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Order Diminution of LTI Systems Using Modified Big Bang Big Crunch Algorithm and Pade Approximation with Fractional Order Controller Design

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Abstract: In this paper, a novel approach is proposed for the reduced order modelling of linear time invariant (LTI) systems. The proposed approach is a combination of modified Big bang big crunch (BBBC) optimization algorithm and Pade approximation technique. The beauty of the proposed approach is that the selection of solution space for BBBC algorithm is not entirely random, but structured via the use of Pade approximation approach. Hence, two principal criticisms of soft computing algorithms, i.e., random choice of solution space and larger simulation time are averted in the proposed technique. The proposed technique is substantiated via four different numerical examples from literature and compared with existing model order reduction (MOR) techniques. The concept of controller design is introduced via application of fractional order internal model control technique for load frequency control of power systems. Further, BBBC algorithm is employed to tune a boiler loop in power station. The results convey the efficiency and powerfulness of the proposed technique.

Keywords: Big Bang Big crunch, model order reduction, Padé approximation, soft computing.

1. INTRODUCTION

Model order reduction is a concept borrowed from mathematics where it is used to reduce the order of a system of ordinary differential equations. In systems and control engineering, it is found that most of the real time systems are complex and difficult to analyse and understand. So, the need to find an equivalent lower order system which preserves the key properties of the original higher order system led to the application of reduced order modelling in control theory. In control systems, model order reduction refers to the task of obtaining a simpler lower order model of the complex higher order systems such that the reduced order model closely resembles the original system with respect to its key properties. It also enables the control engineers to design simpler control laws, thereby making the controllers cost efficient.

In recent years, research in model order reduction (MOR) has led to the development of a large number of methods with an aim to find a technique which works for all types of LTI systems. In the pursuit of this aim, various nature inspired evolutionary optimization techniques were developed. The genetic algorithm (GA) developed by Goldberg [1] and particle swarm optimization (PSO) formulated by Kennedy and Eberhart [2] are two of the widely used global optimization techniques. In 2006, the BBBC algorithm was discovered by Erol and Eksin based on the theory of evolution of the universe. Cuckoo search (CS) is another optimization algorithm developed by Xin-she Yang and Suash Deb in 2008. It was inspired by the breeding behaviour of some cuckoo species which lay their eggs in the nests of host birds of other species [3,4].

In model order reduction theory, the mixed approaches utilising both conventional and evolutionary methods have also been used to minimise an error function based on performance index like the integral square error (ISE). In [5], the combined advantages of the eigen spectrum analysis and the error minimization by particle swarm optimization technique are utilized for model order reduction. Recently, a technique was proposed for MOR which combines the benefits of BBBC optimization and Routh approximation (RA) method [6]. The reduced order model is obtained by the reduction of denominator using RA to preserve the stability. Then, BBBC is applied on the numerator so as to minimise the ISE between the original and the reduced system. In [7], an improved pole clustering technique is proposed via consideration of distance of poles of the system from first pole in the pole clustering procedure. The coefficients of the reduced order denominator are calculated by improved pole clustering technique and the numerator coefficients are obtained by equating the reduced order transfer function with the original higher or-

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der model. Several other mixed techniques are developed by various authors [8–10]. Various other techniques for reduced order modelling have also been reported [11–16].

A literature survey undertaken by the authors reveal that soft computing algorithms are criticised mainly on two counts: random choice of solution space and large simulation time. Random choice on the bounds of solution space lead to the selection of an extremely large area, which leads to longer simulation time. However, in this paper, the lower and upper bounds of the solution space for numerator are chosen around the Pade approximants. The denominator coefficients of the reduced order model are computed based solely on Pade approximation, whereas the numerator coefficients are optimized via BBBC algorithm in the region around its Pade approximants. Hence, the role of Pade approximation in the proposed technique is twofold: computation of denominator in the reduced order model and delineating a compact and a narrow search space for the application of a soft computing algorithm.

Fig. 1 illustrates the proposed solution technique pictorially. Point B $[c_0, c_1]$ on Fig. 1 denotes the numerator coefficients of second order Pade approximant, that are chosen as a basis for the selection of search space for BBBC algorithm. Point A $[\alpha c_0, \alpha c_1]$ represents lower bound and Point C $[\beta c_0, \beta c_1]$ signifies the upper bound. Here, $\alpha < 1$ and $\beta > 1$. Therefore, bounds of solution space are chosen in the concentric circular area between the circles marked by point A and C. The mathematical interpretation of this concept will be explained in an elaborate manner in the upcoming sections.

This paper proposes a new model order reduction method based on the modified BBBC algorithm and Pade approximation technique. The results are compared with a recent paper based on modified cuckoo search (MCS) algorithm [11] and some other well known order reduction techniques. The response of the reduced system is also analysed with respect to time response specifications and four performance indices namely integral square error (ISE), integral absolute error (IAE), integral time absolute error (ITAE) and impulse response energy (IRE). It is found that the reduced system obtained by proposed method is an alternative to the existing methods in the literature.

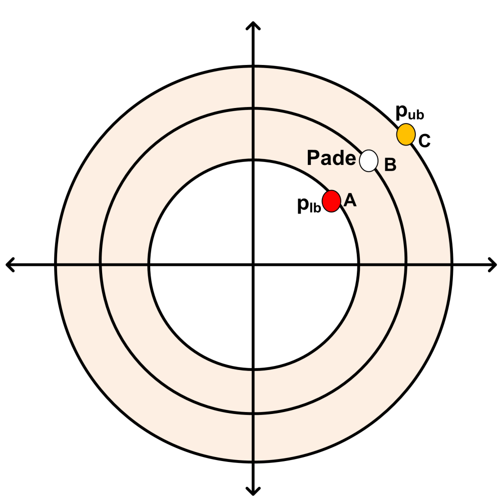

Fig. 1. Proposed solution space bounds ($A = [\alpha c_0, \alpha c_1]$), $B = [c_0, c_1]$, $C = [\beta c_0, \beta c_1]$).

Once a suitable reduced order model of a system is obtained, one can design a controller via the application of control technique on the reduced order model instead of the original higher order plant, leading to a reduction in computational complexity, cost and order of the controller. Broadly, we consider two distinct cases for the controller design. In the first case, both the order reduction and fractional order controller design are undertaken via BBBC algorithm for a boiler loop in a power station. In the second case, we explore fractional order internal model controller (FOIMC) design approach [17,18]. FOIMC is a controller design technique that entails the use of a reduced order mathematical model as one of its constituents. In this case, the advantages of reduced order modelling are delineated via comparison of the case in which model reduction is employed with the scenario when model order reduction is not undertaken. Subsequently, the FOIMC technique is employed for load frequency control of a power system and a comparison is undertaken with the existing techniques in literature. Finally, the FOIMC control procedure is also implemented to investigate the set point tracking behaviour of a higher order system, involving the use of both the minimum phase and non-minimum phase reduced order models in controller design.

The main contributions of our work can thus be recapitulated as follows:

1) A new order reduction technique involving Pade approximation and modified BBBC algorithm is formulated for stable as well as unstable systems.

2) An innovative way for the incorporation of optimal algorithms in order reduction applications via an intermediary reduced order model using Pade approximation is presented. The solution space for optimal control algorithms can be selected in a narrow and compact region around the numerator Pade approximants, leading to a reduction in randomness associated with metaheuristic approaches.

3) The actual application of reduced order modelling in controller design is demonstrated via the design of a fractional order internal model controller (FOIMC) for load frequency control of a power system. A case study is conducted to show that application of reduced order modelling in FOIMC has various advantages, wherein the controller is of a lower order, has a structure similar to that of a ubiquitous PID controller and involves a reduction in cost and computational complexity.

The remainder of the paper is organised into a number of sections as follows: The problem statement is defined in Section 2. The proposed technique is explained in an elaborate manner in Section 3. Simulation studies for four different systems are conducted in Section 4. The fractional order controller design via reduced order modelling is explored in Section 5. Finally the concluding remarks are presented in Section 6.

2. PROBLEM STATEMENT

The principal objective of the paper is to obtain a reduced order model, which is defined as follows:

Definition: Consider an n^{th} order linear time invariant (LTI) single input - single output (SISO) system, represented by the following higher order transfer function:

$$G_n(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{j=0}^{n} b_j s^j}; \;\; n \geq m, \tag{1}$$

where a_i and b_j are scalar constant coefficients of the complex variable s in numerator and denominator respectively.

Then, reduced order modelling is an approach to obtain the k^{th} order system via evaluation of unknown coefficients of the following transfer function:

$$R_k(s) = \frac{\hat{Y}(s)}{U(s)} = \frac{\sum_{i=0}^{k-1} p_i s^i}{\sum_{j=0}^{k} q_j s^j}, \tag{2}$$

where $k < n$ and p_i and q_j are unknown scalar constant coefficients of the complex variable s in numerator and denominator, respectively such that the following performance index is minimised:

$$J = \int_{t=0}^{t=t_{sim}} (y(t) - \hat{y}(t))^2 \, dt, \tag{3}$$

where t_{sim} is simulation time, $y(t) = \mathcal{L}^{-1}(Y(s))$, $\hat{y}(t) = \mathcal{L}^{-1}(\hat{Y}(s))$ and $\mathcal{L}^{-1}$ denotes inverse Laplace transform.

Subsequently, application of reduced order modelling is explored in controller design applications via two different cases, involving BBBC algorithm and FOIMC technique respectively.

3. PROPOSED METHODOLOGY

A mixed technique that combines the advantages of the modified BBBC algorithm and the Pade approximation method is proposed. The denominator polynomial of the original higher order transfer function is reduced using the Pade approximation method. Pade approximation helps in retaining time moments of the impulse response and the steady state output of the original system in the reduced order model obtained in the intermediate step. The i^{th} time moment of the impulse response ($g_n(t)$) of the system $G_n(s)$, about the origin is given as $M_i = \int_0^\infty t^i g_n(t) dt$, where, $i \geq 0$. The numerator coefficients of the reduced order model transfer function are evaluated by the minimisation of the integral square error (ISE) using a modified BBBC algorithm.

