International Journal of Control, Automation and Systems 18(5) (2020) 1103-1115
http://dx.doi.org/10.1007/s12555-018-0841-z

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Interaction Matrix Based Analysis and Asymptotic Cooperative Control of Multi-agent Systems

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Abstract: In this paper, we investigate a decentralized asymptotic cooperative control problem of multi-agent systems with leader-follower configuration. We firstly develop a new method using a proposed "interaction matrix" for the analysis of cooperation convergence of multi-agent systems, i.e. both consensus of the agents states and trajectory tracking of the whole group can be instantaneously concluded only by observing the minimum eigenvalue of the interaction matrix. For a multi-agent system, the external given desired trajectory can be partially obtained (through sensing or detecting) by the leaders, but higher-order derivatives such as acceleration and jerk of the desired trajectory cannot be obtained. In this case, by using some conventional control methods, the trajectory tracking performance is always not satisfactory when a trajectory varies aggressively w.r.t. time. For the sake of asymptotic tracking of an arbitrary given external trajectory of a multi-agent system, we develop a nonlinear cooperative controller based on the robust integral of signum of cooperative error (RISCE) technique, where the interaction matrix is used. The simulation results show asymptotic convergence of cooperation by using the proposed control, and better performance compared to composited nonlinear feedback based PD (CNF-PD) control.

Keywords: Asymptotic tracking of arbitrary trajectory, interaction matrix, multi-agent systems, RISCE, unknown derivatives.

1. INTRODUCTION

The cooperative control of multi-agent systems (MASs) has attracted attentions due to its potential applications, which include multi-vehicle system [1,2], formation flight of unmanned air vehicles [3,4], sensor networks [5], and congestion control in communication networks [6]. Since the dynamics of mobile robots such as spacecraft, underwater vehicles can be represented by double-integrator systems [7], the research on multiple double-integrator systems is progressively increased in such as [2,8–10].

The leader-follower (L-F) is considered as a simple configuration of MAS. In standard L-F configuration, the actions of the following vehicles in the MAS are completely specified by the leader's states [11,12]. The leader can affect the followers whenever it is in their neighboring set but there is no feedback from the followers to the leader [13,14]. Furthermore, the moving trajectory of the flock is clearly given to the leader(s) [15]. The standard L-F configuration is not decentralized, thus it is considered as a strategy lacking robustness, because the L-F configuration is often considered poorly robust with respect to leader's failure [11]. In this paper, the leader(s) moves not only depending on the reference trajectory but also the states of its neighbors. Additionally, the followers do not distinguish leaders from its neighbors. The aim of the cooperative controller design is to achieve consensus incorporate with desired trajectory tracking of the group, which is called the "cooperative task".

In literature, the *Laplacian* is widely used in the analysis such as consensus [15–17], formation control [18,19], switching topologies [20,21], etc. However, the trajectory tracking analysis can not be accomplished by only using *Laplacian*. In this paper, developed from the work [22], in order to analyse the consensus and trajectory tracking of an MAS ensemble, an interaction matrix is proposed based on *Laplacian*. Then, the trajectory error convergence of the agents is proven to be related to the smallest eigenvalue of interaction matrix. Different from [22], the interaction matrix is suitable for the analysis of agents consensus and convergence of trajectory tracking error. Additionally, the nodes of leaders are not necessarily removed from the graph.

For the sake of achieving asymptotic trajectory tracking

Manuscript received November 20, 2018; revised May 13, 2019; accepted September 17, 2019 Recommended by Associate Editor Hyo-Sung Ahn under the direction of Editor Myo Taeg Lim.

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of the MAS, the researchers have proposed many control methods. The distributed average tracking (DAT) problem for a group of agents to track the average of multiple time-varying reference signals is investigated in [23], where each agent has a reference signal. Auxiliary control term is integrated in controller design. The asymptotic formation tracking problem is investigated in [24]. The Lyapunov-like method is used to conclude the asymptotic convergence of the formation error. The Lyapunov-based method is also used in [25], where the asymptotic cooperation can be concluded when the sensing/communication topology is connected.

We consider the scenario such that some states (position and velocity) of the reference trajectory can be obtained only by leader(s). Furthermore, the high-order derivatives cannot be known neither by the followers nor the leaders. In this case, the arbitrary trajectory *asymptotic tracking* of the MAS is very difficult to achieve by using conventional linear cooperative control.

The asymptotic trajectory tracking problem is investigated in literature. A robust integral sliding mode control is proposed in in [26] for networked control systems, where multiple data packet losses is investigated. The integral sliding mode based control strategy is also developed in [18], where the disturbance is rejected by a discontinuous control term. A gradient-based distributed optimization control for handling distributed optimization problem of multi-agent systems with disturbance rejection is presented in [27]. The robust integral of signum of error (RISE) technology is proposed in [28] to compensate for uncertainties in systems. Since it's a continuous control mechanism, the RISE method solves the chattering problem in standard sliding mode control. The authors in [29] design an estimator which estimates the state of the ideal agent without added disturbances. Then, the RISE method is used to eliminate the difference between the estimation and the state of agent, such that the disturbances rejection is achieved. The RISE technique is also implemented in [30], where the external disturbances and neural network approximation errors are suppressed.

The contributions of this paper can be summarized as follows. In order to model the interaction of agents, we first propose a new interaction matrix method, instead of frequently used *Laplacian* method in the literature ([15–17] to name a few). The advantage is that both consensus of agents' states and trajectory tracking of the whole group can be instantaneously concluded only by observing the minimum eigenvalue of the interaction matrix. Under a designed admissible cooperative controller, the analysis of the interaction matrix reveals important properties of the MAS (like convergence and convergence rate). Furthermore, in this paper, limited information (position and velocity) of the arbitrarily given reference trajectory is only available to one or some agents in the group, which is different from [23–25]. In this case, it is more difficult to attain asymptotic convergence of the trajectory tracking error. To this end, we develop a nonlinear cooperative control based on robust integral signum cooperative error (RISCE) method, where the interaction matrix is used. By using the RISCE-based cooperative control, we obtain an asymptotic cooperation of MASs by theoretical proof and simulation illustration.

The rest of the paper is organized as follows: Some preliminaries are introduced in Section 2. The admissible linear cooperative control for the nominal system without uncertain terms is proposed in Section 3. The RISCE-based nonlinear control part and the asymptomatic stability analysis is studied in Section 4. Some simulation results are given in Section 5. Finally, some conclusions are stated in Section 6.

2. PRELIMINARIES

In multi-agent systems, the interaction topologies of agents are represented using a graph $\mathcal{G} = (\mathcal{V},\mathcal{E})$ with the sets of vertices $\mathcal{V}$ and edges $\mathcal{E}$. The set of vertices $\mathcal{V} = \{1,2,\dots,n\}$ is composed of the indices of agents. The symbol $|\mathcal{V}|$ represents the cardinality of the set $\mathcal{V}$, which satisfies $|\mathcal{V}| = n$. The set of edges is represented by $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$. If an edge exists between two vertices, the two vertices are called adjacent. A graph is simple if it has no self-loops or repeated edges. In other words, the edge (i,i) does not exist. The graph $\mathcal{G}$ is said to be undirected if $(i, j) \in \mathcal{E} \Leftrightarrow (j,i) \in \mathcal{E}$. In this work, simple and undirected graphs are considered.

The adjacency matrix of $\mathcal{G}$ is denoted by $G^A = [\omega^a_{ij}] \in R^{n\times n}$, where ω^a_{ij} represents the entry on the ith row and jth column of matrix G^A. Since the simple graph is considered, we have $\omega^a_{ii} = 0$. Since the graph is undirected, we have $\omega^a_{ij} = \omega^a_{ji}$ and $\omega^a_{ij} > 0$ if $(i, j) \in \mathcal{E}$, otherwise, $\omega^a_{ij} = 0$. The degree matrix of $\mathcal{G}$ is denoted by $G^D = \mathrm{diag}\{\sum^n_{j=1}\omega^a_{1j},\dots,\sum^n_{j=1}\omega^a_{nj}\}$.

