International Journal of Control, Automation and Systems 17(9) (2019) 2414-2429
http://dx.doi.org/10.1007/s12555-019-0044-2
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Control of Nonlinear Markovian Jump System with Time Varying Delay via Robust $\mathcal{H}_\infty$ Fuzzy State Feedback Plus State-derivative Feedback Controller

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Abstract: This paper investigates the problem of designing a robust $\mathcal{H}_\infty$ state feedback plus state-derivative feedback control mechanism for a class of uncertain nonlinear markovian jump systems with time varying delay described by a Takagi-Sugeno (T-S) fuzzy model. The linear matrix inequalities (LMIs) approach is applied to derive a robust controller for such a system. The proposed controller satisfies design requirements that ensure that the closed-loop system is asymptotically stable and meets pre-prescribed $\mathcal{H}_\infty$ performance index values. Finally, to illustrate the effectiveness of the design developed in this paper, a numerical example is also provided.

Keywords: Linear matrix inequalities (LMIs), Markovian jump systems, robust $\mathcal{H}_\infty$ control, state-derivative feedback, Takagi-Sugeno (T-S) fuzzy model, time-varying delay systems.

1. INTRODUCTION

During the past two decades, the Markovian jump system has been extensively studied by many researchers [1–3]. The Markovian jump system changes abruptly from one mode to another mode caused by some phenomenon such as environmental disturbances [4], changing subsystem interconnections and fast variations in the operating point of the system plant. The switching between modes is governed by a Markov process with the discrete and finite state space. In other words, Markovian jump systems are referred to as hybrid systems, that is, the state space of the systems contains both continuous (differential equation) and discrete states (Markov process). Due to the growing use of computers in the control of physical plants, manufacturing systems and communication systems, the design of control for Markovian jump nonlinear systems remains an open area [5].

Over the past two decades, $\mathcal{H}_\infty$ theories for nonlinear problems have been extensively studied and developed [6–8]. The aim of $\mathcal{H}_\infty$ methods is to achieve stabilization with the prescribed performance index. Recently, the controller design for the consensus of heterogeneous linear multiagent systems with aperiodic sampled-data have been examined by using the output-feedback procedures [9, 10]. Furthermore, the problem of sensor-network-based distributed control for the large-scale networked control systems and the event-based control for a class of networked markov jump systems with missing measurements have been investigated by using $\mathcal{H}_\infty$ control theories [11, 12]. However, the higher-order nonlinear estimation of real-life dynamical system is an important issue in both the analysis and the design of nonlinear control systems. With highly nonlinear issue, the T-S fuzzy model has been attracted by most researchers due to the fact that the T-S fuzzy model is appropriated for simplifying the dynamics of complex nonlinear systems and has been widely used in many different areas [13–15]. A few years ago, the T-S fuzzy model was employed for reducing the conservatism whilst alleviating the computational burden [16] and also being applied to the discrete-time systems for the relaxed real-time scheduling stabilization [17].

The global behavior of a nonlinear system can be explained by the T-S fuzzy model construction procedures. The T-S fuzzy control design is derived by utilizing the concept of parallel distributed compensation (PDC); i.e., a fuzzy system is represented by each plant rule model [18, 19]. In addition, the T-S fuzzy model based on the LMIs techniques can be used to solve the stability analysis and the control design problems [20, 21]. LMIs based T-S fuzzy model techniques ensure not only stabilization but also important issue of control performance, namely, robustness in fuzzy control system designs. Thus, unquestionably, in recent decades, various robust $\mathcal{H}_\infty$ fuzzy design approaches based on LMIs techniques for uncertain

Manuscript received January 21, 2019; revised March 16, 2019; accepted April 8, 2019. Recommended by Associate Editor Xiangpeng Xie under the direction of Editor Jessie (Ju H.) Park.

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nonlinear systems have been developed in several works [22, 23]. In practicality, the controlled method for many applications cannot easily meet the expected performance index in uncertain nonlinear systems with time varying delay. Together with the high nonlinearities and external disturbance noises, time varying delays are considered as a source of poor control performances and instabilities [24, 25]. Recently, uncertain systems with time-varying delay control designs using the $\mathcal{H}_\infty$ fuzzy approaches have been discussed [26, 27].

In addition, some issues have occurred in practical mechanical control systems, where the obtained measurable signals are the state feedback and the state-derivative feedback signals. For instance, the accelerometers serve as principal sensors of vibration in the control of suppression systems, where [28]. According to several research works [29, 30], it has been found that the state has greatly limited by the need for accurate information about parameters which may be difficult to estimate with high precision while the state derivative is easily obtained. Therefore, for the actual accelerations, it is possible to reconstruct velocities with reasonable accuracy but not displacements [31]. Recently, [32] acquired novel results by designing $\mathcal{H}_\infty$ fuzzy state-derivative feedback control applied using the LMIs technique. Unfortunately, those approach has not been applied to a nonlinear Markovian jump system that includes uncertainties with time-varying delay. As reported in several studies, these designed approaches have not yet been adequately researched, and these design problems are still challenging.

In term of computation viewpoints, the design of robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback controllers for uncertain nonlinear Markovian jump systems with time-varying delay has been aggregated to examine a set of LMIs in conjunction with the T-S fuzzy model approach. The convex optimization algorithm is employed to quickly solve the LMIs problem. The proposed approach can significantly mitigate the computational difficulties; therefore, it reduces the design costs associated with the practical use of theoretical outcomes due to the fact that the T-S fuzzy controller gains are easily acquired and are able to directly apply to the controller for such a system. Therefore, the research on robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback control design for a class of uncertain nonlinear Markovian jump systems with time-varying delay can be conducted on both the theoretical and practical point of view.

In according with the above motivations, the main contributions and novelty of this paper are threefold. First, the definitions of the $\mathcal{H}_\infty$ control problem and asymptotic stability are introduced for the system. Second, the T-S fuzzy model is applied to approximate uncertain nonlinear Markovian jump systems with time-varying delay. Third, the LMIs approach is used to develop a means of designing a robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback controller that adheres to performance and robustness specifications.

This paper is organized as follows. In Section 2, Preliminaries are presented. In Section 3, based on an LMIs approach we develop a technique for designing a robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback controller such that the $\mathcal{L}_2[0,\infty)$ gain derived from mapping from exogenous input noise to the regulated output is less than a prescribed value for the uncertain nonlinear Makovian jump system with time-varying delay as described in Section 2. The validity of this approach is demonstrated by an example from the literature in Section 4. Finally, the conclusion is given in Section 5.

2. PRELIMINARIES

The uncertain nonlinear Markovian jump T-S fuzzy model with time-varying delay is explained by IF-THEN rules that can be used to approximate the nonlinear system by combining the linear models via nonlinear membership functions. An uncertain nonlinear Markovian jump T-S fuzzy model with time-varying delay is examined by the i-th rule as follows:

Plant rule i:

IF $\upsilon_1(t)$ is $M_{i1}(t)$ and...and $\upsilon_\vartheta(t)$ is $M_{i\vartheta}(t)$ THEN

$$\begin{aligned}\dot{x}(t) &= [A_i(\eta(t)) + \Delta A_i(\eta(t))]x(t)\\ &\quad + [B_{1_i}(\eta(t)) + \Delta B_{1_i}(\eta(t))]w(t)\\ &\quad + A_{d_i}(\eta(t)x(t-\tau(t))) + [B_{2_i}(\eta(t))\\ &\quad + \Delta B_{2_i}(\eta(t))]u(t),\\ z(t) &= [C_i(\eta(t)) + \Delta C_i(\eta(t))]x(t),\\ x(t) &= \psi(t),\ \ t \in [-\tau, 0],\ \ \tau(t) \le \tau,\ \ \dot{\tau}(t) \le \tau_d, \end{aligned} \tag{1}$$

where $i = 1, 2, ..., r$, M_{ij} $(j = 1, 2, ..., \vartheta)$ are fuzzy sets j for rule i, r is the number of IF-THEN rules, $\upsilon(t)$ is the premise variables, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $w(t) \in \Re^p$ is the input disturbance belonging to $\mathcal{L}_2[0,\infty)$, $z(t) \in \Re^s$ is the controlled output, matrices $A_i(\eta(t))$, $B_{1_i}(\eta(t))$, $B_{2_i}(\eta(t))$, $A_{d_i}(\eta(t))$ and $C_i(\eta(t))$ are suitable matrices of the system, $0 \le \tau(t) \le \tau$ is the bounded time-varying delay of the state, and $\psi(t)$ is a vector-valued initial continuous function defined based on the interval $[-\tau, 0]$, with τ a real positive constant and the assumption that $\dot{\tau}(t) \le \tau_d < 1$, i.e., the derivative of the time-varying delay function is continuous and bounded to form a natural supplementary condition. In this paper, it is assumed that $\upsilon(t)$ is the vector containing all individual elements $\upsilon_1(t), ..., \upsilon_\vartheta(t)$. $\{\eta(t)\}$, $t \ge 0$ is a continuous-time discrete-state homogeneous Markov process taking values on a finite set $\mathcal{S} = \{1, 2, ..., s\}$ with transition probability matrix $Pr := \{P_{\imath k}(t)\}$ given by

$$P_{\imath k}(t) = Pr(\eta(t+\Delta) = k \mid \eta(t) = \imath),$$

$$= \begin{cases} \lambda_{\imath k}\Delta + O(\Delta) & \text{if} \imath \neq k, \\ 1 + \lambda_{\imath\imath}\Delta + O(\Delta) & \text{if} \imath = k, \end{cases} \tag{2}$$

and $\sum_{k=1}^{s} P_{\imath k}(t) = 1$ where $\Delta > 0$; $\lim_{\Delta\to 0} \frac{O(\Delta)}{\Delta} = 0$; $\lambda_{\imath k} \geq 0, \imath \neq k$ is the transition rate from mode $\imath$ to mode k; $\lambda_{\imath\imath} = -\sum_{k=1,\, k\neq \imath}^{s} \lambda_{\imath k}, \imath,\ k \in \mathcal{S}$ gives the infinitesimal generator of the Markov process $\{\eta(t),\ t \geq 0\}$. The matrices $\Delta A_i\big(\eta(t)\big)$, $\Delta B_{1_i}\big(\eta(t)\big), \Delta B_{2_i}\big(\eta(t)\big)$, and $\Delta C_i\big(\eta(t)\big)$ represent the uncertainties in the system and satisfy the following assumption.

