International Journal of Control, Automation and Systems 17(8) (2019) 1921-1934
http://dx.doi.org/10.1007/s12555-018-0721-6

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Time-varying H_∞ Control for Discrete-time Switched Systems with Admissible Edge-dependent Average Dwell Time

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Abstract: The problem of H_∞ control for discrete-time switched systems is investigated via admissible edge-dependent average dwell time (AED-ADT) method in this paper. By virtue of a convex combination of positive definite matrices, a novel multiple piecewise convex Lyapunov function (MPCLF) is designed, which can relax the restricted conditions of Lyapunov functions at switching points and interval interior points. Based on the MPCLF approach, the time-varying H_∞ state feedback controllers, guaranteeing that the corresponding closed-loop system is globally uniformly exponentially stable (GUES) with a prescribed H_∞ performance, are established for the considered switched system. Finally, three numerical examples are provided to illustrate the effectiveness of the proposed approaches.

Keywords: Admissible edge-dependent average dwell time, discrete-time switched systems, H_∞ state feedback control, multiple piecewise convex Lyapunov function.

1. INTRODUCTION

Switched systems [1] are a special class of hybrid systems, which contain a series of subsystems and a switching signal that schedules the switchings of the subsystems. In the last few decades, switched systems have received considerable attention, not only for their theoretical value [2–12], but also for their widespread practical applications, such as network control systems [13], DC/DC converters [14], oscillators [15], three-phase two-level grid-connected power converters [16], etc. Stability analysis is crucial in the research of switched systems. As is known, the common Lyapunov function [17] is mainly used to investigate the stability of switched systems under arbitrary switching signals. To achieve flexibility, the multiple Lyapunov function (MLF) [18,19] is proposed to study the stability of switched systems with constrained switching signals. Recently, for a class of slowly switched systems, the authors in [20,21] introduced a multiple discontinuous Lyapunov function (MDLF), where the Lyapunov function for each subsystem is piecewise continuous. Based on the MDLF, the stability results under the average dwell time (ADT) or mode-dependent average dwell time (MDADT) with tighter bounds are obtained. However, a series of inequalities $P_{ip} \leq \rho_i P_{i(p-1)}$ of MDLF may lead to the infeasibility of related LMI conditions. This motivates us to design a new Lyapunov function with more degrees of freedom so that larger feasibility regions can be achieved.

As a class of switching signals, ADT switching [22–25] signifies that the switching times in a finite interval is bounded and the average time between consecutive switchings is not less than a constant, which is more general than dwell time (DT) switching [26]. Subsequently, the paper [27] proposed the MDADT switching [28–31] with each mode carrying its own ADT, due to which the MDADT switching is of less restrictiveness than the ADT switching. Recently, a novel notion of AED-ADT was developed in [32,33]. Its switching behavior is represented by a directed graph, where each admissible transition edge (ATE) means a directed switching between subsystems. Owing to the choices of transition weights of ATEs, the AED-ADT switching provides more flexibility compared to the MDADT switching.

Since the disturbance is commonly found in practical situations, H_∞ control or l_2-gain analysis has become an attracting issue [34–38]. The H_∞ control problem was investigated in [34–36] for a class of switched systems with ADT. The authors in [36] studied the asynchronous finite-time H_∞ control problem for a class of switched linear

Manuscript received October 12, 2018; revised January 28, 2019; accepted February 14, 2019. Recommended by Associate Editor Jun Cheng under the direction of Editor Jessie (Ju H.) Park. This work is supported by the National Natural Science Foundation of China (61773235, 61773236), the Project of Shandong Province Higher Educational Science and Technology Program (J18KA324), and the Fundamental Research Funds for the Central Universities (2242019k30017).

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systems with time-varying disturbances. In [37, 38], the finite-time H_∞ control of switched systems was considered under the MDADT switching, where the corresponding closed-loop system is finite-time bounded with a prescribed H_∞ performance. However, there is no result available yet on H_∞ control of discrete-time switched systems with AED-ADT. Moreover, in the extant works, the obtained l_2-gains can not be reduced to low levels, which severely affects the related practical applications.

In this paper, a novel MPCLF is firstly proposed to analyze the problem of H_∞ control for discrete-time switched systems. By employing the MPCLF approach, a time-varying H_∞ controller is designed. Under the AED-ADT and MDADT switching, some sufficient conditions are derived for the switched systems, which can ensure that the resultant closed-loop system is GUES with a prescribed H_∞ performance. It should be pointed out that by using our approach the tighter bounds are provided on the AED-ADT, and the lower l_2-gains can be achieved. The remainder of this paper is organized as follows: Section 2 gives preliminaries and problem formulation. In Section 3, the main results of this paper are put forth. A time-varying H_∞ controller is firstly given, and then H_∞ performance conditions are derived. Section 4 presents three numerical examples to verify the validity of the developed results. In the end, some conclusions are given in Section 5.

Notations: The notations in this paper are fairly standard. We use $A > 0$ ($A < 0$) to stand for a positive definite (negative definite) matrix A. A^T refers to the transpose of a matrix A. Let $\mathbb{R}^n$ and $\mathbb{Z}_{\geq 0}$ denote the n-dimensional Euclidean space and the set of nonnegative integers, respectively. $\|\cdot\|$ is used to denote the vector Euclidean norm. $l_2[0,\infty)$ is the space of square summable infinite sequence and for $\omega = \{\omega(k)\} \in l_2[0,\infty)$, its norm is given by $\|\omega\|_2 = \sqrt{\sum_{k=0}^{\infty}\omega^T(k)\omega(k)}$. As is commonly used in other literature, $*$ denotes the elements below the main diagonal of a symmetric matrix, and max and min, respectively, stand for the maximum and minimum. In addition, matrices, if not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider the following discrete-time switched linear system

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + E_{\sigma(k)}\omega(k), \quad (1)$$

$$z(k) = C_{\sigma(k)}x(k) + F_{\sigma(k)}\omega(k), \quad (2)$$

where $x(k) \in \mathbb{R}^{n_x}$ and $z(k) \in \mathbb{R}^{n_z}$ denote the system state and objective signal, respectively. $\omega(k) \in \mathbb{R}^{n_\omega}$ is the noise input, which belongs to $l_2[0,\infty)$; $\sigma(k)$ is a piecewise constant function of time, called a switching signal, which takes its values in a finite set $\underline{N} = \{1,2,\cdots,n_s\}$, $n_s > 1$ is the number of subsystems. For a switching signal $\sigma(k)$, let $k_1 < k_2 < \cdots < k_m < \cdots$ denote the switching instants of $\sigma(k)$. The switching sequence is defined as $\zeta = \{x(t_0);(i_0,k_0),(i_1,k_1),\cdots,(i_m,k_m),\cdots\}$. The i_m^{th} subsystem is active during the time interval $[k_m,k_{m+1})$. Besides, it is assumed that the switching signal $\sigma(k)$ is known prior to the controller design.

Now, some relevant definitions and lemma are recalled for the derivation of the main results and later discussions.

Definition 1 [39]**:** The equilibrium $x = 0$ of system (1) with $u = 0$ and $\omega = 0$ is GUES under switching signal $\sigma(k)$, if there exist constants $\gamma > 0$, $\lambda > 1$ such that the solution $x(k)$ of system (1) satisfies $\|x(k)\| \leq \gamma\lambda^{-(k-k_0)}\|x(k_0)\|$, $\forall k \geq k_0$.

Definition 2 [2]**:** For $\gamma > 0$, system (1)-(2) with $u = 0$ is said to be GUES with an l_2-gain, if under zero initial condition, it is GUES and the inequality $\sum_{s=k_0}^{\infty} z^T(s)z(s) \leq \sum_{s=k_0}^{\infty}\gamma^2\omega^T(s)\omega(s)$ holds for all nonzero $\omega(k) \in l_2[0,\infty)$.

Definition 3 [27]**:** For a switching signal σ and any interval $[k_1,k_2]$, let $N_{\sigma i}(k_1,k_2)$ be the switching numbers that the i^{th} subsystem is activated over the interval $[k_1,k_2]$, and $T_i(k_1,k_2)$ denote the total running time of the i^{th} subsystem over the interval $[k_1,k_2]$,$\forall i \in \underline{N}$. We say that σ has a mode-dependent average dwell time τ_{ai} if there exist positive numbers N_{0i} and τ_{ai} such that

$$N_{\sigma i}(k_1,k_2) \leq N_{0i} + \frac{T_i(k_1,k_2)}{\tau_{ai}}, \forall k_2 \geq k_1 \geq 0. \quad (3)$$

Definition 4 [33]**:** For a directed switching graph G and $i,j \in \underline{N}$ $(i \neq j)$, if a directed edge from i to j is admissible, then we call $S(i,j)$ as an ATE of G. The set of ATEs is denoted by $S(\underline{N})$. An ATE $S(i,j)$ has an admissible transition edge-dependent weight (ATEDW) $\beta_{i,j}$, which describes the switching property from i to j and the set of which is signified by W.

A directed graph of a switched system with three subsystems is shown in Fig. 1, where the set of ATEs is $S(\underline{N}) = \{S(1,2),S(1,3),S(2,1),S(2,3),S(3,1),S(3,2)\}$, and the set of ATEDWs is $W = \{\beta_{1,2},\beta_{1,3},\beta_{2,1},\beta_{2,3},\beta_{3,1},\beta_{3,2}\}$. In the following, the definition of AED-ADT is introduced on the basis of Definition 4.

Definition 5 [33]**:** For any $i,j \in \underline{N}$ $(i \neq j)$, $S(i,j) \in S(\underline{N})$, and a switching signal $\sigma(k)$, let $N_{i,j}^{\sigma}(k_0,k)$ be the switching count from i to j over the interval $[k_0,k)$, and $T_{i,j}(k_0,k)$ denote the total duration of subsystem j within the interval $[k_0,k)$, where i is the previously active subsystem, and $k \geq k_0 \geq 0$. We say that $\sigma(k)$ has an admissible edge-dependent average dwell time $\tau_{i,j}^a$ if there exist positive numbers $N_{i,j}^0$ and $\tau_{i,j}^a$ such that

$$N_{i,j}^{\sigma}(k_0,k) \leq N_{i,j}^0 + \frac{T_{i,j}(k_0,k)}{\tau_{i,j}^a}, \forall k \geq k_0 \geq 0, \quad (4)$$

where $N_{i,j}^0$ are called as the admissible edge-dependent chatter bounds.

