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Event-triggered Finite-time Consensus with Fully Continuous Communication Free for Second-order Multi-agent Systems

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Abstract: This study deals with finite-time consensus problems of second-order multi-agent systems with intrinsic nonlinear dynamics and external bounded disturbances. First, instead of the time-triggered control algorithm, the event-triggered control algorithm is developed by using integral sliding mode control strategy. Then, a triggering function is explicitly constructed to generate event sequences, and the triggering function is fully continuous communication free. Rigorous proof is given by using Lyapunov stability theory and finite-time stability theory. Several conditions are derived to guarantee the finite-time consensus and exclude Zeno behavior. Finally, a simulation of single-link robotic arms is given to verify the effectiveness of the results.

Keywords: Disturbances, event-triggered control, finite-time consensus, integral sliding mode, nonlinear dynamics.

1. INTRODUCTION

In the past years, consensus of multi-agent systems has attracted an increasing interest due to its range of applications in areas, such as formation control [1], chaotic systems [2, 3], synchronization [4], exploration of hazardous environments and rescue in disaster sites [5]. Consensus is to develop a distributed algorithm such that all agents reach an agreement with local interaction. Much theoretical work with different dynamics and scenarios has been presented on consensus [6–8].

For consensus control, convergence rate [9] is a key performance index in practice which has attracted many researchers to study finite-time consensus [10, 11]. Finite-time consensus of first-order multi-agent systems was extensively investigated with an uncertain leader [12], time-varying reference state [13], disturbances and directed communication graph [14], etc. Later, second-order multi-agent systems were recognized as an important topic due to broad real-word applications [15, 16]. Finite-time consensus of second-order multi-agent systems was extensively investigated with disturbances [17], directed graph [18], intermittent communications [19], and time-varying delay [20, 21], etc.

An important point to note is that all the aforementioned literatures related to time-triggered systems focused only on the desired performance of consensus algorithm. However, such a time-triggered system may require continuous communications and frequent updating of controller, which would result in the waste of network resources and unnecessary energy consumption [22]. To tackle this problem, researchers have done a lot of research on event-triggered consensus problem [23–25].

Recently, some efforts have been made on finite-time consensus via event-triggered control. For first-order multi-agent systems, [26] and [27] investigated finite-time consensus with undirected network topologies via event-triggered method. Dong and Xian [28] took directed network topologies and nonlinear dynamics into consideration on the basis of [26], while the dynamics of agents were one-dimensional. Wang, Li and Xing [29] proposed an event-triggered control algorithm to solve finite-time average consensus problem and presented the relationship between the initial state and the convergence time. For general linear first-order multi-agent systems, [30, 31] investigated event-triggered finite-time control algorithm which can adjust the desired convergence time. By integral sliding-mode control strategy, Zhou *et al*. investigated finite-time consensus of Euler Lagrange systems with Markovian switching topologies [32]. Moreover, for second-order multi-agent systems, Lu *et al*. investigated finite-time consensus of linear system via event-triggered rule [33]. Hu, Lu and Hu [34] took directed network topology into consideration on the basis of [33], while the weight sum of in-edges was larger than that of out-edges for each agent. [33, 34] considered finite-time consensus

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of linear multi-agent systems, but the design of triggering function was time-dependent rather than fully continuous communication free. The control method of [33, 34] brought a paradox to the purpose of saving communication resource by introducing the event-triggered strategy.

Motivated by the above discussions, we consider the finite-time consensus problems of second-order multi-agent systems with intrinsic nonlinear dynamics and external bounded disturbances. The main contribution of this paper can be summarized as follows: 1) To solve the finite-time consensus with intrinsic nonlinear dynamics and external bounded disturbances, an event-triggered control algorithm based on integral sliding mode is developed. 2) A novel threshold is defined, and the triggering function is derived based on the novel threshold. This function does not require continuous communication in both controller update and error measurement, while most previous results have some limitations such as time-dependent triggering function [26,27,29,34], and continuous communication in controller update or error measurement [28, 30–33]. 3) Lower bound for the triggering time is derived to exclude Zeno behavior.

The remainder of this paper is organized as follows: We first address, in Section 2, preliminaries and problem formulation. The finite-time consensus problem of second-order multi-agent systems with nonlinear dynamics and disturbances is studied in Section 3. Section 4 gives the simulation example. The conclusions and future work are provided in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Notations and graph theory

Define $\mathrm{sig}(p)^\gamma = \left[\mathrm{sig}(p_1)^\gamma, \cdots, \mathrm{sig}(p_n)^\gamma\right]^T$, $\mathrm{sig}(p_i)^\gamma = |p_i|^\gamma \mathrm{sgn}(p_i)$, where $\mathrm{sgn}(\cdot)$ is the signum function. I_n denotes the identity matrix. $\|\cdot\|$ denotes the 2-norm. $\otimes$ denotes the Kronecker product. R^n denotes n-dimensional Euclidean space.