Assumption 1:

$$\begin{aligned} \Delta A_i\ &= F\big(x(t),\ \eta(t),\ t\big)H_{1_i}\big(\eta(t)\big), \\ \Delta B_{1_i} &= F\big(x(t),\ \eta(t),\ t\big)H_{2_i}\big(\eta(t)\big), \\ \Delta B_{2_i} &= F\big(x(t),\ \eta(t),\ t\big)H_{3_i}\big(\eta(t)\big), \\ \text{and } \Delta C_i &= F\big(x(t),\ \eta(t),\ t\big)H_{4_i}\big(\eta(t)\big), \end{aligned}$$

where $H_{j_i}\big(\eta(t)\big)$, $j = 1,2,...,4$ are known matrix functions that characterize the structure of uncertainties. Furthermore, the following inequality holds:

$$\|F\big(x(t),\ \big(\eta(t)\big),\ t\big)\| \leq \rho\big(\eta(t)\big) \tag{3}$$

for any known positive constant $\rho\big(\eta(t)\big)$. For any specified state vector and control input, the T-S fuzzy model is inferred as follows:

Let

$$\varpi_i\big(\upsilon(t)\big) = \prod_{j=1}^{\vartheta} M_{ij}\big(\upsilon_j(t)\big),$$

and

$$\mu_i\big(\upsilon(t)\big) = \frac{\varpi_i\big(\upsilon(t)\big)}{\sum_{i=1}^{r} \varpi_i\big(\upsilon(t)\big)},$$

where $M_{ij}\big(\upsilon_j(t)\big)$ is the grade of membership of $\upsilon_j(t)$ in M_{ij}. It is assumed in this paper that

$$\varpi_i\big(\upsilon(t)\big) \geq 0,\quad \sum_{i=1}^{r} \varpi_i\big(\upsilon(t)\big) > 0,\ \ i = 1,2,...,r, \tag{4}$$

where r is the number of local plant rules, for all t. Therefore,

$$\mu_i\big(\upsilon(t)\big) \geq 0,\quad \sum_{i=1}^{r} \mu_i\big(\upsilon(t)\big) = 1,\ \ i = 1,2,...,r, \tag{5}$$

for all t. To keep our notations simple, we use $\varpi_i = \varpi_i\big(\upsilon(t)\big)$, $\mu_i = \mu_i\big(\upsilon(t)\big)$, $\eta = \eta(t)$ and any matrix $N\big(\mu,\ \eta(t) = \imath\big) = N(\mu,\ \imath)$.Thus, we can generalize that the T-S fuzzy models represent the weighted average of the following forms:

$$\begin{aligned} \dot{x}(t) &= [A(\mu,\ \imath) + \Delta A(\mu,\ \imath)]x(t) + [B_1(\mu,\ \imath) \\ &\quad + \Delta B_1(\mu,\ \imath)]w(t) + A_d(\mu,\ \imath)x\big(t-\tau(t)\big) \\ &\quad + [B_2(\mu,\ \imath) + \Delta B_2(\mu,\ \imath)]u(t), \\ z(t) &= [C(\mu,\ \imath) + \Delta C(\mu,\ \imath)]x(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} A(\mu,\ \imath) &= \sum_{i=1}^{r} \mu_i A_i(\imath),\ C(\mu,\ \imath) = \sum_{i=1}^{r} \mu_i C_i(\imath), \\ B_1(\mu,\ \imath) &= \sum_{i=1}^{r} \mu_i B_{1_i}(\imath),\ B_2(\mu,\ \imath) = \sum_{i=1}^{r} \mu_i B_{2_i}(\imath), \\ \Delta A(\mu,\ \imath) &= \sum_{i=1}^{r} \mu_i \Delta A_i(\imath) := F(x(t),\ \imath,\ t)H_1(\mu,\imath), \\ \Delta B_1(\mu,\ \imath) &= \sum_{i=1}^{r} \mu_i \Delta B_{1_i}(\imath) := F(x(t),\ \imath,\ t)H_2(\mu,\imath), \\ \Delta B_2(\mu,\ \imath) &= \sum_{i=1}^{r} \mu_i \Delta B_{2_i}(\imath) := F(x(t),\ \imath,\ t)H_3(\mu,\imath), \\ \Delta C(\mu,\ \imath) &= \sum_{i=1}^{r} \mu_i \Delta C_i(\imath) := F(x(t),\ \imath,\ t)H_4(\mu,\imath), \\ A_d(\mu,\imath) &= \sum_{i=1}^{r} \mu_i A_{d_i}(\imath) \end{aligned}$$

with

$$\begin{aligned} H_1(\mu,\imath) &= \sum_{i=1}^{r} \mu_i H_{1_i}(\imath),\ H_2(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{2_i}(\imath), \\ H_3(\mu,\imath) &= \sum_{i=1}^{r} \mu_i H_{3_i}(\imath) \text{ and } H_4(\mu,\imath) = \sum_{i=1}^{r} \mu_i H_{4_i}(\imath). \end{aligned}$$

Next, let us recall the following definitions.

Definition 1: Suppose γ is a given positive real number. A system of form (6) is said to have an $\mathcal{L}_2[0,T_f]$ gain less than or equal to γ if

$$\mathrm{E}\Big[\int_0^{T_f} \{z^T(t)z(t) - \gamma^2 w^T(t)w(t)\}dt\Big] < 0, \tag{7}$$

where $\mathrm{E}\big[\cdot\big]$ denotes the expectation operator.

Definition 2 (Asymptotic stability)**:** Let $x_e = 0$ be an equilibrium for $\dot{x} = f(x)$. Let $V : R^n \to R$ be a continuously differentiable function such that

- $V(0) = 0$ and $V(x) > 0$ for all $x \neq 0$.
- $\dot{V}(x) < 0$ for all $x \neq 0$, $\dot{V}(0) = 0$.

Then, x_e is asymptotically stable and is the unique equilibrium point.

Note that for the symmetric block matrices, we use (*) as an ellipsis for terms induced by symmetry. Thus, the following results address systems (6).

3. MAIN RESULTS

This section opens by considering the problem of designing an $\mathcal{H}_\infty$ state feedback plus state-derivative feedback controller that guarantees $\mathcal{L}_2$ gains from exogenous input noise to a regulated output of less than or equal to a prescribed value. An LMIs approach is used to derive a fuzzy controller that stabilizes the system (6). Before presenting the next results, the following lemma is recalled.

Lemma 1 [5]**:** Consider system (6). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, the inequality (7) holds if for $\iota = 1,2,\ldots,s$, there exist positive definite symmetric matrices $P(\iota)$, $W(\iota)$ and positive constants $\delta(\iota)$ such that the following condition hold:

$$\Omega_{ii}(\iota) < 0, \qquad i = 1,2,\ldots,r, \tag{8}$$

$$\Omega_{ij}(\iota) + \Omega_{ji}(\iota) < 0, \qquad i < j \le r, \tag{9}$$

where

$$\Omega_{ij}(\iota) = \begin{pmatrix} \Psi_{ij}(\iota) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ \mathcal{R}(\iota)\tilde{B}_{1_i}^T(\iota) + \gamma\mathcal{R}(\iota) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ W(\iota)A_{d_i}(\iota) & 0 & -W(\iota) & (*)^T & (*)^T & (*)^T \\ P(\iota) & 0 & 0 & -W(\iota) & (*)^T & (*)^T \\ \Upsilon_{ij}(\iota) & 0 & 0 & 0 & -\gamma\mathcal{R}(\iota) & (*)^T \\ \mathcal{Z}^T(\iota) & 0 & 0 & 0 & 0 & -\mathcal{P}(\iota) \end{pmatrix}, \tag{10}$$

$$\Psi_{ij}(\iota) = A_i(\iota)P(\iota) + P(\iota)A_i^T(\iota) + B_{2_i}(\iota)Y_j(\iota) + Y_j^T(\iota)B_{2_i}^T(\iota) + \lambda_{\iota\iota}P(\iota), \tag{11}$$

$$\Upsilon_{ij}(\iota) = \tilde{C}_{1_i}(\iota)P(\iota) + \tilde{D}_{12_i}(\iota)Y_j^T(\iota), \tag{12}$$

$$\mathcal{R}(\iota) = diag\{\delta(\iota)I,\ I,\ \delta(\iota)I,\ I\}, \tag{13}$$

$$\mathcal{Z}(\iota) = \Big(\sqrt{\lambda_{\iota 1}}P(\iota),\ldots,\sqrt{\lambda_{\iota(\iota-1)}}P(\iota), \sqrt{\lambda_{\iota(\iota+1)}}P(\iota),\ldots,\sqrt{\lambda_{\iota s}}P(\iota)\Big), \tag{14}$$

$$\mathcal{P}(\iota) = diag\{P(1),\ldots,P(\iota-1),\ P(\iota+1),\ldots,P(s)\}, \tag{15}$$

with

$$\tilde{B}_{1_i}(\iota) = [I \quad I \quad I \quad B_{1_i}(\iota)], \tag{16}$$

$$\tilde{C}_i(\iota) = \Big[\gamma\rho(\iota)H_{1_i}^T(\iota) \;\; \sqrt{2}\aleph(\iota)\rho(\iota)H_{4_i}^T(\iota) \;\; 0 \;\; \sqrt{2}\aleph(\iota)C_{1_i}^T(\iota)\Big]^T, \tag{17}$$

$$\tilde{D}_i(\iota) = \Big[0 \;\; \sqrt{2}\aleph(\iota)\rho(\iota)H_{5_i}^T(\iota) \;\; \gamma\rho(\iota)H_{3_i}^T \;\; \sqrt{2}\aleph(\iota)D_{12_i}^T(\iota)\Big]^T, \tag{18}$$

$$\aleph(\iota) = \Big(1 + \rho^2(\iota)\sum_{i=1}^{r}\sum_{j=1}^{r}\big(\parallel H_{2_i}^T(\iota)H_{2_j}(\iota)\parallel\big)\Big)^{1/2}. \tag{19}$$

Furthermore, a suitable choice of fuzzy controller is

$$u(t) = \sum_{j=1}^{r}\mu_j K_j(\iota)x(t), \tag{20}$$

where $K_j(\iota) = Y_j(\iota)\big(P(\iota)\big)^{-1}$.

Clearly, in real control problems, there have been found that the state has greatly limited by the necessity for accurate information about parameters that may be difficult to estimate with high precision, while the state derivative is easily obtained. Thus two approachs will be studied in this section. Subsection 3.1 considers the fuzzy state-derivative feedback controller, while in Subsection 3.2, the fuzzy state feedback plus state-derivative feedback controller is studied. Before presenting the main results, we describe the problem under our study as follows.

Problem Formulation: Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, design an $\mathcal{H}_\infty$ fuzzy controller of the form of both approaches such that the inequality (7) is guaranteed.

3.1. $\mathcal{H}_\infty$ fuzzy state-derivative feedback controller

In this Subsection, we consider the following $\mathcal{H}_\infty$ fuzzy state-derivative feedback, which is inferred as Fig. 1, the weighted average of the local models of the form:

$$u(t) = -K_d(\mu,\iota)\dot{x}(t), \tag{21}$$

where $K_d(\mu,\iota) = \sum_{j=1}^{r}\mu_j K_{d_j}(\iota)$. The system (6) with the controller (21) shown in Fig. 2 can be rewritten as

$$\dot{x}(t) = A(\mu,\iota)x(t) + B_2(\mu,\iota)u(t) + A_d(\mu,\iota)x\big(t-\tau(t)\big) + B_1(\mu,\iota)w(t). \tag{22}$$

After rearranging (22), we have

$$\big[I + B_2(\mu,\iota)K_d(\mu,\iota)\big]\dot{x}(t) = A(\mu,\iota)x(t) + A_d(\mu,\iota)x\big(t-\tau(t)\big) + B_1(\mu,\iota)w(t). \tag{23}$$

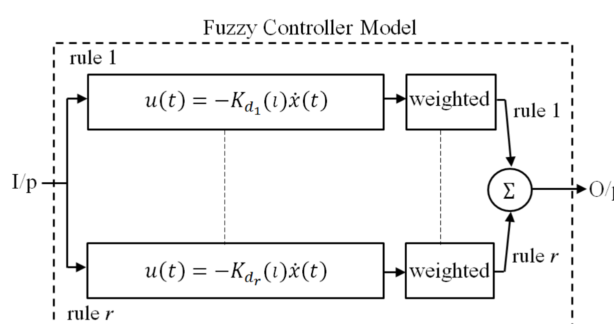

Fig. 1. The weighted average of fuzzy controller model.

