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H_∞ Control of Markovian Jump Systems with Incomplete Knowledge of Transition Probabilities

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Abstract: An H_∞ state-feedback controller for Markovian jump systems with incomplete knowledge of transition probabilities and input quantization is proposed. To derive the less conservative stabilization conditions, the conditions are developed into the second-order matrix polynomials of the unknown transition rate using an appropriate weighting method. Furthermore, the proposed controller not only accomplishes an H_∞ performance but also removes the matched disturbances and the effect of input quantization. Two examples show the effectiveness of the proposed method.

Keywords: H_∞ control, input quantization, linear matrix inequality, Markovian jump system.

1. INTRODUCTION

During the past decades, Markovian jump systems (MJSs) have received a lot of attention because the representation of many dynamics systems subject to random abrupt variations is achieved. Therefore, MJSs with mode transition by a Markov chain taking values in a finite set have been studied extensively [1–8]. Furthermore, MJSs have been widely applied in many practical systems, such as actuator saturation [2] , manufacturing systems [9], networked control systems (NCSs) [9], and economic systems [11], power systems [12]. For these topics, many studies about MJSs have been developed under the assumption that the exact values of transition probabilities are known [1, 4, 6, 8].

However, another issue has been focused on in the MJSs with incomplete knowledge of transition probabilities, because it is difficult to obtain the complete knowledge of transition probabilities in the practical systems. Recently, the research on controller synthesis for such systems employed the free-connections weighting method and the linear matrix inequalities (LMIs) [5,7,13] . The robust stabilization condition for the MJS with known transition probabilities and incomplete transition probabilities with its bounds was introduced [13].

Furthermore, in modern control systems, the plant and the controller are linked through a network. Such a structure is called network control systems (NCSs). NCSs require many kinds of data-processing devices, such as encoders, decoders, A/D converters, and D/A converters. This has such benefits as easy maintenance, simple installation, and flexibility of the system change [14–16]. These systems have many advantages for the control system engineering, but some new issues are found with packet dropout, networked-induced delay, and input quantization. These issues can degrade the performance of the control systems and the stability of the systems [17]. Therefore, a significant problem is not only the model uncertainty, such as incomplete knowledge of the transition probability, but also the uncertainty of the data transfer. In most of the previous research, it has been assumed that the data transfer is ideal without packet dropout, network induced delay, and input quantization. However, it is not a practical system. Among the above issues, input quantization has a serious effect on the performance of the control systems. Recently, many studies have addressed the stabilization problem of the systems with input quantization for NCSs [18] and non-NCSs [19, 20].

To the best of the author's knowledge, intensive studies on H_∞ control of MJSs with incomplete knowledge of transition probabilities and input quantization have not yet been conducted. Even if the state-feedback H_∞ controller and dynamic output-feedback controller for quantized discrete-time linear time-invariant systems are proposed using the dynamic quantizers [21, 22]. The switching controller for Takagi-Sugeno fuzzy systems are introduced [23]. Recently, the stabilization of MJSs with incomplete transition probabilities and input quantization

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was handled [24], it is far from the stabilization of the practical systems, because the disturbances are not considered.

As a result of the above observations, an H_∞ state-feedback controller for MJSs with incomplete knowledge of transition probabilities and input quantization is introduced. The proposed H_∞ controller consists of linear and nonlinear parts: a linear part to accomplish H_∞ performance against the mismatched part of the external disturbances, and a nonlinear part to remove the matched part of the external disturbances and the effect of input quantization. In other words, the nonlinear part is an integer multiple of the quantization level that depends on the magnitude of the matched disturbance. It removes the energy in the sense of Lyapunov caused by quantization errors and the matched part of the external disturbances . The controller is designed for each mode in the MJSs. The main contributions of this research are as follows:

- A state-feedback controller was designed to accomplish asymptotic stability and H_∞ performance for MJSs with incomplete knowledge of transition probabilities and input quantization, where such systems have not yet been introduced.
- To improve the H_∞ performance, the external disturbances are divided into the matched part and the mismatched part using the projection matrix for each mode.
- To derive the less conservative stabilization conditions, the derived conditions are developed into the second-order matrix polynomials of the unknown transition rate using an appropriate weighting method.

The performance of the proposed controller is represented by a numerical example and a practical example.

This work is organized as follows. Section 2 provides a system description and some preliminary results. Section 3 introduces an H_∞ controller for MJSs with incomplete knowledge of transition probabilities and input quantization. Section 4 shows simulation results for verifying the proposed controller. Finally, Section 5 gives a summarization of the study.

