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An Optimal Approach to Online Tuning Method for PID Type Iterative Learning Control

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Abstract: The proportional-integral-derivative (PID) controller is widely used in process control engineering. However, the parameter updating of PID controller has been a challenging issue for control engineers. A new approach to apply iterative learning control (ILC) scheme for updating the PID parameters, is presented in this paper. The quadratic performance index is employed to optimize the parameters of the PID controller and then an optimal PID type iterative learning control (ILC) scheme is established for discrete linear time-invariant (LTI) systems. In addition, the convergence analysis of optimal ILC of PID type is well described by using Lyapunov composite energy function. The tracking performance of the desired output can be enhanced by the proper choice of penalty matrices. The resultant performance using proposed methodology is significantly improved in term of convergence as compared to available methods in the literature. Simulation examples are also given also to demonstrate the effectiveness of the proposed scheme.

Keywords: Discrete-time system, iterative learning control (ILC), parameter optimal ILC, PID iterative learning control.

1. INTRODUCTION

There are varieties of control techniques and methodologies, which have been proposed for industrial processes [1, 2], however PID controller is still considered as a standard tool to solve the automatic control problem of industrial processes, because of their acceptable performance, simple structure and robustness. The performance of the PID controller depends strongly on the proper choice of the PID parameters. Some efforts are made toward to update PID parameters, such as fuzzy logic [3], Adaptive [4], neural network [5] and etc. For batch/repetitive processes, iterative learning control (ILC) is known as a popular control strategy to optimize control performance from iteration to iteration. Hence, it is motivated to apply ILC schemes to update PID parameters for the batch processes.

ILC is one of the preferred control techniques for a special kind of the industrial processes, which performs a repetitive task over a finite duration/interval. The main principle of ILC is to use the error information in the previous iteration to update the control signal in the next trial such that perfect or bounded tracking of the desired trajectory can be achieved. ILC method for computing such input was first addressed in 1978 by Uchiyama [6] and later mathematically formulated by Arimoto *et al*. in 1984 [7]. Afterward, for the improvement of tracking performance, many research effort has been devoted to the ILC design, see the survey papers [8, 9]. In order to deal with linear and non-linear batch processes, researchers have recommended many methods to realize perfect tracking in both continuous- and discrete-time domains. The effectiveness of ILC has been demonstrated in literature with the applications in the field of robotics, mechatronics, manufacturing, building control, nuclear fusion, rehabilitation, and in network control [10–18]. There are varieties of methodologies and techniques in the ILC design, which have been developed to accelerate the convergence rate of tracking error. One of the popular and effective procedures in ILC category is the usage of the optimization theory. To maximize the convergence rate, an optimal ILC is preferred, which could guarantee the exponential convergence rate due to the presence of both feedback and feedforward controllers [19]. The norm optimal ILC (NOILC) framework was designed to minimize a quadratic optimization problem through the selection of weighting matrices targeting convergence and robustness criteria. The NOILC has the ability to deal with linear and nonlinear systems