The topology of n agents is modeled as an undirected graph $G = \{V, \zeta, A\}$, where $\zeta \subseteq \{(i,j), i,j \in V\}$ is the edge set, $V = \{1,2,\cdots,n\}$ is a finite set of nodes, and $A = [a_{ij}]_{n\times n}$ is the associated adjacency matrix, where $a_{ii} = 0$, and $a_{ij} = 1$ is the weight if $(j,i) \in \zeta$ or $a_{ij} = 0$, otherwise. The neighbor set of i is defined as $N_i = \{j \in V : a_{ij} = 1\}$. Denote the matrix $D = diag\{d_{11}, d_{22}, \cdots, d_{nn}\}$ with $d_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$. Then, the Laplacian matrix L can be expressed by $L = D - A$ and L is symmetric.

2.2. Problem formulation

Consider a multi-agent system composed of n agents. The interaction topology among the n agents can be modeled as an undirected graph G with each agent being a vertex. The control objective is to develop a distributed algorithm such that all agents reach an agreement in finite time via local interaction. The dynamics of ith agent is specified by

$$\begin{cases} \dot{r}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t) + d_i(t) + f(r_i, v_i, t), \end{cases} \tag{1}$$

where $r_i \in R^m$, $v_i \in R^m$ denote the position and velocity, respectively, $u_i \in R^m$ is the control input, $d_i(t)$ is the external bounded disturbance, $f(r_i, v_i, t)$ is a continuous vector-valued function, which models the intrinsic nonlinear dynamics of agent i, and $i = 1,2,\cdots,n$.

The following definition, assumption and lemmas are given to derive the results.

Assumption 1: There exist positive constants ρ_1, ρ_2 such that $\|f(r_i, v_i, t)\| < \rho_1$ and $\|d_i(t)\| < \rho_2$, $i = 1,2,\cdots,n$.

Definition 1: The finite-time consensus is achieved for second-order systems if for any initial conditions, there exists a finite time T such that $\lim\limits_{t\to T} \|r_i(t) - r_j(t)\| = 0$, $\lim\limits_{t\to T} \|v_i(t) - v_j(t)\| = 0$ and $\|r_i(t) - r_j(t)\| = 0$, $\|v_i(t) - v_j(t)\| = 0$ if $t \geq T$, where $i,j = 1,2,\cdots,n$.

Lemma 1 [35]**:** Consider the system $\dot{x} = f(x), x \in U \subseteq R^n$ and U is an open neighborhood including the origin. Suppose that $V(x) : U \to R$ is a positive definite continuously differentiable function, which satisfies the condition

$$\dot{V}(x) + cV(x)^\alpha \leq 0, x \in U \setminus \{0\},$$

where $c > 0$ and $0 < \alpha < 1$. Then the origin is a finite-time stable equilibrium. In addition, the finite settling time satisfies $T \leq (V(x_0))^{1-\alpha} \big/ c(1-\alpha)$.

Lemma 2 [36]**:** Consider the system $\dot{x} = f(x), x \in U \subseteq R^n$ and U is an open neighborhood including the origin. Suppose that $V(x) : U \to R$ is a positive definite continuously differentiable function, which satisfies the condition

$$\dot{V}(x) + c_1 V(x)^\alpha + c_2 V(x) \leq 0, x \in U \setminus \{0\},$$

where $c_1, c_2 > 0$ and $0 < \alpha < 1$. Then the origin is a finite-time stable equilibrium. In addition, the finite settling time satisfies $T \leq \frac{1}{c_2(1-\alpha)} \ln\left[\frac{c_2(V(0))^{1-\alpha} + c_1}{c_1}\right]$.

Lemma 3 [37]**:** If the undirected graph of multi-agent system (1) is connected, the Laplacian matrix L is symmetric. $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ are defined as the eigenvalues of L, then $\lambda_1 = 0$ and $\lambda_2 > 0$. Define algebraic connectivity as a, if $\mathbf{1}^T r = 0, r \neq 0$, then $a = \lambda_2 = \min \frac{r^T L r}{r^T r}$ and $r^T L^2 r \geq a r^T L r$.

Lemma 4 [38]**:** Let S be a symmetric matrix. The following linear matrix inequality

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} > 0,$$

holds if and only if $S_{11} > 0$, $S_{22} - S_{21}S_{11}^{-1}S_{12} > 0$ or $S_{22} > 0$, $S_{11} - S_{21}S_{22}^{-1}S_{12} > 0$, where $S_{11}^T = S_{11}$, $S_{22}^T = S_{22}$.

