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Event-triggered Coordination Control for Multi-agent Systems with Connectivity Preservation

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Abstract: This work investigates the connectivity preservation problem of multi-agent systems with event-triggered controllers. The agents in the system have only limited communication ranges, and they are required to achieve rendezvous while preserving the connectivity of the communication graph. To reduce the amount of communication packages, event-triggering mechanism is employed. We propose two kinds of event triggers to realize the connectivity-preserving rendezvous of the multi-agent system, i.e., the connectivity trigger to preserve the network connectivity, and the convergence trigger to drive the agents to achieve rendezvous. By introducing a particular constraint function in the controller design, the control inputs of the agents can be bounded throughout the rendezvous process. This guarantees that the controller can be physically implemented in practice. It is proven that the agent group will achieve rendezvous while all the existing communication links can be preserved under some very mild assumptions on the controller design. Moreover, Zeno behavior can be avoided by using an event/time hybrid triggering approach. The effectiveness of the proposed event-triggered control is illustrated by simulations.

Keywords: Connectivity preserving, event-triggered control, multi-agent systems, Zeno behavior

1. INTRODUCTION

In recent years, multi-agent control systems have been extensively studied due to its significance in various industrial and military applications. In this research area, typical control tasks include multi-agent consensus [1–3], formation control [4], swarming and flocking [5], cooperative tracking [6], coverage and deployment [7, 8], and so on. In all of these coordination control tasks, information exchanging and sharing among agents rely on the communication network, and the accomplishment of the control tasks depends on certain connectivity assumptions of this network. Therefore, preserving network connectivity is critical in achieving all coordination objectives for multi-agent systems [9–11].

The communication and controller actuation schemes are two of the key factors for controller implementation in a multi-agent system. Since many practical systems use digital platforms, traditional periodic sampling techniques are often employed. However, in practical multi-agent systems such as mobile robots and unmanned aerial vehicles, an agent may have only limited onboard energy resources and low-level communication and actuating capabilities. When the periodical sampling scheme is used, the fixed sampling frequency should be chosen as high as possible such that even in the worst case the control performance can be guaranteed. However, a control plant does not always operate at its worst working condition. Consequently, using periodical sampling may waste much energy and shorten the lifespan of the system. To solve this problem, the event-triggered control approach has been utilized in multi-agent systems with the above mentioned limitations. Recently, event-triggered control has been studied in the control of single-loop systems [12] and networked systems [13,14]. In an event-triggered control system, state sampling and controller updating actions will happen when an event occurs. Typically an event is triggered when the realtime state contains sufficient innovation, e.g., the error between the sampled state and the realtime state exceeds a given threshold, and thus such control strategy can be considered as a nonuniform sampling control strategy. Since the energy consumption may be significantly reduced by event-triggered control, this control technique is quite suitable for multi-agent systems, especially for agents with limited resources and capabilities. Using such technique, each agent determines its sam-

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pling time instants, called as event time instants, using prescribed event functions defined based on the neighbors' states. The control input will be held invariant during the uneven sampling time intervals and the inter-agent communication may happen only at the event time instants. By virtue of this, the number of controller update and the communication transmission can be significantly reduced, which will save plenty of energy and lengthen the lifespan of the system.

The event-triggered control approach has been utilized in multi-agent consensus problems and some important results have been obtained [15], including decentralized event-triggered control [16, 17], distributed event-triggered consensus [18–21], self-triggered deployment and robust control [22, 23], and event-triggered coordination with high-order dynamics [24, 25] and sampled-data setup [26, 27]. Since connectivity of the communication network is crucial in achieving coordination tasks for many multi-agent systems, connectivity control using event-triggered scheme is an important problem to be investigated. However, in the open literature there are still very few works studying this problem. On the other hand, if the event-triggered scheme is involved, the multi-agent system may exhibit Zeno behaviors since both continuous dynamics and discrete transitions are involved. If any one of the agents exhibits Zeno behavior by using the event-triggered control, the controller cannot be implemented in practice since a Zeno execution implies that there exists infinitely many transitions in a finite time period, which is not allowed for a real physical system [28]. Thus, excluding the Zeno behavior is a critical requirement for designing and implementation of event-triggered controllers, especially for distributed multi-agent systems [29]. Moreover, for many connectivity control works, potential function based control approach is used. When the distance between agents is close to the communication range, the control input needs to be very large to preserve the connectivity, which is impractical in real applications.

Motivated by these observations, the connectivity preservation task for multi-agent systems using event-triggered controllers is investigated in this work. Two major problems will be solved in this work: driving the agents to rendezvous while preserving connectivity, and excluding the Zeno behavior for each agent. To solve the first problem, two different triggers, i.e., the connectivity trigger and the convergence trigger, are developed to guarantee the group rendezvous while each agent has only a limited communication range. Moreover, by introducing a constraint function in the controller design, the control input can be bounded to facilitate the implementation in practice. We show that to achieve rendezvous while preserving network connectivity, only some very mild assumptions are required for the controller design. To solve the second problem, we provide analysis on the inter-event time generated by the connectivity and convergence triggers and then propose an event/time hybrid trigger to avoid Zeno behavior for each agent. By using the new design, the inter-event time will be lower bounded by a strictly positive time period and thus the proposed controller can be implemented practically. Moreover, since the assumptions are mild, the controller design is quite simple.

The contribution and the novelty of this work lies in the following aspects. Firstly, this work considers the connectivity preserving problem using event-triggered control with bounded inputs. To the best of our knowledge, this has not been considered in the open literature so far. The connectivity event trigger is designed for preserving connectivity while the convergence event trigger is designed for rendezvous. The control input is enforced to be bounded using a function such that when the distance between two agents tends to the communication range, the control input will vanish to provide constraint on the magnitude of control inputs. Secondly, we propose a new design for the event generation such that an event/time hybrid triggering approach is proposed to exclude the Zeno behavior for each agent. Moreover, the proposed hybrid trigger can guarantee asymptotic convergence, while in some existing works such as [24, 30] only convergence to a bounded ball can be guaranteed.

The rest of this paper is organized as follows. Section 2 presents the problem formulation, the controller design, and the event design. Section 3 presents the connectivity and rendezvous analysis of the agent group. In Section 4, the analysis on inter-event time is provided and then new conditions on the event design are proposed to avoid the Zeno behavior. The effectiveness of the proposed approaches are illustrated by numerical simulations in Section 5. Finally, the paper is concluded in Section 6.

2. CONTROLLER DESIGN

In this section the connectivity-preserving rendezvous controller will be presented. Two different event triggers are designed for the purposes of achieving rendezvous and preserving existing communication links.

2.1. Control input

Consider a group of N agents labeled by $i = 1, 2, \dots, N$ in the $\mathbb{R}^n$ space. The position state of agent i at time t is denoted by $x_i(t)$. Each agent has a limited communication range $r > 0$. When the distance between two agents i and j is shorter than r, i.e.,

$$d_{ij}(t) = \|x_i(t) - x_j(t)\| < r, \tag{1}$$

there is a communication link between these two agents and we call them communication neighbors. It should be noted that the distance doesn't need to be strictly shorter than r in this work. The communication neighbor set of agent i is defined as $N_i(t) = \{j | d_{ij}(t) < r\}$, which consists of all the neighbors of agent i at time t. The communication topology can be captured by an undirected graph $G(t) = (V, E(t))$, where V is the vertex set representing the agents and $E(t)$ is the edge set representing the communication links. The dynamics of the agents are

$$\dot{x}_i(t) = u_i(t), \;\; i = 1, \ldots, N. \tag{2}$$

The event-triggered scheme is adopted in agent control and each agent i has an event time sequence denoted by

$$t_0^i = 0, \;\; t_1^i, \;\; \ldots, \;\; t_k^i, \;\; \ldots, \;\; k \in \mathbb{N}_{\geq 0}, \tag{3}$$

where $\mathbb{N}_{\geq 0}$ represents the set of all nonnegative integers. At each event time instant t_k^i, agent i may perform the actions such as communicating with neighbors and updating the control input. We require that the control input of agent i is updated only at t_k^i to reduce the efforts on computation and actuation. Then the control input of agent i has the form

$$u_i(t) = u_i(t_k^i), \;\; t \in [t_k^i, t_{k+1}^i). \tag{4}$$

To preserve the existing links, the constraint function approach is used. Define a function $\psi_r(\lambda) : [0, +\infty) \mapsto [0, 1]$ for each pair of neighboring agents, satisfying the following two assumptions:

Assumption 1: $\psi_r(\lambda)$ is continuous and non-increasing on $[0, +\infty)$, and

Assumption 2: $\psi_r(\lambda)$ is nonnegative and

$$\begin{aligned} &0 < \psi_r(\lambda) \leq 1, \quad \text{if } \lambda \in [0, r), \\ &\psi_r(\lambda) = 0, \qquad\;\; \text{if } \lambda \in [r, +\infty). \end{aligned} \tag{5}$$

Based on $\psi_r(\lambda)$ we define a constraint function for agent i as

$$\Psi_i(t) = \prod_{j \in N_i(t)} \psi_r(d_{ij}(t)). \tag{6}$$

From Assumptions 1 and 2, one notices that if the distance between agent i and any of its neighbor j approaches r, $\Psi_i(t)$ will tends to 0.

