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An Observer and Post Filter Based Scheme for Fault Estimation of Nonlinear Systems

Hai Liu, Maiying Zhong*, Yang Liu, and Zuqiang Yang

Abstract: The problem of actuator fault estimation for a kind of nonlinear discrete time varying (NDTV) systems is investigated. Norm bounded disturbance is considered and the nonlinear function is assumed to fulfill a Lipschitz-like condition. Fault estimation is accomplished by a novel nonlinear state observer and a dynamic post filter. First, the nonlinear state observer is designed with the help of the small gain theorem and the H_∞ filtering approach. Then for the error dynamic of the observer, the dynamic post filter is constructed to estimate the fault. In this scheme, no approximation of the nonlinear function is taken, and the problem of infeasibility resulting from multiple synthesis conditions is considerably improved. Simulation studies are carried out with a nonlinear unmanned aerial vehicle (UAV) model. The turbulent condition is considered and the feed back control loop is employed. Simulation results show that the proposed method can accomplish the estimation work, while the residual evaluation based approach fails to detect the fault.

Keywords: Fault estimation, Lipschitz-like condition, nonlinear observer, post filter.

1. INTRODUCTION

Because of the nonlinear nature of various physical systems, fault diagnosis of nonlinear systems has attracted considerable attention in recent years, see [1–6] and references therein. During the design of the diagnosis observer for nonlinear systems, the nonlinearity must be carefully handled to make the observer stable, and in the meantime, the diagnosis performance is desired. Therefore, fault diagnosis of nonlinear systems is not an easy task. In [7–11], linear parameter varying systems or T-S fuzzy systems are applied to approximate nonlinear systems, then diagnosis observers are designed accordingly. However, modelling errors are inevitable and this may result in degradation of the diagnosis performance. In [12–15], the high order terms in Taylor expansions of the nonlinear functions are neglected, then the error dynamic system of the observer is obtained as a linear system. Thus, the observer gain can be selected in a familiar manner to achieve the desired diagnosis performance. However, when nonlinearities are severe, the neglected terms may lead to stability problems.

Observer or diagnosis observer design for Lipschitz nonlinear systems is widely investigated, because a variety of nonlinearities can be included and the adopted Lipschitz condition makes it easy to deal with the stability problem, see [2, 3, 6, 16–20]. Synthesis of the diagnosis observer is usually accomplished by solving linear matrix inequalities (LMIs). However, as is declared in [21], a general limitation of these approaches is that LMIs are often infeasible because of large Lipschitz constants and multiple synthesis conditions.

In this paper, the problem of actuator fault estimation for a kind of nonlinear discrete time varying (NDTV) systems is considered. The motivation is to take care of the high order terms in Taylor expansions so that the stability problems can be solved. To avoid multiple synthesis conditions, fault estimation is accomplished by a nonlinear state observer and a dynamic post filter. Inspired from the works in [22–24], a Lipschitz-like condition is introduced to characterize the high order terms. Following a similar idea presented in [18, 25, 26], the Lipschitz-like condition is artfully used in nonlinear state observer design with the help of the small gain theorem and the H_∞ filtering approach. It is noted that this observer design approach is applicable for more general systems, while in [18, 25, 26], observer gains are obtained by solving LMIs. After the nonlinear state observer is established, the dynamic post filter can be easily designed for the error dy-

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Hai Liu and Zuqiang Yang are with the Information Science Academy, China Electronics Technology Group Corporation, Beijing 100086, China (e-mails: liuhai036@buaa.edu.cn, gaayzq@163.com). Maiying Zhong and Yang Liu are with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China (e-mails: myzhong@buaa.edu.cn, lianinliyan@163.com).
* Corresponding author.

namic of the observer. The H_∞ filtering approach is used again to achieve robust fault estimation. In this fault estimation scheme, the problem of infeasibility resulting from multiple synthesis conditions is considerably improved. The individual design of the state observer and the post filter are much easier, and a suboptimal fault estimation performance can be reserved.

The rest of the paper is organized as follows: Problem formulation and the basic idea of the propose method are presented in Section 2. Section 3 is dedicated to nonlinear state observer and its post filter design. To illustrate the application of the proposed method, simulation studies are carried out with a nonlinear unmanned aerial vehicle (UAV) model in Section 4. Finally, conclusion is made in Section 5.

2. PROBLEM FORMULATION AND BASIC IDEA

Consider a class of NDTV systems with actuator faults as follows:

$$\begin{cases} x(k+1) = \mathcal{A}(x(k)) + B(k)(u(k)+f(k)) \\ \qquad\qquad\quad + B_w(k)w(k), \\ y(k) = C(k)x(k) + v(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^p$ is the control input and $f(k) \in \mathbb{R}^p$ is the actuator fault, $y(k) \in \mathbb{R}^q$ is the measurement output, $w(k) \in \mathbb{R}^m$ represents disturbances, and $v(k) \in \mathbb{R}^q$ indicates measurement noises. Without loss of generality, $f(k)$, $w(k)$ and $v(k)$ are assumed to be $l_2[0,N]$-norm bounded. $\mathcal{A}(\cdot)$ is a nonlinear function satisfying some Lipschitz-like condition that will be given latter.