Lemma 5 [39]**:** If $\alpha \in (0,1]$, the following inequality holds

$$\left(\sum_{i=1}^{n}|r_i|\right)^{\alpha} \leq \sum_{i=1}^{n}|r_i|^{\alpha} \leq n^{1-\alpha}\left(\sum_{i=1}^{n}|r_i|\right)^{\alpha},$$

where $r_i \in R$.

3. MAIN RESULTS

In this section, finite-time consensus problems with intrinsic nonlinear dynamics and external bounded disturbances are investigated via event-triggered control strategy. In this method, each agent broadcasts its state and updates its control input by its triggering function. The control algorithm is developed on the basis of the broadcast state and the proposed triggering function is fully continuous communication free.

Define $p_i = \sum\limits_{j=1}^{n} a_{ij}\left(r_i(t) - r_j(t)\right)$ and $q_i = \sum\limits_{j=1}^{n} a_{ij}(v_i(t) - v_j(t))$. Let $p = \left[p_1^T, p_2^T, \cdots, p_n^T\right]^T$, $q = \left[q_1^T, q_2^T, \cdots, q_n^T\right]^T$, $r = \left[r_1^T, r_2^T, \cdots, r_n^T\right]^T$ and $v = \left[v_1^T, v_2^T, \cdots, v_n^T\right]^T$, it follows that $p = (L \otimes I_m)\, r$ and $q = (L \otimes I_m)\, v$.

For convenience, we introduce a triggering time sequence $\left\{t_0^i = 0, t_1^i, \cdots, t_{k_i}^i, \cdots\right\}$ for agent i. An auxiliary variable Φ_i is designed for agent i.

$$\begin{aligned}\Phi_i(t) = &-\left(\beta \sum_{j=1}^{n} a_{ij}\left(r_i\left(t_{k_i}^i\right) - r_j\left(t_{k_j}^j\right)\right)\right.\\&\left.+\gamma \sum_{j=1}^{n} a_{ij}\left(v_i\left(t_{k_i}^i\right) - v_j\left(t_{k_j}^j\right)\right)\right)\\&-sig\left(\beta \sum_{j=1}^{n} a_{ij}\left(r_i\left(t_{k_i}^i\right) - r_j\left(t_{k_j}^j\right)\right)\right.\\&\left.+\gamma \sum_{j=1}^{n} a_{ij}\left(v_i\left(t_{k_i}^i\right) - v_j\left(t_{k_j}^j\right)\right)\right)^{\alpha},\end{aligned} \tag{2}$$

where $\beta > 0$, $\gamma > 0$, $\alpha \in (0,1)$, $t \in \left[t_{k_i}^i, t_{k_i+1}^i\right)$, $t_{k_i}^i$ is the latest event-triggered time of agent i, $k_j \triangleq \arg\min_s \left\{t - t_s^j \,\middle|\, t \geq t_s^j, s \in N\right\}$, i.e., $t_{k_j}^j$ is the latest event-triggered time of agent j.

During the time interval $\left[t_{k_i}^i, t_{k_i+1}^i\right)$, the combinational states for agent i are $p_i\left(t_{k_i}^i\right) = \sum\limits_{j=1}^{n} a_{ij}\left(r_i\left(t_{k_i}^i\right) - r_j\left(t_{k_j}^j\right)\right)$ and $q_i\left(t_{k_i}^i\right) = \sum\limits_{j=1}^{n} a_{ij}\left(v_i\left(t_{k_i}^i\right) - v_j\left(t_{k_j}^j\right)\right)$. Define measurement errors as $e_{ir}(t) = r_i\left(t_{k_i}^i\right) - r_i(t)$, $e_{iv}(t) = v_i\left(t_{k_i}^i\right) - v_i(t)$, $t \in \left[t_{k_i}^i, t_{k_i+1}^i\right)$. The combinational measurement errors are defined as $e_i^r = \sum\limits_{j=1}^{n} a_{ij}(e_{ir} - e_{jr})$, $e_i^v = \sum\limits_{j=1}^{n} a_{ij}(e_{iv} - e_{jv})$. Then, auxiliary variable Φ_i can be rewritten as

$$\begin{aligned}\Phi_i(t) = &-\left(\beta p_i(t) + \beta e_i^r(t) + \gamma q_i(t) + \gamma e_i^v(t)\right)\\&- sig\left(\beta p_i(t) + \beta e_i^r(t) + \gamma q_i(t) + \gamma e_i^v(t)\right)^{\alpha}.\end{aligned} \tag{3}$$

Considering the intrinsic nonlinear dynamics and external disturbances in the multi-agent system, an integral sliding mode surface is specified as

$$S_i(t) = v_i(t) - v_i(0) - \int_0^t \Phi_i(t)\, d\tau, \tag{4}$$

where $S_i(t) = \left[s_{i1}(t), s_{i2}(t), \cdots, s_{im}(t)\right]^T$.

