Adaptive Fault-tolerant Neural Control for Large-scale Systems with Actuator Faults

Jian-Ye Gong, Bin Jiang, and Qi-Kun Shen*

Abstract: The active adaptive fault-tolerant neural control problem is discussed for large-scale uncertain systems against actuator faults. The unknown interconnections among subsystems are assumed to be nonlinear, not traditional linear. A general actuator fault model is proposed, which integrates bias and gain time-varying faults. Then, based on Lyapunov stability theory, a novel fault diagnostic algorithm and accommodation scheme are proposed, where the assumptions in the existing works are removed and fault-tolerant controller singularity problem is avoided. Finally, simulation results of near space vehicle show the efficiency of the presented control approach.

Keywords: Adaptive control, fault diagnosis, fault-tolerant control, neural control.

1. INTRODUCTION

Because many practical applications can be modeled as large-scale system, these systems have attracted extensive attention [1–5]. Generally speaking, according to the nominal subsystem's form, the large-scale systems can be classified two categories: linear [3, 4] or nonlinear [6, 7]. For the large-scale systems, the key task is how to handle the interconnection terms among subsystems. As pointed out in [8], the relevant research are developed in two directions. One is to relax the boundedness assumptions on the interconnection terms, namely, the interconnection terms should be bounded by known linear function with/without the unknown gains [3, 5]. In the cases, by using the approximation capability of fuzzy logical systems (FLSs) or neural networks (NNs), the bounded restrictions are further relaxed [9, 10]. The second is to relax the structural constraints imposed on interconnection terms [11, 12].

In [4], for large-scale systems that do not satisfy the above matching condition, a backstepping control was proposed. However, the boundedness assumptions about the interconnections were still necessary. In the practical systems, however, the interconnection terms are often nonlinear and unknown. Furthermore, their bounds can not be described as known functions. It is significant to propose proper control methods for the nonlinear interconnected systems, which motivates us for this paper.

On the other hand, faults may occur in the controlled systems and impose adverse effect on system performance. Hence, many effective fault-tolerant control (FTC) approa-ches have been developed to enhance the reliability and safety of the faulty systems. In general, passive FTC approach [11–15] have more conservatism than active FTC one. Active FTC approach have the following procedures: fault detection and isolation (FDI) [16–21], fault estimation (FE) [22, 23] and fault accommodation (FA) [24–26].

For large-scale systems, many results have been reported as early as 1980s. For the systems, although some results on FTC have been obtained [26], significant developments have not been made [21, 26]. These above works just focused on FD or FDI, and do not consider the fault estimation and active FTC problems. For FTC of interconnected systems, it still needs to be further studied, which is another motivation of this paper.

In this paper, for a class of large-scale systems, we consider the adaptive active FTC problem against actuator faults. The main contributions are given as:

1) In contrast with [5] and [13], the bounds of the interconnection terms considered in our paper are assumed to be the sum of nonlinear functions. The assumption is more reasonable in real cases;

2) Compared with [1–5] where only the normal condition (no fault) were investigated, active adaptive FTC against actuator fault is discussed in our paper. What's more, the fault model in our work can simultaneously handle gain and bias faults. The proposed theoretic results thus have wider applications;

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3) Based on an fault indicator that can be computed online, a FDI decision threshold is given. Hence, the FDI algorithm has more practical interesting;

4) In the FTC scheme presented in this paper, the controller singularity problem is avoided without projection algorithm. Further, some conventional assumptions in the existing works are removes in the scheme.

The rest of this paper is organized as follows: The problem formulation and NNs' description are presented in Section 2. In Section 3, main results are presented, which includes FDI, fault estimation and FA. Section 4 gives simulation results, which show the efficient of the method. Finally, the conclusions are drawn in Section 5.

2. PROBLEM STATEMENT AND DESCRIPTION OF NEURAL NETWORKS

2.1. Problem statement

The uncertain large-scale system Σ composed of N linked subsystems, each subsystem has the following form,

$$\sum : \begin{cases} \dot{x}_i = \bar{\Psi}_i(x) + B_i u_i, \\ y_i = C_i x_i, \end{cases} \quad i = 1, 2 \cdots, N, \tag{1}$$

where $x_i = [x_{i1}, x_{i2}, \dots, x_{i1}^{(n_i-1)}]^T = [x_{i1}, x_{i2}, \dots, x_{in_i}]^T \in \mathbb{R}^{n_i}$, $y_i \in \mathbb{R}^q$, $u_i \in \mathbb{R}^{m_i}$ denote the measurable state, output, input of the *i*th subsystem, respectively; matrices B_i and C_i are of appropriate dimensions; $\bar{\Psi}_i(x) = [\bar{\psi}_{i1}(x), \dots, \bar{\psi}_{in_i}(x)]^T \in \mathbb{R}^{n_i}$ is the uncertain interconnection between the *i*th subsystem and the other subsystems, $\bar{\psi}_{ij}$ $(j = 1, \dots, n_i)$ is an unknown function, which includes the lumped uncertainty of the *i*th subsystem, including unknown parameter variation, model uncertainty, external disturbance, and the *i*th subsystem uncertainty, $x = [x_1^T, \dots, x_N^T]^T \in \mathbb{R}^{\sum_{i=1}^{n_i} n_i}$ denotes the overall system's state variable.

The control objective under fault-free condition is to design suitable controllers such that (1) is stable.

In this paper, actuator fault has the following form,

$$u_{ij}^{f}(t) = (1 - \rho_{ij}(t))u_{ij}(t) + \sum_{k=1}^{p_{ij}} g_{ijk} f_{ijk}(t),$$

$$t > t_j, i = 1, \cdots, N, \quad j = 1, \cdots, m_i,$$
 (2)

where $f_{ijk}(t)$, $k = 1, \dots, p_{ij}$ denote bounded signals, known constant $p_{ij} > 0$, constant g_{ijk} is unknown. With no restriction, let us suppose $p_{11} = \dots = p_{1m_1} = p_{N1} =$ $\dots = p_{Nm_N} = p$ with p being a known positive constant, failure time instant t_j is unknown. Consider the following notation: $a_{ijk}(t) = g_{ijk}f_{ijk}(t)$. Then, (2) becomes the following form,

$$u_{ij}^{f}(t) = (1 - \rho_{ij}(t))u_{ij}(t) + \sum_{k=1}^{p} a_{ijk}(t),$$

$$t > t_{j}, \ i = 1, \cdots, N, \ j = 1, \cdots, m_{i}.$$
(3)

Assumption 1: $|a_{ijk}(t)| \leq \bar{a}_{ijk1}, |\dot{a}_{ijk}(t)| \leq \bar{a}_{ijk2},$ $|\rho_{ij}(t)| \leq \bar{\rho}_{ij1}$ and $|\dot{\rho}_{ij}(t)| \leq \bar{\rho}_{ij2}$, where known real constants $\bar{\rho}_{ij1} > 0, \bar{\rho}_{ij2} > 0, \bar{a}_{ijk1} > 0$ and $\bar{a}_{ijk2} > 0$.

Assumption 2: The interconnection function $|\psi_{ij}| \leq \sum_{k=1}^{N} \varepsilon_{ijk}(x_k)$, $i = 1, \dots, N$, $j = 1, \dots, n_i$, where $\varepsilon_{ijk}(x_k)$ denotes an unknown function, which is dependent on the state of the *k*th subsystem.

In the following, let $\bar{a}_1 = \max_{1 \le i \le N, 1 \le j \le n_i, 1 \le k \le p} \{\bar{a}_{ijk1}\},\ \bar{\rho}_1 = \max_{1 \le i \le N, 1 \le j \le n_i} \{\bar{\rho}_{ij1}\},\ \bar{\rho}_2 = \max_{1 \le i \le N, 1 \le j \le n_i} \{\bar{\rho}_{ij2}\},\ \bar{a}_2 = \max_{1 \le i \le N, 1 \le j \le n_i, 1 \le k \le p} \{\bar{a}_{ijk2}\}.$

In the following, we will use the abbreviations $\overline{\Psi}_i$, ρ_{ij} and a_{ijk} , which denote $\overline{\Psi}_i(\cdot)$, $\rho_{ij}(t)$ and $a_{ijk}(t)$, respectively.

Considering the fault (2), we re-define the control objective: An active FTC scheme is designed to ensure that (1) is stable in all cases. In normal case, $u_i(t)$, $i = 1, \dots, N$ are designed to guarantee that system (1) is stable. At the same time, the FDI algorithm is working. After detecting and isolating a fault, the fault estimation algorithm is activated. using he fault estimation, fault-tolerant control inputs are designed to ensure (1) is stable in faulty case.

Beginning with controller design, we first transform system (1) into the following form,

$$\sum : \begin{cases} \dot{x}_i = A_i x_i + B_i u_i + \Psi_i(x), \\ y_i = C_i x_i, \end{cases} \quad i = 1, 2, \cdots, N, \quad (4)$$

where $\Psi_i = \bar{\Psi}_i - A_i x_i$, matrix A_i is chosen such as the matrix pairs (A_i, B_i) and (A_i, C_i) , $i = 1, \dots, N$, are controllable and observable, respectively. In addition, it is assumed that B_i is of full column rank.

