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A Spiral Curve Gait Design for a Modular Snake Robot Moving on a Pipe

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Abstract: Modular snake robot has the ability to maneuver narrow, complex, and unstructured environments. In recent years, Snake robot with 3-D locomotion has been studied by researchers for inspection of pipes, and stairs climbing etc. One of the effective approaches to accomplishing such complex tasks is by designing gaits for modular snake robot. In this paper, A Spiral curve gait along with rolling motion is proposed to adapt to the changes in the pipe diameter while moving on the pipe, which cannot be overcome by a simple helical rolling motion. The joint angles are calculated using Bellow's model, based on the curvature and torsion of the backbone curve. We validated the proposed gait by simulating thirty degrees of freedom modular snake robot in Open Dynamics Engine simulator.

Keywords: Modular snake robot, pipe inspection, redundant robot, snake-like robot.

1. INTRODUCTION

Hyper-redundant mechanisms such as snake-like robots have been developed in the past decades to meet the mobility requirement in challenging environments such as rescue operations, pipe inspection, and surveillance. Due to a large number of joints, snake robots have the potential of handling such unstructured and complex environments but at the same time, planning and controlling the motion is also very difficult which poses many open research problems.

There is a lot of research has been done on snake robots, with Hirose being the pioneer to study snake-like robots [1]. Different methodologies have been used to control the snake robot based on mathematical modeling, sinusoidal-based or Central Pattern generator (CPG) - based techniques and for a brief overview, the reader is referred to [2], which is a recent review on modeling, development and control of snake robots.

To achieve locomotion in three dimensions, Hirose *et al*. studied on 3-D shape change of Active Cord Mechanism (ACM) [3, 4]. Choset *et al*. also develop modular snake robot with many degrees of freedom (DOF) with alternating lateral and dorsal modules in [5, 6], typically sixteen degrees of freedom, having a single actuator for each module, for a variety of locomotion capability like searching across a gap, and climbing a pipe. In [7], gaits were classified as parameterized and scripted gaits. The classification is based on whether it can be described as a parameterized function or by a series of predefined shapes. The motion for a snake robot can be generated either by using parameterized equations, i.e., gait equations or by designing the shape of a snake robot in the work-space as a spatial curve, called backbone curve [8]. Gait equation technique is simple due to the only a handful of parameters like amplitude, frequency, and phase shift to control but at the expense of flexibility in doing complex tasks. On the other hand, complex tasks can be done using backbone curve needs modeling of the curve in space, and then approximating the joint angles of snake robot by fitting the robot on the continuous curve using methods proposed in [9,10]. The method proposed by [10], to approximate joint angles based on curvature and torsion, gives better results along with low computational cost. The algorithm proposed in [9] is aimed to directly fit the joints position of robot onto a backbone curve, but this algorithm is computationally expensive. Kamegawa *et al*. [11, 12] used the mathematical continuum model to approximate snake robot with wheels to a helical curve to move inside and outside of a pipe. In [13], Kamegawa *et al*. designed the bending helical curve along with a v-shift control method

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to move the snake along a curved pipe. In [14], a branch point on the pipe was overcome, while the robot was moving along the horizontal pipe, by generating a hump and moving it along the body of snake robot from tail to the head. Zhen [15], modeled the rolling motions of a snake robot by Bellows model and simplified the controller in terms of pitch and radius by using a computation reduction technique. The author also introduced the novel rolling hump motion for modular snake robot to overcome an obstacle like stone etc. while rolling on the ground. Takemori *et al.* [16], used the concept of composing simple shapes to design gait for modular snake robot to overcome a flange while moving on the pipe. In [17], Vespignani *et al.* studied the locomotion of snake robot on horizontal pipes using in-body compliant materials to passively adapt to the terrain bumps, and changes in pipe diameter while moving on the pipe with a rolling motion. In this paper, a spiral curve gait is proposed for modular snake robot to adapt to the changes in pipe diameter while moving on the pipe. We validated the proposed gait by simulating the thirty DOF's modular snake robot with an alternating pitch and yaw joints in Open Dynamics Engine (ODE) simulator.

The rest of the paper is structured as follows: In Section 2, we discuss the background of modeling the snake robot motion along with the helical rolling motion of a snake robot using Bellows model. In Section 3, we discuss the proposed spiral curve gait motion. Simulation results of the proposed gait are given in Section 4. In Section 5, we concluded the findings along with future work.