As is presented in [27], when loss of control effectiveness of actuators are considered, the additive fault $f(k)$ can be modeled as unknown parameter $\theta(k) \in \mathbb{R}^p$ in a multiplicative manner for easier estimation, specifically,

$$\begin{aligned} &B(k)(u(k)+f(k)) \\ &= B(k)\cdot \mathrm{diag}\left(1-\theta_1(k) \quad \cdots \quad 1-\theta_p(k) \right)\cdot u(k), \end{aligned}$$

where $\theta_i(k) \in [0,1]$, $i = 1, \cdots, p$ and $\mathrm{diag}(\cdot)$ stands for a block diagonal matrix. Then fault diagnosis work is accomplished by estimation of $\theta(k)$. On the other hand, the work in [21] shows that the detailed description of the nonlinear system is helpful to achieve a less conservative design. Based on the above statement, the nonlinear system (1) is rewritten as follows:

$$\begin{cases} x(k+1) = U(k)x(k) + \Gamma\phi(\Pi x(k)) + B(k)u(k) \\ \qquad\qquad\quad + B_f(k)\theta(k) + B_w(k)w(k), \\ y(k) = C(k)x(k) + v(k), \end{cases} \tag{2}$$

where $U(k)$ is used to describe the linear part of the original system, $\phi(\cdot)$ is the nonlinear function, Γ is the distribution matrix of $\phi(\cdot)$ and Π depends on states that are used in $\phi(\cdot)$. $\theta(k)$, $B_f(k)$ are specified as follows:

$$\begin{aligned} &\theta(k) = \left(\theta_1(k) \quad \cdots \quad \theta_p(k)\right)^T, \\ &B_f(k) = B(k)\cdot \mathrm{diag}\left(u_1(k) \quad \cdots \quad u_p(k)\right), \end{aligned}$$

where $u_i(k)$ represents the ith item of $u(k)$.

Usually, it is assumed that $\phi(\cdot)$ satisfies the following Lipschitz condition:

$$\left\|\phi(\Pi x(k)) - \phi(\Pi\hat{x}(k))\right\| \le \kappa \left\|\Pi\tilde{x}(k)\right\|, \tag{3}$$

where $\kappa > 0$ is the Lipschitz constant, $\hat{x}(k)$ is the state estimate, and $\tilde{x}(k) = x(k) - \hat{x}(k)$. Though matrices Γ and Π are employed for detailed presentation of the nonlinear system, conservativeness may result from a large Lipschitz constant κ. Therefore, in [22–24], a Lipshcitz-like condition is adopted in observer design. In this paper, a similar Lipshcitz-like condition is introduced in the following assumption.

Assumption 1: $\phi(\cdot)$ is a smooth function with respect to $x(k)$, its Taylor expansion is demonstrated as follows:

$$\phi(\Pi x(k)) = \phi(\Pi\hat{x}(k)) + \Lambda(k)\tilde{x}(k) + \tilde{\phi}(k), \tag{4}$$

where

$$\Lambda(k) = \frac{\partial}{\partial x}(\phi(\Pi x(k)))\bigg|_{\hat{x}(k)}, \tag{5}$$

and $\tilde{\phi}(k)$ is the sum of high order terms which satisfies

$$\left\|\tilde{\phi}(k)\right\| \le \kappa_1 \left\|\Pi\tilde{x}(k)\right\| + \kappa_2, \tag{6}$$

where $\kappa_1 > 0$ and $\kappa_2 > 0$ are Lipschitz constants.

When compared with the Lipschitz condition defined in (3), it is observed that the first order term $\Lambda(k)\tilde{x}(k)$ is removed from the left part of (6). This $\Lambda(k)\tilde{x}(k)$ will be used to characterise the propagation of the error dynamic system latter. Moreover, the additional constant κ_2 is included to account for bounded uncertainties, see [22–24]. Then, the problem of fault estimation can be formulated as: consider a class of NDTV systems presented in (2), design a diagnosis observer to get fault estimate $\hat{\theta}(k)$, such that the observer is stable and the following fault estimation performance is fulfilled under Assumption 1.

$$\sup_{\tilde{x}_0,w,v} \frac{\sum_{i=0}^{k} \tilde{\theta}^T(i)\tilde{\theta}(i)}{\tilde{x}_0^T\tilde{x}_0 + \sum_{i=0}^{k}(w^T(i)w(i) + v^T(i)v(i))} \le \gamma_1^2, \tag{7}$$

where $\tilde{\theta}(k) = \theta(k) - \hat{\theta}(k)$, $\tilde{x}_0$ is the initial error, and $\gamma_1 > 0$ is a given constant.

So far, in most of the existing studies, the fault estimation problem will be solved by synthesis of a single fault estimation observer that fulfills both stability and estimation performance conditions. Unfortunately, the multiple synthesis conditions could be infeasible in the general case. Therefore, as is illustrated in Fig. 1, a two step

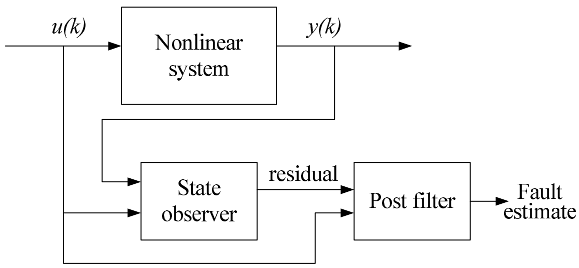


Fig. 1. Fault estimation via state observer and post filter.

design scheme is proposed in this paper. In the first step, regardless of the problem of fault estimation, a nonlinear state observer is designed. Then in the second step, for the error dynamic system of the nonlinear state observer, a fault estimation filter that fulfills a predefined estimation performance is constructed to accomplish the diagnosis work. In this way, the problem of infeasibility resulting from multiple synthesis conditions can be solved. Moreover, individual design of the observer and the post filter can be carried out much easier. In the next section, design of the nonlinear state observer and the post filter will be presented in details.