BBBC is an optimization algorithm discovered by Erol and Eksin based on the theory of the evolution of the universe [19–21]. It models energy dissipation in the big bang phase by the generation of random solutions in the search space. The big crunch phase can be viewed as a convergence operator having a number of random solutions as input and only a single output point which takes all the inputs along with their fitness function values into consideration. With each subsequent iteration of big bang and big crunch, the randomness in the search space decreases and the optimum value of the fitness function is obtained.

The rationale behind the choice of BBBC algorithm lies in its ease of software implementation and quicker convergence as compared to the classical genetic algorithm for various benchmark test functions such as sphere, step, Rastrigin, Rosenbrock and Ackley functions. It can compute an exact optima for step, Rastragin and sphere functions within the maximum allowed iteration count. In this process, it transcends the performance of the genetic algorithm and combat genetic algorithm for numerous benchmark functions [19].

The proposed technique is categorized into broadly four steps, which are explained as given below.

Step 1: Computation of coefficients of the k^{th} order reduced system by using the Pade approximation method [22] on the transfer function of the original system given in (1).

Let the reduced order system using Pade approximation in Step 1 be represented as

$$R_{pade}(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{j=0}^{k} d_j s^j}, \tag{4}$$

where c_i and d_j are scalar constant coefficients of the complex variable s in numerator and denominator respectively.

Step 2: In this step, we retain the denominator Pade coefficients as the denominator coefficients of the reduced order model. So the final reduced order model obtained until now is given by

$$R_k(s) = \frac{\sum_{i=0}^{k-1} p_i s^i}{\sum_{j=0}^{k} d_j s^j}, \tag{5}$$

where $p_i, i \in \{0, 1, \cdots, k-1\}$ are the unknown coefficients. Therefore, the coefficients of the reduced order denominator polynomial $D(s)$ obtained in step 1 are the coefficients of the denominator polynomial of the overall reduced model $R_k(s)$ and the numerator coefficients of $R_{pade}(s)$ will be considered as the base values which will help us in the better choice of candidate solutions as illustrated in the next step.

Step 3: In this step, the numerator Pade approximant is employed to demarcate a compact solution space for the quick implementation of BBBC algorithm in the next step. The lower and upper bounds of the solution space can be selected in a narrow region around the numerator Pade approximants in sharp contrast to the existing schemes, where an extremely large space is delineated to search for optimal parameters of the reduced order model. A compact solution region leads to a reduction in randomness and simulation time of the soft computing algorithm. Therefore, the lower and upper bound for the solution space are given by

$$\begin{aligned} p_{lb} &= \begin{bmatrix} \alpha c_0 & \alpha c_1 & \alpha c_2 & \cdots & \alpha c_{k-1} \end{bmatrix}, \\ p_{ub} &= \begin{bmatrix} \beta c_0 & \beta c_1 & \beta c_2 & \cdots & \beta c_{k-1} \end{bmatrix}, \end{aligned} \tag{6}$$

where $\alpha < 1$ and $\beta > 1$ decide the expansion of search space.

Step 4: Calculation of coefficients of numerator polynomial of the overall reduced order model $R_k(s)$ using modified Big Bang Big Crunch (BBBC) algorithm. In this paper, we present BBBC algorithm in a step by step form to illustrate its applicability in model order reduction problems.

It can be ascertained from (5), that we have k parameters, i.e., $p_0, p_1, \cdots, p_{k-1}$, that are to be computed via the BBBC algorithm. Let p_{lb} and p_{ub} represent vectors that designate lower bound and upper bound respectively for the k decision variables. The N candidate solutions are generated in an initial Big bang - Big crunch phase via the following formula:

$$p_j = p_{lb} - \upsilon \circ (p_{lb} - p_{ub}); \quad j = 1, 2, \cdots, N, \tag{7}$$

where $\circ$ symbolizes Schur multiplication and p_j is a matrix of order $(1 \times k)$ that represents the j^{th} candidate solution. p_{lb} and p_{ub} are also $(1 \times k)$ matrices, which represent lower bound and upper bound for the k decision variables and υ is a set of uniformly distributed random variables, that are bounded in the interval [0,1]. The uniform random parameter υ ensures that the solutions chosen in the initial big bang phase are completely random and are devoid of bias towards any particular area in the search space. Further, the selection of υ in the interval [0,1] enables the solutions to remain within the boundary of the chosen search space. The uniform random numbers can be generated in MATLAB environment via the use of the command $unifrnd(A, B, X, Y)$, which generates a matrix of random numbers comprising of X rows and Y columns, where, the lower limit and the upper limit of the uniformly distributed random numbers is A and B respectively. Simplification of (7) yields

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1,k} \\ p_{21} & p_{22} & p_{23} & \cdots & k_{2,k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{N1} & p_{N2} & p_{N3} & \cdots & p_{N,k} \end{bmatrix}, \tag{8}$$

where $p_{i,j} = p_{lb}(1,j) - \upsilon_{i,j}(p_{lb}(1,j) - p_{ub}(1,j))$; $i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, k$ and, $p_{i,j}$ designates the (i,j) element of p.

Using (6), the estimate of the lower bound and upper bound matrices is given as

$$\begin{aligned} p_{lb} &= \begin{bmatrix} \alpha c_0 & \alpha c_1 & \alpha c_2 & \cdots & \alpha c_{k-1} \end{bmatrix}, \\ p_{ub} &= \begin{bmatrix} \beta c_0 & \beta c_1 & \beta c_2 & \cdots & \beta c_{k-1} \end{bmatrix}, \end{aligned} \tag{9}$$

where $\alpha < 1$ and $\beta > 1$ decide the expansion of search space.

However, it should be noted that there is a definite trade-off involved in limiting the solution space. Limiting the solution space leads to a reduction in simulation time, but also leads to a more sub-optimal solution. In any case, the exact global optimal solution cannot be achieved by most of the soft computing algorithms. Using the proposed technique, we are able to exploit the advantages of sub-optimality, since it leads to a reduction in simulation time and the control objectives can be satisfied with such a solution.

Subsequently, we evaluate the fitness function f_i for all the candidate solutions. In this case, the fitness function is chosen to be ISE function as given below:

$$f_i^{iter} = \int_{t=0}^{t_{sim}} (y(t) - \hat{y}_i(t))^2 dt \quad \forall i = 1, 2, \cdots, N, \tag{10}$$

Here $y(t)$ denote the step response of the original higher order system and $\hat{y}_i(t)$ designates the step response of the reduced order system corresponding to the i^{th} candidate solution. Arrange the fitness function values in ascending order of magnitudes and the minimum fitness value is given as $f_{min}^{iter} = min(f_1^{iter}, f_2^{iter}, \cdots, f_N^{iter})$, where min designates the minimum value.

Next, we represent the N solutions via a single value, namely centre of mass (COM), that is computed as a weighted average of the individual candidate solutions. This step is the big crunch phase and can be viewed as a mapping from a disarrayed state to an ordered state. The COM is calculated as

$$C^{iter} = \frac{\sum_{i=1}^{N} \frac{p_i}{f_i}}{\sum_{i=1}^{N} \frac{1}{f_i}}. \tag{11}$$

Finally, we undertake the iterative Big bang phase. In this step, new candidate solutions, normally distributed around COM are generated as defined below:

$$p_i^{iter+1} = C^{iter} + \frac{r_i \psi (p_{ub} - p_{lb})}{\kappa} \quad \forall i = 1, 2, \cdots, N, \tag{12}$$

where ψ is a constant generally taken in the interval [0.1, 0.3]. r_i is a standard normal random number such that $r_i \in (0, 1]$. p_{ub} and p_{lb} are the upper and lower bound of the variables to be optimised. κ is a variable which increases with a step size of one.

This step is again followed by the evaluation of fitness function values and the iterative process is repeated until the termination criteria is satisfied. Finally, the optimal solution is $p_{optimal} = \begin{bmatrix} c_0 & c_1 & c_2 & \cdots & c_{k-1} \end{bmatrix}$.

The pseudocode for the BBBC algorithm is given as follows: Given $G_n(s)$

1) Input $r, \alpha, \beta, N, iter, \psi$
2) Generate N solutions using (7), (8) and (9)
3) Compute fitness function for candidates using (10)
4) Evaluate the minimum value of fitness function
5) Compute COM using (11)
6) Increment iteration count as $iter = iter + 1$
7) If $iter <$(maximum iteration count), go to 8, else go to 10.
8) Generate new solutions using (12)
9) Loop to 3.
10) Return $p_{optimal}$

Remark 1: The modification regarding the choice of solution space proposed in Step 2 is applicable to any generalised heuristic soft computing technique. It gives a structured representation of the initial choice of solution space and leads to a reduction in randomness for any soft computing algorithm.

Remark 2: The stability of reduced order model is dependent on the roots of denominator polynomial, which in this case is computed via Pade approximation. While, Pade approximation does not guarantee stability, however, it aids in the retention of dominant properties of the original model via preservation of time moments leading to better transient response.

Remark 3: For the case of order reduction of marginally stable and unstable systems, we categorise the system model into a stable part and an unstable part. The unstable part is retained in the reduced order model and the model order reduction is applied on the stable part of the transfer function.

4. NUMERICAL STUDIES AND RESULTS

In this section, we consider four examples from literature to illustrate the application of the proposed approach for widely different systems. The system in Example 1 has real, distinct and stable poles, whereas the system in Example 2 has complex poles too. In Example 3, we consider a system with real and repeated poles, and finally, a system with unstable and marginally stable poles is taken in Example 4. All the simulations are undertaken in MATLAB 2020a environment. The population size is selected as 100 and $\psi = 0.2$. The termination criterion is considered to be as about 100 iterations of the proposed algorithm.

Example 1: Let us consider an eighth order real pole system with poles at $s = -1, -2, -3, -4, -5, -6, -7, -8$ given as

$$G_1(s) = \frac{\begin{matrix} 18s^7 + 514s^6 + 5982s^5 + 36380s^4 \\ +122664s^3 + 222088s^2 + 185760s + 40320 \end{matrix}}{\begin{matrix} s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 \\ +67284s^3 + 118124s^2 + 109584s + 40320 \end{matrix}}. \tag{13}$$

This system was also considered in [23], where a mixed technique comprising of the Factor division algorithm (FDA) and Eigen spectrum analysis (ESA) was used for order diminution. To obtain the reduced order model, we undertake the following steps:

Step 1: In the first step, Pade approximation is used, and the reduced order model via Pade approximation is obtained as

$$R_{pade}(s) = \frac{3.1334s + 1.0003}{0.2075^2 + 1.2436s + 1.0003}. \tag{14}$$

Step 2: In second step, we retain the denominator Pade coefficients as the denominator coefficients of the reduced order model. So the reduced order model obtained until now is given by,

$$R_{pade}(s) = \frac{as + b}{0.2075^2 + 1.2436s + 1.0003}, \tag{15}$$

where a and b are still unknown.

