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Fully Distributed Event-triggered Semi-global Consensus of Multi-agent Systems with Input Saturation and Directed Topology

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Abstract: This paper studies the consensus problem of multi-agent systems with input saturation and directed communication topology, by utilizing the low-gain feedback method and the event-triggered control laws. Two event-triggered control laws, the centralized and the distributed laws, are proposed to guarantee semi-global consensus of the multi-agent systems. Then, a sufficient condition is presented to prove strictly positive low bound of inner-event time, i.e., the Zeno behavior can be avoided. Finally, the effectiveness of the proposed event-triggered laws is further verified by an example.

Keywords: Directed topology, event-triggered control, input saturation, low-gain feedback, multi-agent systems, semi-global consensus.

1. INTRODUCTION

Over the past decade, the coordinated control problems of multi-agent systems have received increasing attention due to its wide applications and great potentials in physical and economical sciences, transport networks, smart grid system, flocking and distributed wireless communication networks [1–10]. With the wide development of coordinated control in various fields and considerations such as reality and environment, the research have been extended from the original ideal model to the more practical dynamic model. Therefore, for the above-mentioned systems, researchers considered the practical dynamic models with delay, saturation, nonlinearity, and further combined the theoretical research and practice closely [11–13], and the application of coordinated control is broadened. Coordination control includes synchronization, clustering, consensus, etc. The consensus is one of the most important problems in coordinated control problems. The purpose of the consensus is to ensure that all agents reach a common value through interactions with their neighboring agents.

Recently, researchers paid more attention to the problem of consensus, because of its widespread existence and application in real life, and they have devoted more and more energy to study it. Many different dynamics have been investigated for the consensus problem, such as single-integrator dynamics [14], double-integrator [15,16], linear dynamics [17], nonlinear dynamics [18,19] continuous time [16], [18] and discrete time [20] and so on. In most of the literatures on consensus problems, each agent needs to measure its state, update its information and send information to its neighboring agents. Generally, we all assume that data packets can be successfully transmitted. However, in actual systems, data packets are lost due to internal system failure or external factors [21–23]. At the same time, in some practical application systems, the state of the system can not be really obtained, so it is necessary to estimate the state information of the system [22]. Because of these factors, researchers have studied different systems and proposed many control methods. Several methods are now available in the existing literatures. These include impulsive control method [24], feedback control method [25], adaptive control method [26], intermittent control method [27] and sampled-data control [19], [21] and [28]. These methods require the continuous communication between the agent and their neighbors or continuously update controllers via current states of agents. However, in the practical applications of multi-agent systems, constrained by resources and technologies, it is necessary to save communication resources as much as possible to prolong the service life of the system when designing the desired controller. To solve the problem, Tabuada [29] presented an event-triggered strategy for a stabilization problem, where the protocol is triggered when error defined exceeds the predetermined

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value. A new combined measurement approach to event-triggered strategy was proposed in [30], so that the control protocol of agents is triggered only at its own event time and not at its neighbor's event time. In our paper, two event-triggered control laws, the centralized and the distributed event-triggered laws, are proposed. Compared to the periodic or aperiodic sampling protocols, the event-triggered protocol does not need to sample and update its information before the next event time.

Event-triggered control strategy is widely used in reality because of its good performance. It can effectively reduce the update frequency of the controller by adjusting the control parameters, so it can be better applied to the system with limited resources. In existing works, there are two kinds of event-triggered control, namely, centralized event-triggered and distributed event-triggered. Centralized event-triggered law means that the updating signals of all agents should be transferred at the same time. However, this centralized event-triggered law will result heavy network congestion, especially when the wireless network bandwidth is limited. Meanwhile, event agent uses the identical event-triggered function condition defined by the information of all agents. Under such a event-triggered mechanism, some unnecessary data may be triggered while the necessary one may not. We know that the distributed event-triggered means that each agent will need an event-triggered condition, which will consume lager costs or other expenditures when plenty of agent nodes exist. Both of these mechanism have their advantages and need to be balanced.

As well known to all, saturated nonlinear dynamics are ubiquitous in physical and engineering systems. Input saturation [12] and [31,32] means that the magnitude of control input is limited in a bounded region which cannot go beyond the boundary of the region. In addition, in existing works such as [30] and [33, 34], the main studies are the undirected connected systems, in which the Laplacian matrix of communication topology is symmetric, so that the communication information between any two agents for the connected topology is bidirectional. However, in many cases, directed information communication is also ubiquitous such as [35–37]. Therefore, consensus of multi-agent systems with input saturation and directed communication not only conforms to the reality but also extends the existing theories.

Motivated by the above-mentioned works and the results of existence, the centralized and distributed event-triggered control strategies are proposed in this paper to solve the consensus problem of multi-agent systems with input saturation and directed topology. The contributions of this paper are three-fold. Firstly, the low-gain feedback method is used to solve the dynamic system with input saturation. Secondly, both the centralized and the distributed event-triggered protocols are designed for the directed communication network with input saturation. Then, the event-triggered schemes are fully distributed, which only need the state information of the agent and its neighbor's. Finally, two methods are used to prove that the Zeno behavior can be avoided in these two event-triggered functions.

The rest of this paper is organized as follows: Section 2 contains preliminaries and model description. Section 3 proposes event-triggered control strategy. Section 4 gives an example to verify the effectiveness and Section 5 is the conclusion of this paper.

Notations: Throughout the paper, R^n and $R^{n\times m}$ denote the space of all n dimensional real column vectors and the space of all $n\times m$ dimensional real matrices and I_n is an $n\times n$-dimensional identity matrix. For a real symmetry matrix $A>0$ $(A<0)$ means A is a positive (negative)-definite matrix and A^T denotes its transpose and $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ represent the maximum and the minimum eigenvalue of A, respectively. $\|\cdot\|$ represents the two-norm of matrix or vector and $\|\cdot\|_\infty$ denotes the infinite-norm of matrix or vector. $\otimes$ denotes the Kronecker product of matrices.

2. PRELIMINARIES

In this section, preliminaries about model formulation, algebraic graph theory, definitions and lemmas are briefly introduced.