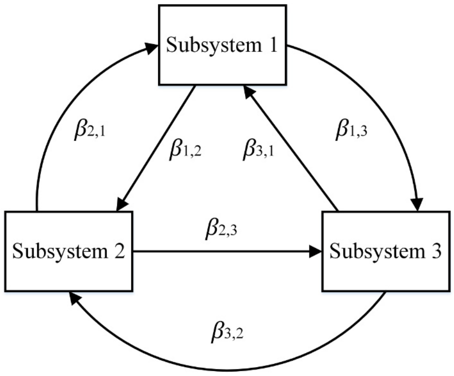

Fig. 1. A directed switching graph G with $\underline{N} = \{1,2,3\}$.

Lemma 1 [40]**:** Let $\xi \in \mathbb{R}^n, \mathcal{P} = \mathcal{P}^T \in \mathbb{R}^{n\times n}$, and $\mathcal{H} \in \mathbb{R}^{m\times n}$ such that $rank(\mathcal{H}) = r < n$, and then the following statements are equivalent:
(i) $\xi^T\mathcal{P}\xi < 0$, for all $\xi \neq 0, \mathcal{H}\xi = 0$;
(ii) $\exists \mathcal{X} \in \mathbb{R}^{n\times m}$ such that $\mathcal{P} + \mathcal{X}\mathcal{H} + \mathcal{H}^T\mathcal{X}^T < 0$.

The objective of this paper is to design an efficient H_∞ controller and find a set of AED-ADT switching signals such that the corresponding closed-loop system is GUES and has a guaranteed H_∞ disturbance attenuation performance, i.e.,, $\|z\|_2^2 \leq \gamma^2\|\omega\|_2^2$ for a constant $\gamma > 0$.

3. MAIN RESULTS

3.1. Time-varying H_∞ controller construction

In this subsection, a MPCLF is firstly designed for studying the H_∞ control of switched system (1)-(2) later. To begin, we divide the switching interval $[k_m, k_{m+1})$ with $\sigma(k_m) = i \in \underline{N}$ into q_i segments: $[k_m, k_{m+1}) = \bigcup_{j=0}^{q_i-1}[k_m + T_{ij}, k_m + T_{i(j+1)})$, where $T_{i0} = 0, k_m + T_{iq_i} = k_{m+1}$. The MPCLF is given as follows:

$$V_{ij}(k) = x^T(k)\sum\nolimits_{l=1}^{L} f_{ijl}(k-k_m)P_{ijl}x(k) \triangleq x^T(k)P_{ij}(k)x(k), \quad (5)$$

where $\forall k \in [k_m + T_{ij}, k_m + T_{i(j+1)})$, $P_{ijl} \in \mathbb{R}^{n\times n}$ are positive definite matrices, and positive integer L denotes the number of matrices P_{ijl}; nonlinear continuous functions $f_{ijl}(k-k_m)$ are defined on the segment $[k_m + T_{ij}, k_m + T_{i(j+1)})$, and satisfy

$$f_{ijl}(k-k_m) \geq 0, \ \sum\nolimits_{l=1}^{L} f_{ijl}(k-k_m) = 1. \quad (6)$$

In order to continue our work, a simple and effective construction method is proposed to construct the above functions $f_{ijl}(k-k_m)$. For any $i \in \underline{N}, j \in \{0,1,\cdots,q_i-1\}, l \in \mathcal{L} = \{1,2,\cdots,L\}$, we define

$$f_{ijl}(k-k_m) = a(k-k_m) + b, \quad (7)$$

where a and b are unknown constants to be determined immediately.

Set

$$f_{ijl}(T_{ij}) = a_{ijl}, \ \ f_{ijl}(T_{i(j+1)}) = b_{ijl}, \quad (8)$$

where $0 \leq a_{ijl} \leq 1, 0 \leq b_{ijl} \leq 1, \sum_{l=1}^{L} a_{ijl} = 1, \sum_{l=1}^{L} b_{ijl} = 1$. By integrating (7) and (8), we can obtain

$$a = \frac{b_{ijl} - a_{ijl}}{T_{i(j+1)} - T_{ij}}, b = \frac{a_{ijl}T_{i(j+1)} - b_{ijl}T_{ij}}{T_{i(j+1)} - T_{ij}}.$$

Thus, we have, $i \in \underline{N}, j \in \{0,1,\cdots,q_i-1\}, l \in \mathcal{L}$,

$$f_{ijl}(k-k_m) = \frac{b_{ijl} - a_{ijl}}{T_{i(j+1)} - T_{ij}}(k-k_m) + \frac{a_{ijl}T_{i(j+1)} - b_{ijl}T_{ij}}{T_{i(j+1)} - T_{ij}}, \quad (9)$$

and it can be checked that

$$f_{ijl}(k-k_m) \geq 0, \sum\nolimits_{l=1}^{L} f_{ijl}(k-k_m) = 1,$$
$$f_{ijl}(k+1-k_m) - f_{ijl}(k-k_m) = \frac{b_{ijl} - a_{ijl}}{T_{i(j+1)} - T_{ij}}. \quad (10)$$

Remark 1: Obviously, larger parameter L yields more degrees of freedom for the MPCLF. Nevertheless, it should also be pointed out that parameter L should not be too large since larger L will bring additional computational burden. Therefore, parameter L must be chosen carefully according to practical situations.

Based on the MPCLF in (5), we provide the following switched state feedback controller

$$u(k) = K_{\sigma(k)j}(k)x(k), \quad (11)$$
$$K_{ij}(k) = \sum\nolimits_{l=1}^{L} f_{ijl}(k-k_m)K_{ijl}, \quad (12)$$

where $K_{ijl}, i \in \underline{N}, j \in \{0,1,\cdots,q_i-1\}, l \in \mathcal{L}$ are controller parameters to be determined afterwards.

Under the controller (11), the corresponding closed-loop switched system becomes

$$x(k+1) = \bar{A}_{\sigma(k)}(k)x(k) + E_{\sigma(k)}\omega(k), \quad (13)$$
$$z(k) = C_{\sigma(k)}x(k) + F_{\sigma(k)}\omega(k), \quad (14)$$

where

$$\bar{A}_{\sigma(k)}(k) = A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)j}(k). \quad (15)$$

Remark 2: Based on the MPCLF, an H_∞ state feedback controller is designed. In this paper, the functions $f_{ijl}(k-k_m)$ of MPCLF are simply constructed as linear and quasi-time-dependent functions. Due to the particularity of functions $f_{ijl}(k-k_m)$, our controller is time-varying and has multiple degrees of freedom, which allows us to obtain more flexibility in designing controller parameters.

3.2. Stability and l_2-gain analysis

Next, the MPCLF (5) will be utilised to deduce the GUES conditions for system (1) with $u = 0$ and $\omega = 0$.

Theorem 1: Consider the switched system

$$x(k+1) = A_{\sigma(k)}x(k). \tag{16}$$

For given scalars $0 < \alpha_i < 1$, $0 < \rho_i \leq 1$, $i \in \underline{N}$, $\beta_{i_1,i_2} > 1$, with $\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2} > 1$, i_1, $i_2 \in \underline{N}$, $i_1 \neq i_2$, suppose that there exist positive definite matrices P_{ijl} and matrices M_{ijl}, $i \in \underline{N}$, $j \in \{0,1,\cdots,q_i-1\}$, $l \in \mathcal{L}$, such that

$$\begin{bmatrix} -\alpha_i P_{ijl} & * \\ M_{ijl}A_i & \Xi - M_{ijl} - M_{ijl}^T \end{bmatrix} < 0, \tag{17}$$

where $\Xi = P_{ijl} + \sum_{r=1}^{L} \frac{b_{ijr}-a_{ijr}}{T_{i(j+1)}-T_{ij}} P_{ijr}$, and the following inequalities can be satisfied

$$\textstyle\sum_{l=1}^{L} a_{ijl}P_{ijl} \leq \rho_i \sum_{l=1}^{L} b_{i(j-1)l}P_{i(j-1)l}, \tag{18}$$

$$\textstyle\sum_{l=1}^{L} a_{i_20l}P_{i_20l} \leq \beta_{i_1,i_2} \sum_{l=1}^{L} b_{i_1(q_{i_1}-1)l}P_{i_1(q_{i_1}-1)l}, \tag{19}$$

where (19) holds whenever the switching from i_1 to i_2 is admissible. Then the switched system (16) is GUES with AED-ADT $\tau^a_{i_1,i_2}$ satisfying

$$\tau^a_{i_1,i_2} > \frac{-\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}{\ln \alpha_{i_2}}, \tag{20}$$

where the parameter β_{i_1,i_2} is ATEDW with respect to ATE $S(i_1,i_2)$.

Proof: Design matrices $M_{ij}(k)$, where $M_{ij}(k) = \sum_{l=1}^{L} f_{ijl}(k-k_m)M_{ijl}$, $M_{ijl} \in \mathbb{R}^{n\times n}$. By (10) and (17), it is clear that

$$\begin{bmatrix} -\alpha_i P_{ij}(k) & * \\ M_{ij}(k)A_i & P_{ij}(k+1) - M_{ij}(k) - M_{ij}^T(k) \end{bmatrix} < 0. \tag{21}$$

From (5), we have $\forall k \in [k_m + T_{ij}, k_m + T_{i(j+1)})$,

$$\begin{aligned} V_{ij}(k+1) - \alpha_i V_{ij}(k) =& x^T(k+1)P_{ij}(k+1)x(k+1) \\ &- \alpha_i x^T(k)P_{ij}(k)x(k). \end{aligned} \tag{22}$$

On the basis of Lemma 1, (21) and (22), we obtain

$$V_{ij}(k+1) \leq \alpha_i V_{ij}(k). \tag{23}$$

At the interval interior points $k_m + T_{ij}$, $j \in \{1,2,\cdots,q_i-1\}$, inequality (18) yields

$$\textstyle\sum_{l=1}^{L} f_{ijl}(T_{ij})P_{ijl} \leq \rho_i \sum_{l=1}^{L} f_{i(j-1)l}(T_{ij})P_{i(j-1)l}. \tag{24}$$

It follows that

$$V_{ij}(k_m + T_{ij}) \leq \rho_i V_{i(j-1)}(k_m + T_{ij}). \tag{25}$$

At the switching points k_m, $m = 1,2,3,\cdots$, assume $\sigma(k_{m-1}) = i_1, \sigma(k_m) = i_2, i_1, i_2 \in \underline{N}$. Inequality (19) brings about

$$\sum_{l=1}^{L} f_{i_20l}(0)P_{i_20l} \leq \beta_{i_1,i_2} \sum_{l=1}^{L} f_{i_1(q_{i_1}-1)l}(T_{i_1q_{i_1}})P_{i_1(q_{i_1}-1)l}. \tag{26}$$