The neighbour set $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$ of agent i, is composed of the indices of the agents j, which has interaction with the agent i. In other words, if $\omega^a_{ij} > 0$, then, agent j is a neighbour of agent i. The number of the neighbours of the agent i is equal to $|\mathcal{N}_i|$. In this paper, $\omega^a_{ij} = 1$, when $(i, j) \in \mathcal{E}$. Then, the degree matrix yields $G^D = \mathrm{diag}\{|\mathcal{N}_1|,|\mathcal{N}_2|,\dots,|\mathcal{N}_n|\}$.

We also define a diagonal matrix $G^L = \mathrm{diag}\{\omega^l_1,\dots,\omega^l_n\}$ representing the role of agents. If $\omega^l_i > 0$, then agent i is a leader. Otherwise, if $\omega^l_i = 0$, agent i is a follower, for $i \in \mathcal{V}$. Then, the leader set is defined as $\mathcal{V}_L = \{i \in \mathcal{V} : \omega^l_i > 0\}$. The leader set $\mathcal{V}_L \subset \mathcal{V}$ is a subset of $\mathcal{V}$, which contains the indices of the leaders. Particularly, all the agents are leaders, when $\mathcal{V}_L = \mathcal{V}$. The indices of the followers are contained in the complementary set of $\mathcal{V}_L$, namely, $\mathcal{V} - \mathcal{V}_L$. In this paper, we assign an agent i as a leader by setting $\omega^l_i = 1$.

We define the interaction matrix G for an MAS as follows

$$G = G^D - G^A + G^L. \tag{1}$$

Obviously, if no leader exists in the group, namely, leaderless formation structure, the matrix G^L will be equal to zero. Therefore, the proposed interaction matrix is an extension of *Laplacian*.

Multiple leaders with different desired trajectory will lead the group to split, in this case, the consensus cannot be achieved. In this paper, we have the following assumption.

Assumption 1: When multiple leaders exist in the MAS, the leaders share the same desired trajectory.

3. COOPERATIVE CONTROL FOR THE NOMINAL SYSTEM

Let us assume a multi-agent system with n agents, each agent i has the following dynamics

$$\ddot{x}_i = u_i, \tag{2}$$

where $u_i = \bar{u}_i + \tilde{u}_i$. The two terms $\bar{u}_i$ and $\tilde{u}_i$ represent the decentralized admissible cooperative control part and the RISCE part respectively.

As proposed in [31], let us denote the *trajectory tracking error* of agent i and the reference trajectory $r(t)$ by

$$e_i = x_i - r - d_{i0}. \tag{3}$$

Remark 1: The assignments of scalars d_{i0} will expand the agents to some special formation shapes. In order to focus on the objective of this paper, we consider a simplest way of expansion, i.e., rigid formation shape, which leads to constant scalars d_{i0}, $i \in \mathcal{V}$.

We first design the admissible cooperative control part $\bar{u}_i$ as follows

$$\begin{aligned}\bar{u}_i = &-k_{i2}\sum_{j\in\mathcal{N}_i}\omega_{ij}^a(\dot{x}_i - \dot{x}_j) - k_{i1}\sum_{j\in\mathcal{N}_i}\omega_{ij}^a(x_i - x_j - d_{ij})\\ &- k_{i2}\omega_i^l(\dot{x}_i - \dot{r}) - k_{i1}\omega_i^l(x_i - r - d_{i0}).\end{aligned} \tag{4}$$

We note that $d_{ij} := d_{i0} - d_{j0}$. Notations k_{i2} and k_{i1}, $i \in \mathcal{V}$ represent some positive gains. For any $i, j \in \mathcal{V}$, the gains satisfies

$$\frac{k_{i2}}{k_{i1}} = \frac{k_{j2}}{k_{j1}} = \frac{1}{\Lambda}, \tag{5}$$

where Λ is a constant scalar. The weights $\omega_i^l = 1$ if $i \in \mathcal{V}_L$, and $\omega_i^l = 0$ if $i \in \mathcal{V} - \mathcal{V}_L$.

Substituting (3) into (4), then, for an agent, we have

$$\begin{aligned}\ddot{e}_i = &-k_{i2}\sum_{j\in\mathcal{N}_i}\omega_{ij}^a(\dot{e}_i - \dot{e}_j) - k_{i1}\sum_{j\in\mathcal{N}_i}\omega_{ij}^a(e_i - e_j)\\ &- k_{i2}\omega_i^l\dot{e}_i - k_{i1}\omega_i^l e_i - \ddot{r}.\end{aligned} \tag{6}$$

Remark 2: In (6), we observe that the second derivative of the reference trajectory appears in the error dynamics of the followers (when $\omega_i^l = 0$). Knowing that the reference trajectory is not available for the follower, the term $\ddot{r}$ is treated as a uncertain term for the agents.

We give a useful lemma as follows

Lemma 1: For a linear system

$$\dot{x} = A_c x + \delta, \tag{7}$$

where A_c is Hurwitz, δ represents a bounded uncertain vector such that $\|\delta\| \le \delta_{\max}$ where $\delta_{\max}$ is a finite scalar. The operator $\|\cdot\|$ represents an $\mathcal{L}_\infty$. Then, the state of system (7) is ultimately bounded.

Let us denote by $e = [e_1, e_2, \ldots, e_n]^T$ a collective tracking error for all the agents. Then, the dynamics of e in state space yields

$$\begin{aligned}\frac{d}{dt}\begin{bmatrix} e \\ \dot{e}\end{bmatrix} = &\begin{bmatrix}\mathbf{0}_{n\times n} & I_n \\ -K_1(G^D - G^A) & -K_2(G^D - G^A)\end{bmatrix}\begin{bmatrix} e \\ \dot{e}\end{bmatrix}\\ &+ \begin{bmatrix}\mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ -K_1 G^L & -K_2 G^L\end{bmatrix}\begin{bmatrix} e \\ \dot{e}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{n\times n} \\ \ddot{r}\mathbf{1}_n\end{bmatrix},\end{aligned}$$

where $K_2 = \mathrm{diag}\{k_{12}, k_{22}, \ldots, k_{n2}\}$ and $K_1 = \mathrm{diag}\{k_{11}, k_{21}, \ldots, k_{n1}\}$.

According to (1), we obtain the collective tracking error dynamics in closed loop as follows

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{n\times n} & I_n \\ -K_1 G & -K_2 G\end{bmatrix}\begin{bmatrix} e \\ \dot{e}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{n\times n} \\ \ddot{r}\mathbf{1}_n\end{bmatrix}, \tag{8}$$

where $\mathbf{1}_n \in \mathbb{R}^n$ represents a vector whose elements are equal to 1.

Remark 3: Note that the second term in the right of the equation (8) is the uncertainty, since $\ddot{r}$ is unknown. Therefore, the nominal system can be represented by

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{n\times n} & I_n \\ -K_1 G & -K_2 G\end{bmatrix}\begin{bmatrix} e \\ \dot{e}\end{bmatrix}.$$

We note that $\|\ddot{r}\mathbf{1}_n\| \le \|\mathbf{1}_n\| \cdot |\ddot{r}|$, where $\|\mathbf{1}_n\|$ is a constant finite scalar. Since $|\ddot{r}|$ is bounded, $\|\ddot{r}\mathbf{1}_n\| \in \mathcal{L}_\infty$.

Let us define

$$A_G = \begin{bmatrix}\mathbf{0}_{n\times n} & I_n \\ -K_1 G & -K_2 G\end{bmatrix}. \tag{9}$$

The solution of the tracking error dynamics in (8) is ultimately bounded, if matrix A_G is Hurwitz and $|\ddot{r}|$ is bounded.

Now we will study when the matrix A_G is Hurwitz. The interaction matrix G in A_G describes the "information exchange" among the agents in a multi-agent system. Since the graph is undirected, G is symmetric.

Since the interaction matrix is constructed by using the *Laplacian*, we will introduce some important properties of the *Laplacian* for future use.