Remark 1: It should be noted that the Lipschitz-like condition presented in Assumption 1 is different from the Lipschitz-like condition adopted in [22–24]. In these studies, Lipschitz-like conditions are adopted with a predefined constant matrix Λ. While in this study, $\Lambda(k)$ is obtained from (5). Similar to the well-known Extended Kalman Filter, $\Lambda(k)\tilde{x}(k)$ is the first order approximation of $\phi(\Pi x(k)) - \phi(\Pi\hat{x}(k))$, and it is fully used during the estimation.

3. MAIN RESULTS

3.1. Nonlinear state observer design

Following the idea about how to stabilize the nonlinear observer presented in [18, 25, 26] , in this section, a nonlinear state observer is proposed for system (2) under Assumption 1. The observer is presented as follows:

$$\begin{cases} \hat{x}(k) = \hat{x}(k|k-1) + K(k)r(k), \\ r(k) = y(k) - C(k)\hat{x}(k|k-1), \\ \hat{x}(k+1|k) = U(k)\hat{x}(k) + \Gamma\phi(\Pi\hat{x}(k)) + B(k)u(k), \\ \hat{x}(0|-1) = \hat{x}_0, \end{cases} \tag{8}$$

where $\hat{x}(k|k-1)$ is the state prediction prior to $y(k)$, and $\hat{x}(k)$ is the state estimation after $y(k)$ is available, $\hat{x}_0$ is the initial guess, $r(k)$ is the residual, $K(k)$ is the observer gain to be designed. Subtracting (8) from (2), the following error dynamic systems is obtained.

$$\begin{cases} \tilde{x}(k+1|k) = A(k)(I - K(k)C(k))\tilde{x}(k|k-1) \\ \qquad\qquad + \Gamma\tilde{\phi}(k) + B_f(k)\theta(k) + B_w(k)w(k) \\ \qquad\qquad - A(k)K(k)v(k), \\ r(k) = C(k)\tilde{x}(k|k-1) + v(k), \end{cases} \tag{9}$$

where $\tilde{x}(k|k-1) = x(k) - \hat{x}(k|k-1)$, $\tilde{x}(k) = x(k) - \hat{x}(k)$ and

$$A(k) = U(k) + \Gamma\Lambda(k). \tag{10}$$

Define $e(k) = \Pi\tilde{x}(k)$ and let $M_{e\tilde{\phi}}$ represent the mapping from $\tilde{\phi}(k)$ to $e(k)$, which can be obtained from the error dynamic system (9). Then a sufficient condition for the error dynamic system (9) being stable is presented in the following lemma.

Lemma 1: Under Assumption 1, if the observer gain $K(k)$ is selected so that

$$\|M_{e\tilde{\phi}}\|_\infty < \frac{1}{\kappa_1}, \tag{11}$$

then the error dynamic system (9) of the nonlinear observer (8) is stable.

Proof: Set $\Delta(k) = \kappa_1\|\Pi\tilde{x}(k)\|$, $\bar{\phi}(k) = \Delta(k) + \kappa_2$. According to (6), there always exists a mapping $M_{\tilde{\phi}\bar{\phi}}$ such that $\tilde{\phi} = M_{\tilde{\phi}\bar{\phi}}\bar{\phi}$ and $\|M_{\tilde{\phi}\bar{\phi}}\|_\infty \leq 1$. Therefore, the structure of the error dynamic system (9) can be illustrated in Fig. 2. It should be noted that, regardless of the presence of an exact expression for $\tilde{\phi}(k)$, $\|\Delta(k)\|_\infty\|M_{\tilde{\phi}\bar{\phi}}\|_\infty \leq \kappa_1$ is true due to the Lipschitz-like condition in (6).

Similar to the proof of Theorem 4 in [26], a straightforward dissipativity argument can then be used to show that $M_{e\tilde{\phi}}$ and $\Delta(k)M_{\tilde{\phi}\bar{\phi}}$ are dissipative with the supply rates $\omega_1 = -e^Te + \|M_{e\tilde{\phi}}\|_\infty^2\tilde{\phi}^T\tilde{\phi}$ and $\omega_2 = -\tilde{\phi}^T\tilde{\phi} + \|\Delta(k)\|_\infty^2\|M_{\tilde{\phi}\bar{\phi}}\|_\infty^2 e^Te$, respectively. Denoting S_1 and S_2 as their corresponding storage functions, it follows that $S_1 + aS_2$, $a > 0$ is a Lyapunov function for the system. Then if $\|M_{e\tilde{\phi}}\|_\infty\|\Delta(k)\|_\infty\|M_{\tilde{\phi}\bar{\phi}}\|_\infty < 1$, which is fulfilled when (11) holds true, the system is asymptotically stable. □

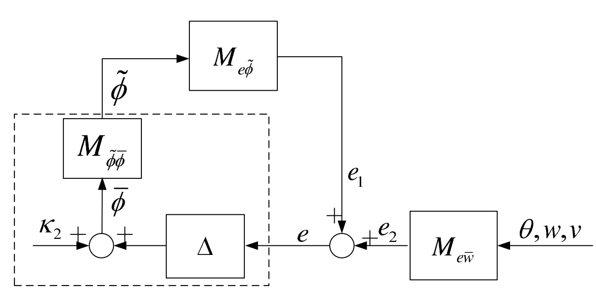


Fig. 2. Structure of the error dynamic system.