Define the relative average position of all the neighbors of agent i as

$$q_i(t) = \frac{1}{n_i(t) + 1} \sum_{j \in N_i(t)} (x_j(t) - x_i(t)), \tag{7}$$

with $n_i(t)$ being the number of neighbors of agent i at time t. To achieve rendezvous of the agent group, each agent will be driven towards this position by its controller. Then we propose the event-triggered connectivity-preserving rendezvous control law as

$$u_i(t) = \xi_i \Psi_i(t_k^i) q_i(t_k^i), \;\; t \in [t_k^i, t_{k+1}^i) \tag{8}$$

with $\xi_i \in [\xi_{min}, \xi_{max}]$ being the feedback gain. Here the feedback gain is only required to be strictly positive. For example, one may set $\xi_{\min} = 0.1$ and $\xi_{\max} = 2$ for simplicity, and such choice will not affect the following results. The closed-loop dynamics of the agents are given by

$$\begin{aligned} \dot{x}_i(t) = &\frac{\xi_i}{n_i(t) + 1} \prod_{j \in N_i(t)} \psi_r(\|x_i(t_k^i) - x_j(t_k^i)\|) \\ &\times \sum_{j \in N_i(t)} (x_j(t_k^i) - x_i(t_k^i)), t \in [t_k^i, t_{k+1}^i) \end{aligned} \tag{9}$$

for all $i = 1, \ldots, N$.

Remark 1: The proposed controller will generally achieve two objectives: preserving existing communication links and driving agents to rendezvous. For the closed-loop dynamics (9), if the product term is removed, it becomes a conventional consensus protocol. Thus the sum term is for the purpose of driving the agents to rendezvous. The product term will be used to preserve existing communication links. From Assumptions 1 and 2, one can notice that the value of the function $\psi_r(d_{ij}(t))$ vanishes to 0 when the distance $d_{ij}(t)$ tends to r. Therefore, if the distance from agent i to any neighbor is close to the communication range r such that this link tends to be lost, the value of the product term will be very small, which results in a small magnitude for the control input. Then under the control law (8), agent i will perform tiny and precise movement to preserve this link. When all the links between agent i and its neighbors have no danger to be lost, the product term will generate a considerable large control input to drive agent i to rendezvous.

Remark 2: It is noted that many existing connectivity-preserving controllers may fail to preserve connectivity under the state sampling and holding scheme since unbounded control inputs are required if the distance between two agents is close to the communication range r. Thus it is challenging to use an event-triggered controller to preserve connectivity. In this work, we propose the following multiple trigger approach to solving this problem. Explicitly, we introduce both the connectivity event and the convergence event to determine the event time instants.

2.2. Connectivity event

As explained in Remark 1, the proposed controller have the function of preserving existing links. However, since the event-triggered scheme is employed, the control input will not be updated when a new event time instant is determined. If these time instants are not properly chosen, the controller may fail to preserve connectivity if the distance between two agents is quite close to the communication range. Thus new strategies should be introduced.

Suppose at time t_k^i agents i and j are neighbors. Then $d_{ij}(t_k^i) < r$. Define the measurement error of the communication link (i, j) for agent i as

$$e_{ij}(t) = d_{ij}(t) - d_{ij}(t_k^i), t \geq t_k^i. \tag{10}$$

If $e_{ij}(t) < 0$, since $d_{ij}(t_k^i) < r$, one has $d_{ij}(t) < r$. Otherwise, if $e_{ij}(t) > 0$ and $e_{ij}(t) < r - d_{ij}(t_k^i)$, one still has $d_{ij}(t) < r$. Then we can conclude that if $e_{ij}(t) < r - d_{ij}(t_k^i)$ holds, one always has $d_{ij}(t) < r$ and thus the link (i, j) will not be lost at time t.

It should be noted that $e_{ij}(t)$ may not be equal to $e_{ji}(t)$ since agents i and j may not be triggered at the same time in general. We propose the condition for connectivity preservation of agent i as a set of inequalities

$$e_{ij}(t) \le \phi_r(d_{ij}(t_k^i)), \forall j \in N_i(t), \tag{11}$$

where $\phi_r(\lambda) : [0, +\infty) \mapsto [0, r)$ is a function satisfying the following two assumptions:

Assumption 3: $\phi_r(\lambda)$ is continuous and non-increasing on $[0, +\infty)$, and

Assumption 4: $\phi_r(\lambda)$ is nonnegative and

$$\begin{aligned} &0 < \phi_r(\lambda) < r - \lambda, && \text{if } \lambda \in [0, r), \\ &\phi_r(\lambda) = 0, && \text{if } \lambda \in [r, +\infty). \end{aligned} \tag{12}$$

Then, a connectivity event occurs whenever

$$e_{ij}(t) - \phi_r(d_{ij}(t_k^i)) = 0 \tag{13}$$

for any $j \in N_i(t)$.

It should be noted that the functions $\psi_r(\lambda)$ and $\phi_r(\lambda)$ are the same for every agent, respectively. Thus in the controller design, we actually require these global knowledge to be shared among the agents.

The determination of event time instants for the connectivity event is as follows. Assume that for agent i, there exists an agent $j' \in N_i(t)$ and a time instant $t' > t_k^i$ such that $e_{ij}(t) < \phi_r(d_{ij}(t_k^i))$ for all $j \in N_i(t) \setminus \{j'\}$ and all $t \in (t_k^i, t')$, but $e_{ij'}(t) < \phi_r(d_{ij'}(t_k^i))$ for $t \in (t_k^i, t')$ and $e_{ij'}(t') = \phi_r(d_{ij'}(t_k^i))$. Then t' will be a candidate choice for t_{k+1}^i. This implies that any agent $j' \in N_i(t)$ can trigger a connectivity event for agent i once (13) is satisfied for j'. It will be proven in the next section that by updating the control input at t' the existing communication links will be preserved. However, this is not sufficient to guarantee the stability and rendezvous of the agent group. To ensure stability and achieve rendezvous, the convergence trigger is needed.

2.3. Convergence event

Define the measurement error of the neighborhood center for agent i as

$$e_i(t) = q_i(t_k^i) - q_i(t). \tag{14}$$

To achieve rendezvous, we propose the condition for convergence as

$$\|e_i(t)\| \le \beta_i \|q_i(t)\|, \tag{15}$$

where $\beta_i \in (0, 1)$ is a constant to be determined. Then a convergence event occurs whenever

$$\|e_i(t)\| - \beta_i \|q_i(t)\| = 0. \tag{16}$$

Now we combine the connectivity trigger and the convergence trigger, and propose the following criteria for event time determination

$$\begin{aligned} t_{k+1}^i = \inf \Big\{ t > t_k^i \Big| & \big[\|e_i(t)\| = \beta_i \|q_i(t)\| \big] \\ & \vee \big[\vee_{j \in N_i(t)} \{ e_{ij}(t) = \phi_r(d_{ij}(t_k^i)) \} \big] \Big\} \end{aligned} \tag{17}$$

with $t_0^i = 0$ being the default event time instant and $\vee$ being the logical "or".

Remark 3: From (17) it should be noticed that with t_k^i being the k-th event time instant for agent i, t_{k+1}^i will be determined by any trigger defined in (13) or (16). Only one of these triggers, i.e., the first one which is triggered after t_k^i, is effective in determining t_{k+1}^i. Actually, since the events in (13) will only be triggered when $d_{ij}(t)$ is increasing and tends to r, there may be quite few events generated by (13) during the rendezvous process.

Remark 4: It is noted that realtime states of the neighbors are still needed to determine the connectivity event and the convergence event. The advantage is that such setting simplifies the design of event functions. However, agents need to continuously acquire neighbors' states and thus the communication scheme cannot be event-triggered. Thus one direction of improvement is to eliminate continuous communication among agents.

3. SYSTEM ANALYSIS

In this section the connectivity and rendezvous analysis will be presented. Firstly we show that any existing communication link will be preserved by the proposed controller. Then we prove that the controllers are effective to drive all the agents to achieve rendezvous. To simplify the analysis, the communication link creation process will not be considered in this work. Thus if the existing links can indeed be preserved by the proposed controller, the communication graph of the agent group and the number of neighbors of each agent will not change during the rendezvous evolution.

3.1. Connectivity analysis

To show the effectiveness of the proposed controller in connectivity preservation, we need to present the following lemma.

Lemma 1: Let $y(t)$ and $\bar{y}(t)$ be two positive, non-decreasing and bounded time-dependent variables defined on $t \in [0, \infty)$. $y(t)$ is differentiable and $\bar{y}(t)$ is piecewise constant, satisfying

$$0 < \bar{y}(t) \le y(t), \tag{18}$$

and

$$y(t)-\bar{y}(t) \leq \phi_r(\bar{y}(t)) \tag{19}$$

for all $t \geq 0$, where $\phi_r(\lambda)$ is a function satisfying assumptions A3 and A4. Assume that $y(t)$ satisfies

$$\dot{y}(t) < K\psi_r(\bar{y}(t)), \tag{20}$$

where K is a finite positive real number and $\psi_r(\lambda)$ satisfies assumptions A1 and A2. Then if $y(0) < r$, $y(t) < r$ for all $t \in (0, +\infty)$.

