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Two-stage Gradient-based Iterative Estimation Methods for Controlled Autoregressive Systems Using the Measurement Data

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Abstract: This paper considers the parameter identification problems of controlled autoregressive systems using observation information. According to the hierarchical identification principle, we decompose the controlled autoregressive system into two subsystems by introducing two fictitious output variables. Then a two-stage gradient-based iterative algorithm is proposed by means of the iterative technique. In order to improve the performance of the tracking the time-varying parameters, we derive a two-stage multi-innovation gradient-based iterative algorithm based on the multi-innovation identification theory. Finally, an example is provided to illustrate the effectiveness of the proposed algorithms.

Keywords: Gradient search, hierarchical identification, iterative technique, mathematical modeling, multi-innovation identification, parameter estimation.

1. INTRODUCTION

Mathematical models are useful for system analysis and control [1,2] and can be applied in many areas such as network model fitting [3, 4]. Basically, there are two ways of constructing mathematical models [5–8]. The first one is the analytic approach, which uses basic laws from physics to describe the dynamical behavior of a process. Due to the difficult of the first way, system identification, which is an experimental way, becomes the common choice to establish mathematical models [9]. Some identification methods have been used for signal modeling [10–12] and time-series modeling [13].

The identification methods can be divided into two main types, one of which is the on-line methods and the other is off-line methods [14–17]. The on-line methods, which are also called the recursive methods, compute the parameter estimates in real-time. The off-line methods, i.e., the iterative methods, update the parameter estimates by using batch data [18, 19]. By eliminating the state variables, Li and Liu gave the input-output representation for a class of bilinear systems and proposed a filtering based least squares iterative algorithm by means of the hierarchical principle [20] and the data filtering technique [21].

The multi-innovation identification is an important branch of system identification [22, 23]. The innovation refers to the useful information that can improve the parameter estimation accuracy. The main idea is to expand the dimension of the innovation and to make full use of the system information. In this literature, for the sine combination signals and periodic signals, a multi-innovation stochastic gradient algorithm was derived by expanding the scalar innovation into the innovation vector [24]. In this paper, we expand the multi-innovation theory into the iterative identification algorithm. In particular, we update the parameter estimates by using the data in a moving data window which moves forward with time increasing.

In this paper, the parameter estimation problems of controlled autoregressive systems are investigated using the gradient search [25] and the hierarchical identification principle [26]. On the basis of the gradient-based iterative parameter estimation algorithms for dynamical systems [27, 28], this work is to divide the linear autoregressive identification model into two sub-identification models, and to identify the parameter vectors of these two models, respectively. The main contributions of this paper are as follows:

- A two-stage identification model is deduced for controlled autoregressive systems by adopting the hierarchical identification principle.
- Based on the gradient search, a two-stage gradient-

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based iterative algorithm is presented for identifying the parameters of the controlled autoregressive systems.

- A two-stage multi-innovation gradient-based iterative algorithm is derived by means of the multi-innovation identification theory.

The rest of this paper is organized as follows: Section 2 offers some notations and derives the two-stage identification model of controlled autoregressive systems. A two-stage gradient-based iterative algorithm is presented in Section 3. By employing the multi-innovation identification theory, Section 4 deduces a two-stage multi-innovation gradient-based iterative algorithm. Section 5 gives the gradient-based iterative algorithm for comparison. Section 6 offers an example to illustrate the effectiveness of the proposed algorithms. Finally, Section 7 gives some concluding remarks.

2. SYSTEM DESCRIPTION AND IDENTIFICATION MODELS

Let us start with the necessary notations. "$A =: X$" or "$X := A$" stands for "A is defined as X"; the superscript T stands for the vector/matrix transpose; The symbol I_n denotes an identity matrix of size $n \times n$; $\mathbf{1}_n$ stands for an n-dimensional column vector whose elements are 1; the norm of a matrix (or a column vector) X is defined by $\|X\|^2 := \mathrm{tr}[XX^{\mathrm{T}}]$. $\lambda_{\max}[X]$ is the largest eigenvalue of the symmetric matrix X.

Consider the following controlled autoregressive (CAR) system:

$$A(z)y(t) = B(z)u(t) + v(t), \tag{1}$$

where $u(t)$ is the input of the system and $y(t)$ is the output of the system, $v(t)$ is a white noise with zero mean, $A(z)$ and $B(z)$ are polynomials in the unit backward shift operator $[z^{-1}y(t) = y(t-1), zy(t) = y(t+1)]$, and defined as

$$\begin{aligned} A(z) &:= 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a}, \\ B(z) &:= b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}. \end{aligned}$$

Assume that n_a and n_b are known, and $y(t) = 0$, $u(t) = 0$ and $v(t) = 0$ for $t \leqslant 0$.

Define the parameter vectors:

$$\begin{aligned} a &:= [a_1, a_2, \cdots, a_{n_a}]^{\mathrm{T}} \in \mathbb{R}^{n_a}, \\ b &:= [b_1, b_2, \cdots, b_{n_b}]^{\mathrm{T}} \in \mathbb{R}^{n_b}, \end{aligned}$$

and the corresponding information vectors:

$$\begin{aligned} \varphi_a(t) &:= [-y(t-1), -y(t-2), \cdots, -y(t-n_a)]^{\mathrm{T}} \in \mathbb{R}^{n_a}, \\ \varphi_b(t) &:= [u(t-1), u(t-2), \cdots, u(t-n_b)]^{\mathrm{T}} \in \mathbb{R}^{n_b}. \end{aligned}$$

Through the above definitions, the system in (1) can be rewritten as

$$\begin{aligned} y(t) &= [1 - A(z)]y(t) + B(z)u(t) + v(t) \\ &= \varphi_a^{\mathrm{T}}(t)a + \varphi_b^{\mathrm{T}}(t)b + v(t). \end{aligned} \tag{2}$$

For the identification model in (2), the commonly used method is to combine the information vectors $\varphi_a(t)$ and $\varphi_b(t)$ into a large information vector $\varphi(t) := \begin{bmatrix} \varphi_a(t) \\ \varphi_b(t) \end{bmatrix}$, and to define a corresponding parameter vector. Then equation (2) can be transformed to the linear autoregressive identification model. Here, we provide another idea to dispose the identification model in (2) by adopting the hierarchical identification principle. Hierarchical identification is the decomposition based identification. The key idea is to decompose the identification model into several subsystems, such that the scale of the optimization problem can be decreased and the computational efficiency of the identification algorithm can be improved.

