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P-type Iterative Learning Control with Initial State Learning for One-sided Lipschitz Nonlinear Systems

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Abstract: In this paper, the problem of iterative learning control is considered for a class of one-sided Lipschitz nonlinear systems. For such nonlinear systems, open-loop and closed-loop P-type learning algorithms with initial state learning are adopted, respectively. Furthermore, the convergence conditions of the P-type learning algorithms are established. It is shown that both algorithms can guarantee the system output converges to the desired one on the whole time interval. A numerical example is constructed to illustrate the effectiveness of the proposed learning algorithms.

Keywords: Iterative learning control, nonlinear systems, one-sided Lipschitz, P-type learning algorithms.

1. INTRODUCTION

Iterative learning control (ILC) is an effective control strategy to achieve perfect trajectory tracking for dynamics with repetitive operation over a finite time interval (see [1, 2]). The basic idea of ILC is to improve the current tracking performance by fully utilizing the past control experience. On account of its simplicity and effectiveness, ILC has attracted extensive attention in the field of both theory and applications, and lots of achievements have been made over the past decades (see, e.g., [3–9] and references therein).

Nonlinearity is a universal phenomenon existing in practical control systems (see [10–13]). Hitherto, there are many significant results which have been reported on the ILC for nonlinear systems. By utilizing the relative degree of the system, paper [14] proposed an ILC law for a class of nonlinear systems. In [15], the D-type ILC algorithm with initial state learning was applied to nonlinear time-varying uncertain systems, then the uniform boundedness of the output tracking error was obtained in the sense of λ-norm. Paper [16] considered the sampled-data ILC problem for a class of nonlinear systems with well-defined relative degree. In [17], a P-type learning algorithm was designed to study the ILC problem for nonlinear systems with random packet dropouts. In order to tracking the desired discontinuous trajectory, open-loop and closed-loop P-type ILC algorithms with initial state learning were developed in [18] for impulsive differential equations. In [19], the sampled-data ILC was applied to a class of continuous-time nonlinear systems with iteration-varying lengths. In [20], the adaptive ILC problem was addressed for a class of MIMO discrete-time nonlinear systems with time-iteration-varying parameters. The fault estimation problem via iterative learning was presented in [21] for a class of nonlinear time-delay systems. However, note that most of the nonlinear ILC designs of above are based on Lipschitz condition. This condition limits the applicability of ILC to some extent, consequently, it motivates us to find some weak condition to replace this constraint condition.

During the past years, one-sided Lipschitz condition has been widely applied in observer design, due to the fact that it provides a less conservative condition than the classical Lipschitz one. The one-sided Lipschitz condition was first introduced in [22] to deal with the observer design problem for a class of nonlinear systems. On this basis, the existence conditions of observers for one-sided Lipschitz nonlinear systems were further investigated in [23, 24]. It is noted that these works utilize a modified one-sided Lipschitz condition in which the nonlinearity is scaled via a fixed symmetric matrix. In [25], the concept of one-sided Lipschitz was extended by introducing the quadratically inner-bounded condition. Actually, such nonlinear systems arise in many practical systems, such as single-link flexible joint robotic system, Lorenz system and electromechanical system (see [26–28]). Recently, the control and observer design problems for such one-sided

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Lipschitz nonlinear systems have attracted considerable attention from researchers (see, e.g., [29–35] and references therein). However, little effort has been made on the design of an efficacious controller to achieve tracking of one-sided Lipschitz nonlinear systems. And fortunately, ILC is a good alternative approach, for repetitive systems, that can offer a systematic design to improve tracking performance by iterations on a finite time interval. This observation motivates our study.

This paper investigates the problem of iterative learning algorithm for a class of nonlinear systems. The nonlinearities considered in this paper satisfy the one-sided Lipschitz condition. Then the open-loop and closed-loop P-type learning algorithms with initial state learning are adopted for such nonlinear systems, and furthermore, the convergence conditions of the algorithms are presented. Under the action of the P-type learning algorithms, the uniform convergence of the output tracking error is guaranteed with the aid of λ-norm. This paper is organized as follows: In Section 2, the basic assumptions of this paper are made and the ILC problem for a class of one-sided Lipschitz nonlinear system is proposed. In Section 3, the open-loop and closed-loop P-type learning algorithms with initial state learning are constructed and the corresponding convergence results are presented. In Section 4, an example is given to illustrate the effectiveness of the presented algorithms. Finally, Section 5 draws the conclusion.