The event-triggered finite-time consensus algorithm based on integral sliding mode is defined as

$$u_i(t) = \Phi_i(t) - (C_1 + C_2)\, sgn\left(S_i\left(t_{k_i}^i\right)\right), \tag{5}$$

where C_1 and C_2 are positive constants.

The triggering function for agent i is defined as

$$\begin{aligned}h_i(t) = &\|L\| \left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\|\\&- \xi \left\|\beta p_i\left(t_{k_i}^i\right) + \gamma q_i\left(t_{k_i}^i\right)\right\|,\end{aligned} \tag{6}$$

where $\xi > 0$ and $\|L\|$ denotes the 2-norm of Laplacian matrix L. Then, the triggering condition is defined as $t_{k_i+1}^i = \inf\left\{t > t_{k_i}^i, h_i(t) > 0\right\}$.

The block schematic of communication and control processes is given in Fig. 1, where agent j represents the neighbors of agent i and each agent generates triggering events at its own event time only.

Remark 1: The triggering function does not require continuous communication with neighbors to check if the

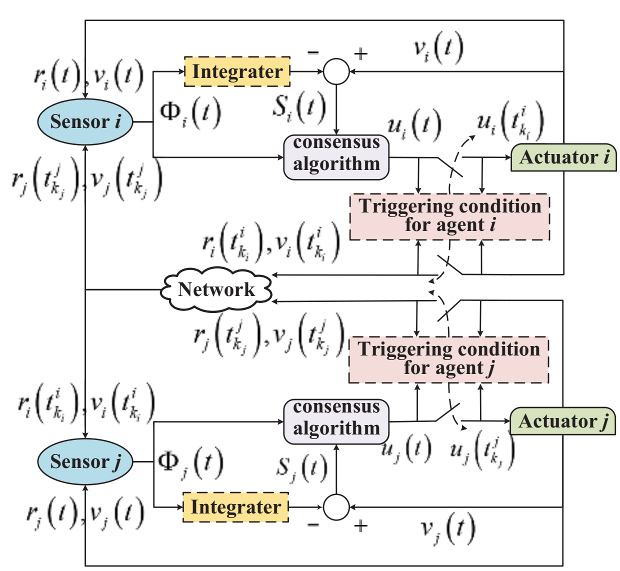

Fig. 1. Communication and control processes of the proposed event-triggered consensus algorithm (5) and triggering function (6).

triggering condition is satisfied. Using the triggering function, the controller of agent i measures its state continuously. If the triggering function does not satisfy the triggering condition, there is no need to communicate. When the measurement error $\|L\|\left\|\beta e_{ir}(t)+\gamma e_{iv}(t)\right\|$ of agent i is greater than the threshold $\xi\left\|\beta p_i\left(t_{k_i}^i\right)+\gamma q_i\left(t_{k_i}^i\right)\right\|$, the controller of agent i updates its control input and broadcasts its current state. When agent i receives the states of its neighbors, its controller also updates its control input. The parameter ξ can be used to adjust the update frequency.

Theorem 1: Suppose that Assumption 1 is satisfied and the undirected graph of multi-agent system (1) is connected. With the event-triggered control algorithm (5) and the triggering function (6), the finite-time consensus problem can be solved if $C_1>\rho_1$, $C_2>\rho_2$, $a\gamma^2>\beta$ and $\xi<\min\left\{\frac{a\gamma^2-\beta}{3a\gamma^2-\beta}, 1\Big/\sqrt{(mn)^{1-\alpha}}\right\}$, irrespective of the nonlinear dynamics and disturbances.

Proof: The proof is divided into two steps: (1) each sliding mode surface $S_i(t)$ with the control algorithm is finite-time stable; (2) Once the sliding-mode surface is reached, i.e., $S_i(t)=\dot{S}_i(t)=0_m$, then we will show that system (1) is finite-time stable.

Step 1: The stability analysis of sliding mode controller is given. Consider the Lyapunov function candidate as

$$V_1(t)=\frac{1}{2}S^TS, \tag{7}$$

where $S=\left[S_1^T, S_2^T, \cdots, S_n^T\right]^T$.