2.2. Description of neural networks

NNs [11] are used to approximate a continuous function $h(Z): R^p \to R$ as follows:

$$h(Z, \theta) = \theta^T \xi(Z),$$

where *p* is the NNs input dimension, $Z = (z_1, \dots, z_p)^T \in R^p$ is the input vector,

$$\theta = (\theta_1, \cdots, \theta_{N_{\theta}})^T, \quad xi(Z) = (\xi_1(Z), \cdots, \xi_N(Z))^T,$$

$$\xi_i(Z) = \exp(-\frac{\sum_{j=1}^p (z_j - a_{i,j})^2}{(c_i)^2}),$$

where N_{θ} is NNs node number, $c_i > 0$ is the width of the receptive field, and $a_{i,j} \in R$, $i = 1, 2, \dots, N$, $j = 1, 2, \dots, i$, are the center of the Gaussian function. Let

$$\begin{split} \Omega_{\theta} &= \{\theta : ||\theta|| \leq \bar{\theta}_m\},\\ \theta^* &= \arg\min_{\theta \in \Omega_{\theta}} [\sup_{z \in \Omega_Z} |h(Z, \theta) - h(Z)|], \end{split}$$

where design parameter $\bar{\theta}_m > 0$, compact set Ω_Z is sufficiently large. For a continuous function h(Z), it can be

obtained $h(Z) = \theta^{*T} \xi(Z) + \varepsilon(Z)$, where $\varepsilon(Z)$ denotes the optimal approximation error.

In this paper, NNs are respectively used to approximate unknown continuous functions $\psi_{ijk}(Z_k)$ in the following form, $\psi_{ijk}(Z_k) = \theta_i^{*T} \xi_i(Z_i) + \varepsilon_i(Z_i)$, where $i = 1, 2, \dots, N$, $j = 1, 2, \dots, n_i$ and $k = 1, 2, \dots, N$.

Assumption 3: $|\varepsilon_{ijk}| \leq \varepsilon_{ijk}^*$, $\varepsilon_{ijk}^* \leq M_{ijk,\varepsilon}$ and $||\theta_{ijk}^*|| \leq M_{ijk,\theta}$, where unknown constants $\varepsilon_{ijk}^* > 0$, $M_{ijk,\varepsilon} > 0$ and $M_{ijk,\theta} > 0$.

3. MAIN RESULTS

3.1. Fault detection

In order to detect the actuator faults, we design an adaptive observer for the *i*th subsystem as follows:

$$\begin{cases} \dot{\hat{x}}_i = A_i \hat{x}_i + B_i u_i + L_i (y_i - \hat{y}_i) + \hat{\Psi}_i \\ + sgn(2e_{ix}^T P_i) \hat{M}_{\varepsilon \theta i}, \\ \hat{y}_i = C_i \hat{x}_i, \end{cases}$$
(5)

where $\hat{\Psi}_i = [\hat{\psi}_{i1}, \dots, \hat{\psi}_{in_i}]^T$, $\hat{\psi}_{ij} = \hat{\theta}_{ijk}^T \xi_{ijk}(\hat{x}_k)$ is the estimate of $\theta_{ijk}^{*T} \xi_{ijk}(x_k)$, $\hat{M}_{\varepsilon\theta i} = [\hat{M}_{\varepsilon\theta i1}, \dots, \hat{M}_{\varepsilon\theta in_i}]^T$ is the estimate of $M_{\varepsilon\theta i} = [\sum_{k=1}^N M_{\varepsilon\theta i1k}, \dots, \sum_{k=1}^N M_{\varepsilon\theta in_ik}]^T$, $M_{\varepsilon\theta ijk} = 2M_{ijk,\theta} + M_{ijk,\varepsilon}$, $sgn(e_{ix}^T P_i) = \{sgn(e_{ix}^T P_i^1), \dots, sgn(e_{ix}^T P_i^{n_i})\}$, P_i^j $(j = 1, \dots, n_i$ denotes the *j*th column of matrix P_i defined later, e_{ix} denotes the observer error defined in (6), L_i $(i = 1, \dots, N)$ denotes observer gain.

Denote

$$e_{ix}(t) = x_i(t) - \hat{x}_i(t), e_{iy}(t) = y_i(t) - \hat{y}_i(t),$$
 (6)

then we have the following error dynamics,

$$\dot{e}_{ix}(t) = (A_i - L_i C_i) e_{ix}(t) + \Psi_i - \hat{\Psi}_i - \operatorname{sgn}(2e_{ix}^T P_i) \hat{M}_{\varepsilon \theta i}.$$
(7)

Now, we will give the condition of the above observer's convergence.

Theorem 1: If there exist matrices $P_i = P_i^T > 0$ and $Q_i > 0$ with appropriate dimensions such that

$$P_i(A_i - L_iC_i) + (A_i - L_iC_i)^T P_i \le -Q_i, \quad i = 1, 2, \cdots, N,$$
(8)

and adaptive laws (9) and (10) are used

$$\dot{\hat{\theta}}_{ijk} = \begin{cases} 0, \text{ if } \hat{\theta}_{ijk} = \bar{M}_{ijk,\theta} \text{ and} \\ 2\eta e_{ix}^T P_i^j \xi_{ijk}(\hat{x}_k) - \eta_{\theta} \hat{\theta}_{ijk} > 0, \\ \text{ or } \hat{\theta}_{ijk} = -\bar{M}_{ijk,\theta} \text{ and} \\ 2\eta e_{ix}^T P_i^j \xi_{ijk}(\hat{x}_k) - \eta_{\theta} \hat{\theta}_{ijk} < 0, \\ 2\eta e_{ix}^T P_i^j \xi_{ijk}(\hat{x}_k) - \eta_{\theta} \hat{\theta}_{ijk}, \text{ otherwise}, \end{cases}$$
(9)

$$\dot{\hat{M}}_{\varepsilon\theta ijk} = \begin{cases} 0, \text{ if } \hat{M}_{\varepsilon\theta ijk} = 2\bar{M}_{ijk,\theta} + \bar{M}_{ijk,\varepsilon} \text{ and} \\ 2\eta | e_{ix}^T P_i^j | + \eta_M \hat{M}_{\varepsilon\theta ijk} > 0, \\ \text{ or } \hat{M}_{\varepsilon\theta ijk} = -2\bar{M}_{ijk,\theta} - \bar{M}_{ijk,\varepsilon} \text{ and} \\ 2\eta | e_{ix}^T P_i^j | + \eta_M \hat{M}_{\varepsilon\theta ijk} < 0, \\ 2\eta | e_{ix}^T P_i^j | - \eta_M \hat{M}_{\varepsilon\theta ijk}, \text{ otherwise}, \end{cases}$$

$$(10)$$

where $\bar{M}_{ijk,\theta}$, $\bar{M}_{ijk,\theta}$, $\bar{M}_{ijk,\varepsilon}$, η , η_{θ} and η_M are design positive parameters, and Assumptions 1-3 hold, **then** (7) is asymptotically stable, $||e_{ix}|| \leq \sqrt{\alpha_{iD}/\lambda_{\min}(P_i)}$, $||\tilde{\theta}_{ijk}|| \leq \sqrt{2\eta_1\alpha_{iD}}$, $||\tilde{M}_{\varepsilon\theta ijk}|| \leq \sqrt{2\eta_1\alpha_{iD}}$, where $\alpha_{iD} = \mu_{iD}/\lambda_{iD} + V_{iD}(0)$, $\lambda_{iD} = \min \left\{ \frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)}, \frac{\eta_{\theta}}{2\eta}, \frac{\eta_M}{2\eta} \right\}$, and $\mu_{iD} = \sum_{j=1}^{n_i} \sum_{k=1}^{N} \left[\frac{\eta_{\theta}}{2\eta} M_{ijk,\theta}^2 + \frac{\eta_M}{2\eta} (2M_{ijk,\theta} + M_{ijk,\varepsilon})^2 \right]$.

Proof: Define

$$V_{iDe} = e_{ix}^T(t)P_ie_{ix}(t).$$
(11)

Differentiating V_{iDe} with respect to time t, it yields

$$\dot{V}_{iDe} = e_{ix}^{T} (P_i (A_i - L_i C_i) + (A_i - L_i C_i)^T P) e_{ix} + 2e_{ix}^{T} P_i (\Psi_i - \hat{\Psi}_i) - 2e_{ix}^{T} P_i sgn(e_{ix}^T P_i) \hat{M}_{\varepsilon \theta i}.$$
(12)

From Assumption 2, it follows that

$$2e_{ix}^{T}P_{i}^{J}(\boldsymbol{\psi}_{ij} - \boldsymbol{\hat{\psi}}_{ij})$$

$$\leq \sum_{k=1}^{N} 2e_{ix}^{T}P_{i}^{J}[\boldsymbol{\tilde{\theta}}_{ijk}^{T}\boldsymbol{\xi}_{ijk}(\hat{x}_{k})$$

$$+ \boldsymbol{\theta}_{ijk}^{*T}(\boldsymbol{\xi}_{ijk}(x_{k}) - \boldsymbol{\xi}_{ijk}(\hat{x}_{k})) + \boldsymbol{\varepsilon}_{ijk}(x_{k})], \qquad (13)$$

where $\tilde{\theta}_{ijk} = \theta^*_{ijk} - \hat{\theta}_{ijk}$, $\hat{\theta}_{ijk}$ is the estimate of θ^*_{ijk} . Further, one has

$$2e_{ix}^{T}P_{i}(\Psi_{i} - sgn(2e_{ix}^{T}P_{i})\hat{\Psi}_{i})$$

$$\leq \sum_{j=1}^{n_{i}} [2e_{ix}^{T}P_{i}^{j}\sum_{k=1}^{N}\tilde{\theta}_{ijk}^{T}\xi(\hat{x}_{k})]$$

$$+ \sum_{j=1}^{n_{i}} [|2e_{ix}^{T}P_{i}^{j}|\sum_{k=1}^{N}M_{\varepsilon\theta ijk}], \qquad (14)$$

where the fact $|\xi_{ijk}(\cdot)| < 1$ is used.