Notations: R^n denotes the n-dimensional real Euclidean space. For a given vector or matrix X, $\|X\|$ denotes its Euclidean norm. $\langle\cdot,\cdot\rangle$ is the inner product in R^n, i.e., given $x,y \in R^n$, then $\langle x,y\rangle = x^{\mathrm{T}}y$, where x^{T} denotes the transpose of the column vector x. For a function $h(t) \in R^n$ and a real number $\lambda > 0$, the λ-norm is defined as $\|h\|_\lambda = \sup_{t\in[0,T]} \mathrm{e}^{-\lambda t}\|h(t)\|$ and the s-norm is defined as $\|h\|_s = \sup_{t\in[0,T]}\|h(t)\|$. I denotes an identity matrix with appropriate dimension.

2. PROBLEM DESCRIPTION

Consider the following nonlinear system:

$$\begin{cases}\dot{x}(t) = f(x(t)) + Bu(t),\\ y(t) = Cx(t) + Du(t),\end{cases} \tag{1}$$

where $t \in [0,T]$ denotes the time index; $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$ represent the state, control input and output, respectively; $f(x(t)) \in R^n$ represents a nonlinear function with respect to $x(t)$; B, C and D are real matrices with appropriate dimensions.

Assume that the system (1) is repeatable over $t \in [0,T]$. Rewrite the system (1) as:

$$\begin{cases}\dot{x}_k(t) = f(x_k(t)) + Bu_k(t),\\ y_k(t) = Cx_k(t) + Du_k(t),\end{cases} \tag{2}$$

where k is the iteration index.

In the following, the basic assumptions are first made throughout this paper.

Assumption 1: The nonlinear function $f(x(t))$ is one-sided Lipschitz, i.e., for $\forall x(t), \hat{x}(t) \in \mathbb{R}^n$,

$$\langle f(x(t)) - f(\hat{x}(t)), x(t) - \hat{x}(t)\rangle \le \alpha\|x(t) - \hat{x}(t)\|^2,$$

where $\alpha \in R$ is the one-sided Lipschitz constant.

Remark 1: It should be noted that the constant α can be positive, zero, or negative, while the Lipschitz constant must be positive. Moreover, the one-sided Lipschitz constant is always less than or equal to the traditional Lipschitz constant.

Assumption 2: The desired output trajectory $y_d(t)$ is realizable, i.e., there exists a unique desired control input $u_d(t)$ such that

$$\begin{cases}\dot{x}_d(t) = f(x_d(t)) + Bu_d(t),\\ y_d(t) = Cx_d(t) + Du_d(t),\end{cases}$$

where $x_d(t)$ is the desired state.

Given a desired output trajectory $y_d(t)$, $t \in [0,T]$, the target of this paper is to apply the ILC method to generate the control sequence $u_k(t)$, such that the system output $y_k(t)$ can track the desired one $y_d(t)$ as $k \to \infty$.

To end this section, we give the following lemma, which will be used in the proof of the main theorems.

Lemma 1 [36]**:** Suppose $\{a_k\}$, $\{b_k\}$ are two nonnegative real sequences satisfying

$$a_{k+1} \le \rho a_k + b_k,\ 0 \le \rho < 1,$$

if $\lim\limits_{k\to\infty} b_k = 0$, then $\lim\limits_{k\to\infty} a_k = 0$.

3. MAIN RESULTS

For the system (2), we first adopt an open-loop P-type learning algorithm with initial state learning as follows:

$$\begin{cases}x_{k+1}(0) = x_k(0) + Le_k(0),\\ u_{k+1}(t) = u_k(t) + \Gamma_1 e_k(t),\end{cases} \tag{3}$$

where $L \in R^{n\times p}$, $\Gamma_1 \in R^{m\times p}$ are the learning gain matrices, and $e_k(t) = y_d(t) - y_k(t)$ is the output tracking error at the kth iteration.

Theorem 1: For the system (2) with the learning algorithm (3), and suppose that Assumptions 1-2 are satisfied. If the gain matrices L and Γ_1 can be chosen such that

$$\rho_1 = \max\{\|I - D\Gamma_1 - CL\|, \|I - D\Gamma_1\|\} < 1, \tag{4}$$

then there exists a constant $\lambda > 0$ such that $\lim\limits_{k\to\infty}\|e_k\|_\lambda = 0$, which means that $\lim\limits_{k\to\infty} y_k(t) = y_d(t), t \in [0,T]$.

