) & * \\ B_{cl}^T(\theta) & -\gamma I \end{pmatrix}<0, \tag{43a}$$

$$\begin{pmatrix} -Q(\theta) & * \\ -C_{cl}(\theta)Q(\theta) & -Z \end{pmatrix}<0, \tag{43b}$$

$$trace(Z)<\gamma. \tag{43c}$$

Since Lemma 7 will be used in this proof, the inequality (42) has been shown as smaller than zero. Since Lemma 7 will be used in this proof, the inequality (42) has been converted into the smaller than zero in the inequality (43b). Now, by defining

$$\bar{T}_1=\begin{pmatrix} He\left(A(\theta)Q(\theta)+B(\theta)VF_{pn}(\theta)\right) & * \\ E^T(\theta)+H^T(\theta)V^TB^T(\theta) & -\gamma I \end{pmatrix},$$
$$\bar{P}_1=\begin{pmatrix} B(\theta)V \\ 0 \end{pmatrix},$$
$$\bar{A}_1=U^{-1}\begin{pmatrix} C_2(\theta)Q(\theta)-UF_{pn}(\theta) & H(\theta)-UH(\theta) \end{pmatrix},$$
$$\bar{U}_1=U,$$
$$\bar{T}_2=\begin{pmatrix} -Q(\theta) & * \\ -C_1(\theta)Q(\theta)-D(\theta)VF_{pn}(\theta) & -Z-He\left(\rho D(\theta)VF_{pq_1}\right) \end{pmatrix},$$
$$\bar{P}_2=\begin{pmatrix} 0 \\ D(\theta)V \end{pmatrix},$$
$$\bar{A}_2=U^{-1}\begin{pmatrix} UF_{pn}(\theta)-C_2(\theta)Q(\theta) & \rho UF_{pq_1} \end{pmatrix},$$
$$\bar{U}_2=U, \tag{44}$$

and by substituting the controller (19) in system (4), the inequalities $\overline{T}_1+\overline{A}_1{}^T\overline{P}_1{}^T+\overline{P}_1\overline{A}_1<0$ and $\overline{T}_2+\overline{A}_2{}^T\overline{P}_2{}^T+\overline{P}_2\overline{A}_2<0$ conclude the inequalities (43a) and (43b), respectively. From Lemma 7, if the following conditions are satisfied, then (43) will also be guaranteed.

$$\begin{pmatrix} \zeta_{11} & * & * \\ \zeta_{21} & -\gamma I & * \\ \zeta_{31} & H(\theta)-UH(\theta) & -\beta\left(U+U^T\right) \end{pmatrix}<0,$$
$$\begin{pmatrix} -Q(\theta) & * & * \\ \sigma_{21} & \sigma_{22} & * \\ \sigma_{31} & \sigma_{32} & -\beta\left(U+U^T\right) \end{pmatrix}<0, \tag{45}$$

where

$$\zeta_{11}=He\left(A(\theta)Q(\theta)+B(\theta)VF_{pn}(\theta)\right),$$
$$\zeta_{21}=E^T(\theta)+H^T(\theta)V^TB^T(\theta),$$
$$\zeta_{31}=\beta\left(B(\theta)V\right)^T+C_2(\theta)Q(\theta)-UF_{pn}(\theta),$$
$$\sigma_{21}=-C_1(\theta)Q(\theta)-D(\theta)VF_{pn}(\theta),$$
$$\sigma_{22}=-Z-He\left(\rho D(\theta)VF_{pq_1}\right),$$
$$\sigma_{31}=-C_2(\theta)Q(\theta)+UF_{pn}(\theta),$$
$$\sigma_{32}=\beta\left(D(\theta)V\right)^T+\rho F_{pq_1}. \tag{46}$$

In addition, by considering the system matrices (2) and $F_{pn}(\theta)=\sum_{j=1}^{\mu}\alpha_jF_{pn}^j$, the inequalities (45) can be represented as follows in which δ_{ij} and Θ_{ij} have been given in (37)-(38).

$$\sum_{i=1}^{\mu}\alpha_i^2\delta_{ii}+\sum_{i=1}^{\mu}\sum_{i<j}^{\mu}\alpha_i\alpha_j(\delta_{ij}+\delta_{ji})<0,$$
$$\sum_{i=1}^{\mu}\alpha_i^2\Theta_{ii}+\sum_{i=1}^{\mu}\sum_{i<j}^{\mu}\alpha_i\alpha_j(\Theta_{ij}+\Theta_{ji})<0,$$
$$trace(Z)<\gamma. \tag{47}$$

