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Passive Fuzzy Control Design for a Class of Nonlinear Distributed Parameter Systems with Time-varying Delay

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Abstract: This paper is devoted to studying the issue of passive fuzzy controller design for a class of nonlinear distributed parameter systems represented by semi-linear parabolic partial differential equations. The main objective of this paper is to develop two kinds of fuzzy controllers, one is static output feedback controller (SOFC), the other is dynamic output feedback controller (DOFC), which can guarantee both the stability and passivity of the designed closed-loop system. For the purpose of achieving the anticipated target, in this paper, the semi-linear parabolic PDE systems are assumed to be exactly represented by a Takagi-Sugeno (T-S) fuzzy parabolic PDE model. Furthermore, two examples are given to demonstrate the effectiveness of the controller design scheme.

Keywords: DOFC, passivity, semi-linear parabolic PDE systems, SOFC, T-S fuzzy model.

1. INTRODUCTION

It is well known that stability problems are often linked to the theory of dissipative systems. The passivity theory is part of dissipativeness [1, 2] that was firstly proposed in the circuit [3], and passivity widely exists in physics, circuit systems, applied mathematics, mechanics and other fields. The physical meaning of passivity is that the system is required to absorb more energy from outside than provides itself. Thus the essential feature of passivity is to maintain the systems internally stable [4–6]. The main reasons for studying passivity are as follows: Firstly, the system can reduce noise only if passivity is satisfied; Secondly, passivity is an effective way to study the stability of nonlinear systems, uncertain systems and high-order systems. Furthermore, passivity theory plays an important role in control theory as well as many other practical systems, such as robotic systems, power systems, chemical processes, etc. As a result, passivity theory has become a hot topic in many research fields [7–10].

On another research front, fuzzy control strategy that is based on fuzzy set theory, fuzzy language variables and computer intelligent control of fuzzy reasoning is able to offer a systematic method to address the complex systems with numerous variables [11], which is essentially a kind of non-linear control scheme. Systematic theory and a large number of practical application background is one of the main characteristics of fuzzy control strategy. In particular, the so-called Takagi-Sugeno (T-S) fuzzy model [12] has been widely employed for the controller design in nonlinear systems. The fundamental idea of T-S fuzzy model is to treat the complex nonlinear dynamical system as a fuzzy approximation of multiple local linear systems. To some degree, it can not only solve the difficulty of direct research on complex non-linear system, but also bring much convenience to controller design and stability analysis of the fuzzy system. Thereby in recent decades, a large number of conclusions have been extended to T-S fuzzy systems [13–19].

For T-S fuzzy systems, a lot of results on passive control are obtained by various methods. For example, in [20], passive control problems of continuous-time T-S fuzzy systems have been discussed, and the problems of discrete-time systems have been solved in [21]. However, most of the above results require measurable state variables, while in practical application, the state of the system is often not measurable, so it is very meaningful to study the output feedback control. There are three kinds of output feedback controllers mentioned in the existing literatures, which are dynamic output feedback controller, observer-based output feedback controller and static output feedback controller. Compared with the other two methods, static output feedback controller has attracted more attention because of its simple structure. For instance, static output feedback controller with time delay was proposed in [22]. In [23], the authors considered the

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passive dynamic output feedback controller design problem of the uncertain T-S fuzzy systems with time-varying delays.

Note that the above discussion is about ODE models, but now many scholars have paid attention to fuzzy controller design problem for PDE systems. More recently, fuzzy controller design methods based on the fuzzy PDE model directly have been proposed for a class of SLP-PDE systems [24–26]. [24] studied the distributed fuzzy P-sI controller design issue with a mixed H_2/H_∞ performance for a class of semi-linear parabolic PDE systems. However, the authors didn't do any work on passivity in [24–26]. To the best of authors' knowledge, the issue of fuzzy controllers design about guaranteeing the stability and passivity of the closed-loop nonlinear parabolic system system has not been studied yet, which greatly motivates this study. Although the authors in [23] and [27, 28] have designed the passive fuzzy controller to guarantee stabilization and passivity of the system, their models are all based on ODE systems.

In this paper, we will investigate the problem of static and dynamic fuzzy controller design for a class of semi-linear PDE systems. Two kinds of feedback controllers are designed here to guarantee the stability and passivity of considered system. By combining the Lyapunov's direct method, integration by parts, Wirting's inequalities and standard linear matrix inequalities, the less conservative results are obtained. The main novelty and contributions of this paper can be summarized as follows:

1) By using the sector nonlinearity approach, a T-S fuzzy model is employed to accurately represent the delayed semi-linear parabolic PDE system, which provides an effective way for fuzzy control design.
2) In fact, fruitful works have been reported for parabolic PDE systems. However, most of the results were achieved based on an assumption that the system's states are measurable, which is difficult to hold in real applications. Thus, this study determines to investigate the dynamic output feedback control problem.
3) It is noteworthy that passivity performance plays an important role in control design. Even though considerable attenuation has been paid to ODE systems, the passivity analysis remains an open issue for PDE systems.

2. PRELIMINARY AND PROBLEM FORMULATION

Consider the semi-linear PDE systems in one spatial dimension:

$$y_t(z,t) = \Theta y_{zz}(z,t) + f_1(y(z,t)) + f_d(y(z,t-\tau(t))) + G_{u1}(y(z,t))u(z,t) + G_{w1}(y(z,t))w(z,t), \tag{1}$$

$$y_c(z,t) = f_2(y(z,t)) + G_{u2}(y(z,t))u(z,t) + G_{w2}(y(z,t))w(z,t), \tag{2}$$

$$y_m(z,t) = f_3(y(z,t)) + G_{w3}(y(z,t))w(z,t), \tag{3}$$

subject to the homogeneous Dirichlet boundary conditions: $y(l_1,t) = y(l_2,t) = 0$, and the initial condition: $y(z,0) = y_0(z)$. $y(z,t) \in R^n$ is the state vector, $z \in [l_1,l_2] \subset R$ and $t \in [0,\infty)$ is the spatial position and time, respectively, $y_c(z,t) \in R^s$ is the control output, $y_m(z,t) \in R^q$ is the measured output. $u(\cdot,t) \in R^m$ is the control input, $w(\cdot,t) \in R^p$ is the exogenous disturbance. $\tau(t)$ is the time delay and satisfies $0 \le \tau(t) \le \tau, 0 \le \dot{\tau}(t) \le d < 1$. Here both τ and d are positive constants. G_{ui}, G_{wi}, are known matrix functions in $y(z,t)$, Θ is real known matrix with appropriate dimensions. For brevity, set

