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Decentralized Fault-tolerant Resilient Control for Fractional-order Interconnected Systems with Input Saturation

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Abstract: This paper investigates the problem of robust decentralized fault-tolerant resilient control for fractional-order large-scale interconnected uncertain system, and the problem considered here is subject to mixed H_∞ and passivity performance constraint, external disturbances, controller perturbations and control input saturation. Based on the Lyapunov approach, the sufficient conditions are derived in terms of linear matrix inequalities to ensure the asymptotic stabilization of the fractional-order large-scale system with a prespecified mixed H_∞ and passivity performance index. The main objective of this work is to design a robust decentralized fault-tolerant resilient controller which compensates both actuator fault and input saturation in its design for obtaining the required result. Finally, a numerical example is included to illustrate the effectiveness of the designed control law. The simulation results reveal that our proposed controller not only can effectively deal with actuator faults, but also has very good robustness for input saturation and external disturbances.

Keywords: Decentralized control, fractional-order large-scale systems, input saturation, mixed H_∞ and passivity performance, nonlinear actuator fault.

1. INTRODUCTION

Fractional calculus is a generalization of traditional integer order calculus. In recent years, it has been proved that the fractional calculus model can better describe many real-world physical systems since it has greater flexibility and accuracy as compared with the conventional integer order calculus. In the last two decades, fractional-order control systems have received significant attention, since it can used to well characterize many industrial problems such as control of autonomous vehicles, flexible robot manipulator, signal processing, thermal-diffusion, non-holonomic systems, chaotic systems and so on [1–3]. Moreover, it is noted that fractional-order controller can enhance the control performance and can maintain strong robustness level of the dynamical control systems. On the other hand, there is an exponential rise in the control of dynamical systems which is composed of large-scale interconnected systems. A large-scale system consists of a set of interconnected subsystems, characterized by a large number of state, input variables and parametric uncertainties [4–7]. It should be mentioned that the interconnections among the subsystems plays a vital role in the dynamics of large-scale systems. It has great importance and wide range of applications in power systems, transportation networks, industrial process systems, multiple aircraft formation systems, economic and social systems. For a fractional-order large-scale interconnected system, maintaining the stability and stabilization of the closed-loop system is an important task. Therefore, the stability and stabilization problem for the fractional-order large-scale interconnected system has received an increasing attention [8]. Three main control schemes are there for the large-scale systems and they are, centralized, decentralized and distributed controls. Precisely, the decentralized control scheme has attracted great attention from researchers, since it reduces the computational burden, storage requirements and easy debugging. There are many valuable results regarding the decentralized control of large-scale systems is reported in [9–11]. Tong *et al.* [12] investigated the adaptive fuzzy decentralized fault-tolerant control of nonlinear large-scale systems subject to actuator failure. The authors in [13], discussed the decentralized sliding mode control for a class of fractional-order large-scale nonlinear systems. In [14], a robust decentralized state feedback controller is proposed for a class of

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perturbed fractional-order linear interconnected systems subject to structure and unstructure perturbations.

In practical large-scale interconnected systems, the components may undergo sudden failures or one of the subsystems may encounter failure. Such failures of one or more subsystems may lead to poor performance or even instability of the overall interconnected large-scale systems. So, it is necessary and important to design a fault-tolerant controller, that will guarantee the reliable performance of the system against the failure of individual subsystems [15–17]. Recently, Li *et al*. [18] developed an adaptive fuzzy output-constrained fault-tolerant control for nonlinear stochastic large-scale systems. However, the number of existing results taking the component failures into account is still limited with the linear fault representation and it is essential to add some nonlinear features to the controller term in order to achieve the better performance. In general, uncertainties are unavoidable on the design of controller due to roundoff errors and unknown noises. If the uncertainties are not handled properly in control design, then they could deteriorate system performance. Therefore, the controller should be designed in such a way that it will be insensitive and robust against its own parameter variations, which is called as non-fragile controller [19–21]. Recently, non-fragile control problem for fractional-order systems has received much attention due to its potential applications. Due to some physical constraints, the saturation in control input is unavoidable and it degrades the system performance or may destabilize the closed-loop system. In [22], the asymptotic stabilization problem of fractional-order linear systems in the presence of input saturation is studied. A decentralized adaptive output feedback controller is designed in [23] for a class of large-scale time-delay systems with input constraints. The authors in [24] proposed a novel decentralized adaptive neural controller for a class of uncertain nonlinear large-scale interconnected time-delay systems with input saturation.

On the other hand, the mixed H_∞ and passivity performance can systematically make the control design when compared with the individual H_∞ and passivity setting. It should be noted that the mixed H_∞ and passive control unify the H_∞ control and passivity control in a single framework. The stabilization problem of fractional-order interconnected uncertain systems with input saturation and nonlinear actuator faults via fault-tolerant resilient controller with prescribed mixed H_∞ and passivity index has not been reported in the literature so far, which motivates the present study. The main contributions of this study is as follows:

- Stabilization of fractional-order large-scale interconnected system is explored subject to external disturbances, controller perturbation and control input saturation.
- A robust decentralized fault-tolerant resilient controller is designed for the asymptotic stability of the proposed system with a desired mixed H_∞ and passivity performance index.
- Sufficient conditions are derived in terms of linear matrix inequalities and by solving those LMIs, a feasible solution can be obtained for the addressed problem.

The rest of this paper is organized as follows: In Section 2, problem formulation and some preliminary results which will be used to prove the main results are given. Section 3 describes the main results that include the asymptotic stabilization of the proposed system through the designed control. A numerical example and conclusion is given in Section 4 and Section 5, respectively.

2. PROBLEM FORMULATION

In this section, we consider the stabilization problem for a class of fractional-order uncertain continuous-time large-scale systems with input saturations and actuator faults, which is composed of N interconnected subsystems. The state dynamics of the i^{th} subsystem S_i of fractional-order uncertain continuous-time large-scale systems is described by

$$\begin{aligned}\frac{d^\alpha x_i(t)}{dt^\alpha} &= (A_{ii}+\Delta A_{ii}(t))x_i(t)+(B_i+\Delta B_i(t))\mathrm{sat}(u_i^F(t)) \\ &\quad + \sum_{j=1,j\neq i}^{\mathcal{N}} (A_{ij}+\Delta A_{ij}(t))x_j(t)+D_{wi}w_i(t), \\ z_i(t) &= C_i x_i(t)+D_i\mathrm{sat}(u_i^F(t)),\ \ i=1,2,\cdots,N, \end{aligned} \tag{1}$$

where $\alpha \in \mathbb{R}$ is the fractional commensurate order and $0<\alpha<1$, $x_i(t)\in\mathbb{R}^{n_i}$ is the state, $u_i^F(t)\in\mathbb{R}^{p_i}$ is the control input, $w_i(t)\in\mathbb{R}^{q_i}$ denotes the disturbance which belongs to $l_2[0,\infty)$ and $z_i(t)\in\mathbb{R}^{m_i}$ is the controlled output of the i^{th} subsystem. A_{ii}, B_i, D_{wi}, C_i and D_i are known system matrices of appropriate dimensions. The interconnection between the i^{th} subsystem to j^{th} subsystem is given by the matrix A_{ij}. Further, $\Delta A_{ii}(t), \Delta B_i(t)$ and $\Delta A_{ij}(t)$ are the norm bounded time-varying uncertainties and are of the form,

$$\begin{aligned}&\Delta A_{ii}(t)=\mathcal{M}_{aii}\mathcal{F}_i(t)\mathcal{N}_{aii},\ \ \Delta B_i(t)=\mathcal{M}_{bi}\mathcal{F}_i(t)\mathcal{N}_{bi},\\ &\Delta A_{ij}(t)=\mathcal{M}_{aij}\mathcal{F}_i(t)\mathcal{N}_{aij},\end{aligned} \tag{2}$$

where $\mathcal{M}_{aii}, \mathcal{M}_{bi}, \mathcal{M}_{aij}, \mathcal{N}_{aii}, \mathcal{N}_{bi}$ and $\mathcal{N}_{aij}$ are appropriate dimensional known constant real matrices. Also, $\mathcal{F}_i(t)$ is the uncertain matrix function which satisfy $\mathcal{F}_i^T(t)\mathcal{F}_i(t)\leq I$. The characteristics of actuators may not be linear always. So, linear controllers are unable to achieve the desired system performance. Also, due to some physical limitations, the saturation in input is unavoidable. Further, sat : $\mathbb{R}^{m_i}\to\mathbb{R}^{m_i}$ are vector-valued saturation functions defined as $\mathrm{sat}(u_i^F(t)) =$

