# A Trajectory Tracking Method for Wheeled Mobile Robots Based on Disturbance Observer

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**Abstract:** In this article, an adaptive tracking control method is proposed for wheeled mobile robots (WMR). In the trajectory tracking control of WMR, skidding and slipping are taken into account, and the slip degree parameters are introduced. The disturbance observer is designed to estimate the total disturbances. Then, we design the adaptive controller to guarantee that the tracking errors asymptotically converge to zero, the stability of the system is proved by Lyapunov theory. Finally, the simulation results are provided to demonstrate the effectiveness of the proposed scheme.

Keywords: Disturbance observer, input torque, kinematic controller, slip degree parameters, tracking error.

# 1. INTRODUCTION

Recently, researches have explored and extensively studied the tracking control of the robot, and lots of results have been achieved [1-7] and the references therein. Scholars have studied the trajectory tracking control of robots under various conditions. In [3], a new controller was designed for dynamic tracking of nonholonomic systems whose constant parameters were unknown. The literature of [4] studied the problem of finite-time trajectory tracking control for mobile robots without measuring velocity. The work [5] considered the problem of adaptive stabilization and tracking control for a two-wheeled mobile robot with input saturation and disturbance. Recently, [6] proposed an adaptive controller to realize the trajectory tracking of WMR with nonholonomic constraints, unknown parameters and external disturbance. The work [7] put forward a method by employing the high-gain observer in kinematic model and output feedback controller in dynamic model to realize trajectory tracking. The work [8] studied how to realize the tracking control of mobile robot by the backstepping method. In [9], a WMR adaptive sliding mode trajectory tracking controller was proposed which could realize finite-time convergence.

However, the above works did not consider the impact of skidding and slipping on WMR. The work [10] presented an improved controller based on neural network online weight tuning algorithm to track the desired trajectory in case of unknown longitudinal slip. [11] used the backstepping method to solve the problem of trajectory tracking control with unknown skidding and slipping. The

work [12] introduced an adaptive control method for trajectory tracking with torque saturation in dynamic model. An adaptive output feedback controller was designed for nonholonomic kinematics and dynamics models in [13]. The work [14] put forward a robust tracking controller based on generalized extended state observer for WMR whose skidding and slipping were unknown. As for unknown nonlinear terms, [15, 16] proposed a homogenous high-gain observer with two novel dynamic gains to estimate the system states. The literature of [17] studied the adaptive tracking system for tracking and obstacle avoidance of WMR with skidding and slipping. The work [19] considered the problem of pre-specified time cluster synchronization of complex networks with a smooth control protocol, and a unified theoretical framework to investigate the finite/fixed-time synchronization of complex networks with stochastic disturbances was proposed in [20]. In this paper, a new method is proposed for trajectory tracking of WMR in presence of skidding and slipping. The main contributions of this paper are summarized: (i) using the slip degree parameters to represent the difference between the reference trajectory and the actual motion trajectory, and obtain the error system; (ii) based on disturbance observer, an adaptive torque controller is designed; (iii) the proposed method requires the least amount of plant information.

# 2. PROBLEM FORMULATION

The nonlinear dynamic model of nonholonomic mobile robot with two driven wheels could be described as [14]

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$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = B(q)\tau - A^{T}(q)\lambda, \qquad (1)$$

where q is generalized coordinates,  $\tau$  is the input vector,  $\lambda$  is the vector of constraint force, M(q) is the symmetric and positive-defined inertia matrix,  $C(q, \dot{q})$  is the centripetal and coriolis matrix, G(q) is the gravitation vector, B(q) is the input transformation matrix, A(q) is the matrix associated with nonholonomic constraints.