Define two fictitious output variables:

$$\begin{aligned} y_1(t) &:= y(t) - \varphi_b^{\mathrm{T}}(t)b \in \mathbb{R}, \\ y_2(t) &:= y(t) - \varphi_a^{\mathrm{T}}(t)a \in \mathbb{R}. \end{aligned}$$

Then the identification model in (2) can be decomposed into two sub-identification models:

$$y_1(t) = \varphi_a^{\mathrm{T}}(t)a + v(t), \tag{3}$$

$$y_2(t) = \varphi_b^{\mathrm{T}}(t)b + v(t). \tag{4}$$

Equations (3)-(4) can also be called the two-stage identification model for the system in (1). Equation (3) contains the parameter vector a and the information vector $\varphi_a(t)$, and (4) includes the parameter vector b and the information vector $\varphi_b(t)$. Here, we can find that there are two coupled variables a and b between these two sub-identification models. The objective of this paper is to use the gradient search principle and the multi-innovation identification theory to propose two-stage algorithms, coordinating the associate items between sub-identification models based on the hierarchical identification principle.

3. THE TWO-STAGE GRADIENT-BASED ITERATIVE ALGORITHM

The iterative algorithm uses batch data to update the parameter estimates. In this section, we define two cost functions to present a two-stage gradient-based iterative algorithm by using the gradient search principle. In addition, we give a brief discussion about how to choose the iterative step-sizes in the algorithm.

Let L be the data length. According to the two-stage identification model in (3)-(4), define two gradient criterion functions as

$$J_1(a) := \frac{1}{2}\sum_{j=1}^{L}[y_1(j) - \varphi_a^{\mathrm{T}}(j)a]^2,$$

$$J_2(b) := \frac{1}{2}\sum_{j=1}^{L}[y_2(j)-\varphi_b^{\mathrm{T}}(j)b]^2.$$

Define the stacked output vector $Y(L)$, the stacked fictitious output vectors $Y_1(L)$ and $Y_2(L)$, and the stacked information matrices $\Phi_a(L)$ and $\Phi_b(L)$ as

$$Y(L) := \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(L) \end{bmatrix} \in \mathbb{R}^L,$$

$$Y_1(L) := \begin{bmatrix} y_1(1) \\ y_1(2) \\ \vdots \\ y_1(L) \end{bmatrix} = Y(L)-\Phi_b(L)b \in \mathbb{R}^L,$$

$$Y_2(L) := \begin{bmatrix} y_2(1) \\ y_2(2) \\ \vdots \\ y_2(L) \end{bmatrix} = Y(L)-\Phi_a(L)a \in \mathbb{R}^L,$$

$$\Phi_a(L) := \begin{bmatrix} \varphi_a^{\mathrm{T}}(1) \\ \varphi_a^{\mathrm{T}}(2) \\ \vdots \\ \varphi_a^{\mathrm{T}}(L) \end{bmatrix} \in \mathbb{R}^{L\times n_a},$$

$$\Phi_b(L) := \begin{bmatrix} \varphi_b^{\mathrm{T}}(1) \\ \varphi_b^{\mathrm{T}}(2) \\ \vdots \\ \varphi_b^{\mathrm{T}}(L) \end{bmatrix} \in \mathbb{R}^{L\times n_b}.$$

Then the cost functions $J_1(a)$ and $J_2(b)$ can be equivalently expressed as

$$J_3(a) := \frac{1}{2}\|Y_1(L)-\Phi_a(L)a\|^2,$$
$$J_4(b) := \frac{1}{2}\|Y_2(L)-\Phi_b(L)b\|^2.$$

Let $k = 1,2,3,\cdots$ be an iterative variable, $\hat{a}_k \in \mathbb{R}^{n_a}$ and $\hat{b}_k \in \mathbb{R}^{n_b}$ be the iterative estimates of a and b at iteration k, and $\mu_1 \geqslant 0$ and $\mu_2 \geqslant 0$ be the step-sizes, which are also known as the convergence factors. Using the negative gradient search to minimize the criterion function $J_3(a)$ and $J_4(b)$ results in the following gradient-based iterative relations,

$$\begin{aligned}
\hat{a}_k =& \hat{a}_{k-1}-\mu_1\mathrm{grad}[J_3(\hat{a}_{k-1})] \\
=& \hat{a}_{k-1}+\mu_1\Phi_a^{\mathrm{T}}(L)[Y_1(L)-\Phi_a(L)\hat{a}_{k-1}] \\
=& \hat{a}_{k-1}+\mu_1\Phi_a^{\mathrm{T}}(L) \\
& \times [Y(L)-\Phi_b(L)b-\Phi_a(L)\hat{a}_{k-1}] \qquad (5) \\
=& [I_{n_a}-\mu_1\Phi_a^{\mathrm{T}}(L)\Phi_a(L)]\hat{a}_{k-1} \\
& +\mu_1\Phi_a^{\mathrm{T}}(L)[Y(L)-\Phi_b(L)b], \qquad (6) \\
\hat{b}_k =& \hat{b}_{k-1}-\mu_2\mathrm{grad}[J_4(\hat{b}_{k-1})] \\
=& \hat{b}_{k-1}+\mu_2\Phi_b^{\mathrm{T}}(L)[Y_2(L)-\Phi_b(L)\hat{b}_{k-1}] \\
=& \hat{b}_{k-1}+\mu_2\Phi_b^{\mathrm{T}}(L) \\
& \times [Y(L)-\Phi_a(L)a-\Phi_b(L)\hat{b}_{k-1}] \qquad (7) \\
=& [I_{n_b}-\mu_2\Phi_b^{\mathrm{T}}(L)\Phi_b(L)]\hat{b}_{k-1} \\
& +\mu_2\Phi_b^{\mathrm{T}}(L)[Y(L)-\Phi_a(L)a]. \qquad (8)
\end{aligned}$$