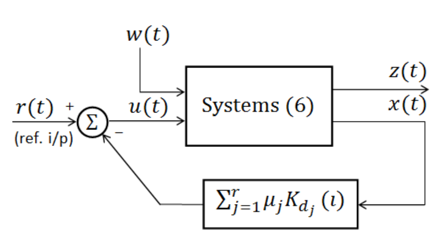

Fig. 2. The closed-loop fuzzy system.

The goal is to obtain state-feedback gains $K(\mu,\iota)$, such that the following conditions hold:

1) Matrices $\big(I+B_{2_i}(\iota)K_{d_j}(\iota)\big)$, $\forall\ i,\ j=1,\ 2,\ 3,\ ...,\ r$ have full rank.

2) The system (6) with the fuzzy controller (21) is asymptotically stable and the $\mathcal{H}_\infty$ performance is satisfied for all admissible values based on the sufficient condition for a prescribed scalary >0.

Remark 1: To establish the proposed results and without sacrificing generality, we apply the following assumption: rank $[I \mid B_i]=n$ exists. Thus, it is easy to conclude that if rank $[I \mid B_i]=n$ holds, then K_d exists such that rank $[I+B_{2_i}(\iota)K_{d_j}(\iota)]=n$ (i.e., matrices $\big(I+B_{2_i}(\iota)K_{d_j}(\iota)\big)$, $\forall i,\ j=1,2,3,...,r$ have full rank). From the above conditions and assumptions, we define

$$E_{ij}(\iota)=\big(I+B_{2_i}(\iota)K_{d_j}(\iota)\big)^{-1}. \tag{24}$$

According to Remark 1, (23) can be written as

$$\begin{aligned}\dot{x}(t)=&E_{ij}(\mu,\iota)A(\mu,\iota)x(t)\\&+E_{ij}(\mu,\iota)A_d(\mu,\iota)x\big(t-\tau(t)\big)\\&+E_{ij}(\mu,\iota)\tilde{B}_1(\mu,\iota)\tilde{w}(t),\end{aligned} \tag{25}$$

where

$$\tilde{B}_1(\mu,\iota)=[I\ \ I\ \ I\ \ B_1(\mu,\iota)], \tag{26}$$

and the disturbance is

$$\tilde{w}(t)=\mathcal{R}^{-1}(\iota)\times\begin{bmatrix}F\big(x(t),\iota,\ t\big)H_1(\mu,\iota)E_{ij}(\mu,\iota)x(t)\\F\big(x(t),\iota,\ t\big)H_2(\mu,\iota)w(t)\\F\big(x(t),\iota,t\big)H_3(\mu,\iota)E_{ij}(\mu,\iota)x(t)\\w(t)\end{bmatrix}. \tag{27}$$

An LMIs approach is applied to derive a fuzzy controller that stabilizes the system (25) and that guarantees the disturbance rejection of level $\gamma>0$ immediately. First, to design the state-drivative feedback controller, the following design objectives must be satisfied:

(a) The closed loop system is asymptotically stable when $w(t)=0$.

(b) Under zero initial conditions, the system (25) satisfies $\|z\|_2\le\gamma\|w\|_2$ for any non-zero $w(t)\in\mathcal{L}_2[0,+\infty)$, where $\gamma>0$ is a prescribed constant.

The following theorem provides sufficient conditions for the existence of a robust $\mathcal{H}_\infty$ fuzzy state-derivative feedback. These sufficient conditions can be derived by the Lyapunov approach.

Theorem 1: Consider system (6). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma>0$ and $0\le\tau_d<1$, the inequality (7) holds if for $\iota=1,2,...,s$, there exist positive definite symmetric matrices $P(\iota)$, $W(\iota)$ and positive constants $\delta(\iota)$ such that the following condition hold:

$$\Xi_{ii}(\iota)<0,\ \ i=1,2,...,r, \tag{28}$$

$$\Xi_{ij}(\iota)+\Xi_{ji}(\iota)<0,\qquad i<j\le r, \tag{29}$$

where

$$\Xi_{ij}(\iota)=\begin{pmatrix}\Phi_{ij}(\iota)&(*)^T&(*)^T&(*)^T&(*)^T&(*)^T\\\mathcal{R}(\iota)\tilde{B}^T_{1_i}(\iota)&-\gamma\mathcal{R}(\iota)&(*)^T&(*)^T&(*)^T&(*)^T\\W(\iota)A_{d_i}(\iota)&0&-W(\iota)&(*)^T&(*)^T&(*)^T\\P(\iota)+\Theta_{ij}(\iota)&0&0&-W(\iota)&(*)^T&(*)^T\\\Upsilon_{ij}(\iota)&0&0&0&-\gamma\mathcal{R}(\iota)&(*)^T\\\mathcal{Z}^T(\iota)+\Theta_{ij}(\iota)&0&0&0&0&-\mathcal{P}(\iota)\end{pmatrix}, \tag{30}$$

$$\begin{aligned}\Phi_{ij}(\iota)=&P(\iota)A^T_i(\iota)+B_{2_i}(\iota)Y_{d_j}(\iota)A^T_i(\iota)\\&+A_i(\iota)P(\iota)+A_i(\iota)Y^T_{d_j}(\iota)B^T_{2_i}(\iota)+\lambda_{\iota\iota}P(\iota),\end{aligned} \tag{31}$$

$$\Upsilon_{ij}(\iota)=\tilde{C}_i(\iota)P(\iota)+\tilde{C}_i(\iota)Y^T_{d_j}(\iota)B^T_{2_i}(\iota), \tag{32}$$

$$\Theta_{ij}(\iota)=Y^T_{d_j}(\iota)B^T_{2_i}(\iota), \tag{33}$$

$$\mathcal{R}(\iota)=diag\{\delta(\iota)I,\ I,\ \delta(\iota)I,\ I\}, \tag{34}$$

$$\begin{aligned}\mathcal{Z}(\iota)=\big(&\sqrt{\lambda_{\iota 1}}P(\iota),...,\sqrt{\lambda_{\iota(\iota-1)}}P(\iota),\\&\sqrt{\lambda_{\iota(\iota+1)}}P(\iota),...,\sqrt{\lambda_{\iota s}}P(\iota)\big),\end{aligned} \tag{35}$$

$$\mathcal{P}(\iota)=diag\{P(1),...,P(\iota-1),\ P(\iota+1),...,P(s)\} \tag{36}$$

with

$$\tilde{B}_{1_i}(\iota)=[I\ \ I\ \ I\ \ B_{1_i}(\iota)], \tag{37}$$

$$\tilde{C}_i(\iota)=\Big[\gamma\rho(\iota)H^T_{1_i}(\iota)\ \sqrt{2}\aleph(\iota)\rho(\iota)H^T_{4_i}(\iota)\ 0\ \sqrt{2}\aleph(\iota)C^T_i(\iota)\Big]^T, \tag{38}$$

$$\aleph(\iota)=\Big(1+\rho^2(\iota)\sum_{i=1}^{r}\sum_{j=1}^{r}\big(\|H^T_{2_i}(\iota)H_{2_j}(\iota)\|\big)\Big)^{1/2}, \tag{39}$$

for any delay $\tau(t)$ satisfying (1), then the inequality (7) holds. Furthermore, a suitable fuzzy controller is determined as

$$u(t)=\sum_{j=1}^{r}\mu_j\big(-K_{d_j}(\iota)\dot{x}(t)\big), \tag{40}$$

where

$$K_{d_j}(\iota)=Y_{d_j}(\iota)\big(P(\iota)\big)^{-1}. \tag{41}$$

Proof: Refer to Appendix A for the proof. □

It is necessary to note that in Theorem 1, the inequalities in (28) and (29) are not only linear with respect to matrix variables, but are also linear with respect to the performance index gamma, which implies that the $\mathcal{H}_\infty$ performance γ_{min} can be optimized by solving a convex optimization algorithm with LMIs solver toolbox.

3.2. $\mathcal{H}_\infty$ fuzzy state plus state-derivative feedback controller

In this Subsection, we consider the following $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback, which is inferred as Fig. 3, the weighted average of the local models of the form:

$$u(t) = K_s(\mu,\imath)x(t) - K_d(\mu,\imath)\dot{x}(t), \tag{42}$$

where $K_s(\mu,\imath) = \sum_{j=1}^{r} \mu_j K_{s_j}(\imath)$ and $K_d(\mu,\imath) = \sum_{j=1}^{r} \mu_j K_{d_j}(\imath)$. The system (6) with the controller shown in Fig. 4 can be rewritten as

$$\begin{aligned}\dot{x}(t) =& A(\mu,\imath)x(t) + B_2(\mu,\imath)u(t)\\ &+ A_d(\mu,\imath)x\big(t-\tau(t)\big) + B_1(\mu,\imath)w(t).\end{aligned} \tag{43}$$

By substituting the controller shown in (42), we have

$$\begin{aligned}\dot{x}(t) =& A(\mu,\imath)x(t) + B_2(\mu,\imath)K_s(\mu,\imath)x(t)\\ &- B_2(\mu,\imath)K_d(\mu,\imath)\dot{x}(t)\\ &+ A_d(\mu,\imath)x\big(t-\tau(t)\big) + B_1(\mu,\imath)w(t).\end{aligned} \tag{44}$$

After rearranging (44), yeilds

$$\begin{aligned}&\big[I + B_2(\mu,\imath)K_d(\mu,\imath)\big]\dot{x}(t)\\ &= A(\mu,\imath)x(t) + B_2(\mu,\imath)K_s(\mu,\imath)x(t)\\ &\quad + A_d(\mu,\imath)x\big(t-\tau(t)\big) + B_1(\mu,\imath)w(t).\end{aligned} \tag{45}$$

The goal is to obtain state-feedback gains and state derivative-feedback gains $K_s(\mu,\imath)$ and $K_d(\mu,\imath)$, respectively, such that the following conditions hold:

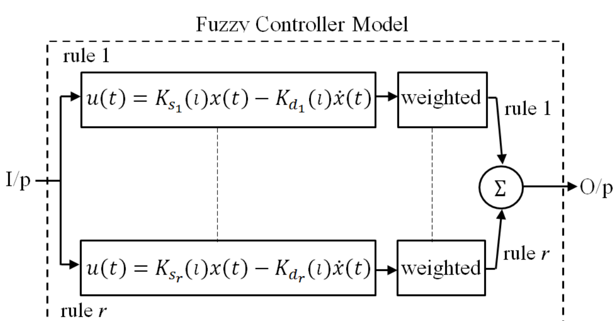

Fig. 3. The weighted average of fuzzy controller model.

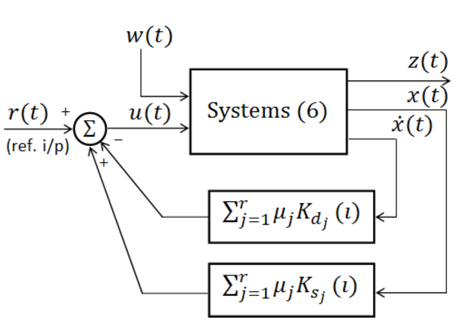

Fig. 4. The closed-loop fuzzy system.

