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A Further Result on Global Stabilization of a Class of Nonlinear Systems by Output Feedback with Unknown Measurement Sensitivity

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Abstract: In this paper, we provide a further result on the output feedback control problem of nonlinear systems with unknown measurement sensitivity studied in [2]. We provide a gain-scaling output feedback controller and a process of determining the allowed measurement sensitivity. As a result, we show that more nonlinearity and measurement sensitivity can be treated by our method. Various examples are presented to illustrate the improved features of our control method.

Keywords: Global stabilization, nonlinear systems, output feedback, perturbed nonlinearity, unknown measurement sensitivity.

1. INTRODUCTION

Output feedback control problems have received much attention and still remain as active research topics as of now [1–11]. For example, in [1], the problem of output feedback stabilization of a class of nonlinear systems which may have unstable zero dynamics was studied. In [3], under a lower triangular linear growth condition, sample-data output feedback controller with an appropriate sampling period guaranteed global asymptotical stabilization. In [5], an adaptive output feedback control scheme was proposed for triangular or feedforward nonlinear systems with unknown linear growth rate. In [8], the problem of global robust stabilization by output feedback was investigated for uncertain systems with polynomial nonlinearity. In [11], the problem of global asymptotic stabilization by sampled-data output feedback was investigated for a class of nonminimum-phase nonlinear systems under Lipschitz condition.

One common feature of all aforementioned results is that they all assumed 'clean' feedback channel, i.e., none considered a case in which there may be some measurement sensitivity, noise, or disturbance in feedback channel. In these regards, there have been several control results considering various uncertainties in feedback. In [12–14], fuzzy controls for nonlinear systems under unreliable communication links and the restricted transmission capacity of communication network were studied. In [12], network-based fuzzy control method for nonlinear Markov jump systems under unreliable communication links was proposed. In [13], the problem of quantized feedback control of a class of nonlinear Markov jump systems was addressed. In [14], the sliding-mode control problem of Takagi-Sugeno fuzzy multiagent systems is studied. However, they did not consider the measurement sensitivity issue and these results are about full state feedback controls. Also, some other results with noise in feedback are reported in [15,16]. In [16], their control purpose was to robustly minimize the upper bound of the estimated error. In [15], they considered a measurement feedback problem for feedforward nonlinear systems. However, their considered cases are about uncertain measurement vector and sensor noise entering the feedback channel in an additive form.

Recently, a new output feedback control result which related to measurement sensitivity appeared in [2]. In [2], they considered a form of $y = \theta(t)x_1$, that is 'normal' state is fluctuated by the uncertain measurement sensitivity function. One limitation of [2] is that the allowed measurement sensitivity is rather conservative due to the use of norm-bound condition in the analysis. We find that this limitation can be much relaxed by utilizing a matrix inequality structure and we will introduce a matrix inequality approach which allows more measurement sensitivity.

In this paper, we significantly extend the result of [2] in mainly two ways: (i) Using a new analysis with a matrix inequality approach, we show that much more uncertain measurement sensitivity can be allowed; (ii) The nonlinearity in consideration is also further extended in an output feedback control scheme. Careful comparisons in terms of

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both analysis and examples are provided in order to illustrate the clear advantage of our control method.

2. PROBLEM STATEMENT

Their considered nonlinear system with measurement sensitivity is expressed as follows.

$$\begin{aligned}\dot{x} &= Ax + Bu + \phi(t,x,u),\\ y &= C^{\theta(t)}x,\end{aligned} \tag{1}$$

where $x = [x_1, \cdots, x_n]^T \in R^n$ is the system state, $u \in R$ is the control input, and $y \in R$ is the measurement output. The perturbed nonlinearity is $\phi(t,x,u) = [\phi_1(t,x,u), \cdots, \phi_n(t,x,u)]^T : R \times R^n \times R \to R^n$ where each $\phi_i(t,x,u)$, $i = 1, \cdots, n$ is continuous, (A, B) is the Brunovsky canonical pair and $C^{\theta(t)} = [\theta(t), 0, \cdots, 0]$. The measurement sensitivity $\theta(t)$ is a bounded and unknown continuous function of time. Notably, $\theta(t)$ does not need to be differentiable. Moreover, the nominal value is $\theta(t) = 1$. So, $\theta(t)$ can be written as $\theta(t) = 1 + \delta\theta(t)$.

Regarding nonlinearity of the system (1), the following two conditions are considered in [2].

(A1) Lower triangular condition: For $i = 1, \cdots, n$, there exists a constant $c \geq 0$ such that

$$|\phi_i(t,x,u)| \leq c(|x_1| + \cdots + |x_i|). \tag{2}$$

(A2) Upper triangular condition : For $i = 1, \cdots, n-2$, there exists a constant $c \geq 0$ such that

$$|\phi_i(t,x,u)| \leq c(|x_{i+2}| + \cdots + |x_n|), \tag{3}$$

and $\phi_{n-1}(t,x,u) = \phi_n(t,x,u) = 0$.

The main result of [2] is: They showed that (i) the system (1) can be globally asymptotically stabilized by an output feedback under either (A1) or (A2); (ii) some nontrivial amount of $\delta\theta(t)$ can be treated as well.

Now, here is the formal statement of our control problem: *We consider the output feedback stabilization of the system* (1) *and we aim to show that*

(i) *More nonlinearity over (A1) and (A2) can be allowed.*

(ii) *Significantly larger magnitude of measurement sensitivity, i.e., $\delta\theta(t)$ can be treated as well.*

In order to solve our control problem, our approaches are: (i) determination process of obtaining a compact set containing the allowed measurement sensitivity is provided by utilizing a Lyapunov equation and matrix inequalities; (ii) a gain-scaling output feedback controller is designed and applied to guarantee a global asymptotic stabilization; (iii) the proposed output feedback control scheme is applied to two numerical examples and one practical example with more measurement sensitivity and nonlinearity for clear illustration.