Proof: Firstly we prove that $y(t) \leq r$ for all $t \in (0, +\infty)$ by contradiction. Assume that there exists a $t_1 > 0$ such that $y(t_1) > r$. Since $y(t)$ is continuous with respect to t and $y(0) < r$, there must exist a $t_2 \in (0, t_1)$ such that $y(t_2^+) = r$ and $\dot{y}(t_2^+) > 0$. Two cases for $\bar{y}(t_2^+)$ need to be investigated here. i) If $\bar{y}(t_2^+) \geq r$, then since $y(t_2^+) - \bar{y}(t_2^+) \leq \phi_r(\bar{y}(t_2^+)) = 0$, one has $\bar{y}(t_2^+) = y(t_2^+) = r$. Then $\dot{y}(t_2^+) < K\psi_r(\bar{y}(t_2^+)) = K\psi_r(r) = 0$, which contradicts $\dot{y}(t_2^+) > 0$. ii) If $\bar{y}(t_2^+) < r$, then since $y(t_2^+) - \bar{y}(t_2^+) \leq \phi_r(\bar{y}(t_2^+)) < r - \bar{y}(t_2^+)$, one has $y(t_2^+) < r$, which contradicts $y(t_2^+) = r$. Thus $y(t) \leq r$ for all $t \in (0, +\infty)$.

Next we show that $y(t)$ cannot be equal to r in finite time, still by contradiction. Consider the dynamic system given by (18), (19) and (20). Assume that there exists a $t_3 > 0$ such that $y(t_3) = r$. Under such assumption, one concludes that $\bar{y}(t_3) = r$. Otherwise from $\bar{y}(t_3) < r$, since $y(t_3) - \bar{y}(t_3) \leq \phi_r(\bar{y}(t_3)) < r - \bar{y}(t_3)$, one has $y(t_3) < r$, which contradicts the assumption that $y(t_3) = r$. Now from (20), one has $\dot{y}(t_3) < K\psi_r(\bar{y}(t_3)) = K\psi_r(r) = 0$. This contradicts the condition that $y(t)$ is non-decreasing. Summarizing all the above arguments, $y(t) < r$ for all $t \in (0, +\infty)$ if $y(0) < r$, which completes the proof. □

Based on the above lemma, we prove that the existing links will be preserved by the proposed control law in (8) in the following lemma.

Lemma 2: Consider a group of N agents with dynamics (2) and an initial communication graph $G(0)$. Let $\psi_r(\lambda)$ satisfy assumptions A1 and A2 and $\phi_r(\lambda)$ satisfy assumptions A3 and A4. Then all the existing links in $G(0)$ will be preserved for any $t > 0$ under the event-triggered control law (8) with the event trigger (17).

Proof: Let agents i and j be neighbors at time t and consider the distance $d_{ij}(t)$ between these two agents after t. For connectivity preservation, we only need to show that $d_{ij}(t) < r$ for all t. Without loss of generality, we assume that $d_{ij}(t)$ is strictly positive. This will not affect the rendezvous result since rendezvous happens only when $d_{ij}(t)$ is strictly smaller than r and thus no link tends to be lost.

Define the subscript of the latest event time instant of agent i at time t as

$$k_i(t) = \arg\max_{k\in\mathbb{N}_{\geq 0}} \{t_k^i | t_k^i \leq t\}. \tag{21}$$

From (9) and the fact that $\psi_r(\lambda) \leq 1$ and $\frac{n_i(t)}{n_i(t)+1} < 1$, one has

$$\begin{aligned}\|\dot{x}_i(t)\| =& \Big\| \frac{\xi_i}{n_i(t)+1} \prod_{l\in N_i(t)} \psi_r(d_{il}(t^i_{k_i(t)})) \\ &\times \sum_{l\in N_i(t)} (x_i(t^i_{k_i(t)}) - x_l(t^i_{k_i(t)})) \Big\| \qquad (22)\\ <& \frac{\xi_{max}}{n_i(t)+1} \psi_r(d_{ij}(t^i_{k_i(t)})) \cdot n_i(t) \cdot r \qquad (23)\\ <& \xi_{max} r \psi_r(d_{ij}(t^i_{k_i(t)})). \qquad (24)\end{aligned}$$

Similarly, for the dynamic of agent j, one has

$$\|\dot{x}_j(t)\| < \xi_{max} r \psi_r(d_{ij}(t^j_{k_j(t)})). \tag{25}$$

Since we consider the case of positive $d_{ij}(t)$, one has

$$\frac{d}{dt} d_{ij}(t) = \frac{(x_i(t)-x_j(t))^T}{d_{ij}(t)} (\dot{x}_i(t) - \dot{x}_j(t)), \tag{26}$$

which implies that

$$\begin{aligned}\|\dot{d}_{ij}(t)\| \leq& \|\dot{x}_i(t)\| + \|\dot{x}_j(t)\| \qquad (27)\\ <& \xi_{max} r \Big(\psi_r\big(d_{ij}(t^i_{k_i(t)})\big) + \psi_r\big(d_{ij}(t^j_{k_j(t)})\big)\Big) \qquad (28)\\ <& 2\xi_{max} r \psi_r(\bar{d}_{ij}(t)), \qquad (29)\end{aligned}$$

where

$$\bar{d}_{ij}(t) = \min\{d_{ij}(t^i_{k_i(t)}), d_{ij}(t^j_{k_j(t)})\}. \tag{30}$$

If $d_{ij}(t)$ is nonincreasing, it is obvious that the connectivity between agents i and j will not be lost. Consider the increasing case of $d_{ij}(t)$. There exists some $t \geq 0$ such that $\dot{d}_{ij}(t) > 0$ and then one has

$$\dot{d}_{ij}(t) < 2\xi_{max} r \psi_r(\bar{d}_{ij}(t)). \tag{31}$$

Consider the connectivity events for agents i and j. One has

$$d_{ij}(t) - d_{ij}(t^i_{k_i(t)}) \leq \phi_r(d_{ij}(t^i_{k_i(t)})), \tag{32}$$

$$d_{ij}(t) - d_{ij}(t^j_{k_j(t)}) \leq \phi_r(d_{ij}(t^j_{k_j(t)})), \tag{33}$$

which implies that

$$d_{ij}(t) - \bar{d}_{ij}(t) \leq \phi_r(\bar{d}_{ij}(t)). \tag{34}$$

From Lemma 1, in the increasing case of $d_{ij}(t)$, $d_{ij}(t)$ satisfies the conditions for $y(t)$. Moreover, since $\bar{d}_{ij}(t)$ is the event-triggered measurement of $d_{ij}(t)$, it satisfies the condition for $\bar{y}(t)$ and one further has $0 < \bar{d}_{ij}(t) \leq d_{ij}(t)$. Also notice that (31) corresponds to (20) with $K = 2\xi_{max} r$ and (34) corresponds to (19). Then by Lemma 1, one concludes that $d_{ij}(t)$ will be smaller than r for all $t > 0$.

The above arguments are valid for any pair of neighboring agents. Thus all the existing links in $G(0)$ will be preserved for all $t > 0$ under the event-triggered control law (8). □

Remark 5: It should be noted that since $\dot{d}_{ij}(t) < 2\xi_{max}r\psi_r(\bar{d}_{ij}(t))$, there exists an upper bound for $d_{ij}(t)$, i.e., the solution to $\dot{y}(t) = 2\xi_{max}r\psi_r(\bar{y}(t))$ with $y(t) - \bar{y}(t) \leq \phi_r(\bar{y}(t))$, who will asymptotically converge to r. However, this does not imply that $d_{ij}(t)$ will asymptotically converge to r because $d_{ij}(t)$ is only upper bounded by $y(t)$. Actually, in the next subsection, it is shown that with proper design of $\psi_r(\lambda)$, the agent group will achieve rendezvous and no d_{ij} approaches r.

3.2. Convergence analysis

In this subsection the rendezvous analysis of the agent group under the proposed event-triggered controller will be provided. Let $x(t) = (x_1^T(t), \ldots, x_N^T(t))^T$ be the compact state of the agent group and $L(t)$ be the Laplacian matrix of the communication graph $G(t)$. It is noted from Lemma 2 that since each existing link is preserved under the proposed controller and also the link creation process is not considered, $G(t)$ becomes a fixed graph and thus $n_i(t)$, $N_i(t)$ and $L(t)$ will not change with time. Then we can choose a Lyapunov functional candidate as

$$\mathbf{V}(x(t)) = \frac{1}{2}x^T(t)(L\otimes I_n)x(t), \tag{35}$$

and we have the following lemma.

Lemma 3: Consider a group of N agents with an initially connected communication graph $G(0)$. The Lyapunov function $\mathbf{V}(x(t))$ defined in (35) is non-increasing along the solution to the dynamic system given by (2) and (8) with the event time instants given by (17).