One can obtain

$$V_{i_20}(k_m) \leq \beta_{i_1,i_2} V_{i_1(q_{i_1}-1)}(k_m). \tag{27}$$

From (23), we get $\forall k \in [k_m + T_{ij}, k_m + T_{i(j+1)})$,

$$\begin{aligned} V_{\sigma(k)}(k) =& V_{\sigma(k)j}(k) \\ \leq& e^{\ln \alpha_{\sigma(k_m)}(k-k_m-T_{\sigma(k_m)j})} V_{\sigma(k_m)j}(k_m + T_{\sigma(k_m)j}). \end{aligned} \tag{28}$$

By integrating (25) with (28), it is directly obtained that

$$\begin{aligned} V_{\sigma(k)}(k) \leq& e^{\ln \alpha_{\sigma(k_m)}(k-k_m-T_{\sigma(k_m)j})} \rho_{\sigma(k_m)} \\ &\times V_{\sigma(k_m)(j-1)}(k_m + T_{\sigma(k_m)j}) \\ \leq& e^{\ln \alpha_{\sigma(k_m)}(k-k_m)} \rho^j_{\sigma(k_m)} V_{\sigma(k_m)0}(k_m). \end{aligned}$$

And then, according to (27), we get

$$\begin{aligned} V_{\sigma(k)}(k) \leq& e^{\ln \alpha_{\sigma(k_m)}(k-k_m)} \rho^j_{\sigma(k_m)} \beta_{\sigma(k_{m-1}),\sigma(k_m)} \\ &\times V_{\sigma(k_{m-1})(q_{\sigma(k_{m-1})}-1)}(k_m). \end{aligned}$$

Via similar steps, one can further obtain

$$\begin{aligned} V_{\sigma(k)}(k) \leq& e^{\ln \alpha_{\sigma(k_m)}(k-k_m)} \rho^j_{\sigma(k_m)} \beta_{\sigma(k_{m-1}),\sigma(k_m)} \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-1} \\ &\times e^{\ln \alpha_{\sigma(k_{m-1})}(k_m-k_{m-1})} V_{\sigma(k_{m-1})0}(k_{m-1}) \\ \leq& e^{\ln \alpha_{\sigma(k_m)}(k-k_m)} e^{\ln \alpha_{\sigma(k_{m-1})}(k_m-k_{m-1})} \rho^j_{\sigma(k_m)} \\ &\times \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-1} \beta_{\sigma(k_{m-1}),\sigma(k_m)} \beta_{\sigma(k_{m-2}),\sigma(k_{m-1})} \\ &\times V_{\sigma(k_{m-2})(q_{\sigma(k_{m-2})}-1)}(k_{m-1}) \\ \leq& \cdots \\ \leq& e^{\ln \alpha_{\sigma(k_m)}(k-k_m)} e^{\ln \alpha_{\sigma(k_{m-1})}(k_m-k_{m-1})} \cdots \\ &\times e^{\ln \alpha_{\sigma(k_1)}(k_2-k_1)} \rho^j_{\sigma(k_m)} \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-1} \cdots \rho_{\sigma(k_1)}^{q_{\sigma(k_1)}-1} \\ &\times \beta_{\sigma(k_{m-1}),\sigma(k_m)} \beta_{\sigma(k_{m-2}),\sigma(k_{m-1})} \cdots \beta_{\sigma(k_0),\sigma(k_1)} \\ &\times V_{\sigma(k_0)(q_{\sigma(k_0)}-1)}(k_1) \\ \leq& e^{\ln \alpha_{\sigma(k_m)}(k-k_m)} e^{\ln \alpha_{\sigma(k_{m-1})}(k_m-k_{m-1})} \cdots \\ &\times e^{\ln \alpha_{\sigma(k_0)}(k_1-k_0)} \rho^j_{\sigma(k_m)} \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-1} \cdots \rho_{\sigma(k_0)}^{q_{\sigma(k_0)}-1} \\ &\times \beta_{\sigma(k_{m-1}),\sigma(k_m)} \beta_{\sigma(k_{m-2}),\sigma(k_{m-1})} \cdots \beta_{\sigma(k_0),\sigma(k_1)} \\ &\times V_{\sigma(k_0)0}(k_0) \\ =& e^{\ln \alpha_{\sigma(k_m)}(k-k_m)} \rho_{\sigma(k_m)}^{j+1-q_{\sigma(k_m)}} \rho_{\sigma(k_0)}^{q_{\sigma(k_0)}-1} \\ &\times \prod_{r=0}^{m-1} (\rho_{\sigma(k_{r+1})}^{q_{\sigma(k_{r+1})}-1} \beta_{\sigma(k_r),\sigma(k_{r+1})}) \\ &\times \prod_{r=0}^{m-1} e^{\ln \alpha_{\sigma(k_r)}(k_{r+1}-k_r)} V_{\sigma(k_0)0}(k_0). \end{aligned} \tag{29}$$

By Definition 5, one gets that

$$\begin{aligned}&\prod_{r=0}^{m-1}(\rho_{\sigma(k_{r+1})}^{q_{\sigma(k_{r+1})}-1}\beta_{\sigma(k_r),\sigma(k_{r+1})})\\&=e^{\sum_{r=0}^{m-1}\ln(\rho_{\sigma(k_{r+1})}^{q_{\sigma(k_{r+1})}-1}\beta_{\sigma(k_r),\sigma(k_{r+1})})}\\&=e^{\sum_{i_2\in\underline{N}}\sum_{r=0}^{m-1}\sum_{i_1\in\underline{N},i_1\neq i_2}\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\\&=e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}N_{i_1,i_2}^{\sigma}(k_0,k)\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})},\end{aligned}\tag{30}$$

$$\begin{aligned}&e^{\ln\alpha_{\sigma(k_m)}(k-k_m)}\prod_{r=0}^{m-1}e^{\ln\alpha_{\sigma(k_r)}(k_{r+1}-k_r)}\\&=e^{\ln\alpha_{\sigma(k_m)}(k-k_m)+\sum_{r=0}^{m-1}\ln\alpha_{\sigma(k_r)}(k_{r+1}-k_r)}\\&=e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}\ln\alpha_{i_2}T_{i_1,i_2}(k_0,k)}.\end{aligned}\tag{31}$$

Substituting (30) and (31) into (29), one can obtain

$$\begin{aligned}V_{\sigma(k)}(k)\le{}&\rho_{\sigma(k_m)}^{j+1-q_{\sigma(k_m)}}\rho_{\sigma(k_0)}^{q_{\sigma(k_0)}-1}e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}\ln\alpha_{i_2}T_{i_1,i_2}(k_0,k)}\\&\times e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}(N_{i_1,i_2}^{0}+\frac{T_{i_1,i_2}(k_0,k)}{\tau_{i_1,i_2}^{a}})\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\\&\times V_{\sigma(k_0)0}(k_0)\\={}&e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}N_{i_1,i_2}^{0}\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\\&\times e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}(\ln\alpha_{i_2}+\frac{\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}{\tau_{i_1,i_2}^{a}})T_{i_1,i_2}(k_0,k)}\\&\times\rho_{\sigma(k_m)}^{j+1-q_{\sigma(k_m)}}\rho_{\sigma(k_0)}^{q_{\sigma(k_0)}-1}V_{\sigma(k_0)0}(k_0).\end{aligned}$$

If the constant τ_{i_1,i_2}^{a} satisfies (20), we have $\ln\alpha_{i_2}+\frac{\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}{\tau_{i_1,i_2}^{a}}<0$. Then, one can obtain

$$\begin{aligned}V_{\sigma(k)}(k)\le{}&\max_{i\in\underline{N}}\{\rho_i^{1-q_i}\}e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}N_{i_1,i_2}^{0}\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\\&\times e^{\max_{i_1,i_2\in\underline{N},i_1\neq i_2}\{\ln\alpha_{i_2}+\frac{\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}{\tau_{i_1,i_2}^{a}}\}(k-k_0)}\\&\times V_{\sigma(k_0)0}(k_0).\end{aligned}$$

Thus, the switched system (16) is GUES. □

Based on Theorem 1, the sufficient conditions will be derived, which guarantee that the resulting closed-loop system is GUES with an H_∞ disturbance attenuation performance.

Theorem 2: Consider the switched system

$$x(k+1)=A_{\sigma(k)}x(k)+E_{\sigma(k)}\omega(k),\tag{32}$$

$$z(k)=C_{\sigma(k)}x(k)+F_{\sigma(k)}\omega(k).\tag{33}$$

For given scalars $0<\alpha_i<1, 0<\rho_i\le 1, i\in\underline{N}, \beta_{i_1,i_2}>1$, with $\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2}>1, i_1,i_2\in\underline{N}, i_1\neq i_2$, suppose that there exist positive definite matrices P_{ijl} and matrices M_{ijl}, $i\in\underline{N}, j\in\{0,1,\cdots,q_i-1\}, l\in\mathcal{L}$, such that

$$\begin{bmatrix}-\alpha_iP_{ijl}&*&*&*\\C_i&-I&*&*\\M_{ijl}A_i&0&\Xi-M_{ijl}-M_{ijl}^T&*\\0&F_i^T&E_i^TM_{ijl}^T&-\gamma^2I\end{bmatrix}<0,\tag{34}$$

where $\Xi=P_{ijl}+\sum_{r=1}^{L}\frac{b_{ijr}-a_{ijr}}{T_{i(j+1)}-T_{ij}}P_{ijr}$, and inequalities (18) and (19) hold. Then system (32)-(33) is GUES with a guaranteed H_∞ performance index γ for any switching signal satisfying (20).

Proof: Firstly, we consider the GUES problem of system (32) with $\omega=0$. Multiplying both sides of the inequality (34) by

$$\begin{bmatrix}I&0&0&0\\0&0&I&0\\0&I&0&0\\0&0&0&I\end{bmatrix},$$

we can rewrite (34) as

$$\begin{bmatrix}-\alpha_iP_{ijl}&*&*&*\\M_{ijl}A_i&\Xi-M_{ijl}-M_{ijl}^T&*&*\\C_i&0&-I&*\\0&E_i^TM_{ijl}^T&F_i^T&-\gamma^2I\end{bmatrix}<0.\tag{35}$$

From (35), we have immediately

$$\begin{bmatrix}-\alpha_iP_{ijl}&*\\M_{ijl}A_i&\Xi-M_{ijl}-M_{ijl}^T\end{bmatrix}<0.\tag{36}$$

Considering (18) and (19), we have system (32) with $\omega=0$ is GUES according to Theorem 1.