Equations (6) and (8) can be seen as the discrete-time systems of the state variables $\hat{a}_k$ and $\hat{b}_k$. In order to make sure of the convergence of the parameter estimation vectors $\hat{a}_k$ and $\hat{b}_k$, all the eigenvalues of matrices $[I_{n_a}-\mu_1\Phi_a^{\mathrm{T}}(L)\Phi_a(L)]$ and $[I_{n_b}-\mu_2\Phi_b^{\mathrm{T}}(L)\Phi_b(L)]$ must be in the unit circle. In other words, μ_1 and μ_2 should satisfy

$$-I_{n_a} \leq I_{n_a}-\mu_1\Phi_a^{\mathrm{T}}(L)\Phi_a(L) \leq I_{n_a},$$
$$-I_{n_b} \leq [I_{n_b}-\mu_2\Phi_b^{\mathrm{T}}(L)\Phi_b(L)] \leq I_{n_b}.$$

Therefore, one conservative choice is

$$\mu_1 \leq \frac{2}{\lambda_{\max}[\Phi_a^{\mathrm{T}}(L)\Phi_a(L)]} = 2\lambda_{\max}^{-1}[\Phi_a^{\mathrm{T}}(L)\Phi_a(L)], \quad (9)$$
$$\mu_2 \leq \frac{2}{\lambda_{\max}[\Phi_b^{\mathrm{T}}(L)\Phi_b(L)]} = 2\lambda_{\max}^{-1}[\Phi_b^{\mathrm{T}}(L)\Phi_b(L)]. \quad (10)$$

Because of the complexity of the eigenvalue computation, the convergence factors can also be simply and conservatively taken as

$$\mu_1 \leq \frac{2}{\|\Phi_a(L)\|^2} = 2\|\Phi_a(L)\|^{-2}, \qquad (11)$$
$$\mu_2 \leq \frac{2}{\|\Phi_b(L)\|^2} = 2\|\Phi_b(L)\|^{-2}. \qquad (12)$$

Remark 1: Compared with that of (9) and (10), the computation of the convergence factors μ_1 and μ_2 in (11) and (12) does not involve the complicated eigenvalues of square matrices. Instead, we use the traces of the matrices $\Phi_a(L)$ and $\Phi_b(L)$, such that the computational cost of finding the convergence factors can be reduced.

However, equations (6), (8) and (9)-(12) still cannot generate the estimates $\hat{a}_k$ and $\hat{b}_k$. Because there are unknown variables b and a on the right-hand side of (6) and (8). To cope with this problem, we use the hierarchical identification principle to coordinate the associated terms. That is, use the estimates $\hat{b}_{k-1}$ and $\hat{a}_{k-1}$ at iteration $k-1$ to replace the unknown variables b and a in (6) and (8). Combining (9)-(12), we can derive a two-stage gradient-based iterative (2S-GI) algorithm to estimate the parameter vectors a and b for the CAR system:

$$\begin{aligned}
\hat{a}_k =& \hat{a}_{k-1}+\mu_1\Phi_a^{\mathrm{T}}(L) \\
& \times [Y(L)-\Phi_a(L)\hat{a}_{k-1}-\Phi_b(L)\hat{b}_{k-1}], \qquad (13) \\
\mu_1 =& \lambda_{\max}^{-1}[\Phi_a^{\mathrm{T}}(L)\Phi_a(L)], \qquad (14) \\
\hat{b}_k =& \hat{b}_{k-1}+\mu_2\Phi_b^{\mathrm{T}}(L) \\
& \times [Y(L)-\Phi_a(L)\hat{a}_{k-1}-\Phi_b(L)\hat{b}_{k-1}], \qquad (15)
\end{aligned}$$

$$\mu_2 = \lambda_{\max}^{-1}[\Phi_b^{\mathrm{T}}(L)\Phi_b(L)], \tag{16}$$
$$Y(L) = [y(1), y(2), \cdots, y(L)]^{\mathrm{T}}, \tag{17}$$
$$\Phi_a(L) = [\varphi_a(1), \varphi_a(2), \cdots, \varphi_a(L)]^{\mathrm{T}}, \tag{18}$$
$$\Phi_b(L) = [\varphi_b(1), \varphi_b(2), \cdots, \varphi_b(L)]^{\mathrm{T}}, \tag{19}$$
$$\varphi_a(t) = [-y(t-1), -y(t-2), \cdots, -y(t-n_a)]^{\mathrm{T}}, \tag{20}$$
$$\varphi_b(t) = [u(t-1), u(t-2), \cdots, u(t-n_b)]^{\mathrm{T}}, \tag{21}$$
$$\hat{a}_k = [\hat{a}_{1,k}, \hat{a}_{2,k}, \cdots, \hat{a}_{n_a,k}]^{\mathrm{T}}, \tag{22}$$
$$\hat{b}_k = [\hat{b}_{1,k}, \hat{b}_{2,k}, \cdots, \hat{b}_{n_b,k}]^{\mathrm{T}}. \tag{23}$$

The steps of computing $\hat{a}_k$ and $\hat{b}_k$ involved in the 2S-GI algorithm in (13)-(23) are summarized in the following:

1) For $t \leqslant 0$, all the variables are set to be zero. Let $k = 1$, give the data length L ($L \gg n_a + n_b$) and set the initial values: $\hat{a}_0 = \mathbf{1}_{n_a}/p_0$, $\hat{b}_0 = \mathbf{1}_{n_b}/p_0$, $p_0 = 10^6$, and the parameter estimation accuracy ε.
2) Collect the input and output data $u(t)$ and $y(t)$, $t = 1, 2, \cdots, L$. Form the information vectors $\varphi_a(t)$ and $\varphi_b(t)$ using (20)-(21).
3) Construct the stacked output vector $Y(L)$ by (17) and the stacked information matrices $\Phi_a(L)$ and $\Phi_b(L)$ by (18)-(19), compute the step-sizes μ_1 and μ_2 according to (14) and (16).
4) Update the parameter estimation vectors $\hat{a}_k$ and $\hat{b}_k$ by using (13) and (15). Read out the estimates $\hat{a}_{i,k}$ ($i = 1, 2, \cdots, n_a$) and $\hat{b}_{j,k}$ ($j = 1, 2, \cdots, n_b$) from (22) and (23).
5) If $\|\hat{a}_k - \hat{a}_{k-1}\| + \|\hat{b}_k - \hat{b}_{k-1}\| > \varepsilon$, increase k by 1 and go to Step 4; otherwise, obtain the iteration k and the parameter estimation vectors $\hat{a}_k$ and $\hat{b}_k$, terminate this computational procedure.

Remark 2: The 2S-GI algorithm in (13)-(23) can also be called the hierarchical gradient-based iterative (HGI) algorithm. The difference is that the 2S-GI algorithm is just one kind of the HGI algorithm. The three-stage gradient-based iterative algorithm and the multi-stage gradient-based iterative algorithm are all known as the HGI algorithm.

4. THE TWO-STAGE MULTI-INNOVATION GRADIENT-BASED ITERATIVE ALGORITHM

The main idea of the multi-innovation identification theory is to use the data in a moving data window to update the parameter estimates. In addition, the moving data window moves forward as time t increases. In this section, we adopt the multi-innovation identification theory to deal with the CAR system in (1) and propose a two-stage multi-innovation gradient-based iterative algorithm which has a constant window length.

Consider the newest p data from $j = t - p + 1$ to $j = t$ ($p \gg n_a + n_b$), and define the stacked output vector $Y(p,t)$, the stacked fictitious output vectors $Y_1(p,t)$ and $Y_2(p,t)$, and the stacked information matrices $\Phi_a(p,t)$ and $\Phi_b(p,t)$:

$$Y(p,t) := \begin{bmatrix} y(t) \\ y(t-1) \\ \vdots \\ y(t-p+1) \end{bmatrix} \in \mathbb{R}^p,$$

$$Y_1(p,t) := \begin{bmatrix} y_1(t) \\ y_1(t-1) \\ \vdots \\ y_1(t-p+1) \end{bmatrix} = Y(p,t) - \Phi_b(p,t)b \in \mathbb{R}^p,$$

$$Y_2(p,t) := \begin{bmatrix} y_2(t) \\ y_2(t-1) \\ \vdots \\ y_2(t-p+1) \end{bmatrix} = Y(p,t) - \Phi_a(p,t)a \in \mathbb{R}^p,$$

$$\Phi_a(p,t) := \begin{bmatrix} \varphi_a^{\mathrm{T}}(t) \\ \varphi_a^{\mathrm{T}}(t-1) \\ \vdots \\ \varphi_a^{\mathrm{T}}(t-p+1) \end{bmatrix} \in \mathbb{R}^{p\times n},$$

$$\Phi_b(p,t) := \begin{bmatrix} \varphi_b^{\mathrm{T}}(t) \\ \varphi_b^{\mathrm{T}}(t-1) \\ \vdots \\ \varphi_b^{\mathrm{T}}(t-p+1) \end{bmatrix} \in \mathbb{R}^{p\times n_b}.$$

According to the two-stage identification models in (3)-(4), define two criterion functions:

$$\begin{aligned} J_5(a) &:= \frac{1}{2}\sum_{j=t-p+1}^{t}[y_1(j) - \varphi_a^{\mathrm{T}}(j)a]^2 \\ &= \frac{1}{2}\|Y_1(p,t) - \Phi_a(p,t)a\|^2, \\ J_6(b) &:= \frac{1}{2}\sum_{j=t-p+1}^{t}[y_2(j) - \psi^{\mathrm{T}}(j)b]^2 \\ &= \frac{1}{2}\|Y_2(p,t) - \Phi_b(p,t)b\|^2. \end{aligned}$$

Let $\mu_1(t) \geqslant 0$ and $\mu_2(t) \geqslant 0$ be the step-sizes. Using the negative gradient search to minimize the criterion functions $J_5(a)$ and $J_6(b)$ gives the following gradient-based iterative relations:

$$\begin{aligned} \hat{a}_k(t) =& \hat{a}_{k-1}(t) - \mu_1(t)\mathrm{grad}[J_5(\hat{a}_{k-1}(t))] \\ =& \hat{a}_{k-1}(t) + \mu_1(t)\Phi_a^{\mathrm{T}}(p,t) \\ & \times [Y_1(p,t) - \Phi_a(p,t)\hat{a}_{k-1}(t)] \\ =& \hat{a}_{k-1}(t) + \mu_1(t)\Phi_a^{\mathrm{T}}(p,t) \\ & \times [Y(p,t) - \Phi_b(p,t)b - \Phi_a(p,t)\hat{a}_{k-1}(t)] \\ =& [I_{n_a} - \mu_1(t)\Phi_a^{\mathrm{T}}(p,t)\Phi_a(p,t)]\hat{a}_{k-1}(t) \end{aligned} \tag{24}$$