Step 3: Before the application of BBBC algorithm, we choose a compact solution space for BBBC algorithm as

$$\begin{aligned} p_{lb} &= \begin{bmatrix} 3.1334\alpha & 1.0003\alpha \end{bmatrix}, \\ p_{ub} &= \begin{bmatrix} 3.1334\beta & 1.0003\beta \end{bmatrix}, \end{aligned} \tag{16}$$

where p_{lb} and p_{ub} denote the upper and lower bound of the solution space and α and β designate the multiplication factors. For this example, we choose $\alpha = 0.5$ and $\beta = 2$. So, the compact solution space for the application of BBBC lies between

$$p_{lb} \in \begin{bmatrix} 1.5667 & 0.50015 \end{bmatrix}, \; p_{ub} = \begin{bmatrix} 6.2668 & 2.0006 \end{bmatrix}. \tag{17}$$

Hence, it can be ascertained that the solution space is not completely random, since the lower and upper bounds are located in a region around the Pade approximants. So, Pade approximation, which is a computationally easy technique, enables us to reduce the randomness associated with soft computing metaheuristic algorithms, thereby leading to the reduction of simulation time and averting the principal criticisms associated with soft computing algorithms.

Step 4: Using the solution space, computed in Step 3, the final reduced order model obtained via the proposed technique is given by

$$R_{prop}(s) = \frac{3.107s + 1}{0.2075s^2 + 1.2436 + 1}. \tag{18}$$

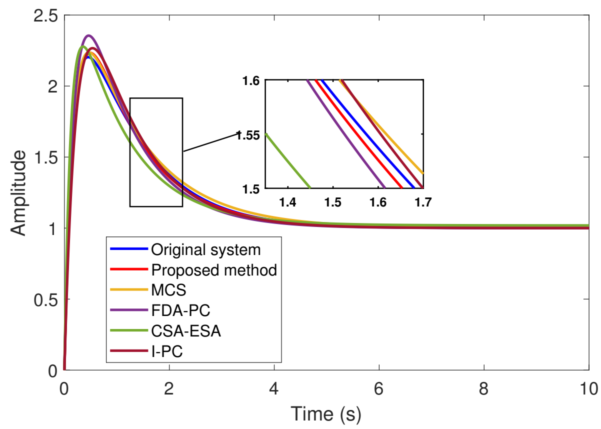

Fig. 2. Comparison of step responses of original and reduced systems for Example 1.

The comparison of proposed technique is carried out with MCS algorithm [11], which gives reduced order model as

$$R_{MCS} = \frac{16.39s + 4.865}{s^2 + 6.627s + 4.865}. \tag{19}$$

This example was also considered in [24] and reduced order model obtained by factor division algorithm-modified pole clustering (FDA-PC) technique (R_{Fp}) is given by

$$R_{Fp} = \frac{16.504 + 5.462}{s^2 + 6.197s + 5.462}. \tag{20}$$

The reduced order model obtained via an improved pole clustering technique (I-PC) [7] is given as

$$R_{I_pC} = \frac{13.4491s + 4.3505}{s^2 + 5.2298s + 4.3505}. \tag{21}$$

Also, the reduced order model obtained by Cuckoo search algorithm - Eigen spectrum analysis (CSA-ESA) [25] is

$$R_{Ce} = \frac{22.51s + 8.151}{s^2 + 9s + 8}. \tag{22}$$

The step responses and Bode plots of the original system, reduced-order model obtained by the proposed technique, MCS [11], I-PC [7], FDA-PC [24] and CSA-ESA [25] are depicted in Fig. 2 and Fig. 3, respectively. Fig. 4 depicts the change in fitness function values with the increase in number of iterations. It is seen that the response of the reduced model obtained by the proposed method is closer to the response of the original model, thus demonstrating the successful applicability of the proposed order reduction scheme. In addition to the plots, the values of different error based performance indices and the time response specifications are tabulated in Table 1. The IAE, ITAE for the proposed scheme is the least as compared to few other methods of reduction.

Thus, we can conclude that for the given system, the proposed method exhibits better performance in comparison to the other methods.

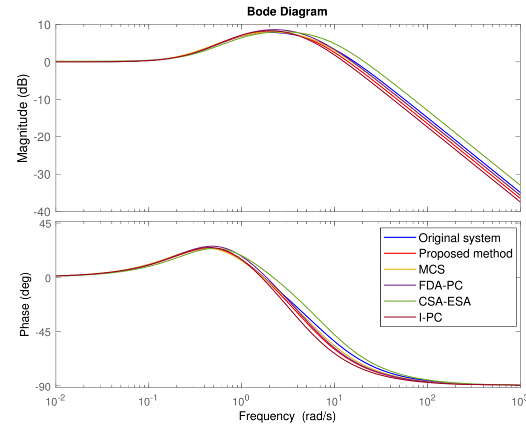

Fig. 3. Bode plot of original and reduced systems for Example 1.

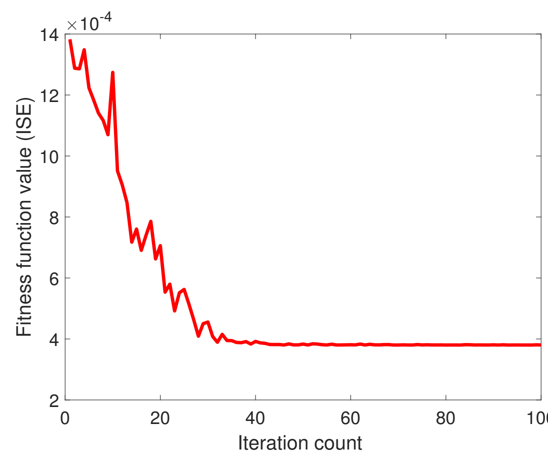

Fig. 4. Variation in fitness function value with iteration count for Example 1.

Example 2: The next system which we consider is a ninth order system having complex poles. It has been taken from [26]. The transfer function of the system is given as

$$G_2(s) = \frac{s^4 + 35s^3 + 291s^2 + 1093s + 1700}{s^9 + 9s^8 + 66s^7 + 294s^6 + 1029s^5 + 2541s^4 + 4684s^3 + 5856s^2 + 4620s + 1700}. \tag{23}$$

On applying the proposed technique, the third order reduced system (R_{prop}) is given by

$$R_{prop} = \frac{0.0258s^2 - 0.3963s + 1}{0.4181s^3 + 1.132s^2 + 1.6684s + 1}, \tag{24}$$

whereas the reduced order model achieved by MCS [11] is given by

Table 1. Analysis of performance parameters of proposed method and other reduction techniques for Example 1.

Performance Parameter	Original model	**Proposed method**	MCS [11]	FDA-PC [24]	CSA-ESA [25]	I-PC [7]
ISE	-	3.8393×10^{-4}	3.257×10^{-4}	1.4×10^{-3}	3.3×10^{-3}	1.5×10^{-3}
IAE	-	0.0743	0.1360	0.1964	0.3682	0.1648
ITAE	-	0.0827	0.3805	0.3488	1.0645	0.2108
IRE	21.739	19.1162	20.6351	22.4176	28.6114	17.7090
Rise time (s)	0.0569	0.0667	0.0614	0.0598	0.0457	0.0738
Settling time (s)	4.8201	4.7973	5.3070	4.3566	4.4003	4.5793
Peak overshoot (%)	120.3504	123.3702	123.6984	135.0533	123.5585	126.6488

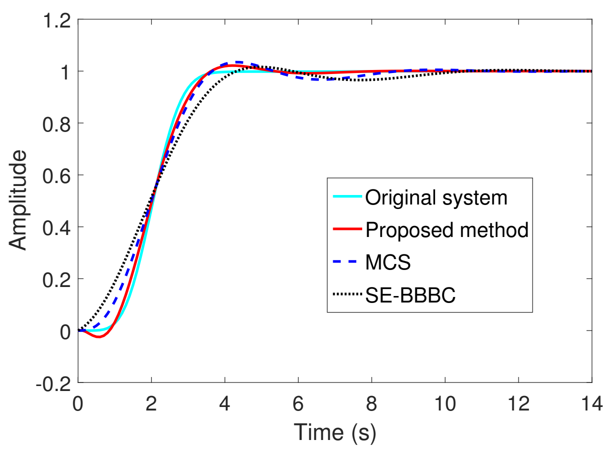

Fig. 5. Comparison of step responses of original and reduced systems for Example 2.

$$R_{MCS} = \frac{0.001935s^2+0.005725s+1.073}{s^3+1.681s^2+2.183s+1.073}. \tag{25}$$

The same example was also taken using the stability equation (SE) method and BBBC algorithm in [26]. The reduced order model (R_{DP}) hence obtained is

$$R_{DP} = \frac{0.0789s^2+0.3142s+0.493}{s^3+1.3s^2+1.34s+0.493}. \tag{26}$$

The comparison of step responses and Bode plots is illustrated in Fig. 5 and Fig. 6, respectively. For comparison, we have considered the MCS method [11] and SE- BBBC method [26]. It is seen from both the plots that the proposed technique gives a closer approximation to the original system as compared to the other recently developed methods of model reduction. Table 2 provides a validation for it. The ISE calculated by the proposed method is 2.5×10^{-3} which is an improvement over other recent methods. Further, the rise time, settling time and peak overshoot of the proposed third order model are much closer to the original system under consideration. Fig. 7 depicts the change in fitness function values with the increase in the number of iterations. Thus, the reduced order model achieved by the proposed technique is a better lower order representation of the original ninth order transfer function.