$[\text{sat}(u^F_{1i}(t))\ \text{sat}(u^F_{2i}(t))\ \cdots\ \text{sat}(u^F_{mi}(t))]^T$ with $\text{sat}(u^F_{ji}(t)) = \text{sign}(u^F_{ji}(t))\min\{1, |u^F_{ji}(t)|\}$, $j = 1, 2, \cdots, m$. In order to tackle the nonlinear variations in actuators, in this paper we will design a fault-tolerant controller for the system (1) in the following form

$$u^F_i(t) = Gu_i(t) + f_i(u_i(t)), \tag{3}$$

where G denotes the actuator fault matrix, $f_i(\cdot)$ represents the nonlinear actuator fault and $u_i(t) = K_i x_i(t)$, where K_i is the control gain matrix of the i^{th} subsystem to be computed. The actuator fault matrix is defined by $G = \text{diag}\{g_1, g_2, \cdots, g_m\}$, satisfying $g_i = [\underline{g}_i, \bar{g}_i]$, $i = 1, 2, \cdots, n$, $0 \le \underline{g}_i \le g_i \le \bar{g}_i \le 1$. Let $\underline{G} = \text{diag}\{\underline{g}_1, \underline{g}_2, \cdots, \underline{g}_m\}$, $G = \text{diag}\{g_1, g_2, \cdots, g_m\}$ and $\bar{G} = \text{diag}\{\bar{g}_1, \bar{g}_2, \cdots, \bar{g}_m\}$. Here, the variables g_k, $k = 1, 2, \cdots, m$ specify the failures of the actuators. Let us define $G_0 = \frac{\bar{G}+\underline{G}}{2}$ and $G_1 = \frac{\bar{G}-\underline{G}}{2}$. Then, the fault matrix G can be expressed as

$$G = G_0 + G_1\Sigma, \tag{4}$$

where $\Sigma = \text{diag}\{\sigma_{1i}, \sigma_{2i}, \cdots, \sigma_{mi}\} \in \mathbb{R}^{m_i \times m_i}$, $-1 \le \sigma_{ji} \le 1$, $j = 1, 2, \cdots, m$. Let $f_i(u_i(t)) = [f_{1i}(u_i(t))\ f_{2i}(u_i(t))\ f_{mi}(u_i(t))]^T$ and it satisfies $|f_{ji}(u_i(t))| \le \sqrt{\beta_j}|u_i(t)|$, $j = 1, 2, \cdots, m$, $\beta_j > 0$ and hence

$$f^T_i(u_i(t))f_i(u_i(t)) \le u^T_i(t)Lu_i(t), \tag{5}$$

where $L = \text{diag}\{\beta_1, \beta_2, \cdots, \beta_m\}$.

Now, we will introduce some basic definitions and lemmas which will be useful in deriving the main results.

Definition 1 [22]**:** The Caputo derivative is defined as $\frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{\frac{d^m f(\tau)}{dt^m}}{(t-\tau)^{\alpha-m+1}}d\tau$, where $m \in \mathbb{N}$ satisfying $m-1 < \alpha < m$, $\alpha \in \mathbb{R}^+$ and $\Gamma(\cdot)$ is the Gamma function defined by $\Gamma(z) = \int_0^t e^{-t}t^{z-1}dt$.

Definition 2 [13]**:** The α^{th} order fractional integral of the function $f(t)$ with initial value t_0 is defined as $I^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^t (t-\tau)^{\alpha-1}f(\tau)d\tau$, $\alpha > 0$.

Definition 3 [25]**:** The uncertain fractional-order large-scale system is asymptotically stable with a mixed H_∞ and passivity performance index γ, if there exist $\theta \in [0,1]$ and under zero initial condition, the output $z_i(t)$ satisfies

$$\begin{aligned}&\int_0^t \left[\gamma^{-1}\theta z^T_i(t)z_i(t) - 2(1-\theta)z^T_i(t)w_i(t)\right]dt\\ &\le \int_0^t w^T_i(t)w_i(t) \ \text{ for all } \ t > 0.\end{aligned} \tag{6}$$

Definition 4 [2]**:** For a linear system $x(t) = h(t)\nu(t)$, where $\nu(t)$ is the input, $x(t)$ is the output and $h(t)$ is the impulse response of the system, then for the elementary frequency ω, $\eta(\omega)$ is called the diffusive representation (or frequency weighting function) of $h(t)$ and it can be expressed as $h(t) = \int_0^\infty \eta(\omega)e^{-\omega t}d\omega$.

Remark 1: The fractional integration $I^n f(t)$ can be interpreted as the convolution of $f(t)$ with the impulse response $h(t)$. That is, $I^n f(t) = h(t) * f(t)$, where $*$ is the convolution operator, $h(t) = t^{\alpha-1}/\Gamma(\alpha)$ and also the diffusive representation $\eta(\omega)$ is defined as $\eta(\omega) = \frac{\sin(\alpha\pi)}{\pi}\omega^{-\alpha}$. Moreover, if $z(\omega,t)$ is the frequency distributed state of the fractional integrator, then it satisfies the equation $\frac{\partial z(\omega,t)}{\partial t} = -\omega z(\omega,t) + \nu(t)$ and the output $x(t)$ of the fractional integrator can be expressed as the weighted integral ranging from 0 to ∞, $x(t) = \displaystyle\int_0^\infty \eta(\omega)z(\omega,t)d\omega$.

Lemma 1 [2]**:** Due to the continuous frequency distributed model of the fractional integrator, the fractional order equation $D^\alpha x(t) = Ax(t)$ can be expressed as, $\frac{\partial z(\omega,t)}{\partial t} = -\omega z(\omega,t)$, $x(t) = \displaystyle\int_0^\infty \eta(\omega)z(\omega,t)d\omega$.

Lemma 2 [26]**:** Let $\mathcal{M}_i$, $\mathcal{N}_i$ and $\mathcal{F}_i(t)$ be real constant matrices of appropriate dimensions with $\mathcal{F}_i(t)$ satisfying $\mathcal{F}^T_i(t)\mathcal{F}_i(t) \le I$, then there exists a scalar $\varepsilon > 0$ such that $\mathcal{M}_i\mathcal{F}_i(t)\mathcal{N}_i + (\mathcal{M}_i\mathcal{F}_i(t)\mathcal{N}_i)^T \le \varepsilon^{-1}\mathcal{M}_i\mathcal{M}^T_i + \varepsilon\mathcal{N}^T_i\mathcal{N}_i$.

Let us define $\mathcal{H}_i \in \mathbb{R}^{m_i \times n_i}$ and a polyhedral $\mathcal{L}(\mathcal{H}_i) = \{x_i(t) \in \mathbb{R}^{n_i} : |h_{ij}(x_i(t))| \le 1, j = 1, 2, \cdots, m\}$, where h_{ij} represents the j^{th} row of $\mathcal{H}_i$. Let $P_i \in \mathbb{R}^{n_i \times n_i}$ be a symmetric matrix, and δ be a positive scalar, then define an ellipsoid $\Omega(P_i, \delta) = \{x_i(t) \in \mathbb{R}^{n_i} : x^T_i(t)P_i x_i(t) \le \delta\}$.

Let $\mathcal{D}$ be the set of $m_i \times m_i$ diagonal matrices with diagonal elements either 1 or 0. If each of its elements is labeled as $\mathcal{D}_s$, $s = 1, 2, \cdots, 2^m$ and denote $\mathcal{D}^-_s = 1 - \mathcal{D}_s$, then both $\mathcal{D}_s, \mathcal{D}^-_s \in \mathcal{D}$.