Next, we will prove that the prescribed l_2-gain of system (32)-(33) can be ensured for all nonzero ω. By (10) and (34), we obtain

$$\begin{bmatrix}-\alpha_iP_{ij}(k)&*&*&*\\C_i&-I&*&*\\M_{ij}(k)A_i&0&\Pi&*\\0&F_i^T&E_i^TM_{ij}^T(k)&-\gamma^2I\end{bmatrix}<0,\tag{37}$$

where $\Pi=P_{ij}(k+1)-M_{ij}(k)-M_{ij}^T(k)$, $M_{ij}(k)=\sum_{l=1}^{L}f_{ijl}(k-k_m)M_{ijl}$, $M_{ijl}\in\mathbb{R}^{n\times n}$. The inequality (37) can be rewritten as follows:

$$\mathcal{P}_{ij}(k)+\mathcal{X}_{ij}(k)\mathcal{H}_i+\mathcal{H}_i^T\mathcal{X}_{ij}^T(k)<0,\tag{38}$$

where

$$\mathcal{P}_{ij}(k)=\begin{bmatrix}-\alpha_iP_{ij}(k)&0&0&0\\0&I&0&0\\0&0&P_{ij}(k+1)&0\\0&0&0&-\gamma^2I\end{bmatrix},$$

$$\mathcal{X}_{ij}(k)=\begin{bmatrix}0&0&M_{ij}^T(k)&0\\0&I&0&0\end{bmatrix}^T,$$

$$\mathcal{H}_i=\begin{bmatrix}A_i&0&-I&E_i\\C_i&-I&0&F_i\end{bmatrix}.\tag{39}$$

Define the augmented signal ξ as

$$\xi=\begin{bmatrix}x^T(k)&z^T(k)&x^T(k+1)&\omega^T(k)\end{bmatrix}^T,\tag{40}$$

and then, system (32)-(33) can be rewritten in the form of

$$\mathcal{H}_i\xi = 0. \tag{41}$$

By Lemma 1 and (38), there holds

$$\xi^T \mathcal{P}_{ij}(k)\xi < 0. \tag{42}$$

Substituting (39) and (40) into (42), and assuming $\Gamma(k) = \gamma^2\omega^T(k)\omega(k) - z^T(k)z(k)$, we have

$$\Delta V_{ij}(k) < (\alpha_i - 1)V_{ij}(k) + \Gamma(k), \tag{43}$$

the above inequality implies that

$$V_{ij}(k) \le \alpha_i^{k-k_0} V_{ij}(k_0) + \textstyle\sum_{s=k_0}^{k-1} \alpha_i^{k-1-s}\Gamma(s). \tag{44}$$

Therefore, from (44), (18) and (19), one gets

$$\begin{aligned}
V_{\sigma(k)}(k) &= V_{\sigma(k)j}(k) \\
&\le e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)j}}} V_{\sigma(k_m)j}(k_m + T_{\sigma(k_m)j}) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
&\le e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)j}}} \rho_{\sigma(k_m)} V_{\sigma(k_m)(j-1)}(k_m + T_{\sigma(k_m)j}) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
&\le e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)j}}} \rho_{\sigma(k_m)} \Bigg(e^{\ln \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)j}-k_m-T_{\sigma(k_m)(j-1)}}} \\
&\quad \times V_{\sigma(k_m)(j-1)}(k_m + T_{\sigma(k_m)(j-1)}) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)(j-1)}}^{k_m+T_{\sigma(k_m)j}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)j}-1-s}\Gamma(s) \Bigg) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
&= e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)(j-1)}}} V_{\sigma(k_m)(j-1)}(k_m + T_{\sigma(k_m)(j-1)}) \\
&\quad \times \rho_{\sigma(k_m)} + e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)j}}} \rho_{\sigma(k_m)} \\
&\quad \times \sum_{s=k_m+T_{\sigma(k_m)(j-1)}}^{k_m+T_{\sigma(k_m)j}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)j}-1-s}\Gamma(s) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
&\le e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)(j-2)}}} V_{\sigma(k_m)(j-2)}(k_m + T_{\sigma(k_m)(j-2)}) \\
&\quad \times \rho_{\sigma(k_m)}^2 + e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)(j-1)}}} \rho_{\sigma(k_m)}^2 \\
&\quad \times \sum_{s=k_m+T_{\sigma(k_m)(j-2)}}^{k_m+T_{\sigma(k_m)(j-1)}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)(j-1)}-1-s}\Gamma(s) \\
&\quad + e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)j}}} \rho_{\sigma(k_m)} \\
&\quad \times \sum_{s=k_m+T_{\sigma(k_m)(j-1)}}^{k_m+T_{\sigma(k_m)j}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)j}-1-s}\Gamma(s) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
&\le \cdots \\
&\le e^{\ln \alpha_{\sigma(k_m)}^{k-k_m}} \rho_{\sigma(k_m)}^j V_{\sigma(k_m)0}(k_m) + e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)1}}} \\
&\quad \times \rho_{\sigma(k_m)}^j \sum_{s=k_m}^{k_m+T_{\sigma(k_m)1}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)1}-1-s}\Gamma(s) \\
&\quad + \cdots + e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)j}}} \rho_{\sigma(k_m)} \\
&\quad \times \sum_{s=k_m+T_{\sigma(k_m)(j-1)}}^{k_m+T_{\sigma(k_m)j}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)j}-1-s}\Gamma(s) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
&= e^{\ln \alpha_{\sigma(k_m)}^{k-k_m}} \rho_{\sigma(k_m)}^j V_{\sigma(k_m)0}(k_m) \\
&\quad + \sum_{t=1}^{j} e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)t}}} \rho_{\sigma(k_m)}^{j+1-t} \\
&\quad \times \sum_{s=k_m+T_{\sigma(k_m)(t-1)}}^{k_m+T_{\sigma(k_m)t}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)t}-1-s}\Gamma(s) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
&\le e^{\ln \alpha_{\sigma(k_m)}^{k-k_m}} \beta_{\sigma(k_{m-1}),\sigma(k_m)} V_{\sigma(k_{m-1})(q_{\sigma(k_{m-1})}-1)}(k_m) \\
&\quad \times \rho_{\sigma(k_m)}^j + \sum_{t=1}^{j} e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)t}}} \rho_{\sigma(k_m)}^{j+1-t} \\
&\quad \times \sum_{s=k_m+T_{\sigma(k_m)(t-1)}}^{k_m+T_{\sigma(k_m)t}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)t}-1-s}\Gamma(s) \\
&\quad + \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
&\le e^{\ln \alpha_{\sigma(k_m)}^{k-k_m}} \rho_{\sigma(k_m)}^j \beta_{\sigma(k_{m-1}),\sigma(k_m)} \Bigg(e^{\ln \alpha_{\sigma(k_{m-1})}^{k_m-k_{m-1}}} \\
&\quad \times \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-1} V_{\sigma(k_{m-1})0}(k_{m-1}) + \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-1} \\
&\quad \times \sum_{s=k_{m-1}}^{k_{m-1}+T_{\sigma(k_{m-1})1}-1} \alpha_{\sigma(k_{m-1})}^{k_{m-1}+T_{\sigma(k_{m-1})1}-1-s}\Gamma(s) \\
&\quad \times e^{\ln \alpha_{\sigma(k_{m-1})}^{k_m-k_{m-1}-T_{\sigma(k_{m-1})1}}} + \cdots \\
&\quad + \sum_{s=k_{m-1}+T_{\sigma(k_{m-1})(q_{\sigma(k_{m-1})}-1)}}^{k_m-1} \alpha_{\sigma(k_{m-1})}^{k_m-1-s}\Gamma(s) \Bigg) \\
&\quad + \sum_{t=1}^{j} e^{\ln \alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)t}}} \rho_{\sigma(k_m)}^{j+1-t}
\end{aligned}$$

$$
\begin{aligned}
&\times \sum_{s=k_m+T_{\sigma(k_m)(t-1)}}^{k_m+T_{\sigma(k_m)t}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)t}-1-s}\Gamma(s) \\
&+ \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
=\,& e^{\ln\alpha_{\sigma(k_m)}^{k-k_m}} e^{\ln\alpha_{\sigma(k_{m-1})}^{k_m-k_{m-1}}} \rho_{\sigma(k_m)}^{j}\rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-1} \\
&\times \beta_{\sigma(k_{m-1}),\sigma(k_m)} V_{\sigma(k_{m-1})0}(k_{m-1}) \\
&+ e^{\ln\alpha_{\sigma(k_m)}^{k-k_m}} \rho_{\sigma(k_m)}^{j}\beta_{\sigma(k_{m-1}),\sigma(k_m)} \\
&\times \sum_{j=1}^{q_{\sigma(k_{m-1})}} e^{\ln\alpha_{\sigma(k_{m-1})}^{k_m-k_{m-1}-T_{\sigma(k_{m-1})j}}} \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-j} \\
&\times \sum_{s=k_{m-1}+T_{\sigma(k_{m-1})(j-1)}}^{k_{m-1}+T_{\sigma(k_{m-1})j}-1} \alpha_{\sigma(k_{m-1})}^{k_{m-1}+T_{\sigma(k_{m-1})j}-1-s}\Gamma(s) \\
&+ \sum_{t=1}^{j} e^{\ln\alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)t}}} \rho_{\sigma(k_m)}^{j+1-t} \\
&\times \sum_{s=k_m+T_{\sigma(k_m)(t-1)}}^{k_m+T_{\sigma(k_m)t}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)t}-1-s}\Gamma(s) \\
&+ \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
\le\,& \cdots \\
\le\,& e^{\ln\alpha_{\sigma(k_m)}^{k-k_m}} \cdots e^{\ln\alpha_{\sigma(k_0)}^{k_1-k_0}} \rho_{\sigma(k_m)}^{j}\cdots\rho_{\sigma(k_0)}^{q_{\sigma(k_0)}-1} \\
&\times \beta_{\sigma(k_{m-1}),\sigma(k_m)}\cdots\beta_{\sigma(k_0),\sigma(k_1)} V_{\sigma(k_0)0}(k_0) \\
&+ e^{\ln\alpha_{\sigma(k_m)}^{k-k_m}} \cdots e^{\ln\alpha_{\sigma(k_1)}^{k_2-k_1}} \rho_{\sigma(k_m)}^{j}\cdots\rho_{\sigma(k_1)}^{q_{\sigma(k_1)}-1} \\
&\times \beta_{\sigma(k_{m-1}),\sigma(k_m)}\cdots\beta_{\sigma(k_0),\sigma(k_1)} \\
&\times \sum_{j=1}^{q_{\sigma(k_0)}} e^{\ln\alpha_{\sigma(k_0)}^{k_1-k_0-T_{\sigma(k_0)j}}} \rho_{\sigma(k_0)}^{q_{\sigma(k_0)}-j} \\
&\times \sum_{s=k_0+T_{\sigma(k_0)(j-1)}}^{k_0+T_{\sigma(k_0)j}-1} \alpha_{\sigma(k_0)}^{k_0+T_{\sigma(k_0)j}-1-s}\Gamma(s)+\cdots \\
&+ e^{\ln\alpha_{\sigma(k_m)}^{k-k_m}} \rho_{\sigma(k_m)}^{j}\beta_{\sigma(k_{m-1}),\sigma(k_m)} \\
&\times \sum_{j=1}^{q_{\sigma(k_{m-1})}} e^{\ln\alpha_{\sigma(k_{m-1})}^{k_m-k_{m-1}-T_{\sigma(k_{m-1})j}}} \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-j} \\
&\times \sum_{s=k_{m-1}+T_{\sigma(k_{m-1})(j-1)}}^{k_{m-1}+T_{\sigma(k_{m-1})j}-1} \alpha_{\sigma(k_{m-1})}^{k_{m-1}+T_{\sigma(k_{m-1})j}-1-s}\Gamma(s) \\
&+ \sum_{t=1}^{j} e^{\ln\alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)t}}} \rho_{\sigma(k_m)}^{j+1-t} \\
&\times \sum_{s=k_m+T_{\sigma(k_m)(t-1)}}^{k_m+T_{\sigma(k_m)t}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)t}-1-s}\Gamma(s) \\
&+ \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s)
\end{aligned}
$$