1) Matrices $\big(I + B_{2_i}(\imath)K_{d_j}(\imath)\big)$, $\forall\, i,\, j = 1, 2, 3, ..., r$ have full rank.

2) The system (6) with the fuzzy controller (42) is asymptotically stable and the $\mathcal{H}_\infty$ performance is satisfied for all admissible values based on the sufficient condition for a prescribed scalary > 0.

From Remark 1, we define

$$E_{ij}(\imath) = \big(I + B_{2_i}(\imath)K_{d_j}(\imath)\big)^{-1}, \tag{46}$$

and thus, (45) can be written as

$$\begin{aligned}\dot{x}(t) =& E_{ij}(\mu,\imath)\big(A(\mu,\imath) + B_2(\mu,\imath)K_s(\mu,\imath)\big)x(t)\\ &+ E_{ij}(\mu,\imath)A_d(\mu,\imath)x\big(t-\tau(t)\big)\\ &+ E_{ij}(\mu,\imath)\tilde{B}_1(\mu,\imath)\tilde{w}(t),\end{aligned} \tag{47}$$

where

$$\tilde{B}_1(\mu,\imath) = [I \;\; I \;\; I \;\; B_1(\mu,\imath)]. \tag{48}$$

and the disturbance is

$$\begin{aligned}&\tilde{w}(t)\\ &= \mathcal{R}^{-1}(\imath)\\ &\times \begin{bmatrix} F\big(x(t),\imath,\, t\big)H_1(\mu,\imath)E_{ij}(\mu,\imath)x(t)\\ F\big(x(t),\imath,\, t\big)H_2(\mu,\imath)w(t)\\ F\big(x(t),\imath,t\big)H_3(\mu,\imath)E_{ij}(\mu,\imath)K_s(\mu,\imath)x(t)\\ w(t)\end{bmatrix}.\end{aligned} \tag{49}$$

An LMIs approach is applied to derive a fuzzy controller that stabilizes the system (47) and that guarantees the disturbance rejection of level $\gamma > 0$ immediately. First, to design the state feedback plus state-derivative feedback controller, the following design objectives must be satisfied:

(a) The closed loop system is asymptotically stable when $w(t) = 0$.

(b) Under zero initial conditions, the system (47) satisfies $\|z\|_2 \le \gamma\|w\|_2$ for any non-zero $w(t) \in \mathcal{L}_2\,[0, +\infty)$, where $\gamma > 0$ is a prescribed constant.

The following theorem provides sufficient conditions for the existence of a robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback. These sufficient conditions can be derived by the Lyapunov approach.

Theorem 2: Consider system (6). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and $0 \le \tau_d < 1$, the inequality (7) holds if for $\imath = 1, 2, \ldots, s$, there exist positive definite symmetric matrices $P(\imath)$, $W(\imath)$ and positive constants $\delta(\imath)$ such that the following condition hold:

$$\Xi_{ii}(\imath) < 0, \quad i = 1, 2, \ldots, r, \tag{50}$$

$$\Xi_{ij}(\imath) + \Xi_{ji}(\imath) < 0, \quad i < j \le r, \tag{51}$$

where

$$\Xi_{ij}(\imath) = \begin{pmatrix} \Phi_{ij}(\imath) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ \mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath) & -\gamma\mathcal{R}(\imath) & (*)^T & (*)^T & (*)^T & (*)^T & (*)^T \\ W(\imath)A_{d_i}(\imath) & 0 & -W(\imath) & (*)^T & (*)^T & (*)^T & (*)^T \\ P(\imath)+\Theta_{ij}(\imath) & 0 & 0 & -W(\imath) & (*)^T & (*)^T & (*)^T \\ \Upsilon_{ij}(\imath) & 0 & 0 & 0 & -\gamma\mathcal{R}(\imath) & (*)^T & (*)^T \\ \Gamma_{ij}(\imath) & 0 & 0 & 0 & 0 & -P(\imath) & (*)^T \\ \mathcal{Z}^T(\imath)+\Theta_{ij}(\imath) & 0 & 0 & 0 & 0 & 0 & -\mathcal{P}(\imath) \end{pmatrix}, \quad (52)$$

$$\Phi_{ij}(\imath) = P(\imath)A_i^T(\imath)+A_i(\imath)P(\imath)+Y_{s_j}^T(\imath)B_{2_i}^T(\imath) + B_{2_i}(\imath)Y_{s_j}(\imath)+B_{2_i}(\imath)Y_{d_j}(\imath)A_i^T(\imath) + A_i(\imath)Y_{d_j}^T(\imath)B_{2_i}^T(\imath)+\lambda_{\imath\imath}P(\imath), \quad (53)$$

$$\Upsilon_{ij}(\imath) = \tilde{C}_i(\imath)P(\imath)+\tilde{C}_i(\imath)Y_{d_j}^T(\imath)B_{2_i}^T(\imath), \quad (54)$$

$$\Theta_{ij}(\imath) = Y_{d_j}^T(\imath)B_{2_i}^T(\imath), \quad (55)$$

$$\Gamma_{ij}(\imath) = \left(Y_{s_j}(\imath)+Y_{d_j}(\imath)\right)^T B_i^T(\imath), \quad (56)$$

$$\mathcal{R}(\imath) = diag\{\delta(\imath)I,\ I,\ \delta(\imath)I,\ I\}, \quad (57)$$

$$\mathcal{Z}(\imath) = \left(\sqrt{\lambda_{\imath 1}}P(\imath),\dots,\sqrt{\lambda_{\imath(\imath-1)}}P(\imath), \sqrt{\lambda_{\imath(\imath+1)}}P(\imath),\dots,\sqrt{\lambda_{\imath s}}P(\imath)\right), \quad (58)$$

$$\mathcal{P}(\imath) = diag\{P(1),\dots,P(\imath-1),\ P(\imath+1),\dots,P(s)\}, \quad (59)$$

$$\text{with } \tilde{B}_{1_i}(\imath) = [I \quad I \quad I \quad B_{1_i}(\imath)], \quad (60)$$

$$\tilde{C}_i(\imath) = \left[\gamma\rho(\imath)H_{1_i}^T(\imath)\ \sqrt{2}\aleph(\imath)\rho(\imath)H_{4_i}^T(\imath)\ 0\ \sqrt{2}\aleph(\imath)C_i^T(\imath)\right]^T, \quad (61)$$

$$\aleph(\imath) = \left(1+\rho^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\left(\| H_{2_i}^T(\imath)H_{2_j}(\imath)\|\right)\right)^{1/2}, \quad (62)$$

for any delay $\tau(t)$ satisfying (1), then the inequality (7) holds. Furthermore, a suitable fuzzy controller is determined as

$$u(t) = \sum_{j=1}^{r}\mu_j\left(K_{s_j}(\imath)x(t)-K_{d_j}(\imath)\dot{x}(t)\right), \quad (63)$$

$$\text{where } K_{s_j}(\imath) = Y_{s_j}(\imath)\left(P(\imath)\right)^{-1}, \quad (64)$$

$$\text{and } K_{d_j}(\imath) = Y_{d_j}(\imath)\left(P(\imath)\right)^{-1}. \quad (65)$$

Proof: Refer to Appendix B for the proof. □

It is necessary to note that in Theorem 2, the inequalities in (50) and (51) are not only linear with respect to matrix variables, but are also linear with respect to the performance index gamma, which implies that the $\mathcal{H}_\infty$ performance γ_{min} can be optimized by solving a convex optimization algorithm with LMIs solver toolbox.

Remark 2: Regarding [5] and Lemma 1, the controller design using $\mathcal{H}_\infty$ fuzzy state feedback for an uncertain nonlinear Markovian jump systems with time-varying delay is developed. Unfortunately, that approach has not been considered regarding some real control problems. Especially, there have been found that the state signal has greatly limited by the necessity for the accurate information about parameters which may be difficult to estimate with high precision, while the state-derivative signal is easily obtained [30]. This issue is frequently encountered in most real dynamical systems and is often found within the complexity of designing the problems. Compared with [5], the advantage of proposed Theorem 1 and Theorem 2 can solve a problem for a class of uncertain nonlinear Markovian jump systems with time-varying delay to achieve both the robust performance and the stability in the presence of bounded modeling errors.

Remark 3: According to computing perspectives, the design of robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback controllers for uncertain nonlinear systems has been aggregated to examine a set of LMIs in conjunction with the T-S fuzzy model approach. The LMIs tool is quickly solved by employing the convex optimization algorithm. The proposed approach in this paper can significantly mitigate computational difficulties since T-S fuzzy controller gains are easily acquired. In Theorem 1 and Theorem 2, the T-S fuzzy controller gain is obtained by using LMIs based solution. The matrices Y and P can be effectively solved by existing numerical software. Hence, our main results have less computation complexity than that of [30,31]. One possible future work is how to choose the optimal approach to reduce the model design conservatism.

4. ILLUSTRATIVE EXAMPLES

Consider a modified nonlinear mass-spring-damper system which is a common control experimental device frequently used in laboratory. The dynamics of the modified nonlinear mass-spring-damper system is governed by the following state equation [5,33,34]:

$$\begin{aligned} \dot{x}_1(t) &= -[0.1125+\Delta R]x_1(t)-\beta x_1\left(t-\tau(t)\right) \\ &\quad -0.02x_2(t)-0.67x_2^3(t)-0.1x_2^3\left(t-\tau(t)\right) \\ &\quad -0.005x_2\left(t-\tau(t)\right)+u(t)+0.1w_1(t), \\ \dot{x}_2(t) &= x_1(t)+0.1w_2(t), \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \end{aligned} \quad (66)$$

Table 1. Notation and definition.

Notation	Definition
$x_1(t)$	State vectors representing velocity
$x_2(t)$	State vectors representing distance
$u(t)$	Control input
$w_1(t)$, $w_2(t)$	Disturbance inputs
$z(t)$	Regulated output
β	Delay parameter
ΔR	Uncertain term
$\tau(t)$	Time-varying delay

where the definitions of $x_1(t)$, $x_2(t)$, $u(t)$, $w_1(t)$, $w_2(t)$, $z(t)$, β, ΔR and $\tau(t)$ are shown in Table 1. It is assumed that ΔR is bounded in $[0\ 0.1125]$, $\tau(t) = 4.5 + 0.5\cos(0.9t)$, $x_1(t) \in [-1.5\ 1.5]$ and $x_2(t) \in [-1.5\ 1.5]$.

Based on [5], the nonlinear term can be written as

$$-0.67x_2^3(t) = M_1 \cdot 0 \cdot x_2(t) - (1 - M_1) \cdot 1.5075x_2(t),$$
$$-0.1x_2^3\big(t-\tau(t)\big) = M_1 \cdot 0 \cdot x_2\big(t-\tau(t)\big) - (1 - M_1) \cdot 0.225x_2\big(t-\tau(t)\big).$$

Upon solving the above equations, M_1 is obtained as follows:

$$M_1\big(x_2(t)\big) = 1 - \frac{x_2^2(t)}{2.25} \text{ and}$$
$$M_2\big(x_2(t)\big) = 1 - M_1\big(x_2(t)\big) = \frac{x_2^2(t)}{2.25}.$$

Note that $M_1\big(x_2(t)\big)$ and $M_2\big(x_2(t)\big)$ can be interpreted as membership functions of the fuzzy sets shown in Fig. 5.