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in the time domain, which has been shown in [10, 11, 15]. The NOILC design in the frequency domain has also been investigated analytically and graphically in [20, 21] Recently, the work contributed in [22, 23] focused on the optimization of weighting matrices and the robustness of the NOILC against model uncertainty. Another optimal ILC technique entitled parameter optimal ILC (POILC) was proposed to find the optimal gain rather than optimal input. The POILC is based on the inversion of the plant and is easy to implement as compared to optimal iterative learning control. The POILC guarantees monotonic convergence which has been discussed in [24]. A multi-parameter optimal ILC algorithm was presented in [25] by using an approximate polynomial representation of the plant inverse. In [26], the authors used optimization techniques to optimize vector learning gain. The effectiveness of POILC has been shown by applying to the robotic arm mentioned in [27, 28]. Therefore, a novel method is suggested to find the optimal gains of the PID type ILC using the optimization techniques. The PID type of ILC is a very effective technique because the P-element of PID plays an important role in achieving monotonic convergence; the D-element helps in minimization of the effect of disturbance inputs, and the I-component has the ability to deal with the non-zero initial error. In order to achieve robustness against uncertainty and improve the convergence rate, PD, PI and PID types of ILC are preferred. These types of ILC possess very simple structure with only requirement of proportional, derivative, integral gains and error from the previous trial to design the P, PD PI, PID types of ILC. Many researchers have recommended different methods for evaluation of the gain values in the continuous time domain, such as [29], wherein an online tuning method for PID type of ILC using the least square method to find the PID gains values is was presented. The author in [29] deliberately uses the excitation signal to ensure the online tuning of the PID gains with requirement of an extra computing effort. Recently in [30] an intelligent tuning method to find the PID parameter for ILC was suggested using the least square fitting method, where convergence rate depends on the initial state deviation with very high cost of time and storage resources for the calculation of the parameters. In the discrete time domain, The author has discussed the method of finding the optimal gains for PID and extended PID types of ILC, which is an offline method and used optimization method to achieve the monotonic convergence [31, 32]. In [33], the linear matrix inequality for robust PID type of ILC was calculated using 2D theory to deal with time-varying uncertainty. For time-delayed system with external disturbance, the PD type ILC is was designed in [30]. To take advantage of the PID type ILC approach, the robustness against initial state, uncertainty, and disturbance for the linear and time delay systems have been discussed [34–38]. The PD and PI types of ILC have been considered, which can guarantee the monotonic convergence [39, 40]. From the literature mentioned above, one can summarize that PID type of ILC has the ability to deal with time-delay systems, and also have robustness against uncertainty and disturbances.Although the above mentioned methods solved the problem of automatic tuning parameters to a certain extent, they still need optimal design method to determine the PID coefficients that can accelerate the convergence rate. Also, The benefits of the proposed method lie in the usage of the PID controller because of its effectiveness, simple structure, robustness and the merits of each control actions (proportional, integral and derivative) of PID in the ILC law, Hence deriving the online tuning mechanism for PID controller is a significant task. Moreover, In the above-discussed work, PID gains have been calculated once, and the same PID parameters have been used for subsequent trials. Whereas, the platform of the ILC provides the way to update the inputs or gains. Therefore, In this paper, a new updating mechanism or online tuning method for the PID gains is suggested using the optimization techniques, which can expedite the convergence rate. Motivated by the aforementioned discussion, an optimal design method for the evaluation of PID gains with application to linear batch process is derived, which take error information of the previous iteration into consideration, due to which PID gains are tuned iteratively. The objective of proposed method is to accelerate the convergence rate while employing the optimal ILC strategy for calculating the PID gains and to guarantee the exponential convergence. Therefore, a cost function consisting of both tracking error and incremental input is considered and is transformed to optimize PID parameters. The updating mechanism for PID type ILC depends on the model and error information from the previous cycle. The error information helps to tune PID gains online, which make PID type ILC robust against model uncertainty. Also, It is proven theoretically that the proposed PID type ILC is convergent under an appropriate condition. The proposed approach illustrates not only its advantage especially in dealing with nonlinear systems, but also the merit of the proposed method over the existing methods in term of the convergence rate through stimulation at the end of the paper. The paper is organized as follows: Section 2 gives the optimal iterative learning control for the discrete-time linear systems. The analysis and convergence of the proposed ILC law are presented in Section 3. Some simulation results are given in Section 4. The discussion about the proposed techniques is addressed in Section 5. Finally; conclusions are drawn in Section 6.

2. OPTIMAL PID TYPE ITERATIVE LEARNING CONTROL

Let 't' be the time sample in the time interval $[0,\ p]$ at any iteration k. Hence, the discrete-time system is assumed to be described here.

$$x_k(t+1) = Ax_k(t) + Bu_k(t),$$
$$y_k(t) = x_k(t). \tag{1}$$

Assumption 1: The matrix pair (A, B) and (A, C) are assumed to be controllable and observable respectively, and CB is a full rank matrix.

Here, $x_k(t)\varepsilon R^n$, $u_k(t)\varepsilon R^m$ and $y_k(t)\varepsilon R^m$ are the state, control input, and output of the system, respectively; $A\varepsilon R^{n\times n}$, $B\varepsilon R^{n\times m}$ and $C\varepsilon R^{m\times n}$ are constant matrices. It is noted that the dimensions of inputs and outputs are assumed to be the same since the PID controller will be employed. In this paper, a new online tuning method for PID gains is suggested for the batch process using iterative learning control. For SISO system, the proposed iterative learning PID controller constitute of the following scheme [31].

$$u_k(t) - u_{k-1}(t) = K_p e_{k-1}(t+1) + K_i \sum_{j=1}^{t+1} e_{k-1}(j) + K_d[e_{k-1}(t+1) - e_{k-1}(t)], \tag{2}$$

where Ks are constants and will be designed in the sequel. Define the super vectors for control input, state vector, system output, and desired output at all sampling times over the interval [0, p] at the kth iteration, as shown below.

$$x_k = \begin{bmatrix} x_k(1) \\ x_k(2) \\ x_k(3) \\ \vdots \\ x_k(p) \end{bmatrix}, \quad u_k = \begin{bmatrix} u_k(0) \\ u_k(1) \\ u_k(2) \\ \vdots \\ u_k(p-1) \end{bmatrix}, \quad y_k = \begin{bmatrix} y_k(1) \\ y_k(2) \\ y_k(3) \\ \vdots \\ y_k(p) \end{bmatrix}.$$

Also, the output can also be represented as

$$y_k = Gu_k + Fx(0), \tag{3}$$

where

$$F = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^p \end{bmatrix}, \quad G = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ CA^{p-1}B & CA^{p-2}B & \cdots & CB \end{bmatrix},$$

where $G\varepsilon R^{p\times p}$ and $F\varepsilon R^{p\times 1}$ are the lifted matrix. It is assumed that the initial states are unknown but repeatable, which means that $\Delta x(0) = 0$. Thereafter, the difference of the error for two consecutive iterations is given as.