Notation: The notations $X \geq Y$ and $X > Y$ mean that $X-Y$ is positive semidefinite and positive definite, respectively. The operator $\lceil \alpha \rceil$ denotes the nearest integer greater than or equal to a scalar α. The notation $diag(X,Y)$ indicates a diagonal matrix with diagonal entries X and Y. In symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry. Furthermore, $\mathbf{He}(X) = X + X^T$ stands for any matrix X. For any matrices S_i and S_{ij},

$$[S_i]_{i\in\{1,2,\ldots,N\}} = [S_{11}, S_{12}, \ldots, S_{1N}],$$

$$[S_{ij}]_{i,j\in\{1,2,\ldots,N\}} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix}.$$

In symmetric block matrices, $(*)$ is used as an ellipsis for therms that are induced by symmetry. We also use $\|x\|_p$ to indicate the p-norm of x, *i.e.*, $\|x\|_p \triangleq (|x_1|^p + \cdots + |x_n|^p)^{\frac{1}{p}}$, $p \geq 1$. When $p = \infty$, $\|x\|_\infty \triangleq \max_{1\leq i\leq n} |x_i|$. For $X \in \mathbf{R}^{m\times n}$, $\|X\|_p$ denotes the matrix p-norm, *i.e.*, $\|X\|_p \triangleq \sup_{x\neq 0} \frac{\|Xx\|_p}{\|x\|_p}$. The space of square-integrable functions is denoted by $\mathcal{L}_2$, that is, for any $x \in \mathcal{L}_2$,

$$\|x\|_2 \triangleq \left(\int_0^\infty x^T(t)x(t)dt\right)^{1/2} < \infty.$$

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following continuous-time MJS with input quantization and external disturbance.

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)Q(u(t)) + D(r_t)\omega(t), \quad (1)$$

$$z(t) = C(r_t)x(t), \quad (2)$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the control input, $\omega(t) \in \Re^q$ is the disturbance, and $z(t) \in \Re^q$ is the controlled output. $Q(\cdot)$ is the quantization operator. Here, $\{r_t, t \geq 0\}$ is a continuous-time Markov jumping process in a finite set $D = \{1,2,3,\ldots,N\}$ and has the mode transition probabilities

$$P(r_{t+\delta} = j | r_t = i) = \begin{cases} \pi_{ij}\delta t + o(\delta t) & \text{if } i \neq j, \\ 1 + \pi_{ij}\delta t + o(\delta t) & \text{if } i = j, \end{cases} \quad (3)$$

where $\delta t > 0$, $\lim_{\delta t\to 0} o(\delta t)/\delta t = 0$, and π_{ij} is the transition rate from mode i to j at time $t + \delta t$. For $r_t = i \in D$, to simplify the notation, $A(r_t) = A_i$, $B(r_t) = B_i$, $D(r_t) = D_i$, and $C(r_t) = C_i$.

Furthermore, the transition rate matrix Π belongs to

$$S_\Pi \triangleq \left\{ [\pi_{ij}]_{i,j\in D} | 0 \leq \pi_{ij} \text{ for } i \neq j, \pi_{ii} = -\sum_{j=1, i\neq j}^{N} \pi_{ij} \right\}. \quad (4)$$

In view of the above relations, π_{ij} accords with the following relationships, for all $i, j \in D$:

$$\mu_{ij}\pi_{ij} \geq 0, \ \sum_{j=1}^{N} \pi_{ij} = 0, \ -\mu_{ij}\pi_{ij}(\pi_{ij} + \pi_{ii}) \geq 0, \quad (5)$$

where

$$\mu_{ij} = \begin{cases} 1, & i \neq j, \\ -1, & i = j. \end{cases}$$

For later convenience, two sets are defined with respect to the measurability of the transition rate for $i, j \in D$:

$$D_i^+ \triangleq \{j \mid \pi_{ij} \text{ is known for } i\}, \tag{6}$$

$$D_i^- \triangleq \{j \mid \pi_{ij} \text{ is unknown for } i\}. \tag{7}$$

For the quantization, it is assumed that the operator $Q(\cdot)$ is defined by a function $round(\cdot)$ that rounds toward the nearest integer, i.e.,

$$Q(u(t)) \triangleq \varepsilon_u\, round\,(u(t)/\varepsilon_u), \tag{8}$$

where $\varepsilon_u(>0)$ is called a *quantizing level* and $Q(\cdot)$ is the uniform quantizer with the fixed ε_u. It is noted that the quantization error $\nabla u(t)$ is defined as

$$\nabla u(t) \triangleq Q(u(t)) - u(t). \tag{9}$$

Based on the quantization in (8) and the definition of $\nabla u(t)$, each component of $\nabla u(t)$ at time t is bounded by the half of the quantizing level ε_u, i.e.,

$$\|\nabla u(t)\|_\infty \le \varepsilon_u/2. \tag{10}$$

The following lemmas are required to prove a main result.

Lemma 1 [5]**:** Suppose that there exist $P_i > 0$, for all $i \in D$, and γ such that

$$\begin{bmatrix} (1,1) & P_iD_i \\ (*) & -\gamma^2 I \end{bmatrix} < 0, \tag{11}$$

where $(1,1) = \mathbf{He}(P_iA_i) + \sum_{j=1}^N \pi_{ij}P_j + C_i^TC_i$. Then the MJS in (1) with $u(t) = 0$ is stochastically stable with H_∞ performance γ.