$$\mathcal{A}y(z,t) = \Theta y_{zz}(z,t).$$

In this study, we assume that the semi-linear system (1)-(3) can be exactly represented by the following T-S fuzzy PDE model:
Plant Rule i: If $\xi_1(z,t)$ is F_{i1} and $\cdots$ and $\xi_l(z,t)$ is F_{il}
Then

$$y_t(z,t) = \mathcal{A}y(z,t) + A_i y(z,t) + A_{d,i} y(z,t-\tau(t)) + B_{1i}u(z,t) + C_{1i}w(z,t), \tag{4}$$

$$y_c(z,t) = D_i y(z,t) + B_{2i}u(z,t) + C_{2i}w(z,t), \tag{5}$$

$$y_m(z,t) = E_i y(z,t) + C_{3i}w(z,t). \tag{6}$$

where $F_{ij}, i \in S = \{1,2,\dots,r\}, j = 1,2,\dots,l$ are fuzzy sets. $A_i, A_{di}, B_{1i}, B_{2i}, C_{1i}, C_{2i}, C_{3i}, D_i, E_i$ are known matrices, r is the number of fuzzy IF-THEN rules. $\xi_j(z,t)$ is the known premise variables. In order to avoid a complicated defuzzification process of fuzzy controller, in this study, these premise variables are assumed to be functions of only the state $y(z,t)$. By applying the center-average defuzzifier, product interference and singleton fuzzifier, the overall dynamics can be expressed as

$$y_t(z,t) = \mathcal{A}y(z,t) + \sum_{i=1}^{r} h_i(\xi(z,t)) \times [A_i y(z,t) + A_{d,i} y(z,t-\tau(t)) + B_{1i}u(z,t) + C_{1i}w(z,t)], \tag{7}$$

$$y_c(z,t) = \sum_{i=1}^{r} h_i(\xi(z,t))[D_i y(z,t) + B_{2i}u(z,t) + C_{2i}w(z,t)], \tag{8}$$

$$y_m(z,t) = \sum_{i=1}^{r} h_i(\xi(z,t))[E_i y(z,t) + C_{3i}w(z,t)]. \tag{9}$$

To this end, the following definition and lemma are useful for the development of control design in this study:

Definition 1: For system (1)-(3), it is called passive if there are constants $\alpha \geq 0$, $\beta \in R$ such that

$$2\int_0^{t_p}\int_{l_1}^{l_2} y_c^T(z,t)w(z,t)dzdt \geq -\beta^2 - \alpha\int_0^{t_p}\int_{l_1}^{l_2} w^T(z,t)w(z,t)dzdt \quad (10)$$

for all $t_p > 0$.

Lemma 1 (Vector-valued Wirtinger's inequalities) [29]**:** Let $y \in \mathcal{W}^{1,2}([l_1,l_2];R^n)$ be a vector function. Then for a matrix $S \geq 0$, we have

$$\int_{l_1}^{l_2} y^T(s)Sy(s)ds \leq (l_2-l_1)^2\pi^{-2}\int_{l_1}^{l_2}(dy(s)/ds)^T S(dy(s)/ds)ds. \quad (11)$$

Remark 1: The symbol * is used as an ellipsis in matrix expressions that are induced by symmetry. e.g.,

$$\begin{bmatrix}[A+B+*]+C & X\\ * & Y\end{bmatrix} = \begin{bmatrix}[A+B+A^T+B^T]+C & X\\ X^T & Y\end{bmatrix}.$$

3. PASSIVITY AND STABILITY

3.1. Stability analysis

Firstly we study stability of the unforced disturbance-free system of (7) (i.e., $u(z,t) = w(z,t) = 0$). Set $u = 0, w = 0$

$$y_t(z,t) = \mathcal{A}y(z,t) + \sum_{i=1}^r h_i(\xi(z,t))[A_i y(z,t) + A_{d,i}y(z,t-\tau(t))]. \quad (12)$$

Theorem 1: Consider the system (12), suppose $\dot{\tau}(t) \leq d < 1$, If there exist positive matrices P, Q such that the following are satisfied: $[P\Theta + *] > 0$,

$$\begin{bmatrix}\Omega_i & PA_{d,i}\\ * & -(1-d)Q\end{bmatrix} < 0, \quad (13)$$

where $\Omega_i = P[\tilde{\Theta} + A_i] + * + Q, \tilde{\Theta} = -\pi^2(l_2-l_1)^{-2}\Theta$, then (12) is asymptotically stable.

Proof: Consider the following Lyapunov functional candidate

$$V(t) = \int_{l_1}^{l_2} y^T(z,t)Py(z,t)dz + \int_{l_1}^{l_2}\int_{t-\tau(t)}^{t} y^T(z,\alpha)Qy(z,\alpha)d\alpha dz. \quad (14)$$

Then, the time derivative of $V(t)$ is

$$\begin{aligned}\dot{V}(t) \leq & 2\int_{l_1}^{l_2} y^T(z,t)Py_t(z,t)dz\\ & + \int_{l_1}^{l_2} y^T(z,t)Qy(z,t)dz\\ & -(1-d)\int_{l_1}^{l_2} y^T(z,t-\tau(t))Qy(z,t-\tau(t))dz\\ = & 2\int_{l_1}^{l_2} y^T(z,t)P\mathcal{A}y(z,t)dz\\ & + \int_{l_1}^{l_2} y^T(z,t)\sum_{i=1}^r h_i(\xi(z,t))(PA_i + *)y(z,t)dz\\ & + \int_{l_1}^{l_2} y^T(z,t)\sum_{i=1}^r h_i(\xi(z,t))\\ & \times(PA_{d,i} + *)y(z,t-\tau(t))dz\\ & + \int_{l_1}^{l_2} y^T(z,t)Qy(z,t)dz\\ & -(1-d)\int_{l_1}^{l_2} y^T(z,t-\tau(t))Qy(z,t-\tau(t))dz.\end{aligned}$$

Utilizing $\mathcal{A}y(z,t) = \Theta y_{zz}(z,t)$ and boundary condition $y(l_1,t) = y(l_2,t) = 0$, integrating by parts, we can find that

$$\begin{aligned}& 2\int_{l_1}^{l_2} y^T(z,t)P\mathcal{A}y(z,t)dz\\ & = 2\int_{l_1}^{l_2} y^T(z,t)P\Theta y_{zz}(z,t)dz\\ & = 2\int_{l_1}^{l_2}\left(y^T(z,t)P\Theta y_z(z,t)\right)_z dz\\ & \quad -2\int_{l_1}^{l_2} y_z^T(z,t)P\Theta y_z(z,t)dz\\ & = 2y^T(z,t)P\Theta y_z(z,t)(z,t)|_{z=l_1}^{z=l_2}\\ & \quad -2\int_{l_1}^{l_2} y_z^T(z,t)P\Theta y_z(z,t)dz\\ & = -\int_{l_1}^{l_2} y_z^T(z,t)[P\Theta + *]y_z(z,t)dz. \quad (15)\end{aligned}$$