$$+\mu_1(t)\Phi_a^{\mathrm{T}}(p,t)[Y(p,t)-\Phi_b(p,t)b], \tag{25}$$

$$\begin{aligned}\hat{b}_k(t)=&\hat{b}_{k-1}(t)-\mu_2(t)\mathrm{grad}[J_6(\hat{b}_{k-1}(t))]\\ =&\hat{b}_{k-1}(t)+\mu_2(t)\Phi_b^{\mathrm{T}}(p,t)\\ &\times[Y_2(p,t)-\Phi_b(p,t)\hat{b}_{k-1}(t)]\\ =&\hat{b}_{k-1}(t)+\mu_2(t)\Phi_b^{\mathrm{T}}(p,t)\\ &\times[Y(p,t)-\Phi_a(p,t)a-\Phi_b(p,t)\hat{b}_{k-1}(t)] \quad (26)\\ =&[I_{n_b}-\mu_2(t)\Phi_b^{\mathrm{T}}(p,t)\Phi_b(p,t)]\hat{b}_{k-1}(t)\\ &+\mu_2(t)\Phi_b^{\mathrm{T}}(p,t)[Y(p,t)-\Phi_a(p,t)a]. \quad (27)\end{aligned}$$

Similarly to the derivation of the 2S-GI algorithm in (13)-(23), Equations (25) and (27) can be seen as two discrete-time systems of the state variables $\hat{a}_k(t)$ and $\hat{b}_k(t)$. In order to make sure of the convergence of the parameter estimation vectors $\hat{a}_k$ and $\hat{b}_k$, all the eigenvalues of matrices $[I_{n_a}-\mu_1(t)\Phi_a^{\mathrm{T}}(p,t)\Phi_a(p,t)]$ and $[I_{n_b}-\mu_2(t)\Phi_b^{\mathrm{T}}(p,t)\Phi_b(p,t)]$ must be in the unit circle, that is to say, $\mu_1(t)$ and $\mu_2(t)$ must satisfy

$$\begin{aligned}&-I_{n_a}\leqslant I_{n_a}-\mu_1(t)\Phi_a^{\mathrm{T}}(p,t)\Phi_a(p,t)\leqslant I_{n_a},\\ &-I_{n_b}\leqslant I_{n_b}-\mu_2(t)\Phi_b^{\mathrm{T}}(p,t)\Phi_b(p,t)\leqslant I_{n_b}.\end{aligned}$$

Therefore, the step-sizes can be conservatively chosen as

$$\begin{aligned}\mu_1(t)&\leqslant\frac{2}{\lambda_{\max}[\Phi_a^{\mathrm{T}}(p,t)\Phi_a(p,t)]}\\ &=2\lambda_{\max}^{-1}[\Phi_a^{\mathrm{T}}(p,t)\Phi_a(p,t)], \quad (28)\\ \mu_2(t)&\leqslant\frac{2}{\lambda_{\max}[\Phi_b^{\mathrm{T}}(p,t)\Phi_b(p,t)]}\\ &=2\lambda_{\max}^{-1}[\Phi_b^{\mathrm{T}}(p,t)\Phi_b(p,t)]. \quad (29)\end{aligned}$$

In consideration of the computational complexity, we take the step-sizes as

$$\mu_1(t)\leqslant\frac{2}{\|\Phi_a(p,t)\|^2}=2\|\Phi_a(p,t)\|^{-2}, \tag{30}$$

$$\mu_2(t)\leqslant\frac{2}{\|\Phi_b(p,t)\|^2}=2\|\Phi_b(p,t)\|^{-2}. \tag{31}$$

Remark 3: Similar to (11) and (12), we use the traces of the matrices $\Phi_a(p,t)$ and $\Phi_b(p,t)$ in (30) and (31) to replace the complicated eigenvalues of square matrices in computing the convergence factors $\mu_1(t)$ and $\mu_2(t)$, so as to reduce the computational cost of finding the convergence factors.

To obtain a realizable algorithm, we replace the unknown parameters b and a in (24) and (26) with their estimates $\hat{b}_{k-1}(t)$ and $\hat{a}_{k-1}(t)$. Combining (28) to (31), we can obtain the two-stage multi-innovation gradient-based iterative (2S-MIGI) algorithm for estimating parameter vectors a and b:

$$\begin{aligned}\hat{a}_k(t)=&\hat{a}_{k-1}(t)+\mu_1(t)\Phi_a^{\mathrm{T}}(p,t)[Y(p,t)\\ &-\Phi_a(p,t)\hat{a}_{k-1}(t)-\Phi_b(p,t)\hat{b}_{k-1}(t)], \quad (32)\end{aligned}$$

$$\begin{aligned}&\mu_1(t)=\lambda_{\max}^{-1}[\Phi_a^{\mathrm{T}}(p,t)\Phi_a(p,t)] \ \text{ or}\\ &\mu_1(t)=\|\Phi_a(p,t)\|^{-2}, \quad (33)\\ &\hat{b}_k(t)=\hat{b}_{k-1}(t)+\mu_2(t)\Phi_b^{\mathrm{T}}(p,t)[Y(p,t)\\ &\qquad-\Phi_a(p,t)\hat{a}_{k-1}(t)-\Phi_b(p,t)\hat{b}_{k-1}(t)], \quad (34)\\ &\mu_2(t)=\lambda_{\max}^{-1}[\Phi_b^{\mathrm{T}}(p,t)\Phi_b(p,t)] \ \text{ or}\\ &\mu_2(t)=\|\Phi_b(p,t)\|^{-2}, \quad (35)\\ &Y(p,t)=[y(t),y(t-1),\cdots,y(t-p+1)]^{\mathrm{T}}, \quad (36)\\ &\Phi_a(p,t)=[\varphi_a(t),\varphi_a(t-1),\cdots,\varphi_a(t-p+1)]^{\mathrm{T}}, \quad (37)\\ &\Phi_b(p,t)=[\varphi_b(t),\varphi_b(t-1),\cdots,\varphi_b(t-p+1)]^{\mathrm{T}}, \quad (38)\\ &\varphi_a(t)=[-y(t-1),\cdots,-y(t-n_a)]^{\mathrm{T}}, \quad (39)\\ &\varphi_b(t)=[u(t-1),u(t-2),\cdots,u(t-n_b)]^{\mathrm{T}}, \quad (40)\\ &\hat{a}_k(t)=[\hat{a}_{1,k}(t),\hat{a}_{2,k}(t),\cdots,\hat{a}_{n_a,k}(t)]^{\mathrm{T}}, \quad (41)\\ &\hat{b}_k(t)=[\hat{b}_{1,k}(t),\hat{b}_{2,k}(t),\cdots,\hat{b}_{n_b,k}(t)]^{\mathrm{T}}. \quad (42)\end{aligned}$$

If we take $p=t=L$, then the 2S-MIGI algorithm in (32)-(42) reduces to 2S-GI algorithm in (13)-(23).