Substituting (14) into (12) and considering Assumption 2 and (13), one has

$$\dot{V}_{iDe} \leq -e_{ix}^{T}Q_{i}e_{ix} + \sum_{j=1}^{n_{i}} [2e_{ix}^{T}P_{i}^{j}\sum_{k=1}^{N}\tilde{\theta}_{ijk}^{T}\xi(\hat{x}_{k})] \\ + \sum_{j=1}^{n_{i}} [|2e_{ix}^{T}P_{i}^{j}|\sum_{k=1}^{N}\tilde{M}_{\varepsilon\theta ijk}],$$
(15)

where $\tilde{\theta}_{ijk} = \theta^*_{ijk} - \hat{\theta}_{ijk}, \tilde{M}_{\varepsilon ijk} = M_{\varepsilon ijk} - \hat{M}_{\varepsilon ijk}, \hat{M}_{\varepsilon \theta ijk}$ is the estimate of $M_{\varepsilon ij} = 2M_{ijk,\theta} + M_{ijk,\varepsilon}$.

Consider the following Lyapunov function

$$V_{iD} = V_{iDe} + \frac{1}{2\eta} \sum_{j=1}^{n_i} \sum_{k=1}^{N} \left[\tilde{\theta}_{ijk}^T \tilde{\theta}_{ijk} + \tilde{M}_{\varepsilon\theta ijk}^2 \right].$$
(16)

Differentiating it with respect to time *t*, it yields

$$\dot{V}_{iD} \leq -e_{ix}^T Q_i e_{ix}$$

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$$+\sum_{j=1}^{n_{i}}\sum_{k=1}^{N} \left[\tilde{\theta}_{ijk}^{T}(2e_{ix}^{T}P_{i}^{j}\xi_{ijk}(\hat{x}_{k})-\frac{1}{\eta}\dot{\theta}_{ijk})\right] \\+\sum_{j=1}^{n_{i}}\sum_{k=1}^{N} \left[\tilde{M}_{\varepsilon\theta ijk}(|2e_{ix}^{T}P_{i}^{j}|-\frac{1}{\eta}\dot{M}_{\varepsilon\theta ijk})\right].$$
(17)

Substituting the adaptive laws (9) and (10) into (17), it yields

$$\begin{split} \dot{V}_{iD} &= -e_{ix}^{T} Q_{i} e_{ix} \\ &+ \sum_{j=1}^{n_{i}} \sum_{k=1}^{N} \left(\frac{\eta_{\theta}}{\eta} \, \tilde{\theta}_{ijk} \hat{\theta}_{ijk} + \frac{\eta_{M}}{\eta} \tilde{M}_{\varepsilon \theta ijk} \hat{M}_{\varepsilon \theta ijk} \right). \end{split}$$
(18)

Since

$$\frac{\eta_{\theta}}{\eta} \tilde{\theta}_{ijk}^{T} \hat{\theta}_{ijk} \leq -\frac{\eta_{\theta}}{2\eta} \tilde{\theta}_{ijk}^{T} \tilde{\theta}_{ijk} + \frac{\eta_{\theta}}{2\eta} M_{ijk,\theta}^{2},$$
(19)
$$\frac{\eta_{M}}{\eta} \tilde{M}_{\varepsilon\theta ijk} \hat{M}_{\varepsilon\theta ijk} \leq -\frac{\eta_{M}}{2\eta} \tilde{M}_{\varepsilon\theta ijk}^{2} + \frac{\eta_{M}}{2\eta} (2M_{ijk,\theta} + M_{ijk,\varepsilon})^{2},$$
(20)

one has

$$\dot{V}_{iD} \le -\lambda_{iD} V_{iD} + \mu_{iD},\tag{21}$$

where $\lambda_{iD} = \min\{\frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)}, \frac{\eta_{\theta}}{2\eta}, \frac{\eta_M}{2\eta}\},\ \mu_{iD} = \sum_{j=1}^{n_i} \sum_{k=1}^{N} [\frac{\eta_{\theta}}{2\eta} M_{ijk,\theta}^2 + \frac{\eta_M}{2\eta} (2M_{ijk,\theta} + M_{ijk,\varepsilon})^2].$ Then, one has $\frac{d}{dt} (V_{iD}(t) e^{\lambda_{iD}t}) \leq e^{\lambda_{iD}t} \mu_{iD}.$ Furthermore,

$$0 \leq V_{iD}(t) \leq rac{\mu_{iD}}{\lambda_{iD}} + V_{iD}(0) = lpha_{iD}.$$

Therefore, the error dynamics (10) is asymptotically stable, $||e_{ix}|| \leq \sqrt{\alpha_{iD} / \lambda_{\min}(P_i)}$, $||\tilde{\theta}_{ijk}|| \leq \sqrt{2\eta \alpha_{iD}}$ and $||\tilde{M}_{\varepsilon \theta ijk}|| \leq \sqrt{2\eta \alpha_{iD}}$.

From Theorem 1, let us define the detection residual as

$$J_i(t) = ||y_i(t) - \hat{y}_i(t)||,$$

and in the healthy case, one has,

$$J_i(t) \leq \|C_i e_{ix}\| \leq \|C_i\| \sqrt{\alpha_{iD}/\lambda_{\min}(P_i)}.$$

Then, the fault detection mechanism is given as:

$$\begin{cases} J_i(t) \le T_{id}, \text{ no fault occurred in the overall system;} \\ J_i(t) > T_{id}, \text{ fault has occurred in the overall system,} \end{cases}$$
(22)

where threshold T_{id} is defined as $T_{id} = ||C_i|| \sqrt{\alpha_{iD}/\lambda_{\min}(P_i)}$.

3.2. Fault isolation

As pointed out in the previous subsection, $J_i(t) > T_{id}$ does not mean that the fault occurs only in the *i*th subsystem because of the existence of the interconnections. So, the problem we meet is how to isolate the fault. Before fault isolation, let us recall the considered system (1),

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i + \Psi_i, \\ y_i = C_i x_i, \end{cases} \quad i = 1, \cdots, N$$

Let $A = diag\{A_1, \dots, A_N\}$, $B = diag\{B_1, \dots, B_N\}$, $C = diag\{C_1, \dots, C_N\}$, $x = [x_1^T, \dots, x_N^T]^T$, $u = [u_1^T, \dots, u_N^T]^T$, $y = [y_1^T, \dots, y_N^T]$, $\Psi = [\Psi_1^T, \dots, \Psi_N^T]^T$, then the whole large-scale system can be written as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu + \Psi, \\ y = Cx. \end{cases}$$
(23)

In order to describe conveniently the following fault isolation design, we re-number the actuators as the 1th, 2th, \dots, \bar{m} th, where $\bar{m} = \sum_{i=1}^{N} m_i$. Correspondingly, the fault model (2) becomes

$$u_i^f(t) = (1 - \rho_i(t))u_i(t) + \sum_{j=1}^p a_{ij}(t),$$

$$i = 1, \cdots, \bar{m}, \ t \ge t_j.$$

Obviously, from Assumption 1, one has

$$\begin{aligned} |\rho_i(t)| &\leq \bar{\rho}_1, \ |\dot{\rho}_i(t)| \leq \bar{\rho}_2, \\ |a_{ij}(t)| &\leq \bar{a}_1, \ |\dot{a}_{ij}(t)| \leq \bar{a}_2. \end{aligned}$$
(24)

Notice that, in this paper, at one time, only single fault is assumed to occurs in one subsystem. Hence, for the overall system (1), there are \bar{m} possible faulty cases.

Without loss of generality, it is assumed that the *l*th actuator becomes faulty. In the faulty case, (1) becomes,

$$\begin{cases} \dot{x} = Ax + Bu - b_l [\rho_l(t)u_l^d - \sum_{j=1}^p a_{lj}(t)] + \Psi, \\ y = Cx, \end{cases}$$
(25)

where $\rho_l(t)$ and $a_{lj}(t)$ denote the *l*th actuator fault defined by (2), u_l^d is the control input when actuator *l* is healthy, $B = [b_1, b_2, \dots, b_{\bar{m}}], b_l \in R^{(\sum_{j=1}^N n_j) \times 1}, 1 \le l \le \bar{m}, j = 1, 2, \dots, p.$

To isolate the fault occurred in the actuator, we propose the following adaptive observers

$$\begin{cases} \dot{x}_{s} = A\hat{x}_{s} + L(y - \hat{y}_{s}) + Bu \\ -b_{s}\mu_{s}[\bar{\rho}_{s}^{u}|u_{s}^{d}| + \sum_{j=1}^{p} \bar{a}_{sj}^{u}] + \hat{\Psi}, \\ \dot{y}_{s} = C\hat{x}_{s}, \end{cases}$$
(26)

where $1 \le s \le \bar{m}$, $\hat{x}_s(t)$ and $\hat{y}_s(t)$ are the state and output of the sth observer, respectively; $\mu_s = [-e_{xs}^T P^1 b_s, \cdots, -e_{xs}^T P^{\sum_{i=1}^N n_i} b_s]^T$, σ_s is a design parameter, P^i is the *i*th column of P satisfying (29); L is the observer gain matrix with appropriate dimensions for the *s*th observer which is chosen to ensure that A - LC is Hurwitz; $\bar{\rho}_s^u = \bar{\rho}_1$ and $\bar{a}_{sj}^u = \bar{a}_1$ denote the upper bounds of the *s*th actuator's gain fault $\bar{\rho}_s$ and bias fault a_{sj} ; $e_{xs} = x_s - \hat{x}_s$ and $e_{ys} = y - \hat{y}_s$. In the following, l is used to denote the practical faulty case, namely, the faulty actuator is actuator l.