Taking the time derivative of $V_1(t)$, we have

$$\begin{aligned}\dot{V}_1(t)=&\sum_{i=1}^n S_i^T\dot{S}_i=\sum_{i=1}^n S_i^T\left(\dot{v}_i(t)-\Phi_i(t)\right)\\ =&\sum_{i=1}^n S_i^T\left(u_i(t)+d_i(t)+f\left(r_i, v_i, t\right)-\Phi_i(t)\right)\\ =&\sum_{i=1}^n S_i^T\left(d_i(t)+f\left(r_i, v_i, t\right)\right.\\ &\left.-\left(C_1+C_2\right)sgn\left(S_i\left(t_{k_i}^i\right)\right)\right)\\ \leq&\left(\rho_1+\rho_2\right)\sum_{i=1}^n\left\|S_i\right\|\\ &-\left(C_1+C_2\right)\sum_{i=1}^n\sum_{j=1}^m s_{ij}(t)\,sgn\left(s_{ij}\left(t_{k_i}^i\right)\right).\end{aligned} \tag{8}$$

It follows from Theorem3.1 [40] that until the system trajectory reaches the sliding manifold, $sgn\left(s_{ij}\left(t_{k_i}^i\right)\right)=sgn\left(s_{ij}(t)\right)$. From Lemma 5, we get

$$\begin{aligned}\dot{V}_1(t)\leq&\left(\rho_1+\rho_2\right)\sum_{i=1}^n\left\|S_i\right\|-\left(C_1+C_2\right)\sum_{i=1}^n\sum_{j=1}^m\left|s_{ij}(t)\right|\\ \leq&-\left(C_1+C_2-\rho_1-\rho_2\right)\sum_{i=1}^n\left\|S_i\right\|\\ \leq&-\left(C_1+C_2-\rho_1-\rho_2\right)\|S\|\\ =&-\sqrt{2}\left(C_1+C_2-\rho_1-\rho_2\right)V_1^{1/2}.\end{aligned} \tag{9}$$

With the conditions $C_1>\rho_1$ and $C_2>\rho_2$, we have $C_1+C_2-\rho_1-\rho_2>0$. It follows from Lemma 1 that each sliding mode $S_i(t)$ with the control algorithm is finite-time stable. The finite time upper bound can be computed as $T_1\leq\sqrt{2V_1(0)}\Big/\left(C_1+C_2-\rho_1-\rho_2\right)$.

Step 2: When the system trajectory reaches the sliding manifold, i.e., $S_i(t)=\dot{S}_i(t)=0_m$, it follows from (4) that $\dot{S}_i(t)=\dot{v}_i(t)-\Phi_i(t)=0$.

Let $\tilde{r}_i=r_i-\bar{r}=r_i-\frac{1}{n}\sum_{j=1}^n r_j$ and $\tilde{v}_i=v_i-\bar{v}=v_i-\frac{1}{n}\sum_{j=1}^n v_j$. Define $\tilde{r}=\left[\tilde{r}_1^T, \tilde{r}_2^T, \cdots, \tilde{r}_n^T\right]^T$, $\tilde{v}=\left[\tilde{v}_1^T, \tilde{v}_2^T, \cdots, \tilde{v}_n^T\right]^T$, then we have $\tilde{r}=r-\frac{1}{n}\left(\mathbf{1}_{n\times n}\otimes I_m\right)r$, $\tilde{v}=v-\frac{1}{n}\left(\mathbf{1}_{n\times n}\otimes I_m\right)v$. According to the property of the Laplacian matrix L, we have $p=\left(L\otimes I_m\right)r=\left(L\otimes I_m\right)\tilde{r}$, $q=\left(L\otimes I_m\right)v=\left(L\otimes I_m\right)\tilde{v}$.

Consider the Lyapunov function candidate as

$$V_2(t)=\frac{1}{2}\begin{pmatrix}\tilde{r}\\ \tilde{v}\end{pmatrix}^T\left(\begin{pmatrix}2\beta\gamma L^2 & \beta L\\ \beta L & \gamma L\end{pmatrix}\otimes I_m\right)\begin{pmatrix}\tilde{r}\\ \tilde{v}\end{pmatrix}. \tag{10}$$

With the condition $a>\frac{\beta}{\gamma^2}$, it follows from Lemma 3, Lemma 4 that $V_2(t)\geq 0$ and $V_2(t)=0$ if and only if $\tilde{r}(t)=0_{mn}$, $\tilde{v}(t)=0_{mn}$. Then, $V_2(t)$ is a valid Lyapunov function. Taking the time derivative of $V_2(t)$, we have

$$\begin{aligned}\dot{V}_2(t)=&2\beta\gamma\tilde{r}^T\left(L^2\otimes I_m\right)\tilde{v}+\beta\tilde{v}^T\left(L\otimes I_m\right)\tilde{v}\\ &+\left(\beta\tilde{r}^T+\gamma\tilde{v}^T\right)\left(L\otimes I_m\right)\left(\dot{v}-\frac{1}{n}\left(\mathbf{1}_{n\times n}\otimes I_m\right)\dot{v}\right)\\ =&2\beta\gamma\tilde{r}^T\left(L^2\otimes I_m\right)\tilde{v}+\beta\tilde{v}^T\left(L\otimes I_m\right)\tilde{v}\\ &+\left(\beta\tilde{r}^T+\gamma\tilde{v}^T\right)\left(L\otimes I_m\right)\Phi(t)\\ \leq&-\beta^2p^Tp-\left(\gamma^2-\frac{\beta}{a}\right)q^Tq\\ &-\left[\beta^2p^Te^r+\beta\gamma q^Te^r+\beta\gamma p^Te^v+\gamma^2q^Te^v\right]\\ &-\left(\beta p^T+\gamma q^T\right)sig(\beta p+\beta e^r+\gamma q+\gamma e^v)^{\alpha},\end{aligned} \tag{11}$$