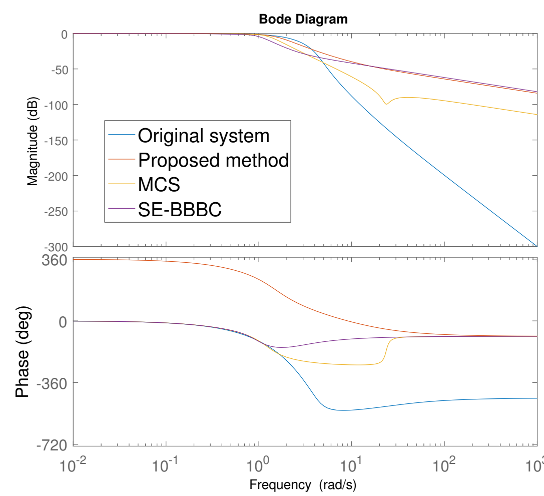

Fig. 6. Bode plot of original and reduced systems for Example 2.

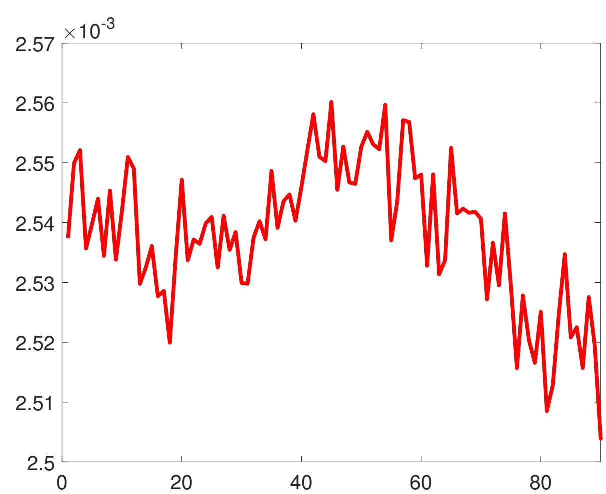

Fig. 7. Variation in fitness function value with iteration count for Example 2.

Example 3: Now, we consider a fourth order system having repeated poles. It has been taken from [11]. The transfer function of the system is given as

Table 2. Analysis of performance parameters of proposed method and other reduction techniques for Example 2.

Performance Parameter	Original model	**Proposed method**	MCS [11]	SE and BBBC [26]	RMT [27]	MPC and GA [8]
ISE	-	2.5×10^{-3}	1.45×10^{-2}	4.54×10^{-2}	8.77×10^{-2}	5.86×10^{-2}
IAE	-	0.1049	0.2980	0.4620	0.9359	0.2060
ITAE	-	0.3712	1.0831	1.6066	14.4252	13.1748
IRE	0.4705	0.4219	0.3466	0.2686	0.5085	0.6974
Rise time (s)	1.539	1.7996	2.1457	2.7708	2.9214	2.6033
Settling time (s)	3.3554	4.3719	7.6152	9.0779	6.9056	5.1518
Peak overshoot (%)	0	2.1413	3.3556	1.6015	0	0

$$G_3(s) = \frac{1}{(s+1)^4}. \tag{27}$$

The second and third order reduced order model obtained by following the steps elaborated in Section 3 are given by

$$G_{Prop2} = \frac{-0.1577s+0.3}{s^2+s+0.3}, \tag{28}$$

$$G_{Prop3} = \frac{0.1241s^2-0.3990s+1}{2s^3+4.5s^2+3.6s+1}. \tag{29}$$

The reduced order model in (28) is a non-minimum phase system. However, the original higher order system is a minimum phase system. On the other hand, when we reduce the model of the original higher order system via the proposed technique and set the lower bounds of the numerator coefficients as equal to zero in order to avoid the non-minimum phase behaviour in the reduced order model, the second order minimum phase reduced order model is obtained as

$$G_{PropMP} = \frac{0.113s+0.3}{s^2+s+0.3}. \tag{30}$$

Fig. 8 depicts the comparison of the step response of the original system and the minimum phase and non-minimum phase reduced order models obtained via the proposed technique. It can be observed that the time response of non-minimum phase reduced order model obtained via the proposed technique in this particular example without consideration of any constraints on the numerator coefficients is closer to the time response of the original system. On the other hand, the minimum phase reduced order system in Example 3 does not present an accurate description of the higher order system model. Therefore, the primary aim in model order reduction problems is to ensure a high degree of matching of the system step response, not just in the initial transient phase, but in the remaining time period as well until steady state of the system is reached. Hence, an accurate reduced order model should be selected, irrespective of the minimum or non-minimum phase nature of the system response.

When the above system is reduced by using MCS technique [11], the reduced order models are given by

$$G_{MCS2} = \frac{0.1s+0.1158}{s^2+0.5202s+0.1158}, \tag{31}$$

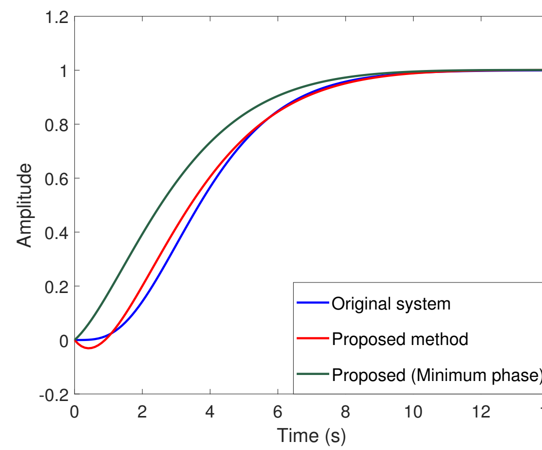

Fig. 8. Comparison of step responses for original and second order minimum and non-minimum phase reduced systems for Example 3.

$$G_{MCS3} = \frac{0.0001064s^2+0.2325}{s^3+1.238s^2+0.9371s+0.2325}. \tag{32}$$

Fig. 9 and Fig. 10 show the step responses and Bode plots of original and second order reduced order models, whereas Fig. 12 and Fig. 13 depict the same for the third order reduced models. Fig. 11 depicts the change in fitness function values with the increase in number of iterations. Table 3 compares the usual performance indices and time response specifications for original system and the second and third order models reduced by proposed procedure and MCS [11]. The reduced third order model almost exactly mimics the original system with respect to time response plot whereas Bode plots also show a much closer matching for the proposed method. The reduced second order system also shows an excellent approximation for the proposed method as compared to the other techniques of model order reduction. The readings of Table 3 further validate the above arguments. Thus we conclude from this example that the proposed method has overshadowed other recent methods in terms of closeness of response matching.

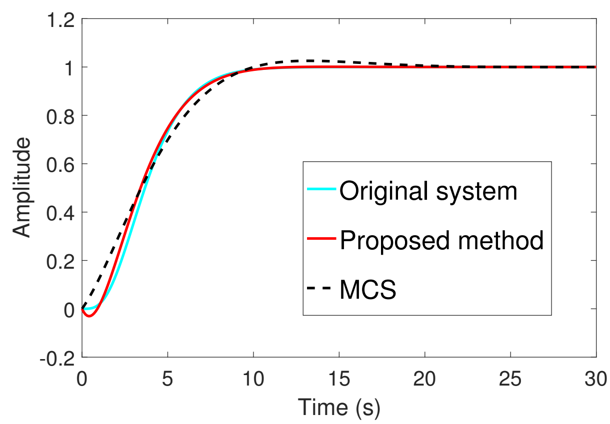

Fig. 9. Comparison of step responses for original and second order reduced systems for Example 3.

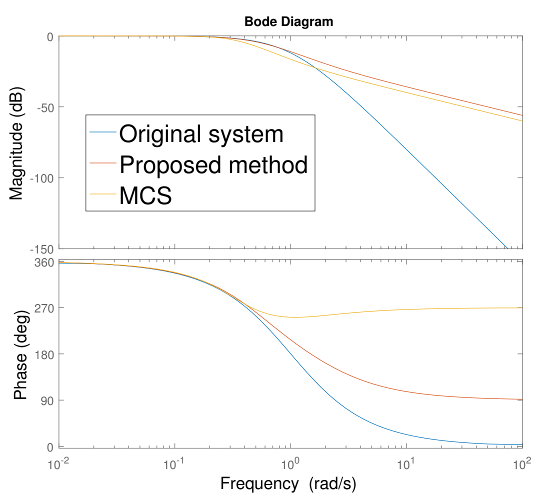

Fig. 10. Bode plot of original and second order reduced systems for Example 3.

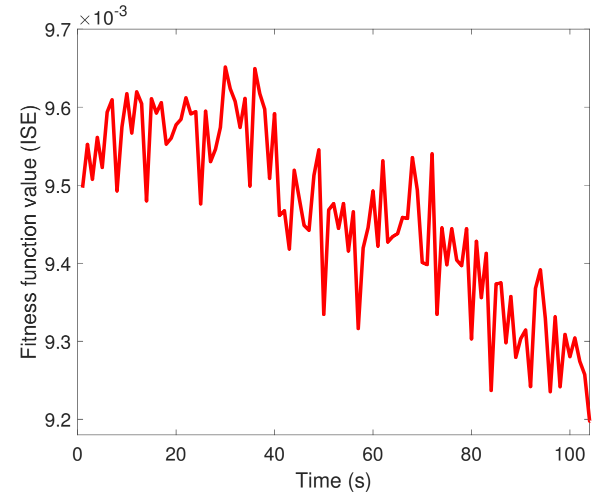

Fig. 11. Variation in fitness function value with iteration count for Example 3.

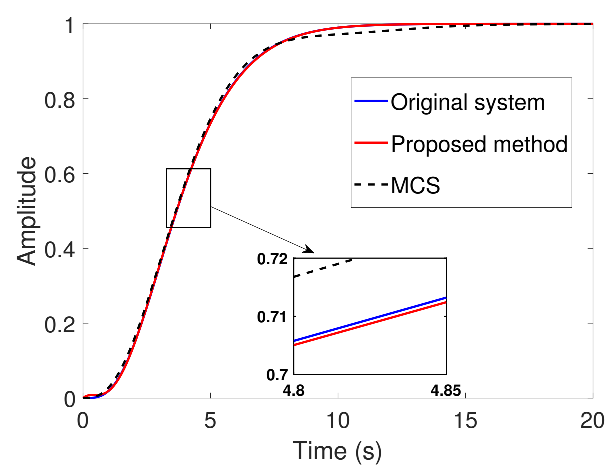

Fig. 12. Comparison of step responses for original and third order reduced systems for Example 3.