Proof: Denote $\delta x_k(t) = x_{k+1}(t) - x_k(t)$, $\delta u_k(t) = u_{k+1}(t) - u_k(t)$, $\delta f_k(t) = f(x_{k+1}(t)) - f(x_k(t))$. Note that

$$\begin{aligned} e_{k+1}(t) &= e_k(t) - (y_{k+1}(t) - y_k(t)) \\ &= e_k(t) - C\delta x_k(t) - D\delta u_k(t) \\ &= (I - D\Gamma_1)e_k(t) - C\delta x_k(t). \end{aligned} \tag{5}$$

It is obvious that

$$\begin{aligned} e_{k+1}(0) &= (I - D\Gamma_1)e_k(0) - C\delta x_k(0) \\ &= (I - D\Gamma_1 - CL)e_k(0). \end{aligned}$$

Taking Euclidean norm on both sides of the above expression and combining with (4), we can get

$$\|e_{k+1}(0)\| \le \|I - D\Gamma_1 - CL\|\|e_k(0)\| \le \rho_1\|e_k(0)\|.$$

Because of $\rho_1 < 1$, then we have

$$\lim_{k\to\infty} \|e_k(0)\| = 0. \tag{6}$$

It follows from (2) and (3) that

$$\delta \dot{x}_k(t) = \delta f_k(t) + B\delta u_k(t) = \delta f_k(t) + B\Gamma_1 e_k(t). \tag{7}$$

Left multiplying both sides of the expression (7) by $(\delta x_k(t))^{\mathrm{T}}$ and combining with Assumption 1, it yields

$$\begin{aligned} &\langle \delta \dot{x}_k(t), \delta x_k(t)\rangle \\ &= \langle \delta f_k(t), \delta x_k(t)\rangle + \langle B\Gamma_1 e_k(t), \delta x_k(t)\rangle \\ &\le \alpha\|\delta x_k(t)\|^2 + (B\Gamma_1 e_k(t))^{\mathrm{T}}\delta x_k(t) \\ &\le |\alpha|\|\delta x_k(t)\|^2 + \|B\Gamma_1\|\|\delta x_k(t)\|\|e_k(t)\|, \end{aligned} \tag{8}$$

while

$$\langle \delta \dot{x}_k(t), \delta x_k(t)\rangle = \frac{1}{2}\frac{\mathrm{d}((\delta x_k(t))^{\mathrm{T}}\delta x_k(t))}{\mathrm{d}t}. \tag{9}$$

From (8) and (9), we know

$$\begin{aligned} &\frac{\mathrm{d}((\delta x_k(t))^{\mathrm{T}}\delta x_k(t))}{\mathrm{d}t} \\ &\le 2|\alpha|\|\delta x_k(t)\|^2 + 2\|B\Gamma_1\|\|\delta x_k(t)\|\|e_k(t)\| \\ &\le (2|\alpha|+1)\|\delta x_k(t)\|^2 + \|B\Gamma_1\|^2\|e_k(t)\|^2 \\ &= c_1\|\delta x_k(t)\|^2 + c_2\|e_k(t)\|^2, \end{aligned} \tag{10}$$

where

$$c_1 = 2|\alpha| + 1, c_2 = \|B\Gamma_1\|^2.$$

Integrating both sides of the expression (10) from 0 to t and combining with (3), we can obtain

$$\begin{aligned} &(\delta x_k(t))^{\mathrm{T}}\delta x_k(t) \\ &\le \int_0^t \{c_1\|\delta x_k(\tau)\|^2 + c_2\|e_k(\tau)\|^2\}\mathrm{d}\tau \\ &\quad + (\delta x_k(0))^{\mathrm{T}}\delta x_k(0) \\ &= \int_0^t \{c_1\|\delta x_k(\tau)\|^2 + c_2\|e_k(\tau)\|^2\}\mathrm{d}\tau \\ &\quad + (Le_k(0))^{\mathrm{T}}Le_k(0), \end{aligned}$$

which implies that

$$\begin{aligned} \|\delta x_k(t)\|^2 \le& \int_0^t \{c_1\|\delta x_k(\tau)\|^2 + c_2\|e_k(\tau)\|^2\}\mathrm{d}\tau \\ &+ \|L\|^2\|e_k(0)\|^2, \end{aligned}$$

Applying the Gronwall lemma and according to the definition of the λ-norm, it gives

$$\begin{aligned} \|\delta x_k(t)\|^2 \le& \int_0^t \mathrm{e}^{c_1(t-\tau)}c_2\|e_k(\tau)\|^2\mathrm{d}\tau \\ &+ \|L\|^2\|e_k(0)\|^2\mathrm{e}^{c_1 t} \\ \le& c_2\int_0^t \mathrm{e}^{c_1(t-\tau)}\mathrm{e}^{2\lambda\tau}\{\mathrm{e}^{-\lambda\tau}\|e_k(\tau)\|\}^2\mathrm{d}\tau \\ &+ \|L\|^2\mathrm{e}^{c_1 T}\|e_k(0)\|^2 \\ \le& c_2\mathrm{e}^{c_1 t}\int_0^t \mathrm{e}^{(2\lambda-c_1)\tau}\mathrm{d}\tau\|e_k\|_\lambda^2 \\ &+ \|L\|^2\mathrm{e}^{c_1 T}\|e_k(0)\|^2. \end{aligned}$$