Lemma 3 [27]**:** Let $K_i, H_i \in \mathbb{R}^{m_i \times n_i}$. Then for any $x_i(t) \in \mathcal{L}(\mathcal{H}_i)$, we have $\text{sat}(u_i(t)) \in Co\{\mathcal{D}_s u_i(t) + \mathcal{D}^-_s H_i x_i(t), s = 1, 2, \cdots, 2^m\}$ or equivalently, $\text{sat}(u_i(t)) = \sum_{s=1}^{2^m}\zeta_s(\mathcal{D}_s u_i(t) + \mathcal{D}^-_s H_i x_i(t))$, where Co represents the convex hull, ζ_s for $s = 1, 2, \cdots, 2^m$ are some scalars which satisfy $0 \le \zeta_s \le 1$ and $\sum_{s=1}^{2^m}\zeta_s = 1$.

Lemma 4 [26]**:** For $x, y \in \mathbb{R}^n$ and for any scalar $\varepsilon > 0$, the inequality $2x^T y \le \varepsilon^{-1}x^T x + \varepsilon y^T y$ holds.

From (3) and Lemma 3, we have, $\text{sat}(u^F_i(t)) = \sum_{s=1}^{2^m}\zeta_s(\mathcal{D}_s u^F_i(t) + \mathcal{D}^-_s H_i x_i(t))$.

Also, by considering the controller gain fluctuations into account, the system (1) can be written as

$$\begin{aligned}\frac{d^\alpha x_i(t)}{dt^\alpha} &= \tilde{A}x_i(t) + \tilde{B}f_i(u_i(t)) + \hat{A}x_j(t) + D_{wi}w_i(t),\\ z_i(t) &= \tilde{C}x_i(t) + \tilde{D}f_i(u_i(t)),\end{aligned} \tag{7}$$

where $\tilde{A} = (A_{ii} + \Delta A_{ii}(t)) + \sum\limits_{s=1}^{2^m}\zeta_s(B_i + \Delta B_i(t))\left[\mathcal{D}_s G\hat{K}_i + \mathcal{D}^-_s\hat{H}_i\right]$, $\tilde{B} = \sum\limits_{s=1}^{2^m}\zeta_s(B_i + \Delta B_i(t))\mathcal{D}_s$, $\hat{A} = \sum\limits_{j=1, j\ne i}^{N}(A_{ij} + \Delta A_{ij}(t))$, $\tilde{C} = C_i + D_i\sum\limits_{s=1}^{2^m}\zeta_s[\mathcal{D}_s G\hat{K}_i + \mathcal{D}^-_s\hat{H}_i]$, $\tilde{D} = D_i\sum\limits_{s=1}^{2^m}\zeta_s\mathcal{D}_s$, $\hat{K}_i = K_i + \Delta K_i(t)$ and $\hat{H}_i = H_i + \Delta H_i(t)$. Here,

$\Delta K_i(t)=\mathcal{M}_{ki}\mathcal{F}_i(t)\mathcal{N}_{ki}$, $\Delta H_i(t)=\mathcal{M}_{hi}\mathcal{F}_i(t)\mathcal{N}_{hi}$, $\mathcal{M}_{ki}$, $\mathcal{N}_{ki}$, $\mathcal{M}_{hi}$, $\mathcal{N}_{hi}$ are known matrices of appropriate dimensions and $\mathcal{F}_i(t)$ is unknown matrix function satisfying $\mathcal{F}_i^T(t)\mathcal{F}_i(t)\le I$.

3. MAIN RESULTS

In this section, a new set of criteria in terms of LMI is developed for the design of decentralized fault-tolerant resilient controller which can guarantee the robust asymptotic stabilization of the fractional-order large-scale interconnected uncertain system with a prespecified mixed H_∞ and passivity performance index. First, when the actuator faults and control gain matrices are known, a set of sufficient conditions is derived to ensure the asymptotic stability of the closed-loop system (7) without considering the gain perturbations. Further, a procedure is developed to design the controller gain parameters of (3).

Theorem 1: For given scalars $\varepsilon_1>0$, $\rho>0$, $\gamma>0$, $\theta>0$, known fault matrix G, known gain matrices K_i and H_i, the fractional-order large-scale uncertain closed-loop system (7) with $0<\alpha<1$ is asymptotically stable with a mixed H_∞ and passivity performance index, if there exist positive definite matrices P_i, $i=1,2,\cdots,N$ and scalars ε_{2i}, ε_{3i} and ε_{4i}, such that, for $s=1,2,\cdots,2^m$ the following LMI together with condition (9) holds:

$$\Psi=[\Psi]_{15\times 15}<0, \tag{8}$$

$$\Omega(P_i,\delta)\subset\mathcal{L}(\mathcal{H}_i), \tag{9}$$

where $\Psi_{1,1}=P_iA_{ii}+P_iB_i\mathcal{K}_i+A_{ii}^TP_i^T+\mathcal{K}_i^TB_i^TP_i^T$, $\Psi_{1,2}=P_iD_{wi}-(1-\theta)(C_i^T+\mathcal{K}_i^TD_i^T)$, $\Psi_{1,3}=P_iB_i\mathcal{D}_s$, $\Psi_{1,4}=P_i^T$, $\Psi_{1,5}=\varepsilon_1\sum_{j=1,j\neq i}^{N}A_{ji}^T$, $\Psi_{1,6}=\sqrt{\theta}(C_i^T+\mathcal{K}_i^TD_i^T)$, $\Psi_{1,7}=\mathcal{K}_i^TL^T\rho$, $\Psi_{1,9}=\varepsilon_{2i}P_i^T\mathcal{M}_{aii}$, $\Psi_{1,10}=\mathcal{N}_{aii}^T$, $\Psi_{1,11}=\varepsilon_{3i}P_i^T\mathcal{M}_{bi}$, $\Psi_{1,12}=\mathcal{K}_i^T\mathcal{N}_{bi}^T$, $\Psi_{1,14}=\varepsilon_1\mathcal{N}_{aij}^T$, $\Psi_{2,2}=-\gamma I$, $\Psi_{2,3}=-(1-\theta)\mathcal{D}_s$, $\Psi_{3,3}=-\rho I$, $\Psi_{3,8}=\sqrt{\theta}\mathcal{D}_s^T$, $\Psi_{3,13}=\mathcal{D}_s^T\mathcal{N}_{bi}^T$, $\Psi_{4,4}=-\varepsilon_1 I$, $\Psi_{5,5}=-\varepsilon_1(N-1)^{-1}$, $\Psi_{5,15}=\varepsilon_{4i}\mathcal{M}_{aij}$, $\Psi_{6,6}=-\gamma I$, $\Psi_{7,7}=-\rho I$, $\Psi_{8,8}=-\gamma I$, $\Psi_{9,9}=-\varepsilon_{2i}I$, $\Psi_{10,10}=-\varepsilon_{2i}I$, $\Psi_{11,11}=-\varepsilon_{3i}I$, $\Psi_{12,12}=-\varepsilon_{3i}I$, $\Psi_{13,13}=-\varepsilon_{3i}I$, $\Psi_{14,14}=-\varepsilon_{4i}I$, $\Psi_{15,15}=-\varepsilon_{4i}I$, and $\mathcal{K}_i=\sum_{s=1}^{2^m}\zeta_s[\mathcal{D}_sG\hat{K}_i+\mathcal{D}_s^-\hat{H}_i]$