3. MAIN RESULTS

Note that we consider the system (1) where the same condition is imposed upon $\theta(t) = 1 + \delta\theta(t)$ which belongs to a compact set Ω_θ. We introduce the following condition on the perturbed nonlinearities $\phi(t,x,u)$.

Assumption 1: [4] There exists a function $\gamma(\varepsilon) \geq 0$ such that for $\varepsilon > 0$,

$$\sum_{i=1}^{n} \varepsilon^{i-1}|\phi_i(t,x,u)| \leq \gamma(\varepsilon)\sum_{i=1}^{n} \varepsilon^{i-1}|x_i|. \tag{4}$$

Remark 1: In [4], it is already shown that Assumption 1 is more general than either (A1) or (A2). Under (A1), it could be that $\gamma(\varepsilon) = c(1 + \varepsilon + \cdots + \varepsilon^{n-1})$. Under (A2), it could be that $\gamma(\varepsilon) = c(\varepsilon^{-2} + \cdots + \varepsilon^{-(n-1)})$. Moreover, Assumption 1 includes ‘other type’ of nonlinearity besides (A1) and (A2). Thus, if the system (1) is shown to be globally asymptotically stabilized by output feedback under Assumption 1, our result naturally includes more nonlinearity over [2].

First, we provide an output feedback controller with a gain-scaling factor ε given by

$$u = K(\varepsilon)z, \tag{5}$$

$$\dot{z} = Az + Bu - L(\varepsilon)(y - Cz), \tag{6}$$

where $z = [z_1, \cdots, z_n]^T \in R^n$, $K(\varepsilon) = [k_1/\varepsilon^n, \cdots, k_n/\varepsilon]$, $L(\varepsilon) = [l_1/\varepsilon, \cdots, l_n/\varepsilon^n]^T$, $\varepsilon > 0$, and $C = [1, 0, \cdots, 0]$.

Before stating the main result, we provide some mathematical setups for clear presentation. Define the observer error $e = [e_1, \cdots, e_n]^T$, $e_i = x_i - z_i$, $1 \leq i \leq n$. By subtracting (6) from (1) and with the controller (5), we have

$$\begin{aligned}\dot{e} &= Ae + L(\varepsilon)Ce + L(\varepsilon)(C^{\theta(t)} - C)x + \phi(t,x,u)\\ &= A_{L(\varepsilon)}e + L(\varepsilon)(C^{\theta(t)} - C)x + \phi(t,x,u),\end{aligned} \tag{7}$$

where $A_{L(\varepsilon)} = A + L(\varepsilon)C$.

From (1) and (5), we obtain

$$\begin{aligned}\dot{x} &= Ax + BK(\varepsilon)(x - e) + \phi(t,x,u)\\ &= A_{K(\varepsilon)}x - BK(\varepsilon)e + \phi(t,x,u),\end{aligned} \tag{8}$$

where $A_{K(\varepsilon)} = A + BK(\varepsilon)$.

From (7) and (8), we have an augmented closed-loop system as

$$\begin{bmatrix}\dot{e}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}A_{L(\varepsilon)} & L(\varepsilon)(C^{\theta(t)} - C)\\ -BK(\varepsilon) & A_{K(\varepsilon)}\end{bmatrix}\begin{bmatrix}e\\ x\end{bmatrix} + \begin{bmatrix}\phi(t,x,u)\\ \phi(t,x,u)\end{bmatrix}. \tag{9}$$

Next, some notations are provided to be used throughout the paper for convenience.

Notations: For any matrix $M^T = M$, $\lambda_{\min}(M)$ denotes the minimum eigenvalue of M. $\|x\|$ denotes the Euclidean norm. Other norms will be denoted by their subscripts. Define $K = K(1)$, $L = L(1)$, $A_K = A_{K(1)}$, $A_L = A_{L(1)}$, and let

$$\tilde{A}^{\theta(t)}_{K(\varepsilon)L(\varepsilon)} = \begin{bmatrix} A_{L(\varepsilon)} & L(\varepsilon)(C^{\theta(t)} - C) \\ -BK(\varepsilon) & A_{K(\varepsilon)} \end{bmatrix} \in R^{2n\times 2n}, \tag{10}$$

$$\tilde{A}_{KL} = \begin{bmatrix} A_L & 0 \\ -BK & A_K \end{bmatrix} \in R^{2n\times 2n}. \tag{11}$$

Now, the following key procedure are introduced with respect to unknown measurement sensitivity.

Determination process of Ω_θ:

(i) Select K and L such that $\tilde{A}_{KL}$ is Hurwitz. Let $M = \text{diag}[m_1, \cdots, m_{2n}] > 0$. There exists a positive definite symmetric matrix P such that $(\tilde{A}_{KL})^T P + P\tilde{A}_{KL} = -M$ where $P = [p_{i,j}] \in R^{2n\times 2n}, i = 1, \cdots, 2n, j = 1, \cdots, 2n$.

(ii) Compute a symmetric matrix $Q(t)$ such that

$$(\tilde{A}^{\theta(t)}_{KL})^T P + P\tilde{A}^{\theta(t)}_{KL} = -Q(t). \tag{12}$$

(iii) Determine the compact set Ω_θ which contains all ranges of $\theta(t)$ as long as $Q(t) > 0$, $\forall t \geq 0$. Then, the allowed measurement sensitivity $\theta(t)$ is contained in Ω_θ.