2.1. Algebraic graph theory

Let $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$ denote a weighted digraph of N nodes with the set of nodes $\mathcal{V}=1,2,\cdots,N$, and the set of edges $\mathcal{E}\subseteq\{(i,j):i,j\in\mathcal{V},j\neq i\}$. $\mathcal{A}=(a_{ij})_{N\times N}$ represents a weighted adjacency matrix, where a_{ij} denotes the weight of edge (j,i). The node indices belong to finite index set $I=\{1,2,\cdots,N\}$. The edge of $\mathcal{E}$ is denoted by $e_{ij}=(v_i,v_j)$. The adjacency elements associated with the edges of the graph are positive and the others are zero, i.e., $e_{ij}\in\mathcal{E}\Longleftrightarrow a_{ij}\neq 0$. Moveover we assume $a_{ii}=0$ for all $i\in I$. Let $\mathcal{D}=\mathrm{diag}(d_1,d_2,\cdots,d_N)$ be a diagonal matrix with elements $d_i=\sum_{j=1,j\neq i}^{N}a_{ij}$. Then the Laplacian matrix of the directed graph $\mathcal{G}$ is defined as $L=\mathcal{D}-\mathcal{A}$, where $\mathcal{A}$ is the adjacent matrix of graph $\mathcal{G}$.

In $\mathcal{G}$ a node $i\in\mathcal{V}$ is reachable from a node $j\in\mathcal{V}$ if there exists a path from node j to node i which respects the direction of the edge. A directed graph $\mathcal{G}$ is called strongly connected if every node is reachable from every other node.

2.2. Problem statement

Consider a linear multi-agent system with N agents. Each agent moves in an n-dimensional Euclidean space and updates itself according to the following dynamics:

$$\dot{x}_i(t)=Ax_i(t)+B\sigma(u_i(t)),\tag{1}$$

where $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are constants matrices, and $x_i(t) \in R^n$ is the state of agent ith and $u_i(t) \in R^m$ is its control input. For all $j = 1,2,\cdots,m$, $\sigma(u_i(t)) = [\sigma(u_{i1}(t)),\cdots,\sigma(u_{im}(t))]^T$ is the saturation function induced by the physical devices of the system and $\sigma(u_{ij}(t)) = \text{sign}(u_{ij}(t))\min\{|u_{ij}(t)|,\varpi\}$ where $\varpi > 0$ an input saturation threshold. Moreover, we assume that the system (1) satisfies the following assumption.

Assumption 1: The pair (A,B) is asymptotically null controllable with bounded control in the sense that:

1) all eigenvalues of matrix A are in the closed left-half s-plane;

2) the pair (A,B) is stabilizable.

Assumption 2: The directed communication graph $\mathcal{G}$ is strongly connected.

Lemma 1 [38]**:** Under Assumption 1, for any $\varepsilon \in (0,1]$, there exists an unique matrix $P(\varepsilon) > 0$, which solves the following algebraic Riccati equation(ARE):

$$A^T P(\varepsilon) + P(\varepsilon)A - P(\varepsilon)BB^T P(\varepsilon) + \varepsilon I_n = 0.$$

Moreover, $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$.

Lemma 2 [39]**:** If $\mathcal{G}$ is a strongly connected graph with Laplacian matrix L, there exists a positive vector ξ such that $\xi^T L = 0$.

Definition 1 [40]**:** For a strongly connected network with Laplacian matrix L, the general algebraic connectivity is defined to be the real number

$$\lambda(L) = \min_{x^T\xi=0, x\neq 0} \frac{X^T \hat{L} X}{X^T \Xi X},$$

where $\hat{L} = (\Xi L + L^T \Xi)/2$, $\Xi = \text{diag}(\xi_1,\xi_2,\cdots,\xi_N)$, $\xi = (\xi_1,\xi_2,\cdots,\xi_N)^T$ with $\xi_i > 0$ for all $i \in I$ and $\sum_{i=1}^{N}\xi_i = 1$.

Lemma 3 [41]**:** The general algebraic connectivity of a connected network can be computed by the following:

$$\begin{aligned} &\max \ \ \delta \\ &\text{subject to } \ Q^T(\hat{L} - \delta\Xi)Q \geq 0, \end{aligned}$$

where $Q = (I_{N-1} - \frac{\hat{\xi}}{\xi_N})^T \in R^{N\times N-1}$ and $\hat{\xi} = (\xi_1,\ \xi_2,\ \cdots,\ \xi_{N-1})^T$.

This paper aims at solving the semi-global consensus problem of multi-agent systems subject to input saturation and directed topology by using event-triggered control strategies. Specially, for agent ith, its input control $u_i(t)$ is computed by monotone increasing sequences of time instants: the event instant sequence $\{t_k\}_{k=0}^{\infty}$. The event instant t_k denotes the kth sampling instant of the control $u_i(t)$. Then, $u_i(t)$ is constant for $t \in [t_k,t_{k+1})$.

Definition 2: For any a priori given bounded set $\chi \in R^n$, the system (1) achieves semi-global consensus, if starting from any $x_i(0) \in \chi$

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0,\ \ i,\ j = 1,2,\cdots,N.$$

To solve this problem, one has to design a fully distributed control algorithm according to the following two steps.

Step 1: Solving the parameterized ARE:

$$A^T P(\varepsilon) + P(\varepsilon)A - \alpha P(\varepsilon)BB^T P(\varepsilon) + 2\varepsilon I_n = 0, \quad (2)$$

where $\varepsilon \in (0,1]$, $0 < \alpha \leq 2\lambda(L)$.

Step 2: Construct control law $u_i(t)$ for system (1):

$$u_i(t) = -B^T P(\varepsilon)\left(\sum_{j=1}^{N} a_{ij}(x_i(t_k) - x_j(t_k))\right), \quad (3)$$

where $P(\varepsilon) \in R^{n\times n}$ is the solution of ARE (2).

3. EVENT-TRIGGERED CONSENSUS OF MULTI-AGENT SYSTEM

In this section, we consider two kinds of event-triggered mechanisms and analyze their characteristics and practical application.