$$e_k = e_{k-1} - G\Delta u_k, \tag{4}$$

where $e_k = \begin{bmatrix} e_k(1) \\ e_k(2) \\ e_k(3) \\ \vdots \\ e_k(p) \end{bmatrix}$, $\Delta u_k = \begin{bmatrix} u_k(0) - u_{k-1}(0) \\ u_k(1) - u_{k-1}(1) \\ u_k(2) - u_{k-1}(2) \\ \vdots \\ u_k(p-1) - u_{k-1}(p-1) \end{bmatrix}$,

where $u_k\varepsilon R^{p\times 1}$ and $e_k\varepsilon R^{p\times 1}$ are the input slew rate and error at iteration k respectively. By the definition of the super vectors, the structure of the PID control law (5) for any k iteration is given as

$$\Delta u_k = (K_p I + K_i F_1 + K_d F_2)\, e_{k-1}, \tag{5}$$

with $F_2 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -1 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & -1 & 1 \end{bmatrix}$, $F_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$.

Using matrix formulation, equation (5) can be written as

$$\Delta u_k = K_F e_{k-1}, \tag{6}$$

where

$$K_F = K_p I + K_i F_i + K_d F_2. \tag{7}$$

Using (6) and (4), one can have

$$e_k = e_{k-1} - GK_F e_{k-1} = (I - GK_F)\, e_{k-1}. \tag{8}$$

Observing (8), the change of input with respect to iteration depends on the information of previous iteration error and Ks. The error information of previous iteration $(k-1)$th can only be used for determining the input for kth iteration, and it cannot be varied. On the other side, the control input merely depends on K_p, K_i and K_d, which can be enhanced at the start of each trial to improve the performance according to the information of error. Hence, it is required to find a way of updatingK_p, K_i and K_d which can adapt to some uncertainties of the plant. To achieve this objective, the following cost function with respect to the change of input needs to be defined by the minimization of the cost function with respect to K_p, K_i and K_d .

$$\min_{K_p,K_i,K_d} J_k(t) = \frac{1}{2}\left[e_k^T Q e_k + \Delta u_k^T R \Delta u_k\right]. \tag{9}$$

Using (6) and (8), the above cost function can be given as

$$\begin{aligned} \min_{K_p,K_i,K_d} J_k(t) =& \frac{1}{2}\left(e_k^T Q e_k + \Delta u_k^T R \Delta u_k\right) \\ =& \frac{1}{2}\left[e_{k-1}^T (I - GK_F)^T Q (I - GK_F)\, e_{k-1} \right. \\ & \left. + e_{k-1}^T K_F^T R K_F e_{k-1}\right] \\ =& \frac{1}{2}\left[e_{k-1}^T Q e_{k-1} - 2e_{k-1}^T QGK_F e_{k-1} \right. \\ & \left. + e_{k-1}^T K_F^T \left(G^T QG + R\right) K_F e_{k-1}\right]. \end{aligned} \tag{10}$$

Then minimizing (10) from (7) and solving $\frac{\partial J_k}{\partial K_p} = 0$, $\frac{\partial J_k}{\partial K_i} = 0$, $\frac{\partial J_k}{\partial K_d} = 0$ gives three equations as

$$-e_{k-1}^T QGe_{k-1}$$

$$
\begin{aligned}
&+e_{k-1}^T(G^TQG+R)(K_pI+K_iF_1+K_dF_2)e_{k-1}\\
&=0,\\
&-e_{k-1}^TQGF_1e_{k-1}\\
&\quad+e_{k-1}^TF_1^T(G^TQG+R)(K_pI+K_iF_1+K_dF_2)e_{k-1}\\
&=0,\\
&-e_{k-1}^TQGF_2^e e_{k-1}\\
&\quad+e_{k-1}^TF_2^T(G^TQG+R)(K_pI+K_iF_1+K_dF_2)e_{k-1}\\
&=0.
\end{aligned}
\tag{11}
$$

Above equation can be given as

$$[K_p, K_i, K_d]^T = M_k^{-1}L_k, \tag{12}$$

with $M_k = \begin{bmatrix} m_1 & m_2 & m_3 \\ m_2^T & m_4 & m_5 \\ m_3^T & m_5^T & m_6 \end{bmatrix}$, $L_k = \begin{bmatrix} e_{k-1}^TQGe_{k-1} \\ e_{k-1}^TQGF_{k-1} \\ e_{k-1}^TQGF_2e_{k-1} \end{bmatrix}$,
where

$$
\begin{aligned}
m_1 &= e_{k-1}^T(G^TQG+R)e_{k-1},\\
m_2 &= e_{k-1}^T(G^TQG+R)F_1e_{k-1},\\
m_3 &= e_{k-1}^T(G^TQG+R)F_2e_{k-1},\\
m_4 &= e_{k-1}^TF_1^T(G^TQG+R)F_1e_{k-1},\\
m_5 &= e_{k-1}^TF_1^T(G^TQG+R)F_2e_{k-1},\\
m_6 &= e_{k-1}^TF_2^T(G^TQG+R)F_2e_{k-1}.
\end{aligned}
$$

Noting that the solution (12) will be obtained at each trail k. The solution (12) will exist iff CB is full rank which satisfies Assumption 1 and error is non-zero. Online updating of PID controller at each trial based on the performance of the last trial will improve the convergence rate.