Proof: To simplify the notations, denote

$$M = diag(n_1+1, n_2+1, \ldots, n_N+1), \tag{36}$$
$$\Xi = diag(\xi_1, \ldots, \xi_N), \tag{37}$$
$$\hat{\Psi}(t) = diag(\hat{\Psi}_1(t), \ldots, \hat{\Psi}_N(t)), \tag{38}$$

with

$$\hat{\Psi}_i(t) = \Psi_i(t_k^i), t \in [t_k^i, t_{k+1}^i). \tag{39}$$

We also omit time (t) for most of the variables if there is no confusion. In addition, let

$$q = (q_1^T, \ldots, q_N^T)^T, \tag{40}$$
$$e = (e_1^T, \ldots, e_N^T)^T. \tag{41}$$

Then the time derivative of $\mathbf{V}(x(t))$ is

$$\dot{\mathbf{V}} = x^T(L\otimes I_n)\dot{x} \tag{42}$$
$$= \{(M\otimes I_n)q\}^T\{-(\Xi\hat{\Psi})\otimes I_n\}(q+e) \tag{43}$$
$$= -\sum_{i=1}^N \xi_i\hat{\Psi}_i(n_i+1)\|q_i\|^2 - \sum_{i=1}^N \xi_i\hat{\Psi}_i(n_i+1)q_i^T e_i. \tag{44}$$

Since $q_i^T e_i \leq \frac{1}{2}\|q_i\|^2 + \frac{1}{2}\|e_i\|^2$ for any q_i and e_i,

$$\dot{\mathbf{V}} \leq -\sum_{i=1}^N \xi_i\hat{\Psi}_i(n_i+1)\|q_i\|^2 + \sum_{i=1}^N \xi_i\hat{\Psi}_i(n_i+1)(\frac{1}{2}\|q_i\|^2 + \frac{1}{2}\|e_i\|^2) \tag{45}$$
$$\leq -\sum_{i=1}^N \xi_i\hat{\Psi}_i(n_i+1)\|q_i\|^2 + \sum_{i=1}^N \xi_i\hat{\Psi}_i(n_i+1)(\frac{1}{2}\|q_i\|^2 + \frac{1}{2}\beta_i^2\|q_i\|^2) \tag{46}$$
$$= -\sum_{i=1}^N \frac{1}{2}\xi_i\hat{\Psi}_i(n_i+1)(1-\beta_i^2)\|q_i\|^2 \tag{47}$$
$$\leq 0, \tag{48}$$

which completes the proof. □

The following theorem presents the main rendezvous result of this work.

Theorem 1: Consider a group of N agents with dynamics (2) under the controller (8) in which the event time is determined by (17). Assume that the communication graph $G(t)$ is initially connected. If no agent exhibits Zeno behavior, then the existing communication links in $G(t)$ will be preserved. Moreover, the agent group will achieve rendezvous asymptotically, i.e., $\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0$ for any $i \neq j$.

Proof: Since the link addition is not taken into account and from Lemma 2, all the existing links will be preserved, then $G(t)$ is fixed and thus $E(t)$ and $L(t)$ will not change. For simplicity, we use E and L instead in the sequel of this proof. It is noted that the Lyapunov function $\mathbf{V}(x)$ is ISS with respect to the measurement error e. Then similar to the analysis in [12, 18], since $\dot{\mathbf{V}} \leq -\sum_{i=1}^N \frac{1}{2}\xi_i\hat{\Psi}_i(n_i+1)(1-\beta_i^2)\|q_i\|^2$ and $1-\beta_i^2 > 0$, one has $\lim_{t\to\infty}\hat{\Psi}_i\|q_i\|^2 = 0$ for all i. We will explain that for all i, $\lim_{t\to\infty}\hat{\Psi}_i \neq 0$ and $\lim_{t\to\infty}\|q_i\| = 0$. Since $\hat{\Psi}_i$ is the event-triggered state of Ψ_i at time t_k^i, from (6) and Assumptions 1 and 2, if we can prove that $\lim_{t\to\infty}d_{ij}(t) \neq r$ for all $(i,j)\in E$, then $\lim_{t\to\infty}\hat{\Psi}_i \neq 0$. Consider the agent group after a sufficiently long time t'. Let $\overline{co}(\cdot)$ represents the convex hull of a position point set. For any agent i' located at the corner position of $\overline{co}(x_1(t'), \ldots, x_N(t'))$, since it will be driven towards the interior of the convex hull $\overline{co}(x_1(t'), \ldots, x_N(t'))$ by the proposed controller (8), $d_{i'j'}(t)$ will decrease for any neighbor j' and thus $\lim_{t\to\infty}d_{i'j'}(t) \neq r$. For the agents who do not locate at the corner position of $\overline{co}(x_1(t'), \ldots, x_N(t'))$, let them be represented by $i'_1, \ldots, i'_l, \ldots, i'_m$ and assume that they have neighbors j'_l such that $\lim_{t\to\infty}d_{i'_l j'_l}(t) = r$. Since all the agents at the corner position will converge to the interior of $\overline{co}(x_1(t'), \ldots, x_N(t'))$, then at least a part of agents in $i'_1, \ldots, i'_l, \ldots, i'_m$ will be located at the corner position of the agent group as time t' goes sufficiently large. For these agents since they are at the corner positions, they will be driven towards the interior of the convex hull $\overline{co}(x_1(t'),\ldots,x_N(t'))$ and thus $\lim_{t\to\infty} d_{i'_l j'_l}(t) \neq r$. Summarize all these arguments, $\lim_{t\to\infty} d_{ij}(t) \neq r$ for all $(i,j) \in E$ and thus $\lim_{t\to\infty} \hat{\Psi}_i \neq 0$. Then one concludes that $\lim_{t\to\infty} \|q_i\| = 0$ for all i, which implies that $\lim_{t\to\infty} q = 0$. From (7) one has $q = ((M^{-1}L)\otimes I_n)x$. Thus $\lim_{t\to\infty}(L\otimes I_n)x = 0$. Since L is the Laplacian matrix of a connected graph, it implies that $\lim_{t\to\infty}(x_i - x_j) = 0$ for any i and j. Thus all the agents will asymptotically achieve rendezvous as time goes to infinity. □

4. ZENO-FREE CONTROLLER DESIGN

Multi-agent systems with event-triggered control can be considered as a special case of sampling control systems in which nonuniform sampling intervals are adopted. For practical systems, digital platforms always have a limit for the sampling frequency. Thus if the inter-event time is too short, the controller can be difficult to be implemented. For event-triggered multi-agent systems, this problem may be even worse: an agent may exhibit Zeno behavior, where infinitely many discrete transitions occur in a finite and bounded length of time. This implies that as k goes to infinity, the time in between two successive event time instants approaches 0. Then the inter-event time will become extremely short so that the sampling frequency is beyond the ability of the working platforms. In this section we will provide analysis for the inter-event time and show that by introducing some mild Assumptions for the controller and the event design, the inter-event time can be bounded from below by a strictly positive time length and thus no agent will exhibit the Zeno behavior.

4.1. Lower time bound for connectivity events

For the connectivity event, if agent i exhibits the Zeno behavior, there must exists a neighbor j such that the distance from agent i to agent j approaches the communication range r. In the proof of Theorem 1, it has been shown that the distance between any pair of neighboring agents cannot tend to the communication range and thus the connectivity event will not generate any Zeno behavior. However, under the connectivity trigger (13), if two neighboring agents move away from each other during the rendezvous evolution, more frequent triggering actions will be required to preserve connectivity and thus the inter-event time determined by (13) may become very short as k increases. This is not expected in practice. In this subsection, we will introduce new assumptions for the controller and event design to solve this problem. Specifically, we introduce the following assumption for design of $\psi_r(\lambda)$ and $\phi_r(\lambda)$.

Assumption 5: There exists a strictly positive number $\zeta \in \mathbb{R}$ such that

$$\frac{\phi_r(\lambda)}{\psi_r(\lambda)} \geq \zeta \tag{49}$$

for all $\lambda \in [0,r)$.

With this assumption, the following lemma guarantees that if any two neighbors move away from each other, the inter-event time of the connectivity trigger cannot be extremely short.

Lemma 4: The inter-event time generated by the connectivity event (13) is bounded from below by a strictly positive value

$$\mu = \frac{\zeta}{2\xi_{max} r} \tag{50}$$

if Assumptions 1 to 5 hold.