According to Lemma 1, we can easily obtain

$$\begin{aligned}& 2\int_{l_1}^{l_2} y^T(z,t)P\mathcal{A}y(z,t)dz\\ & \leq -\pi^2(l_2-l_1)^{-2}\int_{l_1}^{l_2} y^T(z,t)[P\Theta + *]y(z,t)dz\\ & = \int_{l_1}^{l_2} y^T(z,t)[P\tilde{\Theta} + *]y(z,t)dz. \quad (16)\end{aligned}$$

Consequently

$$\dot{V}(t) \leq \sum_{i=1}^r h_i(\xi(z,t))\int_{l_1}^{l_2}\xi^T(z,t)\Phi_{1i}\xi(z,t)dz, \quad (17)$$

where $\xi(z,t) = \begin{bmatrix} y^T(z,t) & y^T(z,t-\tau(t))\end{bmatrix}^T$,

$$\Phi_{1i} = \begin{bmatrix}\Omega_i & PA_{d,i}\\ * & -(1-d)Q\end{bmatrix},$$

where $\Omega_i = P[\tilde{\Theta} + A_i] + * + Q$. From Lyapunov stability theory, we can know the result. This is the end of proof. □

3.2. Passivity analysis

Set $u=0$, then (7)-(8) becomes

$$y_t(z,t)=\mathcal{A}y(z,t)+\sum_{i=1}^{r}h_i(\xi(z,t))[A_iy(z,t)+A_{d,i}y(z,t-\tau(t))+C_{1i}w(z,t)], \quad (18)$$

$$y_c(z,t)=\sum_{i=1}^{r}h_i(\xi(z,t))[D_iy(z,t)+C_{2i}w(z,t)]. \quad (19)$$

Theorem 2: Consider the system (18), suppose $\dot{\tau}(t)\le d<1$, If there exist positive matrices P, Q, and scalar $\alpha\ge 0$ such that the following inequalities are satisfied: $[P\Theta+*]>0$,

$$\begin{bmatrix}\Omega_i & PA_{d,i} & PC_{1i}-D_i^T\\ * & -(1-d)Q & 0\\ * & * & -(C_{2i}+C_{2i}^T+\alpha I)\end{bmatrix}<0, \quad (20)$$

where $\Omega_i=P[\tilde{\Theta}+A_i]+*+Q, \tilde{\Theta}=-\pi^2(l_2-l_1)^{-2}\Theta$, then (18) is asymptotically stable and passive.

Proof: Choose the same Lyapunov function as (14)

$$\begin{aligned}\dot{V}(t)\le&2\int_{l_1}^{l_2}y^T(z,t)Py_t(z,t)dz\\&+\int_{l_1}^{l_2}y^T(z,t)Qy(z,t)dz\\&-(1-d)\int_{l_1}^{l_2}y^T(z,t-\tau(t))Qy(z,t-\tau(t))dz\\&=2\int_{l_1}^{l_2}y^T(z,t)P\mathcal{A}y(z,t)dz\\&+\int_{l_1}^{l_2}y^T(z,t)\sum_{i=1}^{r}h_i(\xi(z,t))(PA_i+*)y(z,t)dz\\&+\int_{l_1}^{l_2}y^T(z,t)\sum_{i=1}^{r}h_i(\xi(z,t))\\&\times(PA_{d,i}+*)y(z,t-\tau(t))dz\\&+\int_{l_1}^{l_2}y^T(z,t)\sum_{i=1}^{r}h_i(\xi(z,t))(PC_{1i}+*)w(z,t)dz\\&+\int_{l_1}^{l_2}y^T(z,t)Qy(z,t)dz\\&-(1-d)\int_{l_1}^{l_2}y^T(z,t-\tau(t))Qy(z,t-\tau(t))dz.\end{aligned}$$

Considering the following

$$\begin{aligned}&2y_c^T(z,t)w(z,t)+\alpha w^T(z,t)w(z,t)\\&=2\sum_{i=1}^{r}h_i(\xi(z,t))\\&\quad\times\left[y^T(z,t)\ \ y^T(z,t-\tau(t))\ \ w^T(z,t)\right]\begin{bmatrix}D_i^T\\0\\C_{2i}^T\end{bmatrix}w(z,t)\\&\quad+\alpha w^T(z,t)w(z,t).\end{aligned}$$

According (16) and above

$$\begin{aligned}&\dot{V}(t)-2\int_{l_1}^{l_2}y_c^T(z,t)w(z,t)dz-\alpha\int_{l_1}^{l_2}w^T(z,t)w(z,t)dz\\&\le\sum_{i=1}^{r}h_i(\xi(z,t))\int_{l_1}^{l_2}\xi^T(z,t)\Phi_{2i}\xi(z,t)dz,\end{aligned} \quad (21)$$

where $\xi(z,t)=\left[y^T(z,t)\quad y^T(z,t-\tau(t))\quad w^T(z,t)\right]^T$,

$$\Phi_{2i}=\begin{bmatrix}\Omega_i & PA_{d,i} & PC_{1i}-D_i^T\\ * & -(1-d)Q & 0\\ * & * & -(C_{2i}+C_{2i}^T+\alpha I)\end{bmatrix}, \quad (22)$$

where $\Omega_i=P[\tilde{\Theta}+A_i]+*+Q$.
It follows that

$$\dot{V}(t)-2\int_{l_1}^{l_2}y_c^T(z,t)w(z,t)dz-\alpha\int_{l_1}^{l_2}w^T(z,t)w(z,t)dz\le 0. \quad (23)$$

By integrating with t over the time period 0 to t_p, we can get

$$2\int_0^{t_p}\int_{l_1}^{l_2}y_c^T(z,t)w(z,t)dzdt\ge V(t_p)-V(0)-\alpha\int_0^{t_p}\int_{l_1}^{l_2}w^T(z,t)w(z,t)dzdt. \quad (24)$$

From the definition of $V(t)$, we have $V(t_p)\ge 0, V(0)\ge 0$. Thus

$$2\int_0^{t_p}\int_{l_1}^{l_2}y_c^T(z,t)w(z,t)dzdt\ge-\beta^2-\alpha\int_0^{t_p}\int_{l_1}^{l_2}w^T(z,t)w(z,t)dzdt \quad (25)$$

for all $t_p\ge 0$, where $\beta=\sqrt{V(0)/2}$. This is the end of proof. □

Remark 2: According to Theorems 1 and 2, it is easy to see that the passive properties of systems can keep the systems internally stable.