Remark 1: The above-mentioned derivation can also be used to derive the gains for PD and PI types of ILC. For PD type, $K_F = (K_pI + K_dF_2)$ and for the *PI* type $K_F = (K_pI + K_iF_1)$ will be considered and then by following similar approach mentioned in Section 2, optimal values of the PD gain and PI gain can be calculated.

Remark 2: The above-stated derivation to find online tuning for the PID controller is applicable if the CB is full rank. In the case of $CB = 0$, PID controller will be zero column vector, which shows that there is no improvement iteratively. Also, In order to find the updated PID controller, the error vector at each trail needs to be a nonzero vector. A termination condition can be used to calculate the updated PID gains, for example, if $\|e_k\|_2 \le \varepsilon$ then PID parameters remain constant.

The proposed method provides the way of online tuning mechanism for gains of the PID-ILC controller, whose block diagram is given in Fig. 1.

Due to the online mechanism, The PID gains need to be calculated at start of the each kth iteration. Also, the

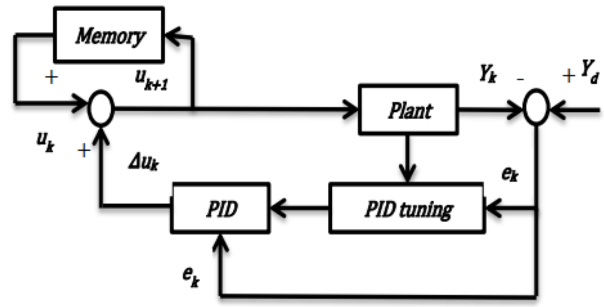

Fig. 1. Block diagram for the PID Type ILC.

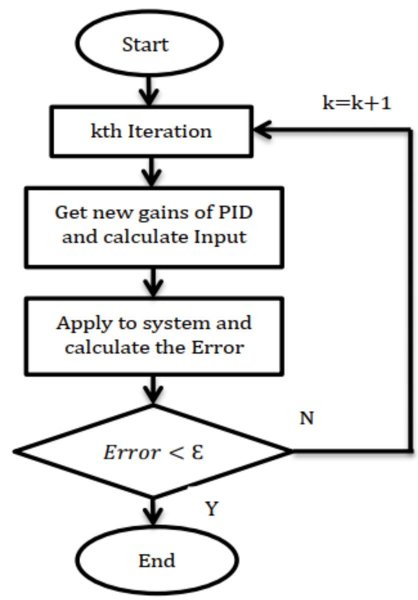

Fig. 2. Flowchart for the proposed online tuning mechanism for PID-ILC.

online tuning method is based on the lifted system, which is computationally expensive than the non-lifted system. However, it is not a big issue due to the availability of high computation machines. The online tuning mechanism requires model and error information from the previous trial for finding the gains of the PID-ILC controller. The whole mechanism for updating the gains is summarized in flowchart mentioned in Fig. 2.

3. CONVERGENCE ANALYSIS

In this section, convergence analysis of currently proposed method is discussed. The similar convergence technique has been used in [23], in which the Parameter Optimal ILC for the discrete-time system has been proposed. In [31], the author used the gain matrix as a parameter that is to be optimized, whereas Ks will be used as a parameter in the proposed technique. The derivation of the convergence for the proposed method is discussed due to the difference between the parameter and the type of ILC. To proves the convergence of the proposed method, let the following candidate of the Lyapunov function as

$$V_k = e_k^T Q e_k,$$

where Q is positive definite and Symmetric matrices, the necessary and sufficient condition for the error to converge is to ΔV_k be the negative definite. Then,

$$\Delta V_k = e_k^T Q e_k - e_{k-1}^T Q e_{k-1},$$

where from (8).

$$e_k = [I - G(K_p I + K_i F_1 + K_d F_2)]e_{k-1}.$$

So,

$$\begin{aligned}\Delta V =& (e_{k-1} - GK_F e_{k-1})^T Q (e_{k-1} - GK_F e_{k-1}) \\ & - e_{k-1}^T Q e_{k-1} \\ =& -2e_{k-1}^T Q (GK_F) e_{k-1} \\ & + e_{k-1}^T (GK_F)^T Q (GK_F) e_{k-1}. \end{aligned} \tag{13}$$