For s = l, the error dynamics between (28) and (29) is:

$$\dot{e}_{xs} = (A - LC)e_{xs} - b_s(\rho_s u_s^d - \mu_s \bar{\rho}_s^u |u_s^d|) + b_s \sum_{j=1}^p (a_{sj} - \mu_s \bar{a}_{sj}^u) + \tilde{\Psi},$$
(27)

and for $s \neq l$, one has

$$\dot{e}_{xs} = (A - LC)e_{xs} - (b_s \rho_s u_s^d - b_l \mu_l \bar{\rho}_l^u | u_l^d |) + \sum_{j=1}^p (b_s a_{sj} - b_l \mu_l \bar{a}_{lj}^u) + \tilde{\Psi},$$
(28)

where $\tilde{\Psi} = \Psi - \hat{\Psi}$, $\bar{\rho}_{l}^{u} = \bar{\rho}_{1}$ and $\bar{a}_{lj}^{u} = \bar{a}_{1}$ denote the upper bounds of the *l*th actuator's gain fault $\bar{\rho}_{l}$ and bias fault a_{lj} .

Theorem 2: If there exist matrices $P = P^T > 0$, L, Q > 0 with appropriate dimensions such that

$$(A - LC)TP + P(A - LC) \le -Q,$$
(29)

and the following adaptive laws are employed

$$\hat{\theta}_{sjk} = \begin{cases}
0, \text{ if } \hat{\theta}_{sjk} = \bar{M}_{sjk,\theta} \text{ and} \\
2\eta e_{xs}^T P^j \xi_{sjk}(\hat{x}_k) - \eta_{\theta} \hat{\theta}_{sjk} > 0, \\
\text{ or } \hat{\theta}_{sjk} = -\bar{M}_{sjk,\theta} \text{ and} \\
2\eta e_{xs}^T P^j \xi_{sjk}(\hat{x}_k) - \eta_{\theta} \hat{\theta}_{sjk} < 0, \\
2\eta e_{xs}^T P^j \xi_{sjk}(\hat{x}_k) - \eta_{\theta} \hat{\theta}_{sjk}, \text{ otherwise}, \\
(30) \\
2\eta e_{xs}^T P^j \xi_{sjk}(\hat{x}_k) - \eta_{\theta} \hat{\theta}_{sjk}, \text{ otherwise}, \\
\hat{M}_{\varepsilon\theta sjk} = \begin{cases}
0, \text{ if } \hat{M}_{\varepsilon\theta sjk} = 2\bar{M}_{sjk,\theta} + \bar{M}_{sjk,\varepsilon} \text{ and} \\
2\eta |e_{xs}^T P^j| + \eta_M \hat{M}_{\varepsilon\theta sjk} > 0, \\
\text{ or } \hat{M}_{\varepsilon\theta sjk} = -2\bar{M}_{sjk,\theta} - \bar{M}_{sjk,\varepsilon} \text{ and} \\
2\eta |e_{xs}^T P^j| + \eta_M \hat{M}_{\varepsilon\theta sjk} < 0, \\
2\eta |e_{xs}^T P^j| - \eta_M \hat{M}_{\varepsilon\theta sjk}, \text{ otherwise},
\end{cases}$$
(31)

and Assumptions 1-3 hold, then, if the faulty actuator is actuator l,

i) for s = l, $e_{xs} \in \Omega_{e_{xs}} =: \{ e_{xs} | ||e_{xs}|| \le \sqrt{\alpha_l / \lambda_{\min}(P)} \}$, $\tilde{\theta}_{ijk} \in \Omega_{\theta_{ijk}} =: \{ \tilde{\theta}_{ijk} | ||\tilde{\theta}_{ijk}|| \le \sqrt{2\eta \alpha_l} \}$ and $\tilde{M}_{\varepsilon ijk} \in \Omega_{M_{\varepsilon ijk}}$ $=: \{ \tilde{M}_{\varepsilon sj} | ||\tilde{M}_{\varepsilon sj}|| \le \sqrt{2\eta \alpha_l} \}$; ii) for $s \ne l$, $e_{xs} \notin \Omega_{e_{xs}}$, $\tilde{\theta}_{sjk} \notin \Omega_{\theta_{sjk}}$ and $\tilde{M}_{\varepsilon sjk} \notin \Omega_{M_{\varepsilon sjk}}$.

Proof: 1) For s = l, according to (27), one has

$$\begin{split} \dot{e}_{xs} = & (A - LC) e_{xs} - b_s (\rho_s u_s^d - \mu_s \bar{\rho}_s^u | u_s^d | \\ & + b_s \sum_{j=1}^p (a_{sj} - \mu_s \bar{a}_{sj}^u) + \tilde{\Psi}. \end{split}$$

Define $V_{Ie} = e_{xs}^T P e_{xs}$. Differentiating V_{Ie} and considering (27), it yields

$$\dot{V}_{le} \leq -e_{xs}^{T}Qe_{xs} + 2e_{xs}^{T}Pb_{s}[(-\rho_{s}u_{s}^{d} + \mu_{s}\bar{\rho}_{s}^{u}|u_{s}^{d}|) + \sum_{j=1}^{p}(a_{sj} + \mu_{s}\bar{a}_{sj}^{u})] + 2e_{xs}^{T}P\tilde{\Psi}.$$
(32)

From the definition of μ_s and Assumption 1, one has

$$2e_{xs}^{T}(t)Pb_{s}[(-\rho_{s}u_{s}^{d}+\mu_{s}\bar{\rho}_{s}^{u}|u_{s}^{d}|)$$

$$= \sum_{i=1}^{\sum n_i} [2e_{xs}^T(t)P^i b_s \rho_s u_s^d - sgn(2e_{xs}^T(t)P^i b_s)\bar{\rho}_s^u | u_s^d |]$$

$$\leq 0.$$
(33)

Similarly, one has

$$2e_{xs}^{T}(t)Pb_{s}\sum_{j=1}^{p}\left(a_{sj}-\mu_{s}\bar{a}_{sj}^{u}\right)\leq0.$$
(34)

Substituting (33) and (34) into (32), it yields

$$\dot{V}_{Ie} \le -e_{xs}^T(t)Qe_{xs}(t) + 2e_{xs}^T(t)Pb_s\tilde{\Psi}.$$
(35)

Similar to (14) in the previous subsection, we have

$$2e_{xs}^{T}P(\Psi - \hat{\Psi}) \leq \sum_{j=1}^{\sum_{i=1}^{N} n_{i}} [2e_{xs}^{T}P^{j} \sum_{k=1}^{N} \tilde{\theta}_{sjk}^{T} \xi(\hat{x}_{k})] \\ + \sum_{j=1}^{\sum_{i=1}^{N} n_{i}} [|2e_{xs}^{T}P^{j}| \sum_{k=1}^{N} M_{\varepsilon \theta sjk}].$$

Further, we have

$$\begin{split} \dot{V}_{Ie} &\leq -e_{xs}^{I}(t)Qe_{xs}(t) \\ &+ \sum_{j=1}^{\sum_{k=1}^{N}n_{i}} [2e_{xs}^{T}P^{j} \sum_{k=1}^{N} \tilde{\theta}_{sjk}^{T} \xi(\hat{x}_{k})] \\ &+ \sum_{j=1}^{\sum_{k=1}^{N}n_{i}} [|2e_{xs}^{T}P^{j}| \sum_{k=1}^{N} M_{\varepsilon\theta sjk}]. \end{split}$$
(36)

Define the following Lyapunov function

$$V_I = V_{Ie} + \frac{1}{2\eta_1} \sum_{j=1}^{\sum_{i=1}^{N} n_i} \sum_{k=1}^{N} \left(\tilde{\theta}_{sjk}^T \tilde{\theta}_{sjk} + \tilde{M}_{\varepsilon \theta sjk}^2 \right),$$

where $\tilde{\theta}_{sjk} = \theta^*_{sjk} - \hat{\theta}_{sjk}$, $\tilde{M}_{\varepsilon\theta sjk} = M_{\varepsilon\theta sjk} - \hat{M}_{\varepsilon\theta sjk}$, $\hat{M}_{\varepsilon\theta sjk}$ is the estimate of $M_{\varepsilon\theta sjk} = 2M_{sjk,\theta} + \varepsilon_{sjk,\varepsilon}$.