Taking skidding and slipping into account, the constraints of the WMR are described by

$$\begin{cases} \dot{x}\sin\theta - \dot{y}\cos\theta = -u, \\ \dot{x}\cos\theta + \dot{y}\sin\theta + b\dot{\theta} = r(\dot{\psi}_1 - \zeta_1), \\ \dot{x}\cos\theta + \dot{y}\sin\theta - b\dot{\theta} = r(\dot{\psi}_2 - \zeta_2), \end{cases}$$
(2)

where  $(x, y, \theta)$  denotes the position and orientation of the platform.  $\psi_1, \psi_2$  indicate the angular velocities of the right and left driving wheels. *u* indicates the lateral skidding velocity while WMR traveling on the wet or icy road,  $\zeta_1$  and  $\zeta_2$  denote disturbance angular velocities caused by two actuated wheels' slipping, respectively. The radius of the driving wheel is *r*, the distance between two wheels is 2*b*.

The constraints (2) can be rewritten as

$$A(q)\dot{q} = R,\tag{3}$$

where  $R = [u, -r\zeta_1, -r\zeta_2]^T$  and

$$A(q) = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 & 0\\ \cos\theta & \sin\theta & b & -r & 0\\ \cos\theta & \sin\theta & -b & 0 & -r \end{bmatrix}.$$
 (4)

Define the matrix  $J^T(q)$  as

$$J^{T}(q) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 1/r & 1/r \\ 0 & 0 & 1 & b/r & -b/r \end{bmatrix},$$
 (5)

which is satisfied with A(q)J(q) = 0. With the help of (2) and (5), the kinematic model of WMR with skidding and slipping can be rewritten as [12]

$$\dot{q} = J(q)(z - \xi) + \varphi(q, u), \tag{6}$$

where  $z = (v, \omega)^T$ ,  $\xi = (\xi_1, \xi_2)^T$ .  $v = r(\dot{\psi}_1 + \dot{\psi}_2)/2$  and  $\omega = r(\dot{\psi}_1 - \dot{\psi}_2)/2b$  are the forward linear velocity and angular velocity of WMR, respectively.  $\xi_1 = r(\zeta_1 + \zeta_2)/2$  is the longitudinal slip velocity;  $\xi_2 = r(\zeta_1 - \zeta_2)/2b$  is the yaw rate perturbation caused by the wheels' slippage.  $\varphi(q, u) = (-u\sin\theta, u\cos\theta, 0, \zeta_1, \zeta_2)^T$  denotes the unmatched disturbance vector induced from the perturbed nonholonomic constraints.

Since the last two components in (6) are  $\dot{\psi}_1 = \dot{\psi}_1$  and  $\dot{\psi}_2 = \dot{\psi}_2$ , we focus on only three elements *x*, *y*,  $\theta$  except  $\psi_1$ ,  $\psi_2$ . Therefore, the vectors *q* and  $\varphi(q, u)$  are redefined as  $q = (x, y, \theta)^T$  and  $\varphi(q, u) = (-u \sin \theta, u \cos \theta, 0)^T$ , respectively.

Assumption 1: The perturbation  $\xi_1$ ,  $\xi_2$  and  $\varphi(q, u)$  are bounded as  $\|\xi_1\| < \varpi_1$ ,  $\|\xi_2\| < \varpi_2$  and  $\|\varphi(q, u)\| = |u| < \varpi_3$ where  $\varpi_i$  (i = 1, 2, 3) are unknown positive constants. In addition, their first derivatives are bounded as  $\|\dot{\xi}\|_1 < \varpi_{d1}$ ,  $\|\dot{\xi}\|_2 < \varpi_{d2}$  and  $\|\dot{\varphi}(q, u)\| < \varpi_{d3}$  with  $|\dot{u}| < \varpi_{d3}$  where  $\varpi_{di}$ (i = 1, 2, 3) are unknown positive constants. Meanwhile, skidding and slipping are extremely small compared with the reference velocity.

# 3. ADAPTIVE TRACKING CONTROLLER DESIGN

We determine smooth velocity control inputs to track the reference velocity and design the input torque to follow smooth velocity control inputs.

The vehicle is described by the following kinematic model

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}.$$
(7)

Let  $(x_r, y_r, \theta_r)^T$  represent the desired pose of the WMR, which is described by

$$\dot{q}_r = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos \theta_r & 0 \\ \sin \theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix},$$
(8)

where  $v_r$  and  $\omega_r$  denote the desired linear and angular velocities of the WMR.