The inequalities (47) are guaranteed if (33)-(34) are feasible. The similar procedure of the proof (33)-(34) should be repeated for the proof of (35)-(36). Therefore, by choosing

$$\bar{T}_1=\begin{pmatrix} t_{11} & * \\ t_{21} & -\gamma I-He(\rho D(\theta)VF_{pq_1})\end{pmatrix},$$
$$t_{11}=He(A(\theta)Q(\theta)+B(\theta)VF_{pn}(\theta)),$$
$$t_{21}=C_1(\theta)Q(\theta)+D(\theta)VF_{pn}(\theta)-(\rho B(\theta)VF_{pq_1})^T,$$
$$\bar{P}_1=\begin{pmatrix} B(\theta)V \\ D(\theta)V\end{pmatrix},$$
$$\bar{A}_1=U^{-1}\begin{pmatrix} C_2(\theta)Q(\theta)-UF_{pn}(\theta) & \rho UF_{pq_1}\end{pmatrix},$$
$$\bar{U}_1=U, \tag{48}$$
$$\bar{T}_2=\begin{pmatrix} -Q(\theta)+\upsilon He(B(\theta)VF_{pn}(\theta)) & * \\ -E^T(\theta)+H^T(\theta)V^TB^T(\theta) & -Z\end{pmatrix},$$
$$\bar{P}_2=\begin{pmatrix} B(\theta)V \\ 0\end{pmatrix},$$
$$\bar{A}_2=U^{-1}\begin{pmatrix} -\upsilon UF_{pn}(\theta) & -H(\theta)+UH(\theta)\end{pmatrix},$$
$$\bar{U}_2=U, \tag{49}$$

and then by replacing (20), the inequalities (11a) and (11b) obtain. Finally, the inequalities (35)-(36) are proved by using Lemma 7 and a similar procedure of the proof (33)-(34). □

Theorem 3 introduces two methods for the SOF H_2 controller design if the system (1) is LTI with polytopic uncertainty (2). However, if the open-loop system is LPV, Theorem 3 cannot be applied because the Lyapunov matrix Q should be considered constant [32]. The following corollary considers the LPV systems.

Corollary 2: Consider the open-loop system (1) as an LPV system. For known scalars ν, ρ, and β, if there exist matrices V, U_i, $i=1, 2, ..., \mu$, and symmetric Lyapunov matrix $Q>0$ such that (33)-(34) are feasible in which δ_{ij} and Θ_{ij} are redefined as

$$\delta_{ij}=\begin{pmatrix} He\left(A_iQ+B_iVF_{pn}^j\right) & * & * \\ E_i^T+H_j^TV^TB_i^T & -\gamma I & * \\ \beta(B_iV)^T+C_{2i}Q-U_iF_{pn}^j & H_i-U_iH_j & -\beta\left(U_i+U_i^T\right)\end{pmatrix}, \tag{50}$$

$$\Theta_{ij}=-\begin{pmatrix} Q & * & * \\ C_{1i}Q+D_iVF_{pn}^j & Z+He(\rho D_iVF_{pq_1}) & * \\ C_{2i}Q-U_iF_{pn}^j & -(D_iV)^T-\rho F_{pq_1} & \beta\left(U_i+U_i^T\right)\end{pmatrix}, \tag{51}$$

or the inequalities (35)-(36) are satisfied in which ψ_{ij} and Ψ_{ij} are represented as

$$\psi_{ij}=\begin{pmatrix} He\left(A_iQ+B_iVF_{pn}^j\right) & * & * \\ C_{1i}Q+D_iVF_{pn}^j-\rho(B_iVF_{pq_1}) & -\gamma I-He(\rho D_iVF_{pq_1}) & * \\ \beta(B_iV)^T+C_{2i}Q-U_iF_{pn}^j & \beta(D_iV)^T+\rho U_iF_{pq_1} & -\beta\left(U_i+U_i^T\right)\end{pmatrix}, \tag{52}$$

$$\Psi_{ij}=\begin{pmatrix} -Q+\upsilon He\left(B_iVF_{pn}^j\right) & * & * \\ -E_i^T-H_j^TV^TB_i^T & -Z & * \\ \beta(B_iV)^T-\upsilon U_iF_{pn}^j & -H_i+U_iH_j & -\beta\left(U_i+U_i^T\right)\end{pmatrix}, \tag{53}$$

then the closed-loop system (4) by using the gain-scheduled controller $K(\theta)=\sum_{i=1}^{\mu}V(\alpha_iU_i)^{-1}$ guarantees both the asymptotical stability and $\|T_2\|_2<\gamma$.

Proof: By defining $Q=\sum_{j=1}^{\mu}\alpha_jQ$, $U=\sum_{i=1}^{\mu}\alpha_iU_i$, $i=1, 2, ..., \mu$ and using the SOF controller (26), the proof will be similar to that of Theorem 3 and is omitted. □

Remark 4: Theorem 3 and Corollary 2 propose two methods for the SOF H_2 controller design. However, by considering the less value of performance index γ, one of two methods should be selected by the designer.

The following theorem shows that the suggested H_2 techniques in Theorem 3 and Corollary 2 can guarantee less or at least the same conservative result as Lemma 5 without constraints on the output matrix.

Theorem 4: If the conditions of Lemma 5 are held, then the proposed conditions of Theorem 3 and Corollary 2 are held, too.