Lemma 2 (Hölder's inequality)**:** For $\alpha, \beta \in \mathbf{R}^n$, $p \ge 1$, and $q \ge 1$, the following inequality holds:

$$|\alpha^T\beta| \le ||\alpha||_p||\beta||_q, \quad p^{-1} + q^{-1} = 1. \tag{12}$$

3. MAIN RESULTS

In this section, the H_∞ state-feedback controller design problem is considered. System (1) yields to

$$\dot{x}(t) = A_ix(t) + B_i\{Q(u(t)) + \tilde{B}_i\omega(t)\} + \bar{B}_i\omega(t), \tag{13}$$

where

$$\tilde{B}_i = (B_i^TB_i)^{-1}B_i^TD_i, \bar{B} = (I - B_i(B_i^TB_i)^{-1}B_i^T)D_i.$$

The external disturbance $D_i\omega(t)$ can be separated into the matched part $\tilde{B}_i\omega(t)$ and the mismatched part $\bar{B}_i\omega(t)$ using the projection matrix. For $\tilde{B}_i$ and $\omega(t)$, it is assumed that the following conditions are valid:

- (A1)

$$\|\omega(t)\|_\infty \le \varepsilon_\omega. \tag{14}$$

- (A2)

$$\|\tilde{B}\|_\infty = \rho. \tag{15}$$

A controller is proposed for the system (1) as

$$u(t) = K(r_t)x(t) + \bar{u}(r_t, x(t)), \tag{16}$$

where $K(r_t)$ is a linear controller part and $\bar{u}(r_t, x(t))$ is a nonlinear controller part to reject the matched disturbance $\tilde{B}_i\omega(t)$ and the effect of quantization errors. In the following, for $r_t = i \in D$, $K(r_t) = K_i$ and $\bar{u}(r_t, x(t)) = \bar{u}_i(x(t))$. Using the system (13) and the proposed controller (16), the resultant closed-loop system is given as follows:

$$\begin{aligned}\dot{x}(t) =& A_ix(t) + B_i\{Q(K_ix(t) + \bar{u}_i(x(t))) + \tilde{B}_i\omega(t)\} \\ &+ \bar{B}_i\omega(t).\end{aligned} \tag{17}$$

Theorem 1: Consider the system (13) with incomplete knowledge of the transition rate. For $i, j \in D$, suppose that there exist symmetric matrices $\bar{P}_i$ and Q_{ij}, matrices $\bar{K}_i$, Λ_{ij}, Y_{ij}, S_{i0}, S_{ij}, Y_{ij}, and a scalar γ such that

$$\bar{P}_i > 0, \tag{18}$$

$$\Lambda_{ij} + \Lambda_{ij}^T > 0, Y_{ij} + Y_{ij}^T > 0, \tag{19}$$

$$\begin{bmatrix} Q_{ij} & \bar{P}_j \\ (*) & \bar{P}_i \end{bmatrix} > 0, \quad i \ne j, \tag{20}$$

$$\begin{bmatrix} \bar{L}_i & [L_j]_{j \in D_i^-} \\ (*) & [L_{jl}]_{j,l \in D_i^-} \end{bmatrix} < 0, \tag{21}$$

where

- $i \in D_i^+$

$$\begin{aligned}\bar{L}_i =& \Omega_i^0 + \Pi_i^+E^T\mathbf{He}(S_{i0})E \\ &+ \sum_{j \in D_i^+} \mu_{ij}\pi_{ij}E^T\mathbf{He}(\Lambda_{ij})E \\ &- \sum_{j \in D_i^+} \mu_{ij}\pi_{ij}(\pi_{ij} + \pi_{ii})E^T\mathbf{He}(Y_{ij})E, \\ L_j =& \frac{1}{2}E^T\Omega_{ij}^1 + E^TS_{i0} + \Pi_i^+E^TS_{ij} + \mu_{ij}E^T\Lambda_{ij} \\ &- \mu_{ij}\pi_{ii}E^TY_{ij}, \\ L_{jj} =& \mathbf{He}(S_{il}) - \mu_{ij}\mathbf{He}(Y_{ij}), \\ L_{jl} =& S_{il} + S_{ij},\end{aligned}$$