3.1. Centralized event-triggered cooperative control

In this part, the consensus problem of multi-agent systems subject to input saturation and directed topology is considered. Then for $t \in [t_k,t_{k+1})$, each agent updates itself according to the following equation:

$$\dot{x}_i(t) = Ax_i(t) - B\sigma\left(B^T P(\varepsilon)\sum_{j=1}^{N} a_{ij}(x_i(t_k) - x_j(t_k))\right). \quad (4)$$

We define the following consensus error and measurable error

$$\tilde{x}_i(t) = x_i(t) - \sum_{k=1}^{N}\xi_k x_k(t), \quad (5)$$

$$w_i(t) = \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)), \quad (6)$$

$$e_i(t) = w_i(t) - w_i(t_k), \quad (7)$$

where $\sum_{k=1}^{N}\xi_k = 1$ and $\xi_k > 0$, the derivative of (5) along with (4), (6) and (7) yields

$$\begin{aligned} \dot{\tilde{x}}_i(t) =& Ax_i(t) - \sum_{k=1}^{N}\xi_k Ax_k(t) \\ &- B\sigma\left(B^T P(\varepsilon)\left(\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) - e_i(t)\right)\right) \\ &+ \sum_{k=1}^{N}\xi_k B\sigma\left(B^T P(\varepsilon)\left(\sum_{j=1}^{N} a_{kj}(x_k(t) - x_j(t)) \right.\right. \\ &\left.\left. - e_k(t)\right)\right). \end{aligned} \quad (8)$$

Theorem 1: Consider the multi-agent system (1) over a directed strongly connected communication graph, if the Assumption 1 holds, then the semi-global consensus of the system can be guaranteed according to the given control law and execution event-triggered function

$$t_{k+1} = \sup\{t \mid \|e(t)\| \leqslant \eta \|w(t)\|\}, \tag{9}$$

where $\eta = \sqrt{\frac{\delta k(2\varepsilon - k\rho)\lambda_{\min}(\Xi)}{\rho\lambda_{\max}(\Xi)}}$ satisfying $0 < \eta < \varepsilon$, with $\rho = \|P(\varepsilon)BB^TP(\varepsilon)\|$, $0 < k < \frac{2\varepsilon}{\rho}$ and $0 < \delta < \min\left\{1, \min\{\rho^2, \frac{1}{\lambda_{\max}(L^TL)}\}\right\}$.

Proof: Define a Lyapunov function as

$$V(t) = \frac{1}{2}\tilde{x}^T(t)(\Xi \otimes P(\varepsilon))\tilde{x}(t), \tag{10}$$

where $\Xi > 0$ and $P(\varepsilon) > 0$.

Due to the existence of the bounded set χ and the facts that $x_i(0) \in \chi$, $i \in I$, and $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$, there exists an $\varepsilon^0 \in (0,1]$, for any given constants $c > 0$, such that

$$c \geq \sup_{\varepsilon^0\in(0,1],\ x_i(0)\in\chi,\ i\in I} V(0). \tag{11}$$

Let $L_V(c) = \{V(t) \leq c\}$. Similarly, there exists an $\varepsilon^* \in (0, \varepsilon^0]$, such that, for all $\varepsilon \in (0, \varepsilon^*]$,

$$\|u_i(t)\|_\infty \leq \varpi, \ \ i \in I. \tag{12}$$

That is, there must exist an ε^* to dominate the nonlinearity induced by the input saturation and $\sigma(u_i(t))$ becomes $u_i(t)$. Then, (1) becomes

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \ \ t \in [t_k, t_{k+1}), \tag{13}$$

and the (8) can be written as

$$\begin{aligned}\dot{\tilde{x}}_i(t) =& Ax_i(t) - \sum_{k=1}^{N}\xi_k Ax_k(t) \\ & - BB^TP(\varepsilon)\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) \\ & + BB^TP(\varepsilon)\sum_{k=1}^{N}\xi_k\sum_{j=1}^{N} a_{kj}(x_k(t) - x_j(t)) \\ & + BB^TP(\varepsilon)e_i(t) - BB^TP(\varepsilon)\sum_{k=1}^{N}\xi_k e_k(t).\end{aligned} \tag{14}$$

Since $\xi^TL = 0$, one has $\sum_{k=1}^{N}\xi_k\sum_{j=1}^{N} l_{kj}x_j(t) = (\xi^TL \otimes I_n)x(t) = 0$. Let $w(t) = (w_1^T(t), \cdots, w_N^T(t))^T$, $e(t) = (e_1^T(t), \cdots, e_N^T(t))^T$ and $\tilde{x}(t) = (\tilde{x}_1^T(t), \cdots, \tilde{x}_N^T(t))^T$. The (14) can be written as

$$\begin{aligned}\dot{\tilde{x}}(t) =& (I_N \otimes A)\tilde{x}(t) - (L \otimes BB^TP(\varepsilon))\tilde{x}(t) \\ & + (I_N \otimes BB^TP(\varepsilon))e(t).\end{aligned} \tag{15}$$

Now, the derivative of $V(t)$ along with (15) yields

$$\begin{aligned}\dot{V}(t) =& \tilde{x}^T(t)\left(\Xi \otimes \frac{(A^TP(\varepsilon) + P(\varepsilon)A)}{2}\right)\tilde{x}(t) \\ & + \tilde{x}^T(t)\left(\Xi \otimes P(\varepsilon)BB^TP(\varepsilon)\right)e(t) \\ & - \tilde{x}^T(t)\left((L^T\Xi + \Xi L) \otimes \frac{P(\varepsilon)BB^TP(\varepsilon)}{2}\right)\tilde{x}(t).\end{aligned}$$

From Definition 1, one has $-\tilde{x}^T(t)(L^T\Xi + \Xi L)\tilde{x}(t) \leq -2\lambda(L)\tilde{x}^T(t)(\Xi \otimes I_n)\tilde{x}(t)$. Let $P(\varepsilon)$ be the solution of the following ARE:

$$A^TP(\varepsilon) + P(\varepsilon)A - \alpha P(\varepsilon)BB^TP(\varepsilon) + 2\varepsilon I_n = 0,$$

where $0 < \alpha \leq 2\lambda(L)$. Then we can further get

$$\begin{aligned}\dot{V}(t) \leq& \tilde{x}^T(t)\bigg(\Xi \otimes \bigg(\frac{A^TP(\varepsilon) + P(\varepsilon)A}{2} \\ & - \lambda(L)(P(\varepsilon)BB^TP(\varepsilon))\bigg)\bigg)\tilde{x}(t) \\ & + \tilde{x}^T(t)\left(\Xi \otimes P(\varepsilon)BB^TP(\varepsilon)\right)e(t).\end{aligned} \tag{16}$$