Proof: According to Lemma 1, the closed-loop system (7) can be rewritten in the following form

$$\begin{aligned}\frac{\partial Z(\omega,t)}{\partial t}&=-\omega Z(\omega,t)+\tilde{A}x_i(t)+\tilde{B}f_i(u_i(t))+\hat{A}x_j(t)\\&\quad+D_{wi}w_i(t),\end{aligned} \tag{10}$$

$$x_i(t)=\int_0^\infty \eta(\omega)Z(\omega,t)d\omega \quad\text{with}\quad \eta(\omega)=\frac{\sin(\alpha\pi)}{\pi}\omega^{-\alpha}.$$

where $Z(\omega,t)=[Z(\omega_1,t),Z(\omega_2,t),\cdots,Z(\omega_n,t)]^T$ is the frequency distributed state, $x_i(t)$ is the output of the fractional weighted integrator with weight frequency $\eta(\omega)$. In order to prove the asymptotic stability of the system (1), let us define two Lyapunov functions, $\nu(\omega,t)=Z^T(\omega,t)P_iZ(\omega,t)$, corresponding to the elementary frequency ω and $V(t)=\int_0^\infty\eta(\omega)\nu(\omega,t)d\omega$, corresponding to all monochromatic $\nu(\omega,t)$ with weighting function $\eta(\omega)$. Then,

$$V(t)=\int_0^\infty \eta(\omega)Z^T(\omega,t)P_iZ(\omega,t)d\omega, \tag{11}$$

where P_i are positive definite matrices. Taking the time derivative of $V(t)$ along the trajectories of (10), we get,

$$\begin{aligned}\dot{V}(t)=&\int_0^\infty \eta(\omega)\Big\{-\omega Z^T(\omega,t)+x_i^T\tilde{A}^T+f_i^T(u_i(t)\tilde{B}^T\\&+x_j^T\hat{A}^T+w_i^T(t)D_{wi}^T\Big\}P_iZ(\omega,t)d\omega\\&+\int_0^\infty\eta(\omega)Z^T(\omega,t)P_i\Big\{-\omega Z(\omega,t)+\tilde{A}x_i(t)\\&+\tilde{B}f_i(u_i(t))+\hat{A}x_j(t)+D_{wi}w_i(t)\Big\}d\omega,\\=&-2\int_0^\infty\omega\eta(\omega)Z^T(\omega,t)P_iZ(\omega,t)d\omega\\&+2x_i^T(t)P_i\tilde{A}x_i(t)+2x_i^T(t)P_i\tilde{B}f_i(u_i(t))\\&+2x_i^T(t)P_i\hat{A}x_j(t)+2x_i^T(t)P_iD_{wi}w_i(t).\end{aligned} \tag{12}$$

According to Lyapunov stability theory, system (10) is asymptotically stable, if $\dot{V}(t)<0$. In order to prove this, we show that,

$$\begin{aligned}\Omega=&2x_i^T(t)P_i\tilde{A}x_i(t)+2x_i^T(t)P_i\tilde{B}f_i(u_i(t))\\&+2x_i^T(t)P_i\hat{A}x_j(t)+2x_i^T(t)P_iD_{wi}w_i(t)<0.\end{aligned} \tag{13}$$

By Lemma 4 and following the procedure as in [8], we can have the following inequality

$$\begin{aligned}&\sum_{i=1}^N\Big\{\Big[\sum_{j=1,j\neq i}^N(A_{ij}+\Delta A_{ij})x_j(t)\Big]^T\Big[\sum_{j=1,j\neq i}^N(A_{ij}+\Delta A_{ij})x_j(t)\Big]\Big\}\\&\le\sum_{i=1}^N\Big\{\Big[\sum_{j=1,j\neq i}^N(A_{ji}+\Delta A_{ji})x_i(t)\Big]^T\Big[\sum_{j=1,j\neq i}^N(A_{ji}+\Delta A_{ji})x_i(t)\Big]\Big\}\\&\le\sum_{i=1}^N\Big\{(N-1)\sum_{j=1,j\neq i}^N x_i^T(t)(A_{ji}+\Delta A_{ji})^T(A_{ji}+\Delta A_{ji})x_i(t)\Big\}.\end{aligned} \tag{14}$$

Given a scalar $\rho>0$, it follows from (5) that

$$\rho[\hat{K}_ix_i]^TL[\hat{K}_ix_i]-\rho f^T(u_i(t))f(u_i(t))\ge 0. \tag{15}$$

From (13)-(15) and the mixed H_∞ and passivity performance index, it is easy to verify that

$$\begin{aligned}\dot{V}(t)<&\Xi^T(t)\hat{\Phi}\Xi(t)+\varepsilon_1^{-1}[P_ix_i(t)]^T[P_ix_i(t)]\\&+\sum_{i=1}^N\Big\{(N-1)\end{aligned}$$

$$\times \sum_{j=1,j\neq i}^{N} x_i^T(t)(A_{ji}+\Delta A_{ji})^T(A_{ji}+\Delta A_{ji})x_i(t)\Big\}, \tag{16}$$

where $\Xi(t) = \begin{bmatrix} x_i(t) & w_i(t) & f(u_i(t)) \end{bmatrix}^T$ and $\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_{1,1} & \hat{\Phi}_{1,2} & P_i\tilde{B} \\ * & -\gamma I & -(1-\theta)\mathcal{D}_s \\ * & * & \hat{\Phi}_{3,3} \end{bmatrix}$, where $\hat{\Phi}_{1,1} = \mathrm{sym}\big(P_i(A_{ii} + \Delta A_{ii}) + P_i(B_i+\Delta B_i)\mathcal{K}_i + \theta\gamma^{-1}(C_i+D_i\mathcal{K}_i)^T(C_i+D_i\mathcal{K}_i) + \rho\hat{K}_i^T L\hat{K}_i\big)$, $\hat{\Phi}_{1,2} = P_iD_{wi} - (1-\theta)(C_i+D_i\mathcal{K}_i)$, $\hat{\Phi}_{3,3} = \theta\gamma^{-1}\mathcal{D}_s^T\mathcal{D}_s - \rho I$. It is obvious that $\dot{V}(t) < 0$ if $\hat{\Phi} < 0$. By decomposing the uncertain parts using Lemma 2 and Schur compliment, it is easy to obtain the LMI in (8). This shows that, the closed-loop system (7) is asymptotically stable with known actuator faults and input saturation. This completes the proof of Theorem 1. □

In Theorem 1, sufficient conditions are derived in terms of LMIs such that, the closed-loop system (7) is asymptotically stable with known actuator faults and without consideration of perturbations in gain. In the following theorem, a non-fragile controller will be designed, which can guarantee the asymptotic stabilization of the system (1).

Theorem 2: For some given positive scalars $\varepsilon_1, \rho, \gamma, \theta$ and known fault matrix G, the uncertain fractional-order large-scale system (1) with $0 < \alpha < 1$ is asymptotically stabilized via robust fault-tolerant non-fragile control law (3), if there exist positive definite matrices $P_i, i = 1, 2, \cdots, N$ and scalars ε_{2i}, ε_{3i}, ε_{4i} and ε_{5i} such that, for $s = 1, 2, \cdots, 2^m$, the following matrix inequality together with the condition (9) holds:

$$\hat{\Psi} = \begin{bmatrix} [\tilde{\Psi}]_{15\times 15} & \Psi_2^T & \varepsilon_{5i}\Psi_1 \\ * & -\varepsilon_{5i}I & 0 \\ * & * & -\varepsilon_{5i}I \end{bmatrix} < 0, \tag{17}$$