Proof: Similar to the proof of Theorem 2, the dependency of system matrices on parameter θ is removed for simplicity. If the inequalities (12) are satisfied, then there exists a sufficiently small scalar $\beta>0$ such that the following LMIs are feasible.

$$\begin{pmatrix} He(AW+BRC_2) & * \\ E^T & -\lambda I\end{pmatrix}+\beta\begin{pmatrix}(RC_2C_2^T)^TB^T & 0\end{pmatrix}^T$$
$$\times(MC_2C_2^T+C_2C_2^TM^T)^{-1}\begin{pmatrix}(RC_2C_2^T)^TB^T & 0\end{pmatrix}$$
$$=\begin{pmatrix} He(AW+BRC_2) & * \\ E^T & -\lambda I\end{pmatrix}+\begin{pmatrix}\beta(RC_2C_2^T)^TB^T & 0\end{pmatrix}^T$$
$$\times\frac{1}{\beta}(MC_2C_2^T+C_2C_2^TM^T)^{-1}\beta\begin{pmatrix}(RC_2C_2^T)^TB^T & 0\end{pmatrix}$$

$$
\begin{aligned}
&< 0,\\
&-\begin{pmatrix} W & * \\ C_1W+DRC_2 & Z \end{pmatrix} + \beta\begin{pmatrix} 0 & (RC_2C_2^T)^T D^T \end{pmatrix}^T\\
&\quad \times \left(MC_2C_2^T + C_2C_2^T M^T\right)^{-1}\begin{pmatrix} 0 & (RC_2C_2^T)^T D^T \end{pmatrix}\\
&= -\begin{pmatrix} W & * \\ C_1W+DRC_2 & Z \end{pmatrix} + \begin{pmatrix} 0 & \beta\,(RC_2C_2^T)^T D^T \end{pmatrix}^T\\
&\quad \times \frac{1}{\beta}(MC_2C_2^T + C_2C_2^T M^T)^{-1}\begin{pmatrix} 0 & \beta(RC_2C_2^T)^T D^T \end{pmatrix}\\
&< 0. \qquad (54)
\end{aligned}
$$

In addition, the following equality can be obtained from the change variable (12c).

$$\beta\left(RC_2C_2^T\right)^T B^T = \beta\left(RC_2C_2^T\right)^T B^T + C_2W - MC_2. \qquad (55)$$

Furthermore, by using (55), the inequality (54) can be rewritten as follows:

$$
\begin{aligned}
&\begin{pmatrix} He\,(AW+BRC_2) & * \\ E^T & -\lambda I \end{pmatrix}\\
&\quad + \begin{pmatrix} \beta\left(RC_2C_2^T\right)^T B^T + C_2W - MC_2 & 0 \end{pmatrix}^T\\
&\quad \times \frac{1}{\beta}\left(MC_2C_2^T + C_2C_2^T M^T\right)^{-1}\\
&\quad \times \begin{pmatrix} \beta\left(RC_2C_2^T\right)^T B^T + C_2W - MC_2 & 0 \end{pmatrix}\\
&< 0, \qquad (56a)\\
&-\begin{pmatrix} W & * \\ C_1W+DRC_2 & Z \end{pmatrix}\\
&\quad + \begin{pmatrix} -C_2W + MC_2 & \beta\left(RC_2C_2^T\right)^T D^T \end{pmatrix}^T\\
&\quad \times \frac{1}{\beta}\left(MC_2C_2^T + C_2C_2^T M^T\right)^{-1}\\
&\quad \times \begin{pmatrix} -C_2W + MC_2 & \beta\left(RC_2C_2^T\right)^T D^T \end{pmatrix}\\
&< 0, \qquad (56b)
\end{aligned}
$$

or

$$
\begin{aligned}
&\begin{pmatrix} He\left(AW + BRC_2C_2^T\left(C_2C_2^T\right)^{-1}C_2\right) & * \\ E^T & -\lambda I \end{pmatrix}\\
&\quad + \left(\beta(RC_2C_2^T)^T B^T + C_2W - MC_2C_2^T(C_2C_2^T)^{-1}C_2 \;\; 0\right)^T\\
&\quad \times \frac{1}{\beta}\left(MC_2C_2^T + C_2C_2^T M^T\right)^{-1}\\
&\quad \times \left(\beta(RC_2C_2^T)^T B^T + C_2W - MC_2C_2^T(C_2C_2^T)^{-1}C_2 \;\; 0\right)\\
&< 0, \qquad (57a)\\
&-\begin{pmatrix} W & * \\ C_1W + DRC_2C_2^T\left(C_2C_2^T\right)^{-1}C_2 & Z \end{pmatrix}\\
&\quad + \left(-C_2W + MC_2C_2^T(C_2C_2^T)^{-1}C_2 \;\; \beta(RC_2C_2^T)^T D^T\right)^T\\
&\quad \times \frac{1}{\beta}\left(MC_2C_2^T + C_2C_2^T M^T\right)^{-1}\\
&\quad \times \left(-C_2W + MC_2C_2^T(C_2C_2^T)^{-1}C_2 \;\; \beta(RC_2C_2^T)^T D^T\right)\\
&< 0. \qquad (57b)
\end{aligned}
$$