Let $2\lambda > c_1$, then

$$\begin{aligned} &\|\delta x_k(t)\|^2 \\ &\le c_2\mathrm{e}^{c_1 t}\frac{\mathrm{e}^{(2\lambda-c_1)t}-1}{2\lambda-c_1}\|e_k\|_\lambda^2 + \|L\|^2\mathrm{e}^{c_1 T}\|e_k(0)\|^2 \\ &= c_2\frac{\mathrm{e}^{2\lambda t}-\mathrm{e}^{c_1 t}}{2\lambda-c_1}\|e_k\|_\lambda^2 + \|L\|^2\mathrm{e}^{c_1 T}\|e_k(0)\|^2, \end{aligned}$$

we further have

$$\begin{aligned} \|\delta x_k\|_\lambda^2 =& \left(\sup_{t\in[0,T]} \mathrm{e}^{-\lambda t}\|\delta x_k(t)\|\right)^2 \\ =& \sup_{t\in[0,T]} \mathrm{e}^{-2\lambda t}\|\delta x_k(t)\|^2 \\ \le& \frac{c_2}{2\lambda-c_1}\|e_k\|_\lambda^2 \sup_{t\in[0,T]}\{1-\mathrm{e}^{(c_1-2\lambda)t}\} \\ &+ \|L\|^2\mathrm{e}^{c_1 T}\|e_k(0)\|^2 \\ \le& \frac{c_2}{2\lambda-c_1}\|e_k\|_\lambda^2 + \|L\|^2\mathrm{e}^{c_1 T}\|e_k(0)\|^2 \\ \le& \left(\sqrt{\frac{c_2}{2\lambda-c_1}}\|e_k\|_\lambda + \|L\|\sqrt{\mathrm{e}^{c_1 T}}\|e_k(0)\|\right)^2. \end{aligned}$$

Therefore,

$$\|\delta x_k\|_\lambda \le \sqrt{\frac{c_2}{2\lambda-c_1}}\|e_k\|_\lambda + \|L\|\sqrt{\mathrm{e}^{c_1 T}}\|e_k(0)\|. \tag{11}$$

It follows from (4) and (5) that

$$\|e_{k+1}\|_\lambda \le \|I - D\Gamma_1\|\|e_k\|_\lambda + \|C\|\|\delta x_k\|_\lambda$$

$$\leq \rho_1 \|e_k\|_\lambda + \|C\|\|\delta x_k\|_\lambda. \tag{12}$$

Substituting (11) into (12) results

$$\|e_{k+1}\|_\lambda \leq \hat{\rho}\|e_k\|_\lambda + c_3\|e_k(0)\|, \tag{13}$$

where

$$\hat{\rho} = \rho_1 + \|C\|\sqrt{\frac{c_2}{2\lambda - c_1}},\ c_3 = \|C\|\|L\|\sqrt{\mathrm{e}^{c_1 T}}.$$

Since $0 \leq \rho_1 < 1$ by (4), it is possible to choose λ large enough so that $\hat{\rho} < 1$. Then, it follows from (6), (13) and Lemma 1 that

$$\lim_{k\to\infty} \|e_k\|_\lambda = 0.$$

Note that $\|e_k\|_s \leq \mathrm{e}^{\lambda T}\|e_k\|_\lambda$, so we have $\lim_{k\to\infty}\|e_k\|_s = 0$, which implies that

$$\lim_{k\to\infty} y_k(t) = y_d(t),\ t \in [0,T].$$

This completes the proof. □

Now, we adopt a closed-loop P-type learning algorithm with initial state learning for the system (2) as follows:

$$\begin{cases} x_{k+1}(0) = x_k(0) + Le_k(0), \\ u_{k+1}(t) = u_k(t) + \Gamma_2 e_{k+1}(t), \end{cases} \tag{14}$$

where $L \in \mathbb{R}^{n\times p}$, $\Gamma_2 \in \mathbb{R}^{m\times p}$ are the learning gain matrices, and $e_{k+1}(t) = y_d(t) - y_{k+1}(t)$ is the output tracking error at the $(k+1)$th iteration.