2. BACKGROUND

In this study, the model of snake robot used is shown in Fig. 1. The snake robot is constructed with alternating pitch-axis and yaw-axis joint without wheels. Each module consists of a single pitch and yaw joint. The total number of joints are thirty. The link length l between each pitch and yaw joint is taken to be 50 mm.

The helical rolling and the proposed spiral curve motion is realized by considering the mathematical continuum model, and it is applied to mechanical discrete modular snake robot. To approximate the continuous curve for modular snake robot, we used the method proposed in [10], which uses the high order information (curvature, torsion) instead of using coordinates of the curve. The motion generation technique used in this study was presented

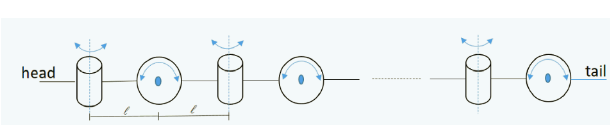

Fig. 1. The Kinematics of snake robot (Alternating pitch and yaw joint configuration).

in previous literature [4, 11–14], and here a brief overview is given below.

Burdick *et al.* [8] introduce a backbone curve to represent the configuration of the redundant robot, which is a simple and intuitive method to design motion for the snake robot. Hyper-redundant manipulators and snakelike robots have been called Active Cord Mechanism (ACM) and in [4], the shape equations of ACM is discussed which is classified into four main groups i.e., Planar, Frenet-Serret, Bellows and Complete model. For 3-D locomotion, Frenet-Serret model is not adequate for snake robots because it cannot copy all backbone curves and also, sometimes target curve has curvature equal to zero but Bellows model is adequate as a model for snake-like robots. In Bellows model, the shape of the backbone curve is represented by decomposing the curvature of the backbone curve into dorsal and lateral curvature. The dorsal and lateral curvature is given as;

$$k_l(s) = k(s)\sin(\phi(s)), \tag{1}$$

$$k_d(s) = k(s)\cos(\phi(s)), \tag{2}$$

where $\phi(s)$ is the rolling offset and can be found as;

$$\phi(s) = \phi_0 + \int_0^s \tau(s)ds, \tag{3}$$

where ϕ_0 is arbitrary integral constant which corresponds the initial angle offset and rolling motion is realized in modular snake robot by setting ϕ_0 as a function of time and $\tau(s)$ is torsion along the backbone curve. By using the information of curvature and torsion, the joint angles of the snake robot are calculated using curvature integration algorithm. The joint angles for each pitch and yaw joint can be calculated by integrating the lateral and dorsal curvature, and given as follows:

$$\alpha(i) = \int_{(i-1)m}^{(i+1)m} k_d(s)ds \qquad dorsal, \tag{4}$$

$$\alpha(i) = \int_{(i-1)m}^{(i+1)m} k_l(s)ds \qquad lateral, \tag{5}$$

where m denotes the length of one module and i denotes the index of joint. Although, the joint angles calculated using curvature integration is fast, but it still slows down the performance in generating the control inputs and in [15], a computation reduction technique is introduced for rolling motion of snake robot to avoid the unnecessary calculations done during motion generation. In the next part of this paper, the helical rolling motion along with computation reduction is briefly over-viewed.

2.1. Helical rolling motion

In Helical rolling motion, the snake robot forms a shape of a helix and twists its body to roll. Using this motion, the snake robot can move both inside and outside of the pipe. The parametric equation of the helix is expressed as;

$$x = r\cos(t),$$
$$y = r\sin(t),$$
$$z = pt, \tag{6}$$

where r is the radius of the helix, and $2\pi p$ denotes the pitch of the helical curve. Generally, curvature $k(t)$ and $\tau(t)$ are derived by geometric calculation given as;

$$k(t) = \frac{\sqrt{(\dot{y}\ddot{z}-\dot{z}\ddot{y})^2+(\dot{z}\ddot{x}-\dot{x}\ddot{z})^2+(\dot{x}\ddot{y}-\dot{y}\ddot{x})^2}}{(\dot{x}^2+\dot{y}^2+\dot{z}^2)^{3/2}}, \tag{7}$$