Proof: Without loss of generality, consider the scenario that the distance $d_{ij}(t)$ between two communication neighbor agents i and j tends to r after a sufficiently long time t during the rendezvous evolution under the proposed controller and event design. For agent i, the index k for event time instants will be sufficiently large after t. For simplicity, denote its connectivity event time instants after t by $\tau_k, \tau_{k+1}, \tau_{k+2}, \ldots$. From the triggering mechanism it is noted that at each event time instant τ_k, the error $e_{ij}(\tau_k)$ will be reset to 0. Since we only consider the increasing case of $d_{ij}(t)$, after τ_k the error $e_{ij}(t)$ will increase at a rate of $\dot{d}_{ij}(t)$. From (31), one notices that such increasing rate is strictly smaller than $2\xi_{max} r \psi_r(d_{ij}(\tau_k))$. On the other hand, under the trigger (11), it can be observed that during the time interval (τ_k, τ_{k+1}) the total increase of $e_{ij}(t)$ is exactly $\phi_r(d_{ij}(\tau_k))$. Thus one lower bound for the inter-event time can be obtained as $\mu_k = \frac{\phi_r(d_{ij}(\tau_k))}{2\xi_{max} r \psi_r(d_{ij}(\tau_k))}$. Then if we can require that $\frac{\phi_r(\lambda)}{\psi_r(\lambda)} \geq \zeta$ for all $\lambda \in [0,r)$ by the appropriate design of $\phi_r(\lambda)$ and $\psi_r(\lambda)$, then the inter-event time for the connectivity event will be strictly bounded from below by $\mu = \frac{\zeta}{2\xi_{max} r}$, which completes the proof. □

4.2. Lower time bound for convergence events

In the previous subsection it is explained that the connectivity trigger will not cause any Zeno behavior under the proposed controller. However, analysis of Zeno behavior for the convergence trigger is more complex. As presented in [31], events are generated when the magnitude of the measurement error reaches the prescribed threshold and generally speaking, the threshold can be state-independent [32] and state-dependent [18, 33]. For the state-independent threshold case, since the threshold is determined by an external variable, it is proven that there always exists a strictly positive lower bound for the event time interval and thus no agent will exhibit the Zeno behavior [32]. However, for the state-dependent threshold case, finding such a lower bound can be quite difficult [29].

To exclude the Zeno behavior for distributed multi-agent system, several approaches have been proposed in recent years. Since there will be no Zeno behavior by using the state-independent threshold in the event design, in [30] a hybrid threshold involving both the state-dependent variable and a small constant has been proposed. A similar design has been reported in [24]. For the event design of this work, using the same idea as in [30] one can develop an improved convergence trigger as $\|e_i(t)\| = \beta_i\|q_i(t)\| + \varepsilon_i$, where ε_i is a small positive real number. It can be proved that this event design can exclude the Zeno behavior for each agent. However, the drawback of such event design is that the agent group can only achieve rendezvous into a closed ball whose size is determined by ε_i [24, 30]. Asymptotic convergence cannot be achieved by this approach. In [31], the authors assumed that no agent will appear at the center position of its neighbors and proved that there will be no accumulation point at each finite time instant. However, this assumption is not reasonable in practice since the final objective of rendezvous is to drive each agent to the center of its neighbors.

To achieve asymptotic rendezvous, we will introduce new design for the convergence trigger while excluding the Zeno behavior for each agent. The design idea is that we embed a strictly positive time ν in the determination of the next event time instant t^i_{k+1}. If the inter-event time $t^i_{k+1} - t^i_k$ with t^i_{k+1} determined by (17) is larger than ν, we use (17) to generate t^i_{k+1}. Otherwise, we abandon the next event time generated by (17) and use $t^i_{k+1} = t^i_k + \nu$ as the next event time. Therefore, the inter-event time will always have a strictly positive lower bound ν and thus Zeno behaviors can be avoided for each agent.

A valid choice of ν requires appropriate design of the controller and event. Before presenting the main convergence result, the following assumption should be introduced.

Assumption 6: The function $\phi_r(\lambda)$ satisfies

$$\phi_r(\lambda) < -\frac{\lambda}{2} + \frac{r}{2}, \ \ \lambda \in [0, r), \tag{51}$$

and there exists a strictly positive real number ρ such that

$$\frac{\psi_r(\lambda + 2\phi_r(\lambda))}{\psi_r(\lambda - 2\phi_r(\lambda))} \geq \rho \tag{52}$$

holds for any $\lambda \in [\lambda^*, r)$, where

$$\lambda^* = \arg\min_{\lambda\in(0,r)} \{\lambda = 2\phi_r(\lambda)\}. \tag{53}$$

Using Assumption 6, we can embed a strictly positive time τ into the convergence trigger to exclude Zeno behaviors for each agent while ensuring the asymptotic rendezvous of the agent group. Before presenting the result for this new event design, we give the following two lemmas. To simplify the notations and for clarity, let $\tilde{M}$ represent $M \otimes I_n$ for a matrix M in these two lemmas.

Lemma 5: Let Assumptions 1 to 6 hold and assume that each inter-event time interval is longer than ν for all the agents, where $\nu \leq \mu$ with μ defined in (50). Consider the evolution of $\hat{\Psi}_i(t)$, defined in (39), in a time interval (t_1, t_2) with a length no longer than ν, i.e., $t_2 - t_1 \leq \nu$. Then under the controller (8),

$$\hat{\Psi}^M_i \rho^{N-1} < \hat{\Psi}^m_i \tag{54}$$

for all the agents $i = 1, \ldots, N$, where

$$\hat{\Psi}^M_i = \max_{t\in(t_1,t_2)} \hat{\Psi}_i(t) \ \text{ and } \ \hat{\Psi}^m_i = \min_{t\in(t_1,t_2)} \hat{\Psi}_i(t), \tag{55}$$

and ρ is defined in A6.

Proof: Let i^* be the agent with the smallest ratio $\frac{\hat{\Psi}^m_i}{\hat{\Psi}^M_i}$ in the agent group, i.e., $i^* = \arg\min_i \{\frac{\hat{\Psi}^m_i}{\hat{\Psi}^M_i}\}$. For agent i^*, we will prove that a lower bound for $\frac{\hat{\Psi}^m_{i^*}}{\hat{\Psi}^M_{i^*}}$ is ρ^{N-1}. Since it is assumed that each inter-event time interval is longer than ν and $t_2 - t_1 \leq \nu$, then in (t_1, t_2), each agent at most triggers for only once. Let $t^{i^*}_k$ be the last event time instant of agent i^* before t_1.

Consider the variation of $\hat{\Psi}_{i^*}$ in (t_1, t_2). Since agent i^* may triggers for one time in (t_1, t_2), we consider two triggering executions after time $t^{i^*}_k$ to cover the interval (t_1, t_2). From (29), the definition of the connectivity event in (11) and (13), and Assumption 5, since $t_2 - t_1 \leq \mu$, one concludes that the total change of $d_{i^*j}(t^{i^*}_k)$, i.e., the change of the distance between agent i^* to any of its neighbor j, cannot be larger than $\phi_r(d_{i^*j}(t^{i^*}_k)) + \phi_r(d_{i^*j}(t^{i^*}_{k+1}))$ in (t_1, t_2).

Now consider the case $d_{i^*j}(t^{i^*}_k) \geq \lambda^*$, where λ^* is defined in Assumption 6. From Assumption 1, since $\psi_r(\lambda)$ is non-increasing, to provide a lower bound for $\hat{\Psi}^m_{i^*}$, assume that $d_{i^*j}(t)$ is increasing during the two triggering executions for all $j \in N_{i^*}$. By Assumption 3, one has $\phi_r(d_{i^*j}(t^{i*}_k)) > \phi_r(d_{i^*j}(t^{i*}_{k+1}))$. Then by Assumption 1, one yields

$$\begin{aligned}\hat{\Psi}^m_{i^*} &> \prod_{j\in N_{i*}} \psi_r\Big(d_{i^*j}(t^{i^*}_k) + \phi_r\big(d_{i^*j}(t^{i*}_k)\big) + \phi_r\big(d_{i^*j}(t^{i*}_{k+1})\big)\Big) \\ &> \prod_{j\in N_{i*}} \psi_r\Big(d_{i^*j}(t^{i^*}_k) + 2\phi_r\big(d_{i^*j}(t^{i*}_k)\big)\Big).\end{aligned} \tag{56}$$

Since $\phi_r(\lambda) < -\frac{\lambda}{2} + \frac{r}{2}$ for any $\lambda \in [0, r)$, functions $\psi_r(\lambda)$ in (56) are well-defined. On the other hand, to provide an upper bound for $\hat{\Psi}^M_{i^*}$, assume that $d_{i^*j}(t)$ is decreasing during the two triggering executions for all $j \in N_{i^*}$. By Assumption 3, one has $\phi_r(d_{i^*j}(t^{i*}_k)) < \phi_r(d_{i^*j}(t^{i*}_{k+1}))$. Then by Assumption 1 one yields

$$\hat{\Psi}^M_{i^*} < \prod_{j\in N_{i*}} \psi_r\Big(d_{i^*j}(t^{i^*}_k) - \phi_r\big(d_{i^*j}(t^{i*}_k)\big) - \phi_r\big(d_{i^*j}(t^{i*}_{k+1})\big)\Big)$$

$$< \prod_{j\in N_{i^*}} \psi_r\Big(d_{i^*j}(t_k^{i^*}) - 2\phi_r\big(d_{i^*j}(t_k^{i^*})\big)\Big). \tag{57}$$

Since $\psi_r(\lambda)$ is non-increasing, $\rho < 1$. Thus from (56) and (57), one has

$$\frac{\hat{\Psi}^m_{i^*}}{\hat{\Psi}^M_{i^*}} > \frac{\prod_{j\in N_{i^*}} \psi_r\Big(d_{i^*j}(t_k^{i^*}) + 2\phi_r\big(d_{i^*j}(t_k^{i^*})\big)\Big)}{\prod_{j\in N_{i^*}} \psi_r\Big(d_{i^*j}(t_k^{i^*}) - 2\phi_r\big(d_{i^*j}(t_k^{i^*})\big)\Big)} \tag{58}$$

$$\geq \prod_{j\in N_{i^*}} \rho = \rho^{n_{i^*}} \tag{59}$$

$$\geq \rho^{N-1}. \tag{60}$$

Consider the case $d_{i^*j}(t_k^{i^*}) < \lambda^*$. In this case $\psi_r\Big(d_{i^*j}(t_k^{i^*}) - 2\phi_r\big(d_{i^*j}(t_k^{i^*})\big)\Big)$ is not well defined. However, to obtain an upper bound of $\hat{\Psi}^M_{i^*}$, we only need to assume that all $d_{i^*j}(t_k^{i^*}), j \in N_{i^*}$ is decreasing. Since $\psi_r(\lambda)$ is non-increasing, one can assume that all $d_{i^*j}(t_k^{i^*})$ have decreased to 0 to obtain the upper bound of $\hat{\Psi}^M_{i^*}$. Then $\frac{\psi_r\Big(d_{i^*j}(t_k^{i^*}) + 2\phi_r\big(d_{i^*j}(t_k^{i^*})\big)\Big)}{\psi_r(0)} > \rho$ for any $j \in N_{i^*}$ and thus $\frac{\hat{\Psi}^m_{i^*}}{\hat{\Psi}^M_{i^*}} > \rho^{N-1}$ still holds.