Using (11), the first term of the above equation can be written as

$$\begin{aligned} e_{k-1}^T Q(GK_F)e_{k-1} =& e_{k-1}^T QG(K_p I + K_i F_1 + K_d F_2)e_{k-1} \\ =& K_p e_{k-1}^T (G^T QG + R) K_F e_{k-1} \\ & + K_i e_{k-1}^T F_1^T (G^T QG + R) K_F e_{k-1} \\ & + K_d e_{k-1}^T F_2^T (G^T QG + R) K_F e_{k-1} \\ =& e_{k-1}^T K_F^T (G^T QG + R) K_F e_{k-1}. \end{aligned} \tag{14}$$

Substituting (18) into (17), one can have

$$\begin{aligned}\Delta V =& (e_{k-1} - GK_F e_{k-1})^T Q (e_{k-1} - GK_F e_{k-1}) \\ & - e_{k-1}^T Q e_{k-1} \\ =& -2e_{k-1}^T QGK_F e_{k-1} + e_{k-1}^T K_F^T G^T QGK_F e_{k-1} \\ =& -2e_{k-1}^T K_F^T (G^T QG + R) K_F e_{k-1} \\ & + e_{k-1}^T K_F^T G^T QGK_F e_{k-1} \\ =& -e_{k-1}^T K_F^T (G^T QG + 2R) K_F e_{k-1}. \end{aligned} \tag{15}$$

It follows from Lyapunov stability theorem that $\Delta V_k$is negative provided that $K_F^T(G^T QG + 2R)K_F$ is the positive definite. Because the weighting matrices Q and R can be chosen such that $G^T QG + 2R$ is positive definite, so the condition of $K_F^T(G^T QG + 2R)K_F$ being the positive definite is equivalent to the matrix K_F is full rank. This conclusion is summarized as the following theorem.

Theorem 1: For plant (1) with repeatable initial states, if the iterative learning PID control law (2) is employed and the controller parameters are chosen at each trail as (12), then the tracking error of closed-loop system can tend to zero if

A1: CB is full rank.

A2: K_F defined as in (7) and the PID parameters chosen at each trail as (12) is full rank.

4. SIMULATION RESULTS

In order to illustrate the applicability and effectiveness of our proposed method, three simulation examples are discussed in this section. In the first example, a comparison of the proposed technique with respect to techniques mentioned in [31, 32] is drawn. The comparison between the proposed method and Owen's paper [25] is given in Example 2. Also, the effects of penalty matrices are well discussed in the second example. Moreover, the proposed method can also be used to implement the P, PD and PI types of iterative learning control which is also demonstrated in Example 2. Finally, the proposed method has the ability to deal with the nonlinear system, which is shown in the third example.

Remark 3: The error e_k is constituted of the vector, whose norms have been calculated by using formulae of the vector norms. As Matlab software has been used to simulate the proposed method, norms of the e_k are calculated using the built-in commands in Matlab.

Example 1: In this example, we consider the DC motor, in which the rotational angle of the motor is controlled using field winding. The details of the model and desire output trajectory are given in [31]. Let us mention the necessary information of the state space model (for details refer to [31]).

$$\begin{aligned} & x_k(t+1) = A_D x_k(t) + B_D u_k(t), \\ & y_k(t) = C_D x_k(t), \\ & t = 1, 2, 3, \ldots, 1200, \text{ and } k = 1, 2, 3, \ldots, \end{aligned}$$

where $x_k(t) = [i_f(t)\ \omega(t)\ \theta(t)]$, $y_k(t) = \theta(t)$, $u_k(t) =$ field winding voltage, $i_f(t) =$ field winding current, $\omega(t) =$ angular velocity, and $\theta(t) =$ angle of the DC motor.

$$A_D = \begin{bmatrix} -\frac{R_f}{L_f} & 0 & 0 \\ \frac{k_m}{J} & \frac{-f}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$B_D = \begin{bmatrix} \frac{1}{L_f} \\ 0 \\ 0 \end{bmatrix}, \quad C_D = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

Choosing the following values of the parameters, Field winding resistance $R_f = 20\Omega$, Field winding inductance $(L_f) = 1H$, Motor torque ratio $(k_m) = 0.5$ Nm/A, Mechanical load inertia momentum $(j) = 2$ Nm/A. Friction ratio $(f) = 2$ Nm/A, sampling period $(T) = 0.01$, and total time span $[0\ \ T] = [0\ \ 12]$. The discrete model is given as

$$A_D = \begin{bmatrix} 0.8187 & 0 & 0 \\ 0.4526 & 0.9975 & 0 \\ 0.0023 & 0.0100 & 1 \end{bmatrix},$$

$$B_D = \begin{bmatrix} 0 \\ 0.0197 \\ 0.0211 \end{bmatrix}, \quad C_D = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}. \tag{16}$$

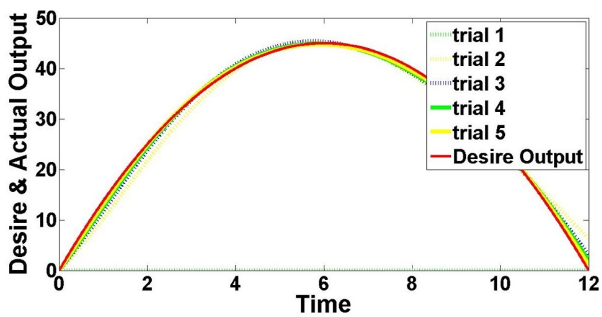

Fig. 3. (Example 1) Output trajectories, when the proposed technique is applied to System (16) when $Q = 10I$ and $R = 1I$.