Lemma 2 [16]**:** Let $\mathcal{G}$ be an undirected graph with *Laplacian* L. Then, $\mathcal{G}$ is connected if and only if $\operatorname{rank}(L) = n-1$.

According to the definition of the degree matrix, the row sum of L satisfies $\sum_{j=1}^{n} L_{(i\cdot j)} = 0$, $i \in \mathcal{V}$. Thus, the *Laplacian* L has a zero eigenvalue corresponding to the eigenvector $\mathbf{1}_n$.

Lemma 3 [32]**:** A *Laplacian* matrix is semi-definite positive.

Now, we give a property of the interaction matrix as follow.

Proposition 1: Let $\mathcal{G} = \bigcup_{1\le j\le|\mathcal{V}|} \mathcal{G}_j$, be an undirected simple graph, where $\mathcal{G}_j$ represents connected subgraphs of $\mathcal{G}$. For any two subgraphs $\mathcal{G}_{j_a}$ and $\mathcal{G}_{j_b}$, their node sets satisfy $\mathcal{V}_{j_a} \cap \mathcal{V}_{j_b} = \Phi$. Then the interconnection matrix G in (1), is positive-definite, if $\mathcal{V}_L^j \neq \Phi$.

Proof: For each subgraph $\mathcal{G}_j$, its interaction matrix can be rewritten as $G_j = L_j + G_j^L$. According to lemma 3, we have $L_j \ge 0$. Considering the definition of G_j^L, we know that $G_j^L \ge 0$. Firstly, we suppose that there exists a nonzero vector $x \in \mathbb{R}^n$, which renders

$$x^T G_j x = x^T (L_j + G_j^L) x = x^T L_j x + x^T G_j^L x = 0.$$

Therefore, we must have $x^T L_j x = 0$ and $x^T G_j^L x = 0$.

We know that $\mathcal{G}_j$ is connected. Then, according to $x^T L_j x = 0$, the eigenvector of L_i satisfies $x = \alpha \mathbf{1}_n$, where α is a nonzero scalar. According to the fact that $\mathcal{V}_L^j \neq \Phi$, then, $G_j^L \ge 0$. As a result, $x^T G_j^L x = \alpha^2 \mathbf{1}_n^T G_j^L \mathbf{1}_n > 0$, which contradicts $x^T G_j^L x = 0$. Therefore, such a nonzero vector x does not exist. Thus, for any nonzero vector x, $x^T G_j x > 0$, namely, G_j is positive-definite for each subgraph $\mathcal{G}_j$.

Since we have $\mathcal{V}_{j_a} \cap \mathcal{V}_{j_b} = \Phi$ for any two subgraphs, thus, we conclude that the interaction matrix is block diagonal. Thus, $G = \operatorname{diag}\{G_1, G_2, \dots\}$. Since $G_j > 0$, then, the interaction matrix is positive-definite. □

In proposition 1, we note that the connectivity property of the graph of an MAS is not required for having a positive definite interaction matrix.

We will investigate as follows the consensus condition and the convergence of the tracking error in system (8) by the following theorem.

Theorem 1: For an agent in a multi-agent system with the controller $u_i = \bar{u}_i$, where $\bar{u}_i$ is in (4) and $\tilde{u}_i = 0$, the interaction topology can be represented by a graph $\mathcal{G}$. The agent dynamics yields double-integrator shown in (2). The dynamics of the collective tracking error (8) of the agents with respect to a sufficiently smooth reference trajectory $r(t)$ is ultimately bounded, if the conditions in proposition 1 are satisfied.

Proof: According to (5), the gain matrices can be represented by $K_1 = k_1 \mathcal{I}_n$ and $K_2 = k_2 \mathcal{I}_n$, where $\mathcal{I}_n$ is a real positive diagonal matrix. Then, we obtain $A_G = \begin{bmatrix} \mathbf{0}_{n\times n} & I_n \\ -k_1 \mathcal{I}_n G & -k_2 \mathcal{I}_n G \end{bmatrix}$. Note that $\mathcal{I}_n G$ may be not symmetric, we carry out a similarity transformation by using matrix $T = \begin{bmatrix} \mathcal{I}_n^{\frac{1}{2}} & 0_{n\times n} \\ 0_{n\times n} & \mathcal{I}_n^{\frac{1}{2}} \end{bmatrix}$, i.e.,

$$\tilde{A}_G = T^{-1} A_G T = \begin{bmatrix} \mathbf{0}_{n\times n} & I_n \\ -k_1 \mathcal{I}_n^{\frac{1}{2}} G \mathcal{I}_n^{\frac{1}{2}} & -k_2 \mathcal{I}_n^{\frac{1}{2}} G \mathcal{I}_n^{\frac{1}{2}} \end{bmatrix}.$$

Then, $\tilde{A}_G$ has the same eigenvalues as the matrix A_G. We observe that $\tilde{G} = \mathcal{I}_n^{\frac{1}{2}} G \mathcal{I}_n^{\frac{1}{2}}$ is symmetric, since G is symmetric. Additionally, $\tilde{G}$ has the same eigenvalues as $\mathcal{I}_n G$.

In view of condition i*)*, the eigenvalues of G are real and positive according to proposition (1). Since $\mathcal{I}_n$ is positive definite, then, $\tilde{G}$ is positive definite. Note that gains k_1 and k_2 are positive scalars. Let us denote $x = [x_1^T, x_2^T]^T$ by the eigenvector with respect to the eigenvalue λ of matrix $\tilde{A}_G$. Then, we have $x_2 = \lambda x_1$ and

$$(\lambda^2 I_n + \lambda k_2 \tilde{G} + k_1 \tilde{G}) x_1 = 0.$$

We left multiply the foregoing equation by x_1^*, where x_1^* represents the conjugate transpose of x_1. Denoting $x_1^* x_1 = a$, $k_2 x_1^* \tilde{G} x_1 = b$ and $k_1 x_1^* \tilde{G} x_1 = c$, we obtain

$$a\lambda^2 + b\lambda + c = 0.$$

The solution of the foregoing matrix yields

$$\lambda = \begin{cases} \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, & \text{if } b^2 - 4ac \ge 0, \\ \dfrac{-b \pm i\sqrt{b^2 - 4ac}}{2a}, & \text{if } b^2 - 4ac < 0. \end{cases} \tag{10}$$

Since $\tilde{G}$ is positive definite, then, $a > 0$, $b > 0$ and $c > 0$. We obtain that the eigenvalues of $\tilde{A}_G$ are negative or have negative real part. Therefore, the matrix A_G is Hurwitz. Then, according to Lemma 1, the solution of the error dynamics (8) is ultimately bounded. Furthermore, if $\ddot{r}$ is available, the tracking error (1) is asymptotically stable. □

Now, we will investigate how the interaction matrix G affects the convergence rate of e. This will be the purpose of Theorem 2.

Theorem 2: In a multi-agent system with agent dynamics (2), the controller referred in theorem 1 are used for each agent. We assume that the conditions in proposition 1 are satisfied such that the corresponding interaction matrix G is positive definite. Let us denote $\lambda_{\min}(\mathcal{I}_n G) = \tilde{\lambda}_{\min}$, then, the convergence rate of the tracking error e is proportional to $\tilde{\lambda}_{\min}$.