Now the observer gain $K(k)$ can be selected accordingly. By applying the H_∞ filtering approach, solution of the observer gain $K(k)$ is presented in the following proposition.

Proposition 1: For the nonlinear system (2) and its observer (8), Assumption 1 is adopted. Choose the observer gain $K(k)$ as

$$K(k)=P(k)C^T(k)(C(k)P(k)C^T(k)+I)^{-1}, \tag{12}$$

and $P(k)$ is obtained from the following Riccati equation

$$\begin{cases} P(k+1)=A(k)P(k)A^T(k)+\Gamma\Gamma^T \\ \qquad\qquad -A(k)P(k)M(k)P(k)A^T(k), \\ P(0)=I, \end{cases} \tag{13}$$

where

$$M(k)=\begin{bmatrix} C^T(k) & \Pi^T \end{bmatrix} R_e^{-1}(k)\begin{bmatrix} C(k) \\ \Pi \end{bmatrix},$$
$$R_e(k)=\begin{bmatrix} C(k) \\ \Pi \end{bmatrix} P(k)\begin{bmatrix} C^T(k) & \Pi^T \end{bmatrix}+\begin{bmatrix} I & 0 \\ 0 & -\gamma_2^2 I \end{bmatrix}.$$

Then if inequalities

$$\Pi(C(k)C^T(k)+P^{-1}(k))^{-1}\Pi^T-\gamma_2^2 I<0, \tag{14}$$
$$(C(k)P(k)C^T(k)+I)>0 \tag{15}$$

hold true for all $k\geq 0$, and $\kappa_1\cdot\gamma_2<1$, it is concluded that the nonlinear observer (8) is stable.

Proof: In [28], the H_∞ filtering problem is solved by looking for the minimum of an indefinite quadratic from in the Krein space. Theorem 1 and the Corollary 1 in [28] show that the inequalities (14) and (15) are the equivalent sufficient and necessary condition for the selected $K(k)$ to fulfill

$$\sup_{\tilde{x}_0,\tilde{\phi}}\frac{\sum_{i=0}^{k}\tilde{x}^T(i)\Pi^T\Pi\tilde{x}(i)}{\tilde{x}_0^T\tilde{x}_0+\sum_{i=0}^{k}\tilde{\phi}^T(i)\tilde{\phi}(i)}\leq\gamma_2^2.$$

That is, $\|M_{e\tilde{\phi}}\|_\infty\leq\gamma_2$.

On the other hand, Lemma 1 shows that the sufficient condition of a stable observer is (11). Together with $\kappa_1\cdot\gamma_2<1$, the proof is completed. □

Remark 2: In existing studies, observer synthesis for Lipschitz nonlinear systems is usually accomplished by solving LMIs. These methods will become invalid when there are time varying terms in systems. While in this paper, observer is designed by combinative application of the small gain theorem and the H_∞ filtering approach. This new method makes observer design for nonlinear time varying systems available.

3.2. Post filter design for fault estimation

Now that a stable error dynamic system of the observer is established, design of the post filter becomes much easier. That is, consider a stable linear time varying system presented in (9), design a fault estimation filter so that some H_∞ performance is fulfilled. Since the actuator fault is modeled as an unknown parameter, a simple way to estimate the fault is to augment the unknown parameter as an additional state, then fault estimate can be obtained by applying standard H_∞ filtering method. In the case that the unknown parameter is time varying, the proportional multiple integral (PMI) observer proposed in [29] can be used and the fault estimation work can be accomplished in a similar way. For easy presentation, the unknown constant parameter model is adopted in this section to illustrate the design of the post filter.

Augment $\theta(k)$ as a new state, then system (9) can be rewritten as

$$\begin{cases} \eta(k+1)=\bar{A}(k)\eta(k)+\bar{B}_w(k)\bar{w}(k), \\ r(k)=\bar{C}(k)\eta(k)+v(k), \end{cases} \tag{16}$$

where

$$\eta(k)=\begin{pmatrix} \tilde{x}^T(k|k-1) & \theta^T(k) \end{pmatrix}^T,$$
$$\bar{w}(k)=\begin{pmatrix} \tilde{\phi}^T(k) & w^T(k) & v^T(k) \end{pmatrix}^T,$$

and

$$\bar{A}(k)=\begin{bmatrix} A(k)(I-K(k)C(k)) & B_f(k) \\ 0 & I \end{bmatrix}, \tag{17}$$
$$\bar{B}_w(k)=\begin{bmatrix} \Gamma & B_w(k) & -A(k)K(k) \\ 0 & 0 & 0 \end{bmatrix}, \tag{18}$$
$$\bar{C}(k)=\begin{bmatrix} C(k) & 0 \end{bmatrix}. \tag{19}$$