$$\tau(t) = \frac{(\dddot{x}(\dot{y}\ddot{z}-\dot{z}\ddot{y})+\dddot{y}(\dot{z}\ddot{x}-\dot{x}\ddot{z})+\dddot{z}(\dot{x}\ddot{y}-\dot{y}\ddot{x}))}{(\dot{y}\ddot{z}-\dot{z}\ddot{y})^2+(\dot{z}\ddot{x}-\dot{x}\ddot{z})^2+(\dot{x}\ddot{y}-\dot{y}\ddot{x})^2}, \tag{8}$$

where $\dot{x}$, $\ddot{x}$, and $\dddot{x}$ represent the first, second and third order of differentiation with respect to t. The curvature and torsion of the helix using (7), and (8) is;

$$k(t) = \frac{r}{r^2+p^2} = \bar{k}, \tag{9}$$

$$\tau(t) = \frac{p}{r^2+p^2} = \bar{\tau}. \tag{10}$$

The curvature and torsion are both constant. It can be used in (4, 5) to calculate the joint angles of the snake robot. As described earlier, the motion generation can be simplified using the computation reduction technique explained in [15], which results in a well-parameterized and smooth controller and is respectively given for dorsal and lateral joints as;

$$\alpha(t,i) = A\cos(t+\bar{\tau}mi), \tag{11}$$
$$\alpha(t,i) = A\sin(t+\bar{\tau}mi), \tag{12}$$

where t is a time parameter, and is a result of setting $\phi_0 = t$, during computation reduction technique and $A = \frac{2\bar{k}}{\bar{\tau}}\sin(\bar{\tau}m)$, is a constant term. By varying the value of r in (9), the snake robot can climb pipes of different sizes.

Although the helical rolling motion can be effectively used to climb pipes of different sizes, it cannot overcome some branch point [14], or gap in continuous pipe [18]. In the next section, we present a spiral curve gait for modular snake robot, which can handle the changes in pipe diameter while moving on the pipe.

3. SPIRAL CURVE GAIT

In this section, the formulation of the spiral curve along with a selection of parameters and its implementation on modular snake robot is discussed. First, the formulation of a spiral curve is presented in Cartesian coordinates and then its implementation on modular snake robot is discussed using the high order (curvature, and torsion) information. In order to adapt to different pipe sizes, the selection of initial parameter values is discussed.

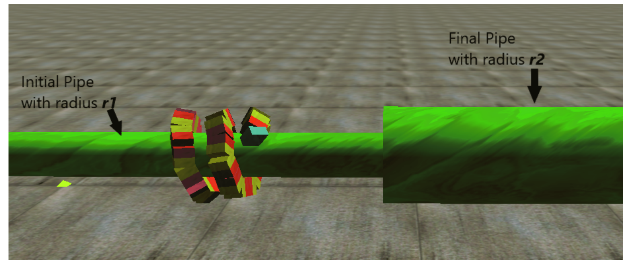

Fig. 2. Overview of Simulation setup

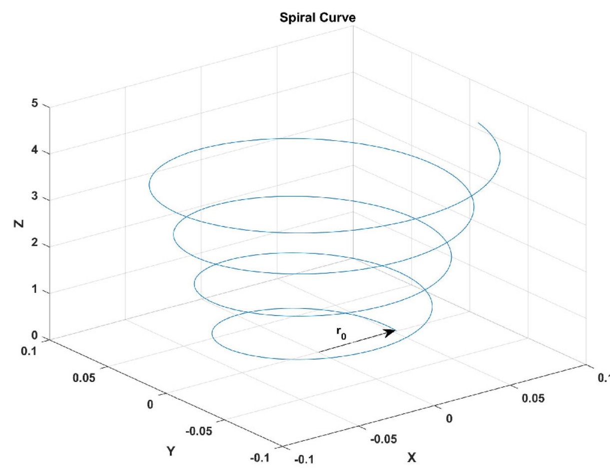

Fig. 3. A Spiral Curve ($r_0 = 0.052$, $p_0 = 0.17$, $\xi = 1.025$).

3.1. Formulation of spiral curve

In this study, initially, it is assumed that a modular snake robot has gripped the initial pipe with radius r_1 using a helical shape (see Fig. 2) and after that, an operator switches to Spiral curve gait motion along with rolling motion. The whole body of the snake robot moves from initial pipe to the final pipe using the spiral curve shape and shifting process which is described later.