Suppose that the system could be aggregated into three modes as shown in Table 2 and the transition probability matrix that relates the three operation. The transition probability matrix that relates the three operation modes is given as follows:

$$P_{\iota k} = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.30 & 0.47 & 0.23 \\ 0.26 & 0.10 & 0.64 \end{bmatrix}.$$

Using two fuzzy sets, the uncertain nonlinear Markovian jump system with time-varying delays can be represented by the following T-S fuzzy model:

Plant rule 1: IF $x_2(t)$ is $M_1(x_2(t))$ THEN

$$\dot{x}(t) = [A_1(\iota) + \Delta A_1(\iota)]x(t) + A_{d_1}(\iota)x\big(t-\tau(t)\big) + B_1(\iota)w(t) + B_2(\iota)u(t),\ \ x(0) = 0,$$
$$z(t) = C_1(\iota)x(t).$$

Plant rule 2: IF $x_2(t)$ is $M_2(x_2(t))$ THEN

$$\dot{x}(t) = [A_2(\iota) + \Delta A_2(\iota)]x(t) + A_{d_2}(\iota)x\big(t-\tau(t)\big) + B_1(\iota)w(t) + B_2(\iota)u(t),\ \ x(0) = 0,$$

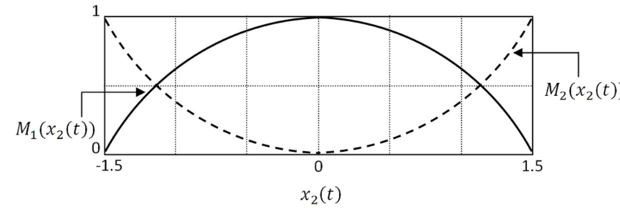

Fig. 5. Membership functions for the two fuzzy sets.

Table 2. System terminology.

Mode ι	$\beta(\iota)$
1	0.0120
2	0.0125
3	0.0130

$$z(t) = C_1(\iota)x(t),$$

where

$$A_1(\iota) = \begin{bmatrix} -0.1125 & -0.02 \\ 1 & 0 \end{bmatrix},$$
$$A_2(\iota) = \begin{bmatrix} -0.1125 & -1.5075 \\ 1 & 0 \end{bmatrix},$$
$$A_{d_1}(\iota) = \begin{bmatrix} -\beta(\iota) & -0.005 \\ 0 & 0 \end{bmatrix},$$
$$A_{d_2}(\iota) = \begin{bmatrix} -\beta(\iota) & -0.225 \\ 0 & 0 \end{bmatrix},$$
$$B_1(\iota) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ B_2(\iota) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C_1(\iota) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\Delta A_1(\iota) = F\big(x(t),t\big)H_{1_1}(\iota),$$
$$\Delta A_2(\iota) = F\big(x(t),t\big)H_{1_2}(\iota),$$
$$x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) \end{bmatrix}^T \text{ and}$$
$$w(t) = \begin{bmatrix} w_1^T(t) & w_2^T(t) \end{bmatrix}^T.$$

Next, by assuming that $\|F\big(x(t),t\big)\| \le \rho = 1$, we have

$$H_{1_1}(\iota) = H_{1_2}(\iota) = \begin{bmatrix} -0.1125 & 0 \\ 0 & 0 \end{bmatrix}$$

from the LMIs optimization algorithm and Theorem 1 with $\gamma = 1$ and $\tau_d = 0.5$, we have

$$P(1) = \begin{bmatrix} 0.2435 & -0.3616 \\ -0.3616 & 0.0356 \end{bmatrix},$$
$$W(1) = \begin{bmatrix} 575.6856 & -30.5913 \\ -30.5913 & 15.1043 \end{bmatrix},$$
$$Y_{s1}(1) = \begin{bmatrix} -0.7781 & -0.1071 \end{bmatrix},$$
$$Y_{s2}(1) = \begin{bmatrix} -1.1794 & -0.0838 \end{bmatrix},$$
$$Y_{d1}(1) = \begin{bmatrix} 0.2170 & 0.4302 \end{bmatrix},$$

$$Y_{d2}(1) = \begin{bmatrix} 0.2683 & 0.7858 \end{bmatrix},$$
$$K_{s1}(1) = \begin{bmatrix} 0.5440 & 2.5178 \end{bmatrix},$$
$$K_{s2}(1) = \begin{bmatrix} 0.5919 & 3.6598 \end{bmatrix},$$
$$K_{d1}(1) = \begin{bmatrix} -1.3372 & -1.5003 \end{bmatrix},$$
$$K_{d2}(1) = \begin{bmatrix} -2.4051 & -2.3611 \end{bmatrix},$$
$$P(2) = \begin{bmatrix} 0.2458 & -0.3110 \\ -0.3110 & 0.0582 \end{bmatrix},$$
$$W(2) = \begin{bmatrix} 69.4223 & -2.6775 \\ -2.6775 & 15.2223 \end{bmatrix},$$
$$Y_{s1}(2) = \begin{bmatrix} -0.7704 & -0.1591 \end{bmatrix},$$
$$Y_{s2}(2) = \begin{bmatrix} -1.1644 & -0.1591 \end{bmatrix},$$
$$Y_{d1}(2) = \begin{bmatrix} 0.2030 & 0.4066 \end{bmatrix},$$
$$Y_{d2}(2) = \begin{bmatrix} 0.3039 & 0.8164 \end{bmatrix},$$
$$K_{s1}(2) = \begin{bmatrix} 1.1449 & 3.3822 \end{bmatrix},$$
$$K_{s2}(2) = \begin{bmatrix} 1.4232 & 4.8693 \end{bmatrix},$$
$$K_{d1}(2) = \begin{bmatrix} -1.6779 & -1.9790 \end{bmatrix},$$
$$K_{d2}(2) = \begin{bmatrix} -3.2960 & -3.5823 \end{bmatrix},$$
$$P(3) = \begin{bmatrix} 0.2314 & -0.3128 \\ -0.3128 & 0.0571 \end{bmatrix},$$
$$W(3) = \begin{bmatrix} 213.8458 & -11.1227 \\ -11.1227 & 16.7639 \end{bmatrix},$$
$$Y_{s1}(3) = \begin{bmatrix} -0.7763 & -0.1612 \end{bmatrix},$$
$$Y_{s2}(3) = \begin{bmatrix} -1.1772 & -0.1632 \end{bmatrix},$$
$$Y_{d1}(3) = \begin{bmatrix} 0.2225 & 0.4106 \end{bmatrix},$$
$$Y_{d2}(3) = \begin{bmatrix} 0.3269 & 0.8219 \end{bmatrix},$$
$$K_{s1}(3) = \begin{bmatrix} 1.1203 & 3.3112 \end{bmatrix},$$
$$K_{s2}(3) = \begin{bmatrix} 1.3985 & 4.7990 \end{bmatrix},$$
$$K_{d1}(3) = \begin{bmatrix} -1.6685 & -1.9460 \end{bmatrix},$$

and

$$K_{d2}(3) = \begin{bmatrix} -3.2597 & -3.4573 \end{bmatrix}.$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^{2} \mu_j \big(K_{s_j}(t)x(t) - K_{d_j}(t)\dot{x}(t)\big), \quad (67)$$

where $\mu_1 = M_1\big(x_2(t)\big)$ and $\mu_2 = M_2\big(x_2(t)\big)$.

Remark 4: The fuzzy controller (67) ensures that the inequality (7) holds. Table 1 shows the system terminology, while Fig. 6 depicts the result of the switching between modes during the simulation with the initial mode 2. Fig. 7 presents the state variables, $x_1(t)$ and $x_2(t)$. The disturbance input signal, $w(t)$, used during the simulation is a rectangular signal with a magnitude of 0.1 and frequency of 1 Hz. As is illustrated in Fig. 8, after 1.8 seconds, the ratio of the regulated output energy to the disturbance input noise energy approaches a constant value of less than the prescribed value of 1.

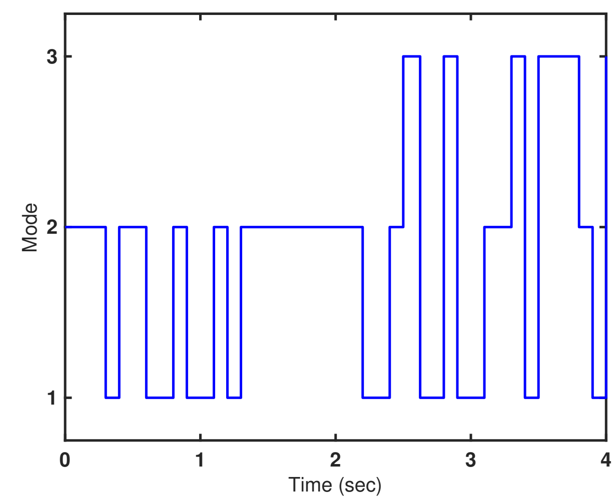

Fig. 6. The result of the switching between modes during the simulation with the initial mode being mode 2.

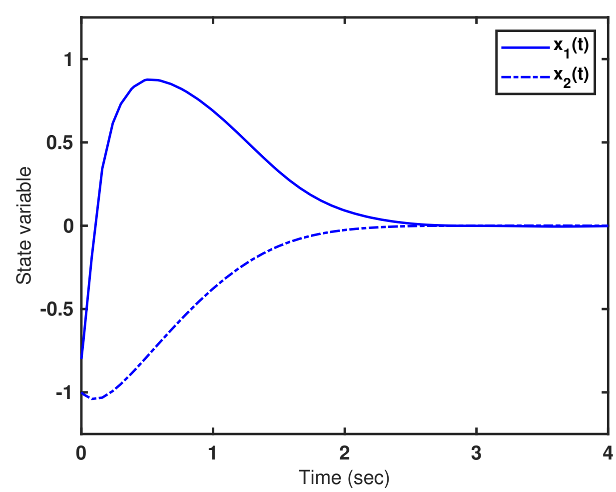

Fig. 7. The histories of the state variables, $x_l(t)$ and $x_2(t)$.

Remark 5: According to Theorem 1 used in [5], Theorem 1 and Theorem 2 used in this paper, Fig. 9 presents comparative results for the state variable $x_2(t)$ at the same $\gamma = 1$ for the allowed delay $\tau = 4.50$ and $\Delta R = 0.05$. Fig. 9 shows that Theorem 2 used in this study generates a response faster than Theorem 1 of this paper and Theorem 1 that shown in [5]. This shows that the uncertain nonlinear Markovian jump system with time-varying delays is effectively controlled using the proposed fuzzy controller.

5. CONCLUSION

This paper has presents a robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback controller design pro-

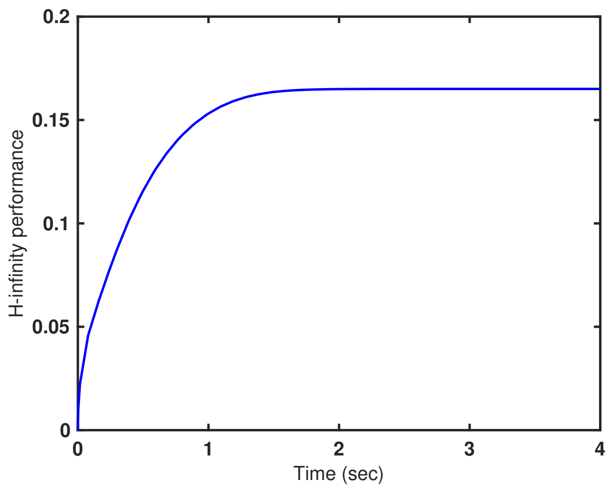

Fig. 8. H_∞ performance, $\left(\sqrt{\frac{\int_0^{T_f} z^T(t)z(t)dt}{\int_0^{T_f} w^T(t)w(t)dt}}\right)$.