The post filter is constructed as follows:

$$\begin{cases} \hat{\eta}(k)=\hat{\eta}(k|k-1)+L(k)\tilde{r}(k), \\ \hat{\theta}(k)=C_\theta\hat{\eta}(k), \\ \tilde{r}(k)=r(k)-\bar{C}(k)\hat{\eta}(k|k-1), \\ \hat{\eta}(k+1|k)=\bar{A}(k)\hat{\eta}(k), \\ \hat{\eta}(0|-1)=0, \end{cases} \tag{20}$$

where $\hat{\eta}(k|k-1)$ is the state prediction prior to $r(k)$, and $\hat{\eta}(k)$ is the state estimation when $r(k)$ is available, $\tilde{r}(k)$ is the prediction error of $r(k)$, $L(k)$ is the filter gain to be designed, and

$$C_\theta=\begin{bmatrix} 0 & I_p \end{bmatrix}. \tag{21}$$

Use the H_∞ filtering approach again, the following proposition can be obtained.

Proposition 2: For the augmented system (16) and its filter (20), the filter gain $L(k)$ is selected as

$$L(k)=\Sigma(k)\bar{C}^T(k)(\bar{C}(k)\Sigma(k)\bar{C}^T(k)+I)^{-1}, \tag{22}$$

and $\Sigma(k)$ is obtained from the following Riccati equation

$$\begin{cases}\Sigma(k+1)=\bar{A}(k)\Sigma(k)\bar{A}^T(k)+\bar{B}_w(k)\bar{B}_w(k)^T\\ \qquad\qquad -\bar{A}(k)\Sigma(k)\bar{M}(k)\Sigma(k)\bar{A}^T(k),\\ \Sigma(0)=I,\end{cases} \tag{23}$$

where

$$\bar{M}(k)=\left[\bar{C}^T(k)\quad C_\theta^T\right]\bar{R}_e^{-1}(k)\begin{bmatrix}\bar{C}(k)\\ C_\theta\end{bmatrix}$$

$$\bar{R}_e(k)=\begin{bmatrix}\bar{C}(k)\\ C_\theta\end{bmatrix}\Sigma(k)\left[\bar{C}^T(k)\quad C_\theta^T\right]+\begin{bmatrix}I & 0\\ 0 & -\gamma_1^2 I\end{bmatrix}.$$

Then if and only if

$$C_\theta(\bar{C}(k)\bar{C}^T(k)+\Sigma^{-1}(k))^{-1}C_\theta^T-\gamma_1^2 I<0, \tag{24}$$

$$(\bar{C}(k)\Sigma(k)\bar{C}^T(k)+I)>0 \tag{25}$$

hold true for all $k\geq 0$, the following H_∞ performance is fulfilled.

$$\sup_{\tilde{x}_0,\tilde{\phi},w,v}\frac{\sum_{i=0}^k\tilde{\theta}^T(i)\tilde{\theta}(i)}{\tilde{x}_0^T\tilde{x}_0+\sum_{i=0}^k(\bar{w}^T(i)\bar{w}(i)+v^T(i)v(i))}\leq\gamma_1^2. \tag{26}$$

Proposition 2 follows from Theorem 1 and Corollary 1 presented in [28] directly. It should be noted that there is a subtle difference between the achieved H_∞ performance specified in (26) and the one presented in (7), because $\tilde{\phi}(k)$ is incorporated in $\bar{w}(k)$. Nevertheless, it can be regarded as a suboptimal solution for the problem of fault estimation. When $\theta(k)$ is time varying, finite time derivatives of the fault should be augmented as the new states together, then by suitable modification of $\bar{A}(k)$, $\bar{B}_w(k)$, $\bar{C}(k)$ and C_θ, the filter gain $L(k)$ and the fault estimate $\hat{\theta}(k)$ can be obtained in a similar way.

3.3. Fault estimation algorithm

As is depicted in Fig. 1, the proposed fault estimation method is divided into two stages. At the first stage, the nonlinear state observer is applied for state estimation for the nonlinear control system. The control input $u(k)$ and system output $y(k)$ are utilized as the input of the observer, and the residual $r(k)$ is generated. At the second stage, the post filter designed for the error dynamic system of the observer is applied to estimate the fault. The residual $r(k)$ and the control input $u(k)$ are used as the input of the filter, then fault estimate $\hat{\theta}(k)$ is obtained. The fault estimation algorithm based on the nonlinear state observer and its post filter is concluded as follows:

Remark 3: The computational complexity of the proposed algorithm is $O\left((n+p)^3\right)$ where n is the dimension of the state and p is the dimension of the fault. The computational load grows as the fault is augmented as the additional state for fault estimation. Fortunately, the comutational load dose not grow in an exponential way, and the algrithm is still applicable. Moreover, when the idea of the PMI observer is used to improve the estimation performance for time varying faults, the growth of the computational load will be significant. A trade-off should be made between the estimation performance and the computational load.