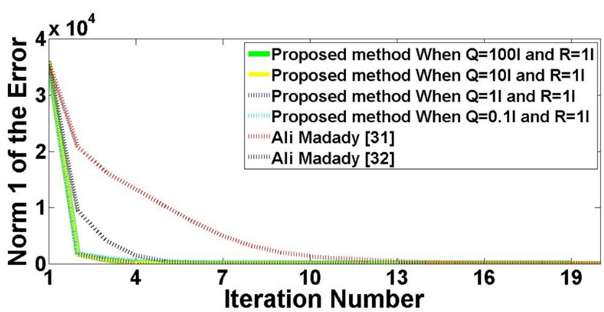

Fig. 4. (Example 1) Comparison of Norm 1 of the Error with respect to iteration.

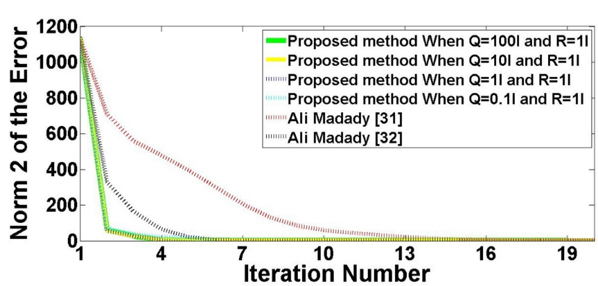

Fig. 5. (Example 1) Comparison of Norm 2 of the Error with respect to iteration.

The desired output trajectory, which is shown as $y_d(t) = 1.25 \times t(t_f - t)$, $0 \leq t \leq t_f$, $t_f = 12$sec.

Fig. 3 shows the simulation result when $[K_p,\ K_i,\ K_d]^T$ and updated input vector are calculated using (12), and (6), respectively, and then applied to the system (16) iteratively, where $Q = 10 \times I$ and $R = 1 \times I$ are chosen as a penalty matrices in the cost function and I is the identity matrix. The input for the first iteration is considered to be zero. The red, green dotted, yellow-dotted, blue-dotted, green and yellow lines represent the desired output, iteration 1, 2, 3, 4, 5 outputs respectively. It can be observed that perfect tracking has been achieved within five iterations.

Example 2: To demonstrate the effect of weighting matrices *R&Q* and applicability of *P*, *PD*, *PI&D* types ILC using the proposed method; let us consider the linear SISO system given in [25].

$$G(s) = \frac{s + 0.75}{s^2 + 1.5s + 0.5}. \tag{17}$$

The discrete-time linear system of the above continuous-time system is achieved at the sampling time of 0.028 sec. The time interval for any iteration is $[0,\ 14]$ with initial condition and the input for the first iteration is considered as zero. Here, $R = r \times I$ and $Q = q \times I$. Fig. 7 shows the Norm 2 of the error for P, PI, PD, and PID type ILC control, when $[K_p,\ K_i,\ K_d]^T$ and updated input is calculated using (12) and (6) respectively and then applied to system

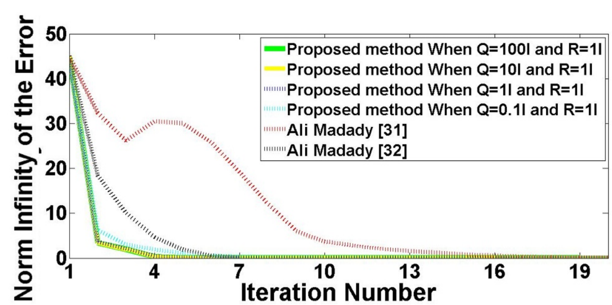

Fig. 6. (Example 1) Comparison of Norm inf of the Error with respect to iteration.

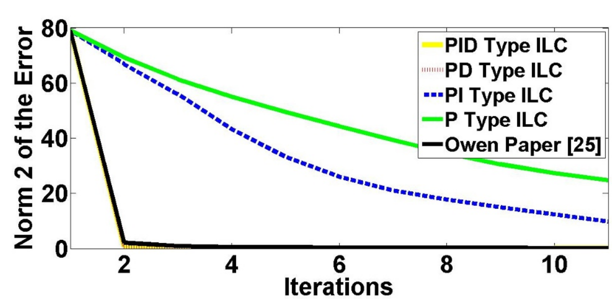

Fig. 7. (Example 2) Norm to the error for PI, PD, and PID type ILC.