where $\Phi(t)=\left[\Phi_1^T(t), \Phi_2^T(t), \cdots, \Phi_n^T(t)\right]^T$, $e_r=\left[e_{1r}^T, e_{2r}^T, \cdots, e_{nr}^T\right]^T$, $e_v=\left[e_{1v}^T, e_{2v}^T, \cdots, e_{nv}^T\right]^T$, $e^r=\left(L\otimes I_m\right)e_r$, $e^v=\left(L\otimes I_m\right)e_v$.

Define $E_i(t)=\beta p_i(t)+\gamma q_i(t)+\beta e_i^r(t)+\gamma e_i^v(t)=\beta p_i\left(t_{k_i}^i\right)+\gamma q_i\left(t_{k_i}^i\right)$, $E=\left[E_1^T, E_2^T, \cdots, E_n^T\right]^T$. From the triggering function (6), we have

$$\begin{aligned}&\left\|\beta e^r(t)+\gamma e^v(t)\right\|\leq\|L\|\left\|\beta e_r(t)+\gamma e_v(t)\right\|\\ &=\sqrt{\|L\|^2\sum_{i=1}^n\left\|\beta e_{ir}(t)+\gamma e_{iv}(t)\right\|^2}\\ &\leq\sqrt{\xi^2\sum_{i=1}^n\left\|\beta p_i\left(t_{k_i}^i\right)+\gamma q_i\left(t_{k_i}^i\right)\right\|^2}\end{aligned}$$

$$= \xi \left\|E(t)\right\|, \tag{12}$$

$$\begin{aligned}\left\|E(t)\right\| &\le \left\|\beta p(t)+\gamma q(t)\right\| + \left\|\beta e^r(t)+\gamma e^v(t)\right\| \\ &\le \left\|\beta p(t)+\gamma q(t)\right\| + \xi\left\|E(t)\right\|.\end{aligned} \tag{13}$$

Then, we have $\|E\| \le \frac{1}{1-\xi}\left\|\beta p(t)+\gamma q(t)\right\|$. It follows that

$$\begin{aligned}&-\left[\beta^2 p^T e^r + \beta\gamma q^T e^r + \beta\gamma p^T e^v + \gamma^2 q^T e^v\right] \\ &\le \|\beta p+\gamma q\| \left\|\beta e^r + \gamma e^v\right\| \\ &\le \|\beta p+\gamma q\| \xi \left\|E(t)\right\| \\ &\le \frac{\xi}{1-\xi}\|\beta p+\gamma q\|^2 \\ &\le 2\frac{\xi}{1-\xi}\left(\beta^2 p^T p + \gamma^2 q^T q\right).\end{aligned} \tag{14}$$

Further, we have

$$\begin{aligned}&-\left(\beta p^T + \gamma q^T\right) sig(\beta p + \beta e^r + \gamma q + \gamma e^v)^\alpha \\ &= (\beta e^r + \gamma e^v)^T sig(E(t))^\alpha - (E(t))^T sig(E(t))^\alpha \\ &\le \|(\beta e^r + \gamma e^v)\| \left\|E^\alpha(t)\right\| - \sum_{i=1}^{n}\sum_{j=1}^{m}\left|E_{ij}(t)\right|^{\alpha+1} \\ &\le \xi (mn)^{\frac{1-\alpha}{2}} \left\|E(t)\right\|^{\alpha+1} - \sum_{i=1}^{n}\left\|E_i(t)\right\|^{\alpha+1} \\ &\le -\left(1-\xi (mn)^{\frac{1-\alpha}{2}}\right)\sum_{i=1}^{n}\left\|E_i(t)\right\|^{\alpha+1}.\end{aligned} \tag{15}$$

By applying (11), we can obtain that

$$\begin{aligned}\dot{V}_2(t) \le &-\beta^2\left(1-2\frac{\xi}{1-\xi}\right)p^T p \\ &-\left(\gamma^2 - \frac{\beta}{a} - 2\frac{\xi\gamma^2}{1-\xi}\right) q^T q \\ &-\left(1-\xi (mn)^{\frac{1-\alpha}{2}}\right)\sum_{i=1}^{n}\left\|E_i(t)\right\|^{\alpha+1}.\end{aligned} \tag{16}$$

If $a\gamma^2 > \beta$ and $\xi < \min\left\{\frac{a\gamma^2-\beta}{3a\gamma^2-\beta}, 1\Big/\sqrt{(mn)^{1-\alpha}}\right\}$, we have $\dot{V}_2(t) < 0$. Hence, consensus can be reached asymptotically.