Remark 2: The actual computation of the above process can easily be done by computer. Of course, it is important to demonstrate that the obtained set Ω_θ is indeed significantly larger than the one shown in [2]. So, in Section 4, direct comparisons will be made for cases of system dimension $n = 2, 3, 4$ as done in [2].

Theorem 1: Suppose (i) Assumption 1 holds; (ii) the determination process of Ω_θ is followed. There exists $\varepsilon > 0$ such that the following holds

$$\Delta(\varepsilon) := \begin{bmatrix} 1 & -\varepsilon\gamma(\varepsilon)\sigma \\ -\varepsilon\gamma(\varepsilon)\sigma & 1 - 2\varepsilon\gamma(\varepsilon)\sigma \end{bmatrix} > 0, \tag{13}$$

where $\sigma > 0$ is a finite constant.

Then, the system (1) is globally asymptotically stable with the controller (5)-(6).

Proof: Define $E_\varepsilon = \text{diag}[1, \varepsilon, \cdots, \varepsilon^{n-1}]$. Then, the following relations hold for all $\varepsilon > 0$:

$$E_\varepsilon^{-1} A_L E_\varepsilon = \varepsilon A_{L(\varepsilon)}, \tag{14}$$

$$E_\varepsilon^{-1} A_K E_\varepsilon = \varepsilon A_{K(\varepsilon)}, \tag{15}$$

$$E_\varepsilon^{-1} L(C^{\theta(t)} - C) E_\varepsilon = \varepsilon L(\varepsilon)(C^{\theta(t)} - C), \tag{16}$$

$$E_\varepsilon^{-1} BK E_\varepsilon = \varepsilon BK(\varepsilon). \tag{17}$$

Define $\tilde{E}_\varepsilon = \text{diag}[E_\varepsilon, E_\varepsilon]$. From (14)-(17), we obtain the equation

$$\tilde{E}_\varepsilon^{-1} \tilde{A}^{\theta(t)}_{KL} \tilde{E}_\varepsilon = \varepsilon \tilde{A}^{\theta(t)}_{K(\varepsilon)L(\varepsilon)}. \tag{18}$$

By substituting (18) into (12), we can derive a Lyapunov equation such as

$$(\tilde{A}^{\theta(t)}_{K(\varepsilon)L(\varepsilon)})^T P_\varepsilon + P_\varepsilon \tilde{A}^{\theta(t)}_{K(\varepsilon)L(\varepsilon)} = -\varepsilon^{-1} \tilde{E}_\varepsilon Q(t) \tilde{E}_\varepsilon, \tag{19}$$

where $P_\varepsilon = \tilde{E}_\varepsilon P \tilde{E}_\varepsilon > 0$.

A Lyapunov function is then set as

$$V(e,x) = \begin{bmatrix} e \\ x \end{bmatrix}^T P_\varepsilon \begin{bmatrix} e \\ x \end{bmatrix}. \tag{20}$$

Along the trajectory of (9), the time derivative of the Lyapunov function using (19) is as

$$\begin{aligned} \dot{V}(e,x) &= -\varepsilon^{-1} \begin{bmatrix} e \\ x \end{bmatrix}^T \tilde{E}_\varepsilon Q(t) \tilde{E}_\varepsilon \begin{bmatrix} e \\ x \end{bmatrix} \\ &\quad + 2\begin{bmatrix} e \\ x \end{bmatrix}^T \tilde{E}_\varepsilon P \tilde{E}_\varepsilon \begin{bmatrix} \phi(t,x,u) \\ \phi(t,x,u) \end{bmatrix} \\ &= -\varepsilon^{-1} \begin{bmatrix} E_\varepsilon e \\ E_\varepsilon x \end{bmatrix}^T Q(t) \begin{bmatrix} E_\varepsilon e \\ E_\varepsilon x \end{bmatrix} \\ &\quad + 2\begin{bmatrix} E_\varepsilon e \\ E_\varepsilon x \end{bmatrix}^T P \begin{bmatrix} E_\varepsilon \phi(t,x,u) \\ E_\varepsilon \phi(t,x,u) \end{bmatrix}. \end{aligned} \tag{21}$$

Note that there is the following relation

$$\begin{bmatrix} E_\varepsilon e \\ E_\varepsilon x \end{bmatrix}^T Q(t) \begin{bmatrix} E_\varepsilon e \\ E_\varepsilon x \end{bmatrix} \geq \alpha \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix}^T \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix}, \tag{22}$$

where $\alpha = \inf_{t\geq 0}\{\lambda_{\min}[Q(t)]\}$ is a positive real constant due to $Q^T(t) = Q(t) > 0$, $\forall t \geq 0$.

With (22), we have

$$\begin{aligned} \dot{V}(e,x) &\leq -\varepsilon^{-1}\alpha \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix}^T \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix} \\ &\quad + 2\|P\| \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix}^T \begin{bmatrix} \|E_\varepsilon \phi(t,x,u)\| \\ \|E_\varepsilon \phi(t,x,u)\| \end{bmatrix}. \end{aligned} \tag{23}$$

Let us investigate the norm bounds of $\|E_\varepsilon \phi(t,x,u)\|$. Under Assumption 1, we have

$$\begin{aligned} &\|E_\varepsilon \phi(t,x,u)\| \\ &\leq \|E_\varepsilon \phi(t,x,u)\|_1 \leq \gamma(\varepsilon)\|E_\varepsilon x\|_1 \leq \sqrt{n}\gamma(\varepsilon)\|E_\varepsilon x\|. \end{aligned} \tag{24}$$