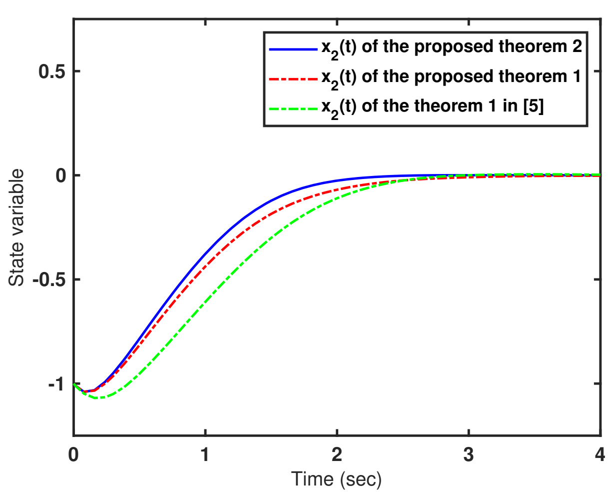

Fig. 9. Comparison of state variable, $x_2(t)$.

cedure for a class of uncertain nonlinear Markovian jump systems with time-varying delays described by the T-S fuzzy model. Based on an LMIs approach, we developed a means of designing a robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative controller that guarantees $\mathcal{L}_2$ gains of mapping from exogenous input noise to the regulated output of less than a prescribed value. In addition, solutions to the designed problem are given in terms of LMIs, rendering this approach more useful. Finally, the illustrative examples are given to describe the synthesis procedure presented in this paper. The proposed controller for uncertain nonlinear Markovian jump systems with time-varying delays is guaranteed to meet design requirements (e.g., the asymptotical stability and $\mathcal{H}_\infty$ performance index of the system). In practice, the failure of components can be easily found in many real physical control problems. Many characteristics of dynamical systems are unable to meet the desired objectives (e.g., the rise time, the settling time and transient oscillations due to poor transient responses). Therefore, motivated by a lack of control characteristics, the robust $\mathcal{H}_\infty$ fuzzy state feedback plus state-derivative feedback controller with $\mathcal{D}$ stability constraints for a nonlinear Markovian jump systems with time-varying delay can be considered in future work. In addition, applications of the proposed approach to uncertain physical systems such as wind energy control systems and photo-voltaic control systems, will be studied in the future work.

APPENDIX A: PROOF OF THEOREM 1

Proof: Consider the quadratic Lyapunov-Krasovskii functional candidate as follows:

$$V(x(t),\iota) = \gamma x^T(t)Q(\iota)x(t) + \gamma\int_{t-\tau(t)}^{t} x^T(v)G(\iota)x(v)dv, \ \ \forall \iota \in \mathcal{S}, \tag{A.1}$$

where $Q(\iota) = P^{-1}(\iota) > 0$ and $G(\iota) = W^{-1}(\iota) > 0$. For this choice, we have $V(0,\iota_0) = 0$ and $V(x(t),\iota) \to \infty$ only when $\|x(t)\| \to \infty$.

Consider the weak infinitesimal operator $\tilde{\Delta}$ of the joint process $\{(x(t),\iota),\ t \geq 0\}$ which is the stochastic analog of the deterministic derivative [35]. $\{(x(t),\iota),\ t \geq 0\}$ is a Markov process with infinitesimal operator given by [36], from (25), we then have

$$\begin{aligned}
&\tilde{\Delta}V(x(t),\iota)\\
&= \gamma x^T(t)\Big[A^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota)\\
&\quad + Q(\iota)E_{ij}(\mu,\iota)A(\mu,\iota) + G(\iota)\Big]x(t)\\
&\quad + \gamma x^T(t-\tau(t))A_d^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota)x(t)\\
&\quad + \gamma x^T(t)Q(\iota)E_{ij}(\mu,\iota)A_d(\mu,\iota)x(t-\tau(t))\\
&\quad - \gamma x^T(t-\tau(t))G(\iota)x(t-\tau(t))\\
&\quad + \gamma x^T(t)\sum_{k=1}^{s}\lambda_{\iota k}Q(k)x(t)\\
&\quad + \gamma\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{B}_1^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota)x(t)\\
&\quad + \gamma x^T(t)Q(\iota)E_{ij}(\mu,\iota)\tilde{B}_1(\mu,\iota)\mathcal{R}(\iota)\tilde{w}(t).
\end{aligned} \tag{A.2}$$

Using the fact that for any vector $x(t)$ and $x(t-\tau(t))$

$$\begin{aligned}
&x^T(t)Q(\iota)A_d(\mu,\iota)x(t-\tau(t))\\
&\quad + x^T(t-\tau(t))A_d^T(\mu,\iota)Q(\iota)x(t)\\
&\leq x^T(t)Q(\iota)A_d(\mu,\iota)G^{-1}(\iota)A_d^T(\mu,\iota)Q(\iota)x(t)\\
&\quad + x^T(t-\tau(t))G(\iota)x(t-\tau(t)).
\end{aligned} \tag{A.3}$$

Equation (A.2) becomes

$$\tilde{\Delta}V(x(t),\iota)$$

$$
\begin{aligned}
&= \gamma x^T(t)\Big[A^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota)+Q(\iota)E_{ij}(\mu,\iota)A(\mu,\iota) \\
&\quad + Q(\iota)E_{ij}(\mu,\iota)A_d(\mu,\iota)G^{-1}(\iota)A_d^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota) \\
&\quad + G(\iota) + \sum_{k=1}^{s} \lambda_{\iota k} Q(k)\Big]x(t) \\
&\quad + \gamma \tilde{w}^T(t)\mathcal{R}(\iota)\tilde{B}_1^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota)x(t) \\
&\quad + \gamma x^T(t)Q(\iota)E_{ij}(\mu,\iota)\tilde{B}_1(\mu,\iota)\mathcal{R}(\iota)\tilde{w}(t). \qquad \text{(A.4)}
\end{aligned}
$$

Adding and subtracting to and from (A.4) by $-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{w}(t)$, we have

$$
\begin{aligned}
&\tilde{\Delta}V(x(t),\iota) \\
&= -\aleph^2(\iota)z^T(t)z(t) + \gamma^2\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{w}(t) \\
&\quad + \aleph^2(\iota)z^T(t)z(t) + \gamma\begin{bmatrix} x^T(t) & \tilde{w}^T(t) \end{bmatrix} \\
&\quad \times \left(\begin{array}{cc} \left(\begin{array}{c} A^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota) \\ +Q(\iota)E_{ij}(\mu,\iota)A(\mu,\iota) \\ +Q(\iota)E_{ij}(\mu,\iota)A_d(\mu,\iota)G^{-1}(\iota) \\ \times A_{d_i}^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota) \\ +G(\iota)+\sum_{k=1}^{s}\lambda_{\iota k}Q(k) \end{array}\right) & (*)^T \\ \mathcal{R}(\iota)\tilde{B}_1^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota) & -\gamma\mathcal{R}(\iota) \end{array}\right) \\
&\quad \times \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}. \qquad \text{(A.5)}
\end{aligned}
$$

Now, let us consider the following terms:

$$
\begin{aligned}
&\gamma^2\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{w}(t) \\
&= \gamma^2 \begin{bmatrix} F\big(x(t),\iota,t\big)H_1(\mu,\iota)E_{ij}(\mu,\iota)x(t) \\ F\big(x(t),\iota,t\big)H_2(\mu,\iota)w(t) \\ F\big(x(t),\iota,t\big)H_3(\mu,\iota)E_{ij}(\mu,\iota)K_s(\mu,\iota)x(t) \\ w(t) \end{bmatrix}^T \mathcal{R}(\iota) \\
&\quad \times \begin{bmatrix} F\big(x(t),\iota,t\big)H_1(\mu,\iota)E_{ij}(\mu,\iota)x(t) \\ F\big(x(t),\iota,t\big)H_2(\mu,\iota)w(t) \\ F\big(x(t),\iota,t\big)H_3(\mu,\iota)E_{ij}(\mu,\iota)K_s(\mu,\iota)x(t) \\ w(t) \end{bmatrix} \\
&\le \frac{\rho^2(\iota)\gamma^2}{\delta(\iota)}x^T(t)\Big[E_{ij}^T(\mu,\iota)H_1^T(\mu,\iota)H_1(\mu,\iota)E_{ij}(\mu,\iota) \\
&\quad + K_s^T(\mu,\iota)E_{ij}^T(\mu,\iota)H_3^T(\mu,\iota)H_3(\mu,\iota)E_{ij}(\mu,\iota) \\
&\quad \times K_s(\mu,\iota)\Big]x(t) + \aleph^2(\iota)\gamma^2 w^T(t)w(t), \qquad \text{(A.6)}
\end{aligned}
$$

and

$$
\begin{aligned}
&\aleph^2(\iota)z^T(t)z(t) \\
&= \aleph^2(\iota)x^T(t)\Big[C(\mu,\iota)+F\big(x(t),\iota,t\big)H_4(\mu,\iota)\Big]^T \\
&\quad \times \Big[C(\mu,\iota)+F\big(x(t),\iota,t\big)H_4(\mu,\iota)\Big]x(t) \\
&\le 2\aleph^2(\iota)x^T(t)\Big[\big(C^T(\mu,\iota)C(\mu,\iota)\big) \\
&\quad + \big[\big(F\big(x(t),\iota,t\big)H_4(\mu,\iota)\big)^T \\
&\quad \times \big(F\big(x(t),\iota,t\big)H_4(\mu,\iota)\big)\big]\Big]x(t) \\
&\le 2\aleph^2(\iota)x^T(t)\Big[\big(C^T(\mu,\iota)C(\mu,\iota)\big) \\
&\quad + \rho^2(\iota)\big[\big(H_4^T(\mu,\iota)\big)\big(H_4(\mu,\iota)\big)\big]\Big]x(t), \qquad \text{(A.7)}
\end{aligned}
$$

where $\aleph(\iota) = \Big(I+\rho^2(\iota)\sum_{i=1}^{r}\sum_{j=1}^{r}\big[\,\|\,H_{2_i}^T(\iota)H_{2_j}(\iota)\,\|\,\big]\Big)^{\frac{1}{2}}$. Hence,

$$
\begin{aligned}
&\gamma^2\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{w}(t) + \aleph^2(\iota)z^T(t)z(t) \\
&\le x^T(t)\tilde{C}^T(\mu,\iota)\mathcal{R}^{-1}(\iota)\tilde{C}(\mu,\iota)x(t) \\
&\quad + \aleph^2(\iota)\gamma^2 w^T(t)w(t), \qquad \text{(A.8)}
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{C}_i(\mu,\iota) = \Big[&\gamma\rho(\iota)H_{1_i}^T(\mu,\iota) \;\; \sqrt{2}\aleph(\iota)\rho(\iota)H_{4_i}^T(\mu,\iota) \;\; 0 \\
&\sqrt{2}\aleph(\iota)C_i^T(\mu,\iota)\Big]^T. \qquad \text{(A.9)}
\end{aligned}
$$