Since $P(\varepsilon)BB^TP(\varepsilon) \geq 0$, it follows from (2) that,

$$\frac{A^TP(\varepsilon) + P(\varepsilon)A}{2} - \lambda(L)(P(\varepsilon)BB^TP(\varepsilon)) \leq -\varepsilon I_n.$$

Equation (16) can be rewritten as

$$\begin{aligned}\dot{V}(t) \leq& \sum_{i=1}^{N} -\varepsilon\xi_i\tilde{x}_i^T(t)\tilde{x}_i(t) \\ & + \sum_{i=1}^{N}\xi_i\tilde{x}_i^T(t)P(\varepsilon)BB^TP(\varepsilon)e_i(t).\end{aligned}$$

By using inequality $\|\xi\| \cdot \|\zeta\| \leq \frac{k}{2}\|\xi\|^2 + \frac{1}{2k}\|\zeta\|^2$ for any $k > 0$ and $\xi, \zeta \in R^n$, one has

$$\begin{aligned}\dot{V}(t) \leq& \sum_{i=1}^{N} -\varepsilon\xi_i\tilde{x}_i^T(t)\tilde{x}_i(t) + \sum_{i=1}^{N}\xi_i\tilde{x}_i^T(t)P(\varepsilon)BB^TP(\varepsilon)e_i(t) \\ \leq& -(\varepsilon - \frac{k\rho}{2})\sum_{i=1}^{N}\xi_i\|\tilde{x}_i(t)\|^2 + \frac{\rho}{2k}\sum_{i=1}^{N}\xi_i\|e_i(t)\|^2 \\ \leq& -(\varepsilon - \frac{k\rho}{2})\lambda_{\min}(\Xi)\|\tilde{x}(t)\|^2 + \frac{\rho}{2k}\lambda_{\max}(\Xi)\|e(t)\|^2.\end{aligned} \tag{17}$$

Then, by choosing $0 < k < \frac{2\varepsilon}{\rho}$ and enforcing the event-triggered function (9), the (17) becomes

$$\begin{aligned}\dot{V}(t) \leq& -(\varepsilon - \frac{k\rho}{2})\lambda_{\min}(\Xi)(1 - \delta\lambda_{\max}(L^TL))\|\tilde{x}(t)\|^2 \\ \leq& 0,\end{aligned}$$

which implies that $V(t) \to 0$ as $t \to \infty$. One can get that $\lim_{t\to\infty}\|x_i(t) - x_j(t)\| = 0$, i, $j = 1, 2, \cdots, N$, i.e., the semi-global consensus of the system (1) can be achieved.

Remark 1: Compared with the results in [7] and [15], the result of this paper can be regarded as an extension from continuous control laws to the event-triggered control laws. By using the event-triggered control law, the update frequency of the controller can be reduced, and the resources can be saved effectively.

Theorem 2: Consider the multi-agent system (1) over a directed strongly connected communication graph, if Assumption 1 holds and all parameters in Theorem 1 are satisfied, then the event-triggered instants $t_k (k \geq 0$ and $t_0 = 0)$ defined by (9) excludes the Zeno behavior for every agent. That is, $t_{k+1} - t_k \geq T$ with $T > 0$.

Proof: For any agent ith, assume its current triggering time is t_k.

Given a sufficient condition to guarantee the event-triggered condition $\|e(t)\| \leq \eta \|w(t)\|$ is

$$\|e(t)\| \leq \frac{\eta}{1+\eta}\|w(t_k)\|, \tag{18}$$

which can be written directly as:

$$\begin{aligned}\|e(t)\| &\leq \frac{\eta}{1+\eta}\|w(t) - e(t)\| \\ &\leq \frac{\eta}{1+\eta}\|w(t)\| + \frac{\eta}{1+\eta}\|e(t)\|.\end{aligned}$$

Since $\|e(t)\| \leq \frac{\eta}{1+\eta}\|w(t_k)\|$, by utilizing $|\|w(t_k)\| - \|w(t)\|| \leq \frac{\eta}{1+\eta}\|w(t_k)\|$, one has

$$\frac{1}{1+\eta}\|w(t_k)\| \leq \|w(t)\| \leq (\frac{\eta}{1+\eta}+1)\|w(t_k)\|. \tag{19}$$

Then, the time derivative of $\|e(t)\|$ over the interval $[t_k, t_{k+1})$ is

$$\frac{d}{dt}\|e(t)\| \leq \frac{\|e^T(t)\|}{\|e(t)\|}\|\dot{e}(t)\| = \|\dot{w}(t)\|. \tag{20}$$

And since

$$\dot{w}(t) = (I_N \otimes A)w(t) - (L \otimes BB^T P(\varepsilon))w(t_k),$$

one has

$$\begin{aligned}\dot{w}(t) =& (I_N \otimes A)e(t) + (I_N \otimes A)w(t_k) \\ &- (L \otimes BB^T P(\varepsilon))w(t_k).\end{aligned}$$

The equality (20) can be written as

$$\frac{d}{dt}\|e(t)\| \leq \|(I_N \otimes A)\|\|e(t)\| + \alpha_k, \tag{21}$$

where $\alpha_k = \|(I_N \otimes A)w(t_k) - (L \otimes BB^T P(\varepsilon))w(t_k)\|$. Then, it follows that:

$$\|e(t)\| \leq \frac{\alpha_k}{\|I_N \otimes A\|}(e^{\|I_N \otimes A\|(t-t_k)} - 1).$$