Proof: Let us set $x = [x_1^T, x_2^T]^T$ by the eigenvector with respect to the eigenvalue λ of matrix $\tilde{A}_G$. Then, we have

$-k_1\tilde{G}x_1 - k_2\tilde{G}x_2 = \lambda x_2$. Since $x_2 = \lambda x_1$, then we obtain

$$\tilde{G}x_2 = -\frac{\lambda^2}{k_2\lambda + k_1}x_2. \tag{11}$$

We denote $\tilde{\lambda} = -\frac{\lambda^2}{k_2\lambda+k_1}$. Equation (11) implies that $\tilde{\lambda}$ is an eigenvalue of matrix $\tilde{G}$ with corresponding eigenvector x_2. It is worth to note that $\tilde{\lambda}$ is real, since $\tilde{G}$ is symmetric matrix. According to $\tilde{\lambda}$, we have

$$\lambda^2 + k_2\tilde{\lambda}\lambda + k_1\tilde{\lambda} = 0.$$

If we define $Re(\lambda_{\max}) := f(\tilde{\lambda})$, according to the solution of the foregoing equation, $f(\tilde{\lambda})$ yields

$$f(\tilde{\lambda}) = \begin{cases} \dfrac{-k_2\tilde{\lambda}+\sqrt{k_2^2\tilde{\lambda}^2-4k_1\tilde{\lambda}}}{2}, \text{ if } k_2^2\tilde{\lambda}^2-4k_1\tilde{\lambda}\geq 0, \\ \dfrac{-k_2\tilde{\lambda}}{2}, \text{ if } k_2^2\tilde{\lambda}^2-4k_1\tilde{\lambda}<0. \end{cases}$$

Let us denote $f_1(\tilde{\lambda}) = -k_2\tilde{\lambda} + \sqrt{k_2^2\tilde{\lambda}^2 - 4k_1\tilde{\lambda}}$ and $f_2(\tilde{\lambda}) = -k_2\tilde{\lambda}$. Since the gains $k_1 > 0$ and $k_2 > 0$, then

$$\begin{aligned} -4k_1^2 < 0 \Rightarrow & k_2^4\tilde{\lambda}^2 - 4k_2^2k_1\tilde{\lambda} - 4k_1^2 < k_2^4\tilde{\lambda}^2 - 4k_2^2k_1\tilde{\lambda} \\ \Rightarrow & (k_2^2\tilde{\lambda} - 2k_1)^2 < k_2^2(k_2^2\tilde{\lambda}^2 - 4k_1\tilde{\lambda}) \\ \Rightarrow & k_2^2\tilde{\lambda} - 2k_1 < k_2\sqrt{k_2^2\tilde{\lambda}^2 - 4k_1\tilde{\lambda}} \\ \Rightarrow & \frac{k_2^2\tilde{\lambda} - 2k_1}{\sqrt{k_2^2\tilde{\lambda}^2 - 4k_1\tilde{\lambda}}} < k_2 \\ \Rightarrow & -k_2 + \frac{k_2^2\tilde{\lambda} - 2k_1}{\sqrt{k_2^2\tilde{\lambda}^2 - 4k_1\tilde{\lambda}}} < 0 \\ \Rightarrow & f_1'(\tilde{\lambda}) < 0, \end{aligned}$$

and

$$f_2'(\tilde{\lambda}) = -k_2 < 0.$$

Therefore, $f_1(\tilde{\lambda})$ and $f_2(\tilde{\lambda})$ are decreasing functions.

When $k_2^2\tilde{\lambda}^2 - 4k_1\tilde{\lambda} = 0$, since $\tilde{\lambda} \neq 0$, this implies $\tilde{\lambda} = \frac{4k_1}{k_2^2}$, since the graph is connected and a leader exists, i.e., $\tilde{\lambda} > 0$. We assume $\tilde{\lambda}_- < \frac{4k_1}{k_2^2}$. Then,

$$\lim_{\tilde{\lambda}_- \to \frac{4k_1}{k_2^2}} \frac{f_1(\tilde{\lambda}_-)}{2} = \frac{2k_1}{k_2} = \frac{f_2(\frac{4k_1}{k_2^2})}{2},$$

which indicates that $f(\tilde{\lambda})$ is continuous at point $\tilde{\lambda} = \frac{4k_1}{k_2^2}$. Therefore, $f(\tilde{\lambda})$ is decreasing.

Then, we can conclude that $Re(\lambda_{\max})$ is equal to $f(\tilde{\lambda}_{\min})$. Note that $\tilde{\lambda}_{\min}$ represents the minimum eigenvalue of the interaction matrix $\tilde{G}$. Therefore, the multi-agent system with a relatively high $\tilde{\lambda}_{\min}$ will have a relatively small $\lambda_{\max}$, i.e., the maximum eigenvalue of A_G will be farer away from the imaginary axis. Then, the "worst-case speed of convergence" (defined in [16] and used in our multi-agent system) will increase. □

In Theorem 2, the convergence rate of the tracking error is proven to be related to the minimum eigenvalue of matrix $\mathcal{I}_n G$. Particularly, if the gains are selected as $K_1 = k_1 I_n$ and $K_2 = k_2 I_n$, the convergence rate is uniquely related to the minimum eigenvalue of matrix G. In the sequel, we will investigate the cases that cause the minimum eigenvalue of G increasing.

Proposition 2: The minimum eigenvalue of the interaction matrix G for an undirected connected graph $\mathcal{G}$ will increase, if the edge set $\mathcal{E}$ is augmented.

Proof: The graph can be represented by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The corresponding interaction matrix is G. We construct another graph $\mathcal{G}_e = (\mathcal{V}, \mathcal{E}_e)$, where the vertices set is the same as the graph $\mathcal{G}$, while the edge set $\mathcal{E}_e$ represents some new links between some agents. Its interaction matrix is denoted by G_e. Then, the graph $\mathcal{G}$ becomes $\mathcal{G}' = (\mathcal{V}, \mathcal{E} \cup \mathcal{E}_e)$ after extra edges have been added. The interaction matrix for graph $\mathcal{G}'$ is denoted by G', then, we have

$$G' = G + G_e.$$

According to Courant-Fischer theorem, $\lambda_{\min}(G') = \min_{\|x\|=1} x^T G' x = \min_{\|x\|=1} x^T (G+G_e) x > \min_{\|x\|=1} x^T G x + \min_{\|x\|=1} x^T G_e x = \lambda_{\min}(G) + \lambda_{\min}(G_e) \geq \lambda_{\min}(G)$. Then, the result is obtained. □

Corollary 1: The minimum eigenvalue of the interaction matrix for an undirected connected graph $\mathcal{G}$ will increase, if the leader set $\mathcal{V}_L$ is augmented.

Proof: The proof is similar to the proof of Proposition 2. □

According to Proposition 1, the graph connectivity is not required for achieving cooperation convergence.

In this section, we show that the convergence of cooperation is only related to the smallest eigenvalue of the interaction matrix, rather than the connectivity of the graph. The development of the interaction matrix extends the function of *Laplacian*. In practice, the numerical calculation of smallest eigenvalue of a matrix is usually simpler than the calculation of second smallest eigenvalue. Thus, the development of interaction matrix has also practical significance.

4. ASYMPTOMATIC STABILITY ANALYSIS

In the analysis shown above, we note that there exists unknown term $\ddot{r}\mathbf{1}_n$ which makes the system tracking error ultimately bounded but not converging to zero. In this section, we design the $\tilde{u}_i$ based on RISCE method to attain asymptotic convergence.

Considering that the reference trajectory $r(t)$ should be bounded in actual system,we have the follow assumption.

Assumption 2: The second-order, third-order, fourth-order derivatives of reference trajectory $r(t)$ are included in $\mathcal{L}_\infty$.

$$\ddot{r},\ \dddot{r},\ r^{(4)} \in \mathcal{L}_\infty, \tag{12}$$

where $\mathcal{L}_\infty$ indicates the space of bounded signal.

With the assumption, we then design the $\tilde{u}_i$ as follow.

$$\tilde{u}_i = \int_0^t (\alpha\bar{u}_i + \beta \cdot \mathrm{sgn}(\bar{u}_i))d\tau, \tag{13}$$

where α and β are positive scalars and sgn$(\cdot)$ denotes the standard signum function.