Next, we will prove that the $\left(0_{mn}^T, 0_{mn}^T\right)^T$ is a finite-time-stable equilibrium. Define $\theta_1 = \min\{\beta^2 - \frac{2\xi\beta^2}{1-\xi}, \gamma^2 - \frac{\beta}{a} - \frac{2\xi\gamma^2}{1-\xi}, 1-\xi (mn)^{\frac{1-\alpha}{2}}\}$, we have

$$\begin{aligned}&\frac{\left(1-\xi (mn)^{\frac{1-\alpha}{2}}\right)\sum\limits_{i=1}^{n}\left\|E_i(t)\right\|^{\alpha+1}}{V(t)^{\frac{1+\alpha}{2}}} \\ &\ge \frac{\theta_1 \sum\limits_{i=1}^{n}\left\|E_i(t)\right\|^{\alpha+1}}{\mu_1^{\frac{1+\alpha}{2}}\left\|\tilde{r}^T\tilde{r} + \tilde{v}^T\tilde{v}\right\|^{\frac{1+\alpha}{2}}} \ge \frac{a^{1+\alpha}\theta_1 \sum\limits_{i=1}^{n}\left\|E_i(t)\right\|^{\alpha+1}}{\mu_1^{\frac{1+\alpha}{2}}\left\|p^T p + q^T q\right\|^{\frac{1+\alpha}{2}}} \\ &= \theta_2,\end{aligned} \tag{17}$$

$$\begin{aligned}&\beta^2\left(1-2\frac{\xi}{1-\xi}\right)p^T p + \left(\gamma^2 - \frac{\beta}{a} - 2\frac{\xi\gamma^2}{1-\xi}\right) q^T q \\ &\ge \theta_1\left(p^T p + q^T q\right) \ge \theta_1 a^2\left(\tilde{r}^T\tilde{r} + \tilde{v}^T\tilde{v}\right) \ge \frac{\theta_1 a^2}{\mu_1}V(t),\end{aligned} \tag{18}$$

where μ_1 is the maximum eigenvalue of the matrix $\begin{pmatrix} \beta\gamma L^2 & \beta L/2 \\ \beta L/2 & \gamma L/2 \end{pmatrix}$.

Before consensus is achieved, we have $\tilde{r}(t) \ne 0_{nm}$, $\tilde{v}(t) \ne 0_{nm}$. It follows that $E^T(t)E(t) > 0$. If $E(t) = 0_{nm}$, $t \in \left[t_{k_i}^i, t_{k_i+1}^i\right)$, it follows that $\dot{\tilde{v}}_i(t) = 0_m$ and $\dot{q}_i(t) = 0_m$, $i = 1, 2, \cdots, n$. We further get $\dot{e}_{iv}(t) = 0_m$ and $\dot{e}_i^v(t) = 0_m$. Meanwhile, $\dot{E}_i(t) = \beta\dot{p}_i\left(t_{k_i}^i\right) + \gamma\dot{q}_i\left(t_{k_i}^i\right) = \beta\left(q_i(t) + e_i^v(t)\right) + \gamma\left(\dot{q}_i(t) + \dot{e}_i^v(t)\right) = 0_m$, then, we have $q_i\left(t_{k_i}^i\right) = q_i(t) + e_i^v(t) = 0_m$. We further get $\beta p_i\left(t_{k_i}^i\right) = -\gamma q_i\left(t_{k_i}^i\right) = 0_m$. Define $\varepsilon_i(t) = \left(\beta p_i^T\left(t_{k_i}^i\right), \gamma q_i^T\left(t_{k_i}^i\right)\right)^T$, we have $E_i(t) = \begin{pmatrix} 1 & 1 \end{pmatrix} \otimes I_m \begin{pmatrix} \beta p_i\left(t_{k_i}^i\right) \\ \gamma q_i\left(t_{k_i}^i\right) \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \otimes I_m \varepsilon_i(t)$. Before consensus is achieved, it follows from Lemma 6 [34] that

$$\begin{aligned}E^T(t)E(t) &= \sum_{i=1}^{n} E_i^T(t)E_i(t) \\ &\ge \vartheta\sum_{i=1}^{n}\left(\beta^2 p_i^T\left(t_{k_i}^i\right)p_i\left(t_{k_i}^i\right) + \gamma^2 q_i^T\left(t_{k_i}^i\right)q_i\left(t_{k_i}^i\right)\right),\end{aligned} \tag{19}$$

where $\vartheta = \min_{\frac{\varepsilon(t)}{\|\varepsilon(t)\|}}\left(\frac{\varepsilon(t)}{\|\varepsilon(t)\|}\right)^T\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\otimes I_{mn}\left(\frac{\varepsilon(t)}{\|\varepsilon(t)\|}\right) > 0$, $\varepsilon(t) = \left[\varepsilon_1^T(t), \varepsilon_2^T(t), , \varepsilon_n^T(t)\right]^T$.