Algorithm 1: Nonlinear state observer and dynamic post filter based fault estimation

Step 1: Let $k=0$, initialize the nonlinear state observer and the post filter, set $\hat{x}(0|-1)=\hat{x}_0$, $\hat{\eta}(0)=0$, $P(0)=I$, $\Sigma(0)=I$. Choose a suitable γ_1, and a large enough γ_2 such that $\gamma_2\cdot\kappa_1<1$

Step 2: Check whether inequalities (14) and (15) are fulfilled. If yes, goto **Step 3**, otherwise stop the algorithm.

Step 3: Calculate $K(k)$ according to (12).

Step 4: Calculate $r(k)$, $\hat{x}(k)$ and $\hat{x}(k+1|k)$ according to the nonlinear observer (8).

Step 5: Construct $A(k)$ according to (10) and (5), then update $P(k+1)$ according to (13).

Step 6: Construct $\bar{C}(k)$, C_θ according to (19) and (21), respectively, and check whether inequalities (24) and (25) are fulfilled. If yes, goto **Step 7**, otherwise stop the algorithm, choose a larger γ_1 and restart the algorithm.

Step 7: Calculate $L(k)$ according to (22).

Step 8: Construct $\bar{A}(k)$, $\bar{B}_w(k)$ according to (17) and (18), respectively, then update $\Sigma(k+1)$ according to (23).

Step 9: Calculate $\tilde{r}(k)$, $\hat{\eta}(k)$, $\hat{\eta}(k+1|k)$, and get the fault estimate $\hat{\theta}(k)$ according to the filter (20).

Step 10: Let $k=k+1$, goto **Step 2** until the end of the algorithm.

4. SIMULATION STUDY

4.1. Description of UAV model

To evaluate the performance of the proposed method, the longitudinal model of a small UAV is employed. In the model, V, α, q, ψ and H represent the air speed, angle of attack, pitch rate, pitch angle and altitude of the UAV. δ_e and δ_p are the elevator deflection and throttle setting. w_x, w_z are the wind speed in the horizonal and vertical direction, and g_x, g_z are the wind gradients. Set

$$x(k)=[V\quad\alpha\quad q\quad\psi\quad H]^T,\ u(k)=[\delta_e\quad\delta_p]^T,$$
$$w(k)=[w_z\quad g_x\quad g_z]^T,\ y(k)=[V\quad q\quad\psi\quad H]^T,$$

a NDTV model in a similar form with (2) is established, specifically,

$$U(k)=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0.0141T_s & 1 & 1.0201T_s & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & T_s & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix},$$

$$B_w(k)=T_s\begin{bmatrix}0 & -\cos(\psi-\alpha) & -\sin(\psi-\alpha)\\ 0 & -\frac{\sin(\psi-\alpha)}{V} & \frac{\cos(\psi-\alpha)}{V}\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 1 & 0 & 0\end{bmatrix},$$

$$\Gamma=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&1\end{bmatrix},\quad \Pi=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\end{bmatrix},$$

$$C(k)=\begin{bmatrix}1&0&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix},\quad B(k)=T_s\begin{bmatrix}0 & B_3\\ B_1 & B_4\\ B_2 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix},$$

$$\phi(\Pi x(k))=T_s[\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4 \quad \phi_5]^T,$$

where $B_1=-0.0009V$, $B_2=-0.0115V^2\alpha^2+0.0002V^2\alpha-0.00005V^2$, $B_3=1.4191\cos\alpha$, $B_4=1.4191\frac{\sin\alpha}{V}$, and $\phi_1=(-0.0251\alpha^2-0.0013\alpha-0.0014)V^2$, $\phi_2=-0.1069V\alpha+9.8\frac{\cos(\psi-\alpha)}{V}$, $\phi_3=-0.0101V^2\alpha-0.0372Vq$, $\phi_4=0$, $\phi_5=V\sin(\psi-\alpha)$, and T_s is the sampling time. It should be noted that during the flight, $V(k)\neq 0$, and $B(k)$, $B_w(k)$ in the above model are nonlinear functions of the state. To simplify the investigation, they are treated as know matrices and identified with the state estimates.

Simulations are carried out under closed-loop control and the UAV model reaches a steady state. The simulation time is 100s, and the sampling time $T_s=0.1$s. Turbulent condition is considered and shown in Fig. 3. The measurement noises are simulated as Gaussian signals with zero means, and their standard deviations are set to be 0.01. The initial condition is $x_0=[24.2 \quad 0.1 \quad 0.2 \quad 0.01 \quad 201]^T$, $\hat{x}_0=[24 \quad 0 \quad 0 \quad 0 \quad 200]^T$. Moreover, the Lipschitz constants are estimated with simulation data, specifically, we chose $\kappa_1=0.5$ and $\kappa_2=0.1$. γ_1 and γ_2 are set to be $\gamma_1^2=8$, $\gamma_2^2=3$.