Similar to Theorem 1, differentiating V_I and considering (30), (31) and (36), it yields

$$\dot{V}_I \le -\lambda_I V_I + \mu_I, \tag{37}$$

where $\lambda_I = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{\eta_{\theta}}{2\eta_1}, \frac{\eta_M}{2\eta_1}\},\ \mu_I = \{\sum_{j=1}^{\sum_{i=1}^{N} n_i} \sum_{k=1}^{N} [\frac{\eta_{\theta}}{2\eta_1} M_{sjk,\theta}^2 + \frac{\eta_M}{2\eta_1} (2M_{sjk,\theta} + \bar{M}_{sjk,\theta})^2]\}.$ Then, one has $\frac{d}{dt} (V_I(t) e^{\lambda_I t}) \leq e^{\lambda_I t} \mu_I$. Furthermore,

$$0 \leq V_I(t) \leq \frac{\mu_I}{\lambda_I} + [V_I(0) - \frac{\mu_I}{\lambda_I}]e^{-\lambda_I t} \leq \frac{\mu_I}{\lambda_I} + V_I(0)$$

= α_I .

Therefore, $||e_{xs}|| \leq \sqrt{\alpha_l / \lambda_{\min}(P)}$, $||\tilde{\theta}_{sjk}|| \leq \sqrt{2\eta_1 \alpha_l}$ and $||\tilde{M}_{\varepsilon \theta s jk}|| \leq \sqrt{2\eta_1 \alpha_l}$. 2) For $s \neq l$, from (25) and (26), we have,

$$\dot{e}_{xs}(t) = (A - LC)e_{xs} - (b_s \rho_s u_s^d - b_l \mu_l \bar{\rho}_l^u | u_l^d |) \\ + \sum_{j=1}^p (b_s a_{sj} - b_l \mu_l \bar{a}_{lj}^u) + \tilde{\Psi}.$$

Since $B = diag(B_1, \dots, B_N) = [b_1, b_2, \dots, b_{\bar{m}}]$ and B_i is of full column rank, B is of full column rank. Further, b_s and b_l are linearly independent. So, the following inequality does not hold

$$2e_{xs}^T P(b_s \rho_s u_s^d - b_l \mu_l \bar{\rho}_l^u | u_l^d |)$$

$$+2e_{xs}^{T}P\sum_{j=1}^{p}(b_{s}a_{sj}-b_{l}\mu_{l}\bar{a}_{lj}^{u})\leq0.$$
(38)

What's more, it is noted that $2e_{xs}^T P(b_s \rho_s u_s^d - b_l \mu_l \bar{\rho}_l^u |u_l^d|) + 2e_{xs}^T P \sum_{j=1}^p (b_s a_{sj} - b_l \mu_l \bar{a}_{lj}^u)$ varies infinitely since $s \neq l, u_s^d \neq u_l^d, \rho_s(t) \neq \rho_l(t)$ and $a_{sj}(t) \neq a_{lj}(t)$. This further causes that $V_I(t)$ varies infinitely. Hence, by using the above adaptive laws (30) and (31) under the condition (29), the observer error e_{xs} does not converge $\Omega_{e_{xs}}$, namely, $e_{xx} \notin \Omega_{e_{xs}}$.

From 1) and 2), the conclusion is easily obtained. \Box

Now, let us define the following residuals,

$$J_s(t) = \|\hat{y}_s(t) - y(t)\|, \ 1 \le s \le \bar{m}.$$

From Theorem 2, we know, if the faulty actuator is the *l*th one, namely, s = l, $J_s(t)$ must converge to $\Omega_{e_{xx}}$; if $s \neq l$, $J_s(t)$ does not basically converge to $\Omega_{e_{xx}}$. Therefore, we can design the actuator fault isolation law in this paper as

$$\begin{cases} J_s(t) > T_I, \ l \neq s, \\ J_s(t) \le T_I, \ l = s \Rightarrow \text{the } l \text{th actuator is faulty }, \end{cases}$$
(39)

where
$$T_I = ||C|| \sqrt{\frac{\alpha_l}{\lambda_{\min}(P)}}$$
 is a threshold.

3.3. Fault estimation

In this section, we will estimate the occurred fault by constructing a adaptive fault estimation observer. Assume the faulty actuator is the *s*th one, and the faulty system has the following form,

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) - b_s \rho_s(t) u_s(t) \\ + \sum_{j=1}^p a_{sj}(t) + \Psi, \\ y(t) = Cx(t). \end{cases}$$
(40)

An adaptive fault estimation observer is designed as:

$$\begin{cases} \dot{x}(t) = A\hat{x}(t) + L(y(t) - \hat{y}(t)) + Bu(t) \\ -b_s[\hat{\rho}_s u_s - \sum_{j=1}^p \hat{a}_{sj}] + \hat{\Psi}, \\ \hat{y}(t) = C\hat{x}(t), \end{cases}$$
(41)

where state estimation error $e_x = x - \hat{x}$, $\hat{\rho}_s$ and $\hat{a}_{s,j}$ are the estimations of $\rho_s(t)$ and $a_{sj}(t)$.

Using (40) and (41), we have

$$\dot{e}_x = (A - LC)e_x - b_s(\tilde{\rho}_s u_s - \sum_{j=1}^p \tilde{a}_{sj}) + \tilde{\Psi}, \qquad (42)$$

where $\tilde{\rho}_s = \rho_s - \hat{\rho}_s$, $\tilde{a}_{sj} = a_{sj} - \hat{a}_{sj}$, and $\tilde{\Psi} = \Psi - \hat{\Psi}$.

Theorem 3: If there exist real matrices $P = P^T > 0$, L, Q > 0 with appropriate dimensions such that

$$(A - LC)^T P + P(A - LC) \le -Q, \tag{43}$$

and adaptive laws (44)-(47) are employed

$$\dot{\hat{P}}_{s} = \begin{cases} 0, \text{ if } \hat{p}_{s} = \bar{p}_{1} \text{ and } -2\eta_{1}e_{x}^{T}Pu_{s} - \hat{p}_{s} > 0 \text{ or} \\ \hat{p}_{s} = -\bar{p}_{1} \text{ and } -2\eta_{1}e_{x}^{T}Pu_{s} - \hat{p}_{s} < 0; \quad (44) \\ -2\eta_{1}e_{x}^{T}Pu_{s} - \eta_{\rho}\hat{p}_{s}, \text{ otherwise}, \end{cases} \\ \dot{\hat{a}}_{sj} = \begin{cases} 0, \text{ if } \hat{a}_{sj} > \bar{a}_{1} \text{ and } 2\eta_{2}e_{x}^{T}P - \hat{a}_{sj} > 0 \text{ or} \\ \hat{a}_{sj} < -\bar{a}_{1} \text{ and } 2\eta_{2}e_{x}^{T}P - \hat{a}_{sj} < 0; \quad (45) \\ 2\eta_{2}e_{x}^{T}P - \eta_{\alpha}\hat{a}_{sj}, \text{ otherwise}, \quad j = 1, 2, \cdots, p, \end{cases} \\ \dot{\hat{\theta}}_{jk} = \begin{cases} 0, \text{ if } \hat{\theta}_{jk} = \bar{M}_{jk,\theta} \text{ and} \\ 2\eta_{3}e_{x}^{T}P^{j}\xi(\hat{x}_{k}) - \eta_{\theta}\hat{\theta}_{jk} > 0, \\ \text{ or } \hat{\theta}_{jk} = -\bar{M}_{jk,\theta} \text{ and} \\ 2\eta_{3}e_{x}^{T}P^{j}\xi(\hat{x}_{k}) - \eta_{\theta}\hat{\theta}_{jk} < 0, \\ 2\eta_{3}e_{x}^{T}P^{j}\xi(\hat{x}_{k}) - \eta_{\theta}\hat{\theta}_{jk}, \text{ otherwise}, \end{cases} \\ \dot{\hat{M}}_{\varepsilon\theta jk} = \begin{cases} 0, \text{ if } \hat{M}_{\varepsilon\theta jk} = 2\bar{M}_{jk,\theta} \text{ and} \\ 2\eta_{3}|e_{x}^{T}P^{j}| + \eta_{M}\hat{M}_{\varepsilon\theta jk} > 0, \\ \text{ or } \hat{M}_{\varepsilon\theta jk} = -2\bar{M}_{jk,\theta} - \bar{M}_{jk,\varepsilon} \text{ and} \\ 2\eta_{3}|e_{x}^{T}P^{j}| + \eta_{M}\hat{M}_{\varepsilon\theta jk} < 0, \\ 2\eta_{3}|e_{x}^{T}P^{j}| - \eta_{M}\hat{M}_{\varepsilon\theta jk}, \text{ otherwise}, \end{cases}$$

and Assumptions 1-3 hold, **then** the error system (42) is asymptotically stable, $||e_x|| \leq \sqrt{\alpha_E/\lambda_{\min}(P)}$, $|\tilde{\rho}_s| \leq \sqrt{2\eta_1\alpha_E}$, $|\tilde{a}_{sj}| \leq \sqrt{2\eta_2\alpha_E}$, $||\theta_{jk}|| \leq \sqrt{2\eta_3\alpha_E}$ and $|\tilde{M}_{\epsilon\theta_{jk}}| \leq \sqrt{2\eta_3\alpha_E}$, $\alpha_E = \frac{\mu_E}{\lambda_E} + V_E(0)$ and $\lambda_E = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{\eta_\rho}{2\eta_1}, \frac{\eta_\alpha}{\eta_2}, \frac{\eta_\theta}{2\eta_3}\}$, where P^j is the *j*th row of P; $\eta_1 > 0$, $\eta_2 > 0$, $\eta_3 > 0$, $\eta_\rho > 0$, $\eta_\alpha > 0$, $\eta_\theta > 0$ and $\eta_M > 0$ denote design parameters, respectively; $P = diag\{P_1, \dots, P_N\}, L = diag\{L_1, \dots, L_N\},$ $Q = diag\{Q_1, \dots, Q_N\}.$