Motivated by [10], we introduce the slip degree parameters as  $s_L$  and  $s_R$ .  $v'_L$  and  $v'_R$  represent the current velocity of left and right wheels while skidding or slipping;  $v_L$  and  $v_R$  are the driving velocities of the right and left wheel, which can be measured by encoders. Then the two slip degree parameters can be expressed as

$$s_L = \frac{v_L - v'_L}{v_L}, \quad s_R = \frac{v_R - v'_R}{v_R}.$$
 (9)

Then, the linear and angular velocities of the WMR can be expressed as

$$z = \begin{bmatrix} v & \boldsymbol{\omega} \end{bmatrix}^T = A \begin{bmatrix} v_L & v_R \end{bmatrix}^T, \tag{10}$$

where  $A = \frac{1}{2} \begin{bmatrix} 1 - s_L & 1 - s_R \\ (s_L - 1)/b & (1 - s_R)/b \end{bmatrix}$ . Substituting (10) into (7), it yields

$$s_L = 1 - \frac{\dot{x} - \dot{\theta}b\cos\theta}{v_L\cos\theta}, \ s_R = 1 - \frac{\dot{x} + \dot{\theta}b\cos\theta}{v_R\cos\theta}.$$
 (11)

From (9), it can be seen that  $s_L$ ,  $s_R \in [-1, 1]$  when slipping or skidding occurs. The slip degree parameters become  $s_L, s_R = \pm 1$  when WMR are not able to move.

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When considering the tracking problem, we assume that  $s_L, s_R \neq \pm 1$ . Thus, *A* is non-singular. From (10), one has

$$\begin{bmatrix} v_L \\ v_R \end{bmatrix} = \begin{bmatrix} \frac{1}{1-s_L} & -\frac{b}{1-s_L} \\ \frac{1}{1-s_R} & \frac{b}{1-s_R} \end{bmatrix} \begin{bmatrix} v \\ \boldsymbol{\omega} \end{bmatrix} = A^{-1} \begin{bmatrix} v \\ \boldsymbol{\omega} \end{bmatrix}.$$
(12)

Define the tracking error as

$$e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}.$$
 (13)

Then, the time derivative of (13) becomes

$$\dot{e} = \begin{bmatrix} \dot{x_e} \\ \dot{y_e} \\ \dot{\theta_e} \end{bmatrix} = \begin{bmatrix} \omega y_e + v_r \cos \theta_e - v \\ -\omega x_e + v_r \sin \theta_e \\ \omega_r - \omega \end{bmatrix}.$$
 (14)

An auxiliary velocity control input which achieves tracking for (6) is given as

$$\begin{bmatrix} v_c \\ \boldsymbol{\omega}_c \end{bmatrix} = \begin{bmatrix} v_r \cos \theta_e + k_x x_e \\ k_y \sin \frac{\theta_e}{2} + \boldsymbol{\omega}_r + 2y_e v_r \cos \frac{\theta_e}{2} \end{bmatrix},$$
(15)

where  $k_x$  and  $k_y$  are positive constants,  $v_r \neq 0$ .

**Remark 1:** In actual WMR system, we only need to consider the range of  $\theta_e$  is  $(-\pi, \pi)$ . When  $\theta_e = \pm \pi$ , the WMR will move in the opposite direction of the desired direction. The velocity control law  $\omega_c$  shown in (15) could be well defined in  $(-\pi, \pi)$ . The proof was shown in [8].

However, we need to compensate for the loss of velocity caused by skidding and slipping while designing the controller. According to (10), one has

$$\begin{bmatrix} f_{Lc} \\ f_{Rc} \end{bmatrix} = \begin{bmatrix} \frac{1}{1-s_L} & -\frac{b}{1-s_L} \\ \frac{1}{1-s_R} & \frac{b}{1-s_R} \end{bmatrix} \begin{bmatrix} v_c \\ \boldsymbol{\omega}_c \end{bmatrix} = A^{-1} \begin{bmatrix} v_c \\ \boldsymbol{\omega}_c \end{bmatrix}, \quad (16)$$

where  $f_{Lc}$ ,  $f_{Rc}$  are the auxiliary velocity control law input of the left and right wheels.  $s_L$ ,  $s_R$  could be obtained by (11). After the slip compensation, the auxiliary velocity tracking errors of wheels are defined as

$$e_c = A^{-1} \begin{bmatrix} v - v_c \\ \omega - \omega_c \end{bmatrix} = A^{-1} e_v, \tag{17}$$

where  $e_v = \begin{bmatrix} v - v_c \\ \boldsymbol{\omega} - \boldsymbol{\omega}_c \end{bmatrix}$ .