It is noted that i^* is the agent with the smallest ratio $\frac{\hat{\Psi}^m_i}{\hat{\Psi}^M_i}$. Thus we have $\frac{\hat{\Psi}^m_i}{\hat{\Psi}^M_i} > \rho^{N-1}$ for all agents $i = 1,\dots,N$, which is equivalent to $\hat{\Psi}^M_i \rho^{N-1} < \hat{\Psi}^m_i$. □

Lemma 6: Denote $\nu = \min\{\mu, \tau\}$ with μ being defined in (50) and

$$\tau = \frac{\gamma\rho^{N-1}}{\xi^2_{max} N\sqrt{nN}(1+\gamma\sqrt{\rho^{N-1}})}. \tag{61}$$

Assume that each inter-event time interval is longer than ν for all agents. Then under the proposed controller (8), one always has

$$\xi_i(n_i+1)\hat{\Psi}_i\|e_i\|^2 < \gamma^2 \sum_{j=1}^N \xi_j(n_j+1)\hat{\Psi}_j\|q_j\|^2 \tag{62}$$

during the time interval $(t^i_k, t^i_k+\nu)$ with $\gamma > 0$.

Proof: Denote $\hat{\Psi}^M_i = \max_{t\in(t^i_k,t^i_k+\nu)} \hat{\Psi}_i(t)$ and $\hat{\Psi}^m_i = \min_{t\in(t^i_k,t^i_k+\nu)} \hat{\Psi}_i(t)$, and further define matrices $\hat{\Psi}_M = diag(\hat{\Psi}^M_1,\dots,\hat{\Psi}^M_N)$ and $\hat{\Psi}_m = diag(\hat{\Psi}^m_1,\dots,\hat{\Psi}^m_N)$. Let A, B, C, R and S be matrices defined as

$$A = diag\Big(\sqrt{\xi_1(n_1+1)\hat{\Psi}^M_1},\dots,\sqrt{\xi_N(n_N+1)\hat{\Psi}^M_N}\Big), \tag{63}$$

$$B = diag\Big(\sqrt{\xi_1(n_1+1)\hat{\Psi}^m_1},\dots,\sqrt{\xi_N(n_N+1)\hat{\Psi}^m_N}\Big), \tag{64}$$

$$C = diag\Big(\sqrt{\xi_1(n_1+1)\hat{\Psi}_1(t)},\dots,\sqrt{\xi_N(n_N+1)\hat{\Psi}_N(t)}\Big),$$

$$R = diag\Big(\sqrt{\xi_1(n_1+1)},\dots,\sqrt{\xi_N(n_N+1)}\Big), \tag{65}$$

$$S = diag\Big(\sqrt{n_1+1},\dots,\sqrt{n_N+1}\Big). \tag{66}$$

Then one has $\hat{\Psi}_M = A^2$, $\hat{\Psi}_m = B^2$, $\hat{\Psi}(t) = C^2$, and $M = S^2$. Denote $e_A = \tilde{A}e$ and $q_B = \tilde{B}q$ to simplify the presentation. From the controller design (8) and the definitions of e_i and q_i, one has $\tilde{M}q = \tilde{L}x$, $\dot{e} = -\dot{q}$, and $\dot{x} = -\tilde{\Xi}\tilde{\hat{\Psi}}(q+e)$. Then by denoting $H = M^{-1}L\Xi\hat{\Psi}$ one yields $\dot{e} = -\dot{q} = \tilde{H}(q+e)$.

To prove (62), we firstly provide an estimation of the evolution time for $\|e_A\|$ from 0 to $\gamma\|q_B\|$ by computing the time derivative of $\frac{\|e_A\|}{\|q_B\|}$.

$$\frac{d}{dt}\frac{\|e_A\|}{\|q_B\|} = \frac{e_A^T\dot{e}_A}{\|q_B\|\cdot\|e_A\|} - \frac{\|e_A\|q_B^T\dot{q}_B}{\|q_B\|^3} \tag{67}$$

$$\leq \frac{\|e_A\|\cdot\|\dot{e}_A\|}{\|q_B\|\cdot\|e_A\|} + \frac{\|e_A\|\cdot\|q_B\|\cdot\|\dot{q}_B\|}{\|q_B\|^3} \tag{68}$$

$$= \frac{\|\dot{e}_A\|}{\|q_B\|} + \frac{\|e_A\|\cdot\|\dot{q}_B\|}{\|q_B\|^2}. \tag{69}$$

Notice that $B \preceq A$. Then

$$\frac{d}{dt}\frac{\|e_A\|}{\|q_B\|} \leq \frac{\|\dot{q}_A\|}{\|q_B\|} + \frac{\|e_A\|\cdot\|\dot{q}_A\|}{\|q_B\|^2} \tag{70}$$

$$= (1+\frac{\|e_A\|}{\|q_B\|})\frac{\|\tilde{A}\dot{q}\|}{\|q_B\|} \tag{71}$$

$$= (1+\frac{\|e_A\|}{\|q_B\|})\frac{\|\tilde{A}\tilde{H}(q+e)\|}{\|q_B\|} \tag{72}$$

$$= (1+\frac{\|e_A\|}{\|q_B\|})\frac{\|\tilde{A}\tilde{H}\tilde{A}^{-1}(q_A+e_A)\|}{\|q_B\|} \tag{73}$$

$$\leq \|\tilde{A}\tilde{H}\tilde{A}^{-1}\|\cdot(1+\frac{\|e_A\|}{\|q_B\|})\frac{\|q_A\|+\|e_A\|}{\|q_B\|}. \tag{74}$$

From Lemma 5, it can be obtained that

$$\sqrt{\rho^{N-1}}\tilde{A} \preceq \tilde{B}, \tag{75}$$

and thus

$$\|q_A\| = \frac{1}{\sqrt{\rho^{N-1}}}\|\sqrt{\rho^{N-1}}\tilde{A}q\| \leq \rho^{\frac{1-N}{2}}\|q_B\|. \tag{76}$$

For $\|\tilde{A}\tilde{H}\tilde{A}^{-1}\|$ one has

$$\|\tilde{A}\tilde{H}\tilde{A}^{-1}\| = \|(AM^{-1}L\Xi\hat{\Psi}A^{-1})\otimes I_n\| \tag{77}$$

$$= \|(AM^{-1}L\Xi C(CA^{-1}))\otimes I_n\| \tag{78}$$

$$\leq \|(AM^{-1}L\Xi C)\otimes I_n\| \tag{79}$$

$$\leq \|(RM^{-1}L\Xi R)\otimes I_n\|. \tag{80}$$

Then since $\rho < 1$, one yields

$$\frac{d}{dt}\frac{\|e_A\|}{\|q_B\|} \leq \alpha(1+\frac{\|e_A\|}{\|q_B\|})(\rho^{\frac{1-N}{2}} + \frac{\|e_A\|}{\|q_B\|}) \tag{81}$$

$$< \alpha(\kappa + \frac{\|e_A\|}{\|q_B\|})^2 \tag{82}$$

with $\alpha \geq \|(RM^{-1}L\Xi R)\otimes I_n\|$ and $\kappa = \rho^{\frac{1-N}{2}}$. Thus the evolution time for $\|e_A\|$ from 0 to $\gamma\|q_B\|$ is lower bounded by the time for $y(t)$ evolution from 0 to γ, with $y(t)$ being the solution of $\dot{y}(t) = \alpha(y(t)+\kappa)^2$, $y(0)=0$. By solving this differential equation, one obtains that $y(t) = \frac{\alpha\kappa^2 t}{1-\alpha\kappa t}$. Thus one concludes that the evolution time for $\|e_A\|$ from 0 to $\gamma\|q_B\|$ is lower bounded by $\tau = \frac{\gamma}{\alpha\kappa^2+\alpha\kappa\gamma}$. By simple computation and observing that the 2-norm $\|\cdot\|_2$ is upper bounded by the Frobenius norm $\|\cdot\|_F$, one has

$$\|(RM^{-1}L\Xi R)\otimes I_n\| \leq \xi_{max}^2\|(SM^{-1}LS)\otimes I_n\| \quad (83)$$
$$<\xi_{max}^2 N\|(M^{-1}L)\otimes I_n\|_F \quad (84)$$
$$<\xi_{max}^2 N\sqrt{nN}. \quad (85)$$