The radius and pitch of spiral curve change along the length of a curve as shown in Fig. 3, which makes it suitable to be applied on modular snake robot to negotiate with changes in pipe diameter while moving on the pipe. The parametric equation for the spiral curve is expressed in Cartesian coordinates as follow [19]:

$$x = r_0\xi^t\cos(t),$$
$$y = r_0\xi^t\sin(t),$$
$$z = p_0\xi^t, \tag{13}$$

where r_0 is the initial radius of the spiral curve, and $2\pi p_0$ denotes the initial pitch and t is a parameter. ξ is a constant which effects the rate of change of curvature and torsion along the length of the curve. The exponential function is smooth, concise and differentiable and it allows the operator to control the rate of change of curvature and torsion along the length of the snake robot to effectively

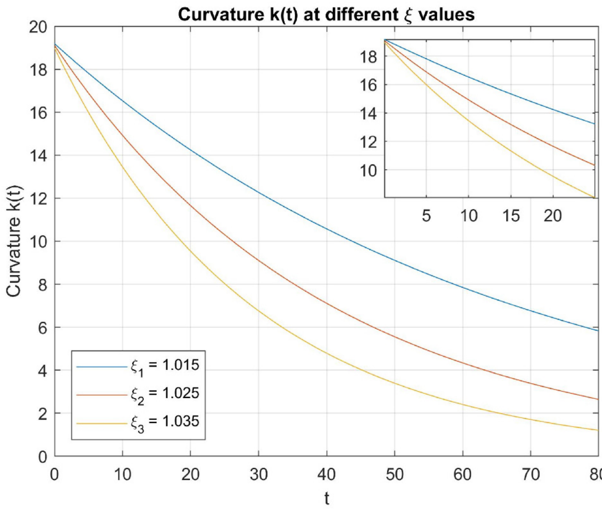

Fig. 4. Curvature of Spiral Curve at different values of ξ ($r_0 = 0.052$, $p_0 = 0.17$).

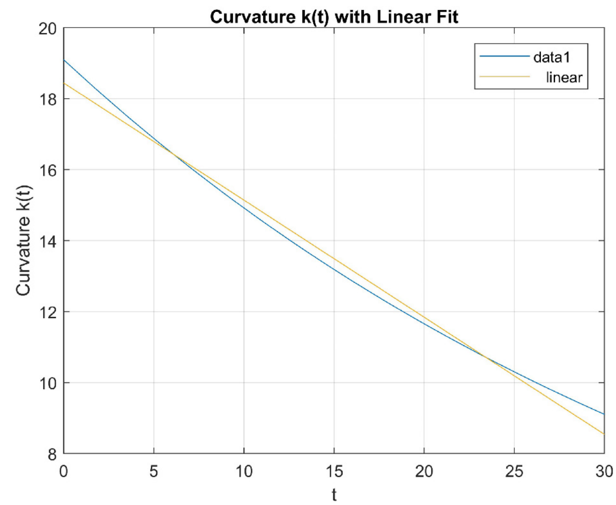

Fig. 5. Curvature of Spiral Curve with Linear Fit ($r_0 = 0.052$, $p_0 = 0.17$, $\xi = 1.025$).

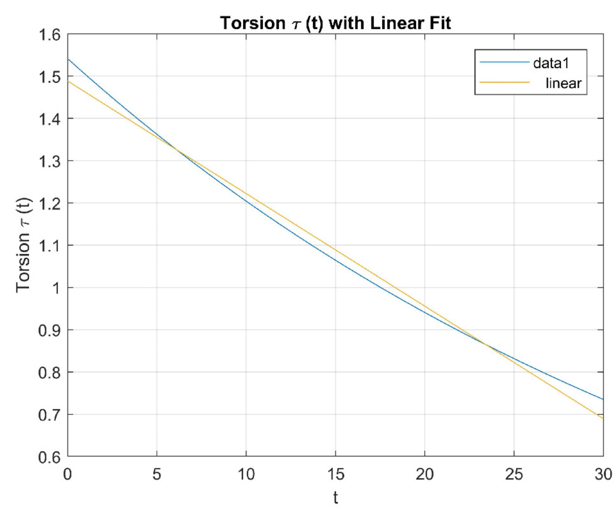

Fig. 6. Torsion of Spiral Curve with Linear Fit ($r_0 = 0.052$, $p_0 = 0.17$, $\xi = 1.025$).

move on both pipes.