3.3. Static passivity feedback control

In this subsection, we will design static feedback controllers to guarantee the stabilization and passivity of the system. Firstly, we consider the following fuzzy controller which is called memoryless state feedback control law

$$u(z,t)=\sum_{i=1}^{r}h_i(\xi(z,t))K_iy(z,t). \quad (26)$$

From the above (7)-(8), we have the following fuzzy PDE

$$y_t(z,t)=\mathcal{A}y(z,t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\bar{h}_{ij}(\xi(z,t))[A_i+B_{1j}K_i]y(z,t)$$

$$+\sum_{i=1}^{r} h_i(\xi(z,t))A_{d,i}y(z,t-\tau(t))$$
$$+\sum_{i=1}^{r} h_i(\xi(z,t))C_{1i}w(z,t), \quad (27)$$
$$y_c(z,t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \bar{h}_{ij}(\xi(z,t))[D_i+B_{2j}K_i]y(z,t)$$
$$+\sum_{i=1}^{r} C_{2i}w(z,t), \quad (28)$$

where $\bar{h}_{ij}(\xi(z,t)) \triangleq h_i(\xi(z,t))h_j(\xi(z,t))$.

Theorem 3: Consider the system (27), suppose $\dot{\tau}(t) \le d < 1$, If there exist scalar $\alpha \ge 0$, positive matrices X, $\tilde{Q}$, and Y_i such that the following are satisfied: $[\Theta X + *] > 0$,

$$\begin{bmatrix} \tilde{\Omega}_{ij} & A_{d,i}X & C_{1i}-XD_i^T-Y_i^TB_{2j}^T \\ * & -(1-d)\tilde{Q} & 0 \\ * & * & -(C_{2i}+C_{2i}^T+\alpha I) \end{bmatrix} < 0, \quad (29)$$

where $\tilde{\Omega}_{ij} = [\tilde{\Theta}X + A_iX + B_{1j}Y_i] + * + \tilde{Q}, \tilde{\Theta} = -\pi^2(l_2 - l_1)^{-2}\Theta$, then (27) is asymptotically stable and passive. In this case, the control gain matrices $K_i = Y_iX^{-1}$.

Proof: Using the same method as Theorem 2, we can get

$$\dot{V}(t) - 2\int_{l_1}^{l_2} y_c^T(z,t)w(z,t)dz$$
$$-\alpha\int_{l_1}^{l_2} w^T(z,t)w(z,t)dz$$
$$\le \sum_{i=1}^{r}\sum_{j=1}^{r} \bar{h}_{ij}(\xi(z,t))\int_{l_1}^{l_2} \xi^T(z,t)\Phi_{ij}\xi(z,t)dz,$$

where $\xi(z,t) = \begin{bmatrix} y^T(z,t) & y^T(z,t-\tau(t)) & w^T(z,t)\end{bmatrix}^T$,

$$\Phi_{ij} = \begin{bmatrix} \Omega_{ij} & PA_{d,i} & PC_{1i}-D_i^T-K_i^TB_{2j}^T \\ * & -(1-d)Q & 0 \\ * & * & -(C_{2i}+C_{2i}^T+\alpha I) \end{bmatrix}, \quad (30)$$

where $\Omega_{ij} = P[\tilde{\Theta} + A_i + B_{1j}K_i] + * + Q$
Let $X = P^{-1}, \tilde{Q} = XQX, K_i = Y_iX^{-1}$, pre- and post-multiplying the matrices Φ_{ij} with diag$\{X,X,I\}$, we have

$$\Psi ij = \begin{bmatrix} \tilde{\Omega}_{ij} & A_{d,i}X & C_{1i}-XD_i^T-Y_i^TB_{2j}^T \\ * & -(1-d)\tilde{Q} & 0 \\ * & * & -(C_{2i}+C_{2i}^T+\alpha I) \end{bmatrix}, \quad (31)$$

where $\tilde{\Omega}_{ij} = [\tilde{\Theta}X + A_iX + B_{1j}Y_i] + * + \tilde{Q}$. According to theorem 2, we get the result. This is the end of proof. □

In order to reduce the conservativeness of the system, we consider the following fuzzy controller which is called memorial state feedback control law

$$u(z,t) = \sum_{i=1}^{r} h_i(\xi(z,t))[K_iy(z,t) + K_{d,i}y(z,t-\tau(t))]. \quad (32)$$

From the above (7)-(8), we have the following fuzzy PDE

$$y_t(z,t) = \mathcal{A}y(z,t) + \sum_{i=1}^{r}\sum_{j=1}^{r} \bar{h}_{ij}(\xi(z,t))[A_i + B_{1j}K_i]y(z,t)$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r} \bar{h}_{ij}(\xi(z,t))[A_{d,i} + B_{1j}K_{d,i}]y(z,t-\tau(t))$$
$$+\sum_{i=1}^{r} h_i(\xi(z,t))C_{1i}w(z,t), \quad (33)$$
$$y_c(z,t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \bar{h}_{ij}(\xi(z,t))[D_i + B_{2j}K_i]y(z,t)$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r} \bar{h}_{ij}(\xi(z,t))B_{2j}K_{d,i}y(z,t-\tau(t))$$
$$+\sum_{i=1}^{r} h_i(\xi(z,t))C_{2i}w(z,t). \quad (34)$$

Theorem 4: Consider the system (33), suppose $\dot{\tau}(t) \le d < 1$, If there exist scalar $\alpha \ge 0$, positive matrices X, $\tilde{Q}$ and $Y_i, Y_{d,i}$ such that the following are satisfied: $[\Theta X + *] > 0$,

$$\begin{bmatrix} \tilde{\Omega}_{ij} & A_{d,i}X + B_{1j}Y_{d,i} & C_{1i}-XD_i^T-Y_i^TB_{2j}^T \\ * & -(1-d)\tilde{Q} & -Y_{d,i}^TB_{2j}^T \\ * & * & -(C_{2i}+C_{2i}^T+\alpha I) \end{bmatrix} < 0, \quad (35)$$

where $\tilde{\Omega}_{ij} = [\tilde{\Theta}X + A_iX + B_{1j}Y_i] + * + \tilde{Q}$, then (33) is asymptotically stable and passive. In this case, the control gain matrices $K_i = Y_iX^{-1}, K_{d,i} = Y_{d,i}X^{-1}$.