Theorem 2: For the system (2) with the learning algorithm (14), and suppose that Assumptions 1-2 are satisfied. If the gain matrices L and Γ_2 can be chosen such that

$$\begin{aligned}\rho_2 = &\max\{\|(I+D\Gamma_2)^{-1}(I-CL)\|,\ \|(I+D\Gamma_2)^{-1}\|\} \\ &< 1,\end{aligned} \tag{15}$$

then there exists a constant $\lambda > 0$ such that $\lim_{k\to\infty}\|e_k\|_\lambda = 0$, which means that $\lim_{k\to\infty} y_k(t) = y_d(t), t \in [0,T]$.

Proof: From (2) and (14), we have

$$\begin{aligned} e_{k+1}(t) &= e_k(t) - (y_{k+1}(t) - y_k(t)) \\ &= e_k(t) - C\delta x_k(t) - D\delta u_k(t) \\ &= e_k(t) - C\delta x_k(t) - D\Gamma_2 e_{k+1}(t), \end{aligned}$$

that is,

$$(I + D\Gamma_2)e_{k+1}(t) = e_k(t) - C\delta x_k(t).$$

On account of the matrix $I + D\Gamma_2$ is nonsingular, then we can get

$$e_{k+1}(t) = (I + D\Gamma_2)^{-1}(e_k(t) - C\delta x_k(t)). \tag{16}$$

Furthermore, we can derive

$$\begin{aligned} e_{k+1}(0) &= (I + D\Gamma_2)^{-1}(e_k(0) - C\delta x_k(0)) \\ &= (I + D\Gamma_2)^{-1}(I - CL)e_k(0). \end{aligned}$$

Taking Euclidean norm on both sides of the above expression and combining with (15), it becomes

$$\begin{aligned} \|e_{k+1}(0)\| &\leq \|(I + D\Gamma_2)^{-1}(I - CL)\|\|e_k(0)\| \\ &\leq \rho_2\|e_k(0)\|. \end{aligned}$$

Since $\rho_2 < 1$, we further have

$$\lim_{k\to\infty} \|e_k(0)\| = 0. \tag{17}$$

It follows from (2) and (14) that

$$\begin{aligned} \delta\dot{x}_k(t) &= \delta f_k(t) + B\delta u_k(t) \\ &= \delta f_k(t) + B\Gamma_2 e_{k+1}(t). \end{aligned} \tag{18}$$

Left multiplying both sides of the expression (18) by $(\delta x_k(t))^{\mathrm{T}}$ and combining with Assumption 1, it gives

$$\begin{aligned} &\langle \delta\dot{x}_k(t), \delta x_k(t)\rangle \\ &= \langle \delta f_k(t), \delta x_k(t)\rangle + \langle B\Gamma_2 e_{k+1}(t), \delta x_k(t)\rangle \\ &\leq \alpha\|\delta x_k(t)\|^2 + (B\Gamma_2 e_{k+1}(t))^{\mathrm{T}}\delta x_k(t) \\ &\leq |\alpha|\|\delta x_k(t)\|^2 + \|B\Gamma_2\|\|\delta x_k(t)\|\|e_{k+1}(t)\|. \end{aligned} \tag{19}$$

Obviously, (19) together with (9) implies

$$\begin{aligned} &\frac{\mathrm{d}((\delta x_k(t))^{\mathrm{T}}\delta x_k(t))}{\mathrm{d}t} \\ &\leq 2|\alpha|\|\delta x_k(t)\|^2 + 2\|B\Gamma_2\|\|\delta x_k(t)\|\|e_{k+1}(t)\| \\ &\leq (2|\alpha|+1)\|\delta x_k(t)\|^2 + \|B\Gamma_2\|^2\|e_{k+1}(t)\|^2 \\ &= c_1\|\delta x_k(t)\|^2 + c_4\|e_{k+1}(t)\|^2, \end{aligned}$$

where $c_4 = \|B\Gamma_2\|^2$. Similar to the procedure as that (10) to (11) of Theorem 1, we can obtain

$$\|\delta x_k\|_\lambda \leq \sqrt{\frac{c_4}{2\lambda - c_1}}\|e_{k+1}\|_\lambda + \|L\|\sqrt{\mathrm{e}^{c_1 T}}\|e_k(0)\|. \tag{20}$$