By defining the change variables $V = R\ C_2C_2^T$, $U = MC_2C_2^T$, $Q = W$, and $F_{pn} = (C_2C_2^T)^{-1}C_2$, the inequalities (57) will be

$$
\begin{aligned}
&\begin{pmatrix} He\,(AQ + BVF_{pn}) & * \\ E^T & -\lambda I \end{pmatrix}\\
&\quad + \begin{pmatrix} \beta V^T B^T + C_2Q - UF_{pn} & 0 \end{pmatrix}^T\\
&\quad \times \frac{1}{\beta}\left(U + U^T\right)^{-1}\begin{pmatrix} \beta V^T B^T + C_2Q - UF_{pn} & 0 \end{pmatrix}\\
&< 0, \qquad (58a)\\
&-\begin{pmatrix} Q & * \\ C_1Q + DVF_{pn} & Z \end{pmatrix}\\
&\quad + \begin{pmatrix} -C_2Q + UF_{pn} & \beta V^T D^T \end{pmatrix}^T\\
&\quad \times \frac{1}{\beta}\left(U + U^T\right)^{-1}\begin{pmatrix} -C_2Q + UF_{pn} & \beta V^T D^T \end{pmatrix}\\
&< 0. \qquad (58b)
\end{aligned}
$$

Finally, by applying the schur complement of $\beta U + \beta U^T$ to (58), the inequalities (45) will be obtained in which the matrix H and scalar ρ are zero. If the inequality (45) is held, Theorem 3 using the controller (19) and symmetric matrices $Q = \sum_{j=1}^{\mu} \alpha_j Q_j$ is proved. Therefore, the inequalities (33)-(34) are concluded from (47). Further, the inequalities (35)-(36) can be proved similar to the above procedure from inequalities (13). Furthermore, by considering $U = \sum_{i=1}^{\mu} \alpha_i U_i$, $i = 1, 2, \ldots, \mu$, $Q = Q^T > 0$, and the controller (26), the proof can be done for Corollary 2. □

5. SOF H_2/H_∞ CONTROL SYNTHESIS WITH POLE PLACEMENT CONSTRAINTS

In this section, the gain-scheduled SOF H_∞ controller is proposed for the LPV system (1). In addition, the problem of H_2/H_∞ with pole placement constraints will be proposed.

Lemma 8: Consider the system (1) as an LTI system with polytopic uncertainty. For known scalar parameters ρ and β, if there exist matrices U, V, and $Q_j = Q_j^T > 0$, $i = 1, 2, \ldots, \mu$ such that the following conditions are satisfied

$$
\begin{aligned}
&\lambda_{ii} < 0, \;\; i = 1, 2, \ldots, \mu,\\
&\lambda_{ij} + \lambda_{ji} < 0, \;\; i < j = 1, 2, \ldots, \mu, \qquad (59)
\end{aligned}
$$

where

$$
\lambda_{ij} = \left(\begin{matrix} He\left(A_iQ_j + B_iVF_{pn}^j\right) & * \\ E_i^T + H_j^T V^T B_i^T & -\gamma I \\ C_{\infty 1i}Q_j + \rho F_{pq_2}^T V^T B_i^T + D_{\infty i}VF_{pn}^j & F_{\infty i} + D_{\infty i}VH_j \\ \beta V^T B_i^T + C_{2i}Q_j - UF_{pn}^j & H_i - UH_j \end{matrix} \right.
$$

$$\begin{matrix} * & * \\ * & * \\ -\gamma I + He(\rho D_{\infty i} V F_{pq_2}) & * \\ \beta V^T D_{\infty i}^T - \rho U F_{pq_2} & -\beta\left(U+U^T\right) \end{matrix}\Bigg),$$

$$F_{pq_2} = \begin{cases} I, & p=q, \\ \left(I_p \ \ 0_{p\times(q-p)}\right), & p<q, \ \ q=q_z+q_e, \\ \begin{pmatrix} I_q \\ 0_{(p-q)\times q} \end{pmatrix}, & p>q, \end{cases} \quad (60)$$

then the closed-loop system (4) by applying $K = VU^{-1}$ satisfies the H_∞ performance γ as well as the asymptotic stability.