In [12], along with simple helical rolling motion, a conical spiral form is also considered to move the snake robot with wheels inside the pipe. The conical spiral form given in [12] is of similar form as proposed in our method but there is a lack of insight on the selection of parameters of conical spiral form. Also, the curvature and torsion are important factors while moving from the initial pipe to the final pipe. By using the exponential function, the curvature and torsion can easily be controlled along the length of the robot using parameter ξ and r_0 to negotiate with initial and final pipe simultaneously. The curvature and torsion is also smooth and changes with a same exponential pattern which is not the case when using the conical spiral form mentioned in [12], (i.e., sometimes curvature increases and then decreases, or decreases very rapidly or exponentially) when tested with different values of parameters of the conical spiral curve.

The curvature of the spiral curve is shown in Fig. 4 for different values of ξ. The curvature and torsion decreases non-linearly with parameter t, but it is almost linear with in the certain range of parameter t, as shown is Figs. 5 and 6, and in this paper, we used this region because the whole length of the modular snake robot can easily be fitted within this region. The selection of the appropriate value of ξ is important to adapt to the changes in pipe diameter which is described in the next subsection.

3.2. Selection of parameters

In (13), the value of r_0 is selected based on the radius of the initial pipe (i.e., r_1 in Fig. 2). By selecting r_0 based on the initial pipe radius along with the appropriate value of ξ, the snake robot would have enough curvature to grip the initial pipe near the tail part of the snake robot to move forward using rolling motion, while the head part of the snake robot will have less curvature to grip the final pipe (i.e., r_2 in Fig. 2)

As described earlier, ξ is a constant which effects the rate of change of curvature and torsion along the length of the robot. In Figs. 5 and 6, the curvature and torsion of the the spiral curve is shown along with linear fitting. In order to derive the relation between ξ and given pipe radii, it is assumed that curvature decreases linearly, and based on this assumption ξ can be taken as (see Appendix A for derivation);

$$\xi \approx \sqrt[n-1]{\frac{r_2}{r_1}}, \tag{14}$$

where n is the no. of joints, and r_1 is the radius of the initial pipe, and r_2 is the radius of the final pipe as shown in Fig. 2. Due to the linear assumption, the value needs to be tuned for the snake robot to adapt to the pipe diameter but (14) is helpful for the operator in initially selecting the value of parameter ξ. (14) can be used for a higher number of joints (i.e., more than thirty as simulated in this paper) and more joints will help the robot to grip the initial pipe more effectively.

3.3. Computing curvature and torsion

Unlike the helical rolling, the curvature and torsion for spiral curve gait are not constant and a function of parameter t, when calculated using (7), and (8). In order to use curvature $k(t)$ and torsion $\tau(t)$ to calculate joint angles for the snake robot, we need to find curvature and torsion in terms of arc length parameter s. The curvature $k(t)$, and torsion $\tau(t)$ is converted to $k(s(t))$, and $\tau(s(t))$ using path integral formula given below:

$$s(t) = \int_0^s \sqrt{\left(\frac{dx(t)}{dt}\right)^2 + \left(\frac{dy(t)}{dt}\right)^2 + \left(\frac{dz(t)}{dt}\right)^2}. \tag{15}$$

After numerically computing the curvature and torsion, the joint angles are computed using (11), and (12). The curvature and torsion need to be recomputed only when r_0, p_0 or ξ are changed. Also, ϕ_0 is taken as $\phi_0 = t$, a function of time during simulation, to move the robot forward using rolling motion while having the shape of a spiral curve. The curvature and torsion values of the head part is shifted to the tail part by combining shifting and rolling which is described in the next section to move the robot from a smaller radius pipe to a larger radius pipe. The rolling motion is also slowed down by scaling down the parameter ϕ_0 during the shifting process to avoid a collision.

3.4. Combining shifting and rolling

In order for the snake robot to adapt to the larger diameter pipe, a method that is based on Follow the leader (FTL) approach is used [20, 21]. In FTL method, the head (leader) movement is shifted from head to tail and 3-D operations can be performed by designing the snake robots head motion. The curvature and torsion value used to calculate the joint angle of the head part is shifted up to the tail part. This shifting process is done sequentially when snake robot joints reach near to larger diameter pipe (i.e., within some radius Δ_a of the larger diameter pipe). The shifting process can be started using operator command along with controlling the rolling speed. In the simulation, we used the joint position information of snake robot and distance information between snake robot and larger diameter pipe to trigger the shifting process. Let d_i be the distance between i_{th} joint position and larger diameter pipe. The shifting process starts for each joint when $d_i < \Delta_a$. The value of Δ_a depends upon diameter variation between the initial pipe and final pipe as well as the module width of the snake robot. The operator can command the desired parameter values to form a proper spiral curve to negotiate with pipe diameter.