In what follows, we should design  $\tau$  to make  $e_c$  and  $e_v$  asymptotically converge to zero. Then, the time derivative of (6) becomes

$$\ddot{q} = \dot{J}(q)(z - \xi) + J(q)(\dot{z} - \dot{\xi}) + \dot{\phi}(q, u).$$
(18)

Substituting (6) and (18) into (1), one has

$$\overline{M}\dot{z} + \overline{L}z + \overline{F} = \overline{B}\tau \tag{19}$$

where  $\overline{M} = J^T M J$ ,  $\overline{L} = J^T M J^{\dagger} = 0$ ,  $\overline{F} = -J^T M J \dot{\xi} - J^T M J \dot{\xi} + J^T M \dot{\phi} + J^T C + J^T G$  and  $\overline{B} = J^T B$ .

Assumption 2: The mass *m* and the moment of inertia *I* of the robot are difficult to measure but can be estimated.

**Remark 2:** By the definition of  $\|\overline{F}\|$ , it is obviously that  $\|\overline{F}\|$  is bounded. Since the WMR is a large inertia system, it is insensitive to the fast time-varying disturbance. Thus, it is reasonable to suppose that  $\lim_{t\to\infty} \|\overline{F}\| = 0$ .

Letting  $\psi(e_c) = -\Gamma Y^T e_c$ ,  $\Gamma = \gamma diag\{1,1\}$ ,  $Y = \begin{bmatrix} \dot{v}_c & 0\\ 0 & \dot{\omega}_c \end{bmatrix}$ ,  $\gamma > 0$ ,  $\Phi = (\Phi_1, \Phi_2)^T = (m, I)^T$ , where *I* is the total moment of inertia of the system,  $\widehat{\Phi}$  is the estimated value of  $\Phi$ . As for  $\Phi_i$ , there exist maximum value  $\Phi_i max$  and minimum value  $\Phi_i min$  which satisfy  $0 < \Phi_i min < \Phi_i < \Phi_i max$ , and  $\overline{\Phi}_i min = \frac{2-\sqrt{2}}{2} \Phi_i min$ ,  $\overline{\Phi}_i max = \frac{2+\sqrt{2}}{2} \Phi_i max$ .

**Lemma 1:** The uncertain parameters of the robot are all in a limited set. The adaptive law was taken as [13]:

$$\hat{\Phi}_{i} = \begin{cases} \psi_{i}(e_{c}), & \Phi_{i}min < \widehat{\Phi}_{i} < \Phi_{i}max, \\ \psi_{i}(e_{c}) + (1 - \frac{\widehat{\Phi}_{i}}{\Phi_{i}min})^{2}[1 + \psi_{i}^{2}(e_{c})], \\ & \widehat{\Phi}_{i} \leq \Phi_{i}min, \\ \psi_{i}(e_{c}) - (1 - \frac{\widehat{\Phi}_{i}}{\Phi_{i}max})^{2}[1 + \psi_{i}^{2}(e_{c})], \\ & \widehat{\Phi}_{i} \geq \Phi_{i}max. \end{cases}$$

$$(20)$$

The above adaptive law is supposed to satisfy the following conditions:

(i)  $\widehat{\Phi}_i$  is continuous; (ii) If  $\overline{\Phi}_i min < \widehat{\Phi}_i(0) \le \overline{\Phi}_i max$ , then  $\overline{\Phi}_i min \le \widehat{\Phi}_i \le \overline{\Phi}_i max$ ; (iii)  $\widetilde{\Phi}^T (\Gamma^{-1} \dot{\widehat{\Phi}} + Y^T e_c) > 0$ , where  $\widetilde{\Phi} = \Phi - \widehat{\Phi}$ .