Proof: The condition (59) has been proved in [26]. However, the proof of this lemma has an important difference from the proof suggested in Theorem 1 of [26]. The matrix T from Theorem 1 of [26] should be changed as

$$T = \begin{pmatrix} He\left(A_iQ_j + B_iV F_{pn}^j\right) & * & * \\ E_i^T + H_j^T V^T B_i^T & -\gamma I & * \\ C_{\infty 1i}Q_j + \rho F_{pq_2}^T V^T B_i^T + D_{\infty i} V F_{pn}^j & (F_{\infty i} + D_{\infty i} V H_j) & -\gamma I + He(\rho D_{\infty i} V F_{pq_2}) \end{pmatrix}. \quad (61)$$

By applying the above definition, the inequalities (60) will be linear with respect to γ. □

Theorem 5: Consider system (1) as an LPV system. For known scalar parameters ρ and β, if there exist matrices V, U_i, $i = 1, 2, ..., \mu$, and symmetric matrix $Q > 0$ such that the following LMIs are guaranteed,

$$\begin{aligned} &\omega_{ii} < 0, \ \ i = 1,2,...,\mu, \\ &\omega_{ij} + \omega_{ji} < 0, \ \ i<j = 1,2,...,\mu, \end{aligned} \quad (62)$$

where

$$\omega_{ij} = \begin{pmatrix} He\left(A_iQ + B_iVF_{pn}^j\right) & * & * & * \\ E_i^T + H_j^T V^T B_i^T & -\gamma I & * & * \\ C_{\infty 1i}Q + \rho F_{pq_2}^T V^T B_i^T + D_{\infty i} V F_{pn}^j & F_{\infty i} + D_{\infty i} V H_j & -\gamma I + He(\rho D_{\infty i} V F_{pq_2}) & * \\ \beta V^T B_i^T + C_{2i}Q - U_i F_{pn}^j & H_i - U_i H_j & \beta V^T D_{\infty i}^T - \rho U_i F_{pq_2} & -\beta\left(U_i + U_i^T\right) \end{pmatrix}, \quad (63)$$

then by defining the gain-scheduled controller $K(\theta) = \sum_{i=1}^{\mu} V(\alpha_i U_i)^{-1}$, the closed-loop LPV system (4) is asymptotically stable with the H_∞ performance γ.

Proof: By considering $Q = \sum_{j=1}^{\mu} \alpha_j Q$, $U = \sum_{i=1}^{\mu} \alpha_i U_i$, $i = 1, 2, ..., \mu$, and the controller (28), the proof of this theorem will be similar to that of Theorem 4 from [26]. Therefore, it is omitted. □

In the previous sections, it was shown that the robust H_2 or H_∞ SOF controller can be designed to locate the closed-loop poles in the desired region of the complex plane. The following theorem presents the LMI conditions satisfying the H_2 and H_∞ performance with the pole location constraints.

Theorem 6: Consider the open-loop system (1).

a) If the system (1) is to be LTI with polytopic uncertainty: For known scalar parameters ν, ρ, and β, if there exist matrices U, V, and symmetric matrices $Q_j > 0$, $i = 1$, $2, \ldots, \mu$ such that the inequalities (16), (33)-(34) or (35)-(36), and (59) are satisfied simultaneously, the SOF controller (19) satisfies the mixed H_2/H_∞ performance as well as the pole placement constraints.

b) If the system (1) is to be LPV: For known scalar parameters ν, ρ, and β, if there exist matrices U, V, and symmetric matrix $Q > 0$ such that the inequalities (24), (50)-(51) or (52)-(53), and (62) are satisfied simultaneously, the gain-scheduled SOF controller (26) guarantees the mixed H_2/H_∞ performance as well as the pole placement constraints.

Proof: By using the proposed theorems in the previous sections and Lemma 8, if all of the inequalities for the pole placement constraints, H_2 problem, and H_∞ problem are satisfied, then the conditions of Theorem 6 will be concluded. □

Remark 5: By using Corollary 1, Corollary 2, and Theorem 5, the SOF controller will be LPV. However, if the scheduling parameters α_i is not available, the LPV controller as shown in (26), cannot be implemented. Therefore, it should be designed one fixed controller by assuming $K_i = K$, i.e., $U_i = U$, $i = 1, 2, ..., \mu$. Consequently, the inaccessibility of the scheduling parameters α_i is not a limiting condition for the proposed techniques. Further, if the inequality (16), (24), (33)-(36), (59), or (62) is satisfied, then matrix U will be nonsingular. Therefore, the controller can be calculated. Furthermore, the scalar parameters ν, ρ, and β are the degrees of freedom. The proposed inequalities by considering these parameters will be nonlinear. Therefore, they should be defined before solving the inequalities. These parameters can also be found in the previous SOF controller design methods, e.g. the method proposed in [26]. In the following remark is proposed an optimal algorithm to obtain the scalar parameters.

Remark 6: In the proposed methods, the scalar parameters such as ν, ρ, and β are the degrees of freedom. However, the matrix inequalities will be nonlinear if these parameters are unknown. Therefore, before solving the proposed inequalities, these scalar variables should be determined using an optimization algorithm such as Genetic Algorithm (GA). In order to reduce conservatism, the flowchart in Fig. 1 suggests an optimal algorithm to