Example 4: Finally, let us consider the application of the proposed scheme on an unstable system, that can be modelled by the following transfer function:

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^6 + 8s^5 + 15s^4 - 20s^3 - 76s^2 - 48s}. \quad (33)$$

The system in (33) has stable poles at $s = -1, -2, -3, -4$, an unstable pole at $s = 2$ and a marginally stable pole at $s = 0$. We decompose the system model into a stable part and an unstable or marginally stable part. The unstable part is to be retained in the reduced order model and model order reduction is applied on stable part of the transfer

Table 3. Analysis of performance parameters of proposed method and other reduction techniques for Example 3.

Performance Parameter	Original model	**Proposed method 2nd order**	**Proposed method 3rd order**	MCS [11] 2nd order	MCS [11] 3rd order
ISE	-	0.0092	8.9976×10^{-6}	0.0459	0.002
IAE	-	0.2070	0.0079	0.6758	0.1679
ITAE	-	0.8003	0.0413	4.3462	1.4330
IRE	0.1562	0.1624	0.1562	0.1209	0.1552
Rise time (s)	4.9361	5.2783	4.9481	6.5553	4.8681
Settling time (s)	9.0842	9.2542	9.0982	15.6311	11.6137
Peak overshoot (%)	0	0.0917	0	2.5588	0

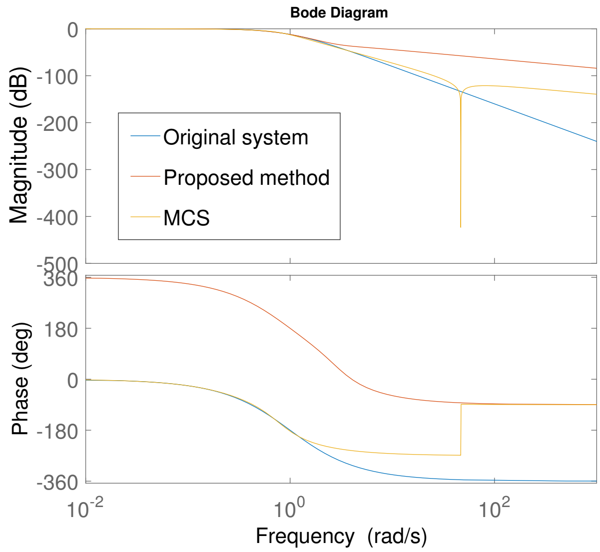

Fig. 13. Bode plot of original and third order reduced systems for Example 3.

function (Refer Remark 3). Hence,

$$G_4^{stable}(s) = \frac{s^3+7s^2+24s+24}{s^4+10s^3+35s^2+50s+24}, \tag{34}$$

$$G_4^{unstable}(s) = \frac{1}{s(s-2)}. \tag{35}$$

The reduced order model for the stable part via proposed approach is obtained as

$$R_4^{stable}(s) = \frac{0.2861s+1}{0.3993s^2+1.375s+1}. \tag{36}$$

On appending the unstable part given in (35) to the reduced order model in (36), we obtain the final reduced order model as

$$G_{prop}(s) = \frac{0.2861s+1}{0.3993s^4+0.5764s^3-1.75s^2-2s}. \tag{37}$$

Equation (37) is the fourth order reduced order model for the sixth order system given in (33). On the other hand, the second order reduced order model is given by only the unstable part as

$$G_{prop2}(s) = \frac{1}{s(s-2)}. \tag{38}$$

Fig. 14 and Fig. 15 illustrate the corresponding step response and Bode plot for the unstable system. It can be deduced that the proposed scheme depicts a high degree of coincidence among the original and reduced order models. Hence, the proposed technique is applicable to an unstable system as well.

Limitations: As discussed in Remark 2, Pade approximation does not guarantee the stability of the reduced order model during order reduction of a stable higher order system via proposed approach. In such a case, when the poles of the reduced order model of a higher order stable system lie on the right half complex plane, model order reduction does not give desired results. Consider an original higher order system given by $G(s) = \frac{8s^2+6s+2}{s^3+4s^2+5s+2}$. The reduced order model obtained via Pade approximation in Step 1 of the proposed technique is given by $R(s) = \frac{-1.778s-0.2222}{s^2-1.667s-0.2222}$, which has poles at $s = -0.1241, 1.7908$, therefore, it is an unstable system. In such a case, we cannot proceed further, since a stable original higher order system cannot be represented by an unstable reduced order model. One possible solution to ameliorate the aforementioned drawback is to employ another stability guaranteeing criterion for order reduction of denominator such as Routh approximation, etc. and choose the solution space for BBBC algorithm around the numerator coefficients

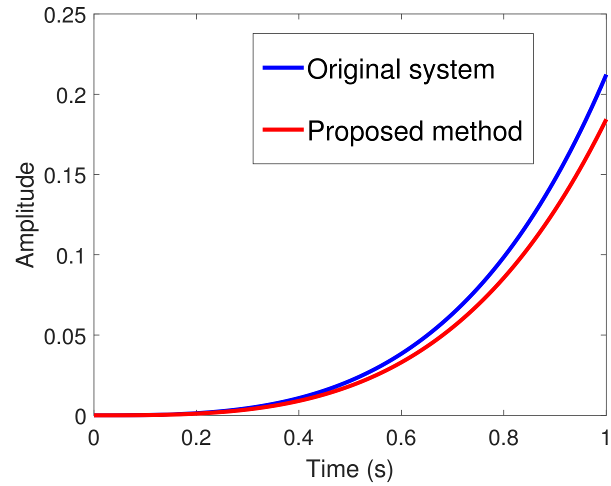

Fig. 14. Step response for original and reduced unstable system in Example 4.

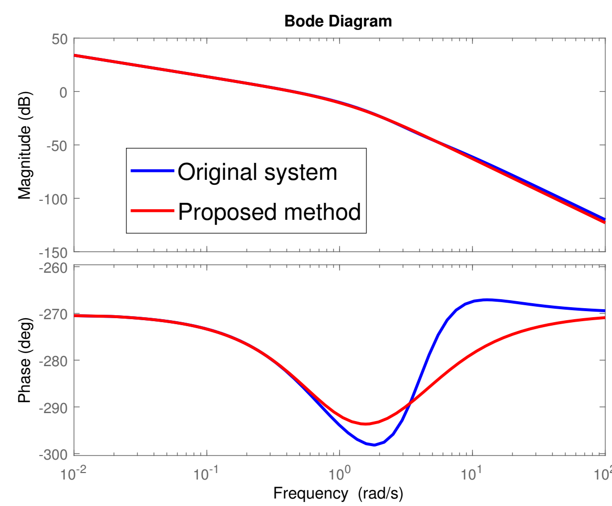

Fig. 15. Bode plot of original and reduced order unstable system in Example 4.

obtained by order reduction using Routh approximation. However, such a technique will not be able to preserve time moments of original system in reduced order model.

5. PID AND FOPID CONTROLLER DESIGN

In this section, we explore the application of reduced order modelling for controller design applications. Broadly, we consider two distinct cases, which are enlisted as: Case I: Order reduction and fractional order controller design by BBBC algorithm. Case II: Order reduction by BBBC algorithm and controller design via fractional order internal model control (FOIMC) technique. In Case I, we consider a boiler loop of a power station and undertake both model reduction and controller design via the BBBC algorithm. On the other hand, in Case II, order diminution is undertaken by BBBC algorithm and controller design is performed via FOIMC technique. This case examines the practical applicability of model reduction in control applications. It is validated via a study of problem of load frequency control in power systems. Both the case studies are delineated individually as follows:

Case I: In this case study, we examine the application of BBBC algorithm for the design of fractional order PID (FOPID) controller. A PID controller is an indispensable part of the modern control industry and has emerged as a widely used control scheme in academia and industry. The primary reason behind the success of PID control scheme lies in its simple structure, ease of tuning and optimal functionality in a wide range of operating conditions. A classical PID controller has three tuning parameters, namely, the proportional gain (K_p), integral gain (K_i) and the derivative gain (K_d). It is a rudimentary control algorithm that employs a linear weighted combination of the error between the reference and the desired value, its integration and differentiation to formulate the control law. Its popularity can be gauged from the fact that about 90% of the control loops in the industry use the PI/PID control approach. However, if the plant model is of a higher order, the design of controller becomes computationally complex and difficult to analyze. Hence, using the model order reduction concept, it is plausible to design the controller for the reduced order model of the plant, instead of the original higher order plant. In this section, we consider the design of PID controller on the reduced order model of the plant and implement it on the original higher order plant. The transfer function of a PID controller is given as

$$C(s) = K_p + \frac{K_i}{s} + K_d s. \tag{39}$$

Further, the PID controller uses only integer order tools, hence the range to which system performance can be enhanced is limited. Using the tools imparted by fractional order calculus, it is plausible to improve the system performance even further. Fractional calculus generalizes the order of integration and differentiation to arbitrary non-integer orders. There are several definitions of fractional calculus elaborated in literature like Grunwald Letnikov (GL), Caputo, Riemann-Liouville (RL), etc. Using the notion of fractional calculus in control theory and applications, one can furnish an accurate and precise control of the plant in comparison to the integer order schemes. A fractional order PID (FOPID) controller has five tuning parameters, which are greater than those available in conventional PID controller by two. The two extra tuning parameters aid us in achieving more accurate performance, which was not possible to obtain using only the integer order tools [28–30]. The transfer function of a fractional order PID controller is given by

$$C_f(s) = K_p + \frac{K_i}{s^{\lambda_f}} + K_d s^{\mu_f}; \mu_f,\ \lambda_f \in (0,2). \tag{40}$$

In this section, the procedure adopted for controller design is as follows. First, a reduced order model of the original plant is obtained via Big Bang Big crunch (BBBC) optimization algorithm. Subsequently, a PID and FOPID controller are tuned for the reduced order model of the plant, by using BBBC algorithm, the corresponding performance index being integral square error (ISE). Finally, the control is implemented on the original plant and the simulation results are shown.