- $i \in D_i^-$

$$\begin{aligned}\bar{L}_i =& \Omega_i^0 + \Pi_i^+E^T\mathbf{He}(S_{i0})E \\ &+ \sum_{j \in D_i^+} \mu_{ij}\pi_{ij}E^T\mathbf{He}(\Lambda_{ij})E \\ &- \sum_{j \in D_i^+} \mu_{ij}\pi_{ij}^2E^T\mathbf{He}(Y_{ij})E, \\ L_j =& \frac{1}{2}E^T\Omega_{ij}^1 + E^TS_{i0} + \Pi_i^+E^TS_{ij} + \mu_{ij}E^T\Lambda_{ij}\end{aligned}$$

$$- \sum_{j\in D_i^+} c_{ij}\mu_{ij}\pi_{ij}E^T Y_{ij},$$

$$L_{jj} = \mathbf{He}(S_{il}) - 2\mu_{ij}\mathbf{He}(Y_{ij}),$$

$$L_{jl} = \begin{cases} S_{il}+S_{ij}, & l \neq i, \\ S_{il}+S_{ij}-\mu_{ij}Y_{ij}, & l = i, \end{cases}$$

$$\Pi_i^+ = \sum_{j\in D^+} \pi_{ij},$$

$$\Omega_i^0 \triangleq \begin{bmatrix} \mathbf{He}(A_i\bar{P}_i+B_i\bar{K}_i)+\sum_{j\in D_i^+}\pi_{ij}G_{ij} & \bar{B}_i & \bar{P}_iC_i^T \\ (*) & -\gamma^2 I & 0 \\ (*) & (*) & -I \end{bmatrix},$$

$$\Omega_{ij}^1 \triangleq \kappa_{ij}Q_{ij} + (1-\kappa_{ij})\bar{P}_i, \ \kappa_{ij} = \begin{cases} 1, & i\neq j, \\ 0, & i=j, \end{cases}$$

$$E = \begin{bmatrix} I & 0 & 0 \end{bmatrix} \in \Re^{n\times(n+p+q)}.$$

Then, the proposed system (1) is stochastically stable with γ-disturbance attenuation. Furthermore, the proposed controller is constructed as $u(t) = K_i x(t) + \bar{u}_i(x(t))$ for the mode i, where $K_i = \bar{K}_i\bar{P}_i^{-1}$ and each component of $\bar{u}_i(x(t))$ is defined as

$$\bar{u}_{i,k}(x(t)) = -\varepsilon_u N \mathrm{sgn}(\sigma_{i,k}(x(t))), \tag{22}$$

where

$$\bar{u}_i(x(t)) = \begin{bmatrix} \bar{u}_{i,1}(x(t)) & \bar{u}_{i,2}(x(t)) & \cdots & \bar{u}_{i,m}(x(t)) \end{bmatrix}^T,$$

$$N = \left\lceil \frac{\varepsilon_u/2+\rho\varepsilon_\omega}{\varepsilon_u} \right\rceil, \ \sigma_i(x(t)) \triangleq B_i^T P_i x(t).$$

Proof: Choose $V(x(t)) = x^T(t)P(r_t)x(t)$ as a Lyapunov function, where $P(r_t)$ is a positive definite matrix. Then, the weak infinitesimal operator $\mathfrak{L}$ of the stochastic process $x(t)$ acting on $V(x(t))$ is given by

$$\begin{aligned}\mathfrak{L}V =& 2x^T(t)P_i\dot{x}(t) + x^T\sum_{j=1}^N \pi_{ij}P_jx(t) \\ =& 2x^T(t)P_iA_ix(t) + 2x^T(t)P_iB_i\{Q(u(t))+\tilde{B}_i\omega(t))\} \\ &+ 2x^T(t)P_i\bar{B}_i\omega(t) + x^T(t)\sum_{j=1}^N \pi_{ij}P_jx(t).\end{aligned}$$

Using the quantization error (9), the second term of $\mathfrak{L}V$ can be rewritten as

$$\begin{aligned}&2x^T(t)P_iB_i\{Q(u(t))+\tilde{B}_i\omega(t))\} \\ &= 2x^T(t)P_iB_i\{\nabla u(t)+u(t)+\tilde{B}_i\omega(t)\} \\ &= 2\sigma_i^T(x(t))\{\nabla u(t)+K_ix(t)+\bar{u}_i(x(t))+\tilde{B}_i\omega(t)\}.\end{aligned}$$

Therefore, $\mathfrak{L}V$ can be written as follows:

$$\begin{aligned}\mathfrak{L}V =& 2x^T(t)\{P_iA_i+P_iB_iK_i\}x(t) \\ &+ 2\sigma_i^T(x(t))(\nabla u(t)+\bar{u}_i(x(t))+\tilde{B}_i\omega(t)) \\ &+ 2x^T(t)P_i\bar{B}_i\omega(t) + x^T(t)\sum_{j=1}^N \pi_{ij}P_jx(t).\end{aligned}$$