Proof: Define

$$V_{E} = e_{x}^{T} P e_{x} + \frac{1}{2\eta_{1}} \tilde{\rho}_{s}^{2} + \sum_{j=1}^{p} \frac{1}{2\eta_{2}} \tilde{a}_{sj}^{2} + \frac{1}{2\eta_{3}} \sum_{j=1}^{\sum_{i=1}^{N} n_{j}} \sum_{k=1}^{N} (\tilde{\theta}_{jk}^{T} \tilde{\theta}_{jk} + \tilde{M}_{\varepsilon\theta jk}^{2}).$$
(48)

Differentiating V_E , it yields

$$\begin{split} \dot{V}_{E} &= -e_{x}^{T}Qe_{x} + 2e_{x}^{T}P\tilde{\Psi} \\ &- \frac{1}{\eta_{3}}\sum_{j=1}^{\sum_{i=1}^{n_{j}}}\sum_{k=1}^{N}\left(\tilde{\theta}_{jk}^{T}\dot{\theta}_{jk} + \tilde{M}_{\varepsilon\theta jk}\dot{M}_{\varepsilon\theta jk}\right) \\ &+ \tilde{\rho}_{s}\left(-2e_{x}^{T}Pb_{s}u_{s} + \frac{1}{\eta_{1}}\dot{\rho}_{s} - \frac{1}{\eta_{1}}\dot{\hat{\rho}}_{s}\right) \\ &+ \sum_{j=1}^{p}\tilde{a}_{sj}\left(2e_{x}^{T}Pb_{s} + \frac{\dot{a}_{sj}}{\eta_{2}} - \frac{\dot{a}_{sj}}{\eta_{2}}\right). \end{split}$$
(49)

Since $|\hat{\rho}_s(t)| \leq \bar{\rho}_1$ and $|\hat{a}_{sj}(t)| \leq \bar{a}_1$, which can be guaranteed by adaptive laws (48) and (49), and Assumptions 1-3 (i.e. $|\rho_s(t)| \leq \bar{\rho}_1$, $|\dot{\rho}_s(t)| \leq \bar{\rho}_2$, $|a_{sj}(t)| \leq \bar{a}_1$, and $|\dot{a}_s(t)| \leq \bar{a}_2$) are satisfied, one has

$$\frac{\tilde{\rho}_{s}\dot{\rho}_{s}}{\eta_{1}} = \frac{(\rho_{s} - \hat{\rho}_{s})\dot{\rho}_{s}}{\eta_{1}} \le \frac{(|\rho_{i}| + |\hat{\rho}_{i}|)|\dot{\rho}_{i}|}{\eta_{1}} \le \frac{2\bar{\rho}_{1}\bar{\rho}_{2}}{\eta_{1}},$$
(50)

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$$\sum_{j=1}^{p} \frac{\tilde{a}_{sj} \dot{a}_{sj}}{\eta_2} \le \sum_{j=1}^{p} \frac{2\bar{a}_1 \bar{a}_2}{\eta_2}.$$
(51)

Similar to Theorem 1, from adaptive laws (49) and (50), it follows

$$2e_{x}^{T}P\tilde{\Psi} - \frac{1}{\eta_{3}}\sum_{j=1}^{\sum_{i=1}^{N}n_{j}}\sum_{k=1}^{N}\left(\tilde{\theta}_{jk}^{T}\dot{\hat{\theta}}_{jk} + \tilde{M}_{\varepsilon\theta jk}\dot{\hat{M}}_{\varepsilon\theta jk}\right)$$
$$\leq -\sum_{j=1}^{\sum_{i=1}^{N}n_{i}}\sum_{k=1}^{N}\left[\frac{\eta_{\theta}}{2\eta_{3}}\tilde{\theta}_{jk}^{2} + \frac{\eta_{M}}{2\eta_{3}}\tilde{M}_{\varepsilon\theta jk}^{2}\right] + \Delta_{1}, \quad (52)$$

where $\Delta_1 = \sum_{j=1}^{\sum_{k=1}^{n} n_j} \sum_{k=1}^{N} \left[\frac{\eta_{\theta}}{2\eta_3} M_{jk,\theta}^2 + \frac{\eta_M}{2\eta_3} (2M_{jk,\theta} + M_{jk,\varepsilon})^2 \right]$ From (49)-(52), it follows

$$egin{aligned} \dot{V}_E &= -\,e_x^T Q e_x - \sum_{j=1}^{\sum_{i=1}^{n}n_i} \sum_{k=1}^{N} \left[rac{\eta_\theta}{2\eta_3} ilde{ heta}_{jk}^2 + rac{\eta_M}{2\eta_3} ilde{M}_{arepsilon jk}^2
ight] \ &+ rac{2ar{
ho}_1ar{
ho}_2}{\eta_1} + \sum_{j=1}^{p} rac{2ar{a}_1ar{a}_2}{\eta_2} + ar{
ho}_s(-2e_x^T P b_s u_s - rac{1}{\eta_1} \dot{eta}_s) \ &+ \sum_{j=1}^{p} ar{a}_{sj}(2e_x^T P b_s - rac{\dot{a}_{sj}}{\eta_2}) + \Delta_1. \end{aligned}$$

Considering adaptive laws (43) and (44), we further have

$$\dot{V}_{E} = -e_{x}^{T}Qe_{x} - \sum_{j=1}^{\sum_{i=1}^{n_{i}}n_{i}}\sum_{k=1}^{N}\left[\frac{\eta_{\theta}}{2\eta_{3}}\tilde{\theta}_{jk}^{2} + \frac{\eta_{M}}{2\eta_{3}}\tilde{M}_{\varepsilon jk}^{2}\right] \\ + \frac{2\bar{\rho}_{1}\bar{\rho}_{2}}{\eta_{1}} + \sum_{j=1}^{p}\frac{2\bar{a}_{1}\bar{a}_{2}}{\eta_{2}} + \Delta_{1} + \frac{1}{\eta_{1}}\tilde{\rho}_{s}\hat{\rho}_{s} \\ + \sum_{j=1}^{p}\frac{\tilde{a}_{sj}\hat{a}_{sj}}{\eta_{2}}.$$
(53)

Applying Young's inequality, one has

$$\frac{\eta_{\rho}}{\eta_1}\tilde{\rho}_s\hat{\rho}_s \leq -\frac{\eta_{\rho}}{2\eta_1}\tilde{\rho}_s^2 + \frac{\eta_{\rho}}{2\eta_1}\bar{\rho}_1^2.$$
(54)

Similarly, one has

$$\sum_{j=1}^{p} \frac{\eta_{\alpha}}{\eta_{2}} \tilde{a}_{sj} \hat{a}_{sj} \leq -\sum_{j=1}^{p} \frac{\eta_{\alpha}}{\eta_{2}} \tilde{a}_{sj}^{2} + \sum_{j=1}^{p} \frac{\eta_{\alpha}}{\eta_{2}} \tilde{a}_{1}^{2}.$$
 (55)

Substituting (53) and (55) into (53), it yields

$$\dot{V}_E \leq -\lambda_E V(t) + \mu_E,$$

where $\mu_E = \frac{\eta_{\rho}}{2\eta_1} \bar{\rho}_1^2 + \frac{\eta_{\alpha}}{2\eta_2} p \bar{a}_1^2 + \frac{2}{\eta_1} \bar{\rho}_1 \bar{\rho}_2 + \frac{2}{\eta_2} p \bar{a}_1 \bar{a}_2 + \Delta_1,$ $\lambda_E = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{\eta_{\rho}}{2\eta_1}, \frac{\eta_{\alpha}}{\eta_2}, \frac{\eta_{\theta}}{2\eta_3}, \frac{\eta_M}{2\eta_3}\}.$ Then, one has $\frac{d}{dt} (V_E(t) e^{\lambda_E t}) \leq e^{\lambda_E t} \mu_E.$ Furthermore,

$$0 \leq V_E(t) \leq \frac{\mu_E}{\lambda_E} + [V(0) - \frac{\mu_E}{\lambda_E}]e^{-\lambda_E t} \leq \frac{\mu_E}{\lambda_E} + V_E(0).$$

Let $\alpha_E = \frac{\mu_E}{\lambda_E} + V_E(0)$, one has $||e_x|| \leq \sqrt{\frac{\alpha_E}{\lambda_{\min}(P)}}$, $|\tilde{\rho}_s| \leq \sqrt{2\eta_1 \alpha_E}, |\tilde{a}_{sj}| \leq \sqrt{2\eta_2 \alpha_E}, ||\theta_{jk}|| \leq \sqrt{2\eta_3 \alpha_E}$ and $|\tilde{M}_{\varepsilon\theta ijk}| \leq \sqrt{2\eta_3 \alpha_E}.$ \square

3.4. Fault accommodation

After fault estimation, the FTC problem of the faulty system (40) will be considered, and a FTC law is designed to recover the system performance when an actuator fault of a subsystem occurs. Let us firstly consider the following nominal system (fault-free):

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i + \Psi_i \\ y_i = C_i x_i, \end{cases}$$

where $i = 1, \dots, N$. Let us design the controller for the system:

$$u_i(t) = K_i x(t) - B_i^+ \hat{\Psi}_i, \qquad (56)$$

where K_i denotes gain matrix, which will be defined in the following theorem, and $\hat{\Psi}_i$ denotes the estimate of Ψ_i, B_i^+ is the generalized inverse matrix of B_i that has the property: $B_i B_i^+ = I_{n_i \times n_i}$.