Let us define a cooperative error for an agent i as follow

$$e_{ci} = -\sum_{j\in\mathcal{N}_i} \omega_{ij}^a(x_i - x_j - d_{ij}) - \omega_i^l(x_i - r - di0). \tag{14}$$

We denote by $u^T = [u_1, u_2, \dots, u_n]$ and $e_c^T = [e_{c1}, \dots, e_{cn}]$ the collective vectorial form of the control inputs and cooperative error of all agents. According to (4) and (3), we can rewrite the closed-loop multi-agent system in the following form.

$$\ddot{x} = u, \tag{15}$$
$$u = \bar{u} + \tilde{u}, \tag{16}$$
$$e_c = -Ge, \tag{17}$$
$$\bar{u} = K_2\dot{e}_c + K_1 e_c, \tag{18}$$
$$\tilde{u} = \int_0^t (\alpha\bar{u} + \beta \cdot \mathrm{sgn}(\bar{u}))d\tau. \tag{19}$$

The interaction matrix G is defined in (1).

Remark 4: We note that the higher-order derivatives (greater than one) are not needed in the controller (16).

Let us define a new variable by

$$e_r = \dot{\bar{u}} + \alpha\bar{u}. \tag{20}$$

We now differentiate e_r and substituting from the second derivative of (19), then obtain

$$\dot{e}_r = K_2\dddot{e}_c + K_1\ddot{e}_c + \alpha\ddot{\bar{u}}. \tag{21}$$

Applying the third derivative of (17) and the derivative of (15) and (16), we get

$$\dot{e}_r = (K_2 G\cdot\dddot{r}\mathbf{1}_n + K_1\ddot{e}_c + \alpha\ddot{\bar{u}}) - K_2Ge_r - \beta K_2 G\mathrm{sgn}(\bar{u}).$$

Using (19) and (20), we eliminate all the derivatives of e_c and $\bar{u}$ in the above equation, we can get the auxiliary system state equations from (22) to (24).

$$\dot{e}_c = K_2^{-1}\bar{u} - \Lambda e_c, \tag{22}$$
$$\dot{\bar{u}} = e_r - \alpha\bar{u}, \tag{23}$$
$$\begin{aligned}\dot{e}_r =& K_2G\cdot\dddot{r}\mathbf{1}_n + (\Lambda+\alpha)e_r - (\alpha\Lambda + \Lambda^2 + \alpha^2)\bar{u} \\ &+ K_1\Lambda^2 e_c - K_2Ge_r - \beta K_2G\cdot\mathrm{sgn}(\bar{u}).\end{aligned} \tag{24}$$

To prove stability of the system, we state one lemma which will be invoked later.

Lemma 4: Let the function $L(t) \in \mathcal{R}$ be defined as:

$$L := e_r^T K_2(G\cdot\dddot{r}\mathbf{1}_n - \beta G\mathrm{sgn}(\bar{u})), \tag{25}$$

if the control gain β satisfies the following condition:

$$\beta \geq \frac{\|\dddot{r}\|_{\mathcal{L}_\infty}}{\lambda_{\min}(G)} + \frac{\|r^{(4)}\|_{\mathcal{L}_\infty}}{\alpha\cdot\lambda_{\min}(G)}, \tag{26}$$

where $\|\cdot\|_{\mathcal{L}_\infty}$ denotes the $\mathcal{L}_\infty$ norm, then we have

$$P(t) := \zeta_b - \int_0^t L(\tau)d\tau \geq 0, \tag{27}$$

where ζ_b is a constant scalar defined as

$$\zeta_b := \bar{u}^T(0)G\beta\mathrm{sgn}(\bar{u}(0)) - \bar{u}^T(0)G\ddot{r}(0). \tag{28}$$

Proof: Substituting (20) into (25) and integrating in time, we get

$$\begin{aligned}&\int_0^t L(\tau)d\tau \\ &= \int_0^t \dot{\bar{u}}^T(\tau)G\dddot{r}(\tau)\mathbf{1}_n - \dot{\bar{u}}^T(\tau)G\beta\mathrm{sgn}(\bar{u}(\tau)) \\ &\quad + \alpha\bar{u}^T(\tau)\left(G\dddot{r}(\tau) - G\beta\mathrm{sgn}(\bar{u}(\tau))\right)d\tau.\end{aligned} \tag{29}$$

Then we integrate the first integral on the right hand side by part, noticing that $G\cdot r\mathbf{1}_n = G^L r\mathbf{1}_n$, it becomes

$$\begin{aligned}&\int_0^t L(\tau)d\tau \\ &= \bar{u}^T(\tau)G\cdot\ddot{r}(\tau)\mathbf{1}_n|_0^t - \int_0^t \bar{u}^T(\tau)G\cdot r^{(4)}(\tau)\mathbf{1}_n d\tau \\ &\quad - \bar{u}^T(\tau)G\beta\mathrm{sgn}(\bar{u}(\tau))|_0^t + \int_0^t \alpha\bar{u}^T(\tau)\cdot \\ &\quad (G\dddot{r}(\tau) - G\beta\mathrm{sgn}(\bar{u}(\tau)))d\tau \\ &= \bar{u}^T(t)G^L\ddot{r}(t) - \bar{u}^T(t)G\beta\mathrm{sgn}(\bar{u}(t)) \\ &\quad - \bar{u}^T(0)G\ddot{r}(0) + \bar{u}^T(0)G\beta\mathrm{sgn}(\bar{u}(0)) \\ &\quad + \int_0^t \alpha\bar{u}^T(\tau)(G^L\dddot{r}(\tau) - G\beta\mathrm{sgn}(\bar{u}(\tau))) \\ &\quad - \bar{u}^T(\tau)G^L r^{(4)}(\tau)d\tau \\ &\leq \|\bar{u}(t)\|\cdot\|\ddot{r}(t)\| - \beta\lambda_{\min}(G)\|\bar{u}(t)\| \\ &\quad - \bar{u}^T(0)G\ddot{r}(0) + \bar{u}^T(0)G\beta\mathrm{sgn}(\bar{u}(0)) \\ &\quad + \int_0^t \alpha\|\bar{u}(\tau)\|\cdot\|\dddot{r}(\tau)\| \\ &\quad - \alpha\beta\lambda_{\min}(G)\|\bar{u}(\tau)\| + \|\bar{u}(\tau)\|\cdot\|r^{(4)}\|d\tau \\ &= \|\bar{u}(t)\|\cdot(\|\ddot{r}(t)\| - \beta\lambda_{\min}(G))\end{aligned}$$

$$+\int_0^t \|\bar{u}(\tau)\|\cdot(\alpha\|\dddot{r}(\tau)\| - \alpha\beta\lambda_{\min}(G)) + \|r^{(4)}\|)d\tau$$
$$-\bar{u}^T(0)G\dddot{r}(0) + \bar{u}^T(0)G\beta\mathrm{sgn}(\bar{u}(0)). \quad (30)$$

According to (30), we can find that when (26) is satisfied, equation (27) holds. □

With the lemma 4, we now propose the theorem about the stability of the multi-agent system with the proposed cooperative control.

Theorem 3: For a multi-agent system described by (15) with the cooperative controller (16), where assumption 2 is satisfied, the system states will asymptotically track the reference trajectory, if the control gains are selected to satisfy (26) and the following inequalities

$$\begin{aligned}&2K_1K_2 > K_1^4 + K_2,\\&(\alpha K_2 + K_1)^2 > \frac{1}{2\alpha}I_n + \alpha K_1K_2,\\&\lambda_{\min}(K_2)\cdot\lambda_{\min}(G) > \alpha + \Lambda + \frac{1}{2}\Lambda^2.\end{aligned} \quad (31)$$

Proof: Let us choose a positive-definite function as

$$V := \frac{1}{2}e_c^Te_c + \frac{1}{2}(\alpha\Lambda + \Lambda^2 + \alpha^2)\bar{u}^T\bar{u} + \frac{1}{2}e_r^Te_r + P, \quad (32)$$

where P is defined in (27). Since Λ and α are positive scalars, the function is positive definite. The derivative of (32) yields

$$\begin{aligned}\dot{V} =& e_c^TK_2^{-1}\bar{u} - \Lambda e_c^Te_c + (\alpha\Lambda + \Lambda^2 + \alpha^2)\bar{u}^T\dot{\bar{u}}\\&+ e_r^T\dot{e}_r + \dot{P}.\end{aligned} \quad (33)$$