Using the bound of the quantization error (10), the assumption (14), (15), the relations $|\alpha^T\beta| \leq ||\alpha||_1||\beta||_\infty$ from Lemma 2, and $||X\alpha||_\infty \leq ||X||_\infty||\alpha||_\infty$ from the definition of the matrix p-norm, it is shown that $\bar{u}_i(x(t))$ in (22) ensures that the second term of $\mathfrak{L}$ is negative, i.e.,

$$\begin{aligned}&2\sigma_i^T(x(t))(\nabla u(t)+\bar{u}_i(x(t))+\tilde{B}\omega(t)) \\ &= 2\sigma_i^T(x(t))\nabla u(t) + 2\sigma_i^T(x(t))\bar{u}_i(x(t)) + 2\sigma_i^T(x(t))\tilde{B}\omega(t) \\ &\leq 2\|\sigma_i(x(t))\|_1\|\nabla u(t)\|_\infty + 2\sigma_i^T(x(t))\bar{u}_i(x(t)) \\ &\quad + 2\|\sigma_i(x(t))\|_1\|\tilde{B}_i\|_\infty\|\omega\|_\infty \\ &\leq \varepsilon_u\|\sigma_i(x(t))\|_1 - 2\varepsilon_uN\|\sigma_i(x(t))\|_1 + 2\rho\varepsilon_\omega\|\sigma_i(x(t))\|_1 \\ &= 2\left(\frac{\varepsilon_u}{2} - \varepsilon_uN + \rho\varepsilon_\omega\right)\|\sigma_i(x(t))\|_1 \\ &= 2\varepsilon_u\left(\frac{\varepsilon_u/2+\rho\varepsilon_\omega}{\varepsilon_u} - N\right)\|\sigma_i(x(t))\|_1,\end{aligned}$$

where by choosing N as $\lceil \frac{\varepsilon_u/2+\rho\varepsilon_\omega}{\varepsilon_u} \rceil$ and using the relation $-\lceil a \rceil \leq -a$ for a scalar $a \geq 0$, one can make the second term of $\mathfrak{L}V(x(t))$ negative. Then, $\mathfrak{L}V(x(t))$ can be rewritten as

$$\begin{aligned}\mathfrak{L}V(x(t)) \leq& 2x^T(t)\{P_iA_i+P_iB_iK_i\}x(t) \\ &+ 2x^T(t)P_i\bar{B}_i\omega(t) + x^T(t)\sum_{j=1}^N \pi_{ij}P_jx(t).\end{aligned} \tag{23}$$

Using Lemma (1), equation (23) can be converted into the following condition:

$$\begin{bmatrix} \mathbf{He}(P_iA_i+P_iB_iK_i)+\sum_{j=1}^N\pi_{ij}P_j+C_i^TC_i & P_i\bar{B}_i \\ (*) & -\gamma^2 I \end{bmatrix} < 0. \tag{24}$$

By pre- and post-multiplying (24) with $diag\{P_i^{-1}, I\}$ and its transpose,

$$\begin{bmatrix} \mathbf{He}(A_i\bar{P}_i+B_i\bar{K}_i)+\sum_{j=1}^N\pi_{ij}\bar{P}_iP_j\bar{P}_i+\bar{P}_iC_i^TC_i\bar{P}_i & \bar{B}_i \\ (*) & -\gamma^2 I \end{bmatrix} < 0, \tag{25}$$

where $\bar{P}_i = P_i^{-1}$ and $\bar{K}_i = K_i\bar{P}_i$.

Using Schur's complement to (25) results in

$$\begin{bmatrix} \mathbf{He}(A_i\bar{P}_i+B_i\bar{K}_i)+\sum_{j=1}^N\pi_{ij}\bar{P}_iP_j\bar{P}_i & \bar{B}_i & \bar{P}_iC_i^T \\ (*) & -\gamma^2 I & 0 \\ (*) & (*) & -I \end{bmatrix} < 0. \tag{26}$$

Note that for $i = j$, $\bar{P}_iP_j\bar{P}_i = \bar{P}_i$ and for $i \neq j$, equation (20) leads to $\bar{P}_iP_j\bar{P}_i \leq Q_{ij}$, equation (26) holds by the following condition:

$$\begin{bmatrix} \mathbf{He}(A_i\bar{P}_i+B_i\bar{K}_i)+\sum_{j=1}^N\pi_{ij}G_{ij} & \bar{B}_i & \bar{P}_iC_i^T \\ (*) & -\gamma^2 I & 0 \\ (*) & (*) & -I \end{bmatrix} < 0, \tag{27}$$

where $G_{ij} \triangleq \kappa_{ij}Q_{ij} + (1-\kappa_{ij})\bar{P}_i$.