$$
\begin{aligned}
=\,& e^{\ln\alpha_{\sigma(k_m)}^{k-k_m}} \rho_{\sigma(k_m)}^{j+1-q_{\sigma(k_m)}} \rho_{\sigma(k_0)}^{q_{\sigma(k_0)}-1}\prod_{r=0}^{m-1} e^{\ln\alpha_{\sigma(k_r)}^{k_{r+1}-k_r}} \\
&\times \rho_{\sigma(k_{r+1})}^{q_{\sigma(k_{r+1})}-1}\beta_{\sigma(k_r),\sigma(k_{r+1})}V_{\sigma(k_0)0}(k_0) \\
&+ e^{\ln\alpha_{\sigma(k_m)}^{k-k_m}} \rho_{\sigma(k_m)}^{j}\beta_{\sigma(k_{m-1}),\sigma(k_m)} \\
&\times \sum_{h=1}^{m-1}\left(\prod_{p=h}^{m-1} e^{\ln\alpha_{\sigma(k_p)}^{k_{p+1}-k_p}}\rho_{\sigma(k_p)}^{q_{\sigma(k_p)}-1}\beta_{\sigma(k_{p-1}),\sigma(k_p)}\right. \\
&\times \sum_{j=1}^{q_{\sigma(k_{h-1})}} e^{\ln\alpha_{\sigma(k_{h-1})}^{k_h-k_{h-1}-T_{\sigma(k_{h-1})j}}}\rho_{\sigma(k_{h-1})}^{q_{\sigma(k_{h-1})}-j} \\
&\times \left.\sum_{s=k_{h-1}+T_{\sigma(k_{h-1})(j-1)}}^{k_{h-1}+T_{\sigma(k_{h-1})j}-1}\alpha_{\sigma(k_{h-1})}^{k_{h-1}+T_{\sigma(k_{h-1})j}-1-s}\Gamma(s)\right) \\
&+ e^{\ln\alpha_{\sigma(k_m)}^{k-k_m}} \rho_{\sigma(k_m)}^{j}\beta_{\sigma(k_{m-1}),\sigma(k_m)} \\
&\times \sum_{j=1}^{q_{\sigma(k_{m-1})}} e^{\ln\alpha_{\sigma(k_{m-1})}^{k_m-k_{m-1}-T_{\sigma(k_{m-1})j}}} \rho_{\sigma(k_{m-1})}^{q_{\sigma(k_{m-1})}-j} \\
&\times \sum_{s=k_{m-1}+T_{\sigma(k_{m-1})(j-1)}}^{k_{m-1}+T_{\sigma(k_{m-1})j}-1} \alpha_{\sigma(k_{m-1})}^{k_{m-1}+T_{\sigma(k_{m-1})j}-1-s}\Gamma(s) \\
&+ \sum_{t=1}^{j} e^{\ln\alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)t}}} \rho_{\sigma(k_m)}^{j+1-t} \\
&\times \sum_{s=k_m+T_{\sigma(k_m)(t-1)}}^{k_m+T_{\sigma(k_m)t}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)t}-1-s}\Gamma(s) \\
&+ \sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1} \alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s) \\
\le\,& \max_{i\in\underline{N}}\{\rho_i^{1-q_i}\} e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2} N_{i_1,i_2}^0 \ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})} \\
&\times e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}\left(\ln\alpha_{i_2}+\frac{\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}{\tau_{i_1,i_2}^a}\right)T_{i_1,i_2}(k_0,k)} \\
&\times V_{\sigma(k_0)0}(k_0)+\max_{i\in\underline{N}}\{\rho_i^{1-q_i}\} \\
&\times \sum_{h=1}^{m}\left(e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2} N_{i_1,i_2}^0 \ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\right. \\
&\times e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}\left(\ln\alpha_{i_2}+\frac{\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}{\tau_{i_1,i_2}^a}\right)T_{i_1,i_2}(k_h,k)} \\
&\times \sum_{j=1}^{q_{\sigma(k_{h-1})}} e^{\ln\alpha_{\sigma(k_{h-1})}^{k_h-k_{h-1}-T_{\sigma(k_{h-1})j}}}\rho_{\sigma(k_{h-1})}^{q_{\sigma(k_{h-1})}-j} \\
&\times \left.\sum_{s=k_{h-1}+T_{\sigma(k_{h-1})(j-1)}}^{k_{h-1}+T_{\sigma(k_{h-1})j}-1}\alpha_{\sigma(k_{h-1})}^{k_{h-1}+T_{\sigma(k_{h-1})j}-1-s}\Gamma(s)\right) \\
&+ \sum_{t=1}^{j} e^{\ln\alpha_{\sigma(k_m)}^{k-k_m-T_{\sigma(k_m)t}}} \rho_{\sigma(k_m)}^{j+1-t} \\
&\times \sum_{s=k_m+T_{\sigma(k_m)(t-1)}}^{k_m+T_{\sigma(k_m)t}-1} \alpha_{\sigma(k_m)}^{k_m+T_{\sigma(k_m)t}-1-s}\Gamma(s)
\end{aligned}
$$

$$+\sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1}\alpha_{\sigma(k_m)}^{k-1-s}\Gamma(s). \quad (45)$$

For $i_1,\ i_2\in\underline{N},\ i_1\neq i_2$, from $0<\alpha_i<1,\ 0<\rho_i\leq 1$, $\beta_{i_1,i_2}>1$ with $\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2}>1$, and inequality (20), we have

$$\max_{i\in\underline{N}}\{\rho_i^{1-q_i}\}\geq 1, e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}N_{i_1,i_2}^0\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\geq 1,$$

$$0<e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}\left(\ln\alpha_{i_2}+\frac{\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}{\tau_{i_1,i_2}^a}\right)T_{i_1,i_2}(k_h,k)}\leq 1.$$

Hence, under zero initial condition, the above inequality gives

$$\begin{aligned}V_{\sigma(k)}(k)&\leq\max_{i\in\underline{N}}\{\rho_i^{1-q_i}\}e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}N_{i_1,i_2}^0\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\sum_{h=1}^{m}\\&\quad\sum_{j=1}^{q_{\sigma(k_{h-1})}}\sum_{s=k_{h-1}+T_{\sigma(k_{h-1})(j-1)}}^{k_{h-1}+T_{\sigma(k_{h-1})j}-1}\Gamma(s)+\sum_{t=1}^{j}\sum_{s=k_m+T_{\sigma(k_m)(t-1)}}^{k_m+T_{\sigma(k_m)t}-1}\\&\quad\times\Gamma(s)+\sum_{s=k_m+T_{\sigma(k_m)j}}^{k-1}\Gamma(s)\\&=\max_{i\in\underline{N}}\{\rho_i^{1-q_i}\}e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}N_{i_1,i_2}^0\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\\&\quad\times\sum_{s=k_0}^{k_m-1}\Gamma(s)+\sum_{s=k_m}^{k-1}\Gamma(s)\\&\leq\max_{i\in\underline{N}}\{\rho_i^{1-q_i}\}e^{\sum_{i_2\in\underline{N}}\sum_{i_1\in\underline{N},i_1\neq i_2}N_{i_1,i_2}^0\ln(\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2})}\\&\quad\times\sum_{s=k_0}^{k-1}\Gamma(s).\end{aligned}\quad (46)$$

Due to $V_{\sigma(k)}(k)\geq 0$, we can obtain

$$\sum_{s=k_0}^{k-1}\Gamma(s)\geq 0, \quad (47)$$

i.e.,

$$\sum_{s=k_0}^{\infty}z^T(s)z(s)\leq\sum_{s=k_0}^{\infty}\gamma^2\omega^T(s)\omega(s). \quad (48)$$

Thus, the system (32)-(33) has an l_2-gain γ and the proof is completed. □

In Theorem 2, if the MPCLF is replaced by the MLF, we can obtain the following result.

Corollary 1: Consider the switched system (32)-(33). For given scalars $0<\alpha_i<1, i\in\underline{N}, \beta_{i_1,i_2}>1, i_1,i_2\in\underline{N}, i_1\neq i_2$, suppose that there exist positive definite matrices P_i and matrices $M_i,\ i\in\underline{N}$, such that

$$\begin{bmatrix}-\alpha_iP_i & * & * & *\\ C_i & -I & * & *\\ M_iA_i & 0 & P_i-M_i-M_i^T & *\\ 0 & F_i^T & E_i^TM_i^T & -\gamma^2I\end{bmatrix}<0, \quad (49)$$

$$P_{i_2}\leq\beta_{i_1,i_2}P_{i_1}, \quad (50)$$

where (50) holds whenever the switching from i_1 to i_2 is admissible. Then system (32)-(33) is GUES with a guaranteed H_∞ performance index γ for any switching signal satisfying

$$\tau_{i_1,i_2}^a>\frac{-\ln\beta_{i_1,i_2}}{\ln\alpha_{i_2}}. \quad (51)$$

3.3. H_∞ controller design

Now, we are in a position to deal with the design of H_∞ controller for switched system (1)-(2).