Using (24), we obtain the following inequality

$$\begin{aligned} \dot{V}(e,x) &\leq -\varepsilon^{-1}\alpha \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix}^T \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix} \\ &\quad + 2\sqrt{n}\gamma(\varepsilon)\|P\| \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix}^T \begin{bmatrix} \|E_\varepsilon x\| \\ \|E_\varepsilon x\| \end{bmatrix} \\ &\leq -\varepsilon^{-1}\alpha \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} 1 & -\varepsilon\gamma(\varepsilon)\sigma \\ -\varepsilon\gamma(\varepsilon)\sigma & 1 - 2\varepsilon\gamma(\varepsilon)\sigma \end{bmatrix} \begin{bmatrix} \|E_\varepsilon e\| \\ \|E_\varepsilon x\| \end{bmatrix}, \end{aligned} \tag{25}$$

where $\sigma = \sqrt{n}\alpha^{-1}\|P\|$ which is a constant independent of ε.

Selecting ε as

$$\Delta(\varepsilon) := \begin{bmatrix} 1 & -\varepsilon\gamma(\varepsilon)\sigma \\ -\varepsilon\gamma(\varepsilon)\sigma & 1-2\varepsilon\gamma(\varepsilon)\sigma \end{bmatrix} > 0 \tag{26}$$

assures the global asymptotic stability of the controlled system. □

Remark 3: Recalling Remark 1, it can be seen that there always exist finite constants $\varepsilon_1, \varepsilon_2 > 0$ such that $\Delta(\varepsilon) > 0$ in (26) is satisfied for $0 < \varepsilon < \varepsilon_1$ for (A1) and $\varepsilon_2 < \varepsilon < \infty$ for (A2). For nonlinearity other than (A1) or (A2), we present Example 2 in Section 5. This clarifies that our result includes more nonlinearity over [2].

4. RELATION BETWEEN ALLOWED MEASUREMENT SENSITIVITY AND THE VALUES OF K, L, AND M

In this section, we show the conditions that $Q(t)$ remains positive definite following the selection of K, L, and M through cases of system dimension $n = 2,3,4$. Note that for comparison with [2], the selected K and L are as similar values to ones in [2] as possible even though the controller structure is different.

($n = 2$ case): From (12), $Q(t)$ is obtained as follows.

$$Q(t) = \begin{bmatrix} m_1 & 0 & -\rho_1\delta\theta(t) & 0 \\ * & m_2 & -\rho_2\delta\theta(t) & 0 \\ * & * & m_3 - 2\rho_3\delta\theta(t) & -\rho_4\delta\theta(t) \\ * & * & * & m_4 \end{bmatrix}, \tag{27}$$

where $\rho_1 = l_1p_{1,1} + l_2p_{1,2}$, $\rho_2 = l_1p_{1,2} + l_2p_{2,2}$, $\rho_3 = l_1p_{1,3} + l_2p_{2,3}$, and $\rho_4 = l_1p_{1,4} + l_2p_{2,4}$.

For $Q(t) > 0$ in (27), the following conditions must be satisfied.

- $m_1m_2m_3 - m_2\rho_1^2\delta\theta(t)^2 - m_1\rho_2^2\delta\theta(t)^2 - 2m_1m_2\rho_3\delta\theta(t) =: \star_1 > 0. \quad (28)$
- $m_4\star_1 - m_1m_2\rho_4^2\delta\theta(t)^2 > 0. \quad (29)$

In order to obtain the allowed measurement sensitivity using (28)-(29), the values of K and L are selected as $K = [-0.3, -1.5]$ and $L = [-18, -1.3]^T$ such that $\tilde{A}_{KL}$ is Hurwitz. Then, using (28)-(29), the following Table 1 is obtained depending on M. So, when $M = \text{diag}[0.1, 7, 0.4, 0.5]$ is selected from Table 1, the following range of $\theta(t)$ is obtained such as

$$0.3992(-60\%) < \theta(t)(\delta\theta(t)\%) < 3.4964(249\%). \tag{30}$$

Thus, the allowed measurement sensitivity is $\theta(t) = 1 + \delta\theta(t) \in [1-\underline{\theta}, 1+\overline{\theta}]$ where $\underline{\theta} = 0.6007$, $\overline{\theta} = 2.4963$.

Table 1. The allowed $\delta\theta(t)$ depending on M under K, L for system dimension $n = 2$.

$M = \text{diag}[1,1,1,1]$
$-0.3470 < \delta\theta(t) < 0.6180$
$M = \text{diag}[0.1,7,1,1]$
$-0.5400 < \delta\theta(t) < 1.7007$
$M = \text{diag}[0.1,7,0.4,0.5]$
$-0.6008 < \delta\theta(t) < 2.4964$
$M = \text{diag}[0.1,7,0.9,0.7]$
$-0.5433 < \delta\theta(t) < 1.7345$
$M = \text{diag}[1,1,0.9,0.7]$
$-0.3569 < \delta\theta(t) < 0.6502$

($n = 3$ case): From (12), $Q(t)$ is obtained as follows.

$$Q(t) = \begin{bmatrix} m_1 & 0 & 0 & -\rho_1\delta\theta(t) & 0 & 0 \\ * & m_2 & 0 & -\rho_2\delta\theta(t) & 0 & 0 \\ * & * & m_3 & -\rho_3\delta\theta(t) & 0 & 0 \\ * & * & * & m_4 - 2\rho_4\delta\theta(t) & -\rho_5\delta\theta(t) & -\rho_6\delta\theta(t) \\ * & * & * & * & m_5 & 0 \\ * & * & * & * & * & m_6 \end{bmatrix}, \tag{31}$$

where $\rho_1 = l_1p_{1,1} + l_2p_{1,2} + l_3p_{1,3}$, $\rho_2 = l_1p_{1,2} + l_2p_{2,2} + l_3p_{2,3}$, $\rho_3 = l_1p_{1,3} + l_2p_{2,3} + l_3p_{3,3}$, $\rho_4 = l_1p_{1,4} + l_2p_{2,4} + l_3p_{3,4}$, $\rho_5 = l_1p_{1,5} + l_2p_{2,5} + l_3p_{3,5}$, and $\rho_6 = l_1p_{1,6} + l_2p_{2,6} + l_3p_{3,6}$.