To derive the linear matrix inequality (LMI) conditions, equation (27) can be written as follows:

$$0 > \Omega_i \triangleq \Omega_i^0 + \sum_{j\in D_i^-} \pi_{ij}E^T\Omega_{ij}^1 E. \tag{28}$$

In addition, from the condition (5), it follows under (19) that

$$\begin{aligned} C_i^1 &\triangleq \mathbf{He}\left(\left(\Pi_i^+ + \sum_{j\in D_i^-}\pi_{ij}\right)E^T\left(S_{i0} + \sum_{j\in D_i^-}\pi_{ij}S_{ij}\right)E\right) \\ &= 0, \end{aligned} \tag{29}$$

$$C_i^2 \triangleq \sum_{j=1}^{N}\mu_{ij}\pi_{ij}E^T\mathbf{He}(\Lambda_{ij})E \geq 0, \tag{30}$$

$$C_i^3 \triangleq -\sum_{j=1}^{N}\mu_{ij}\pi_{ij}(\pi_{ij}+\pi_{ii})E^T\mathbf{He}(Y_{ij})E \geq 0. \tag{31}$$

Then, the positive definite matrix N_i is constructed by (30) and (31) as the following form.

$$\begin{aligned} N_i \triangleq& C_i^1 + C_i^2 + C_i^3 \\ =& N_{i0} + \sum_{j\in D_i^-}\pi_{ij}\mathbf{He}(N_{ij}E) \\ &+ \sum_{j\in D_i^-}\sum_{l\in D_i^-, l\geq j}\pi_{ij}\pi_{il}E^T\mathbf{He}(N_{i,jl})E \geq 0, \end{aligned}$$

where

- $i \in D_i^+$

$$\begin{aligned} N_{i0} =& \Pi_i^+E^T\mathbf{He}(S_{i0})E + \sum_{j\in D_i^+}\mu_{ij}\pi_{ij}E^T\mathbf{He}(\Lambda_{ij})E \\ &- \sum_{j\in D_i^+}\mu_{ij}\pi_{ij}(\pi_{ij}+\pi_{ii})E^T\mathbf{He}(Y_{ij})E, \\ N_{ij} =& E^TS_{i0} + \Pi_i^+E^TS_{ij} + \mu_{ij}E^T\Lambda_{ij} - \mu_{ij}\pi_{ii}E^TY_{ij}, \\ N_{i,jl} =& \begin{cases} S_{il}+S_{ij}, & j\neq l, \\ S_{il} - \mu_{ij}Y_{ij}, & j = l, \end{cases} \end{aligned}$$

- $i \in D_i^-$

$$\begin{aligned} N_{i0} =& \Pi_i^+E^T\mathbf{He}(S_{i0})E + \sum_{j\in D_i^+}\mu_{ij}\pi_{ij}E^T\mathbf{He}(\Lambda_{ij})E \\ &- \sum_{j\in D_i^+}\mu_{ij}\pi_{ij}^2E^T\mathbf{He}(Y_{ij})E, \\ N_{ij} =& E^TS_{i0} + \Pi_i^+E^TS_{ij} + \mu_{ij}E^T\Lambda_{ij} \\ &- \sum_{j\in D_i^+}c_{ij}\mu_{ij}\pi_{ij}E^TY_{ij}, \\ N_{i,jl} =& \begin{cases} S_{il}+S_{ij}, & j\neq l,\ l\neq i, \\ S_{il}+S_{ij} - \mu_{ij}Y_{ij}, & j\neq l,\ l = i, \\ S_{il} - 2\mu_{ij}Y_{ij}, & j = l,\ l = i, \end{cases} \end{aligned}$$

where

$$c_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

By the S-procedure, if $\Omega_i < 0$ whenever $N_i \geq 0$, then the following sufficient condition is formulated as

$$N_i + \Omega_i < 0, \tag{32}$$

which can be converted into the following LMI condition.

$$\begin{bmatrix} I \\ [\pi_{ij}E]^T_{j\in D_i^-} \end{bmatrix}^T \begin{bmatrix} \bar{L}_i & [L_j]_{j\in D_i^-} \\ (*) & [L_{jl}]_{j,l\in D_i^-} \end{bmatrix} \begin{bmatrix} I \\ [\pi_{ij}E]^T_{j\in D_i^-} \end{bmatrix} < 0, \tag{33}$$

where

$$\bar{L}_i = \Omega_i^0 + N_{i0},\ \ L_j = \frac{1}{2}E^T\Omega_{ij}^1 + N_{ij},\ \ L_{jl} = N_{i,jl}.$$

Then, (33) holds by the LMI conditions (18)-(21). □

4. NUMERICAL EXAMPLES

In this section, the H_∞ performance of the proposed controller is considered to verify the effectiveness of the proposed method.