It follows from (15) and (16) that

$$\begin{aligned} \|e_{k+1}\|_\lambda \leq &\|(I + D\Gamma_2)^{-1}\|\|e_k\|_\lambda \\ &+ \|(I + D\Gamma_2)^{-1}C\|\|\delta x_k\|_\lambda \\ \leq &\rho_2\|e_k\|_\lambda + \|(I + D\Gamma_2)^{-1}C\|\|\delta x_k\|_\lambda. \end{aligned} \tag{21}$$

Substituting (20) into (21) yields

$$\begin{aligned} &\|e_{k+1}\|_\lambda \\ &\leq \rho_2\|e_k\|_\lambda + \|(I + D\Gamma_2)^{-1}C\|\sqrt{\frac{c_4}{2\lambda - c_1}}\|e_{k+1}\|_\lambda \\ &\quad + \|(I + D\Gamma_2)^{-1}C\|\|L\|\sqrt{\mathrm{e}^{c_1 T}}\|e_k(0)\|. \end{aligned}$$

Take the λ of above such that

$$\|(I+D\Gamma_2)^{-1}C\|\sqrt{\frac{c_4}{2\lambda-c_1}}<1,$$

then we have

$$\|e_{k+1}\|_\lambda \le \bar{\rho}\|e_k\|_\lambda + c_5\|e_k(0)\|, \tag{22}$$

where

$$\bar{\rho}=\frac{\rho_2}{1-\|(I+D\Gamma_2)^{-1}C\|\sqrt{\frac{c_4}{2\lambda-c_1}}},$$
$$c_5=\frac{\|(I+D\Gamma_2)^{-1}C\|\|L\|\sqrt{e^{c_1T}}}{1-\|(I+D\Gamma_2)^{-1}C\|\sqrt{\frac{c_4}{2\lambda-c_1}}}.$$

Due to $0\le\rho_1<1$, so we can choose λ large enough so that $\bar{\rho}<1$. From (17), (22) and Lemma 1, we can obtain

$$\lim_{k\to\infty}\|e_k\|_\lambda=0.$$

Since $\|e_k\|_s\le e^{\lambda T}\|e_k\|_\lambda$, we know $\lim_{k\to\infty}\|e_k\|_s=0$, it is obvious that

$$\lim_{k\to\infty}y_k(t)=y_d(t),\ t\in[0,T].$$

This completes the proof. □

Remark 2: In the process of ILC design, the P-type learning algorithm is needed for systems with direct feed-through term. For the open-loop P-type learning algorithm (3), the last cycle's input is updated by the last cycle's information to generate the current cycle's input. And for the closed-loop P-type learning algorithm (14), the last cycle's input is updated by the measurement data from the current cycle to generate the current cycle's input. On account of the causality, the output of the system is unavailable and only be replaced by its estimated value, which results in the degradation of tracking performance. However, the closed-loop learning algorithms have the features such as wide ranges in choosing learning gain and the improvement of convergence rate, in contrast to their open-loop counterparts.

4. NUMERICAL EXAMPLE

In this section, an example is given to demonstrate the effectiveness of the proposed P-type learning algorithms with initial state learning. Construct the following nonlinear system:

$$\begin{cases}\dot{x}_k(t)=f(x_k(t))+Bu_k(t),\\ y_k(t)=Cx_k(t)+Du_k(t),\end{cases}$$

where $t\in[0,1]$, and $x_k(t)=[x_{1k}^{\mathrm{T}}(t)\quad x_{2k}^{\mathrm{T}}(t)]^{\mathrm{T}}$,

$$B=\begin{bmatrix}1&1\\3&1\end{bmatrix},\ C=\begin{bmatrix}1&0\\0&2\end{bmatrix},\ D=\begin{bmatrix}1&0\\0&1\end{bmatrix},$$

$$f(x_k(t))=\begin{bmatrix}-x_{1k}(t)(x_{1k}^2(t)+x_{2k}^2(t))\\-x_{2k}(t)(x_{1k}^2(t)+x_{2k}^2(t))\end{bmatrix}.$$

We know from [18] that the nonlinear function $f(x_k(t))$ is one-sided Lipschitz in R^2 with $\alpha=0$. Take the given desired output trajectory as:

$$y_d(t)=\begin{bmatrix}\sin(2\pi t)\\t^2(5t^2-1)\end{bmatrix}.$$

Furthermore, set the initial condition and the initial control as:

$$x_0(0)=\begin{bmatrix}1\\2\end{bmatrix},\ u_0(t)=\begin{bmatrix}0\\0\end{bmatrix}.$$

i) For the open-loop P-type learning algorithm with initial state learning, take the gain matrices

$$L=\begin{bmatrix}1&0\\0&0.5\end{bmatrix},\ \Gamma_1=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},$$

then we can compute that

$$\rho_1=\max\{\|I-D\Gamma_1-CL\|,\|I-D\Gamma_1\|\}=0.5<1.$$

Figs. 1 and 2 give the tracking situations of the system outputs $y_k^{(1)}(t)$ and $y_k^{(2)}(t)$ to the desired trajectories at the 4th, 6th and 12th iterations, respectively. From Fig. 3, we know that the maximum output tracking error is tend to zero as the iteration number increases by using the learning algorithm (3).