Substituting (A.8) into (A.5) yields

$$
\begin{aligned}
&\tilde{\Delta}V(x(t),\iota) \\
&\le -\aleph^2(\iota)z^T(t)z(t) + \gamma^2\aleph^2(\iota)w^T(t)w(t) \\
&\quad + \gamma\begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \Psi(\mu,\iota)\begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}, \qquad \text{(A.10)}
\end{aligned}
$$

where

$$
\begin{aligned}
&\Psi(\mu,\iota) \\
&= \left(\begin{array}{cc} \left(\begin{array}{c} A^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota) \\ +Q(\iota)E_{ij}(\mu,\iota)A(\mu,\iota) \\ +\frac{1}{\gamma}\big(\tilde{C}^T(\mu,\iota)\mathcal{R}^{-1}(\iota)\tilde{C}(\mu,\iota)\big)+ \\ Q(\iota)E_{ij}(\mu,\iota)A_d(\mu,\iota)G^{-1}(\iota)\times \\ A_d^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota) \\ +G(\iota)+\sum_{k=1}^{s}\lambda_{\iota k}Q(k) \end{array}\right) & (*)^T \\ \mathcal{R}(\iota)\tilde{B}_1^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota) & -\gamma\mathcal{R}(\iota) \end{array}\right). \qquad \text{(A.11)}
\end{aligned}
$$

Using the fact that

$$
\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n M_{ij}^T(\iota)N_{mn}(\iota) \\
&\le \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\left[M_{ij}^T(\iota)M_{ij}(\iota)+N_{ij}(\iota)N_{ij}^T(\iota)\right], \qquad \text{(A.12)}
\end{aligned}
$$

we can rewrite (A.10) as follows:

$$
\begin{aligned}
&\tilde{\Delta}V(x(t),\iota) \\
&\le -\aleph^2(\iota)z^T(t)z(t) + \gamma^2\aleph^2(\iota)w^T(t)w(t) \\
&\quad + \gamma\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \Psi_{ij}(\iota)\begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&= -\aleph^2(\iota) z^T(t) z(t) + \gamma^2 \aleph^2(\iota) w^T(t) w(t) \\
&\quad + \gamma \sum_{i=1}^{r} \mu_i{}^2 \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \Psi_{ii}(\iota) \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix} \\
&\quad + \gamma \sum_{i=1}^{r} \sum_{i<j}^{r} \mu_i \mu_j \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \left(\Psi_{ij}(\iota) + \Psi_{ji}(\iota)\right) \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix},
\end{aligned} \tag{A.13}
$$

where

$$
\Psi_{ij}(\iota) = \begin{pmatrix} \begin{pmatrix} A_i^T(\iota) E_{ij}^T(\iota) Q(\iota) \\ + Q(\iota) E_{ij}(\iota) A_i(\iota) \\ + \frac{1}{\gamma}\left(\tilde{C}_i^T(\iota) \mathcal{R}^{-1}(\iota) \tilde{C}_i(\iota)\right) \\ + Q(\iota) E_{ij}(\iota) A_{d_i}(\iota) G^{-1}(\iota) \\ \times A_{d_i}^T(\iota) E_{ij}^T(\iota) Q(\iota) \\ + G(\iota) + \sum_{k=1}^{s} \lambda_{\iota k} Q(k) \end{pmatrix} & (*)^T \\ \mathcal{R}(\iota) \tilde{B}_{1_i}^T(\iota) E_{ij}^T(\iota) Q(\iota) & -\gamma \mathcal{R}(\iota) \end{pmatrix}. \tag{A.14}
$$

Pre and post multiplying (A.14) by $\begin{pmatrix} P(\iota) & 0 \\ 0 & I \end{pmatrix}$, we obtain

$$
\Psi_{ij}(\iota) = \begin{pmatrix} \begin{pmatrix} P(\iota) A_i^T(\iota) E_{ij}^T(\iota) + E_{ij}(\iota) A_i(\iota) P(\iota) \\ + \frac{1}{\gamma}\left(\tilde{C}_i^T(\iota) P(\iota) \mathcal{R}^{-1}(\iota) \tilde{C}_i(\iota) P(\iota)\right) \\ + E_{ij}(\iota) A_{d_i}(\iota) G^{-1}(\iota) A_{d_i}^T(\iota) E_{ij}^T(\iota) \\ + P(\iota) G(\iota) P(\iota) \\ + \sum_{k=1}^{s} \lambda_{\iota k} P(\iota) P^{-1}(k) P(\iota) \end{pmatrix} & (*)^T \\ \mathcal{R}(\iota) \tilde{B}_{1_i}^T(\iota) E_{ij}^T(\iota) & -\gamma \mathcal{R}(\iota) \end{pmatrix}. \tag{A.15}
$$

Pre and post multiplying $\begin{pmatrix} E_{ij}^{-1}(\iota) & 0 \\ 0 & I \end{pmatrix}$ and $\begin{pmatrix} E_{ij}^{-T}(\iota) & 0 \\ 0 & I \end{pmatrix}$, respectively, to (A.15), we have

$$
\Psi_{ij}(\iota) = \begin{pmatrix} \begin{pmatrix} E_{ij}^{-1}(\iota) P(\iota) A_i^T(\iota) + A_i(\iota) P(\iota) E_{ij}^{-T}(\iota) \\ + \frac{1}{\gamma} E_{ij}^{-1}(\iota)\left(\tilde{C}_i^T(\iota) P(\iota) \right. \\ \left. \times \mathcal{R}^{-1}(\iota) \tilde{C}_i(\iota) P(\iota)\right) E_{ij}^{-T}(\iota) \\ + A_{d_i}(\iota) G^{-1}(\iota) A_{d_i}^T(\iota) \\ + E_{ij}^{-1}(\iota) P(\iota) G(\iota) P(\iota) E_{ij}^{-T}(\iota) \\ + E_{ij}^{-1}(\iota)\left(\sum_{k=1}^{s} \lambda_{\iota k} P(\iota) P^{-1}(k) P(\iota)\right) \\ \times E_{ij}^{-T}(\iota) \end{pmatrix} & (*)^T \\ \mathcal{R}(\iota) \tilde{B}_{1_i}^T(\iota) & -\gamma \mathcal{R}(\iota) \end{pmatrix}. \tag{A.16}
$$

Using (24) and (41), we obtain

$$
\Psi_{ij}(\iota) = \begin{pmatrix} \begin{pmatrix} P(\iota) A_i^T(\iota) + A_i(\iota)) P(\iota) \\ + B_{2_i}(\iota) Y_{d_j}(\iota) P^{-1}(\iota) P(\iota) A_i^T(\iota) \\ + A_i(\iota) P(\iota) P^{-1}(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota) \\ + \frac{1}{\gamma}\left(\tilde{C}_1(\iota) P(\iota)\right. \\ \left. + \tilde{C}_1(\iota) P(\iota) P^{-1}(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right)^T \\ \times \mathcal{R}^{-1}(\iota)\left(\tilde{C}_1(\iota) P(\iota)\right. \\ \left. + \tilde{C}_1(\iota) P(\iota) P^{-1}(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right) \\ + A_{d_i}(\iota) G^{-1}(\iota) A_{d_i}^T(\iota) \\ + \left(P(\iota) + P(\iota) P^{-1}(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right)^T \\ \times G(\iota)\left(P(\iota) + P(\iota) P^{-1}(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right) \\ + \sum_{k=1}^{s} \lambda_{\iota k}\left(P(\iota)\right. \\ \left. + P(\iota) P^{-1}(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right)^T \\ \times P^{-1}(k)\left(P(\iota)\right. \\ \left. + P(\iota) P^{-1}(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right) \end{pmatrix} & (*)^T \\ \mathcal{R}(\iota) \tilde{B}_{1_i}^T(\iota) & -\gamma \mathcal{R}(\iota) \end{pmatrix}. \tag{A.17}
$$

Rearranging (A.17), we have

$$
\Psi_{ij}(\iota) = \begin{pmatrix} \begin{pmatrix} P(\iota) A_i^T(\iota) + A_i(\iota) P(\iota) \\ + B_{2_i}(\iota) Y_{d_j}(\iota) A_i^T(\iota) + A_i(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota) \\ + \frac{1}{\gamma}\left(\tilde{C}_1(\iota) P(\iota) + \tilde{C}_1(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right)^T \\ \times \mathcal{R}^{-1}(\iota) \\ \times\left(\tilde{C}_1(\iota) P(\iota) + \tilde{C}_1(\iota) Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right) \\ + A_{d_i}(\iota) G^{-1}(\iota) A_{d_i}^T(\iota) \\ + \left(P(\iota) + Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right)^T \\ \times G(\iota)\left(P(\iota) + Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right) \\ + \sum_{k=1}^{s} \lambda_{\iota k}\left(P(\iota) + Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right)^T \\ \times P^{-1}(k)\left(P(\iota) + Y_{d_j}^T(\iota) B_{2_i}^T(\iota)\right) \end{pmatrix} & (*)^T \\ \mathcal{R}(\iota) \tilde{B}_{1_i}^T(\iota) & -\gamma \mathcal{R}(\iota) \end{pmatrix}. \tag{A.18}
$$

Note that (A.18) is the Schur complement of $\Xi_{ij}(\iota)$, defined in (30). Using (28), (29) and (A.18), we learn that

$$
\Psi_{ii}(\iota) < 0, \tag{A.19}
$$

$$
\Psi_{ij}(\iota) + \Psi_{ji}(\iota) < 0. \tag{A.20}
$$

Following from (3), (A.19) and (A.20), we know that

$$
\tilde{\Delta} V(x(t), \iota) < -\aleph^2(\iota) z^T(t) z(t) + \gamma^2 \aleph^2(\iota) w^T(t) w(t). \tag{A.21}
$$

Applying the operator $\mathrm{E}[\int_0^{T_f}(\cdot)dt]$ to both sides of (A.21), we obtain

$$\mathrm{E}\Big[\int_0^{T_f}\tilde{\Delta}V(x(t),\iota)dt\Big] < \mathrm{E}\Big[\int_0^{T_f}\big(-\aleph^2(\iota)z^T(t)z(t) + \gamma^2\aleph^2(\iota)w^T(t)w(t)\big)dt\Big]. \tag{A.22}$$

From the Dynkin's formula [37], it follows that

$$\mathrm{E}\Big[\int_0^{T_f}\tilde{\Delta}V\big(x(t),\iota\big)dt\Big] = \mathrm{E}\big[V\big(x(T_f),\iota(T_f)\big)\big] - \mathrm{E}\big[V\big(x(0),\iota(0)\big)\big]. \tag{A.23}$$

Substitution of (A.23) into (A.22) yields

$$0 < \mathrm{E}\Big[\int_0^{T_f}\big(-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\aleph^2(\iota)w^T(t)w(t)\big)dt\Big] - \mathrm{E}\big[V\big(x(T_f),\iota(T_f)\big)\big] + \mathrm{E}\big[V\big(x(0),\iota(0)\big)\big]. \tag{A.24}$$

Using (A.21) and the fact that $V\big(x(0)=0,\iota(0)\big)=0$ and $V\big(x(T_f),\iota(T_f)\big)\geq 0$, we have

$$\mathrm{E}\Big[\int_0^{T_f}\{z^T(t)z(t)-\gamma^2 w^T(t)w(t)\}dt\Big] < 0. \tag{A.25}$$

Hence the inequality (7) holds. This is the case when $w(t)=0$, and (A.21) becomes $\tilde{\Delta}V\big(x(t),\iota\big) < -z^T(t)z(t) \leq 0$. Therefore, the closed-loop system (25) is asymptotically stable, and (b) is achieved. This completes the proof of Theorem 1. □

APPENDIX B: PROOF OF THEOREM 2

Proof: Consider the quadratic Lyapunov-Krasovskii functional candidate as follows:

$$V(x(t),\iota) = \gamma x^T(t)Q(\iota)x(t) + \gamma\int_{t-\tau(t)}^{t} x^T(v)G(\iota)x(v)dv, \ \ \forall \iota\in\mathcal{S}, \tag{B.1}$$

where $Q(\iota)=P^{-1}(\iota)>0$ and $G(\iota)=W^{-1}(\iota)>0$. For this choice, we have $V(0,\iota_0)=0$ and $V\big(x(t),\iota\big)\to\infty$ only when $\|x(t)\|\to\infty$.