For $Q(t) > 0$ in (31), the following conditions must be satisfied.

- $m_1m_2m_3m_4 - m_2m_3\rho_1^2\delta\theta(t)^2 - m_1m_3\rho_2^2\delta\theta(t)^2 - m_1m_2\rho_3^2\delta\theta(t)^2 - 2m_1m_2m_3\rho_4\delta\theta(t) =: \star_1 > 0. \quad (32)$
- $m_5\star_1 - m_1m_2m_3\rho_5^2\delta\theta(t)^2 =: \star_2 > 0. \quad (33)$
- $m_6\star_2 - m_1m_2m_3m_5\rho_6^2\delta\theta(t)^2 > 0. \quad (34)$

In order to obtain the allowed measurement sensitivity using (32)-(34), the values of K and L are selected as $K = [-0.04, -1.6, -3]$ and $L = [-14.02, -20, -0.48]^T$ such that $\tilde{A}_{KL}$ is Hurwitz. Then, using (32)-(34), the following Table 2 is obtained depending on M. So, when $M = \text{diag}[0.3, 0.2, 12, 0.7, 1, 0.3]$ is selected from Table 2, the following range of $\theta(t)$ is obtained such as

$$0.7125(-28\%) < \theta(t)(\delta\theta(t)\%) < 1.7019(70\%). \tag{35}$$

Thus, the allowed measurement sensitivity is $\theta(t) = 1 + \delta\theta(t) \in [1-\underline{\theta}, 1+\overline{\theta}]$ where $\underline{\theta} = 0.2874$, $\overline{\theta} = 0.7018$.

Table 2. The allowed $\delta\theta(t)$ depending on M under K, L for system dimension $n = 3$.

$M = \text{diag}[1,1,1,1,1,1]$
$-0.2480 < \delta\theta(t) < 0.5050$
$M = \text{diag}[0.3,0.2,12,1,1,1]$
$-0.2720 < \delta\theta(t) < 0.6156$
$M = \text{diag}[0.3,0.2,12,0.7,1,0.3]$
$-0.2875 < \delta\theta(t) < 0.7019$
$M = \text{diag}[0.3,0.2,12,0.8,0.8,0.6]$
$-0.2788 < \delta\theta(t) < 0.6506$
$M = \text{diag}[1,1,1,0.8,0.8,0.6]$
$-0.2521 < \delta\theta(t) < 0.5219$

($n = 4$ case): From (12), $Q(t)$ is obtained as follows:

$$Q(t) = \begin{bmatrix} m_1 & 0 & 0 & 0 & -\rho_1\delta\theta(t) & 0 & 0 & 0 \\ * & m_2 & 0 & 0 & -\rho_2\delta\theta(t) & 0 & 0 & 0 \\ * & * & m_3 & 0 & -\rho_3\delta\theta(t) & 0 & 0 & 0 \\ * & * & * & m_4 & -\rho_4\delta\theta(t) & 0 & 0 & 0 \\ * & * & * & * & m_5 - 2\rho_5\delta\theta(t) & -\rho_6\delta\theta(t) & -\rho_7\delta\theta(t) & -\rho_8\delta\theta(t) \\ * & * & * & * & * & m_6 & 0 & 0 \\ * & * & * & * & * & * & m_7 & 0 \\ * & * & * & * & * & * & * & m_8 \end{bmatrix}, \tag{36}$$

where $\rho_1 = l_1p_{1,1} + l_2p_{1,2} + l_3p_{1,3} + l_4p_{1,4}$, $\rho_2 = l_1p_{1,2} + l_2p_{2,2} + l_3p_{2,3} + l_4p_{2,4}$, $\rho_3 = l_1p_{1,3} + l_2p_{2,3} + l_3p_{3,3} + l_4p_{3,4}$, $\rho_4 = l_1p_{1,4} + l_2p_{2,4} + l_3p_{3,4} + l_4p_{4,4}$, $\rho_5 = l_1p_{1,5} + l_2p_{2,5} + l_3p_{3,5} + l_4p_{4,5}$, $\rho_6 = l_1p_{1,6} + l_2p_{2,6} + l_3p_{3,6} + l_4p_{4,6}$, $\rho_7 = l_1p_{1,7} + l_2p_{2,7} + l_3p_{3,7} + l_4p_{4,7}$, and $\rho_8 = l_1p_{1,8} + l_2p_{2,8} + l_3p_{3,8} + l_4p_{4,8}$.

For $Q(t) > 0$ in (36), the following conditions must be satisfied.