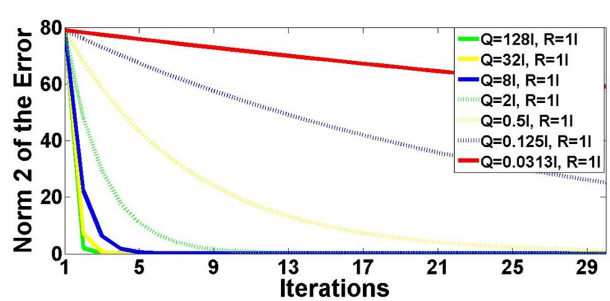

Fig. 8. (Example 2) Norm 2 to the error at different values of Q when $R = 1I$.

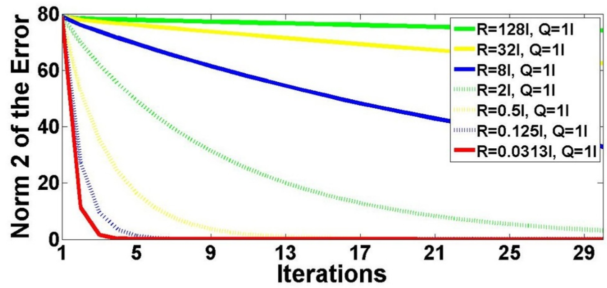

Fig. 9. (Example 2) Norm 2 to the error at different values of R when $Q = 1I$.

Table 1. (Example 2) Norm 2 the error for P, PI, PD and PID type ILC from iteration 1 to 11.

k	Result [25]	P Type	PI Type	PD type	PID type
1	79.056	79.056	79.056	79.056	79.056
2	2.075	42.082	36.851	0.705	0.697
3	0.845	23.459	17.526	0.012	0.011
4	0.540	12.909	7.922	4.03e-04	3.70e-04
5	0.399	7.135	4.1242	1.07e-05	9.23e-06
6	0.316	4.184	2.3320	3.07e-07	2.47e-07
7	0.262	2.520	1.4073	8.49e-09	5.31e-09
8	0.223	1.607	0.9340	2.39e-10	1.09e-10
9	0.192	1.139	0.7331	6.75e-12	2.63e-12
10	0.169	0.869	0.603	1.64e-13	6.14e-14
11	0.149	0.706	0.509	1.60e-14	1.72e-14

(17) iteratively, where $Q = 10 \times I$ and $R = 0.01 \times I$ are chosen as penalties in the cost function.

Example 3: To demonstrate the ability of the proposed method to deal with the nonlinear system, consider a single-link manipulator (for detail, please refer to [41,42].

$$\tau(t) = ml^2\ddot{\theta}(t) + v\dot{\theta}(t) + mgl\cos(t). \tag{18}$$

Let the real length, mass and friction coefficient be $l = 1$ m, $m = 2.0$ kg, and $v = 1.0\ \mathrm{kgm}^2/s$, respectively. Letting h be a sampling period, one can discretize the model by using the Euler method, as follows:

$$\begin{aligned}\tau(ih) =& \frac{ml^2}{h^2}\theta(ih+2h) + \left(\frac{v}{h} - \frac{2ml^2}{h^2}\right)\theta(ih+h)\\ &+ \left(\frac{2ml^2}{h^2} - \frac{v}{h}\right)\theta(ih) + mgl\cos\theta(ih).\end{aligned} \tag{19}$$

Let $x_1(t) = \theta(ih), x_2(t) = \theta(ih+h)$ and $x_3(t) = \theta(ih+h)$ then above equation can be written as in state-space form as

$$\begin{aligned}&\begin{bmatrix} x_1(i+1) \\ x_2(i+1) \end{bmatrix}\\ &= \begin{bmatrix} 0 \\ \frac{h^2}{ml^2} \end{bmatrix} u_1(i)\\ &+ \begin{bmatrix} x_2(i) \\ (2 - \frac{vh}{ml^2})x_2(i) + (\frac{vh}{ml^2} - 1)x_1(i) - (\frac{gh^2}{l}\cos x_1(i)) \end{bmatrix}.\end{aligned} \tag{20}$$

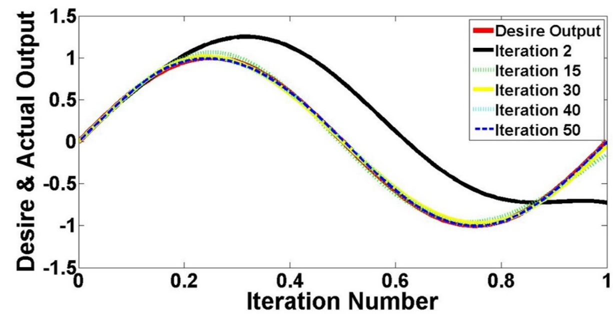

Fig. 10. (Example 3) Output trajectories, when the proposed method is applied to System (19) when $Q = 100I$ and $R = 0.01I$.