3.4. Dynamic passivity feedback control

Considering the following dynamic output feedback controller

$$\hat{y}_t(z,t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(z,t))h_j(\xi(z,t))$$
$$\times [A_{kij}\hat{y}(z,t) + B_{kj}y_m(z,t)], \quad (36)$$
$$u(z,t) = \sum_{j=1}^{r} h_j(\xi(z,t))C_{kj}\hat{y}(z,t). \quad (37)$$

Here, $\hat{y}(z,t)$ is state vector of controller, A_{kij}, B_{kj}, C_{kj} to be determined. From (7)-(9) and the above, we have

$$y_t(z,t) = \mathcal{A}y(z,t) + \sum_{i=1}^{r}\sum_{j=1}^{r} \bar{h}_{ij}(\xi(z,t))\{A_iy(z,t)$$
$$+B_{1i}C_{kj}\hat{y}(z,t) + A_{d,i}y(z,t-\tau(t)) + C_{1i}w(z,t)\},$$

$$y_c(z,t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\bar{h}_{ij}(\xi(z,t))[D_i y(z,t)+B_{2i}C_{kj}\hat{y}(z,t)+C_{2i}w(z,t)],$$
$$\hat{y}_t(z,t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\bar{h}_{ij}(\xi(z,t))[B_{kj}E_i y(z,t)+A_{kij}\hat{y}(z,t)+B_{kj}C_{3i}w(z,t)].$$

Set

$$e(z,t)=\begin{bmatrix}y(z,t)\\ \hat{y}(z,t)\end{bmatrix}, \tag{38}$$
$$e_t(z,t)=\bar{\Theta}He_{zz}(z,t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\bar{h}_{ij}(\xi(z,t))\{A_{ij}e(z,t)+\bar{A}_{d,i}He(z,t-\tau(t))+B_{kij}w(z,t)\}, \tag{39}$$

where

$$A_{ij}=\begin{bmatrix}A_i & B_{1i}C_{kj}\\ B_{kj}E_i & A_{kij}\end{bmatrix},\ \bar{A}_{d,i}=\begin{bmatrix}A_{d,i}\\ 0\end{bmatrix},$$
$$B_{kij}=\begin{bmatrix}C_{1i}\\ B_{kj}C_{3i}\end{bmatrix},\ H=\begin{bmatrix}I & 0\end{bmatrix},\ \bar{\Theta}=\begin{bmatrix}\Theta\\ 0\end{bmatrix}. \tag{40}$$

Next we give passivity and stability analysis.

Theorem 5: Consider the system (39), suppose $0\le\tau(t)\le\tau,\dot{\tau}(t)\le d<1$. If there exist scalar $\alpha\ge 0$, positive matrices P, Q, and R such that the following are satisfied: $[P\Theta+*]>0$,

$$\begin{bmatrix}\hat{\Omega}_{ij} & P\bar{A}_{d,i} & 0 & PB_{kij}-D_{kij}^T\\ * & -(1-d)Q & 0 & 0\\ * & * & \hat{\mu}R & 0\\ * & * & * & \hat{C}_{2i}\end{bmatrix}<0, \tag{41}$$

where $\hat{\Omega}_{ij}=P[A_{ij}+\hat{\Theta}H]+*+H^TQH+\mu\tau H^TRH, \hat{\Theta}=-\pi^2(l_2-l_1)^{-2}\bar{\Theta}, \hat{\mu}=-\mu\tau(1-d), \hat{C}_{2i}=-(C_{2i}+C_{2i}^T+\alpha I)$, then (39) is asymptotically stable and passive.

Proof: Consider the following Lyapunov functional candidate

$$\begin{aligned}V(t)=&V_1+V_2+V_3\\ =&\int_{l_1}^{l_2}e^T(z,t)Pe(z,t)dz\\ &+\int_{l_1}^{l_2}\int_{t-\tau(t)}^{t}e^T(z,\alpha)H^TQHe(z,\alpha)d\alpha dz\\ &+\mu\int_{l_1}^{l_2}\int_{-\tau(t)}^{0}\int_{t+l}^{t}e^T(z,\beta)H^TRHe(z,\beta)d\beta dldz.\end{aligned}$$

The time derivative of $V(t)$ along the solution to the system (39)

$$\begin{aligned}\dot{V}_1(t)=&2\int_{l_1}^{l_2}e^T(z,t)Pe_t(z,t)dz\\ =&2\int_{l_1}^{l_2}e^T(z,t)P\bar{\Theta}He_{zz}(z,t)dz\\ &+2\int_{l_1}^{l_2}\sum_{i=1}^{r}\sum_{j=1}^{r}\bar{h}_{ij}(\xi(z,t))e^T(z,t)P\{A_{ij}e(z,t)\\ &+\bar{A}_{di}He(z,t-\tau(t))+B_{kij}w(z,t)\}dz.\end{aligned}$$

Considering $0\le\tau(t)\le\tau, 0\le\dot{\tau}\le d<1$

$$\begin{aligned}\dot{V}_2(t)=&\int_{l_1}^{l_2}e^T(z,t)H^TQHe(z,t)dz-(1-\dot{\tau}(t))\\ &\times\int_{l_1}^{l_2}e^T(z,t-\tau(t))H^TQHe(z,t-\tau(t))dz\\ \le&\int_{l_1}^{l_2}e^T(z,t)H^TQHe(z,t)dz-(1-d)\\ &\times\int_{l_1}^{l_2}e^T(z,t-\tau(t))H^TQHe(z,t-\tau(t))dz,\end{aligned}$$
$$\begin{aligned}\dot{V}_3(t)=&\mu\int_{l_1}^{l_2}\int_{-\tau(t)}^{0}e^T(z,t)H^TRHe(z,t)dldz\\ &-\mu\int_{l_1}^{l_2}\int_{-\tau(t)}^{0}e^T(z,t+l)H^TRHe(z,t+l)dldz\\ &+\mu\dot{\tau}(t)\int_{l_1}^{l_2}\int_{t-\tau(t)}^{t}e^T(z,\beta)H^TRHe(z,\beta)d\beta dz.\end{aligned}$$

Considering

$$\begin{aligned}&\int_{l_1}^{l_2}\int_{-\tau(t)}^{0}e^T(z,t+l)H^TRHe(z,t+l)dlds\\ &=\int_{l_1}^{l_2}\int_{t-\tau(t)}^{t}e^T(z,\beta)H^TRHe(z,\beta)d\beta dz.\end{aligned} \tag{42}$$

Consequently

$$\begin{aligned}&\dot{V}_3(t)\\ &\le\mu\tau\int_{l_1}^{l_2}e^T(z,t)H^TRHe(z,t)dz\\ &\quad-\mu(1-d)\int_{l_1}^{l_2}\int_{t-\tau(t)}^{t}e^T(z,\beta)H^TRHe(z,\beta)d\beta dz.\end{aligned}$$

Integrating by parts and taking into account of boundary conditions

$$\begin{aligned}&2\int_{l_1}^{l_2}e^T(z,t)P\bar{\Theta}He_{zz}(z,t)dz\\ &=-2\int_{l_1}^{l_2}e_z^T(z,t)P\bar{\Theta}He_z(z,t)dz\\ &\le-\pi^2(l_2-l_1)^{-2}\int_{l_1}^{l_2}e^T(z,t)[P\bar{\Theta}H+*]e(z,t)dz\\ &=\int_{l_1}^{l_2}e^T(z,t)[P\hat{\Theta}H+*]e(z,t)dz.\end{aligned}$$