Theorem 3: Consider the switched system (1)-(2). For given scalars $0<\alpha_i<1, 0<\rho_i\leq 1, i\in\underline{N}, \beta_{i_1,i_2}>1$, with $\rho_{i_2}^{q_{i_2}-1}\beta_{i_1,i_2}>1, i_1,i_2\in\underline{N}, i_1\neq i_2$, suppose that there exist positive definite matrices N_{ijl}, matrices $Y_{ijl}, i\in\underline{N}, j\in\{0,1,\cdots,q_i-1\}, l\in\mathcal{L}$, and symmetric invertible matrix X, such that

$$\begin{bmatrix}-\alpha_iN_{ijl} & * & * & *\\ C_iX & -I & * & *\\ A_iX+B_iY_{ijl} & 0 & \Theta-2X & *\\ 0 & F_i^T & E_i^T & -\gamma^2I\end{bmatrix}<0, \quad (52)$$

where $\Theta=N_{ijl}+\sum_{r=1}^{L}\frac{b_{ijr}-a_{ijr}}{T_{i(j+1)}-T_{ij}}N_{ijr}$, and the following inequalities can be satisfied

$$\sum_{l=1}^{L}a_{ijl}N_{ijl}\leq\rho_i\sum_{l=1}^{L}b_{i(j-1)l}N_{i(j-1)l}, \quad (53)$$
$$\sum_{l=1}^{L}a_{i_20l}N_{i_20l}\leq\beta_{i_1,i_2}\sum_{l=1}^{L}b_{i_1(q_{i_1}-1)l}N_{i_1(q_{i_1}-1)l}, \quad (54)$$

where (54) holds whenever the switching from i_1 to i_2 is admissible. Then there exists a time-varying state feedback controller in the form of (11) such that the system (13)-(14) is GUES with a guaranteed H_∞ performance index γ for any switching signal satisfying (20). Moreover, a suitable controller realization is given as follows:

$$K_{ij}(k)=\sum_{l=1}^{L}f_{ijl}(k-k_m)K_{ijl},\ \ K_{ijl}=Y_{ijl}X^{-1}. \quad (55)$$

Proof: According to (10) and (52), we get

$$\begin{bmatrix}-\alpha_iN_{ij}(k) & * & * & *\\ C_iX & -I & * & *\\ A_iX+B_iY_{ij}(k) & 0 & N_{ij}(k+1)-2X & *\\ 0 & F_i^T & E_i^T & -\gamma^2I\end{bmatrix}<0. \quad (56)$$

Denote $N_{ij}(k)=X^TP_{ij}(k)X,\ Y_{ij}(k)=K_{ij}(k)X$, and $X=M^{-1}$, it is clear that

$$\begin{bmatrix}-\alpha_iM^{-1}P_{ij}(k)M^{-1} & * & * & *\\ C_iM^{-1} & -I & * & *\\ A_iM^{-1}+B_iK_{ij}(k)M^{-1} & 0 & \Phi-2M^{-1} & *\\ 0 & F_i^T & E_i^T & -\gamma^2I\end{bmatrix}<0, \quad (57)$$

where $\Phi = M^{-1}P_{ij}(k+1)M^{-1}$. Multiplying both sides of (57) by diag$\{M,I,M,I\}$, we have

$$\begin{bmatrix} -\alpha_i P_{ij}(k) & * & * & * \\ C_i & -I & * & * \\ M\bar{A}_i(k) & 0 & P_{ij}(k+1)-2M & * \\ 0 & F_i^T & E_i^T M & -\gamma^2 I \end{bmatrix} < 0, \tag{58}$$

which can ensure (37). Besides, inequalities (53) and (54) guarantee inequalities (18) and (19), respectively. Hence, the proof is completed. □

From Theorem 3, the following Corollary 2 can be directly obtained under the MDADT switching.

Corollary 2: Consider the switched system (1)-(2). For given scalars $0 < \alpha_i < 1, 0 < \rho_i \leq 1, \beta_i > 1$, with $\rho_i^{q_i-1}\beta_i > 1, i \in \underline{N}$, suppose that there exist positive definite matrices N_{ijl}, matrices $Y_{ijl}, i \in \underline{N}, j \in \{0,1,\cdots,q_i-1\}, l \in \mathcal{L}$, and symmetric invertible matrix X, such that

$$\begin{bmatrix} -\alpha_i N_{ijl} & * & * & * \\ C_i X & -I & * & * \\ A_i X + B_i Y_{ijl} & 0 & \Theta - 2X & * \\ 0 & F_i^T & E_i^T & -\gamma^2 I \end{bmatrix} < 0, \tag{59}$$

where $\Theta = N_{ijl} + \sum_{r=1}^{L} \frac{b_{ijr}-a_{ijr}}{T_{i(j+1)}-T_{ij}} N_{ijr}$, and the following inequalities can be satisfied

$$\sum_{l=1}^{L} a_{ijl} N_{ijl} \leq \rho_i \sum_{l=1}^{L} b_{i(j-1)l} N_{i(j-1)l}, \tag{60}$$

$$\sum_{l=1}^{L} a_{i_2 0 l} N_{i_2 0 l} \leq \beta_{i_2} \sum_{l=1}^{L} b_{i_1(q_{i_1}-1)l} N_{i_1(q_{i_1}-1)l}, \tag{61}$$

where (61) holds whenever the switching from i_1 to i_2 is admissible. Then there exists a state feedback controller in the form of (11) such that the system (13)-(14) is GUES with a guaranteed H_∞ performance index γ for any switching signal satisfying

$$\tau_{ai} > \frac{-\ln(\rho_i^{q_i-1}\beta_i)}{\ln \alpha_i}, \quad \forall i \in \underline{N}. \tag{62}$$

Moreover, a suitable controller realization is given by (55).

Remark 3: It is apparent that our H_∞ controller design method is based on the known switching signals. Once the switching signals are unknown, our method can not be applied. In the future work, we will consider the prediction algorithms of switching signals, or develop new controller design methods with no need for switching information.

4. NUMERICAL EXAMPLES

Now, we provide three examples to show the effectiveness of the main results in this paper.

Example 1: Consider switched system (16) composed of three subsystems:

$$A_1 = \begin{bmatrix} 0.89 & 0.48 \\ 0 & 0.48 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.35 & 0.15 \\ 0.79 & 0.46 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.18 & 0.5 \\ 0.61 & 0.15 \end{bmatrix}.$$

For the MPCLF, we choose parameters:

$$\begin{aligned} & q_1 = q_2 = q_3 = 2, \quad L = 2, \\ & a_{101} = a_{102} = a_{111} = a_{112} = a_{201} = a_{202} = a_{211} \\ & \quad = a_{212} = a_{301} = a_{302} = a_{311} = a_{312} = 0.5, \\ & b_{101} = b_{111} = b_{201} = b_{211} = b_{301} = b_{311} = 0.4, \\ & b_{102} = b_{112} = b_{202} = b_{212} = b_{302} = b_{312} = 0.6. \end{aligned}$$

The directed switching graph of the above switched system is shown in Fig. 1. In order to compare the results under MPCLF with the ones under MLF in [33], the relevant parameters and the corresponding results for Theorem 2 in [33] and our Theorem 1 are listed in Table 1. It can be derived from Table 1 that the AED-ADTs obtained by our Theorem 1 are smaller than the ones obtained by Theorem 2 in [33]. This is because the MPCLF is piecewise continuous during the dwell time on an activated system mode so that the restrictions of Lyapunov function at switching points and interval interior points can be relaxed. As a result, tighter bounds on AED-ADT can be achieved.

Example 2: Next, we compare the minimum H_∞ performance index γ_{min} feasible for Theorem 2 and Corollary 1. Consider switched system (32)-(33) including two subsystems:

$$A_1 = \begin{bmatrix} -0.1 & 0.5 \\ 0 & -0.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.4 & 0.3 \\ 0.5 & -0.7 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.2 & 0.2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.1 & 0.15 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.15 \\ 0.6 \end{bmatrix},$$
$$F_1 = 0.4, \quad F_2 = -0.3.$$

The parameters of MPCLF are the same as those given in Example 1. For the usage of Theorem 2, set $\rho_1 = \rho_2 = 0.9$. The corresponding comparison results are shown in Table 2, from which we can see that the γ_{min} can be selected to be smaller via Theorem 2 (MPCLF) than that by Corollary 1 (MLF). It is obvious that MPCLF outperforms MLF. The MPCLF helps to achieve a better disturbance attenuation performance and reduces the l_2-gains to lower levels.

Example 3: Through the comparison of MDADT switching and AED-ADT switching, we demonstrate the superiority of AED-ADT switching. Consider the switched system given by (1) and (2), where

$$A_1 = \begin{bmatrix} -0.4 & 0.5 \\ 0.8 & -2.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.4 & 1.3 \\ 0.5 & -2.7 \end{bmatrix},$$

Table 1. Comparison results under two different Lyapunov function approaches.

Criteria	Theorem 2 in [33]	Theorem 1 in this paper
Parameters	$\beta_{2,1}$=2.1, $\beta_{3,1}$=2.3, $\beta_{1,2}$=2.2 $\beta_{3,2}$=2.4, $\beta_{1,3}=2.5$, $\beta_{2,3}=2.2$ α_1=0.8, α_2=0.78, α_3=0.76	$\beta_{2,1}$=2.1, $\beta_{3,1}$=2.3, $\beta_{1,2}$=2.2 $\beta_{3,2}$=2.4, $\beta_{1,3}=2.5$, $\beta_{2,3}=2.2$ α_1=0.8, α_2=0.78, α_3=0.76 ρ_1=0.62, ρ_2=0.6, ρ_3=0.6
Switching signal	$\tau^{a*}_{2,1}$=3.3249, $\tau^{a*}_{3,1}$=3.7326 $\tau^{a*}_{1,2}$=3.1734, $\tau^{a*}_{3,2}$=3.5236 $\tau^{a*}_{1,3}$=3.3388, $\tau^{a*}_{2,3}$=2.8730	$\tau^{a*}_{2,1}$=1.1827, $\tau^{a*}_{3,1}$=1.5903 $\tau^{a*}_{1,2}$=1.1174, $\tau^{a*}_{3,2}$=1.4676 $\tau^{a*}_{1,3}$=1.4774, $\tau^{a*}_{2,3}$=1.0116
Positive definite matrices	$P_1 = \begin{bmatrix} 25.1486 & 24.8655 \\ 24.8655 & 49.5807 \end{bmatrix}$ $P_2 = \begin{bmatrix} 37.6171 & 15.9693 \\ 15.9693 & 24.2868 \end{bmatrix}$ $P_3 = \begin{bmatrix} 28.8311 & 13.1975 \\ 13.1975 & 30.7291 \end{bmatrix}$	$P_{101} = \begin{bmatrix} 18.0290 & 15.6095 \\ 15.6095 & 18.1244 \end{bmatrix}$ $P_{102} = \begin{bmatrix} 5.5205 & -1.0457 \\ -1.0457 & 2.8735 \end{bmatrix}$ $P_{111} = \begin{bmatrix} 10.3641 & 8.4025 \\ 8.4025 & 10.2779 \end{bmatrix}$ $P_{112} = \begin{bmatrix} 2.6600 & -1.4377 \\ -1.4377 & 0.8336 \end{bmatrix}$ Due to the limit of the space, the rest of P_{ijl} are omitted.

Table 2. Comparison results of minimum H_∞ performance index γ_{min}.