The length of the snake robot and also the number of modules will affect the variation of pipe diameter which can be overcome using spiral curve and research will be conducted and described in future papers on this work.

4. SIMULATION

In this section, simulation results are presented to verify the proposed spiral curve gait. The simulation was carried out in Open Dynamics Engine (ODE) [22], which is an open-source library for simulating the dynamics of rigid body. ODE allows fast simulations for articulated rigid body structures along with the desired re-configurable environment.

The wheel-less modular snake robot with alternating pitch and yaw joint was simulated with thirty degrees of freedom. The online library for the snake robot [23] is used for simulation. The library was modified to validate the rolling spiral motion. The number of joints was 30, along with the tail and head module. The link length was taken to be 50 mm. The weight of each module, containing one pitch and one yaw joint, was 0.2 kg. The friction coefficient between the robot and ground surface as well as the pipe is set to be 1.

4.1. Horizontal pipe motion

The spiral curve gait motion executed on modular snake robot for horizontal pipe is shown in Fig. 7. The outside diameter of the initial pipe was 120 mm, and of the final pipe was 260 mm. The values of parameters used for the spiral motion were $r_0 = 0.052$, $\xi = 1.027$ and $p_0 = 0.17$. In the simulation, the snake robot successfully moved from initial pipe to the final pipe. In the simulation, the shifting process is done using joint position information. The operator can also control the parameter values, and can trigger the shifting process at an appropriate time.

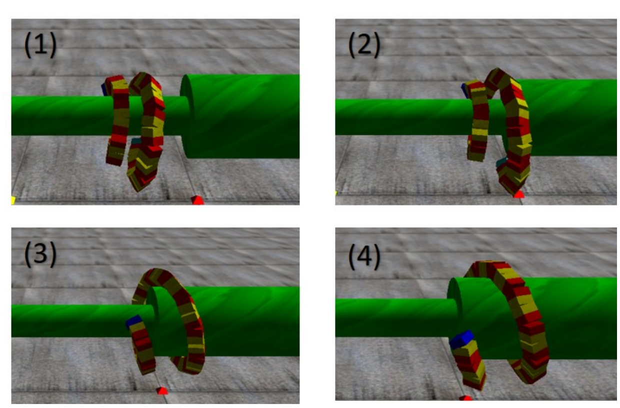

Fig. 7. Spiral Curve gait simulation for Horizontal pipe ($r_0 = 0.052$, $p_0 = 0.17$, $\xi = 1.027$).

Table 1. The Spiral Curve Gait Simulation trials for different Horizontal pipe sizes.

	Initial pipe diameter	Final pipe diameter	Parameter Values		
			r_0	p_0	ξ
(1)	120 mm	260 mm	0.052	0.17	1.027
(2)	120 mm	280 mm	0.052	0.17	1.029
(3)	120 mm	300 mm	0.052	0.17	1.032

In addition, we performed the simulation for three different pipe sizes. The modular snake robot successfully adapts to the changes in pipe diameter. The value of ξ is calculated using (14) in these trials. The results of successful trials are given in Table 1. Also, we performed the simulation for more number of joints (i.e., 40 joints). The parameters used in the simulation are $r_0 = 0.049$, $\xi = 1.024$, and $p_0 = 0.21$ and the initial and final pipe diameter are 120 mm and 260 mm respectively. The simulations can be viewed in the attached videos. The range of pipe diameter variation which snake robot can overcome using spiral curve gait is one of the future work. This mainly depends on the length of the modular snake robot. In order to successfully adapt to the pipe diameter, the initial state of the snake robot is also important. The head part must be in an appropriate position to grip the final pipe along with the support of the modules following the head part to avoid losing the grip. The simulation is performed while having the head part at top and bottom of the pipe which can be viewed in the attached videos. In order to successfully negotiate with pipe diameter variation, the head part should be in an appropriate position (i.e., the head part along with modules next to the head part should cover more than half area of the final pipe, while approaching from the top or the bottom) to adapt the pipe without any potential risk of failure.