The procedure of computing the parameter estimation vectors $\hat{a}_k(t)$ and $\hat{b}_k(t)$ by the 2S-MIGI algorithm in (32)-(42) is listed as follows:

1) For $t\leqslant 0$, all the variables are set to to be zero. Let $k=1$, give the moving data window length p ($p\gg n_a+n_b$) and set the initial values: $\hat{a}_0=\mathbf{1}_{n_a}/p_0$, $\hat{b}_0=\mathbf{1}_{n_b}/p_0$, $p_0=10^6$, the maximum iteration $k_{\max}$ and ε.
2) Let $k=1$, collect the input and output data $u(t)$ and $y(t)$, form the information vectors $\varphi_a(t)$ and $\varphi_b(t)$ using (39)-(40).
3) Construct the stacked output vector $Y(p,t)$ by (36) and the stacked information matrices $\Phi_a(p,t)$ and $\Phi_b(p,t)$ by using (37)-(38).
4) Compute the step-sizes $\mu_1(t)$ and $\mu_2(t)$ according to (33) and (35).
5) Update the parameter estimation vectors $\hat{a}_k(t)$ and $\hat{b}_k(t)$ by using (32) and (34). Read out the estimates $\hat{a}_{i,k}(t)$ and $\hat{b}_{j,k}(t)$ from (41) and (42).
6) If $k<k_{\max}$, increase k by 1 and go to Step 5; otherwise, proceed to the next step.
7) Compare $\hat{a}_k(t)$ and $\hat{b}_k(t)$ with $\hat{a}_{k-1}(t)$ and $\hat{b}_{k-1}(t)$: If $\|\hat{a}_k(t)-\hat{a}_{k-1}(t)\|+\|\hat{b}_k(t)-\hat{b}_{k-1}(t)\|>\varepsilon$, then set $\hat{a}_0(t+1):=\hat{a}_k(t)$ and $\hat{b}_0(t+1):=\hat{b}_k(t)$, increase t by 1 and go to Step 2; otherwise, obtain the parameter estimation vectors $\hat{a}_k(t)$ and $\hat{b}_k(t)$, terminate this computational procedure.

Remark 4: The 2S-MIGI algorithm both have the time variable t and the iterative variable k. At time t, we collect p data in a moving data window with the fixed length and compute the parameter estimates $\hat{a}_k(t)$ and $\hat{b}_k(t)$ with k increasing. If there is no significant change in the parameter estimates with increasing k, we increase t rather than k and introduce new data into the algorithm.

Remark 5: Compared with the 2S-GI algorithm in (13)-(23), the 2S-MIGI algorithm can track time-varying parameters. Because the 2S-MIGI expands the dimensions of the innovation and makes full use of the system information. For $k=1,2,\cdots,k_{\max}-1$, we use the newest p data from $j=t-p+1$ to $j=t$ to estimate the parameter vectors. For $k=k_{\max}$, we move the data window forward to the next moment, introduce new observation data and eliminate the oldest data to keep p data in the data window. Based on the above idea, we repeat the iterative process until we obtain the satisfactory parameter estimates.

At each iterative estimation, both the 2S-GI and the 2S-MIGI algorithms use the data with the fixed length to identify the unknown parameters, and the iterative computing continues with increasing iteration variable k. The difference is that, the iterative computing of the 2S-GI algorithm stops when $k=k_{\max}$, where $k_{\max}$ is the iterations for desirable parameter estimation accuracy; for the 2S-MIGI algorithm, when $k=k_{\max}$, the iterative estimation will continue with the data window moved forward to the next moment. This is also the reason why the 2S-MIGI algorithm can track time-varying parameters compared with the 2S-GI algorithm. The 2S-MIGI algorithm is the extension of the 2S-GI algorithm, or the 2S-GI algorithm is the special case of the 2S-MIGI algorithm.

5. THE GRADIENT-BASED ITERATIVE ALGORITHM

In order to show that the proposed 2S-GI algorithm can generate highly accurate parameter estimates, the following gives the gradient-based iterative (GI) algorithm for estimating the parameter vector $\vartheta:=\begin{bmatrix} a \\ b \end{bmatrix}$ of the CAR system in (1):

$$\hat{\vartheta}_k=\hat{\vartheta}_{k-1}+\mu\Phi^{\mathrm{T}}(L)[Y(L)-\Phi(L)\hat{\vartheta}_{k-1}], \tag{43}$$

$$\mu=\lambda_{\max}^{-1}[\Phi^{\mathrm{T}}(L)\Phi(L)], \quad \text{or} \quad \mu=\|\Phi(L)\|^{-2}, \tag{44}$$

$$Y(L)=[y(1),y(2),\cdots,y(L)]^{\mathrm{T}}, \tag{45}$$

$$\Phi(L)=[\varphi(1),\varphi(2),\cdots,\varphi(L)]^{\mathrm{T}}, \tag{46}$$

$$\varphi(t)=\begin{bmatrix} \varphi_a(t) \\ \varphi_b(t) \end{bmatrix}, \tag{47}$$

$$\varphi_a(t)=[-y(t-1),-y(t-2),\cdots,-y(t-n_a)]^{\mathrm{T}}, \tag{48}$$

$$\varphi_b(t)=[u(t-1),u(t-2),\cdots,u(t-n_b)]^{\mathrm{T}}. \tag{49}$$

$$\hat{\vartheta}_k=\begin{bmatrix} \hat{a}_k \\ \hat{b}_k \end{bmatrix}, \tag{50}$$

$$\hat{a}_k=[\hat{a}_{1,k},\hat{a}_{2,k},\cdots,\hat{a}_{n_a,k}]^{\mathrm{T}}, \tag{51}$$

$$\hat{b}_k=[\hat{b}_{1,k},\hat{b}_{2,k},\cdots,\hat{b}_{n_b,k}]^{\mathrm{T}}. \tag{52}$$