Now, suppose we have a linear system model (21) as

$$\begin{aligned}&\begin{bmatrix} \tilde{x}_1(i+1) \\ \tilde{x}_2(i+1) \end{bmatrix}\\ &= \begin{bmatrix} 0 \\ \frac{h^2}{\tilde{m}\tilde{l}^2} \end{bmatrix} u_1(i)\\ &+ \begin{bmatrix} x_2(i) \\ (2 - \frac{\tilde{v}h}{\tilde{m}\tilde{l}^2})x_2(i) + (\frac{\tilde{v}h}{\tilde{m}\tilde{l}^2} - 1)x_1(i) \end{bmatrix}.\end{aligned} \tag{21}$$

Here, modeled parameters are $\tilde{l} = 0.8$ m, $\tilde{m} = 1.5$ m, and $\tilde{v} = 0.8\ \mathrm{kgm}^2$/s. The desired output trajectory, which is shown in $y_d = \sin(2 \times \pi \times t)$ for $t = (0,1)$. The output trajectories at iteration 2, 15, 30, 40 , 50 and reference signal are shown in Fig. 10, when PID gains for ILC are evaluated using the linear model and PID ILC scheme is applied to control the nonlinear system. Fig. 11 shows the Norm 2 of the error for PID type ILC control, when $[K_p, K_i, K_d]^T$ are calculated using (12). Linear nominal model (21) is used to calculate the PID gains, Afterward, updated input is calculated using (6) and applied to the nonlinear system (20) iteratively, where $Q = 100 \times I$ and $R = 0.01 \times I$ are chosen as weighting matrices in the cost function.

5. DISCUSSION

Firstly, it can be observed that the proposed technique calculates the $[K_p, K_i, K_d]$ at the start of each kth iteration by using error information of the $(k-1)$th iteration. Whereas, [31,32] proposed the technique to calculate the optimal gain $[K_p, K_i, K_d]$ based on the information of the model. The performances of norm 1, 2 and infinity of the error with respect to iteration are shown in Figs. 4, 5, and 6, respectively. The green, yellow, blue dotted and green

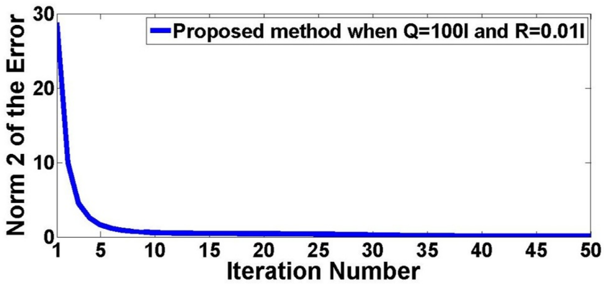

Fig. 11. (Example 3) Norm 2 of the Error with respect to iteration.

dotted lines show the norm of the Error when the proposed method is applied. The red and black dotted lines show the norms of error, when Madady approaches [31, 32] are applied. It can be observed that the proposed technique performs better than the approach mentioned in [31, 32].

Secondly, In the proposed technique, finding the control input at kth iteration requires information of error at iteration $(k-1)$th and also the new set of proportional, Integral and derivative gains. Hence, the parameter $[K_p, K_i, K_d]$ is updated at each trial according to the information of an error of the previous iteration, due to which performance of the tracking trajectory is improved.

Thirdly, the proposed approach can also be used to find optimal gains for P, PD and PI types of ILC. Fig. 5 shows the norm 2 of the error, when PID, PI, PD and P types of ILC have been applied to the system (22). Error converges faster in case of PID type ILC as compare to P, PD, and PI types ILC. Also, it can be observed from Fig. 7 that P type ILC performs better than P, PI types of ILC. Furthermore, Table 1 compares the Norm 2 of the error obtained from Owen's paper [25] with respect to our proposed method.

Fourthly, due to the presence of two penalties matrices R and Q, the performance of the tracking error can be enhanced if R and Q are chosen appropriately. The proposed technique uses the matrices Q and R to penalize the error and incremental input respectively. Figs. 8 and 9 show the behavior of norm 2 of the error with respect to iteration for different values of R and Q. It can be observed that the performance to tracking the desired output trajectory is improved if the ratio of R/Q is decreased or approaches to zero.

Finally, the proposed method has the ability to deal with the nonlinear system, which is demonstrated in Example 3. It is worth noting that the proposed method used the information of the nominal model only for finding the gains of the *PID* type *ILC*. Also, parameter uncertainties are considered between the real model and the nominal model. Using the information of the nominal model, nonlinear system is controlled. From Figs. 10 and 11, it is demonstrated that the proposed method can deal with the nonlinear system.

6. CONCLUSION

In this brief, a new approach for the estimation of the optimal gains of PID types of ILC is proposed. The convergence of the proposed ILC is given using Lyapunov composite function. The performance of the proposed approach can be further enhanced by the proper choice of the weighting matrices. The effectiveness of the proposed technique has been demonstrated through simulations of threee numerical examples. Furthermore, the proposed technique has shown better performance in terms of fast convergence as compared to available methods in the literature.

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