Considering

$$y_c(z,t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\bar{h}_{ij}(\xi(z,t))\{D_{kij}e(z,t)+C_{2i}w(z,t)\}.$$

Here $D_{kij} = \begin{bmatrix} D_i & B_{2i}C_{kj} \end{bmatrix}$, so

$$
\begin{aligned}
&2y_c^T(z,t)w(z,t) + \alpha w^T(z,t)w(z,t) \\
&= 2\sum_{i=1}^{r}\sum_{j=1}^{r}\bar{h}_{ij}(\xi(z,t))e^T(z,t)D_{kij}^T w(z,t) \\
&\quad + \sum_{i=1}^{r} h_i(\xi(z,t))w^T(z,t)(C_{2i}+C_{2i}^T+\alpha I)w(z,t). \quad (43)
\end{aligned}
$$

Let

$$
\begin{aligned}
&\xi^T(z,t,\beta) \\
&= \begin{bmatrix} e^T(z,t) & e^T(z,t-\tau(t))H^T & e^T(z,\beta)H^T & w^T(z,t) \end{bmatrix}.
\end{aligned}
$$

Consequently

$$
\begin{aligned}
&\dot{V}(t) - 2\int_{l_1}^{l_2} y_c^T(z,t)w(z,t)dz \\
&\quad - \int_{l_1}^{l_2} \alpha w^T(z,t)w(z,t)dz \\
&\le \frac{1}{\tau}\sum_{i=1}^{r}\sum_{j=1}^{r}\bar{h}_{ij}(\xi(z,t)) \\
&\quad \times \int_{l_1}^{l_2}\int_{t-\tau(t)}^{t} \xi^T(z,t,\beta)\Upsilon_{ij}\xi(z,t,\beta)d\beta dz,
\end{aligned}
$$

where

$$
\Upsilon_{ij} = \begin{bmatrix} \hat{\Omega}_{ij} & P\bar{A}_{d,i} & 0 & PB_{kij} - D_{kij}^T \\ * & -(1-d)Q & 0 & 0 \\ * & * & \hat{\mu}R & 0 \\ * & * & * & \hat{C}_{2i} \end{bmatrix},
$$

where $\hat{\Omega}_{ij}, \hat{\mu}, \hat{C}_{2i}$ are stated as (41). This is the end of proof. □

The following is the solvable of passive dynamic output feedback control problem.

Theorem 6: Consider the system (39) and controller (36), (37), suppose $0 \le \tau(t) \le \tau$, $\dot{\tau}(t) \le d < 1$, If there exist scalar $\alpha \ge 0$, positive matrices X, Y, Q, R, Ω_i, Φ_i, and Ψ_i $(1 \le i \le r)$ such that the following inequalities hold:

$$
\begin{bmatrix} -Y & -I \\ -I & -X \end{bmatrix} < 0, \quad (44)
$$

$$
\begin{bmatrix} X & S \\ S^T & -S^TYW^{-T} \end{bmatrix}\Theta + * > 0, \quad (45)
$$

$$
\begin{bmatrix} \gamma_{11ij} & \gamma_{12i} & 0 & \gamma_{14ij} \\ * & -(1-d)Q & 0 & 0 \\ * & * & \hat{\mu}R & 0 \\ * & * & * & \hat{C}_{2i} \end{bmatrix} < 0, \quad (46)
$$

where

$$
\gamma_{11ij} = \begin{bmatrix} Z_{11ij} & Z_{12i} \\ Z_{21j} & Z_{22ij} \end{bmatrix}, \quad \gamma_{12i} = \begin{bmatrix} A_{d,i} \\ XA_{d,i} \end{bmatrix},
$$

$$
\begin{aligned}
&\gamma_{14ij} = \begin{bmatrix} C_{1i} - Y^TD_i^T - \Psi_i^T \\ XC_{1i} + \Phi_iC_{3i} - D_i^T \end{bmatrix}, \\
&Z_{11ij} = [A_iY + B_{1i}\Psi_j + \Theta Y] + * + Y^TQ_1Y, \\
&Z_{12i} = [A_i + \Theta] + *Y^TQ_1, \\
&Z_{21j} = [\Omega_j + X\Theta W^T + * + Q_1Y, \\
&Z_{22ij} = [XA_i + \Phi_jE_i + X\Theta] + * + Q_1, \\
&Q_1 = Q + \mu\tau R,
\end{aligned}
$$

then (39) is asymptotically stable and passive. In this case, the control gain matrices A_{ki}, B_{ki}, C_{ki} are given as $A_{ki} = S^{-1}(\Omega_i - XA_iY - XB_{1i}\Psi_i - \Phi_iE_iY)W^{-T}, B_{ki} = S^{-1}\Phi_i$, $C_{ki} = \Psi_iW^{-T}$, S and W satisfy $SW^T = I - XY$.

Proof: From (44), we can know $I - XY$ is nonsingular. So there exist nonsingular matrix S and W, such that $SW^T = I - XY$. Set

$$
\Pi_1 = \begin{bmatrix} Y & I \\ W^T & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} I & X \\ 0 & S^T \end{bmatrix}, \quad (47)
$$

we choose

$$
P = \Pi_2\Pi_1^{-1} = \begin{bmatrix} X & S \\ S^T & -S^TYW^{-T} \end{bmatrix}. \quad (48)
$$

After some derivation, we can obtain $P > 0$. Pre- and post-multiplying the matrix (46) with diag$\{\Pi_1^{-T}, I, I, I\}$ and diag$\{\Pi_1^{-1}, I, I, I\}$, we can get (41). According to Theorem 5, we can see the result is correct. This is the end of proof. □

4. TWO NUMERICAL EXAMPLES

This section provides two simulation examples to illustrate the effectiveness of proposed dynamic output feedback control design method.

Example 1: Consider a T–S fuzzy system with the form of (39), the corresponding parameters are given as follows:

$$
\begin{aligned}
&\Theta = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \; A_1 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \; A_2 = \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}, \\
&B_{11} = B_{12} = I, \; B_{21} = B_{22} = I, \; C_{11} = C_{12} = 0.1I, \\
&D_1 = D_2 = 0.1I, \; E_1 = E_2 = 0.1I, \; C_{21} = C_{22} = 10I, \\
&C_{31} = C_{32} = 0.1I, \; A_{d,1} = A_{d,2} = 0.1I, \; \tau(t) = 0.1, \\
&h_1(y(z,t)) = \frac{y_2(z,t) + 1.2}{1.7}, \\
&h_2(y(z,t)) = 1 - \frac{y_2(z,t) + 1.2}{1.7}.
\end{aligned}
$$

The states of the open-loop system are presented in Figs. 1-2, it is obvious that the system is unstable. It is necessary to stabilize the system.