According to the sufficiency of the trigger condition, one has

$$\begin{aligned}\|e(t_{k+1})\| &= \frac{\eta}{1+\eta}\|w(t_k)\| \\ &\leq \frac{\alpha_k}{\|I_N \otimes A\|}(e^{\|I_N \otimes A\|(t_{k+1}-t_k)} - 1),\end{aligned} \tag{22}$$

which yields $t_{k+1} - t_k \geq \frac{1}{\|I_N \otimes A\|}\ln(\frac{\|I_N \otimes A\|\|w(t_k)\|\eta}{(1+\eta)\alpha_k} + 1)$.
To prove that the inner-event interval is strictly positive, we first consider the case when $w(t_k) > 0$. Since $w(t_k) > 0$, one has

$$\begin{aligned}t_{k+1} - t_k &\geq \frac{1}{\|I_N \otimes A\|}\ln\left(\frac{\|I_N \otimes A\|\|w(t_k)\|\eta}{(1+\eta)\alpha_k} + 1\right) \\ &\geq \frac{1}{\|I_N \otimes A\|}\ln\left(\frac{\|I_N \otimes A\|\|w(t_k)\|\eta}{\prod} + 1\right) \\ &\geq \frac{1}{\|I_N \otimes A\|}\ln\left(\frac{\|I_N \otimes A\|\eta}{\mho} + 1\right) > 0,\end{aligned} \tag{23}$$

where $\prod = (1+\eta)\Big(\|I_N \otimes A\|\|w(t_k)\| + \|L \otimes BB^T P(\varepsilon)\|$ $\|w(t_k)\|\Big)$ and $\mho = (1+\eta)\Big(\|I_N \otimes A\| + \|L \otimes BB^T P(\varepsilon)\|\Big)$.

Next, we consider the case when $w(t_k) = 0$ as $k \to \infty$. Then it follows from (19) that $w(t) = 0$, and thus:

$$\dot{w}(t) = (I_N \otimes A)w(t) - (L \otimes BB^T P(\varepsilon))w(t_k) = 0.$$

Simple transposition of (19) leads to

$$\lim_{k\to\infty}\frac{\|w(t)\|}{\|w(t_k)\|} \leq \frac{\eta}{1+\eta} + 1. \tag{24}$$

Then, one has

$$\begin{aligned}\alpha_k &\leq \|(L \otimes BB^T P(\varepsilon))w(t_k)\| + \|I_N \otimes A\|w(t_k)\| \\ &\leq \max_{t' \in [t_k, t_{k+1}]}\|I_N \otimes A\|\|w(t')\| + \|I_N \otimes A\|\|w(t_k)\|.\end{aligned} \tag{25}$$

Together with (24) and (25), we have

$$\begin{aligned}T &= \lim_{k\to\infty}(t_{k+1} - t_k) \\ &\geq \frac{1}{\|I_N \otimes A\|}\ln\left(\frac{\|I_N \otimes A\|\|w(t_k)\|\eta}{(1+\eta)\alpha_k} + 1\right) \\ &\geq \frac{1}{\|I_N \otimes A\|}\ln\left(\frac{\|w(t_k)\|\eta}{(1+\eta)(\|w(t_k)\| + \|w(t')\|)} + 1\right) \\ &\geq \frac{1}{\|I_N \otimes A\|}\ln\left(\frac{\eta}{2+3\eta} + 1\right) > 0,\end{aligned} \tag{26}$$

which is strictly positive.

Remark 2: In order to prove Theorem 2, a sufficient condition is required when $0 < \eta < \varepsilon$ is satisfied. It is noted that $\eta = \sqrt{\frac{\delta\varepsilon^2\lambda_{\min}(\Xi)}{\rho^2\lambda_{\max}(\Xi)}}$ (when $k = \frac{\varepsilon}{\rho}$), so $0 < \eta < \varepsilon$ can be guaranteed. The event-triggered function is reduced to this sufficient condition at the event time instants, i.e., t_{k+1}. We can know from (23) and (26) that the Zeno behavior can be excluded, and the lower bound value and the rate of convergence are related to η. Therefore, the trigger frequency and the update frequency of the controller can be changed by adjusting the parameter η to achieve the purpose of saving resources.

3.2. Distributed event-triggered cooperative control

Under distributed event-triggered control strategy, event instants could be different for individual agents and the sequence of event instants for agent ith is denoted by $t_0^i,\ t_1^i, \cdots$. Then for $t \in [t_k^i, t_{k+1}^i)$, $(k = 0, 1, \cdots$ and $t_0^i = 0)$, each agent updates itself according to the following equation:

$$\dot{x}_i(t) = Ax_i(t) - B\sigma\left(B^T P(\varepsilon)\sum_{j=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_{\acute{k}(t)}^j))\right),$$

where $\acute{k}(t) = \arg\max_{l\in I}\{l \mid t_l^j \le t\}$.

Define

$$\begin{aligned}
&E_i(t) = x_i(t_k^i) - x_i(t),\\
&E_j(t) = x_j(t_{\acute{k}(t)}^j) - x_j(t),\\
&\Delta_i(t) = \sum_{j=1}^{N} a_{ij}(E_i(t) - E_j(t)).
\end{aligned} \tag{27}$$

The derivative of (5) along with (28) yields

$$\begin{aligned}
\dot{\tilde{x}}_i(t) =& Ax_i(t) - \sum_{k=1}^{N} \xi_k A x_k(t)\\
&- B\sigma\left(B^T P(\varepsilon)\left(\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) + \Delta_i(t)\right)\right)\\
&+ \sum_{k=1}^{N} \xi_k B\sigma\left(B^T P(\varepsilon)\left(\sum_{j=1}^{N} a_{kj}(x_k(t) - x_j(t))\right.\right.\\
&\left.\left. + \Delta_k(t)\right)\right).
\end{aligned} \tag{28}$$

Theorem 3: Consider a multi-agent system (1) over a directed strongly connected communication graph, if the Assumption 1 holds, then, the event-triggered function

$$t_{k+1}^i = \sup\{t \mid \|\Delta_i(t)\|^2 \leqslant \eta_i \|w_i(t)\|^2\} \tag{29}$$

can guarantee system to achieve the semi-global consensus, where $\eta_i = \frac{\delta_i k(2\lambda_{\min}(\Xi)\varepsilon - \lambda_{\max}(\Xi)k\rho)}{\rho\lambda_{\max}(\Xi)}$ satisfying $0 < \eta_i < \varepsilon^2$, with $0 < \delta_i < \min\left\{1, \min\{\frac{\lambda_{\max}^2(\Xi)\rho^2}{\lambda_{\min}^2(\Xi)}, \frac{1}{a_i\lambda_{\max}(L^T L)}\}\right\}$, and $a_i > 0$ such that $\|w_i(t)\|^2 \le a_i \lambda_{\max}(L^T L)\|\tilde{x}_i(t)\|^2$, $i \in I$, $\rho = \|P(\varepsilon)BB^T P(\varepsilon)\|$, and $0 < k < \frac{2\varepsilon\lambda_{\min}(\Xi)}{\lambda_{\max}(\Xi)\rho}$.