ii) For the closed-loop P-type learning algorithm with initial state learning, take the gain matrices

$$L=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},\ \Gamma_2=\begin{bmatrix}1&0\\0&1\end{bmatrix},$$

then we have

$$\begin{aligned}\rho_2&=\max\{\|(I+D\Gamma_2)^{-1}(I-CL)\|,\|(I+D\Gamma_2)^{-1}\|\}\\&=0.5<1.\end{aligned}$$

From Figs.4 and 5, it is obvious that the output trajectories $y_k^{(1)}(t)$ and $y_k^{(2)}(t)$ at the 9th iteration are close to the desired ones. From Fig. 6, we can see that the uniform convergence of the output tracking error is guaranteed under the action of the learning algorithm (14).

From the above simulation results, we know that both algorithms (3) and (14) are effective for the one-sided Lipschitz nonlinear system (2). At the 9th iteration, the values of $||e_k^{(i)}||_s$ $(i=1,2)$ are 2.46×10^{-2}, 1.14×10^{-2} by using the open-loop P-type learning algorithm (3), and the values of $||e_k^{(i)}||_s$ $(i=1,\ 2)$ are 3.3×10^{-3}, 6.1×10^{-3} by employing the closed-loop P-type learning algorithm (14). It can be seen from Figs. 3 and 6 that the closed-loop P-type learning algorithm performs better than the open-loop one in the speed of convergence.

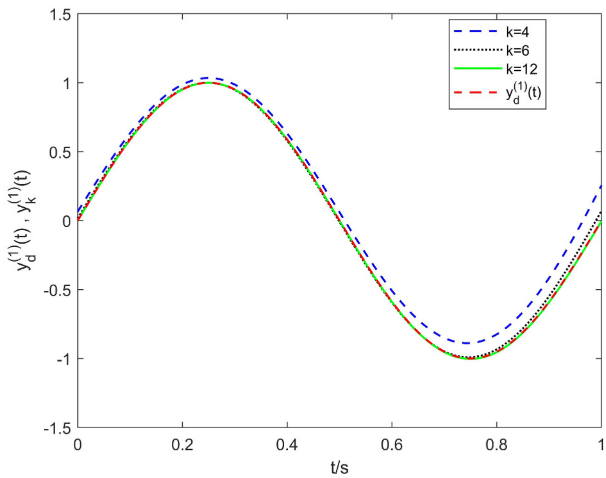

Fig. 1. The outputs $y_k^{(1)}(t)$ and $y_d^{(1)}(t)$ under the action of the open-loop P-type learning algorithm (3).

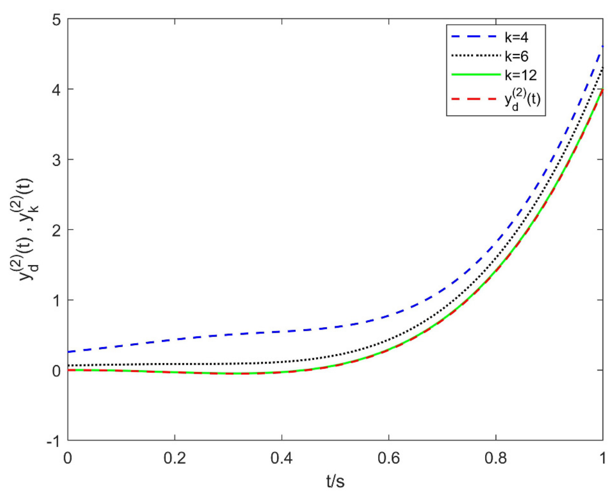

Fig. 2. The outputs $y_k^{(2)}(t)$ and $y_d^{(2)}(t)$ under the action of the open-loop P-type learning algorithm (3).