4.1. Example 1

Consider an MJS with four modes($N = 4$), whose systems matrices are

$$A_1 = \begin{bmatrix} 0.35 & -7.30 \\ 1.48 & 0.81 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.89 & -3.11 \\ 1.48 & 0.21 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.11 & -0.85 \\ 2.31 & -0.10 \end{bmatrix},\ A_4 = \begin{bmatrix} -0.17 & -1.48 \\ 1.59 & -0.27 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.57 \\ 1.23 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.78 \\ -0.49 \end{bmatrix},\ B_3 = \begin{bmatrix} 1.34 \\ 0.39 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} -0.38 \\ 1.07 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.0 \\ -0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.1 \\ 0.0 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 0.0 \\ 0.1 \end{bmatrix},\ C_4 = \begin{bmatrix} 0.1 \\ 0.0 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.15 \\ 0.0 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0.0 \\ 0.4 \end{bmatrix},\ D_4 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} -1.3 & 0.2 & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & 0.3 & 0.3 \\ 0.6 & \pi_{32} & -1.5 & \pi_{34} \\ 0.4 & \pi_{42} & \pi_{43} & \pi_{44} \end{bmatrix},\ \ \varepsilon_u = 0.1,$$

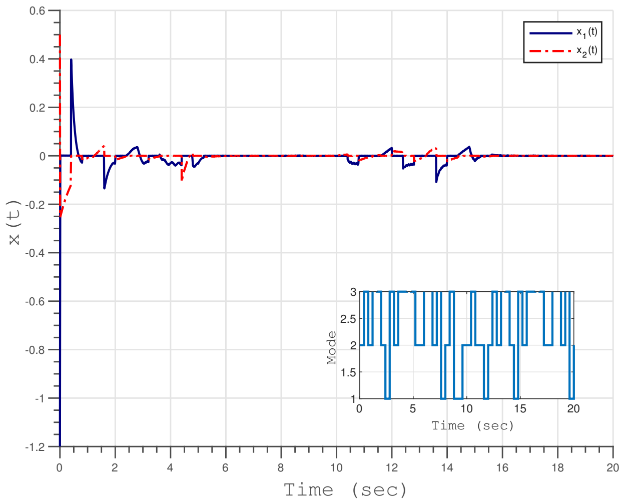

Fig. 1. State trajectories for Example 1.

where π_{13}, π_{14}, π_{21}, π_{22}, π_{32}, π_{34}, π_{42}, π_{43}, and π_{44} are unknown transition probabilities. Using the above transition probability matrix, the following sets can be obtained.

$$D_1^+ = \{1,2\},\ D_2^+ = \{3,4\},$$
$$D_3^+ = \{1,3\},\ D_4^+ = \{1\},$$
$$D_1^- = \{3,4\},\ D_2^- = \{1,2\},$$
$$D_3^- = \{2,4\},\ D_4^- = \{2,3,4\}.$$

Letting the initial condition $x(0) = [-1.2\ 0.5]^T$, the state trajectories of the closed-loop system shown in Fig. 1 is stochastically stable with incomplete knowledge of transition rates under the input quantization and the external disturbances.

By Theorem 1, the H_∞ performance $\gamma = 1.6634 \times 10^{10}$ and the proposed controller gains are obtained as follows.

$$K_1 = \begin{bmatrix} -4.1749 \times 10^{-1} & -3.3041 \times 10^3 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -1.2015 \times 10^3 & 1.4393 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -3.6401 \times 10^{-1} & -1.9419 \times 10^3 \end{bmatrix},$$
$$K_4 = \begin{bmatrix} 1.5652 \times 10^4 & 1.6342 \end{bmatrix}.$$

Fig. 2 shows the control input, where $r_0 = 2$ and for $10 \le t \le 15$, $\omega(t) = 0.1\sin(3t^2 + 0.8) + 0.2$ and otherwise, $\omega(t) = 0$.

4.2. Example 2

Consider the following inverted pendulum system [25] with two modes.

$$\dot{x}_1(t) = x_2(t),$$
$$\dot{x}_2(t) = \frac{g}{l}\sin x_1(t) + \frac{NK_m}{ml^2}x_3(t),$$
$$L_a\dot{x}_3(t) = K_bNx_2(t) - R(r_t)x_3(t) + Q(u(t)), \qquad (34)$$

where $x_1(t)$ is an angle of the inverted pendulum, $x_2(t)$ is an angular velocity, $x_3(t)$ is an input current, $u(t)$ is the control input voltage, g is the gravity acceleration of gravity, m and l are the mass and length of the inverted pendulum, K_b is the back emf constant, K_m is the motor torque constant, and N is the gear ratio. Here, $R(r_t)$ is the resistance in the DC motor, which is defined as

$$R(r_t) = \begin{cases} R_a & \text{if } r_t = 1, \\ R_b & \text{if } r_t = 2. \end{cases}$$

Let $L_a = 1$, $g = 9.8$ m/s^2, $l = 1$ m, $m = 1$ kg, $N = 10$, $K_m = 0.1$ Nm/A, $K_b = 0.1$ Vs/rad, $R_a = 1\ \Omega$, and $R_b = 0.5\ \Omega$.

Using the above parameters, the system (34) is the following linearized model.