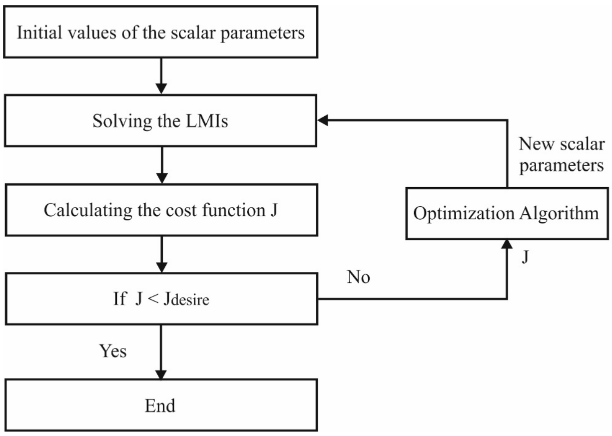

Fig. 1. The proposed algorithm for calculating the scalar parameters.

define the scalar parameters. In Fig. 1, the cost function J, the desired cost Jdesire, and the optimization algorithm should be defined by the designer. For example, the feasibility problem and the H_2 performance γ are good choices for the cost function. As shown in Fig. 1, the proposed algorithm can determine the scalar parameters such that the cost function J is minimized. However, some conservatism issues might arise because solving the LMIs and determining the scalar parameters are not synchronous.

6. SIMULATION RESULTS

In this section, the performance and effectiveness of the proposed methods are compared with those suggested in the recent studies based on two examples. In addition, the scalar parameters ν, ρ, and β obtain by the algorithm proposed in Remark 6. Further, the Sedumi solver of Yalmip software is used to solve LMIs.

Example 1: This example shows the proposed algorithms in Theorem 1 and Corollary 1 are more general than the previous works. Consider the system (1) with $w(t) = 0$ and two vertices as follows:

$$A_1 = \begin{bmatrix} -0.9896 & 17.41 & 96.15 \\ 0.2648 & -0.8512 & -11.39 \\ 0 & 0 & -30 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -97.78 \\ 0 \\ 3 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1.702 & 50.72 & 263.5 \\ 0.2201 & -1.418 & -31.99 \\ 0 & 0 & -30 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -85.09 \\ 0 \\ 3 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{64}$$

The matrices A and B have been given from [26]. Further, the output matrix C_2 has been chosen such that C_{22} is not full rank. It means that the second sensor has failed to measure data. By considering system (64) as an LTI system with polytopic uncertainty and by assuming $H = 0$, the robust SOF has been obtained from Theorem 1 as follows:

$$K = \begin{bmatrix} 0.0665 & 0.629 \end{bmatrix}, \tag{65}$$

where $\overline{\alpha} = 0$ and $\overline{\beta} = 1$ have been selected to satisfy the robust stability. In addition, by using Theorem 8 of [15], the static controller will be $K = \begin{bmatrix} 0.1126 & 0.3887 \end{bmatrix}$ in which the scalar parameter τ has been selected 0.1. Now, if either the system (64) is LPV or the pole placement constraint $Real\,(Acl) < -1$ is considered, the SOF controller can be designed by using Theorem 1 and Corollary 1. Table 1 shows the SOF results with $\beta = 0.0075$. Furthermore, if the parameters α_i are not available in the LPV model, the gain-scheduled SOF cannot be implemented. However, by using Remark 3, one SOF controller can be designed as shown in Table 1. Based on the results obtained in this example, the following points can be concluded:

Applicability: For system (64), the proposed techniques in [12] and [16] are not applicable because C_{22} is not invertible. Also, if $H \neq 0$ or system is LPV, the method presented in [15] cannot be used.

Performance: The performance is defined as the time-domain characteristics in this example. In [12, 15, 16], the robust stability is assumed. However, Theorem 1 and Corollary 1 can satisfy the pole placement constraints in addition to the robust stability. Furthermore, in order to

Table 1. The SOF results.

	Theorem 1	Corollary 1	
	K	$K = \sum_{i=1}^{\mu} \alpha_i K_i$	$K_1 = K_2 = K_3$
$\bar{\alpha} = 0$, $\bar{\beta} = 1$ or $Real(Acl) < 0$	[0.067 0.63]	$K_1 =$ [0.05 0.59] $K_2 =$ [0.06 0.60]	[0.06 0.59]
$\bar{\alpha} = 2$, $\bar{\beta} = 1$ or $Real(Acl) < -1$	[0.09 0.80]	$K_1 =$ [0.06 1.00] $K_2 =$ [0.09 1.04]	[0.09 0.82]

Table 2. The results of robust SOF H_2 controller while system (66) is LTI with polytopic uncertainty.