where $\tilde{\Psi}_{1,1} = \mathrm{sym}(A_{ii}X_i + B_i\mathcal{D}_sGY_i + B_i\mathcal{D}_s^- Z_i)$, $\tilde{\Psi}_{1,2} = D_{wi} - (1-\theta)X_iC_i^T - (1-\theta)[\mathcal{D}_sGY_i + \mathcal{D}_s^- Z_i]^T D_i^T$, $\tilde{\Psi}_{1,3} = B_i\mathcal{D}_s$, $\tilde{\Psi}_{1,4} = I$, $\tilde{\Psi}_{1,5} = \varepsilon_1 X_i \sum_{j=1,j\neq i}^{N} A_{ji}^T$, $\tilde{\Psi}_{1,6} = \sqrt{\theta}X_iC_i^T + \sqrt{\theta}[\mathcal{D}_sGY_i + \mathcal{D}_s^- Z_i]^T D_i^T$, $\tilde{\Psi}_{1,7} = \rho[\mathcal{D}_sGY_i + \mathcal{D}_s^- Z_i]^T L^T$, $\tilde{\Psi}_{1,9} = \varepsilon_{2i}\mathcal{M}_{aii}$, $\tilde{\Psi}_{1,10} = X_i\mathcal{N}_{aii}^T$, $\tilde{\Psi}_{1,11} = \varepsilon_{3i}\mathcal{M}_{bi}$, $\tilde{\Psi}_{1,12} = [\mathcal{D}_sGY_i + \mathcal{D}_s^- Z_i]^T\mathcal{N}_{bi}^T$, $\tilde{\Psi}_{1,14} = \varepsilon_1 X_i\mathcal{N}_{aij}^T$, $\tilde{\Psi}_{2,2} = -\gamma I$, $\tilde{\Psi}_{2,3} = -(1-\theta)\mathcal{D}_s$, $\tilde{\Psi}_{3,3} = -\rho I$, $\tilde{\Psi}_{3,8} = \sqrt{\theta}\mathcal{D}_s^T$, $\tilde{\Psi}_{3,13} = D_s^T\mathcal{N}_{bi}^T$, $\tilde{\Psi}_{4,4} = -\varepsilon_1 I$, $\tilde{\Psi}_{5,5} = -\varepsilon_1(N-1)^{-1}$, $\tilde{\Psi}_{5,15} = \varepsilon_{4i}\mathcal{M}_{aij}$, $\tilde{\Psi}_{6,6} = -\gamma I$, $\tilde{\Psi}_{7,7} = -\rho I$, $\tilde{\Psi}_{8,8} = -\gamma I$, $\tilde{\Psi}_{9,9} = -\varepsilon_{2i}I$, $\tilde{\Psi}_{10,10} = -\varepsilon_{2i}I$, $\tilde{\Psi}_{11,11} = -\varepsilon_{3i}I$, $\tilde{\Psi}_{12,12} = -\varepsilon_{3i}I$, $\tilde{\Psi}_{13,13} = -\varepsilon_{3i}I$, $\tilde{\Psi}_{14,14} = -\varepsilon_{4i}I$, $\tilde{\Psi}_{15,15} = -\varepsilon_{4i}I$, $\Psi_1 = \big[B_i\Lambda \;\; (1-\theta)D_i\Lambda \;\; \underbrace{0\ldots 0}_{3} \;\; \sqrt{\theta}D_i\Lambda \;\; \rho L\Lambda \;\; \underbrace{0\ldots 0}_{4} \;\; \mathcal{N}_{bi}\Lambda \;\; \underbrace{0\ldots 0}_{3}\big]$, $\Psi_2 = \big[(\mathcal{N}_{ki}+\mathcal{N}_{hi})X_i^T \;\; \underbrace{0\ldots 0}_{14}\big]^T$, $\Lambda = \sum_{s=1}^{2^m}\zeta_s[\mathcal{D}_sG\mathcal{M}_{ki} + \mathcal{D}_s^-\mathcal{M}_{hi}]$. Moreover, the fault-tolerant controller gain and the auxiliary gain matrices can be computed as $K_i = Y_iX_i^{-1}$ and $H_i = Z_iX_i^{-1}$.

Proof: The same steps as in Theorem 1 are followed to prove Theorem 2. By taking the gain perturbations into considerations, we obtain $\hat{\Phi} = \tilde{\Phi}_{15\times 15} + (\Phi_1\mathcal{F}\Phi_2) + (\Phi_1\mathcal{F}\Phi_2)^T$, where $\tilde{\Phi}_{1,1} = sym(P_iA_{ii} + P_iB_i\mathcal{D}_sGK_i + P_iB_i\mathcal{D}_s^- H_i)$, $\tilde{\Phi}_{1,2} = P_iD_{wi} - (1-\theta)C_i^T - (1-\theta)[\mathcal{D}_sGK_i + \mathcal{D}_s^- H_i]^T D_i^T$, $\tilde{\Phi}_{1,3} = P_iB_i\mathcal{D}_s$, $\tilde{\Phi}_{1,4} = P_i^T$, $\tilde{\Phi}_{1,5} = \varepsilon_1 \sum_{j=1,j\neq i}^{N} A_{ji}^T$, $\tilde{\Phi}_{1,6} = \sqrt{\theta}C_i^T + \sqrt{\theta}[\mathcal{D}_sGK_i + \mathcal{D}_s^- H_i]^T D_i^T$, $\tilde{\Phi}_{1,7} = \rho[\mathcal{D}_sGK_i + \mathcal{D}_s^- H_i]^T L^T$, $\tilde{\Phi}_{1,9} = \varepsilon_{2i}P_i\mathcal{M}_{aii}$, $\tilde{\Phi}_{1,10} = \mathcal{N}_{aii}^T$, $\tilde{\Phi}_{1,11} = \varepsilon_{3i}P_i\mathcal{M}_{bi}$, $\tilde{\Phi}_{1,12} = [\mathcal{D}_sGK_i + \mathcal{D}_s^- H_i]^T\mathcal{N}_{bi}^T$, $\tilde{\Phi}_{1,14} = \varepsilon_1\mathcal{N}_{aij}^T$, $\tilde{\Phi}_{2,2} = -\gamma I$, $\tilde{\Phi}_{2,3} = -(1-\theta)\mathcal{D}_s$, $\tilde{\Phi}_{3,3} = -\rho I$, $\tilde{\Phi}_{3,8} = \sqrt{\theta}\mathcal{D}_s^T$, $\tilde{\Phi}_{3,13} = D_s^T\mathcal{N}_{bi}^T$, $\tilde{\Phi}_{4,4} = -\varepsilon_1 I$, $\tilde{\Phi}_{5,5} = -\varepsilon_1(N-1)^{-1}$, $\tilde{\Phi}_{5,15} = \varepsilon_{4i}\mathcal{M}_{aij}$, $\tilde{\Phi}_{6,6} = -\gamma I$, $\tilde{\Phi}_{7,7} = -\rho I$, $\tilde{\Phi}_{8,8} = -\gamma I$, $\tilde{\Phi}_{9,9} = -\varepsilon_{2i}I$, $\tilde{\Phi}_{10,10} = -\varepsilon_{2i}I$, $\tilde{\Phi}_{11,11} = -\varepsilon_{3i}I$, $\tilde{\Phi}_{12,12} = -\varepsilon_{3i}I$, $\tilde{\Phi}_{13,13} = -\varepsilon_{3i}I$, $\tilde{\Phi}_{14,14} = -\varepsilon_{4i}I$, $\tilde{\Phi}_{15,15} = -\varepsilon_{4i}I$, $\Phi_1 = \big[B_i\Lambda \;\; (1-\theta)D_i\Lambda \;\; \underbrace{0\ldots 0}_{3} \;\; \sqrt{\theta}D_i\Lambda \;\; \rho L\Lambda \;\; \underbrace{0\ldots 0}_{4} \;\; \mathcal{N}_{bi}\Lambda \;\; \underbrace{0\ldots 0}_{3}\big]$, $\Phi_2 = \big[(\mathcal{N}_{ki}+\mathcal{N}_{hi})X_i^T \;\; \underbrace{0\ldots 0}_{14}\big]^T$. Applying Lemma. (2), the above expression can be rewritten as $\hat{\Phi} = \tilde{\Phi}_{15\times 15} + \varepsilon_{5i}\Phi_1\Phi_1^T + \varepsilon_{5i}^{-1}\Phi_2^T\Phi_2$. In order to design the controller gains, pre and post-multiplying the previous expression by $\mathrm{diag}\big\{X_i, \underbrace{I\ldots I}_{14}\big\}$, where $X_i = P_i^{-1}$. Further, by using Schur complement and letting $Y_i = K_iX_i$ and $Z_i = H_iX_i$, we can easily obtain the LMI (17). This completes the proof of Theorem 2. □

In Theorem 2, sufficient conditions are derived by considering gain perturbations in the controller design such that the closed-loop system (7) is asymptotically stable with known actuator faults. In the following theorem, a fault-tolerant resilient controller will be designed, which can guarantee the asymptotic stabilization of the system (1) with unknown actuator failures.