Before control designing, we choose $\alpha = 1$, $\beta = 3$, $\mu = 1$, $\tau = 0.2$, $d = 0.5$, $Q = 2I$, $W = 0.1I$, and $R = 0.1I$. Then

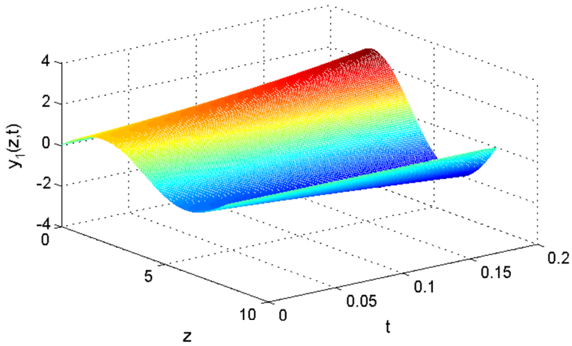

Fig. 1. Open-loop trajectory of $y_1(z,t)$.

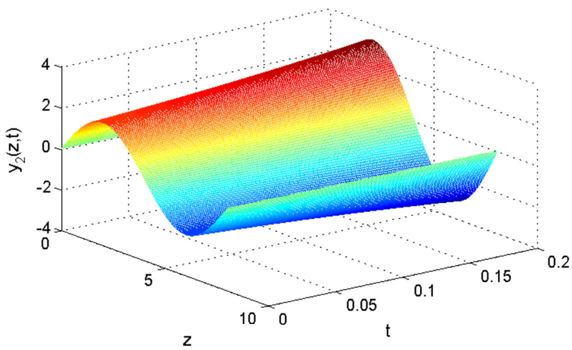

Fig. 2. Open-loop trajectory of $y_2(z,t)$.

based on Theorem 6, we can obtain the controller as

$$A_{k11} = \begin{bmatrix} -285.5200 & -254.2706 \\ -255.6626 & -347.1601 \end{bmatrix},$$

$$A_{k12} = \begin{bmatrix} -285.5200 & -254.2706 \\ -255.6626 & -347.1601 \end{bmatrix},$$

$$A_{k21} = \begin{bmatrix} -301.9097 & -250.3246 \\ -269.0234 & -344.0255 \end{bmatrix},$$

$$A_{k22} = \begin{bmatrix} -301.9097 & -250.3246 \\ -269.0234 & -344.0255 \end{bmatrix},$$

and

$$B_{k1} = \begin{bmatrix} 117.6636 & 118.0398 \\ 121.6092 & 142.7050 \end{bmatrix},$$

$$B_{k2} = \begin{bmatrix} 117.6636 & 118.0398 \\ 121.6092 & 142.7050 \end{bmatrix},$$

$$C_{k1} = \begin{bmatrix} -207.3755 & -0.9285 \\ -0.8382 & -207.2884 \end{bmatrix},$$

$$C_{k2} = \begin{bmatrix} -207.3755 & -0.9285 \\ -0.8382 & -207.2884 \end{bmatrix}.$$

Set the initial condition as $y_1(z,0) = \sin(0.25\pi z)$, $y_2(z,0) = 2\sin(0.25\pi z)$, where $z \in [0, 8]$ and the perturbation input is chosen as $w(z,t) = [\frac{0.1}{30t+50}, \frac{0.1}{30t+50}]^T$. Then

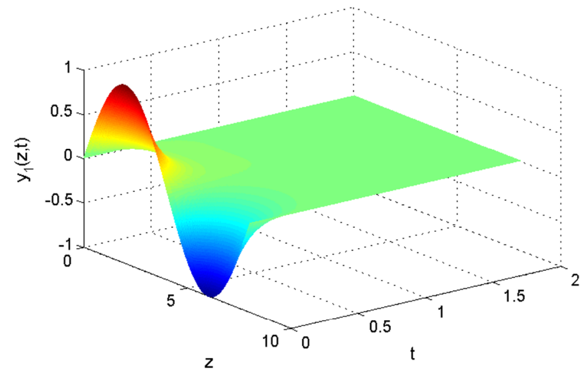

Fig. 3. closed-loop trajectory of $y_1(z,t)$.

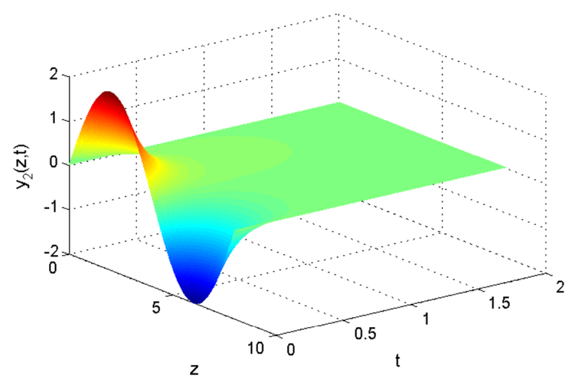

Fig. 4. closed-loop trajectory of $y_2(z,t)$.

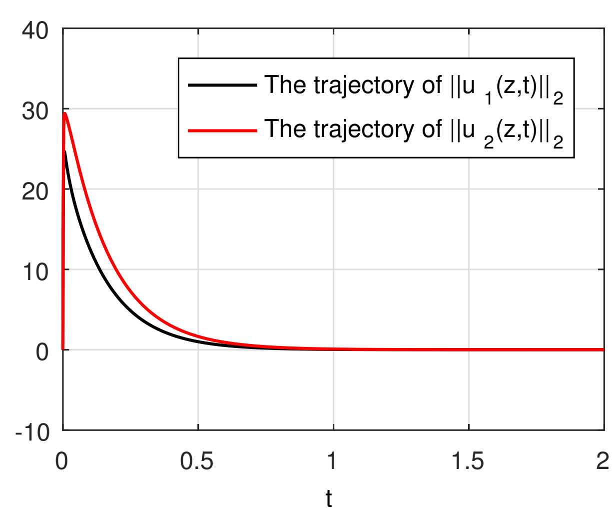

Fig. 5. The trajectories of $\|u_1(z,t)\|_2$ and $\|u_2(z,t)\|_2$.

the simulation results are given as follows: the trajectories of states are presented in Figs. 3-4, the control input are given in Fig. 5. From the figures above, we can conclude that our proposed method is effectiveness.