- $$m_1m_2m_3m_4m_5 - m_2m_3m_4\rho_1^2\delta\theta(t)^2 - m_1m_3m_4\rho_2^2\delta\theta(t)^2 - m_1m_2m_4\rho_3^2\delta\theta(t)^2 - m_1m_2m_3\rho_4^2\delta\theta(t)^2 - 2m_1m_2m_3m_4\rho_5\delta\theta(t) =: \star_1 > 0. \tag{37}$$
- $$m_6\star_1 - m_1m_2m_3m_4\rho_6^2\delta\theta(t)^2 =: \star_2 > 0. \tag{38}$$
- $$m_7\star_2 - m_1m_2m_3m_4m_6\rho_7^2\delta\theta(t)^2 =: \star_3 > 0. \tag{39}$$
- $$m_8\star_3 - m_1m_2m_3m_4m_6m_7\rho_8^2\delta\theta(t)^2 > 0. \tag{40}$$

In order to obtain the allowed measurement sensitivity using (37)-(40), the values of K and L are selected as $K = [-0.04, -0.5, -3.2, -2]$ and $L = [-40, -80, -100, -9.3925]^T$ such that $\tilde{A}_{KL}$ is Hurwitz. Then, using (37)-(40), the following Table 3 is obtained depending on M. So, when $M = \text{diag}[2, 2.1, 0.6, 21, 2.2, 0.3, 0.2, 0.1]$ is selected from Table 3, the following range of $\theta(t)$ is obtained such as

$$0.8497(-15\%) < \theta(t)(\delta\theta(t)\%) < 1.2205(22\%). \tag{41}$$

Table 3. The allowed $\delta\theta(t)$ depending on M under K, L for system dimension $n = 4$.

$M = \text{diag}[1,1,1,1,1,1,1,1]$
$-0.0514 < \delta\theta(t) < 0.0580$
$M = \text{diag}[2,2.1,0.6,21,1,1,1,1]$
$-0.0797 < \delta\theta(t) < 0.0968$
$M = \text{diag}[2,2.1,0.6,21,2.2,0.3,0.2,0.1]$
$-0.1503 < \delta\theta(t) < 0.2205$
$M = \text{diag}[2,2.1,0.6,21,1.2,0.8,0.3,0.6]$
$-0.1047 < \delta\theta(t) < 0.1354$
$M = \text{diag}[1,1,1,1,1.2,0.8,0.3,0.6]$
$-0.0633 < \delta\theta(t) < 0.0733$

Table 4. Allowed measurement sensitivity.

System dimension n	2
$1+\delta\theta(t)$ [2]	$-43.83\% \le \delta\theta(t) \le 43.83\%$
$1+\delta\theta(t)$ (our case)	$-60\% \le \delta\theta(t) \le 249\%$
System dimension n	3
$1+\delta\theta(t)$ [2]	$-15.89\% \le \delta\theta(t) \le 15.89\%$
$1+\delta\theta(t)$ (our case)	$-28\% \le \delta\theta(t) \le 70\%$
System dimension n	4
$1+\delta\theta(t)$ [2]	$-5.04\% \le \delta\theta(t) \le 5.04\%$
$1+\delta\theta(t)$ (our case)	$-15\% \le \delta\theta(t) \le 22\%$

Thus, the allowed measurement sensitivity is $\theta(t) = 1 + \delta\theta(t) \in [1-\underline{\theta}, 1+\overline{\theta}]$ where $\underline{\theta} = 0.1502$, $\overline{\theta} = 0.2204$.

From the previous three cases, the allowed magnitude of measurement sensitivity with comparison is summarized in the following Table 4. Obviously, our result significantly enlarges the allowed measurement sensitivity over [2].

In summary, the advantages of our results are two-fold: (i) enlarged nonlinearity; (ii) enlarged allowed measurement sensitivity.

Remark 4: The values of K, L, and M in the case study are selected in order to maximize the allowed measurement sensitivity. With these extreme choices, however, α of (22) can be small, which possibly leads to very conservative choice of ε in using (26). In this regard, if the bound of $\theta(t)$ is somewhat known beforehand, different values of K, L, and M can be selected so that the considered $\theta(t)$ can be covered, and at the same time, less conservative value of ε can be selected.

Remark 5: Although our result shows that much more measurement sensitivity can be allowed over [2], our stability condition of (26) uses the term σ which is derived by calculating the norm value. So, this causes some limitations in our approach and there is a room for improvement if this norm-term σ can be removed or replaced by developing a new matrix inequality.

5. ILLUSTRATIVE EXAMPLES

Example 1: The following system is taken from [2].

$$\begin{aligned}\dot{x}_1 &= x_2 + \sin(x_1),\\ \dot{x}_2 &= u + d(t)\ln(1+x_1^2),\\ y &= \theta(t)x_1,\end{aligned} \tag{42}$$

where $\theta(t) = 1 + 0.63|\sin(10t)|$ and $d(t) = \cos(t)$. Here, $\theta(t)$ is modified such that $\delta\theta(t)$ changes now up to 63% which cannot be handled by [2].

The values of K and L are selected as $K = [-0.74, -1.64]$, $L = [-7, -5.3]^T$. Let $M = \text{diag}[0.6,\ 1.57,\ 0.79,\ 1.1]$. With these choices, using the conditions (28)-(29), the allowed range of $\theta(t)$ is

$$1(0\%) \le \theta(t)(\delta\theta(t)\%) \le 2.1469(114\%). \tag{43}$$

Using (26), we obtain that $\gamma(\varepsilon) = 1+\varepsilon$ and $\sigma = \sqrt{2}\alpha^{-1}\|P\| = 13.1135$. This yields the selection range of ε as $0 < \varepsilon < 0.0306$ such that $\Delta(\varepsilon) > 0$ in (26). We pick $\varepsilon = 0.03$. Then, this system can certainly be stabilized by our method. For simulation, the initial conditions are set as $[x_1(0), x_2(0), z_1(0), z_2(0)]^T = [-0.5, 4, 0, 8]^T$. The simulation result in Fig. 1 agrees with our analysis.