Example 5: Consider a boiler loop of power station having the following transfer function [31]:

$$G(s) = \frac{s+1.5}{5s^4 + 40s^3 + 56.5s^2 + 58.5s + 5}. \tag{41}$$

The reduced order model using the proposed BBBC algorithm is

$$R_e(s) = \frac{0.01661s + 0.02827}{s^2 + 1.089s + 0.0943}. \tag{42}$$

Using the modified Big Bang Big crunch algorithm, the transfer function of the PID controller is

$$C_1(s) = 34.04 + \frac{5.23}{s} + 22.78s. \tag{43}$$

Subsequently, FOPID controller is tuned by keeping the parameters of PID controller as in (43), and tuning the remaining two parameters by BBBC optimization algorithm. The transfer function of FOPID controller, so obtained can be expressed as

$$C_2(s) = 34.04 + \frac{5.23}{s^{0.812}} + 22.78s^{1.246}. \tag{44}$$

On the other hand, the transfer function of PID controller obtained by Ziegler Nichols rules [31] is given by

$$C_3(s) = 18.18 + \frac{16.899}{s} + 29.633s. \tag{45}$$

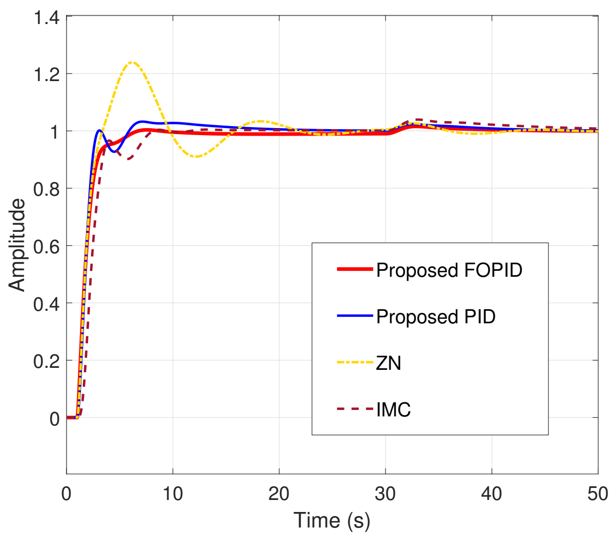

Fig. 16. Comparison of PID controller for boiler system.

The transfer function of a one degree of freedom internal model controller for the proposed boiler loop system [31] can be computed as

$$C_4(s) = \frac{10s+1}{0.3(0.3s+1)^2}. \tag{46}$$

The comparison of various controller design techniques is illustrated in Fig. 16. For the investigation of set point tracking and disturbance rejection properties of the controller, a step input and a step disturbance are introduced at $t = 1s$ and $t = 30s$, respectively. The controller tuned via Ziegler Nichols technique exhibits a large undershoot during set point tracking and IMC controller has a comparatively slow response and shows an overshoot during disturbance rejection. On the other hand, the proposed technique exhibits excellent set point tracking and disturbance rejection with minimal overshoot and undershoot. The fractional order PID controller based on BBBC outperforms the integer order BBBC-PID controller, whereas both outstrip the Ziegler Nichols (ZN) and Internal model control (IMC) technique. Hence, BBBC algorithm is a better alternative to conventional methods of PID design.

Case II: In this case study, we consider the application of the notion of reduced order modelling to the design of FOIMC controllers for load frequency control in power systems [17, 18, 32]. We will undertake the procedure of controller design via MOR and without MOR. The underlying difficulties without using order reduction will be demonstrated and will be ameliorated by employing MOR via BBBC algorithm. Internal model control (IMC) is a model predictive control (MPC) approach based on the principle of Youla parameterization. It can be contemplated as a special case of classical feedback structure, that explicitly includes a plant model in the control procedure. The block diagram of the IMC scheme is shown in Fig. 17. The outputs from the plant $G_p(s)$ and the reduced order plant model $G_{rm}(s)$ are subtracted and represent the effect of external disturbances and internal mismatch between the plant and the model. This difference signal is fed back to IMC controller $Q_{imc}(s)$. It is known that the model plant mismatch (MPM) plays a principal role in the determination of the overall strength of the IMC controller. The lower the mismatch, the better is the control. The plant is designated by the higher order transfer function and the model is represented via the reduced order model. Hence, the closer the reduced order model to the actual plant, the better is control that we achieve. Therefore, IMC control technique is apt for demonstration of the practical application of the reduced order modelling in controller design. Further, the IMC structure can also be altered into the classical feedback structure via the application of Q parameterization or Youla parameterization formula, which is given as $C(s) = \frac{Q_{imc}(s)}{1-G_{rm}(s)Q_{imc}(s)}$.

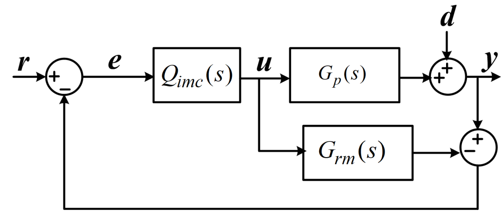

Fig. 17. Block diagram of IMC scheme.

The primary advantages of IMC are dual stability, ability to achieve perfect control, zero steady state error and fewer tuning parameters. With the advent of fractional order calculus in control applications, an innovative technique that amalgamated the concept of CRONE principle for fractional order systems formulated by Oustaloup and IMC principle for integer order systems was developed. An extensive explanation regarding IMC and FOIMC technique is given in [17,18,31,33]. The FOIMC approach of controller design is a generalized technique that is applicable to both minimum as well as non-minimum phase reduced order plant models.

Stability analysis: The output of IMC controlled plant with respect to the reference signal can be described by the following transfer function [18, 34]:

$$\frac{Y(s)}{R(s)} = \frac{G_p(s)Q_{imc}(s)}{1+Q_{imc}(s)(G_p(s)-G_{rm}(s))}. \tag{47}$$

Under the assumption that the reduced order plant model approximates the original higher order system, i.e., $G_p(s) \approx G_{rm}(s)$, (47) reduces to

$$\frac{Y(s)}{R(s)} = G_p(s)Q_{imc}(s). \tag{48}$$

In the FOIMC control technique elaborated in this paper, $G_{rm}(s)$ represents the second order reduced order model of the original plant and $Q_{imc}(s)$ denotes the FOIMC controller. Therefore, $G_{rm}(s)$ can be expressed as

$$G_{rm}(s) = \frac{p_0(1+p_1 s)}{s^2+q_1 s+q_2}, \tag{49}$$

where $q_1, q_2 > 0$ for a stable reduced order model. The expression of $Q_{imc}(s)$ depends on the sign of p_1. If $p_1 < 0$, the FOIMC controller is given by

$$Q_{imc1}(s) = \frac{s^2+q_1 s+q_2}{p_0(1+\sigma s^{\gamma+1})}. \tag{50}$$

On the other hand, if the plant model is a minimum phase system ($p_1 > 0$), the FOIMC controller is expressed as

$$Q_{imc2}(s) = \frac{s^2+q_1 s+q_2}{p_0(1+p_1 s)(1+\sigma s^{\gamma+1})}, \tag{51}$$

where σ and γ are constants of fractional order filter and $\gamma > 0$ [18]. Using (49)-(50), equation (48) is simplified as

$$\frac{Y(s)}{R(s)} = \frac{1+p_1 s}{1+\sigma s^{\gamma+1}}, \tag{52}$$

whereas using (49) and (51), equation (48) is written as

$$\frac{Y(s)}{R(s)} = \frac{1}{1+\sigma s^{\gamma+1}}. \tag{53}$$

The poles of the overall transfer function in (52) and (53) are at $s = \left(-\frac{1}{\sigma}\right)^{\frac{1}{\gamma+1}}$, that can also be written on the principal Riemann sheet as $s = |\frac{1}{\sigma}|^{\frac{1}{\gamma+1}} e^{\frac{i\pi}{\gamma+1}}$. It is the roots that are located on the principal branch of the Riemann sheet, which are responsible for various dynamics such as monotonically increasing oscillations, damped oscillations, etc. However, the roots that are placed on the secondary Riemann sheets represent the solutions, which are monotonically decreasing functions and do not create any issues with regards to the stability of the system [29]. Since the argument of the pole $s = |\frac{1}{\sigma}|^{\frac{1}{\gamma+1}} e^{\frac{i\pi}{\gamma+1}}$, i.e., $\frac{\pi}{2} < \frac{\pi}{\gamma+1} < \pi$ holds true when $\gamma \in (0,1)$ and γ is always a positive quantity in the transfer function of fractional order IMC filter, therefore the overall FOIMC controlled system is stable [35, 36]. In a similar manner, the output of the IMC controlled plant with respect to the disturbance is given as [34, 38]

$$\frac{Y(s)}{D(s)} = \frac{1-G_{rm}(s)Q_{imc}(s)}{1+Q_{imc}(s)(G_p(s)-G_{rm}(s))}. \tag{54}$$

Under the assumption $G_p(s) \approx G_{rm}(s)$, (54) reduces to

$$\frac{Y(s)}{D(s)} = 1-G_{rm}(s)Q_{imc}(s). \tag{55}$$

Using (49) and (50), (55) reduces to

$$\frac{Y(s)}{D(s)} = \frac{\sigma s^{\gamma+1}-p_1 s}{1+\sigma s^{\gamma+1}}, \tag{56}$$

whereas using (49) and (51), (55) is simplified as

$$\frac{Y(s)}{D(s)} = \frac{\sigma s^{\gamma+1}}{1+\sigma s^{\gamma+1}}. \tag{57}$$

Using a similar reasoning as employed in the stability analysis with respect to reference signal, the poles of the overall transfer function in (56) and (57) lie on the left half plane and the system is stable. Further, using the final value Laplace transform, the response of the system to the unit step disturbance is given as

$$y(t) = \lim_{s\to 0} sY(s) = 0. \tag{58}$$

Further, we analyze the response of the system to ramp disturbance as well. The Laplace transform of the ramp disturbance is given as $D(s) = \frac{1}{s^2}$. Substituting the expression of $D(s)$ in (57), the output of the system to ramp disturbance, when the reduced order model is a minimum phase system is obtained as

$$Y(s) = \frac{\sigma s^{\gamma+1}}{1+\sigma s^{\gamma+1}}\left(\frac{1}{s^2}\right). \tag{59}$$

On the other hand, for the case of non-minimum phase reduced order models ($p_1 < 0$), using (56), the system response to a ramp disturbance is computed as

$$Y(s) = \frac{\sigma s^{\gamma+1}-p_1 s}{1+\sigma s^{\gamma+1}}\left(\frac{1}{s^2}\right). \tag{60}$$

Using (59) and the final value theorem of Laplace transforms, the response of the system to ramp disturbance, for the case of minimum-phase reduced order model is calculated as

$$y(t) = \lim_{s\to 0} sY(s) = \frac{\sigma s^{\gamma}}{1+\sigma s^{\gamma+1}} = 0, \tag{61}$$

whereas using (60), the system response to ramp disturbance, for the case of non-minimum-phase reduced order model is obtained as

$$y(t) = \lim_{s\to 0} sY(s) = \frac{\sigma s^{\gamma}-p_1}{1+\sigma s^{\gamma+1}} = -p_1. \tag{62}$$

Hence, it can be deduced from the aforementioned analysis that FO-IMC technique ensures an effective rejection of the step disturbance with zero steady state error. Moreover, the FO-IMC technique also ensures an effective rejection of ramp disturbance with zero steady state error, when reduced order model exhibits minimum phase dynamics. However, when there are non-minimum phase dynamics in the reduced order model, the response to ramp disturbance will exhibit a constant steady state error.