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)Q(u(t)) + D(r_t)\omega(t), \qquad (35)$$
$$y(t) = C(r_t)x(t), \qquad (36)$$

where

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^T,$$
$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & 1 & -0.5 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.2 & 0 & 0 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix},\ D_2 = \begin{bmatrix} 0 & 0 & 0.4 \end{bmatrix},\ \varepsilon_u = 0.1,$$
$$\Pi = \begin{bmatrix} -0.6127 & 0.6127 \\ \pi_{21} & \pi_{22} \end{bmatrix},\ \omega(t) = e^{-0.6t}\sin 100t^2,$$

where π_{21} and π_{22} are unknown transition rates. By Theorem 1, the proposed controller gains are obtained as fol-

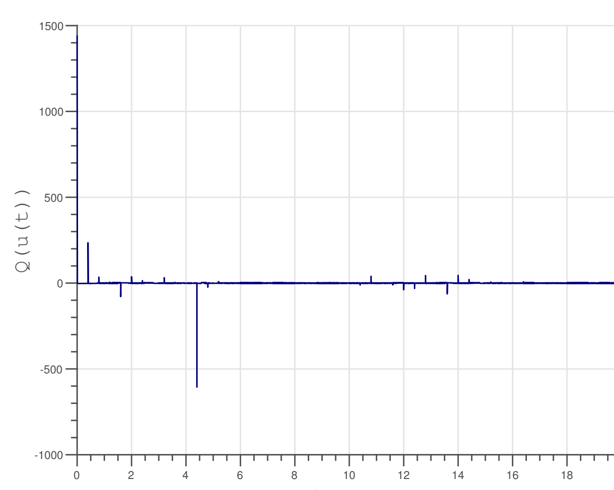

Fig. 2. Trajectory of the quantized input for Example 1.

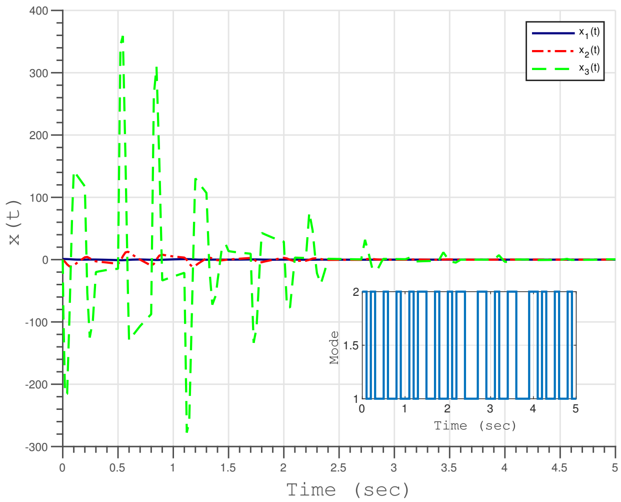

Fig. 3. State trajectories for Example 2.

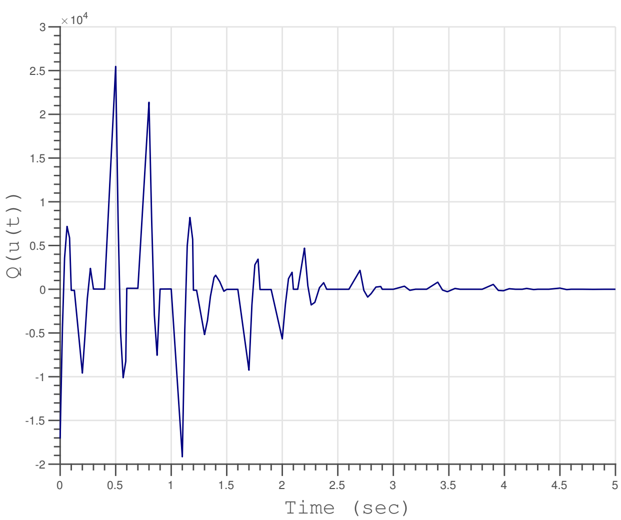

Fig. 4. Trajectory of the quantized input for Example 2.

lows.

$$K_1 = \begin{bmatrix} 18.564 & -0.7979 & -0.9476 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -1.5811 \times 10^4 & -1.4393 \times 10^3 & -38.554 \end{bmatrix}.$$

Also, the H_∞ performance γ is 3.5037×10^{-13}.

Fig. 3 shows the state trajectories and the mode evolution obtained by the above controller gains and Fig. 4 shows the quantized control input, where $x(0) = [1 \ \ -1 \ \ 0]^T$ and $r_0 = 2$. These figures show that the proposed controller effectively stabilizes the MJS with incomplete knowledge of the transition rates under input quantization and the external disturbance.

5. CONCLUSION

In this research, an H_∞ state-feedback controller for MJSs with incomplete knowledge of transition probabilities and input quantization was proposed. To do so, the stabilization conditions are developed into the second-order matrix polynomials of the unknown transition rate using an appropriate weighting method obtained by all possible slack variables for the transition rates. Also, the proposed controller guarantees an H_∞ performance and eliminates the matched disturbances and the effect of input quantization. Two examples illustrated the effectiveness of the proposed controller. The future work is to study the design of the output-feedback controller for the proposed system with system uncertainties, which will guarantee the desired performance.

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