Parameters	α_1=0.9, $\alpha_2=0.9$ $\beta_{2,1}$=2, $\beta_{1,2}$=3	α_1=0.8, $\alpha_2=0.7$ $\beta_{2,1}$=2, $\beta_{1,2}$=3	α_1=0.8, $\alpha_2=0.9$ $\beta_{2,1}$=3, $\beta_{1,2}$=2.5
Theorem 2	0.8370	1.8197	0.8363
Corollary 1	0.9711	6.4046	0.9711
Parameters	α_1=0.6, $\alpha_2=0.8$ $\beta_{2,1}$=3, $\beta_{1,2}$=2.5	α_1=0.85, $\alpha_2=0.76$ $\beta_{2,1}$=3.4, $\beta_{1,2}$=4	α_1=0.75, $\alpha_2=0.88$ $\beta_{2,1}$=3.4, $\beta_{1,2}$=4
Theorem 2	1.0589	1.2293	0.8680
Corollary 1	1.4836	2.0328	1.0333

$$A_3 = \begin{bmatrix} 0.2 & -1 \\ 1.5 & -1.4 \end{bmatrix}, \ B_1 = \begin{bmatrix} -2.2 & 1.5 \\ 1 & 1.8 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 3.1 & 2 \\ 1 & 2.4 \end{bmatrix}, \ B_3 = \begin{bmatrix} 1.5 & 0.8 \\ 2 & 2.6 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \ E_2 = \begin{bmatrix} 0.35 \\ 0.15 \end{bmatrix}, \ E_3 = \begin{bmatrix} 0.15 \\ 0.6 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.2 & 0.2 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0 & 0.15 \end{bmatrix},$$
$$C_3 = \begin{bmatrix} -0.1 & 0.15 \end{bmatrix}, \ F_1 = 0.4, \ F_2 = F_3 = -0.3.$$

The MPCLF parameters are the same as in Example 1.

The corresponding results under MDADT switching and AED-ADT switching with $\gamma = 0.43$ are shown in Table 3, from which we can conclude that the H_∞ performance index $\gamma = 0.43$ obtained by MDADT $\tau^*_{ai}, i \in \underline{N}$ can also be guaranteed by selecting a smaller AED-ADT $\tau^{a*}_{i_1,i_2}$, $i_1, i_2 \in \underline{N}, i_1 \neq i_2$.

The controller parameters Λ_1 and Λ_2 obtained by Theorem 3 and Corollary 2, respectively, are listed as follows:

Λ_1:

$$K_{101} = \begin{bmatrix} -0.1993 & 0.6519 \\ 0.3290 & 0.2044 \end{bmatrix},$$
$$K_{102} = \begin{bmatrix} -0.2000 & 0.6518 \\ 0.3337 & 0.2050 \end{bmatrix},$$
$$K_{111} = \begin{bmatrix} -0.2021 & 0.6517 \\ 0.3146 & 0.2183 \end{bmatrix},$$
$$K_{112} = \begin{bmatrix} -0.2027 & 0.6520 \\ 0.3034 & 0.2223 \end{bmatrix},$$
$$K_{201} = \begin{bmatrix} 0.8153 & -1.5232 \\ -0.6224 & 1.8122 \end{bmatrix},$$
$$K_{202} = \begin{bmatrix} 0.8096 & -1.5193 \\ -0.5960 & 1.7897 \end{bmatrix},$$
$$K_{211} = \begin{bmatrix} 0.8115 & -1.5228 \\ -0.6032 & 1.8109 \end{bmatrix},$$
$$K_{212} = \begin{bmatrix} 0.8115 & -1.5231 \\ -0.6107 & 1.8162 \end{bmatrix},$$
$$K_{301} = \begin{bmatrix} 0.0994 & 0.6209 \\ -0.8274 & 0.1695 \end{bmatrix},$$

Table 3. Comparison results under MDADT and AED-ADT with $\gamma = 0.43$.

Switching schemes	MDADT	AED-ADT
Criteria	Corollary 2	Theorem 3
Parameters	β_1=2.2, β_2=3, β_3=2.5 α_1=0.8, $\alpha_2 = 0.7$, $\alpha_3 = 0.8$ ρ_1=0.68, ρ_2=0.9, ρ_3=0.7	$\beta_{2,1}$=1.9, $\beta_{3,1}$=2.2, $\beta_{1,2}$=3 $\beta_{3,2}$=2.1, $\beta_{1,3} = 2$, $\beta_{2,3} = 2.5$ α_1=0.8, α_2=0.7, α_3=0.8 ρ_1=0.68, ρ_2=0.9, ρ_3=0.7
Switching signal	τ_{a1}^*=1.8051 τ_{a2}^*=2.7848 τ_{a3}^*=2.5079	$\tau_{2,1}^{a*}$=1.1481, $\tau_{3,1}^{a*}$=1.8051 $\tau_{1,2}^{a*}$=2.7848, $\tau_{3,2}^{a*}$=1.7848 $\tau_{1,3}^{a*}$=1.5079, $\tau_{2,3}^{a*}$=2.5079

$$K_{302} = \begin{bmatrix} 0.1078 & 0.6242 \\ -0.8282 & 0.1685 \end{bmatrix},$$
$$K_{311} = \begin{bmatrix} 0.0797 & 0.6596 \\ -0.8126 & 0.1569 \end{bmatrix},$$
$$K_{312} = \begin{bmatrix} 0.0855 & 0.6352 \\ -0.8200 & 0.1654 \end{bmatrix}.$$

Λ_2:

$$K_{101} = \begin{bmatrix} -0.1959 & 0.6489 \\ 0.3270 & 0.2000 \end{bmatrix},$$
$$K_{102} = \begin{bmatrix} -0.1964 & 0.6488 \\ 0.3303 & 0.2008 \end{bmatrix},$$
$$K_{111} = \begin{bmatrix} -0.1990 & 0.6487 \\ 0.3149 & 0.2140 \end{bmatrix},$$
$$K_{112} = \begin{bmatrix} -0.1991 & 0.6487 \\ 0.3036 & 0.2182 \end{bmatrix},$$
$$K_{201} = \begin{bmatrix} 0.8071 & -1.5185 \\ -0.5910 & 1.7854 \end{bmatrix},$$
$$K_{202} = \begin{bmatrix} 0.8062 & -1.5174 \\ -0.5818 & 1.7767 \end{bmatrix},$$
$$K_{211} = \begin{bmatrix} 0.8072 & -1.5196 \\ -0.5858 & 1.7948 \end{bmatrix},$$
$$K_{212} = \begin{bmatrix} 0.8079 & -1.5202 \\ -0.5924 & 1.7993 \end{bmatrix},$$
$$K_{301} = \begin{bmatrix} 0.0785 & 0.6362 \\ -0.8252 & 0.1654 \end{bmatrix},$$
$$K_{302} = \begin{bmatrix} 0.0844 & 0.6374 \\ -0.8259 & 0.1650 \end{bmatrix},$$
$$K_{311} = \begin{bmatrix} 0.0637 & 0.6693 \\ -0.8118 & 0.1545 \end{bmatrix},$$
$$K_{312} = \begin{bmatrix} 0.0595 & 0.6617 \\ -0.8175 & 0.1580 \end{bmatrix}.$$

Set the initial value $x(0) = \begin{bmatrix} 3 & -1 \end{bmatrix}^T$, and the periodic switching path $1 \to 3 \to 2 \to 1 \to 3 \to 2 \cdots$. Based on the solutions of Table 3, Fig. 2 and Fig. 4 with $\tau_{a1} = 2, \tau_{a3} = 3, \tau_{a2} = 3$ are given to show the corresponding

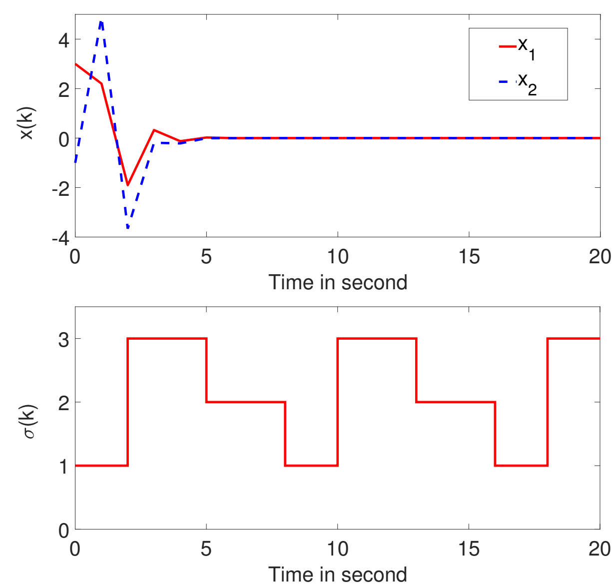

Fig. 2. State response $x(k)$ and switching signal $\sigma(k)$ under MDADT switching.

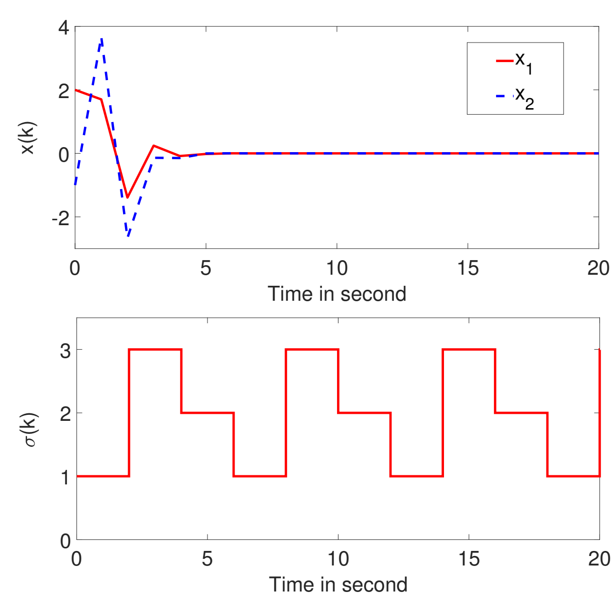

Fig. 3. State response $x(k)$ and switching signal $\sigma(k)$ under AED-ADT switching.

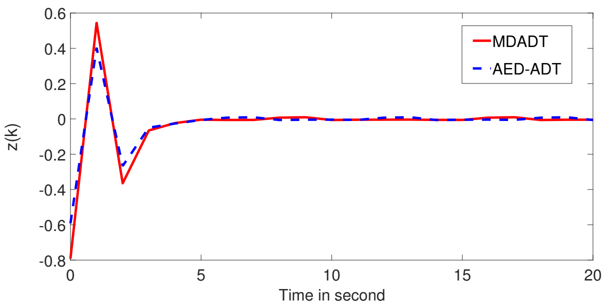

Fig. 4. Controlled output response $z(k)$ under MDADT and AED-ADT switching.

state response $x(k)$, switching signal $\sigma(k)$ and controlled output response $z(k)$ under the MDADT switching. From Fig. 2 and Fig. 4, we can get that the switched system is stable under MDADT switching.