Example 2: Consider a non-triangular system as

$$\begin{aligned}\dot{x}_1 &= x_2 + 0.1x_1 + 0.01x_2\cos t,\\ \dot{x}_2 &= u + \frac{x_1^2 x_2}{10(3+x_1^2)},\\ y &= \theta(t)x_1,\end{aligned} \tag{44}$$

where $\theta(t) = 1 + 0.5\cos(5t)$. Here, the nonlinearity violates both (A1) and (A2). The change of measurement sensitivity is $\pm 50\%$, which is beyond the bound given in [2].

The values of K and L are selected as $K = [-4.7, -3]$ and $L = [-10, -6]^T$. Let $M = \text{diag}[0.4, 3, 2.6, 1]$. With these choices, using the conditions (28)-(29), the allowed range of $\theta(t)$ is

$$0.4484(-55\%) \le \theta(t)(\delta\theta(t)\%) \le 2.1889(118\%). \tag{45}$$

We obtain that $\gamma(\varepsilon) = \max\{\frac{1}{100\varepsilon}, \frac{1}{10}\} = 0.1$ when $0.1 \le \varepsilon$ under Assumption 1 and using (26), $\sigma = \sqrt{2}\alpha^{-1}\|P\| = 40.9628$. This yields the selection range of ε as $0.1 \le \varepsilon < 0.1011$ such that $\Delta(\varepsilon) > 0$ in (26). We select $\varepsilon = 0.101$. For initial conditions $[x_1(0), x_2(0), z_1(0), z_2(0)]^T = [1, -3, 0, 0]^T$, the simulation result is shown in Fig. 2.

Example 3: The state equation of DC motor with parameter uncertainties is given below [17]

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= -\frac{B+\bar{B}}{J_m}x_2 + \frac{K_t}{J_m}x_3,\\ \dot{x}_3 &= -\frac{K_e}{L_a}x_2 - \frac{R_a+\bar{R}_a}{L_a}x_3 + \frac{1}{L_a}u,\\ y &= \theta(t)x_1,\end{aligned} \tag{46}$$

where x_1 is the position of motor, x_2 is the velocity of motor, and x_3 is the current, u is the input voltage, and

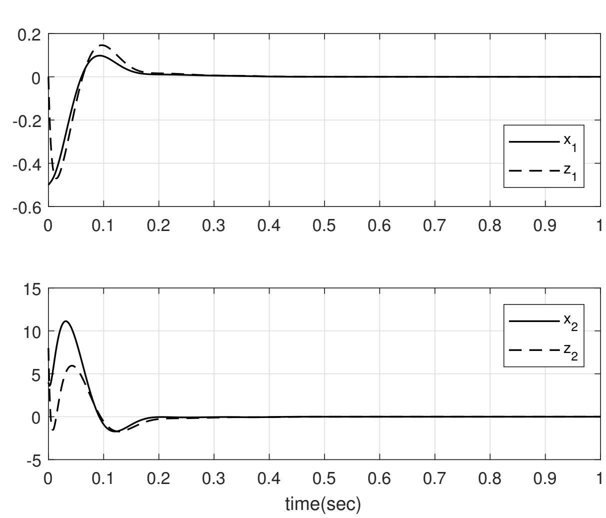

Fig. 1. Trajectories of the closed-loop system in Example 1.

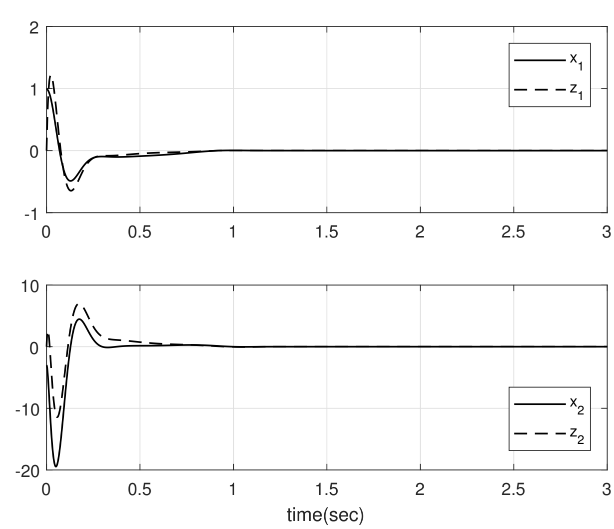

Fig. 2. Trajectories of the closed-loop system in Example 2.

$\theta(t) = 1 + 0.18\sin(3t)$. The change of measurement sensitivity is $\pm 18\%$, which is beyond the bound given in [2].

The motor parameters are that $B = 0.047 \cdot 10^{-4}$ [Nms/rad] is the friction constant, $|\bar{B}| \le 0.005 \cdot 10^{-4}$ [Nms/rad] is the uncertainty of friction constant, $J_m = 0.226 \cdot 10^{-3}$ [Nms^2/rad] is the moment of inertia, $K_t = 0.0346$ [Nm/A] is the torque constant, $K_e = 0.0156$ [Vs/rad] is the back EMF constant, $R_a = 0.1$ [Ω] is the armature resistance, $|\bar{R}_a| \le 0.01$ [Ω] is the uncertainty of resistance, $L_a = 0.92$ [H] is the armature inductance. Define $\chi_1 = x_1$, $\chi_2 = x_2$, $\chi_3 = -\frac{B}{J_m}x_2 + \frac{K_t}{J_m}x_3$, and $u = \frac{JL_a}{K_t}v$. Then, we obtain the transformed system as

$$\begin{aligned}\dot{\chi}_1 &= \chi_2,\\ \dot{\chi}_2 &= \chi_3 + \phi_2(t,\chi,v),\\ \dot{\chi}_3 &= v + \phi_3(t,\chi,v),\\ y &= \theta(t)\chi_1,\end{aligned} \tag{47}$$

where

$$\begin{aligned}\phi_2(t,\chi,v) &= -\frac{\bar{B}}{J_m}\chi_2,\\ \phi_3(t,\chi,v) &= \left(\frac{B\bar{B}}{J_m^2} - \frac{K_tK_e + B(R_a+\bar{R}_a)}{J_mL_a}\right)\chi_2\\ &\quad - \left(\frac{B}{J_m} + \frac{R_a+\bar{R}_a}{L_a}\right)\chi_3.\end{aligned} \tag{48}$$

Note that this system (47) belongs to (A1) case.