Consider the system with (59), we give the following theorem, which guarantees the closed-loop system stability.

Theorem 4: If there exist matrices $P_i = P_i^T > 0$ and $Q_i > 0$ with appropriate dimensions such that

$$P_i(A_i + B_i K_i) + (A_i + B_i K_i)^T P_i \le -Q_i, \ i = 1, \cdots, N,$$
(57)

and consider the following adaptive laws

$$\dot{\hat{\theta}}_{ijk} = \begin{cases} 0, \text{ if } \hat{\theta}_{ijk} = \bar{M}_{ijk,\theta} \text{ and} \\ 2\eta_1 x_i^T P_i^j \xi(\hat{x}_k) - \eta_\theta \hat{\theta}_{ijk} > 0, \\ \text{ or } \hat{\theta}_{ijk} = -\bar{M}_{ijk,\theta} \text{ and} \\ 2\eta_1 x_i^T P_i^j \xi(\hat{x}_k) - \eta_\theta \hat{\theta}_{ijk} < 0, \\ 2\eta_1 x_i^T P_i^j \xi(\hat{x}_k) - \eta_\theta \hat{\theta}_{ijk}, \text{ otherwise}, \\ 0, \text{ if } \hat{M}_{\varepsilon\theta ijk} = 2\bar{M}_{ijk,\theta} + \bar{M}_{ijk,\varepsilon} \text{ and} \\ 2\eta_1 |x_i^T P_i^j| + \eta_M \hat{M}_{\varepsilon\theta ijk} > 0, \\ \text{ or } \hat{M}_{\varepsilon\theta ijk} = -2\bar{M}_{ijk,\theta} - \bar{M}_{ijk,\varepsilon} \text{ and} \\ 2\eta_1 |x_i^T P_i^j| - \eta_M \hat{M}_{\varepsilon\theta ijk} < 0, \\ 2\eta_1 |x_i^T P_i^j| - \eta_M \hat{M}_{\varepsilon\theta ijk}, \text{ otherwise}, \end{cases}$$

and Assumptions 1-4 hold, then the healthy system (1) under the controller (56) is asymptotically stable with all closed-loop system signals asymptotically converge to a neighborhood of the origin, namely $||x_i|| \leq \sqrt{\alpha_i/\lambda_{\min}(P_i)}$, $||\tilde{\theta}_{ijk}|| \leq \sqrt{2\eta_1 \alpha_i}$ and $||\tilde{M}_{\epsilon\theta ijk}|| \leq \sqrt{2\eta_1 \alpha_i}$, where $\alpha_i =$ $\mu_i/\lambda_i + V_i(0), \ \lambda_i = \min\{\frac{\lambda_{\min}(\underline{Q}_i)}{\lambda_{\max}(P_i)}, \frac{\eta_\theta}{2\eta_1}, \frac{\eta_M}{2\eta_2}\}, \ \mu_i = \sum_{j=1}^{n_i} \sum_{k=1}^{N}$ $[\frac{\eta_{\theta}}{2\eta_1}M_{ijk,\theta}^2 + \frac{\eta_M}{2\eta_2}(2\cdot M_{ijk,\theta} + \overline{M}_{ijk,\varepsilon})^2].$

From the proof of the previous Theorems, the conclusion is easily obtained. To save pages, the detailed derivation is omitted.

By using the fault estimation, FTC input is designed as

$$u_{s} = \frac{(1-\hat{\rho}_{s})(u_{s}^{d} - \sum_{j=1}^{p_{i}} \hat{a}_{i,j})}{(1-\hat{\rho}_{s})^{2} + \sigma},$$
(60)

where $\sigma > 0 \in R$, $\hat{\rho}_s$, $\hat{a}_{i,j}$ are the estimations of ρ_s , $a_{i,j}$, and u_s^d is the *s*th desired control input.

Theorem 5: For the faulty system (40), if there exist matrices $P_i = P_i^T > 0$, L_i and $Q_i > 0$, $i = 1, 2, \dots, N$ such that

$$\begin{split} P_{i}(A_{i}+B_{i}K_{i})+(A_{i}+B_{i}K_{i})^{T}P_{i} \leq -Q_{i}, \quad (61) \\ \dot{\rho}_{s} &= \begin{cases} 0, \text{ if } \hat{\rho}_{s} = \bar{\rho}_{1} \text{ and } -2\eta_{1}x_{i}^{T}Pu_{s} - \hat{\rho}_{s} > 0 \text{ or} \\ \hat{\rho}_{s} &= -\bar{\rho}_{1} \text{ and } -2\eta_{1}x_{i}^{T}Pu_{s} - \hat{\rho}_{s} < 0; \quad (62) \\ -2\eta_{1}x_{i}^{T}Pu_{s} - \eta_{\rho}\hat{\rho}_{s}, \text{ otherwise}, \end{cases} \\ \dot{a}_{sj} &= \begin{cases} 0, \text{ if } \hat{a}_{sj} > \bar{a}_{1} \text{ and } 2\eta_{2}x_{i}^{T}P - \hat{a}_{sj} > 0 \text{ or} \\ \hat{a}_{sj} < -\bar{a}_{1} \text{ and } 2\eta_{2}x_{i}^{T}P - \hat{a}_{sj} < 0; \quad (63) \\ 2\eta_{2}x_{i}^{T}P - \eta_{\alpha}\hat{a}_{sj}, \text{ otherwise}, \quad j = 1, 2, \cdots, p, \end{cases} \\ \dot{\theta}_{ijk} &= \begin{cases} 0, \text{ if } \hat{\theta}_{ijk} = \bar{M}_{ijk,\theta} \text{ and} \\ 2\eta_{1}xe_{i}^{T}P_{i}^{j}\xi(\hat{x}_{k}) - \eta_{\theta}\hat{\theta}_{ijk} > 0, \\ \text{ or } \hat{\theta}_{ijk} = -\bar{M}_{ijk,\theta} \text{ and} \\ 2\eta_{1}x_{i}^{T}P_{i}^{j}\xi(\hat{x}_{k}) - \eta_{\theta}\hat{\theta}_{ijk} < 0, \\ 2\eta_{1}x_{i}^{T}P_{i}^{j}|\xi(\hat{x}_{k}) - \eta_{\theta}\hat{\theta}_{ijk}, \text{ otherwise}, \end{cases} \\ \dot{M}_{\varepsilon\theta ijk} &= \begin{cases} 0, \text{ if } \hat{M}_{\varepsilon\theta ijk} = 2\bar{M}_{ijk,\theta} \text{ and} \\ 2\eta_{1}|x_{i}^{T}P_{i}^{j}| + \eta_{M}\hat{M}_{\varepsilon\theta ijk} > 0, \\ \text{ or } \hat{M}_{\varepsilon\theta ijk} = -2\bar{M}_{ijk,\theta} - \bar{M}_{ijk,\varepsilon} \text{ and} \\ 2\eta_{1}|x_{i}^{T}P_{i}^{j}| + \eta_{M}\hat{M}_{\varepsilon\theta ijk} > 0, \\ \text{ or } \hat{M}_{\varepsilon\theta ijk} = -2\bar{M}_{ijk,\theta} - \bar{M}_{ijk,\varepsilon} \text{ and} \end{cases}$$

and Assumptions 1-3 hold, **then** the faulty system is asymptotically stable, $||x|| \leq \sqrt{\alpha/\lambda_{\min}(P)}$, $|\tilde{\rho}_s| \leq \sqrt{2\eta_1 \alpha}$, $|\tilde{a}_{sj}| \leq \sqrt{2\eta_2 \alpha}$, $||\theta_{ijk}|| \leq \sqrt{2\eta_3 \alpha}$ and $|\tilde{M}_{ijk}| \leq \sqrt{2\eta_3 \alpha}$, where $\alpha = \frac{\mu}{\lambda} + V(0)$ and $\lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{\eta_{\rho}}{2\eta_1}, \frac{\eta_{\alpha}}{\eta_2}, \frac{\eta_{\theta}}{2\eta_3}, \frac{\eta_M}{2\eta_3}\}$, $\mu = \frac{\eta_{\rho}}{2\eta_1} \bar{\rho}_1^2 + \frac{\eta_{\alpha}}{2\eta_2} p \bar{a}_1^2 + \frac{2}{\eta_1} \bar{\rho}_1 \bar{\rho}_2 + \frac{2}{\eta_2} p \bar{a}_1 \bar{a}_2 + \Delta_1, \lambda = \min\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{\eta_{\rho}}{2\eta_1}, \frac{\eta_{\alpha}}{\eta_2}, \frac{\eta_{\theta}}{2\eta_1}, \frac{\eta_M}{2\eta_2}\}$, $P = diag\{P_1, \cdots, P_N\}$, $L = diag\{L_1, \cdots, L_N\}$, $Q = diag\{Q_1, \cdots, Q_N\}$, P^j is the *j*th row of P, $\eta_1 > 0$, $\eta_2 > 0$, $\eta_3 > 0$, $\eta_{\rho} > 0$, $\eta_{\alpha} > 0$, $\eta_{\theta} > 0$ and $\eta_M > 0$ are design parameters, respectively.