Then one can set $\alpha = \xi_{max}^2 N\sqrt{nN}$ and thus $\tau = \frac{\gamma\rho^{N-1}}{\xi_{max}^2 N\sqrt{nN}(1+\gamma\sqrt{\rho^{N-1}})}$. From the definitions of e_A and q_B, one notices that

$$\|e_A\|^2 = \|\tilde{A}e\|^2 = e^T\Xi\hat{\Psi}_M Me > e^T\Xi\hat{\Psi}Me \quad (86)$$
$$>\xi_i(n_i+1)\hat{\Psi}_i\|e_i\|^2, \quad (87)$$

and

$$\gamma^2\|q_B\|^2 = \gamma^2\|\tilde{B}q\|^2 = \gamma^2 q^T\Xi\hat{\Psi}_m Mq \quad (88)$$
$$<\gamma^2 q^T\Xi\hat{\Psi}Mq \quad (89)$$
$$=\gamma^2\sum_{j=1}^N \xi_j(n_j+1)\hat{\Psi}_j\|q_j\|^2. \quad (90)$$

Then since the evolution time for $\|e_A\|$ from 0 to $\gamma\|q_B\|$ is lower bounded by τ, one concludes that (62) holds during the time interval $(t_k^i, t_k^i+\nu)$. □

Now we are at the position to present the result for the convergence event.

Lemma 7: Consider a group of N agents with an initially connected communication graph $G(0)$. The Lyapunov function $\mathbf{V}(x(t))$ defined in (35) is non-increasing along the solution to the dynamic system given by (2) and (8) with the event time instants given by

$$t_{k+1}^i = \max\{t_k^i+\nu, \inf\{t>t_k^i \big| \|e_i(t)\| = \beta_i\|q_i(t)\|\}. \quad (91)$$

Here, $\nu = \min\{\mu,\tau\}$ with μ defined in (50) and

$$\tau = \frac{\rho^{N-1}\sqrt{\eta_2}}{\xi_{max}^2 N\sqrt{nN}(\sqrt{N}+\sqrt{\eta_2\rho^{N-1}})}, \quad (92)$$

where β_i, η_1 and η_2 are positive real numbers satisfying $\beta_i^2 \leq \eta_1$ and $\eta_1+\eta_2<1$.

Proof: Firstly we consider the triggering behaviors for each agent at time t. The agents can be divided into two sets. The agents whose next event instants are determined by $\inf\{t>t_k^i \big| \|e_i(t)\| = \beta_i\|q_i(t)\|\}$ constitute a set $S_1(t)$. And the agents whose next event instants are determined by $t_k^i+\nu$ constitute a set $S_2(t)$.

For agents in $S_1(t)$, since $\|e_i(t)\| \leq \beta_i\|q_i(t)\|$ for each $i\in S_1(t)$ and $\beta_i^2<\eta_1$, one has

$$\sum_{i\in S_1(t)}\xi_i\hat{\Psi}_i(n_i+1)\|e_i\|^2 \leq \sum_{i\in S_1(t)}\beta_i^2\xi_i\hat{\Psi}_i(n_i+1)\|q_i\|^2 \leq \eta_1\sum_{j=1}^N\xi_j\hat{\Psi}_j(n_j+1)\|q_j\|^2. \quad (93)$$

For agents in $S_2(t)$, consider the evolution of $e_i(t)$ in the time interval $(t_k^i, t_k^i+\nu)$. Let $\gamma = \sqrt{\frac{\eta_2}{N}}$ in (61) and one obtains τ as given in (92). From Lemma 6 one concludes that letting the evolution time for $\|e_i\|$ from 0 be exactly ν yields

$$\xi_i\hat{\Psi}_i(n_i+1)\|e_i\|^2 \leq \frac{\eta_2}{N}\sum_{j=1}^N\xi_j(n_j+1)\hat{\Psi}_j\|q_j\|^2, \quad (94)$$

which leads to

$$\sum_{i\in S_2(t)}\xi_i\hat{\Psi}_i(n_i+1)\|e_i\|^2 \leq \eta_2\sum_{j=1}^N\xi_j\hat{\Psi}_j(n_j+1)\|q_j\|^2. \quad (95)$$

Then adding (93) and (95) yields

$$\sum_{i=1}^N\xi_i\hat{\Psi}_i(n_i+1)\|e_i\|^2 \leq \eta\sum_{i=1}^N\xi_i\hat{\Psi}_i(n_i+1)\|q_i\|^2, \quad (96)$$

where $\eta = \eta_1+\eta_2 \in (0,1)$. With the observation of (48) and (96), one has

$$\dot{\mathbf{V}} \leq -\sum_{i=1}^N\xi_i\hat{\Psi}_i(n_i+1)\|q_i\|^2 + \sum_{i=1}^N\xi_i\hat{\Psi}_i(n_i+1)(\frac{1}{2}\|q_i\|^2+\frac{\eta}{2}\|q_i\|^2) \quad (97)$$
$$= -\sum_{i=1}^N\frac{1}{2}\xi_i\hat{\Psi}_i(n_i+1)(1-\eta)\|q_i\|^2 \quad (98)$$
$$\leq 0. \quad (99)$$

Thus from all the above arguments one concludes that to let $t_{k+1}^i = \max\{t_k^i+\nu, \inf\{t>t_k^i \big| \|e_i(t)\| = \beta_i\|q_i(t)\|\}$ is sufficient to guarantee that $\mathbf{V}(x(t))$ is non-increasing, which completes the proof. □

Remark 6: Actually, the main convergence result of this section only requires Lemma 7. However, Lemmas 5 and 6 are necessary for Lemma 7. The relationships among Lemmas 5, 6 and 7 are as follows. To obtain (75) in the proof of Lemma 6, Lemma 5 should be used. To obtain (94) in the proof of Lemma 7, Lemma 6 should be employed.

The overall rendezvous controller can be summarized as control input (8) with event time determined by

$$t_{k+1}^i = \max\Big\{t_k^i+\nu, \inf\Big\{t>t_k^i\Big|\big[\|e_i(t)\|=\beta_i\|q_i(t)\|\big] \vee \big[\vee_{j\in N_i(t)}\{e_{ij}(t)=\phi_r(d_{ij}(t_k^i))\}\big]\Big\}\Big\} \quad (100)$$

under Assumptions 1-6, where $\nu=\min\{\mu,\tau\}$ with μ and τ being given by (50) and (92), respectively.

Now we are at the position to present the main result for the new connectivity-preserving rendezvous controller.

Theorem 2: Consider a group of N agents with dynamics (2). Assume that the communication graph $G(t)$ is initially connected and Assumptions 1-6 hold. Then the existing communication links of the agent group will be preserved and no agent will exhibit the Zeno behavior under the controller (8) with the event time determined by (100). Moreover, all the agents will asymptotically achieve rendezvous as time goes to infinity.

Proof: From Lemma 2, it is noted that any existing communication link will be preserved. From (100), one also notices that the inter-event time is bounded from below by a strictly positive value $\nu=\min\{\mu,\tau\}$. Thus no agent in the group will exhibit the Zeno behavior during the rendezvous evolution. For the rendezvous process, since $\mathbf{V}(x(t))$ is non-increasing from Lemma 7, the rest of the proof is similar to that of Theorem 1. □

Remark 7: Sampling data based event-triggered scheme is a natural way to avoid Zeno behavior [34]. By using the sampling data, this work may also avoid Zeno behavior without the design of Assumptions 5 and 6. However, this is not easy because connectivity preserving depends on exquisite designs of the sampling scheme. On the other hand, although the Zeno-free analysis is complex in this work, the controller design is simple. This is illustrated in the next simulation section.

5. SIMULATIONS

In this section a rendezvous example of a multi-agent system with limited communication ranges will be presented to show the effectiveness of the proposed controller. Although it seems that the controller as well as the triggers are complex, it is easy to implement them since there is no priority of these triggers. Once one of any trigger works, the control input is updated. Each agent obtains its neighbors' position states by communication.