The values of K and L are selected as $K = [-9.5, -14, -5.9]$ and $L = [-11.8, -12, -6]^T$. Let $M = \text{diag}[2.6, 3.2, 2.5, 3.4, 1, 0.5]$. With these choices, using the conditions (32)-(34), the allowed range of $\theta(t)$ is

$$0.7734(-22\%) \le \theta(t)(\delta\theta(t)\%) \le 1.3892(38\%). \tag{49}$$

Note that measurement sensitivity of $\pm 18\%$ can be obviously covered under the condition (49). Using (26), we obtain that $\gamma(\varepsilon) = \frac{B+|\bar{B}|}{J_m} + \frac{R_a+|\bar{R}_a|}{L_a} + \varepsilon\left(\frac{B|\bar{B}|}{J_m^2} + \frac{K_tK_e+B(R_a+|\bar{R}_a|)}{J_mL_a}\right)$ under Assumption 1 and $\sigma = \sqrt{3}\alpha^{-1}\|P\| = 122.6590$. This yields the selection range of ε as $0 < \varepsilon < 0.0179$ such that $\Delta(\varepsilon) > 0$ in (26). We select $\varepsilon = 0.0178$. For initial conditions $[x_1(0), x_2(0), x_3(0), z_1(0), z_2(0), z_3(0)]^T = [-0.6, 2, 0.3, 0, 0, 0]^T$, the trajectories of transformed states χ_1, χ_2, χ_3 and observer states z_1, z_2, z_3 are shown in Fig. 3. Also, the trajectories of original states x_1, x_2, x_3 are shown in Fig. 4. Simulation results agree with our theoretical analysis.

Remark 6: The advantages of our control scheme over [2] are demonstrated as follows:

(i) In example 1, the method of [2] can cover the measurement sensitivity $\delta\theta(t)$ up to 43.83%, but our proposed method can cover 63% of $\delta\theta(t)$ for the same considered system.

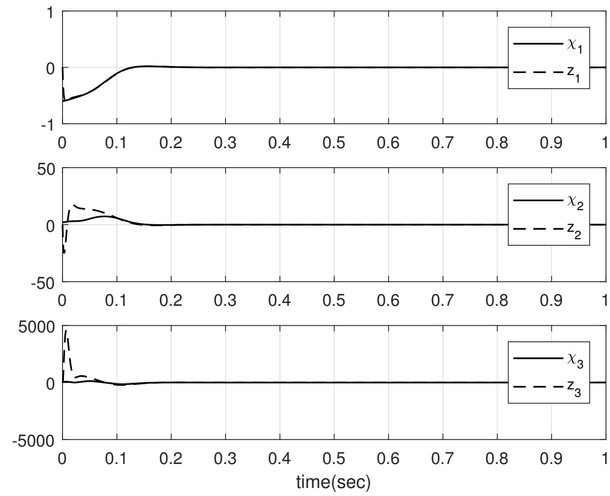

Fig. 3. Trajectories of transformed states χ_1, χ_2, χ_3 and observer states z_1, z_2, z_3 in Example 3.

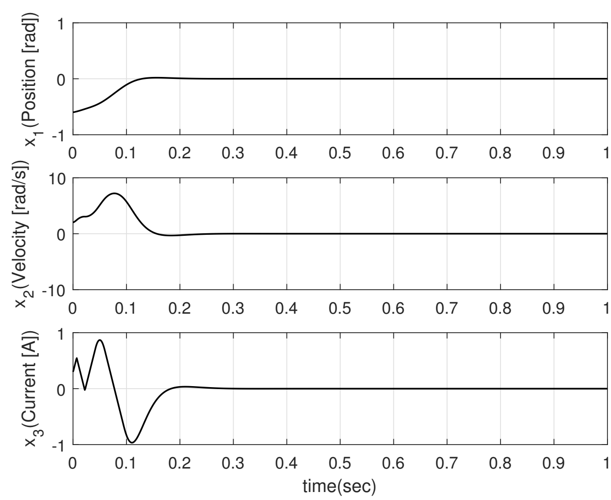

Fig. 4. Trajectories of original states x_1, x_2, x_3 in Example 3.

(ii) In example 2, we can handle the non-triangular nonlinearity that is not treatable by [2].

(iii) In example 3, the proposed method is applied to the position control of DC motor with the measurement sensitivity $\delta\theta(t)$ of 18% that is beyond the bound given in [2].

6. CONCLUSIONS

We have considered an output feedback control problem for a class of nonlinear systems where there is an unknown measurement sensitivity in output feedback. We have shown that more nonlinearity and the significantly larger amount of measurement sensitivity can be handled over the existing result. Analysis and examples are given for clear illustration. The proposed control approach may have a potential to be extended for further generalization of the systems by considering (i) input time-delay, (ii) uncertain time-varying parameters, (iii) more relaxed matrix inequality condition, and all these can be interesting future research topics.

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