Remark 4: The reduced order model $G_{rm}(s)$ in (49) will have poles in the right half plane, when the plant $G_p(s)$ is unstable. In such a case, the IMC structure is internally stable if (a) $Q_{imc}(s)$ is stable and (b) (1 −

$G_{rm}(s)Q_{imc}(s) = 0$) has right half plane zeros at the unstable poles of $G_{rm}(s)$ [37–39].

To investigate the robust stability of the IMC technique, consider a set μ of possible real plants G_p that are defined via a norm bounded multiplicative error ζ and the model of plant G_{rm} such that $\mu = \{G_p : |\zeta| \leq \zeta_m\}$ and $\zeta = \frac{G_p(i\omega)-G_{rm}(i\omega)}{G_{rm}(i\omega)}$, where ζ_m is the upper limit of uncertainty. Then, the closed loop IMC system is said to be robustly stable if and only if [38]

$$|Q_{imc}G_{rm}\zeta_m| < 1 \ \ \forall\omega, \tag{63}$$

where the reduced order model of the system can be expressed as $G_{rm}(s) = G^+_{rm}(s)G^-_{rm}(s)$ such that $G^+_{rm}(s)$ and $G^-_{rm}(s)$ denotes the non-minimum phase and the minimum phase components of the reduced order model, respectively. Further, IMC controller is formulated as $Q_{imc}(s) = (G^-_{rm}(s))^{-1}F(s)$. Substituting expressions of $G_{rm}(s)$ and $Q_{imc}(s)$ in (63) yields robust stability condition as

$$|\zeta_m| < \left|\frac{1}{F(s)G^+_{rm}(s)}\right|. \tag{64}$$

Hence, (64) gives the relation between an estimate of the upper limit of uncertainty and the reduced order model of the system. Simplifying further by substituting $F(s) = \frac{1}{1+\sigma s^{\gamma+1}}$ and $G^+_{rm}(s) = (1+p_1 s)$ from (49) and (50), where, $p_1 < 0$, (64) is obtained as

$$|\zeta_m| < \sqrt{\frac{1+\sigma^2\omega^{2\gamma+2}+2\sigma\omega^{\gamma+1}\cos\left(\frac{\pi(\gamma+1)}{2}\right)}{1+{p_1}^2\omega^2}} \ \ \forall\omega. \tag{65}$$

On the other hand, when $p_1 > 0$, $G^+_{rm}(s) = 1$. In such a case, (64) is simplified as

$$|\zeta_m| < \sqrt{1+\sigma^2\omega^{2\gamma+2}+2\sigma\omega^{\gamma+1}\cos\left(\frac{\pi(\gamma+1)}{2}\right)} \ \ \forall\omega. \tag{66}$$

Now, the application of reduced order modelling for the design of FOIMC controller is considered via the following examples.

Example 6: Consider the transfer function of a single area power system with droop characteristics [40] as follows:

$$G_p(s) = \frac{\Delta f(s)}{\Delta u(s)} = \frac{250}{s^3+15.88s^2+42.46s+106.2}, \tag{67}$$

where $\Delta f(s)$ and $\Delta u(s)$ designates the deviation in frequency. The reduced order model obtained via BBBC algorithm (G_{rm}) is given by

$$G_{rm}(s) = \frac{-5s+94.56}{5s^2+13.5s+40.17}. \tag{68}$$

The transfer function of fractional order low pass filter employed in FOIMC design is given as

$$F(s) = \frac{1}{1+\sigma s^{\gamma+1}}, \tag{69}$$

where σ and γ are computed via gain crossover frequency and phase margin requirements. Using FOIMC technique, we obtain the FOIMC controller in classical feedback configuration as

$$C_{IMC1}(s) = \left(0.148+0.057s+0.434\left(\frac{1}{s}\right)\right) \times\left(\frac{1}{0.017s^{0.111}+0.052}\right), \tag{70}$$

which is a series combination of ubiquitous PID controller ($K_p = 0.148$, $K_i = 0.434$, $K_d = 0.057$) and a low pass fractional order filter. On the other hand, if we do not integrate the concepts of reduced order modelling in FOIMC scheme, the final FOIMC controller in classical feedback configuration will be given by

$$C_{IMC2}(s) = \frac{s^3+15.88s^2+42.46s+106.2}{(250)(\sigma^2 s^{2\gamma+2}+2\sigma s^{\gamma+1})}, \tag{71}$$

where σ and γ are the constants to be chosen randomly.

Hence, the advantages of adopting reduced order model instead of employing the original higher order system are enlisted as follows:

- Using the notion of reduced order modelling, the controller is of a lower order, leading to a reduction of cost and reduction of analog hardware complexity. The order of the controller via the use of reduced order modelling is 1.111 and without the use of reduced order modelling is $2\gamma+2$, that can have a value as large as 4.
- It can be noticed that the final FOIMC controller obtained using the concept of reduced order modelling is a series combination of the ubiquitous PID controller and fractional order filter. On the other hand, if we do not integrate the concepts of reduced order modelling in FOIMC scheme, the final FOIMC controller in classical feedback configuration will be given by

$$C_{IMC2}(s) = \frac{s^3+15.88s^2+42.46s+106.2}{(250)(\sigma^2 s^{2\gamma+2}+2\sigma s^{\gamma+1})}, \tag{72}$$

where σ and γ are the constants to be chosen randomly. It can be ascertained clearly from (72), that the FOIMC controller obtained in (72) exhibits higher order dynamics and cannot be put in the form of a conventional PID controller. Therefore, reduced order modelling has led to a reduction of complexity for this control strategy.

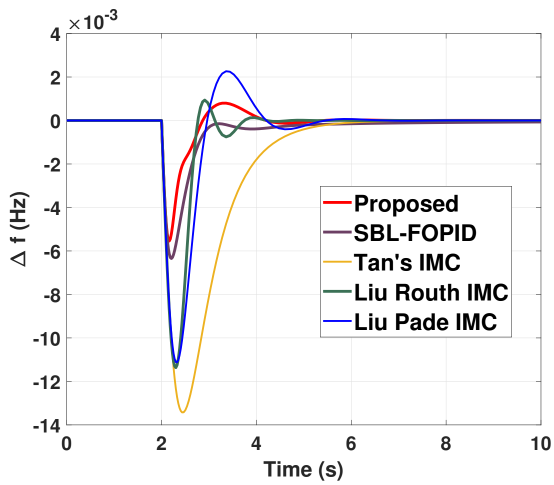

Fig. 18. Frequency deviation comparison with existing techniques.

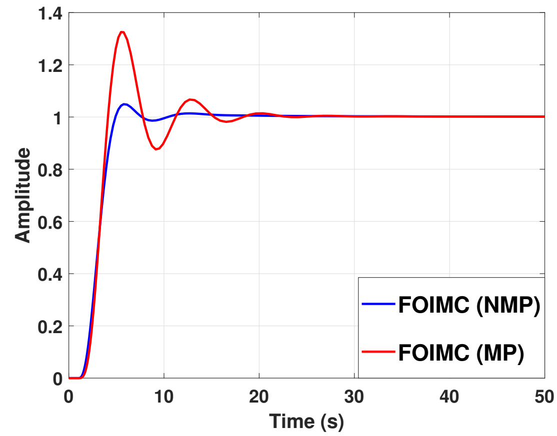

Fig. 19. Comparison of step responses for FO-IMC controlled systems using second order minimum and non-minimum phase reduced systems as system models in Example 7.

- Hence, if we do not employ the notion of reduced order modelling in controller design, we encounter two principal disadvantages: higher order controller leading to an increase in cost and hardware complexity and inability to transform the FOIMC controller into PID form, leading to increase in implementation complexity. Therefore, this example clearly demonstrates the practical advantages of reduced order modelling.

Fig. 18 depicts the comparison of frequency deviation response for the proposed FOIMC approach with existing techniques in literature such as stability boundary locus (SBL-FOPID) [41], Tan's IMC [42], Liu Routh IMC and Liu Pade IMC [40], when a step disturbance of 0.01 p.u is introduced into the system at time $t = 2s$. It can be ascertained that the frequency deviation response obtained via the proposed approach converges to zero quickly with minimal undershoot in comparison to existing techniques in literature.

Example 7: To investigate the applicability of the FOIMC technique for the set point tracking of a system involving a non-minimum phase reduced order model, let us consider the same transfer function as taken in Example 3 of Section 4. A comparative study is undertaken involving the use of both the minimum phase and non-minimum phase reduced order models obtained via the proposed technique in (28) and (30), respectively. For the sake of fair comparison, all the remaining parameters of FOIMC, except the plant model are considered to be same.