Proof: Define a Lyapunov function as

$$V(t) = \frac{1}{2}\tilde{x}^T(t)(\Xi \otimes P(\varepsilon))\tilde{x}(t), \tag{30}$$

where $\Xi > 0$ and $P(\varepsilon) > 0$.

Being similar to the analysis in Theorem 1, for any $c > 0$ and $L_V(c) = \{V(t) \le c\}$, there exists an $\varepsilon^* \in (0, 1]$, such that, for $\varepsilon \in (0, \varepsilon^*]$, $\|u_i(t)\|_\infty \le \varpi$, $i \in I$. That is, there must exist an ε^* to dominate the nonlinearity induced by the input saturation and $\sigma(u_i(t))$ becomes $u_i(t)$. Thus, one gets

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t). \tag{31}$$

Define $\Delta(t) = (\Delta_1^T(t), \cdots, \Delta_N^T(t))^T$. For $t \in [t_k^i, t_{k+1}^i)$, similar to the analysis in Theorem 1, we can get compact style

$$\begin{aligned}
\dot{\tilde{x}}(t) =& (I_N \otimes A)\tilde{x}(t) - (L \otimes BB^T P(\varepsilon))\tilde{x}(t)\\
&- (I_N \otimes BB^T P(\varepsilon))\Delta(t).
\end{aligned} \tag{32}$$

The derivative of $V(t)$ along with (32) yields

$$\begin{aligned}
\dot{V}(t) =& \tilde{x}^T(t)\left(\Xi \otimes \frac{(A^T P(\varepsilon) + P(\varepsilon)A)}{2}\right)\tilde{x}(t)\\
&- \tilde{x}^T(t)\left(\Xi \otimes P(\varepsilon)BB^T P(\varepsilon)\right)\Delta(t)\\
&- \tilde{x}^T(t)\left((L^T\Xi + \Xi L) \otimes \frac{P(\varepsilon)BB^T P(\varepsilon)}{2}\right)\tilde{x}(t).
\end{aligned}$$

From definition 1 and ARE, we have

$$\dot{V}(t) \le -\varepsilon\lambda_{\min}(\Xi)\|\tilde{x}(t)\|^2 - \tilde{x}^T(t)\left(\Xi \otimes P(\varepsilon)BB^T P(\varepsilon)\right)\Delta(t).$$

By using Young's inequality, we have

$$\begin{aligned}
\dot{V}(t) \le& -\left(\varepsilon\lambda_{\min}(\Xi) - \frac{\lambda_{\max}(\Xi)k\rho}{2}\right)\|\tilde{x}(t)\|^2\\
&+ \frac{\lambda_{\max}(\Xi)\rho}{2k}\|\Delta(t)\|^2.
\end{aligned} \tag{33}$$

Then, by choosing $0 < k < \frac{2\varepsilon\lambda_{\min}(\Xi)}{\lambda_{\max}(\Xi)\rho}$ and enforcing the event-triggered function (29), the (33) can be written as

$$\begin{aligned}
\dot{V}(t) \le& -\left(\varepsilon\lambda_{\min}(\Xi) - \frac{\lambda_{\max}(\Xi)k\rho}{2}\right)\\
&\times \sum_{i=1}^{N}\left(1 - \delta_i a_i \lambda_{\max}(L^T L)\right)\|\tilde{x}_i(t)\|^2\\
\le& 0,
\end{aligned}$$

which means consensus can be achieved.

Remark 3: In this part, the state of each agent is regulated by a single event-triggered condition. Therefore, compared with the centralized event-triggered control strategy, the trigger time of each agent is different in this method. At each triggered instant, agent i will update its controller $u_i(t)$ using the current state $x_i(t_k^i)$ and send the current state to neighbors. The neighbors will thereby update their controllers, but they will not transmit their information at this moment unless their event-triggered conditions are violated. Different from the reference [30], the event-triggered function with multiple control parameters is adopted in this paper, so there are more choices of control parameters in adjusting convergence rate and updating frequency.

Now, we prove that there is no Zeno behavior for each agent given the event-triggered function. We consider agent $h \in I$ that satisfies

$$h \triangleq \arg\max_{i\in I}\{i \mid \|w_i(t)\|\}.$$

Since $\|\Delta_h(t)\| \leq \|\Delta(t)\|$, the following inequality holds

$$\frac{\|\Delta_h(t)\|}{N\|w_h(t)\|} \leq \frac{\|\Delta(t)\|}{N\|w_h(t)\|} \leq \frac{\|\Delta(t)\|}{\|w(t)\|},$$

which is equivalent to $\frac{\|\Delta_h(t)\|}{\|w_h(t)\|} \leq N\frac{\|\Delta(t)\|}{\|w(t)\|}$.

Denote $y_h(t) = \frac{\|\Delta_h(t)\|}{\|w_h(t)\|}$, then $y_h(t) \leq N\frac{\|\Delta(t)\|}{\|w(t)\|} = Ny(t)$, where $y(t) = \frac{\|\Delta(t)\|}{\|w(t)\|}$. For any interval $t \in [t_k^h, t_{k+1}^h)$, $\frac{\|\Delta(t)\|}{\|w(t)\|}$ is continuous. We have

$$\dot{y}(t) \leq \frac{\|\dot{\Delta}(t)\|}{\|w(t)\|} + y(t)\frac{\|\dot{w}(t)\|}{\|w(t)\|}. \tag{34}$$