Theorem 3: Consider the uncertain fractional-order large-scale system (1) with $0 < \alpha < 1$. For some positive scalars $\varepsilon_1, \rho, \gamma, \theta$ and for unknown fault matrix G, system (1) is asymptotically stabilized via the controller (3), if there exist scalars ε_{2i}, ε_{3i}, ε_{4i}, ε_{5i}, ε_{6i}, ε_{7i} and positive definite matrices $P_i, i = 1, 2, \cdots, N$ such that, for $s = 1, 2, \cdots, 2^m$, the following matrix inequality together with the condition (9) holds:

$$\hat{\Psi}_1 = \begin{bmatrix} [\bar{\Psi}]_{15\times 15} & \hat{\Psi}_1 \\ * & \hat{\Psi}_2 \end{bmatrix} < 0, \tag{18}$$

where $\hat{\Psi}_1 = \big[\bar{\Psi}_2^T \;\; \varepsilon_{5i}\bar{\Psi}_1 \;\; \bar{\Psi}_3^T \;\; \varepsilon_{6i}\bar{\Psi}_4 \;\; \varepsilon_{5i}\bar{\Psi}_5^T \;\; \varepsilon_{7i}\bar{\Psi}_6\big]$, $\hat{\Psi}_2 = \mathrm{diag}\{-\varepsilon_{5i}I, -\varepsilon_{5i}I, -\varepsilon_{6i}I, -\varepsilon_{6i}I, -\varepsilon_{6i}I, -\varepsilon_{6i}I\}$, $\bar{\Psi}_{1,1} = sym(A_{ii}X_i + B_i\mathcal{D}_sG_0Y_i + B_i\mathcal{D}_s^- Z_i)$, $\bar{\Psi}_{1,2} = D_{wi} - (1-\theta)X_iC_i^T - (1-\theta)[\mathcal{D}_sG_0Y_i + \mathcal{D}_s^- Z_i]^T D_i^T$, $\bar{\Psi}_{1,6} = \sqrt{\theta}X_iC_i^T + \sqrt{\theta}[\mathcal{D}_sG_0Y_i + \mathcal{D}_s^- Z_i]^T D_i^T$, $\bar{\Psi}_{1,7} = \rho[\mathcal{D}_sG_0Y_i + \mathcal{D}_s^- Z_i]^T L^T$, $\bar{\Psi}_{1,12} = [\mathcal{D}_sG_0Y_i + \mathcal{D}_s^- Z_i]^T\mathcal{N}_{bi}^T$, $\Psi_1 = \big[B_i\Lambda \;\; (1-\theta)D_i\Lambda \;\; \underbrace{0\ldots 0}_{3} \;\; \sqrt{\theta}D_i\Lambda \;\; \rho L\Lambda \;\; \underbrace{0\ldots 0}_{4} \;\; \mathcal{N}_{bi}\Lambda$

$\underbrace{0 \dots 0}_{3}]$, $\Lambda = \sum_{s=1}^{2^m} \zeta_s[\mathcal{D}_s G_0 \mathcal{M}_{ki} + \mathcal{D}_s^- \mathcal{M}_{hi}]$, $\bar{\Psi}_3 = [B_i X_i \quad (1-\theta)X_i^T D_1^T \quad \underbrace{0 \dots 0}_{3} \quad \sqrt{\theta} X_i^T D_1^T \quad \rho L X_i^T D_1^T \quad \underbrace{0 \dots 0}_{4} \quad X_i^T N_{bi}^T \quad \underbrace{0 \dots 0}_{5}]$, $\bar{\Psi}_4 = [\zeta_s \mathcal{D}_s G_1 \quad \underbrace{0 \dots 0}_{17}]$, $\bar{\Psi}_5 = [\zeta_s G_1^T \mathcal{D}_s^T \quad -(1-\theta)\zeta_s G_1^T \mathcal{D}_s^T D_1^T \quad \underbrace{0 \dots 0}_{3} \quad \sqrt{\theta}\zeta_s G_1^T \mathcal{D}_s^T D_1^T \quad \zeta_s \rho L G_1^T \mathcal{D}_s^T \quad \underbrace{0 \dots 0}_{4} \quad \zeta_s G_1^T \mathcal{D}_s^T N_{b1}^T \quad \underbrace{0 \dots 0}_{7}]$, $\bar{\Psi}_6 = [\underbrace{0 \dots 0}_{16} \quad \mathcal{M}_{ki}^T \quad \underbrace{0 \dots 0}_{3}]$. Further, the fault-tolerant controller gain and the auxiliary gain matrices can be computed as $K_i = Y_i X_i^{-1}$ and $H_i = Z_i X_i^{-1}$.

Proof: By using the relation (4) in (17), with the aid of Lemma 2 and Schur compliment Lemma, the LMI in Theorem (3) can be easily obtained. Hence, it can be concluded that, uncertain fractional-order large-scale system (1) with $0 < \alpha < 1$ is asymptotically stabillized through the controller (3) in the presence of nonlinear actuator faults and input saturation. The proof is completed. □

Remark 2: In the derivation of main results, we have employed Lyapunov stability theory to ensure the asymptotic stabilization of the considered system where the number of variables in the obtained LMIs plays a crucial role. Further, no free-weighting matrices are introduced in the proofs of the theorems, so the structure of the obtained LMIs is simpler and hence the computational burden is reduced significantly. However, the conservatism of proposed results could be further reduced by developing control algorithms with the use of some advanced integral inequalities.

4. NUMERICAL EXAMPLE

In this section, a numerical example is presented to validate the effectiveness of the proposed robust decentralized fault-tolerant resilient controller design. Consider a large-scale system consisting of 2 subsystems ($N = 2$) and the system parameters associated with that are,

$$A_{11} = \begin{bmatrix} -3 & 1 \\ 2 & -3 \end{bmatrix}, \; A_{12} = \begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}, \; B_1 = \begin{bmatrix} 1.6 \\ 1.8 \end{bmatrix},$$

$$B_{w1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \; C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}, \; A_{22} = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix}, \; B_2 = \begin{bmatrix} 1.4 \\ 1.8 \end{bmatrix},$$

$$B_{w2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; D_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Further, the uncertain matrices are chosen as

$$\mathcal{M}_{a11} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \; \mathcal{N}_{a11} = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}, \; \mathcal{M}_{a12} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$\mathcal{N}_{a12} = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}, \; \mathcal{M}_{b1} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \; \mathcal{N}_{b1} = \begin{bmatrix} 0.6 \end{bmatrix},$$

$$\mathcal{M}_{a21} = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \; \mathcal{N}_{a21} = \begin{bmatrix} 0.4 & 0.2 \end{bmatrix}, \; \mathcal{M}_{a22} = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix},$$

$$\mathcal{N}_{a22} = \begin{bmatrix} 0.4 & 0.2 \end{bmatrix}, \; \mathcal{M}_{b2} = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \; \mathcal{N}_{b2} = \begin{bmatrix} 0.1 \end{bmatrix},$$

$$\mathcal{M}_{k1} = \begin{bmatrix} 0.2 \end{bmatrix}, \; \mathcal{N}_{k1} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \; \mathcal{M}_{h1} = \begin{bmatrix} 0.1 \end{bmatrix},$$

$$\mathcal{N}_{h1} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \; \mathcal{M}_{k2} = \begin{bmatrix} 0.2 \end{bmatrix}, \; \mathcal{N}_{k2} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$\mathcal{M}_{h2} = \begin{bmatrix} 0.2 \end{bmatrix}, \; \mathcal{N}_{h2} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}.$$

The remaining parameters are taken as $\varepsilon_1 = 0.5$, $L = 0.01$, $\rho = 3$, $\mathcal{D}_1 = 1$, $\mathcal{D}_1^- = 0$, $\mathcal{D}_2 = 0$, $\mathcal{D}_2^- = 1$ and the actuator fault matrix G is assumed to lie in the interval $[0.6 \; 0.8]$. Further, the nonlinear function is chosen as $f(u_i(t)) = \sin(u_i(t)) - 0.15u_i(t)\cos(u_i(t))$. Also, we take the external disturbance as $w_i(t) = 0.03\sin t$. Using the above parameters and solving the LMIs obtained in Theorem (3), the minimum H_∞ and passivity performance index is obtained as $\gamma = 0.75$ and the corresponding state feedback controller gain matrices are obtained as, $K_1 = [0.0007 \; -4.3378]$, $H_1 = [0.8688 \; -8.6117]$, $K_2 = [0.7411 \; -6.0163]$ and $H_2 = [1.9345 \; -11.1401]$.