Meanwhile, choose initial value $x(0) = \begin{bmatrix} 2 & -1 \end{bmatrix}^T$ and periodic switching path $1 \to 3 \to 2 \to 1 \to 3 \to 2 \cdots$. Under the AED-ADT switching with $\tau^a_{1,3} = 2, \tau^a_{3,2} = 2, \tau^a_{2,1} = 2$, the corresponding state response $x(k)$, switching signal $\sigma(k)$ and controlled output response $z(k)$ are displayed in Fig. 3 and Fig. 4, respectively, which illustrate that the switched system under the AED-ADT switching also has a good performance even if the dwell time is smaller than the MDADT switching. Hence, we can summarize that AED-ADT switching provides better flexibility than MDADT switching, and can further relax the constraints of MDADT switching.

5. CONCLUSIONS

This paper concerns the problem of H_∞ control for discrete-time switched systems. By the aid of the MPCLF approach combined with AED-ADT switching, a time-varying H_∞ state feedback controller has been designed such that the corresponding closed-loop system is GUES with a guaranteed H_∞ performance. Eventually, three numerical examples have also been given to illustrate the effectiveness of the developed results.

REFERENCES

[1] Z. D. Sun and S. S. Ge, *Stability Theory of Switched Dynamical Systems*, Springer, London, 2011.

[2] L. X. Zhang, N. G. Cui, M. Liu, and Y. Zhao, "Asynchronous filtering of discrete-time switched linear systems with average dwell time," *IEEE Trans. on Circuits Systems I: Regular Papers*, vol. 58, no. 5, pp. 1109-1118, May 2011.

[3] X. M. Liu, K. J. Zhang, S. T. Li, S. M. Fei, and H. K. Wei, "Time optimisation problem for switched stochastic systems with multi-switching times," *IET Control Theory and Applications*, vol. 8, no. 16, pp. 1732-1740, November 2014.

[4] G. D. Zong, R. H. Wang, and W. X. Zheng, "Finite-time H_∞ control for discrete-time switched nonlinear systems with time delay," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 6, pp. 914-936, April 2015.

[5] X. M. Liu, S. T. Li, and K. J. Zhang, "Optimal control of switching time in switched stochastic systems with multi-switching times and different costs," *International Journal of Control*, vol. 90, no. 8, pp. 1604-1611, August 2017.

[6] W. H. Qi, J. H. Park, J. Cheng, Y. G. Kao, and X. W. Gao, "Anti-windup design for stochastic Markovian switching systems with mode-dependent time-varying delays and saturation nonlinearity," *Nonlinear Analysis: Hybrid Systems*, vol. 26, pp. 201-211, November 2017.

[7] H. Gao, J. W. Xia, and G. M. Zhuang, "Robust finite-time extended dissipative control for a class of uncertain switched delay systems," *International Journal of Control, Automation and Systems*, vol. 16, no. 3, pp. 1459-1468, June 2018.

[8] Y. H. Liu, "Sliding mode control for a class of uncertain discrete switched systems," *International Journal of Control, Automation and Systems*, vol. 16, no. 4, pp. 1716-1723, August 2018. 10.1007/s12555-017-0468-5

[9] Z. Y. Jiang and P. Yan, "Asynchronous switching control for continuous-time switched linear systems with output-feedback," *International Journal of Control, Automation and Systems*, vol. 16, no. 5, pp. 2082-2092, October 2018.

[10] J. Cheng, J. H. Park, J. D. Cao, and D. Zhang, "Quantized H_∞ filtering for switched linear parameter-varying systems with sojourn probabilities and unreliable communication channels," *Information Sciences*, vol. 466, pp. 289-302, October 2018.

[11] J. Cheng, J. H. Park, H. R. Karimi, and H. Shen, "A flexible terminal approach to sampled-data exponentially synchronization of Markovian neural networks with time-varying delayed signals," *IEEE Trans. on Cybernetics*, vol. 48, no. 8, pp. 2232-2244, August 2018.

[12] J. Cheng, C. K. Ahn, H. R. Karimi, J. D. Cao, and W. H. Qi, "An event-based asynchronous approach to markov jump systems with hidden mode detections and missing measurements," *IEEE Trans. on Systems, Man, and Cybernetics: Systems*, pp. 1-10, September 2018. DOI: 10.1109/TSMC.2018.2866906

[13] Z. D. Sun and S. S. Ge, *Switched Linear Systems: Control and Design*, Springer, Berlin, 2004.

[14] F. Guerin, D. Lefebvre, and S. B. Mboup, "Hybrid modeling for performance evaluation of multisource renewable energy systems," *IEEE Trans. Automation Science and Engineering*, vol. 8, no. 3, pp. 532-539, July 2011.

[15] H. Torikai and T. Saito, "Synchronization of chaos and its itinerancy from a network by occasional linear connection," *IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications*, vol. 45, no. 4, pp. 464-472, April 1998.

[16] J. X. Liu, S. Vazquez, L. G. Wu, A. Marquez, and H. J. Gao, "Extended state observer-based sliding-mode control for three-phase power converters," *IEEE Trans. on Industrial Electronics*, vol. 64, no. 1, pp. 22-31, January 2017.

[17] D. Liberzon, *Switching in Systems and Control*, Springer Science and Business Media, 2012.

[18] M. S. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems," *IEEE Trans. on Automatic Control*, vol. 43, no. 4, pp. 475-782, April 1998.

[19] S. Yuan, L. Zhang, and B. D. Schutter, "A novel Lyapunov function for a non-weighted L_2 gain of asynchronously switched linear systems," *Automatica*, vol. 87, pp. 310-317, January 2018.

[20] X. D. Zhao, P. Shi, Y. F. Yin, and S. K. Nguang, "New results on stability of slowly switched systems: a multiple discontinuous Lyapunov function approach," *IEEE Trans. on Automatic Control*, vol. 57, no. 7, pp. 1809-1815, July 2017.

[21] L. J. Li, L. Liu, and Y. F. Yin, "Stability analysis for discrete-time switched nonlinear system under MDADT switching," *IEEE Access*, vol. 5, pp. 18646-18653, September 2017.

[22] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Systems Magazine*, vol. 19, no. 5, pp. 59-70, November 1999.

[23] B. Niu, D. Wang, N. D. Alotaibi, and F. E. Alsaadi, "Adaptive neural state-feedback tracking control of stochastic nonlinear switched systems: an average dwell-time method," *IEEE Trans. on Neural Networks and Learning Systems*, vol. 30, no. 4, pp. 1076-1087, April 2019.

[24] B. Niu, H. R. Karimi, H. Wang, and Y. Liu, "Adaptive output-feedback controller design for switched nonlinear stochastic systems with a modified average dwell-time method," *IEEE Trans. on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 7, pp. 1371-1382, July 2017.

[25] W. H. Qi, J. H. Park, J. Cheng, Y. G. Kao, and X. W. Gao, "Exponential stability and L_1-gain analysis for positive time-delay Markovian jump systems with switching transition rates subject to average dwell time," *Information Sciences*, vol. 424, pp. 224-234, January 2018.

[26] A. S. Morse, "Supervisory control of families of linear set-point controllers. Part I: exact matching," *IEEE Trans. on Automatic Control*, vol. 41, no. 10, pp. 1413-1431, April 1996.

[27] X. D. Zhao, L. X. Zhang, P. Shi, and M. Liu, "Stability and stabilization of switched linear systems with mode-dependent average dwell time," *IEEE Trans. on Automatic Control*, vol. 57, no. 7, pp. 1809-1815, July 2012.

[28] R. H. Wang and S. M. Fei, "New stability and stabilization results for discrete-time switched systems," *Applied Mathematics and Computation*, vol. 238, no. 1, pp. 358-369, July 2014.

[29] X. D. Zhao, Y. F. Yin, and L. X. Zhang, "Control of switched nonlinear systems via T-S fuzzy modeling," *IEEE Trans. on Fuzzy Systems*, vol. 24, no. 1, pp. 235-241, February 2016.

[30] H. Zheng, G. H. Sun, Y. Ren, and C. C. Tian, "Quasi-time-dependent H_∞ controller for discrete-time switched linear systems with mode-dependent average dwell-time," *Asian Journal of Control*, vol. 20, no. 1, pp. 263-275, January 2018.

[31] R. H. Wang, L. L. Hou, G. D. Zong, S. M. Fei, and D. Yang, "Stability and stabilization of continuous-time switched systems: a multiple discontinuous convex Lyapunov function approach," *International Journal of Robust and Nonlinear Control*, vol. 20, no. 5, pp. 1499-1514, December 2018.

[32] J. Q. Yang, X. D. Zhao, X. H. Bu, and W. Qian, "Stabilization of switched linear systems via admissible edge-dependent switching signals," *Nonlinear Analysis: Hybrid Systems*, vol. 29, pp. 100-109, August 2018.

[33] L. L. Hou, X. D. Zhao, H. B. Sun, and G. D. Zong, "l_2-l_∞ filtering of discrete-time switched systems via admissible edge-dependent switching signals," *Systems and Control Letters*, vol. 113, pp. 17-26, March 2018.

[34] L. X. Zhang and P. Shi, "Stability, l_2-gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time," *IEEE Trans. on Automatic Control*, vol. 54, no. 9, pp. 2193-2200, September 2009.

[35] Y. E. Wang, X. M. Sun, and J. Zhao, "Asynchronous H_∞ control of switched delay systems with average dwell time," *Journal of the Franklin Institute*, vol. 349, no. 10, pp. 3159-3169, December 2012.

[36] H. Liu, Y. Shen, and X. D. Zhao, "Asynchronous finite-time H_∞ control for switched linear systems via mode-dependent dynamic state-feedback," *Nonlinear Analysis: Hybrid Systems*, vol. 8, pp. 109-120, May 2013.

[37] H. Liu and X. D. Zhao, "Finite-time H_∞ control of switched systems with mode-dependent average dwell time," *Journal of the Franklin Institute*, vol. 351, no. 3, pp. 1301-1315, March 2014.

[38] S. Shi, Z. Y. Fei, and J. C. Li, "Finite-time H_∞ control of switched systems with mode-dependent average dwell time," *Journal of the Franklin Institute*, vol. 353, no. 1, pp. 221-234, January 2016.

[39] G. Zhai, B. Hu, K. Yasuda, and A. N. Michel, "Qualitative analysis of discrete-time switched systems," *Proc. of the American Control Conference*, pp. 1880-1885, 2002.

[40] D. W. Ding and G. H. Yang, "H_∞ static output feedback control for discrete-time switched linear systems with average dwell time," *Proc. of the American Control Conference*, pp. 2356-2361, 2009.

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