Proof: To save pages, the detailed derivation is omitted. In fact, from the proof of the previous Theorems, the conclusion is easily obtained. \Box

4. SIMULATION

4.1. Near space vehicle dynamics

In re-entry phase, near space vehicle (NSV) attitude dynamics has the following form [21]:

$$\begin{cases} \dot{\gamma} = R(\cdot)\omega, \\ J\dot{\omega} = -\Omega J\omega + \delta, \end{cases}$$
(66)

where angular rate vector $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T = [p, q, r]^T$, r, qand p and respectively denote the yaw rate, roll and pitch of NSV; $\boldsymbol{\gamma} = [\phi, \beta, \alpha]^T$, α, β and ϕ respectively are the attack angles, sideslip, and bank of NSV; $J = J^T > 0 \in R^{3 \times 3}$



Fig. 1. The diagram of near space vehicle.

denotes moment of inertia tensor; control surface deflection $\delta = [\delta_e, \delta_\alpha, \delta_r]^T \in R^{3 \times 1}$, δ_α, δ_e and δ_r respectively denote the aileron deflection, elevator deflection and rudder deflection of NSV; $R(\cdot)$ and Ω respectively defines as:

$$R(\cdot) = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ \sin \alpha & 0 & -\cos \alpha \\ 0 & 1 & 0 \end{bmatrix},$$

and

$$\Omega = \left[\begin{array}{ccc} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ \omega_2 & \omega_1 & 0 \end{array} \right].$$

From [21], equation (66) can be divided into the outer loop γ (slow loop) and inner loop ω (fast loop) shown in Fig. 1. Correspondingly, (66) can be expressed by a interconnected system consisting of slow subsystem (67) and fast subsystem (68).

$$\begin{cases} \dot{x}_{\gamma} = f(x_{\gamma}, t) y_{\omega}, \\ y_{\gamma} = x_{\gamma}, \end{cases}$$
(67)

$$\begin{cases} \dot{x}_{\omega} = f(x_{\omega}) + g(x_{\omega})u(t), \\ y_{\omega} = x_{\omega}, \end{cases}$$
(68)

where $f(x_{\gamma}) = R(\cdot), x_{\omega} = \omega, x_{\gamma} = \gamma, g(x_{\omega}) = J^{-1}, f(x_{\omega}) = J^{-1}\Omega(\omega)Jx_{\omega}.$

The control objectives is to design u(t) to ensure that $\lim_{t\to\infty}(x_{\omega} - \omega_d) = 0 \Rightarrow \lim_{t\to\infty}(\gamma_{\gamma} - \gamma_d) = 0$, where $y_{\omega}(=\omega_d)$ and y_d respectively are the ideal angular rate and the desired reference signal.

In this paper, it is assumed that the slow subsystem is always healthy, and has been stable, which means $\omega_d = [0,0,0]^T$. Hence, our main task is, for the fast subsystem, how to design the proper fault-tolerant controller such that can guarantees the tracking performance in normal and faulty conditions, namely, the fast subsystem's output can tracks the desired command $y_{\omega} (= \omega_d = [0,0,0]^T)$ in spit of actuator fault.

In this simulation, the case is considered, where altitude H = 40km and speed V = 2500 m/s. Further, we have

$$J = \begin{bmatrix} 554486 & 0 & -23002 \\ 0 & 1136949 & 0 \\ -23002 & 0 & 1376852 \end{bmatrix} = \begin{bmatrix} \bar{a} & 0 & \bar{b} \\ 0 & \bar{d} & 0 \\ \bar{b} & 0 & \bar{c} \end{bmatrix}.$$

Further, one has

$$J^{-1} = 1.0e - 005 \times \begin{bmatrix} 0.1805 & 0 & 0.0030 \\ 0 & 0.0880 & 0 \\ 0.0030 & 0 & 0.0727 \end{bmatrix}$$

$$= \left[\begin{array}{rrr} a & 0 & b \\ 0 & d & 0 \\ b & 0 & c \end{array} \right],$$

where $a = 0.1805 \times 10^{-5}, b = 0.0030 \times 10^{-5}, c = 0.0727 \times 10^{-5}, d = 0.0880 \times 10^{-5}, \ \bar{a} = 554486, \bar{b} = -23002, \ \bar{c} = 1376852, \ \bar{d} = 1136949.$ Denote $x = [x_1, x_2, x_3]^T = [\omega_1, \omega_2, \omega_3]^T$, then

$$\Omega = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ x_2 & x_1 & 0 \end{bmatrix}.$$

Further, one has

$$\dot{x} = \begin{bmatrix} f_{11}x_1x_2 + f_{11}x_2x_3\\ f_{21}x_1x_3 + f_{22}x_1^2 + f_{23}x_3^2\\ f_{31}x_1x_2 + f_{32}x_2x_3 \end{bmatrix} + \begin{bmatrix} au_1 + bu_3\\ du_2\\ bu_1 + cu_3 \end{bmatrix}.$$

Now, we can transform the fast subsystem into interconnected systems form.

$$\begin{cases} \dot{\bar{x}}_1 = A_1 + B_1 \bar{u}_1 + \Psi_1(\bar{x}_1, \bar{x}_2), \\ \dot{\bar{x}}_2 = A_2 + B_2 \bar{u}_2 + \Psi_2(\bar{x}_1, \bar{x}_2), \\ \bar{x}_1 = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}, \ \bar{x}_2 = x_2, \ \bar{u}_1 = \begin{bmatrix} u_1 \\ u_3 \end{bmatrix}, \ \bar{u}_2 = u_2, \\ A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ A_2 = 1, B_1 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}, \ B_2 = d, \\ \Psi_1(\bar{x}_1, \bar{x}_2) = \begin{bmatrix} f_{11}x_2 - 11 & f_{12}x_2 \\ f_{31}x_2 & f_{32}x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}, \\ \Psi_2(\bar{x}_1, \bar{x}_2) = f_{21}x_1x_3 - x_1 + f_{22}x_1^2 + f_{23}x_3^2, \\ \Psi_1(\bar{x}_1, \bar{x}_2) = \begin{bmatrix} f_{11}x_2 - 11 & f_{12}x_2 \\ f_{31}x_2 & f_{32}x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}, \\ \Psi_2(\bar{x}_1, \bar{x}_2) = f_{21}x_1x_3 - x_1 + f_{22}x_1^2 + f_{23}x_3^2, \\ \Psi_1(\bar{x}_1, \bar{x}_2) = f_{21}x_1x_3 - x_1 + f_{22}x_1^2 + f_{23}x_3^2. \end{cases}$$

4.2. Simulation results

In this simulation, $\omega_d = [0,0,0]^T$, $\gamma(0) = [0,0,0]^T$, $\omega(0) = [0.01,0.02,-0.01]^T$. The parameters in (66) are taken as in [21]. It also is assumed that only one actuator becomes faulty at one time. The faulty case can be described as:

$$u_1^f(t) = \begin{cases} u_1(t), & t < 5, \\ g_1(t)u_1(t) + f_1(t), & t \ge 5, \end{cases}$$
$$u_2^f(t) = u_2(t), & u_3^f(t) = u_3(t), \end{cases}$$

where $g_1(t) = 0.4$, $f_1(t) = (0.5 + \cos(t)) \times 10^5$. By using Matlab LMI control toolbox, the matrices inequalities (8) and (29) are solved, *L* and *L_i* thus are obtained, and the corresponding fault diagnostic observers (5) and (26) further are designed. Then, By solving (43), the fault estimation observer (41) is constructed, and the fault information is obtained by (44)-(47). Further, by solving (57), we can



Fig. 2. The state responses without fault.



Fig. 3. Detection residual.

obtain P_1 and K_i . Hence, normal control (56) is designed. Based on (56) and the obtain fault information, FTC input (60) can be designed.

The simulation results are presented in Figs. 2-5. From Fig. 2, it is seen that, under normal operating condition, system states are asymptotically bounded and converge to the small neighborhood of the origin. If a fail occurs in the first actuator at t = 5s, the detection residual deviates significantly from the small neighborhood of the origin, as shown in Fig. 3, while that the isolation residual signals shows in Fig. 4. As shown in Fig. 5, using the proposed FTC (60), the system states become asymptotically bounded, again.

5. CONCLUSIONS

In this paper, the problem of adaptive FTC for a class of large-scale systems with nonlinear interconnections is investigated. A general actuator fault model is proposed,



Fig. 4. Isolation residuals.



Fig. 5. The state response with FTC.

which integrates varying bias and gain faults, and a bank of adaptive fuzzy observers are designed to provide a bank of residuals for FDI. Based on Lyapunov stability theory, a novel fault diagnostic algorithm is proposed, which removes some classical assumptions.

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