Moreover, for simulation purposes, we choose the initial conditions of the two subsystems as $[0.2 \; -0.2]^T$ and $[0.3 \; -0.4]^T$ respectively. Based on the obtained gain values, Figs. 1 and 7 depict the state trajectories of the two subsystems in the presence of nonlinear actuator faults. Figs. 2 and 8 show the trajectories of the system states when there is no nonlinear faults in the actuators. Further, the state responses of the two subsystems with unknown actuator fault under H_∞ and passivity performances are shown in Figs. 5, 6, 11, and 12.

The response of the proposed fault-tolerant resilient controller with and without nonlinear actuator faults are shown in Figs. 3, 9, 4, and 10 respectively. Also, Figs. 13 and 14 show the maximal invariant ellipsoids of the subsystems with input saturation. It is evident from these figures that, for different initial conditions, the trajectories of the states remain inside the ellipsoids, which proves the efficiency of the designed fault-tolerant resilient controller.

Hence, the simulation results concludes that, the fractional-order uncertain large-scale system with input saturation and unknown actuator faults is asymptotically stabilized via the designed robust decentralized fault-tolerant resilient controller even in the presence of nonlinear term in the control input.

5. CONCLUSION

In this paper, the robust decentralized fault-tolerant resilient control problem for fractional-order large-scale interconnected uncertain systems with input saturation and nonlinear actuator faults is studied. By developing suit-

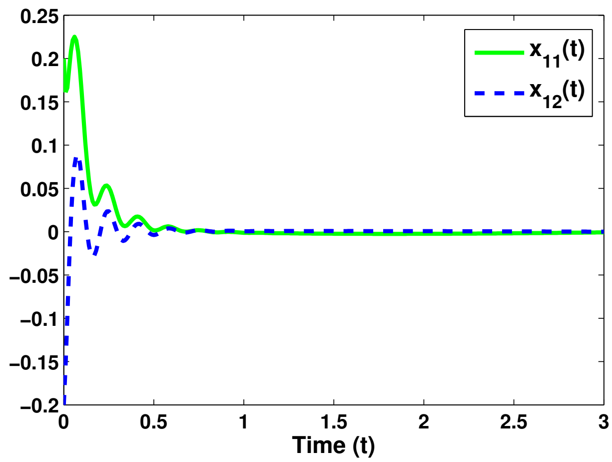

Fig. 1. State responses of subsystem 1 when $f(u_i(t)) \neq 0$.

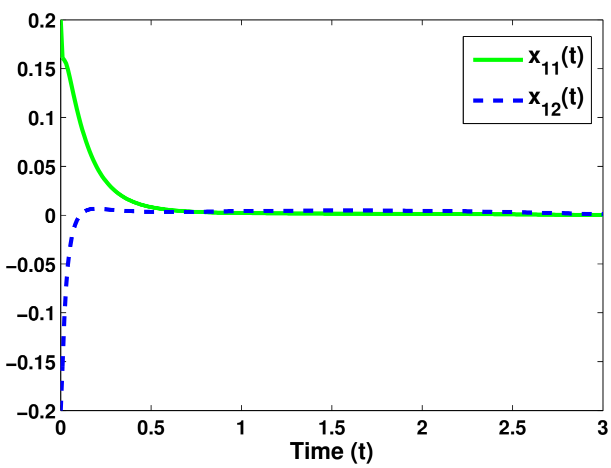

Fig. 2. State responses of subsystem 1 when $f(u_i(t)) = 0$.

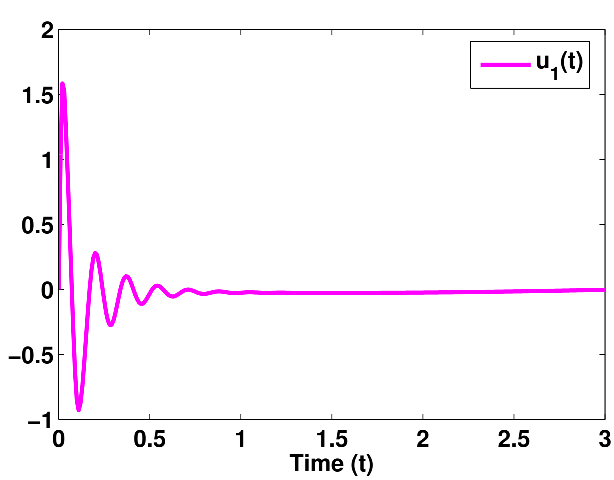

Fig. 3. Control response of subsystem 1 when $f(u_i(t)) \neq 0$.

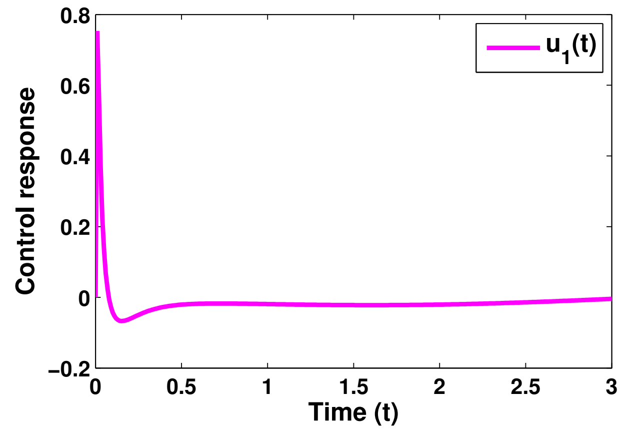

Fig. 4. Control response of subsystem 1 when $f(u_i(t)) = 0$.

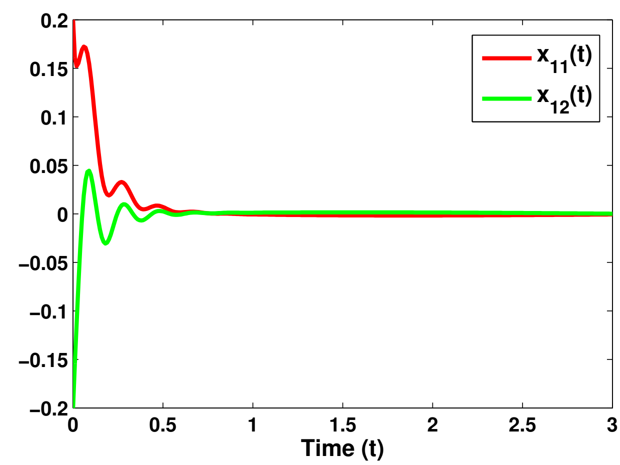

Fig. 5. State responses of subsystem 1 under H_∞ performance.

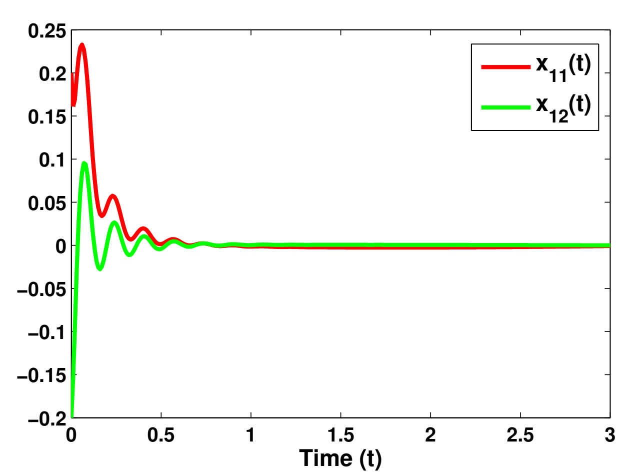

Fig. 6. State responses of subsystem 1 under passivity performance.

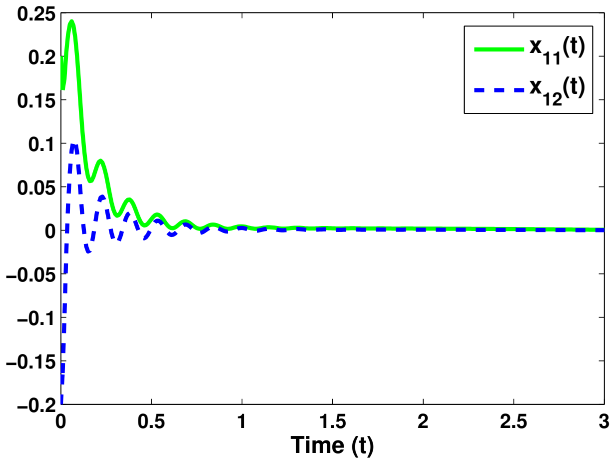

Fig. 7. State responses of subsystem 2 when $f(u_i(t)) \neq 0$.