Consider a group of $N=5$ agents in a $\mathbb{R}^3$ space. The communication range of each agent is $r=8$. The initial positions are selected such that the r-disk graph of the agent group is connected. A variety forms of functions $\phi_r(\lambda)$ and $\phi_r(\lambda)$ satisfy Assumptions 1 to 6. A relatively simple way is to choose piecewise line forms for them. The constraint function $\psi_r(\lambda)$ is selected as follows to satisfy Assumptions 1 and 2

$$\psi_r(\lambda)=\begin{cases}1, & \text{if } 0\le\lambda<\frac{r}{2},\\ -\frac{2}{r}\lambda+2, & \text{if } \frac{r}{2}\le\lambda<r,\\ 0, & \text{if } \lambda\ge r.\end{cases} \quad (101)$$

To satisfy assumptions A3 to A6 and also to simplify the computation, $\phi_r(\lambda)$ is chosen as

$$\phi_r(\lambda)=\begin{cases}-\frac{1}{8}\lambda+\frac{r}{8}, & \text{if } 0\le\lambda<r,\\ 0, & \text{if } \lambda\ge r.\end{cases} \quad (102)$$

Then one has $\frac{\phi_r(\lambda)}{\psi_r(\lambda)}>0.5$ if $0\le\lambda<\frac{r}{2}$ and $\frac{\phi_r(\lambda)}{\psi_r(\lambda)}=0.5$ if $\frac{r}{2}\le\lambda<r$, which implies that A5 is satisfied. For Assumption A6, we define a function $\rho_r(\lambda)$ as follows to obtain a valid ρ

$$\rho_r(\lambda)=\begin{cases}\frac{\psi_r(\lambda+2\phi_r(\lambda))}{\psi_r(0)}, & 0\le\lambda<\lambda^*,\\ \frac{\psi_r(\lambda+2\phi_r(\lambda))}{\psi_r(\lambda-2\phi_r(\lambda))}, & \lambda^*\le\lambda<r.\end{cases} \quad (103)$$

Functions $y=\psi_r(\lambda)$, $y=\phi_r(\lambda)$, and $y=\rho_r(\lambda)$ are shown in Fig. 1. To show that $\phi_r(\lambda)$ satisfies Assumptions 4 and 6, $y=-\frac{\lambda}{2}+\frac{r}{2}$ is also plotted in this figure.

For the feedback gains, the requirement is only to be strictly positive. Considering the results in the existing literature, an appropriate choice is a gain in $[0.1,2]$. Large gain will lead to fast convergence and more triggering events, and small gain leads to the opposite. For simplicity, let the feedback gains in (8) be $\xi_i=0.3$ for all the agents. Then from (50) one has $\mu=0.1043$. Choose $\rho=\min_{0<\lambda<r}\rho_r(\lambda)=0.6$. Other parameters will be chosen according to Lemma 7. Let $\eta_1=0.49$ and $\eta_2=0.5$. Then $\eta_1+\eta_2<1$ holds. Since $\beta_i^2\le\eta_1$, one may simply choose $\beta_i=0.7$ for all the agents in the convergence event design (16). Furthermore from (92) one can compute $\tau=0.0211$. This implies that if the inter-event time

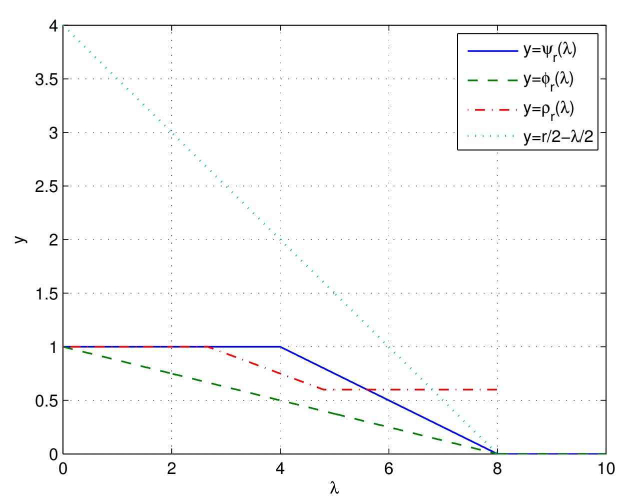

Fig. 1. Illustration of the functions $\psi_r(\lambda)$, $\phi_r(\lambda)$ and $\rho_r(\lambda)$.

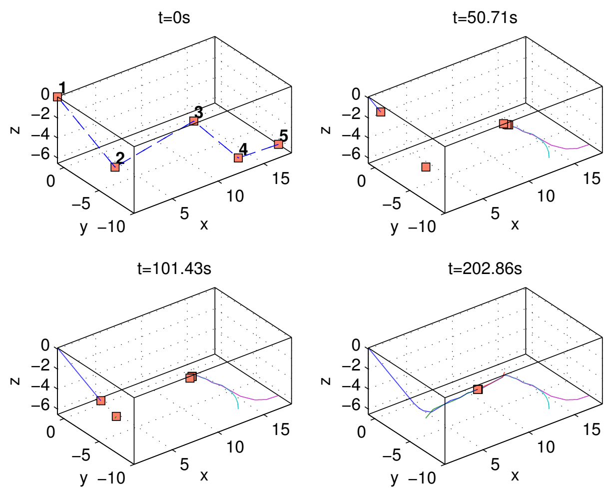

Fig. 2. Initial conditions and rendezvous evolution of agents.

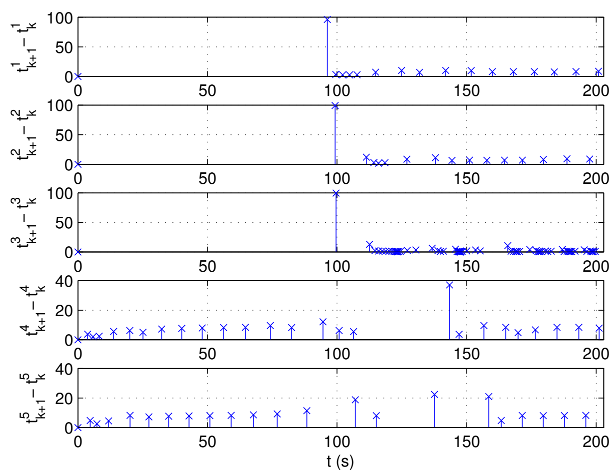

Fig. 3. Event time instants of agents.

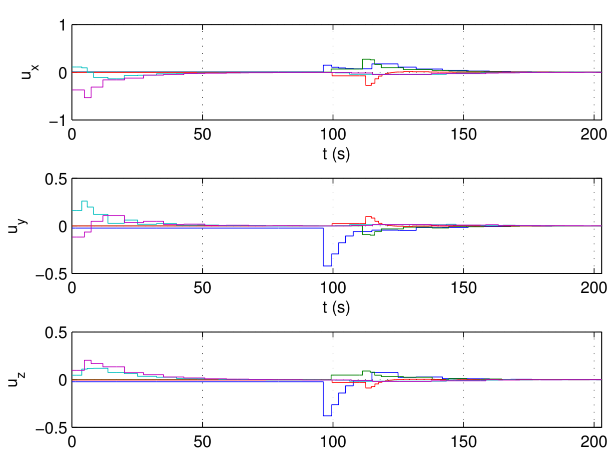

Fig. 4. Control input of each agent.

determined by the convergence event for any agent is shorter than $\nu = \min\{\mu, \tau\} = 0.0211$, then the agent will wait and trigger its next event at $t^i_{k+1} = t^i_k + \nu$. Finally, since the agent group will achieve asymptotic rendezvous, one needs to choose a termination condition for the simulation. In this example, when the sum of the distances from agents to the group center is smaller than 0.1, i.e., $\sum_{i=1}^N \|x_i - \frac{1}{N}\sum_{i=1}^N x_i\| < 0.1$, the agent group is considered to have achieved rendezvous.

The initial positions and communication graph are shown in the first subfigure of Fig. 2. To validate the connectivity preservation, let the distance between agents 2 and 3 be close to the communication range. The rendezvous evolution of the agent group under the proposed controller with the improved event design (100) is shown in the rest subfigures of Fig. 2. It can be observed that since the communication link $(2,3)$ may be lost, the motions of agents 2 and 3 must be delicate and precise, involving tiny movements at the beginning to preserve existing links. The event time instants of each agent are captured in Fig. 3 while the control inputs are given in Fig. 4. The total numbers of events for agents 1 to 5 are 17, 16, 67, 26, and 22, respectively. From these two figures one can notice that the control input is only updated at the event time instants, which indicates that the controllers are indeed event-triggered.

There are very few results regarding event-triggered connectivity control in literature [35, 36]. Compared with these existing results, the proposed approach has several unique features. Firstly, it is noted that, with the potential function like approach, the control input in [35,36] cannot always be bounded, especially when the distance between agents is close to the communication range. While in this work, the control input is bounded anytime. This is quite critical since in practice the saturation limitation of the actuator will lead to link failure and connectivity lost under the controller in [35,36] if the distance between two agents approaches the communication range. Secondly, in the event design in [35], the threshold is an exponential function, while in this work we use state-dependent threshold. As time goes, event-triggered control degenerates into continuous control when using exponential function threshold. There is no such problem in the state-dependent design in the proposed design. Thirdly, the trigger in [36] is centralized and all agents should be triggered at the same event time, while in this work each agent has its own event time and the controller and the event trigger can be implemented in a distributed way.

6. CONCLUSIONS

In this paper the rendezvous problem of multi-agent systems with limited communication ranges is studied. Event-triggered controllers with bounded inputs are developed to preserve existing communication links while driving the agent group to achieve rendezvous. To avoid extremely high event triggering frequency for each agent, new designs for both the connectivity trigger and the convergence trigger are proposed. Future research directions of this work include extending the proposed controller to self-triggered setup and developing connectivity-preserving event-triggered control for agent groups with more general dynamics.

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