(iii)  $\Phi^{T}(1^{-1}\Phi + Y^{T}e_{c}) \geq 0$ , where  $\Phi = \Phi - \Phi$ .

According to the analysis, one has  $\overline{M} = diag\{\Phi_1, \Phi_2\}$ . Define  $\widehat{\overline{M}} = diag\{\widehat{\Phi}_1, \widehat{\Phi}_2\}$ , where  $\widehat{\overline{M}}$  is the estimated value of  $\overline{M}$ . Considering (20),  $\widehat{\overline{M}}$  is a positive definite matrix.

According to the dynamic model (19), taking the time derivative of (17), one has

$$\widehat{\overline{M}}\dot{z} = \overline{B}\tau - \overline{F} - \begin{bmatrix} \widetilde{\Phi}_1 & 0\\ 0 & \widetilde{\Phi}_2 \end{bmatrix} \dot{z} = \overline{B}\tau - \eta, \qquad (21)$$

where  $\eta = \overline{F} + \begin{bmatrix} \Phi_1 & 0 \\ 0 & \Phi_2 \end{bmatrix} \dot{z}$  is the sum of all the factors causing the errors to be estimated.

Design the disturbance observer as [18]:

$$\begin{cases} \dot{\beta} = -L\widehat{\overline{M}}^{-1}\beta - L(\widehat{\overline{M}}^{-1}Lz + \widehat{\overline{M}}^{-1}\overline{B}\tau), \\ \widehat{\eta} = \beta + Lz, \end{cases}$$
(22)

where  $\hat{\eta}$  is the estimation of  $\eta$ , *L* is the observer gain matrix which could adjust the convergence performance of the disturbance observer.

Then, based on the disturbance observer (22), the adaptive controller for WMR is designed as

$$\tau = \overline{B}^{-1} (Y \widehat{\Phi} - k_1 e_v - k_2 sgn(e_v) + \widehat{\eta}), \qquad (23)$$

where  $k_1, k_2 > 0, sgn(\cdot)$  is the symbolic function.

# 4. STABILITY ANALYSIS

In this section, the stability of the closed-loop system is proved based on Lyapunov stability theory. Considering the Lyapunov functional candidate  $V_1$  as

$$V_1 = \frac{1}{2} \left( e_{\nu}^T \widehat{\overline{M}} e_{\nu} + \widetilde{\Phi}^T \Gamma^{-1} \widetilde{\Phi} \right).$$
(24)

According to (21), one has

$$\widehat{\overline{M}}\begin{bmatrix} \dot{v} - \dot{v}_c \\ \dot{\omega} - \dot{\omega}_c \end{bmatrix} = \widehat{\overline{M}}\dot{e}_v = \overline{B}\tau - \eta - \widehat{\overline{M}}\begin{bmatrix} \dot{v}_c \\ \dot{\omega}_c \end{bmatrix}$$

$$= \overline{B}\tau - \eta - Y\widehat{\Phi}.$$
(25)

Combining (20) with (25) into (24) yields

$$\dot{V}_1 = e_v^T (\overline{B}\tau - \eta - Y\widehat{\Phi}) - \widetilde{\Phi}^T (\Gamma^{-1}\widehat{\Phi} + Y^T e_c).$$
(26)

By Lemma 1, one has  $-\widetilde{\Phi}^T (\Gamma^{-1}\widehat{\Phi} + Y^T e_c) \leq 0$ . Substituting the adaptive controller (23) into (26) yields

$$\begin{aligned} \dot{V}_1 \leq & e_v^T (-k_1 e_v - k_2 sgn(e_v) + \widetilde{\eta}) \\ \leq & -k_1 e_v^T e_v - (k_2 - \|\widetilde{\eta}\|) \|e_v\|, \end{aligned}$$

$$(27)$$

where  $\tilde{\eta} = \hat{\eta} - \eta$ .