		Theorem 3 using (33)-(34)	Theorem 3 using (35)-(36)	Theorem 3 of [27]	Theorem 5 of [16] without I_{ij}	Theorem 5 of [16] with I_{ij}
Scalar parameters		$\beta = 0.0468$, $\rho = 0.4648$	$\beta = 0.3721$, $\rho = 10^{-4}$, $\nu = 1$	$\beta_1 = .25$, $\beta_2 = .05$ $\beta_3 = 1$, $\rho = 10^8$		
$H = 0$	γ	13.24	7.11	9.97	14.38	8.03
	K	−1.88	−3.81	−4.14	−2.75	−2.93
$H = .1$	γ	12.61	6.04	Infeasible	Not applicable	Not applicable

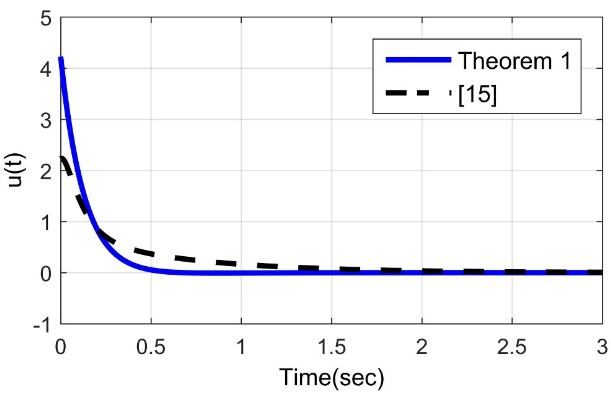

Fig. 2. The control input of the closed-loop system.

compare the controller obtained from Theorem 1 considering constraint $Real\,(Acl) < -1$ with the controller obtained from [15], the closed-loop system has been simulated with initial conditions $\alpha_1 = 1$, $\alpha_2 = 0$, and $x(0) = [2.5\ 5\ 3]$. The control input $u(t)$ in Fig. 2 shows that applying the pole placement constraint results in a faster closed-loop response and improves the time-domain characteristics.

Example 2: In this example, the robust H_2 and H_∞ controller with pole placement constraints are designed and compared with recent studies. Consider the following system with two vertices [16]:

$$A_1 = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix},\ A_2 = \begin{bmatrix} 2 & -1 \\ 0 & -5 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ E_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ E_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$D_1 = 1,\ D_2 = 2,\ C_{11} = \begin{bmatrix} 1 & 2 \end{bmatrix},\ C_{12} = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$C_{\infty 11} = \begin{bmatrix} 1 & 2 \end{bmatrix},\ C_{\infty 12} = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$D_{\infty 1} = 1,\ D_{\infty 2} = 2,\ C_{21} = \begin{bmatrix} 1 & 0 \end{bmatrix},\ C_{22} = \begin{bmatrix} 2 & 1 \end{bmatrix}, \tag{66}$$

where the other system matrices are zero. By using the proposed methods in Theorem 3 and Theorem 6, the controller results have been shown in Tables 2-3 for LTI systems with polytopic uncertainty. Furthermore, the system matrix $H \neq 0$ is also considered for more comparison with recent studies. Based on the results obtained in this example, the following points can be concluded:

Performance: The higher performance in this example is defined as the lower value of γ in H_2 and H_∞ control problems. Table 2 shows that the proposed SOF H_2 algorithms in Theorem 3 guarantee better performance compared with the methods suggested [27] and [16] since the value of γ for the proposed methods is lower. Furthermore, Theorem 6 achieves the performance level $\gamma = 8.25$ in the SOF H_2/H_∞ controllers while Theorem 3 and Theorem 4 in [27] obtain the performance level $\gamma = 13.57$.

Applicability: The results in Tables 2-3 show that if the matrix H is not zero, the methods presented in [27] and [16] are not applicable while Theorem 6 in the proposed method can be applied. Besides, if either pole placement constraint is considered in the design procedure or system is LPV, the methods suggested in [27] and [16] cannot be used while the proposed methods can be applied in these cases. In the pole placement problem and the H_2/H_∞ inequalities, if the scalar parameter β is selected different, the performance will be better. Therefore, this parameter has been defined as β_{td} in the pole placement constraints as shown in Tables 4-6. In Fig. 3, considering the initial conditions $\alpha_1 = 1$, $\alpha_2 = 0$, $x(0) = [2\ \ -40]$, and $w(t) = 0.1\sin(20\pi t)$, the closed-loop system using the proposed SOF H_2/H_∞ controller in Table 4, i.e. $K = -4.41$, has been compared with the controller obtained from [27] in Table 3, i.e., $K = -3.13$. Fig. 3 shows that the proposed method yields better time-domain performance for the closed-loop system when the pole placement constraint $Real\,(Acl) <\ -1$ is also considered.

Table 3. The results of robust SOF H_2/H_∞ controller where system (66) is LTI with polytopic uncertainty.

		Theorem 6: Using (33)-(34) in H_2 design	Theorem 6: Using (35)-(36) in H_2 design	Theorems 3-4 of [27]
Scalar parameters		$\beta = 0.037$, $\rho = 0.333$	$\beta = 0.345$, $\rho = 0.057$, $\nu = 1$	$\beta_1 = 0.25$, $\beta_2 = 0.0002$, $\beta_3 = 1$, $\rho = 10^8$
$H = 0$	γ	13.45	8.25	13.57
	K	−20.7	−4.05	−3.13
$H = .1$	γ	12.82	7.19	Infeasible
	K	−2.09	−4.58	