According to (22), (23) and (24), the derivative $\dot{V}$ satisfies

$$\begin{aligned}\dot{V} =& e_c^TK_2^{-1}\bar{u} - \Lambda e_c^Te_c - (\alpha^2\Lambda + \alpha\Lambda^2 + \alpha^3)\bar{u}^T\bar{u}\\&- e_r^T(K_2G - \Lambda I_n - \alpha I_n)e_r + \Lambda e_r^TK_1e_c\\\le& -e_c^T(\Lambda I_n - \frac{1}{2}K_2^{-1} - \frac{1}{2}K_1^2\Lambda^2)e_c\\&- \bar{u}^T(\alpha^2\Lambda I_n + \alpha\Lambda^2 I_n + \alpha^3 I_n - \frac{1}{2}K_2^{-2})\bar{u}\\&- e_r^T(K_2G - \Lambda I_n - \alpha I_n - \frac{1}{2}\Lambda^2 I_n)e_r.\end{aligned} \quad (34)$$

According to the first two inequalities in (31), we conclude that the first two terms in the right-hand side of the inequality (34) are negative definite. For the third term, we have

$$\begin{aligned}&-e_r^T(K_2G - \Lambda I_n - \alpha I_n - \frac{1}{2}\Lambda^2 I_n)e_r\\&\le -\underline{\lambda}(K_2)\cdot\underline{\lambda}(G)\cdot\|e_r\|^2 + (\Lambda I_n + \alpha I_n + \frac{1}{2}\Lambda^2 I_n)\cdot\|e_r\|^2\\&\le 0.\end{aligned} \quad (35)$$

We can observe that $\dot{V} \le 0$, if and only if $e_c = 0$, $\bar{u} = 0$ and $e_r = 0$. According to LaSalle's invariant principal, error e_c converges to origin asymptotically. If the conditions in proposition 1 is satisfied, the trajectory tracking error e converges to origin asymptotically, i.e. the agents' states track the reference asymptotically. □

5. SIMULATION RESULTS

We first interpret the relation between convergence speed and $\lambda_{\min}(G)$ in subsection 5.1. Then, the asymptotic converging of the trajectory tracking error using RISCE is shown in subsection 5.2

5.1. Relation of convergence speed and $\lambda_{\min}(G)$

In Section 3, we demonstrate that the convergence rate of the tracking errors of agents is proportional to the minimum eigenvalue of the interaction matrix $\lambda_{\min}(G)$. In Example 1, we first show some cases where $\lambda_{\min}(G)$ augments, and in the sequel, the convergence rate increases. In Example 2, we show a special case of the convergence of tracking error of a multi-agent system with disconnected graph.

Since the added desired inter-distances of agents have no effects on the convergence rate, we omit the terms of d_{ij} in (4) in this subsection for the sake of simplicity.

Example 1: Consider a multi-agent system with four agents whose dynamics is shown in (2), the admissible cooperative control in (4) is used, where $k_{i1} = 1$ and $k_{i2} = 1$, for all $i \in \mathcal{V}$. The reference is a stationary point at origin. The interaction matrix of the left graph in Fig. 2 is as follows:

$$G = G^D - G^A + G^L = \begin{bmatrix} 2 & -1 & 0 & 0\\ -1 & 2 & 0 & -1\\ 0 & 0 & 1 & -1\\ 0 & -1 & -1 & 2\end{bmatrix}. \quad (36)$$

Its smallest eigenvalue is $\lambda_{\min}(G) = 0.1206$. The consensus of the agents states is shown in Fig. 1. The transient time represents the time when the response of the agents enters the error band $[-0.05, 0.05]$.

Then, we consider the following two cases.

Augmented edge set $\mathcal{E}$: We assume that four extra edges $(1,4)$, $(4,1)$, $(2,3)$ and $(3,2)$ are added in edge set $\mathcal{E}$, then the graph is represented by Fig. 2 (right).

The added four edges and the same vertices constitute the graph $\mathcal{G}_e = (\mathcal{V}, \mathcal{E}_e)$. Then, G_e yields

$$G_e = \begin{bmatrix} 1 & 0 & 0 & -1\\ 0 & 1 & -1 & 0\\ 0 & -1 & 1 & 0\\ -1 & 0 & 0 & 1\end{bmatrix},$$

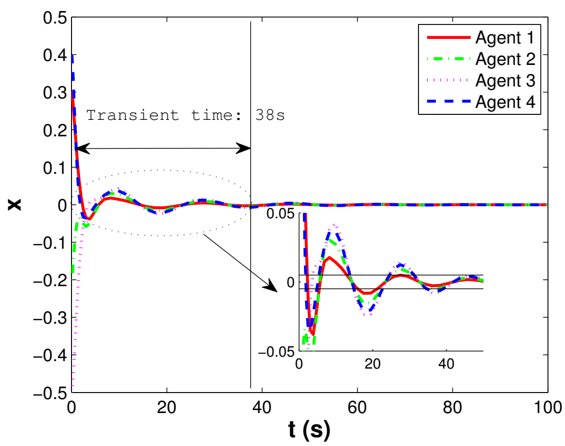


Fig. 1. The consensus result with interaction matrix G in (36). The transient time represents the time when the response of the agents enters the error band $[-0.05, 0.05]$. The transient time is 38 s.

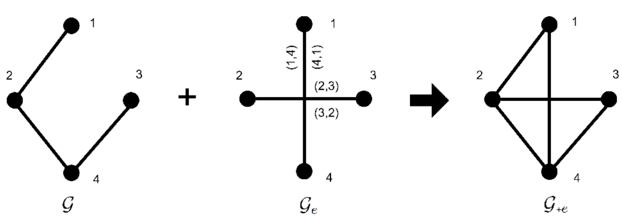


Fig. 2. The graphs of a multi-agent system with $\mathcal{V}_l = \{1\}$. The interaction topologies are represented by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ (on the right) and $\mathcal{G}_e = (\mathcal{V}, \mathcal{E}_{+e})$ (on the left). The new edge set $\mathcal{E}_{+e} = \mathcal{E} \cup \mathcal{E}_e$, where $\mathcal{E}_e = \{(1,4),(4,1),(2,3),(3,2)\}$

and G_{+e} yields

$$G_{+e} = G + G_e = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}. \tag{37}$$

Then, we can calculate that $\lambda_{\min}(G_{+e}) = 0.1864 > \lambda_{\min}(G)$. The simulation result is shown in Fig. 3. We observe that the convergence rate with extra edges is faster than the MAS with original topology in Fig. 1.

Remark 5: In fact, adding new edges may increase the total input energy of the MAS, because the trace of the interaction matrix is augmented (Trace(G_{+e}) >Trace(G)). Similarly, the method such as augmented leader set $\mathcal{V}_L$ will also increase the convergence rate of the MAS, since the total input energy is increased.

We introduce the second case where the total input energy maintains constant, but the minimum eigenvalue of interaction matrix augments.

Weighted-neighbor-based protocol: In the foregoing stated case, the neighbors of an agent are equally weighted. For example, in the multi-agent system with interaction matrix G_{+e} (37), the agent 2 has three neighbors, which are agents 1, 3, and 4. Their weights are "1". If we use the weighted-neighbor-based protocol [33] to assign the weights in controller 4. We get the following interaction matrix G_w in (38), where Trace(G_w)=Trace(G_{+e}).