4.2. Vertical pipe motion

The spiral curve gait was also executed for vertical pipe scenario. The outside diameter of the initial pipe was 120 mm, and of the final pipe was 260 mm. During the simulation, the operator adjusted the parameter values to have enough grip and avoid slipping for the snake robot to move forward while having the shape of a spiral curve. The parameter values of spiral motion were $r_0 = 0.044$, $\xi = 1.027$ and $p_0 = 0.17$. In the simulation, the snake robot successfully climbed from an initial pipe to the final pipe using the proposed method. In Fig. 8, the spiral curve motion is shown at different time intervals during the simulation. The value of r_0 is reduced as compared to the value used for horizontal pipe motion to have sufficient grip to execute the proposed gait for vertical pipe motion. The simulation can be viewed in the attached videos.

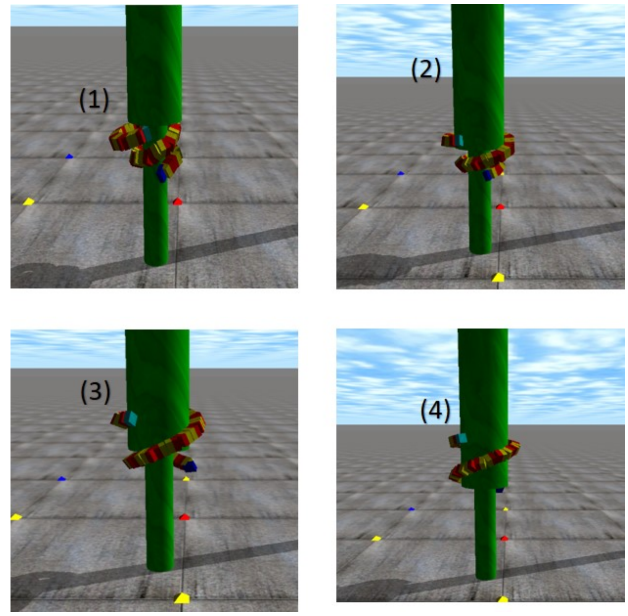

Fig. 8. Spiral Curve gait simulation for Vertical pipe ($r_0 = 0.044$, $p_0 = 0.17$, $\xi = 1.027$).

5. CONCLUSION

In this paper, a spiral curve gait is proposed for a modular snake robot without wheels to adapt to the changes in pipe diameter while moving on the pipe with a rolling motion. The target joint angles are calculated using continuous curve model. The selection of appropriate values to adapt to the pipe of different sizes, along with the shifting process that is based on following the leader approach is also discussed.

Furthermore, the simulations are conducted in Open Dynamics Engine simulator to verify the proposed gait for the horizontal and vertical pipe. The parameters are tuned to form a proper curve and then shifting process and rolling motion is combined to adapt the changes in pipe diameter. The simulation results verified the proposed gait. Future work will be devoted to applying the proposed gait on a real snake robot.

APPENDIX A

In order to select an appropriate value of ξ, for a given pipe radii and number of joints, a relation is derived based on the assumption of linear decrease in the curvature of the robot as shown in Fig. 5. So, assuming that curvature decreases by approximately by a fixed ratio ξ, we have;

$$\frac{k_1}{k_2} \approx \xi \;\Rightarrow\; \frac{k_1}{\xi} \approx k_2, \tag{A.1}$$

where k_1 and k_2 are the curvature of the first and second joint. Similarly, for the next joint:

$$\frac{k_2}{k_3} \approx \xi \Rightarrow \frac{k_2}{\xi} \approx k_3.$$

Using (A.1) we have

$$k_3 \approx \frac{k_1}{\xi^2}.$$

Repeating the procedure for next joints, we have

$$k_4 \approx \frac{k_1}{\xi^3} \quad \dots \quad k_n \approx \frac{k_1}{\xi^{n-1}},$$

$$\frac{k_1}{k_n} \approx \xi^{n-1},$$

$$\xi \approx \sqrt[n-1]{\frac{k_1}{k_n}}.$$

As curvature is inversely proportional to the radius, $k = \frac{1}{r}$. In order to handle the radius of the initial and final pipe simultaneously to form a proper spiral curve. By taking $k_1 = \frac{1}{r_1}$, and $k_n = \frac{1}{r_2}$, where r_1 and r_2 are initial pipe and final pipe radius, ξ can be taken as

$$\xi \approx \sqrt[n-1]{\frac{r_2}{r_1}}.$$

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