It follows from (27) that $\dot{\Delta}(t) = -\dot{w}(t)$, and we can get $\|\dot{\Delta}(t)\| = \|\dot{w}(t)\|$. The (34) becomes

$$\dot{y}(t) \leq (1+y(t))\frac{\|\dot{w}(t)\|}{\|w(t)\|}. \tag{35}$$

It follows from (6) that

$$\begin{aligned}\dot{w}(t) =&(L\otimes I_n)\dot{x}(t)\\ =&(I_N\otimes A - L\otimes BB^TP(\varepsilon))w(t)\\ &-(L\otimes BB^TP(\varepsilon))\Delta(t).\end{aligned} \tag{36}$$

Consequently,

$$\begin{aligned}\|\dot{w}(t)\| \leq&\left(\|I_N\otimes A\| + \|L\otimes BB^TP(\varepsilon)\|\right)\|w(t)\|\\ &+\|L\otimes BB^TP(\varepsilon)\|\|\Delta(t)\|,\end{aligned}$$

then, we can conclude that

$$\begin{aligned}\frac{\|\dot{w}(t)\|}{\|w(t)\|} \leq&\|I_N\otimes A\| + \|L\otimes BB^TP(\varepsilon)\|\\ &+\|L\otimes BB^TP(\varepsilon)\|\frac{\|\Delta(t)\|}{\|w(t)\|}.\end{aligned} \tag{37}$$

Substituting (37) into (35), one has

$$\dot{y}(t) \leq (1+y(t))(a+by(t)), \tag{38}$$

where $a = \|I_N\otimes A\| + \|L\otimes BB^TP(\varepsilon)\|$ and $b = \|L\otimes BB^TP(\varepsilon)\|$. Thus, the evolution of $y(t)$ for $t \in [t_k^h, t_{k+1}^h)$ satisfies that $y(t) \leq \phi(t,\phi_0)$, where $\phi(t,\phi_0)$ is the solution of

$$\dot{\phi}(t) = (1+\phi(t))(a+b\phi(t)). \tag{39}$$

Assume that for agent hth, an event instant at times t_k^h, before the next event instant, the following inequality:

$$y_h(t) \leq N\int_{t_k^h}^t \dot{\phi}(s)ds = N\frac{ae^{(a-b)(s+c)}-1}{1-be^{(a-b)(s+c)}}\Big|_{s=t_k^h}^t$$

holds, where $c = \frac{1}{b-a}\ln a$. The next event will not be triggered before (29) is crossing zero, i.e., before

$$\frac{ae^{(a-b)(T+c)}-1}{1-be^{(a-b)(T+c)}} = d, \tag{40}$$

where $d = \sqrt{\frac{1}{N}\frac{\delta_h k(2\lambda_{\min}(\Xi)\varepsilon - k\rho\lambda_{\max}(\Xi))}{\lambda_{\max}(\Xi)\rho}}$. By solving (40), one has

$$T = \frac{1}{a-b}\ln\frac{a(d+1)}{a+bd}. \tag{41}$$

Because a is bigger than b, $ad+a > bd+a$ and one has $T > 0$. This ensures that the Zeno behavior is excluded.

Remark 4: In [23], the consensus of multiagent systems was considered by constructing event-triggered function. The convergence rate is mainly regulated by the eigenvalues of matrices. However, in our paper, it can be adjusted by the control parameters of the event-triggered function. Hence, the proposed method in our paper is easier to adjust the convergence rate.

4. NUMERICAL EXAMPLE

In this section, a numerical example is simulated to verify the theoretical analysis.

Consider the multi-agent system (1) consisting of four nodes. The dynamics of each agent is described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t),\ \ i = 1,\cdots,4, \tag{42}$$

where $x_i(t) = (x_{i1}, x_{i2})^T$, matrices A and B are described by

$$A = \begin{pmatrix}-1 & 1\\ 0.1 & -0.2\end{pmatrix},\ \ B = \begin{pmatrix}0.4\\ 0.2\end{pmatrix}.$$

The directed communication topology of multi-agent system (42) is given by Fig. 1.

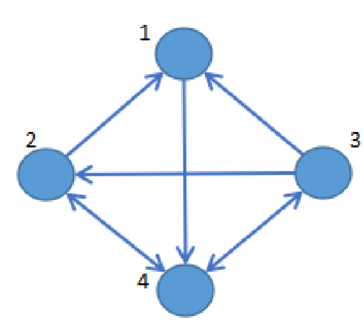

Fig. 1. Communication topology.

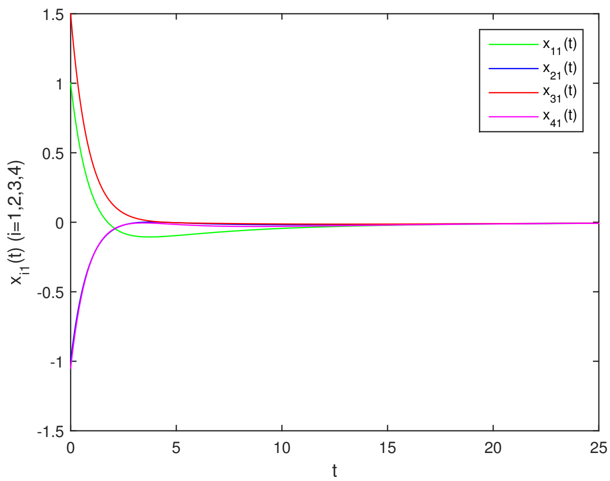

Fig. 2. The state trajectories of x_{i1}.

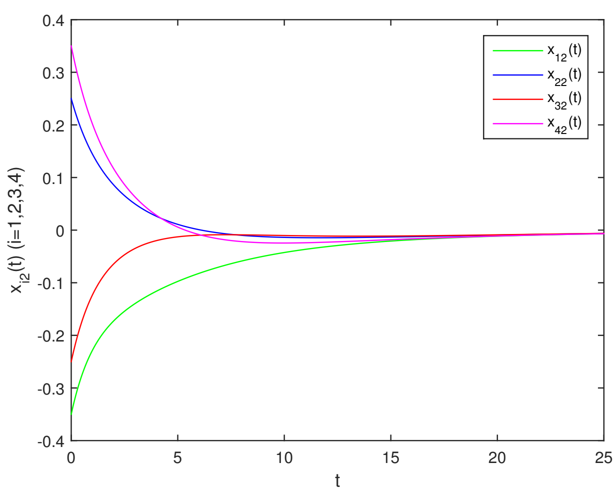

Fig. 3. The state trajectories of x_{i2}.