4.2. Fault diagnosis results

To compare the fault diagnosis performance, both of the proposed method and the residual evaluation based approach are carried out in the simulation study. In the residual evaluation based approach, $r(k)$ generated from the state observer (8) is employed together with the following evaluation function.

$$J_N(k)=\frac{1}{N+1}\sum_{j=k-N}^{k} r^T(j)r(j),$$

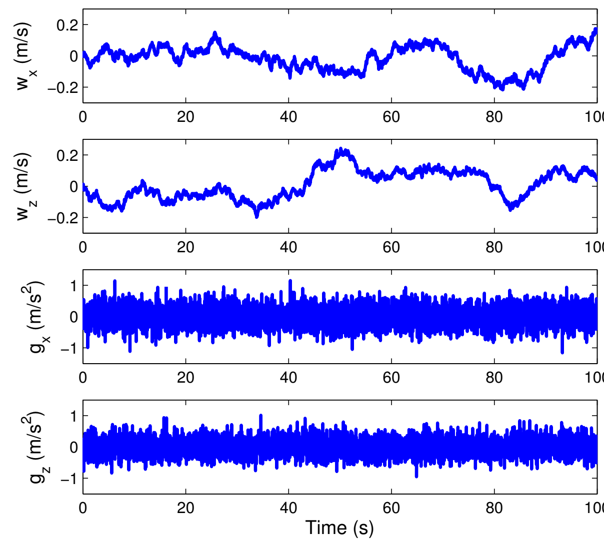


Fig. 3. Simulated turbulent condition.

where $N=50$ is the length of the sliding window. The Threshold is set as $J_{th}=\sup J_{N,0}(k)$, where $J_{N,0}(k)$ is $J_N(k)$ obtain from fault free cases. Fault alarms are triggered according to the following logic:

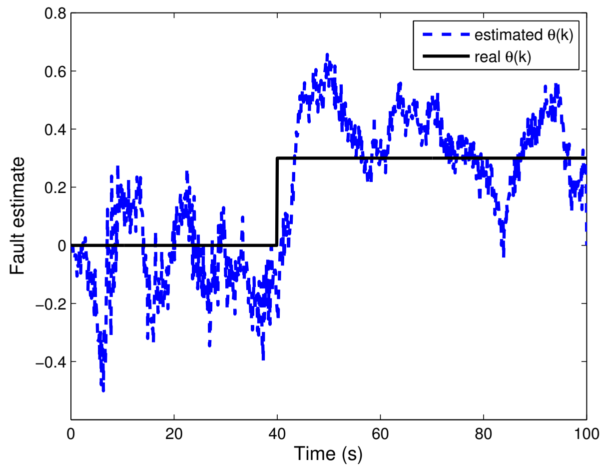
$$\begin{cases} J_N(k)>J_{th} \Longrightarrow \text{fault alarm},\\ J_N(k)\leq J_{th} \Longrightarrow \text{fault free}.\end{cases}$$

Two actuator fault scenarios are considered. In the first scenario, loss of control effectiveness of the elevator is modeled as $\delta_{e,f}(t)=0.7\cdot\delta_{e,o}(t)$ when $t>40s$, where the subscript f and o indicate the faulty value and the nominal value, respectively. Diagnosis results of the elevator fault are show in Fig. 4. It is depicted in Fig. 4: (a) that the fault estimation method proposed in this paper can accomplish the estimation work and the elevator fault is successfully detected. While in Fig. 4: (b), the residual evaluation based approach almost fails to detect the fault. Moreover, the calculation result shows

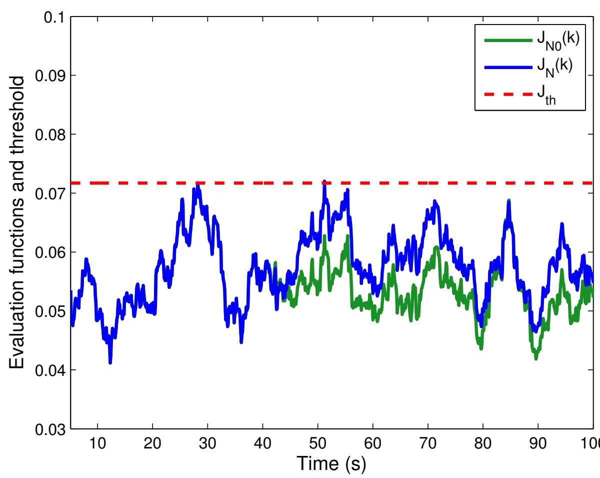
$$\frac{\sum_{i=0}^{k}\tilde{\theta}^T(i)\tilde{\theta}(i)}{\tilde{x}_0^T\tilde{x}_0+\sum_{i=0}^{k}(w^T(i)w(i)+v^T(i)v(i))}=1.1168\leq\gamma_1^2.$$

Loss of control effectiveness of the throttle is considered in the second scenario. The throttle fault is modeled as $\delta_{p,f}(t)=0.85\cdot\delta_{p,o}(t)$ when $t>40s$. Fault diagnosis results are show in Fig. 5. Similar to the elevator fault case, it is depicted in Fig. 5: (a) that the throttle fault is successfully detected from the fault estimates. While in Fig. 5: (b), the detection performance of the residual evaluation based method is poor. Only a transient alarm is triggered at about 40s after the fault occurs. Moreover, the calculation result shows

$$\frac{\sum_{i=0}^{k}\tilde{\theta}^T(i)\tilde{\theta}(i)}{\tilde{x}_0^T\tilde{x}_0+\sum_{i=0}^{k}(w^T(i)w(i)+v^T(i)v(i))}=0.5656\leq\gamma_1^2.$$