Combining (21) and (22), one has

$$\begin{aligned} \dot{\widetilde{\eta}} &= \dot{\beta} + L\dot{z} - \dot{\eta} = -L\widehat{\overline{M}}^{-1}(\beta + Lz) - L\widehat{\overline{M}}^{-1}\eta - \dot{\eta} \\ &= -L\widehat{\overline{M}}^{-1}(\widehat{\eta} - \eta) - \dot{\eta} = -L\widehat{\overline{M}}^{-1}\widetilde{\eta} - \dot{\eta}. \end{aligned}$$
(28)

From (27), it can be obtained that  $\dot{V}_1 \leq 0$  in the condition of  $k_2 \geq ||\tilde{\eta}||$ . By Assumptions 1 and 2, choose a suitable value of *L* to make  $-L\widehat{\overline{M}}^{-1}$  be Hurwitz matrix. Then, the disturbance error  $\tilde{\eta}$  would asymptotically converge to zero.

From (17), one has  $e_c = A^{-1}e_v$ . By Assumption 1,  $s_L, s_R$  are small constants, therefore  $A^{-1}$  is bounded. From (27), it can be concluded that  $e_v$  is asymptotical stability, thus  $e_c$  asymptotically converges to zero.

Choose the Lyapunov functional candidate  $V_2$  as

$$V_2 = \frac{1}{2}(x_e^2 + y_e^2) + 2(1 - \cos\frac{\theta_e}{2}).$$
<sup>(29)</sup>

Then, the time derivative of (29), one has

$$\dot{V}_2 = x_e \dot{x}_e + y_e \dot{y}_e + \sin \frac{\theta_e}{2} \dot{\theta}_e$$
$$= -k_x x_e^2 - k_y \sin^2 \frac{\theta_e}{2} \le 0.$$
(30)

Letting  $\dot{V}_2 = 0$ , one has  $x_e = 0$ ,  $\theta_e = 0$ . According to LaSalle's invariance principle, it can be seen that  $y_e = 0$ . Due to  $V_2 \ge 0$  and  $\dot{V}_2 \le 0$ , it can be concluded that  $x_e \to 0$ ,  $y_e \to 0$  and  $\theta_e \to 0$ , as  $t \to \infty$ .

# 5. SIMULATION

In this section, the effectiveness of the controller is provided through a series of simulation experiments. The parameters of WMR are chosen as: the radius of the wheels is r = 0.2 m, the distance is b = 0.45 m, total mass of the robot and total moment of inertia are m = 5 kg, I = 4 kg.m<sup>2</sup>. Parameters' selection for disturbance observer and controller are  $k_x = k_y = k_1 = k_2 = 5$ ,  $\gamma = 0.5$ ,  $L = diag\{l_1, l_2\}$ , where  $l_1 = l_2 = 30$ . The initial and reference pose of WMR are  $(x(0), y(0), \theta(0)) = (3, 0.4, 0.4)$  and  $(x_r(0), y_r(0), \theta_r(0)) = (2, 1, 0.2\pi)$ . The reference velocities are  $v_r = 4$  m/s,  $\omega_r = 3$  rad/s. The disturbance could be expressed as:  $\overline{F} = \begin{bmatrix} 0.3\dot{v} + \omega + e^{-t} \cos t \\ 0.2\dot{\omega} + v + e^{-t} \sin t \end{bmatrix}$ .

Fig. 1 shows WMR's actual trajectory gradually coincides with the reference trajectory. Fig. 2 exhibits the control torques and the velocities. The tracking error signals and slip parameters are shown in Fig. 3.



Fig. 1. The robot tracking performance with expected trajectory.



Fig. 2. The input torque and velocity.



Fig. 3. The trajectory error and slip parameters.

### 6. CONCLUSION

In this paper, an adaptive tracking control method has been proposed for tracking the reference trajectory of WMR with the consideration of skidding and slipping. Slip degree parameters are important for slipcompensation, and the disturbance observer could estimate the total disturbances. The adaptive controller is designed based on the disturbance observer. From the simulation results, it can be seen that the proposed control scheme is effective. In the future work, we might consider the Magic formula and Fiala tire model in the process for designing the controller of WMR.

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