The FOIMC controller obtained via consideration of the minimum phase (MP) reduced order system $G_{rm}(s) = \frac{0.113s+0.3}{s^2+s+0.3}$ as the plant model and $G_p(s) = \frac{1}{(s+1)^4}$ in FOIMC procedure is

$$C_{IMC3}(s)=\left(1+\frac{0.3}{s}+s\right)\left(\frac{1}{1.71s^{0.056}}\right)\left(\frac{1}{0.11s+0.3}\right). \quad (73)$$

On the other hand, the FOIMC controller formulated via consideration of the non-minimum phase (NMP) reduced order system $G_{rm}(s) = \frac{-0.1577s+0.3}{s^2+s+0.3}$ as the plant model and $G_p(s) = \frac{1}{(s+1)^4}$ is

$$C_{IMC3}(s)=\left(\frac{1}{0.3}+\frac{1}{s}+\frac{1}{0.3}s\right)\left(\frac{1}{1.7146s^{0.0556}+0.5257}\right). \quad (74)$$

It can be observed from Fig. 19 that the FOIMC controller designed via a non-minimum phase reduced order model shows the least overshoot and quick settling times in comparison to the response obtained by consideration of the minimum phase plant as the plant model. For this particular example, it is shown in Fig. 8 that the non-minimum phase reduced order model approximates the original system more closely in comparison to the minimum phase reduced order model. Therefore, it can be concluded that a more accurate reduced order model should be chosen as a plant model for the formulation of controller via FOIMC technique, irrespective of whether it is minimum phase or non-minimum phase in nature.

Hence, it can be concluded from the aforementioned analysis that the BBBC algorithm is a viable alternative for controller design and the application of reduced order modelling greatly simplifies the structure of the final controller.

6. CONCLUSIONS

In this paper, we have proposed a model order reduction technique, that combines the advantages of Pade approximation for computation of numerator of the reduced order model and to delineate a solution space for the application of Big bang big crunch algorithm (BBBC) to obtain the numerator coefficients of the reduced order model. An extensive comparative analysis with respect to simulation plots and different numerous performance indices conducted on both stable and unstable system reveals that the proposed technique outperforms various existing techniques in literature. The reduced order model is further employed for simplification of controller design via BBBC algorithm and fractional order internal model control (FOIMC) technique, respectively. The concepts of reduced order modelling and FOIMC controller design are integrated for application to load frequency control of power systems, which reveals that the proposed FOIMC design greatly simplifies the controller design and exhibits minimum undershoot and a quick settling time in comparison to existing integer order and fractional order controller design techniques in literature.

REFERENCES

[1] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, 1st ed., Addison-Wesley Longman Publishing CO., Inc., Boston, MA, USA, 1989.

[2] J. Kennedy and R. Eberhart, "Particle swarm optimization," *Proc. of IEEE Int. Conf. on Neural Networks*, vol. 4, Perth, WA, Australia, Nov 1995, pp. 1942-1948.

[3] X.-S. Yang and S. Deb, "Engineering optimisation by cuckoo search," *Int. J. Mathematical Modelling and Numerical Optimisation*, vol. 1, no. 4, pp. 330-343, 2010.

[4] ——, "Cuckoo search via lévy flights," *Proc. of World Congr. on Nature Biologically Inspired Computing (NaBIC)*, Coimbatroe, INdia, IEEE, 2009, pp. 210-214.

[5] G. Parmar, S. Mukherjee, and R. Prasad, "Reduced order modelling of linear dynamic systems using particle swarm optimized eigen spectrum analysis," *International Journal of Electrical and Computer Engineering*, vol. 1, no. 1, pp. 73-80, 2007.

[6] S. Desai and R. Prasad, "A novel order diminution of LTI systems using Big Bang Big Crunch optimization and Routh Approximation," *Applied Mathematical Modelling*, vol. 37, no. 16, pp. 8016-8028, 2013.

[7] R. Komarasamy, N. Albhonso, and G. Gurusamy, "Order reduction of linear systems with an improved pole clustering," *Journal of Vibration and Control*, vol. 18, no. 12, pp. 1876-1885, 2012.

[8] C. B. Vishwakarma and R. Prasad, "Mimo system reduction using modified pole clustering and genetic algorithm," *Modelling and Simulation in Engineering*, vol. 2009, 2009.

[9] A. Sikander and R. Prasad, "Soft computing approach for model order reduction of linear time invariant systems," *Circuits, Systems, and Signal Processing*, vol. 34, no. 11, pp. 3471-3487, Nov 2015.

[10] A. B. H. Adamou-Mitiche and L. Mitiche, "Multivariable systems model reduction based on the dominant modes and genetic algorithm," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 2, pp. 1617-1619, Feb 2017.

[11] A. Sikander and P. Thakur, "Reduced order modelling of linear time-invariant system using modified cuckoo search algorithm," *Soft Computing*, vol. 22, no. 10, pp. 3449-3459, May 2018.

[12] G. Parmar, S. Mukherjee, and R. Prasad, "System reduction using eigen spectrum analysis and Padé approximation technique," *International Journal of Computer Mathematics*, vol. 84, no. 12, pp. 1871-1880, 2007.

[13] S. Biradar, Y. V. Hote, and S. Saxena, "Reduced-order modeling of linear time invariant systems using big bang big crunch optimization and time moment matching method," *Applied Mathematical Modelling*, vol. 40, no. 15, pp. 7225 - 7244, 2016.

[14] A. K. Prajapati and R. Prasad, "Reduced-order modelling of LTI systems by using routh approximation and factor division methods," *Circuits, Systems, and Signal Processing*, vol. 38, no. 7, pp. 3340-3355, Jul 2019.

[15] D. Casagrande, W. Krajewski, and U. Viaro, "On the asymptotic accuracy of reduced-order models," *International Journal of Control, Automation and Systems*, vol. 15, no. 5, pp. 2436-2442, 2017.

[16] E. Malekshahi and S. Mohammad, "The model order reduction using LS, RLS and MV estimation methods," *International Journal of Control, Automation and Systems*, vol. 12, no. 3, pp. 572-581, 2014.

[17] B. Maâmar and M. Rachid, "IMC-PID-fractional-order-filter controllers design for integer order systems," *ISA Transactions*, vol. 53, no. 5, pp. 1620-1628, 2014.

[18] S. Saxena, "Load frequency control strategy via fractional-order controller and reduced-order modeling," *International Journal of Electrical Power & Energy Systems*, vol. 104, pp. 603 - 614, 2019.

[19] O. K. Erol and I. Eksin, "A new optimization method: Big bang-big crunch," *Advances in Engineering Software*, vol. 37, no. 2, pp. 106 -111, 2006.

[20] B. Xing and W. Gao, *Innovative Computational Intelligence: A Rough Guide to 134 Clever Algorithms*, ser. Intelligent Systems Reference Library.Springer International Publishing, 2013.

[21] S. Jain, Y. V. Hote, and S. Saxena, "Model order reduction of commensurate fractional-order systems using big bang-big crunch algorithm," *IETE Technical Review*, vol. 37, no. 5, pp. 453-464, 2020.

[22] Y. Shamash, "Stable reduced-order models using Padé-type approximations," *IEEE Transactions on Automatic Control*, vol. 19, no. 5, pp. 615-616, October 1974.

[23] G. Parmar, S. Mukherjee, and R. Prasad, "System reduction using factor division algorithm and eigen spectrum analysis," *Applied Mathematical Modelling*, vol. 31, no. 11, pp. 2542 - 2552, 2007.

[24] A. Sikander and R. Prasad, "A new technique for reduced-order modelling of linear time-invariant system," *IETE Journal of Research*, vol. 63, no. 3, pp. 316-324, 2017.

[25] ——, "New technique for system simplification using cuckoo search and esa," *Sādhanā*, vol. 42, no. 9, pp. 1453-1458, Sep 2017.

[26] S. R. Desai and R. Prasad, "A new approach to order reduction using stability equation and big bang big crunch optimization," *Systems Science & Control Engineering*, vol. 1, no. 1, pp. 20-27, 2013.

[27] S. Mukherjee, Satakshi, and R. Mittal, "Model order reduction using response-matching technique," *Journal of the Franklin Institute*, vol. 342, no. 5, pp. 503 - 519, 2005.

[28] C. A. Monje, Y. Chen, B. M. Vinagre, D. Xue, and V. Feliu-Batlle, *Fractional-order Systems and Ccontrols: Fundamentals and Applications*, Springer-Verlag London, 2010.

[29] Y. Chen, I. Petras, and D. Xue, "Fractional order control - A tutorial," *Proc. of American Control Conference*, pp. 1397-1411, 2009.

[30] F. M. Bayat and A. Mahdi, "Extending the root-locus method to fractional-rder systems," *Journal of Applied Mathematics*, vol. 2008, Article ID 528934, 2008.

[31] S. Saxena and Y. V.Hote, "Advances in internal model control technique: A review and future prospects," *IETE Technical Review*, vol. 29, no. 6, pp. 461-472, 2012.

[32] S. Jain and Y. V. Hote, "Fractional order imcc ontroller via order reduction and crone principle for load frequency control," *Proc. of 8th International Conference on Power and Energy Systems (ICPES)*, pp. 135-140, Dec 2018.

[33] S. Saxena and Y. V. Hote, "Internal model control based pid tuning using first-order filter," *International Journal of Control, Automation and Systems*, vol. 15, no. 1, pp. 149-159, Feb 2017.

[34] Shivam Jain, Y. V. Hote, P. Dehuri, D. Mittal, and V. Siddhartha, "Experimental validation of fractional order internal model controller design on buck and boost converter," *Measurement and Control*, 2020. DOI: 10.1177/0020294020922264

[35] D. Matignon, "Stability results for fractional differential equations with applications to control processing," *In Computational Engineering in Systems Applications*, pp. 963-968, 1996.

[36] I. Petráš, "Stability of fractional-order systems with rational orders: A survey," *Fractional Calculus and Applied Analysis*, vol. 12, no. 3, pp. 269-298, 2009.

[37] S. Saxena and Y. V. Hote, "Stabilization of perturbed system via imc: An application to load frequency control," *Control Engineering Practice*, vol. 64, pp. 61-73, 2017.

[38] M. Morari and E. Zafiriou, *Robust Process Control.* Prentice Hall, 1989.

[39] Q.-G. Wang, T.-H. Lee, and J.-B. He, "Internal stability of interconnected systems," *IEEE Transactions on Automatic Control*, vol. 44, no. 3, pp. 593-596, 1999.

[40] S. Saxena and Y. V. Hote, "Load frequency control in power systems via internal model control scheme and model-order reduction," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2749-2757, Aug 2013.

[41] S. Sondhi and Y. V. Hote, "Fractional order PID controller for load frequency control," *Energy Conversion and Management*, vol. 85, pp. 343-353, 2014.

[42] W. Tan, "Unified tuning of PID load frequency controller for power systems via IMC," *IEEE Transactions on Power Systems*, vol. 25, no. 1, pp. 341-350, Feb 2010.

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