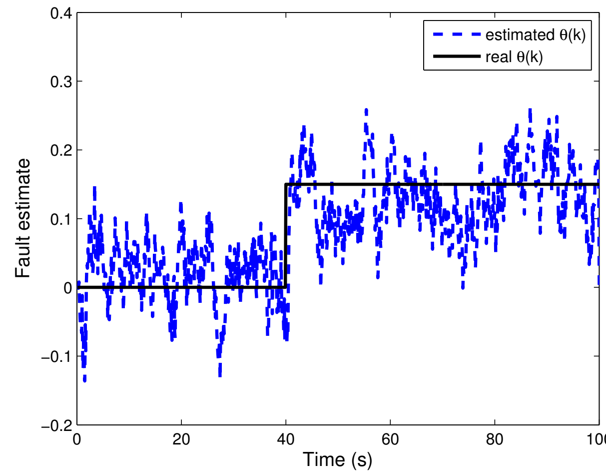


(a) Diagnosis result based on fault estimation.

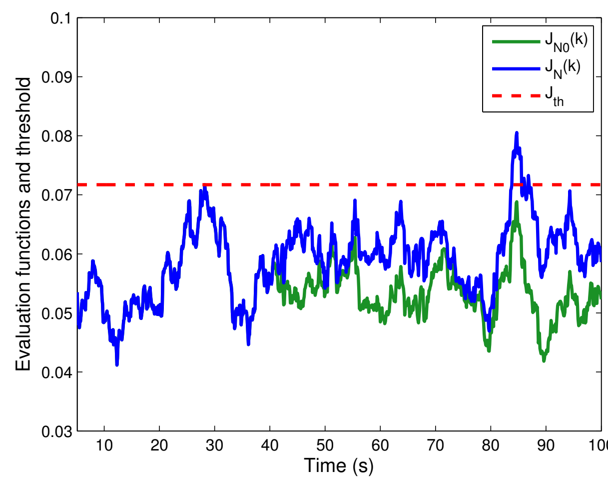


(b) Detection result based residual evaluation.

Fig. 4. Diagnosis results of the elevator fault.



(a) Diagnosis result based on fault estimation.



(b) Detection result based residual evaluation.

Fig. 5. Diagnosis results of the throttle fault.

The above results show that the fault estimation method outperforms the residual evaluation based approach. The reason of the advantage of the fault estimation method is interesting. In [30], it is stated that closed-loop systems are naturally robust to small faults. System responses to these faults are usually covered by controllers, thus small faults in closed-loop systems are hardly detectable. For a system reaching steady state under feedback control, the control input $u(k)$ is usually very small. When loss of control effectiveness of the actuator occurs, the equivalent additive fault $\Delta_u(k)$ (the fault is modeled in the additive manner) will be even smaller. The residual evaluation based approach tries to evaluate the change of the residual energy resulting from the tiny term $\Delta_u(k)$, thus often fails. Differently, in the fault estimation manner, actuator faults are modeled as unknown parameters to identify the scale of the ineffective part and the additional knowledge of the fault distribution matrices is used. Therefore, the fault estimation method is preferable for diagnosis of closed-loop systems.

5. CONCLUSION

The problem of actuator fault estimation for a kind of nonlinear control systems is investigated. A Lipschitz-like condition is introduced and the nonlinear state observer is designed with the help of the small gain theorem and the H_∞ filtering approach. No approximation is needed and this design approach is applicable for more general systems. Fault estimation is accomplished by a dynamic post filter. The problem of infeasibility resulting from multiple synthesis conditions is considerably improved. Moreover, the simulation results indicate that for the purpose of actuator fault diagnosis in closed-loop systems, the fault estimation method outperforms the residual evaluation based approach.

On the other hand, the introduced global Lipschitz-like condition is restrictive in applications. The local Lipschitz-like conditions is preferable and works to estimate Lipschitz constants should be done in future studies.

Furthermore, this work is limited to estimation of the actuator fault. Diagnosis of the sensor fault would be included in our future work.

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Hai Liu received his Ph.D. degree in navigation instrument and system technology from Beihang University, Beijing, China, in 2018. He is currently a research engineer in Information Science Academy of China Electronics Technology Group Corporation, Beijing, China. His research interests include model-based fault-diagnosis and application, unmanned systems, and multi-agent cooperation.

Maiying Zhong received her Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 1999. From 2002 to July 2008, she was a Professor in the School of Control Science and Engineering, Shandong University. From 2006 to 2007, she was a Postdoctoral Researcher Fellow with the Central Queensland University, Australia. From 2009 to 2016, she was a Professor in the School of Instrument Science and Opto-Electronics Engineering, Beihang University. In March 2016, she joined Shandong University of Science and Technology, Qingdao, China, where she is currently a Professor in the College of Electrical Engineering and Automation. Her research interests include model-based fault-diagnosis, fault-tolerant systems, and their applications.

Yang Liu received his B.E. and Ph.D. degrees in automation from Tsinghua University, Beijing, China, in 2010 and 2016, respectively. He is currently a Postdoctoral Research Fellow with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, China. His research interests include robust optimal fltering, closed-loop systems, and fault detection and diagnosis for modern systems.

Zuqiang Yang received his Ph.D. from Zhejiang University in Navigation, Guidance, and Control. Since 2016, he has become a research engineer in Information Science Academy of China Electronics Technology Group Corporation, Beijing, China. His interested research areas have been directed to bionic engineering, multi-agent cooperation, machine learning, etc.