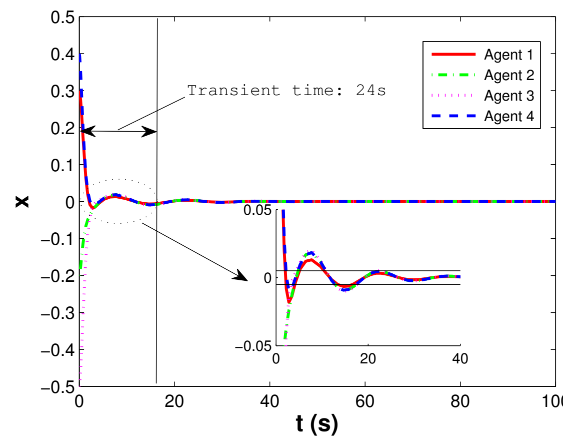
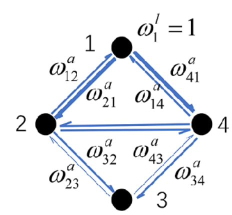


Fig. 3. The consensus result with interaction matrix G_{+e} in (37). The transient time is 24 s.



$$G_w = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -\frac{3}{2} & 3 & -\frac{2}{2} & -\frac{1}{2} \\ 0 & -1 & 2 & -1 \\ -\frac{3}{2} & -\frac{2}{2} & -\frac{1}{2} & 3 \end{bmatrix}. \tag{38}$$

By reassigning the weights of agents, the new interaction matrix has greater smallest eigenvalue, i.e., $\lambda_{\min}(G_w) = 0.2759 > \lambda_{\min}(G_{+e}) = 0.1864$. The consensus result using G_w is shown in Fig. 4. We can observe that the convergence rate is increased, compared to Fig. 3. We note that the reassignment of the weights does not change the trace of the interaction matrix.

Therefore, we conclude that some special assignments of the weights in (4) can augment the smallest eigenvalue of a multi-agent system without increasing the total energy input of an MAS.

The example 1 shows that the convergence speed of the trajectory tracking error of a MAS is proportional to the smallest eigenvalue of the interaction matrix.

The following example show that the cooperation convergence can be achieved, although the graph is disconnected. This results can be predicted by observing the positive-definite interaction matrix.

Example 2: Consider the MAS introduced in example 1. Among the four agents, there are two connected sub-

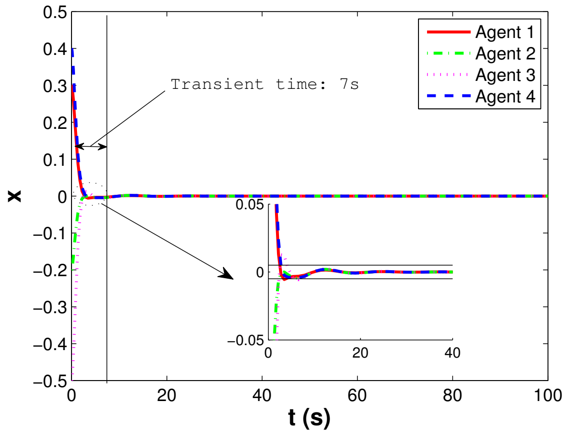


Fig. 4. The consensus result with interaction matrix G_w in (38). The transient time is 7 s.

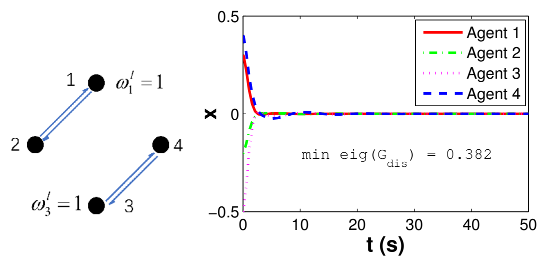


Fig. 5. Multi-agent system achieve consensus with non-singular interaction matrix, even the graph is disconnected. Left: the disconnected graph; Right: the consensus of agents states.

groups $\mathcal{V}_1 = \{1,2\}$ and $\mathcal{V}_2 = \{3,4\}$. In each of them, there exists $\mathcal{V}_L^1 = \{1\}$ and $\mathcal{V}_L^2 = \{3\}$, such that the conditions in Proposition 1 is satisfied. Then we can write the interaction matrix as follows

$$G_{dis} = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

We observe that $\lambda_{\min}(G_{dis}) = 0.382 > 0$. Therefore, the outputs of the agents are able to achieve the convergence of cooperation, as shown in Fig. 5, even though the graph is disconnected.

5.2. Asymptotic convergence of the formation

In this subsection, we compare the trajectory tracking of the multi-agent systems using respectively the RISCE based cooperative control and the composited nonlinear feedback based PD (CNF-PD) cooperative control [31, 34]. The CNF-PD method has shown very satisfactory performance in aggressive formation of MASs. It is very robust w.r.t. the change of unknown external navigation trajectories.

Let us reconsider the MAS with information exchanging graph $\mathcal{G}$ in the left of Fig. 2. Its interaction matrix G is shown in equation (36), whose smallest eigenvalue is $\lambda_{\min}(G) = 0.1206$. Then, according to (26) and (31), the control gains are selected as $\Lambda = 1$, $\alpha = 3$, $\beta = 180$, $k_{1i} = 40$, $k_{2i} = 40$, $i \in \{1,2,3,4\}$ for each agent. We suppose that the agents should maintain the inter-distances $d_{ij} = 1$, $j \in \mathcal{N}_i$.

In the MAS with CNF-PD cooperative control, the control inputs for the leader and follower can be written as follows:

When $i \in \mathcal{V}_L$,

$$\begin{aligned} u_i = & -k_{i2} \sum_{j\in\mathcal{N}_i} (\dot{x}_i - \dot{x}_j) - k_{i1} \sum_{j\in\mathcal{N}_i} (x_i - x_j - d_{ij}) \\ & - k_{i2}(\dot{x}_i - \dot{r}) - k_{i1}(x_i - r - d_{0i}) \\ & - \eta_1 \exp^{-\eta_2 e_{iCNF}^2} \dot{e}_{iCNF}, \end{aligned} \tag{39}$$

where we denote $e_{iCNF} = \dot{x}_i - \dot{x}_j - d_{ij} + \dot{x}_i - \dot{r} - d_{0i}$.

When $i \in \mathcal{V} - \mathcal{V}_L$,

$$\begin{aligned} u_i = & -k_{i2} \sum_{j\in\mathcal{N}_i} (\dot{x}_i - \dot{x}_j) - k_{i1} \sum_{j\in\mathcal{N}_i} (x_i - x_j) \\ & - \eta_1 \exp^{-\eta_2 e_{iCNF}^2} \dot{e}_{iCNF}, \end{aligned} \tag{40}$$

where $e_{iCNF} = \dot{x}_i - \dot{x}_j - d_{ij}$. We select $k_{1i} = 150$ and $k_{2i} = 100$, $i \in \{1,2,3,4\}$. The selection protocol of the CNF gains is given in [31]. Here, we select $\eta_1 = 100$ and $\eta_2 = 1$.

Three tests, where the reference trajectories are $r_1 : x = 3\sin(t)$, $r_2 : x = 3\sin(2t)$ and $r_3 : x = 3\sin(3t)$, are given respectively. The objective is that the four agents cooperate to maintain the given inter-distances d_{ij}, meanwhile, the centre of the formation (COF) should track the external given trajectories r_1, r_2 or r_3.

The aim of giving three sinusoidal trajectories of different frequencies is to show if the proposed cooperation control has good robustness w.r.t. to different external trajectories. Same tests are also processed with using the CNF-PD method for the purpose of comparing.

The initial states are $x_0 = [-1,-2,-1,-2]^T$ for all these tests. The simulation results are shown in figures from Fig. 6 to Fig. 8.

In Fig. 6, we observe that the states of agents can track the given trajectory r_1 using both CNF-PD (on top of Fig. 6) and RISCE (on middle of Fig. 6) based cooperation controllers. The foregoing selected control gains make satisfactory performance. According to the sub-figure on bottom of Fig. 6, the error of COF w.r.t. the given trajectory converges asymptotically to origin by using RISCE based cooperation control.