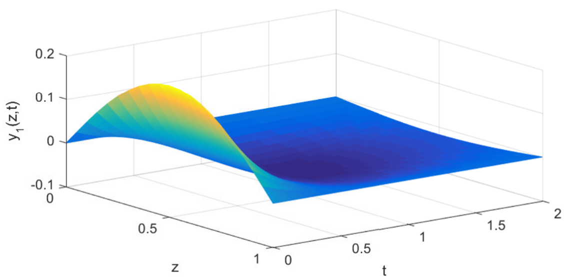

Fig. 6. Open-loop trajectory of $y_1(z,t)$.

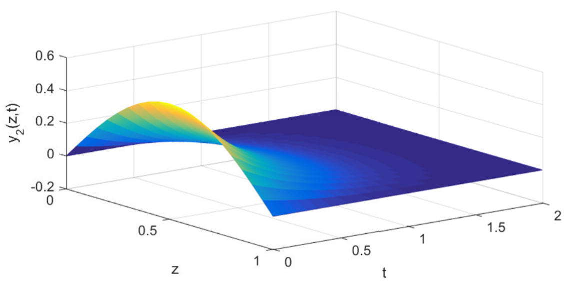

Fig. 7. Open-loop trajectory of $y_2(z,t)$.

Example 2: Consider the following delayed reaction-diffusion equation:

$$\begin{aligned} y_{1,t}(z,t) &= 0.1y_{1,zz}(z,t) + y_1(z,t) - y_1(z,t-\tau(t)) \\ &\quad - y_2(z,t) + u_1(z,t) + 0.1w_1(z,t), \\ y_{2,t}(z,t) &= 0.1y_{1,zz}(z,t) - 0.1y_2(z,t) - y_2(z,t-\tau(t)) \\ &\quad + 0.45y_1(z,t) + u_2(z,t) + 0.1w_2(z,t), \end{aligned}$$

with $y_i(0,t) = y_i(1,t) = 0$, $(i=1,2)$, $y_1(z,0) = 0.2\sin(\pi z)$, and $y_2(z,0) = 0.5\sin(\pi z)$. The open-loop trajectory of $y(z,t)$ is given in Figs. 6–7, from which we can observe $y(z,t) \in [0,\ \alpha]$, $\alpha = 0.5$.

Define $\upsilon(z,t) = y_1^2(z,t)$. Based on [30], the nonlinear system can be represented by the following T–S fuzzy model:

$$\begin{aligned} y_t(z,t) =& \Theta y_{zz}(z,t) + \sum_{i=1}^{2} h_i(\upsilon(z,t))[A_i y(z,t) \\ &+ A_{d,i} y(z,t-\tau(t)) + B_i u(z,t) + C_{1,i} w(z,t)], \end{aligned}$$

where $h_1(\upsilon(z,t)) = \alpha^{-2}\upsilon(z,t)$, $h_2(\upsilon(z,t)) = 1 - h_1(\upsilon(z,t))$, and

$$\Theta = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ A_1 = \begin{bmatrix} 1-\alpha^2 & -1 \\ 0.45 & -0.1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & -1 \\ 0.45 & -0.1 \end{bmatrix},\ A_{d,i} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$
$$B_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \mathrm{C}_{1,i} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ i = 1,2.$$

Choosing the same control and measured output in Example 1, one can obtain

$$A_{k11} = A_{k12} = \begin{bmatrix} -58.2111 & -3.9846 \\ -0.9009 & -28.4839 \end{bmatrix},$$
$$A_{k21} = A_{k22} = \begin{bmatrix} -57.7335 & -3.9880 \\ -0.8912 & -28.4840 \end{bmatrix},$$
$$B_{k1} = B_{k2} = \begin{bmatrix} 16.2609 & 2.3622 \\ 1.2385 & 2.2402 \end{bmatrix},$$
$$C_{k1} = C_{k2} = \begin{bmatrix} -136.0627 & 0.0285 \\ -0.0605 & -132.4640 \end{bmatrix}.$$

Then, the corresponding simulation results are presented in Figs. 8-10, where Figs. 8 and 9 show the closed-loop trajectory of $y(z,t)$ and Fig. 10 presents the control input.

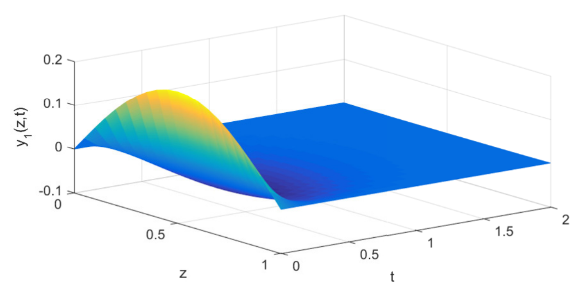

Fig. 8. closed-loop trajectory of $y_1(z,t)$.

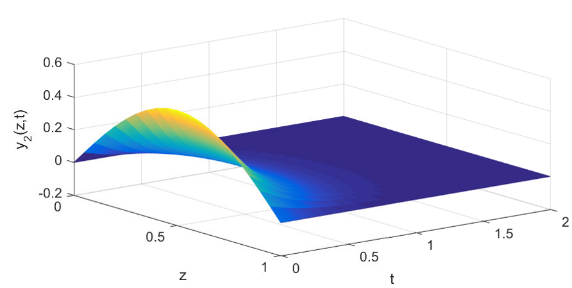

Fig. 9. closed-loop trajectory of $y_2(z,t)$.

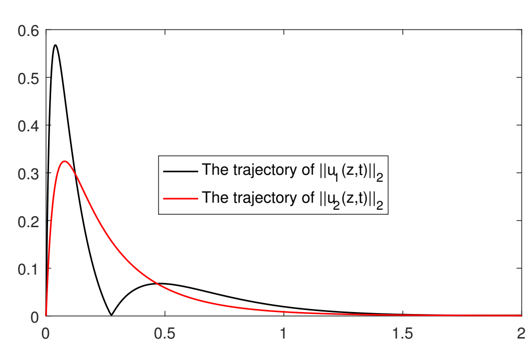

Fig. 10. The trajectories of $\|u_1(z,t)\|_2$ and $\|u_2(z,t)\|_2$.

Remark 3: It is note worthy that one can easily observe the unexpected states' evolutions from the open-loop trajectories. Therefore, a dynamic controller is designed. From the closed-loop trajectories, we find that the stabilization time is smaller than the open-loop system. Furthermore, the norm of the control input is also shown in Fig. 10 to understand the system design.

5. CONCLUSION

In this paper, static and dynamic fuzzy controllers are designed for a class of nonlinear distributed parameter systems with time-varying delay. They both guarantee the stability and passivity of the closed-loop system. Our work is based on the assumption that the semi-linear systems can be represented by the T-S fuzzy PDE model. We investigate stability, passivity, static fuzzy controller and dynamic fuzzy controller. Finally, two numerical examples have been given to show the correctness of the derived results.

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