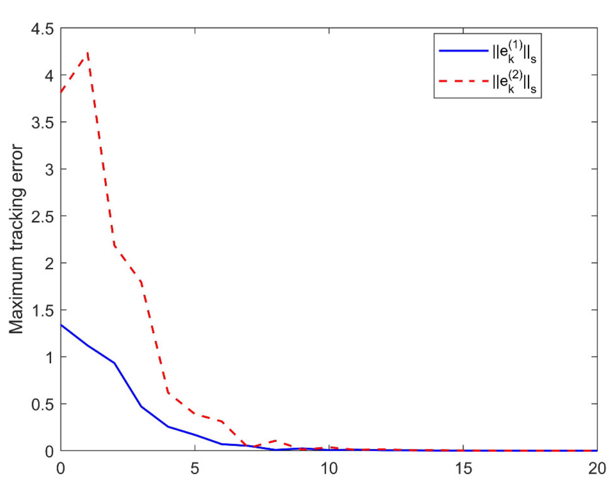

Fig. 3. The asymptotic convergence of tracking error with iterations.

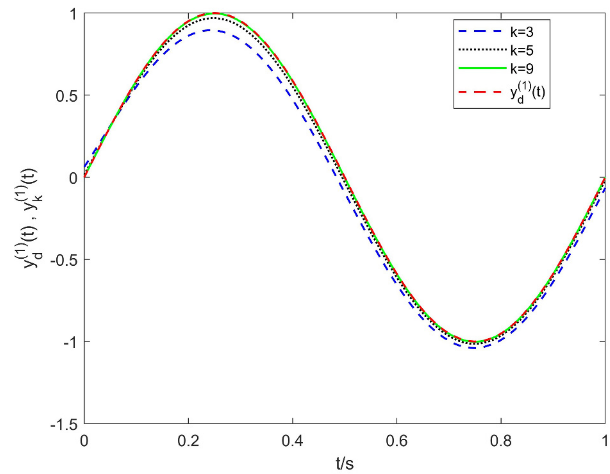

Fig. 4. The outputs $y_k^{(1)}(t)$ and $y_d^{(1)}(t)$ under the action of the closed-loop P-type learning algorithm (14).

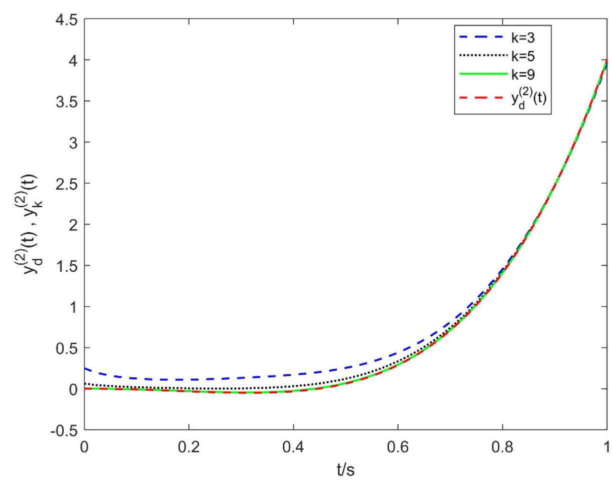

Fig. 5. The outputs $y_k^{(2)}(t)$ and $y_d^{(2)}(t)$ under the action of the closed-loop P-type learning algorithm (14).

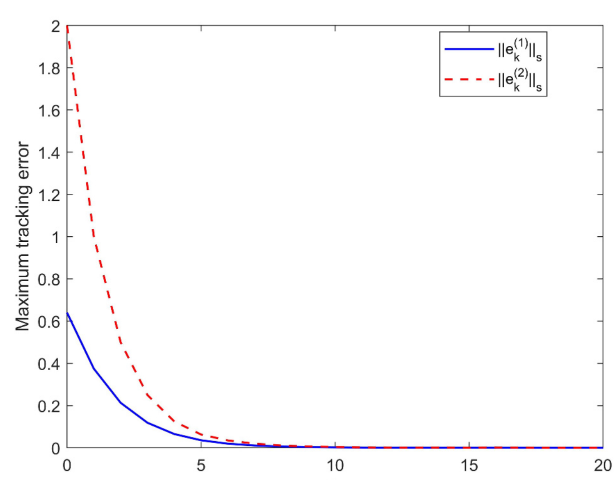

Fig. 6. The asymptotic convergence of tracking error with iterations.

5. CONCLUSION

This paper deals with the iterative learning control problem for a class of nonlinear systems with one-sided Lipschitz condition. Then the open-loop and closed-loop P-type learning algorithms with initial state learning are adopted for such nonlinear systems, and the corresponding convergence conditions of the P-type learning algorithms are established. We show that both algorithms can ensure the output tracking error converges to zero on the whole time interval. As well known, time-delay is often encountered in many practical control problems. In the next research phase, we will investigate the ILC problem for time-delay systems with one-sided Lipschitz nonlinearity.

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