Let us redo the test by giving different trajectories r_2 and r_3 in Fig. 7 and Fig. 8. With the same control gains and initial conditions, the tracking accuracy is getting worse when CNF-PD method is used (see the sub-figure on top

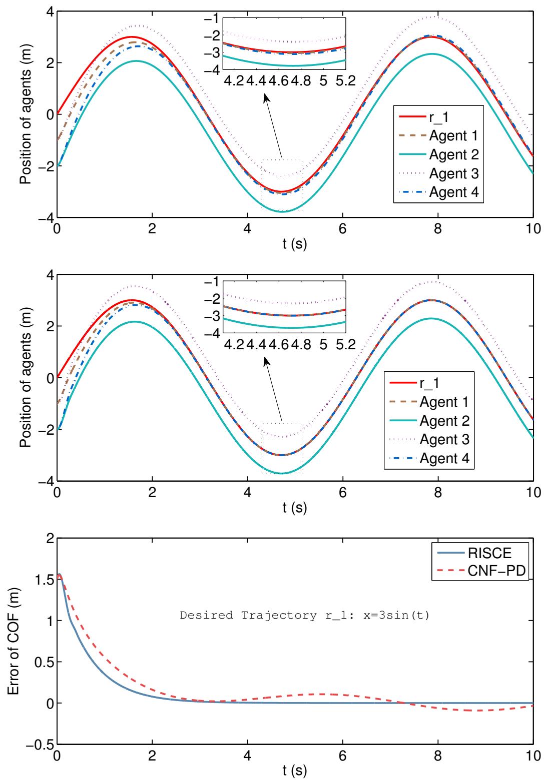


Fig. 6. Responses of agents states (x_i, $i = \{1,2,3,4\}$, when $r_1 : x = 3\sin(t)$) with CNF-PD (top) and RISCE (middle) based cooperative control methods. The sub-figure on the bottom represents the errors of the centre of formation (COF) w.r.t the trajectory r_1 by using these two controllers.

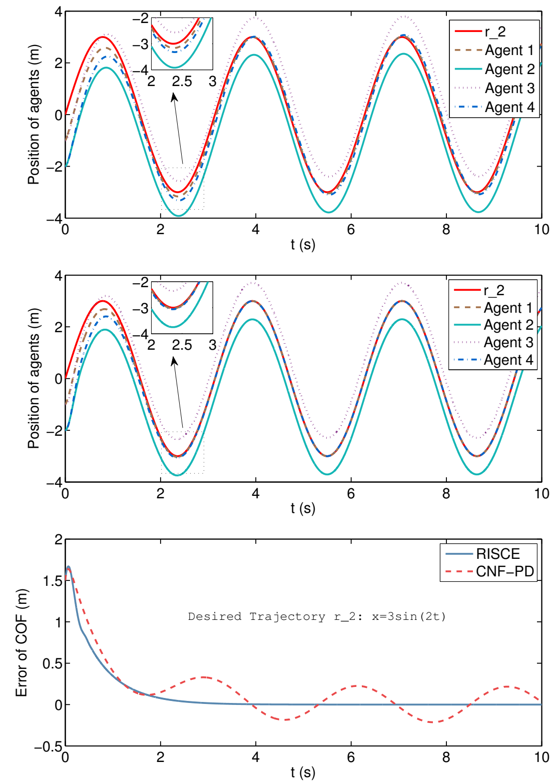


Fig. 7. Responses of agents states (x_i, $i = \{1,2,3,4\}$, when $r_2 : x = 3\sin(2t)$) with CNF-PD (top) and RISCE (middle) based cooperative control methods. The sub-figure on the bottom represents the errors of the centre of formation (COF) w.r.t the trajectory r_2 by using these two controllers.

of Fig. 7 and Fig. 8). Furthermore, the error of COF has greater oscillation around the origin (see the red dashed lines in the sub-figure on bottom of Fig. 7 and Fig. 8).

Nevertheless, by using the RISCE based cooperative control, not only the agents keep the desired inter-distance (sub-figure on middle of Fig. 7 and Fig. 8), but also the error of COF converges to the origin asymptotically (solid lines in sub-figures on bottom of Fig. 7 and Fig. 8), even different trajectories are given.

A planar formation of four agents using our proposed cooperation controller is given in Fig. 9, where we assume that the motion on x and y directions are decoupled into two second-order systems, such assumption is practical in the control of robots like quadrotors and AGVs.

We select the same control gains as foregoing mentioned. The desired trajectory r is represented by $r_x : x = 3\sin(t)$ and $r_y : y = -3\cos(0.25t) - 1.5\cos(t) - 0.75\sin(t)$ respectively. The initial states are $x_0 = [-1,-2,-1,-2]^T$ and $y_0 = [-1,-2,-4,-3]^T$. Fig. 9 shows that the agents form a rectangular pattern, and the COF asymptotically track the desired trajectory.

In this section, we first introduce some cases where the convergence rate increases in terms of the increase of the minimum eigenvalue of interaction matrix. Then, the simulation results validate that the proposed cooperative control can guarantee an asymptotic convergence of an MAS. Furthermore, it shows better performance than the CNF-PD cooperative control.

6. CONCLUSION

In this paper, the interaction matrix is first proposed to analyse the consensus of agents states and the convergence of desired trajectory tracking error. Some important properties, such as the convergence and convergence rate of the desired trajectory tracking error of an MAS are proven to be related to the smallest eigenvalue of the interaction matrix. Then, in order to obtain an asymptotic convergence, we develop a nonlinear cooperative control based on robust integral signum cooperative error (RISCE) method, where the interaction matrix is used. Both the theoretical proof and the simulation results show that the proposed controller make the multi-agent system attaining asymptotic convergence to arbitrary trajectory tracking error. A

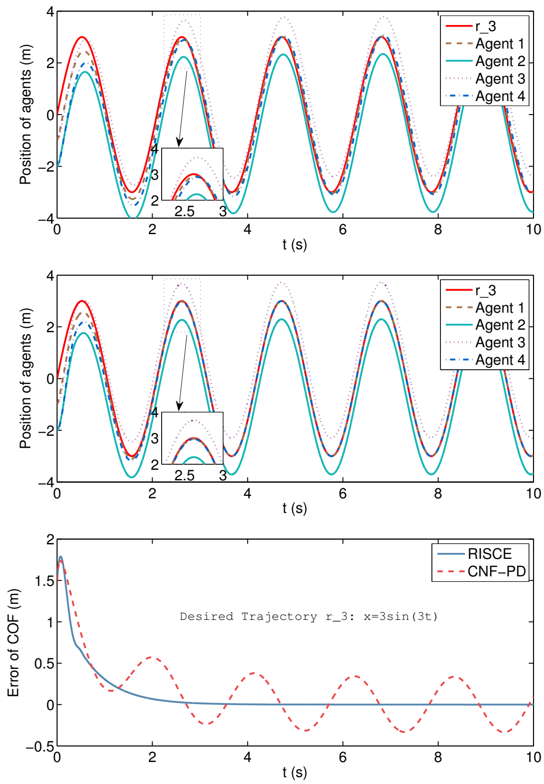


Fig. 8. Responses of agents states (x_i, $i = \{1,2,3,4\}$, when $r_3 : x = 3\sin(3t)$) with CNF-PD (top) and RISCE (middle) based cooperative control methods. The sub-figure on the bottom represents the errors of the centre of formation (COF) w.r.t the trajectory r_3 by using these two controllers.

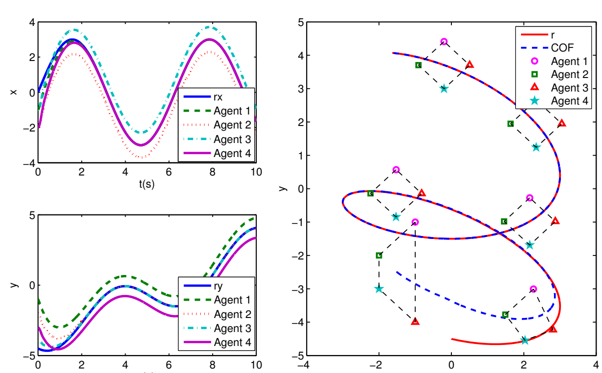


Fig. 9. The formation of four agents with their COF tracking the desired trajectory, using the RISCE based cooperative control.

comparison of the proposed control approach is given to show the improvements on the trajectory tracking w.r.t the CNF-PD controller.

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