The 2S-GI algorithm can generate more accurate parameter estimates than the GI algorithm. The proposed 2s-GI algorithm and 2S-MIGI algorithm for controlled autoregressive systems can combine other estimation algorithms [29–31] to explore new identification methods of different systems with colored noises [32–37] and can be applied to other fields such as information processing and communication [38–44].

6. EXAMPLE

Consider the following second-order controlled autoregressive system:

$$\begin{aligned} &A(z)y(t)=B(z)u(t)+v(t),\\ &A(z)=1+a_1z^{-1}+a_2z^{-2}=1+1.35z^{-1}+0.75z^{-2},\\ &B(z)=b_1z^{-1}+b_2z^{-2}=1.68z^{-1}+2.32z^{-2}. \end{aligned}$$

The parameter vector to be identified is given by

$$\vartheta=[a_1,a_2,b_1,b_2]^{\mathrm{T}}=[1.35,0.75,1.68,2.32]^{\mathrm{T}}.$$

In simulation, the input $\{u(t)\}$ is taken as an uncorrelated stochastic signal sequence with zero mean and unit variance, $\{v(t)\}$ is taken as a white noise sequence with zero mean and variance σ^2. We use the example parameters and the input signal generates the output signal sequence $\{y(t)\}$.

Taking the noise variances $\sigma^2=0.50^2$ and $\sigma^2=1.50^2$, respectively, the corresponding noise-to-signal ratios are $\delta_{\mathrm{ns}}=33.78\%$ and $\delta_{\mathrm{ns}}=101.33\%$. Taking the data length $L=3000$, applying the GI and 2S-GI algorithms and the input-output data $\{u(t),y(t):\ t=1,2,\cdots,L\}$ to estimate the parameters of this example system, the GI and 2S-GI parameter estimates and errors versus k are shown in Tables 1 to 4, the GI and 2S-GI estimation errors $\delta:=\|\hat{\vartheta}(t)-\vartheta\|/\|\vartheta\|$ versus k are shown in Figs. 1 and 2 with $\sigma^2=0.50^2$ and $\sigma^2=1.50^2$, and the GI and 2S-GI estimates of the parameters a_1, a_2, b_1 and b_2 versus k are shown in Figs. 3 and 4 for $\sigma^2=1.50^2$.

From Tables 1-4 and Figs. 1-4, we draw the following conclusions:

- As the noise levels decrease, the GI and 2S-GI algorithms can give more accurate parameter estimates - see and compare the parameter estimation errors in the last columns in Tables 1 and 2, and in Tables 3 and 4.
- The 2S-GI parameter estimation errors tend to zero faster than the GI parameter estimation errors - see the GI and 2S-GI estimation error curves in Figs. 1 and 2.
- Under the same noise levels, the 2S-GI algorithm can give more accurate parameter estimates than the GI algorithm - see the parameter estimation errors in the last columns in Tables 1 and 3 and in Tables 2 and 4, and compare the parameter estimation error curves in Figs. 1 and 2.

Table 1. The GI estimates and their errors with $\sigma^2 = 0.50^2$.

k	a_1	a_2	b_1	b_2	δ (%)
1	0.37465	-0.04311	0.07643	0.00366	94.80048022
2	0.50048	0.08091	0.15063	0.03563	90.77808869
5	0.73119	0.30193	0.35429	0.17235	81.03294080
10	0.89670	0.44666	0.63673	0.43991	68.16606805
20	1.03276	0.54489	1.03527	0.91685	48.85155929
50	1.22057	0.66760	1.53018	1.74466	18.86833601
100	1.31967	0.73236	1.67003	2.18910	4.17567060
150	1.34189	0.74693	1.68257	2.28945	0.97914391
200	1.34690	0.75022	1.68366	2.31211	0.28382179
True values	1.35000	0.75000	1.68000	2.32000	

Table 2. The GI estimates and their errors with $\sigma^2 = 1.50^2$.

k	a_1	a_2	b_1	b_2	δ (%)
1	0.40419	-0.09774	0.03871	0.00127	95.62556223
2	0.52849	0.02605	0.07719	0.01802	92.53172774
5	0.77934	0.27426	0.18776	0.09185	85.51106304
10	0.97476	0.46353	0.35621	0.25142	76.85043759
20	1.09838	0.57410	0.63948	0.58087	62.98815992
50	1.20883	0.65557	1.17798	1.31009	35.04771968
100	1.29146	0.71446	1.53679	1.91203	13.45227896
150	1.32465	0.73817	1.64518	2.15478	5.25923540
200	1.33802	0.74773	1.67770	2.25281	2.09961774
True values	1.35000	0.75000	1.68000	2.32000	

Table 3. The 2S-GI estimates and their errors with $\sigma^2 = 0.50^2$.