The general algebraic connectivity of the above graph can be computed by Lemma 3, then one has $\lambda(L) = 0.4033$. When $\varepsilon = 0.1$, solve the ARE: $A^T P(\varepsilon) + P(\varepsilon)A - 2\lambda(L)P(\varepsilon)BB^T P(\varepsilon) + 2\varepsilon I_n = 0$, one has

$$P(\varepsilon) = \begin{pmatrix} 0.1152 & 0.1787 \\ 0.1787 & 1.1984 \end{pmatrix}.$$

Using the event-triggered function (29), the simulation results are shown in Figs. 2-5. Figs. 2-5 show simulation results in the case of $\eta_1 = 0.0008$, $\eta_2 = 0.0006$, $\eta_3 = 0.0004$, $\eta_4 = 0.0002$ and $\eta_1 = 0.008$, $\eta_2 = 0.006$, $\eta_3 = 0.004$, $\eta_4 = 0.002$. It can be seen from Fig. 2 and Fig. 3 that the consensus of MASs (1) is achieved asymptotically. And the event-triggered instants for each agent are marked in Fig. 4 and Fig. 5, where each symbol represents the event time instants.

Under the condition of the same event-triggered, for each agent to select different parameters and select the same parameters, some agent's trigger instants will change.

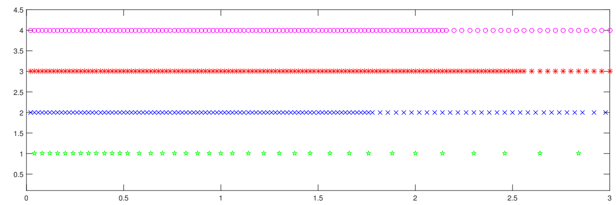

Fig. 4. The event-triggered instants of each agent.

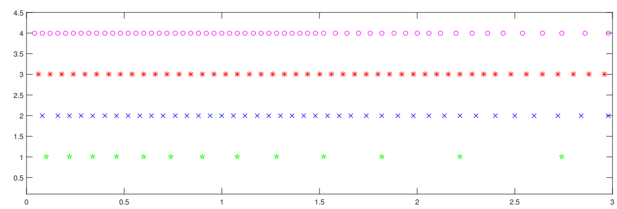

Fig. 5. The event-triggered instants of each agent.

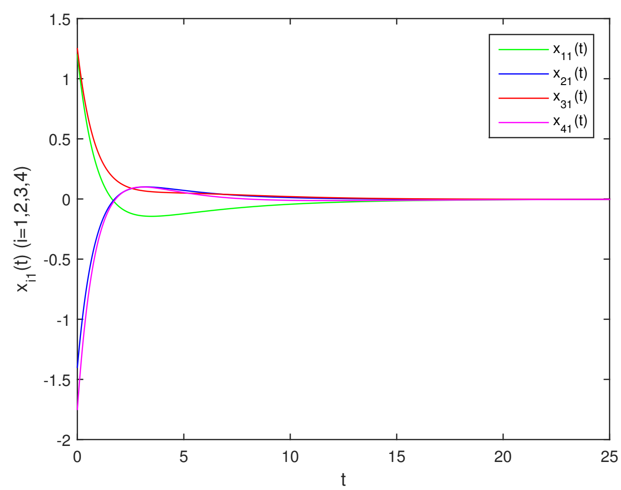

Fig. 6. The state trajectories of x_{i1}.

Using the event-triggered function (9), the simulation results are shown in Figs. 6-8. Figs. 6-8 show simulation results in the case of $\eta = 0.02$. From Figs. 6-7 show results of that the consensus of multi-agent system (1) is achieved. And the event-triggered instants of the whole system as shown in Fig. 8.

Under the conditions of two kinds of event-triggered, the consensus of MASs (1) is achieved. However, due to different triggering mechanism, it is concluded that the simulation diagrams are different. One case is to control each agent, and the other one is to control the whole system.

Remark 5: According to the comparison between Figs. 4-5, triggering frequency of event-triggered function can be directly changed by choosing different parameters. In addition, from Figs. 4-5 and Fig. 8, the difference between centralized and decentralized trigger mechanisms can be seen directly. Both of them have their own advantages, depending on the situation in the actual system.

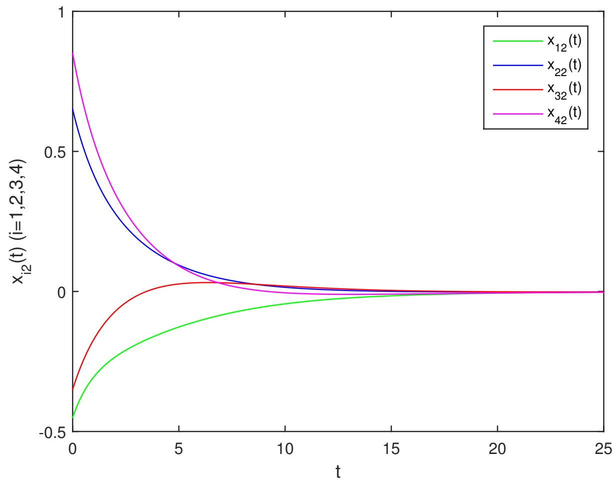

Fig. 7. The state trajectories of x_{i2}.

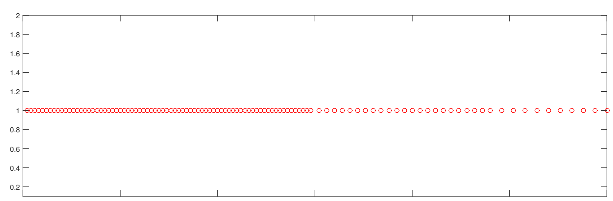

Fig. 8. The event-triggered instants of the whole system.

5. CONCLUSION

In this paper, the consensus problem of multi-agent system with input saturation and directed strongly topology is studied. We propose two novel event-triggered strategies with distributed protocols and use low-gain feedback method to solve the consensus problem of multi-agent systems. Finally, an example is presented to illustrate the effectiveness of the theoretical results. In the future work, we will extend our results to the general directed topology with external disturbance, delay and the nonlinear dynamic system.

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