Consider the weak infinitesimal operator $\tilde{\Delta}$ of the joint process $\{\big(x(t),\iota\big),\ t\geq 0\}$ which is the stochastic analog of the deterministic derivative [35]. $\{\big(x(t),\iota\big),\ t\geq 0\}$ is a Markov process with infinitesimal operator given by [36], from (46), we then have

$$\begin{aligned}
&\tilde{\Delta}V\big(x(t),\iota\big)\\
&= \gamma x^T(t)\Big[\Big(A(\mu,\iota)+B_2(\mu,\iota)K_s(\mu,\iota)\Big)^T E_{ij}^T(\mu,\iota)Q(\iota)\\
&\quad + Q(\iota)E_{ij}(\mu,\iota)\Big(A(\mu,\iota)+B_2(\mu,\iota)K_s(\mu,\iota)\Big)\\
&\quad + G(\iota)\Big]x(t) + \gamma x^T\big(t-\tau(t)\big)A_d^T(\mu,\iota)E_{ij}^T(\mu,\iota)\\
&\quad \times Q(\iota)x(t) + \gamma x^T(t)Q(\iota)E_{ij}(\mu,\iota)A_d(\mu,\iota)\\
&\quad \times x\big(t-\tau(t)\big) - \gamma x^T\big(t-\tau(t)\big)G(\iota)x\big(t-\tau(t)\big)\\
&\quad + \gamma x^T(t)\sum_{k=1}^{s}\lambda_{\iota k}Q(k)x(t)\\
&\quad + \gamma\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{B}_1^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota)x(t)\\
&\quad + \gamma x^T(t)Q(\iota)E_{ij}(\mu,\iota)\tilde{B}_1(\mu,\iota)\mathcal{R}(\iota)\tilde{w}(t).
\end{aligned} \tag{B.2}$$

Using the fact that for any vector $x(t)$ and $x\big(t-\tau(t)\big)$

$$\begin{aligned}
&x^T(t)Q(\iota)A_d(\mu,\iota)x\big(t-\tau(t)\big)\\
&\quad + x^T\big(t-\tau(t)\big)A_d^T(\mu,\iota)Q(\iota)x(t)\\
&\leq x^T(t)Q(\iota)A_d(\mu,\iota)G^{-1}(\iota)A_d^T(\mu,\iota)Q(\iota)x(t)\\
&\quad + x^T\big(t-\tau(t)\big)G(\iota)x\big(t-\tau(t)\big).
\end{aligned} \tag{B.3}$$

Equation (B.2) becomes

$$\begin{aligned}
&\tilde{\Delta}V\big(x(t),\iota\big)\\
&= \gamma x^T(t)\Big[\Big(A(\mu,\iota)+B_2(\mu,\iota)K_s(\mu,\iota)\Big)^T E_{ij}^T(\mu,\iota)Q(\iota)\\
&\quad + Q(\iota)E_{ij}(\mu,\iota)\Big(A(\mu,\iota)+B_2(\mu,\iota)K_s(\mu,\iota)\Big)\\
&\quad + Q(\iota)E_{ij}(\mu,\iota)A_d(\mu,\iota)G^{-1}(\iota)A_d^T(\mu,\iota)\\
&\quad \times E_{ij}^T(\mu,\iota)Q(\iota) + G(\iota) + \sum_{k=1}^{s}\lambda_{\iota k}Q(k)\Big]x(t)\\
&\quad + \gamma\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{B}_1^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota)x(t)\\
&\quad + \gamma x^T(t)Q(\iota)E_{ij}(\mu,\iota)\tilde{B}_1(\mu,\iota)\mathcal{R}(\iota)\tilde{w}(t).
\end{aligned} \tag{B.4}$$

Adding and subtracting to and from (B.4) by $-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{w}(t)$, we have

$$\begin{aligned}
&\tilde{\Delta}V(x(t),\iota)\\
&= -\aleph^2(\iota)z^T(t)z(t)+\gamma^2\tilde{w}^T(t)\mathcal{R}(\iota)\tilde{w}(t)\\
&\quad + \aleph^2(\iota)z^T(t)z(t)+\gamma\begin{bmatrix} x^T(t) & \tilde{w}^T(t)\end{bmatrix}\\
&\quad \times\begin{pmatrix}
\begin{pmatrix}\Big(A(\mu,\iota)+B_2(\mu,\iota)K_s(\mu,\iota)\Big)^T\\ \times E_{ij}^T(\mu,\iota)Q(\iota)\\ +Q(\iota)E_{ij}(\mu,\iota)\\ \times\Big(A(\mu,\iota)+B_2(\mu,\iota)K_s(\mu,\iota)\Big)\\ +Q(\iota)E_{ij}(\mu,\iota)A_d(\mu,\iota)G^{-1}(\iota)\\ \times A_{d_i}^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota)\\ +G(\iota)+\sum_{k=1}^{s}\lambda_{\iota k}Q(k)\end{pmatrix} & (*)^T\\
\mathcal{R}(\iota)\tilde{B}_1^T(\mu,\iota)E_{ij}^T(\mu,\iota)Q(\iota) & -\gamma\mathcal{R}(\iota)
\end{pmatrix}\\
&\quad \times\begin{bmatrix} x(t)\\ \tilde{w}(t)\end{bmatrix}.
\end{aligned} \tag{B.5}$$

Now, by employing the same technique used in the proof for Theorem 1, we obtain

$$\Psi_{ij}(\iota)$$

$$
= \begin{pmatrix} \begin{pmatrix} \begin{array}{c} P(\iota)A_i^T(\iota)+A_i(\iota)P(\iota) \\ +B_{2_i}(\iota)Y_{s_j}(\iota)+Y_{s_j}^T(\iota)B_{2_i}^T(\iota) \\ +B_{2_i}(\iota)Y_{d_j}(\iota)A_i^T(\iota)+A_i(\iota)Y_{d_j}^T(\iota)B_{2_i}^T(\iota) \\ B_i(\iota)(Y_{s_j}(\iota)+Y_{d_j}(\iota))P^{-1} \\ \times(Y_{s_j}(\iota)+Y_{d_j}(\iota))^T B_i^T(\iota) \\ +\frac{1}{\gamma}\left(\tilde{C}_1(\iota)P(\iota)+\tilde{C}_1(\iota)Y_{d_j}^T(\iota)B_{2_i}^T(\iota)\right)^T \\ \times\mathcal{R}^{-1}(\iota) \\ \times\left(\tilde{C}_1(\iota)P(\iota)+\tilde{C}_1(\iota)Y_{d_j}^T(\iota)B_{2_i}^T(\iota)\right) \\ +A_{d_i}(\iota)G^{-1}(\iota)A_{d_i}^T(\iota) \\ +\left(P(\iota)+Y_{d_j}^T(\iota)B_{2_i}^T(\iota)\right)^T \\ \times G(\iota)\left(P(\iota)+Y_{d_j}^T(\iota)B_{2_i}^T(\iota)\right) \\ +\sum_{k=1}^{s}\lambda_{\iota k}\left(P(\iota)+Y_{d_j}^T(\iota)B_{2_i}^T(\iota)\right)^T \\ \times P^{-1}(k)\left(P(\iota)+Y_{d_j}^T(\iota)B_{2_i}^T(\iota)\right) \end{array} \end{pmatrix} & (*)^T \\ \mathcal{R}(\iota)\tilde{B}_{1_i}^T(\iota) & -\gamma\mathcal{R}(\iota) \end{pmatrix}. \tag{B.6}
$$

Note that (B.6) is the Schur complement of $\Xi_{ij}(\iota)$, defined in (51). Using (49), (50) and (B.6), we learn that

$$\Psi_{ii}(\iota) < 0, \tag{B.7}$$
$$\Psi_{ij}(\iota)+\Psi_{ji}(\iota) < 0. \tag{B.8}$$

Following from (3), (B.7) and (B.8), we know that

$$
\begin{aligned}
&\tilde{\Delta}V(x(t),\iota) \\
&< -\aleph^2(\iota)z^T(t)z(t)+\gamma^2\aleph^2(\iota)w^T(t)w(t).
\end{aligned} \tag{B.9}
$$

Applying the operator $\mathrm{E}[\int_0^{T_f}(\cdot)dt]$ to both sides of (B.9), we obtain

$$
\begin{aligned}
\mathrm{E}\Big[\int_0^{T_f}\tilde{\Delta}V(x(t),\iota)dt\Big] <&\, \mathrm{E}\Big[\int_0^{T_f}\big(-\aleph^2(\iota)z^T(t)z(t) \\
&+\gamma^2\aleph^2(\iota)w^T(t)w(t)\big)dt\Big].
\end{aligned} \tag{B.10}
$$

From the Dynkin's formula [37], it follows that

$$
\begin{aligned}
&\mathrm{E}\Big[\int_0^{T_f}\tilde{\Delta}V\big(x(t),\iota\big)dt\Big] \\
&= \mathrm{E}\big[V\big(x(T_f),\iota(T_f)\big)\big]-\mathrm{E}\big[V\big(x(0),\iota(0)\big)\big].
\end{aligned} \tag{B.11}
$$

Substitution of (B.11) into (B.10) yields

$$
\begin{aligned}
0 <&\, \mathrm{E}\Big[\int_0^{T_f}\big(-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\aleph^2(\iota)w^T(t)w(t)\big)dt\Big] \\
&-\mathrm{E}\big[V\big(x(T_f),\iota(T_f)\big)\big]+\mathrm{E}\big[V\big(x(0),\iota(0)\big)\big].
\end{aligned} \tag{B.12}
$$

Using (B.9) and the fact that $V\big(x(0)=0,\iota(0)\big)=0$ and $V\big(x(T_f),\iota(T_f)\big)\geq 0$, we have

$$\mathrm{E}\Big[\int_0^{T_f}\{z^T(t)z(t)-\gamma^2 w^T(t)w(t)\}dt\Big] < 0. \tag{B.13}$$

Hence the inequality (7) holds. This is the case when $w(t)=0$, and (B.9) becomes $\tilde{\Delta}V\big(x(t),\iota\big) < -z^T(t)z(t) \leq 0$. Therefore, the closed-loop system (47) is asymptotically stable, and (b) is achieved. This completes the proof of Theorem 1. □

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