k	a_1	a_2	b_1	b_2	δ (%)
1	0.37572	-0.04323	1.61535	0.07731	79.01908360
2	0.51210	0.07990	1.66516	0.66077	60.71998715
5	0.85747	0.34711	1.67069	1.32367	36.32947056
10	1.14245	0.58174	1.67830	1.90179	15.25053891
20	1.31214	0.72137	1.68279	2.24539	2.71856140
50	1.34816	0.75101	1.68374	2.31832	0.14149581
100	1.34836	0.75117	1.68374	2.31872	0.13649796
150	1.34836	0.75117	1.68374	2.31872	0.13649723
200	1.34836	0.75117	1.68374	2.31872	0.13649723
True values	1.35000	0.75000	1.68000	2.32000	

Table 4. The 2S-GI estimates and their errors with $\sigma^2 = 1.50^2$.

k	a_1	a_2	b_1	b_2	δ (%)
1	0.40448	-0.09781	1.64023	0.05364	79.84208003
2	0.53307	0.02506	1.69099	0.68996	60.29629113
5	0.84810	0.29961	1.68855	1.30870	37.35311552
10	1.12344	0.55047	1.69009	1.86655	16.74281499
20	1.30215	0.71329	1.69107	2.22842	3.38681701
50	1.34670	0.75388	1.69132	2.31863	0.38362505
100	1.34707	0.75421	1.69132	2.31937	0.38237428
150	1.34707	0.75421	1.69132	2.31937	0.38237419
200	1.34707	0.75421	1.69132	2.31937	0.38237419
True values	1.35000	0.75000	1.68000	2.32000	

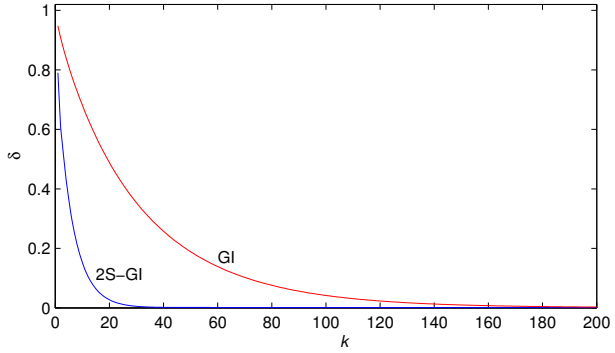

Fig. 1. The GI and 2S-GI estimation errors δ versus k with $\sigma^2 = 0.50^2$.

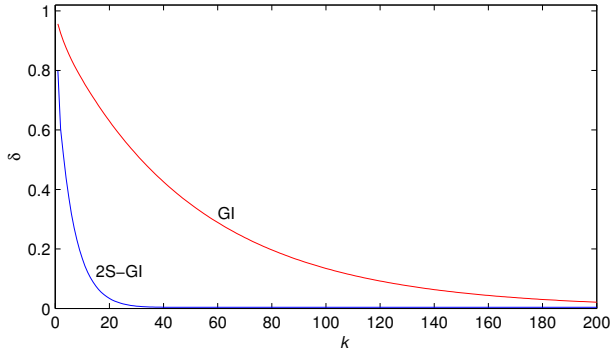

Fig. 2. The GI and 2S-GI estimation errors δ versus k with $\sigma^2 = 1.50^2$.

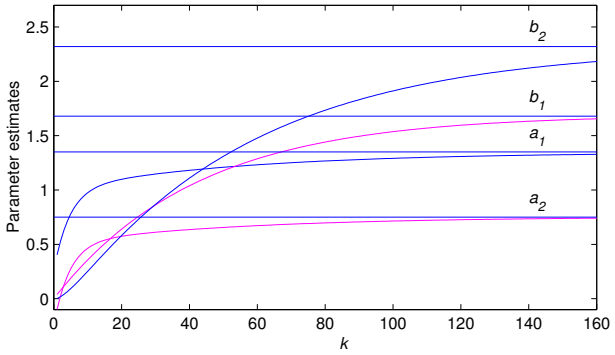

Fig. 3. The GI estimates versus k with $\sigma^2 = 1.50^2$.

- The 2S-GI parameter estimates approach to their true values faster the GI parameter estimates - see and compare the parameter estimates versus k in Figs. 3 and 4.

7. CONCLUSIONS

In this paper, the identification problems of controlled autoregressive systems have been studied. By adopting the hierarchical identification principle, we define two fictitious output variables and decompose the original system into two subsystems. Then a two-stage gradient-based iterative algorithm is proposed based on the gradient search principle. By means of the multi-innovation identification theory, we propose a two-stage multi-innovation gradient-based iterative algorithm. The simulation results indicate that the proposed algorithms are effective. In addition, the 2S-MIGI algorithm can track time-varying parameters. The proposed methods proposed in this paper can combine other tools and strategies [45–54] to study the parameter estimation problems of time-varying systems, nonlinear systems and multivariable systems, and can be applied to other literatures [55–65] such as system identification [66-69].

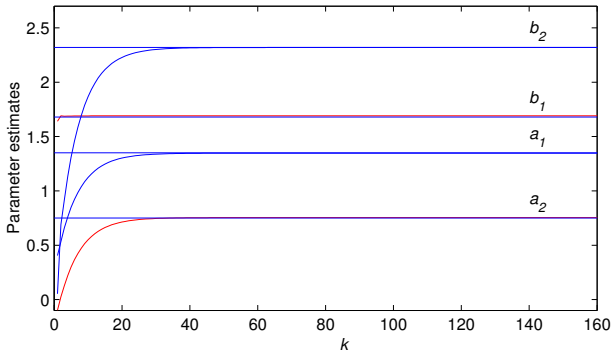

Fig. 4. The 2S-GI estimates versus k with $\sigma^2 = 1.50^2$.

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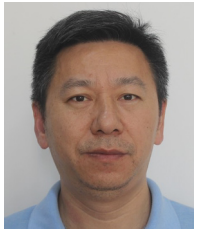

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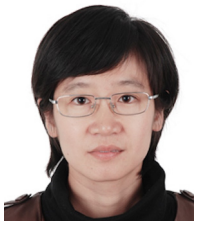

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