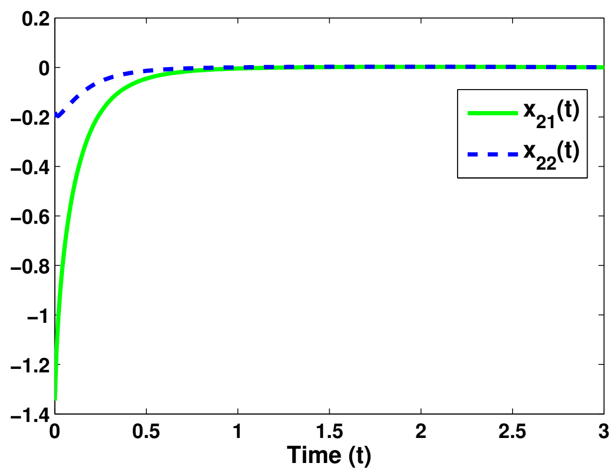

Fig. 8. State responses of subsystem 2 when $f(u_i(t)) = 0$.

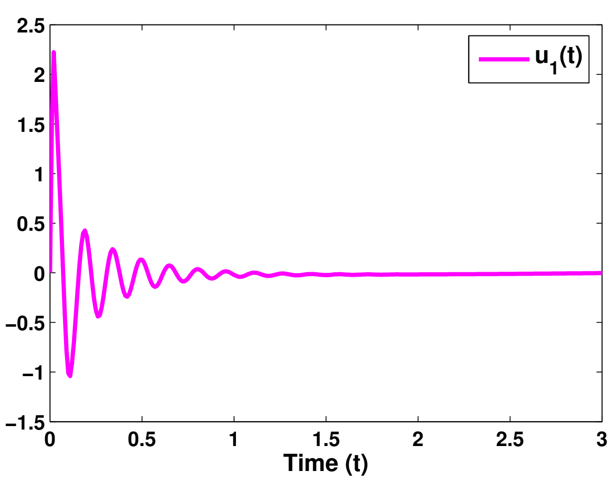

Fig. 9. Control response of subsystem 2 when $f(u_i(t)) \neq 0$.

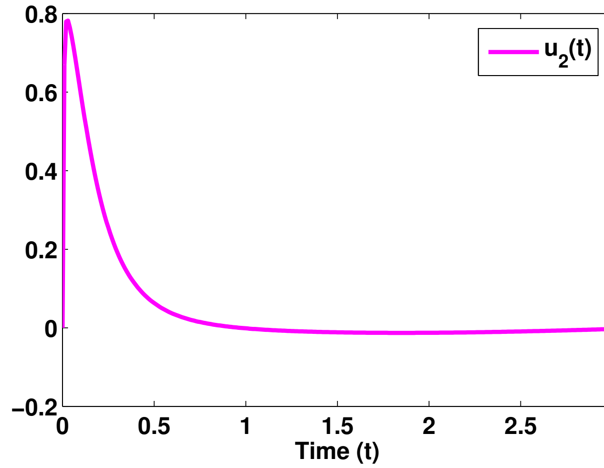

Fig. 10. Control response of subsystem 2 when $f(u_i(t)) = 0$.

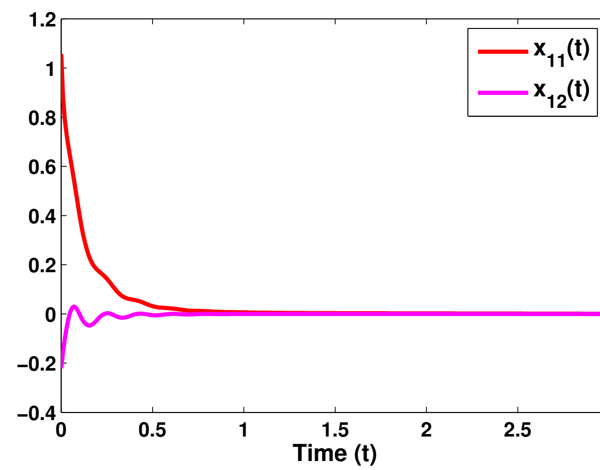

Fig. 11. State responses of subsystem 2 under H_∞ performance.

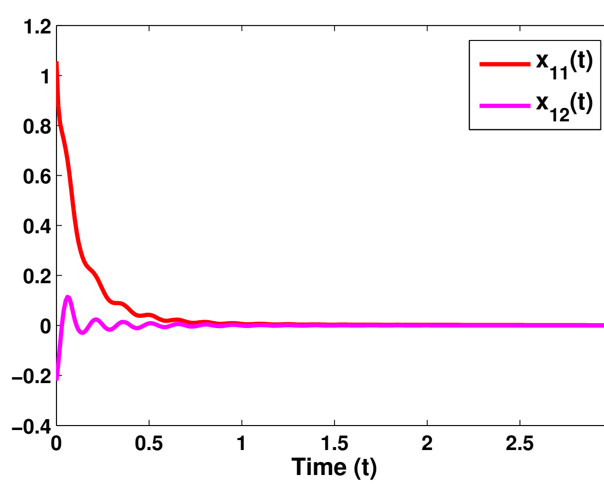

Fig. 12. State responses of subsystem 2 under passivity performance.

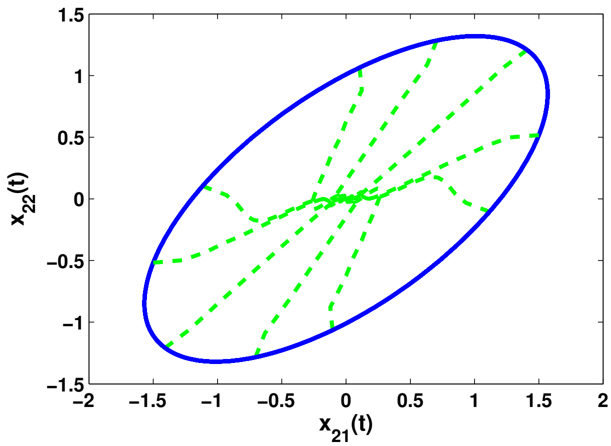

Fig. 13. State Trajectories of subsystem 1.

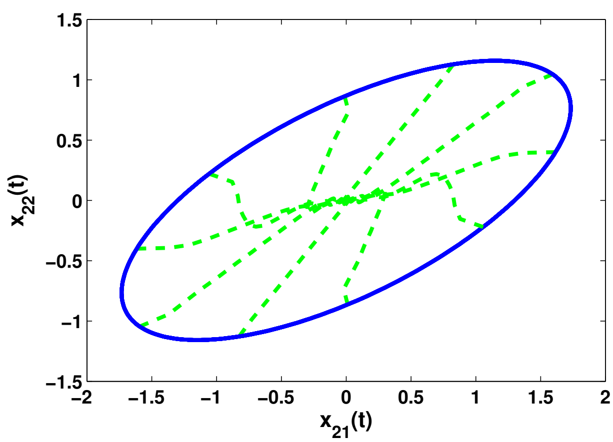

Fig. 14. State Trajectories of subsystem 2.

able Lyapunov functional together with LMI technique, a set of sufficient conditions is derived in terms of linear matrix inequalities which ensures the asymptotic stabilization of the considered system with prescribed mixed H_∞ and passivity performance index. Moreover, the fault-tolerant resilient control gain matrices are obtained by solving the developed LMIs. Finally, a numerical example with simulation results is given to validate the efficiency of the proposed controller design technique. Further, the problem of fault-tolerant resilient control for stochastic large-scale fractional-order interconnected systems with nonlinearities, quantization and energy constraints is an untreated area. These issues will be our future research topic.

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