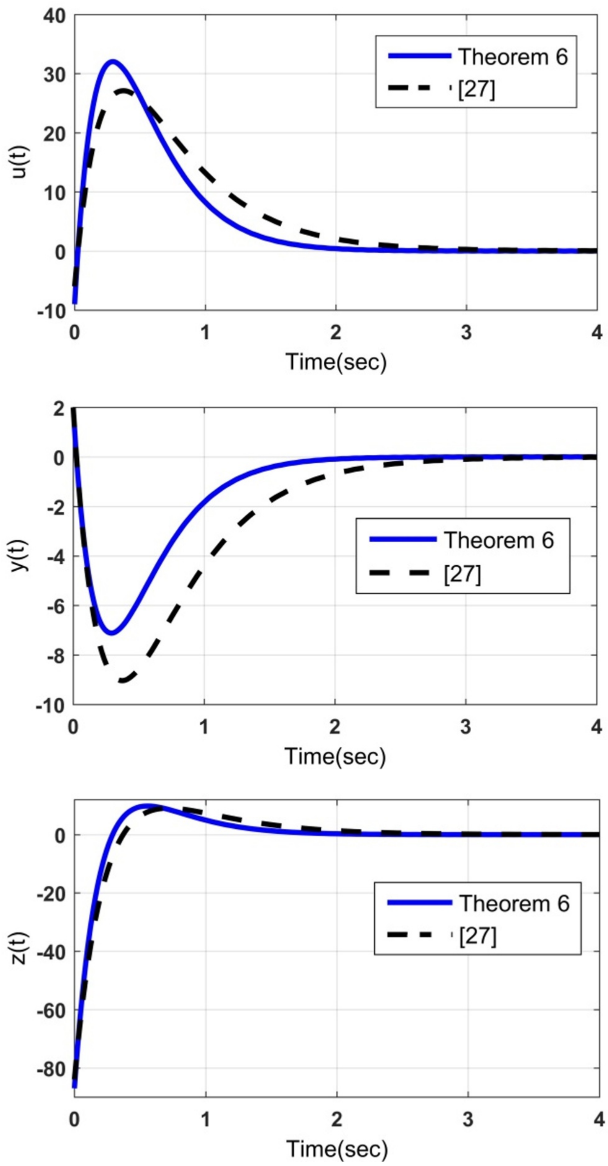

Fig. 3. The input signal $u(t)$, the output signal $y(t)$ and the controlled output signal $z(t)$.

Table 4. SOF H_2/H_∞ control synthesis with pole location constraint $Real\,(Acl) < -1$ while system (66) is LTI with polytopic uncertainty.

		H_2 problem from (33)-(34)	H_2 problem from (35)-(36)
Scalar parameters		$\beta = 0.0585$, $\rho = 0.3426$, $\beta_{td} = 0.0041$	$\beta = 0.3176$, $\rho = 0.0001$, $\beta_{td} = 0.2143$ $\nu = 1$
$H = 0$	γ	16.31	12.47
	K	-4.0	-4.41
$H = 0.1$	γ	16.25	9.55
	K	-4.02	-4.55

Table 5. Gain-scheduled SOF H_2/H_∞ control synthesis with pole location constraint $Real\,(Acl) < -1$.

		H_2 problem from (50)-(51)	H_2 problem from (52)-(53)
Scalar parameters		$\beta = 0.0760$, $\rho = 0.3400$, $\beta_{td} = 0.0787$	$\beta = 0.2597$, $\rho = 0.0001$, $\beta_{td} = 0.1876$, $\nu = 1$
$H = 0$	γ	16.23	11.64
	K_1	-4.53	-5.34
	K_2	-4.01	-4.65
$H = 0.1$	γ	15.96	9.00
	K_1	-3.96	-5.10
	K_2	-4.00	-4.55

Table 6. One Fixed SOF H_2/H_∞ control synthesis with pole location constraint $Real\,(Acl) < -1$ while system (66) is LPV system and $K_i = K$, $\forall i = 1, \ldots, \mu$.

		H_2 problem from (50)-(51)	H_2 problem from (52)-(53)
Scalar parameters		$\beta = 0.0650$, $\rho = 0.3632$, $\beta_{td} = 0.0044$	$\beta = 0.2456$, $\rho = 0.0001$, $\beta_{td} = 0.1893$, $\nu = 1$
$H = 0$	γ	16.35	15.35
	K	-4.00	-5.58
$H = 0.1$	γ	16.30	10.34
	K	-4.02	-5.73

7. CONCLUSION

The SOF H_2/H_∞ controller design with pole placement constraints for both linear systems with polytopic uncertainty and LPV systems has been proposed in this paper. The LMI-based approaches use an optimal line search to obtain some scalar variables. In addition, the H_2 control problem has been proposed by two different methods. It has been shown that the gain-scheduled or the fixed SOF can be designed for an LPV system. However, the gain-scheduled ones can result in better performance. Furthermore, the proposed methods have been applicable for general linear systems with less conservatism without any constraints on the output matrix. Finally, by two examples, the performance and the effectiveness of the proposed methods have been compared with recent related studies.

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