We have proved that the state $\begin{pmatrix} p^T & q^T \end{pmatrix}^T$ can asymptotically converge to $\left(0_{mn}^T, 0_{mn}^T\right)^T$. Then, we can get $\left\|p_i\left(t_{k_i}^i\right)\right\| \ge \left\|p_i(t)\right\|$ and $\left\|q_i\left(t_{k_i}^i\right)\right\| \ge \left\|q_i(t)\right\|$, for each time interval $\left[t_{k_i}^i, t_{k_i+1}^i\right)$. It is obvious that

$$\begin{aligned}\theta_2 &\ge \frac{\theta_1\left(a^2\vartheta\mu_2^2\sum\limits_{i=1}^{n}\left(p_i^T p_i + q_i^T q_i\right)\right)^{\frac{1+\alpha}{2}}}{\mu_1^{\frac{1+\alpha}{2}}\left\|p^T p + q^T q\right\|^{\frac{1+\alpha}{2}}} \\ &= \frac{\theta_1\left(a^2\vartheta\mu_2^2\right)^{\frac{1+\alpha}{2}}}{\mu_1^{\frac{1+\alpha}{2}}} = \theta_3,\end{aligned} \tag{20}$$

where $\mu_2 = \min\{\beta, \gamma\}$.

Hence, we have $\dot{V}_2(t) + \theta_3 V_2(t)^{\frac{1+\alpha}{2}} + \frac{\theta_1 a^2}{\mu_1}V_2(t) \le 0$. Using Lemma 2, $\left(0_{mn}^T, 0_{mn}^T\right)^T$ is a finite-time stable equilibrium, and the settling time is given by $T_2 \le \frac{2\mu_1}{\theta_1 a^2(1-\alpha)}\ln\left[\frac{\theta_1 a^2}{\mu_1\theta_3}V_2(0)^{\frac{1-\alpha}{2}} + 1\right]$.

According to Step 1 and Step 2, finite-time consensus can be reached with the settling time $T \le \frac{\sqrt{2V_1(0)}}{C_1+C_2-\rho_1-\rho_2} + \frac{2\mu_1}{\theta_1 a^2(1-\alpha)}\ln\left[\frac{\theta_1 a^2}{\mu_1\theta_3}V_2(0)^{\frac{1-\alpha}{2}} + 1\right]$. Then, we have $\lim_{t\to T}\tilde{r} = 0_{mn}, \lim_{t\to T}\tilde{v} = 0_{mn}$ and $\tilde{r} = 0_{mn}, \tilde{v} = 0_{mn}$ when $t \ge T$. It is easy to obtain that $r_1 = r_2 = \cdots = r_n = \bar{r}$, $v_1 = v_2 = \cdots = v_n = \bar{v}$. We can conclude that the proposed event-triggered control algorithm can solve finite-time consensus problem of second-order multi-agent system with intrinsic nonlinear dynamics and external bounded disturbances. This is the end of proof. □

Remark 2: From the triggering function (6), ξ is used to adjust the allowable error. When the parameter ξ is large, the update frequency is low. However, increasing parameter ξ will increase the convergence time. In practical applications, ξ needs be adjusted according to convergence time and updated frequency.

Remark 3: From the proposed consensus algorithm (5), β, γ are used to adjust the effect of combinational position and velocity states between the ith agent and its neighbors. According to Theorem 1, if β, γ satisfy the condition $a\gamma^2 > \beta$, then multi-agent system (1) is finite-time stable with the proposed control algorithm. Obviously, the parameter β makes the parameter γ have a flexible adjustment range. In the case that the parameter β remains unchanged, increasing γ will reduce the convergence rate, so γ can be set over the interval $\left(\sqrt{\beta/a}, \sqrt{\beta/a}+1\right]$. Increasing the parameter β will result in a fast convergence rate, but due to the input saturation constraint, the parameter β should not be chosen very large. In simulations and applications, the parameter β is usually selected in the interval $[0.5, 4]$. In addition, when the number of agents is large, according to $\xi < 1 \Big/ \sqrt{(mn)^{1-\alpha}}$, α needs to take a larger value to avoid the value of ξ being too small. Meanwhile, decreasing α will result in fast convergence rate. But, when $\alpha \in (0, 0.5)$, the effect on convergence rate is not obvious. In simulations and applications, the parameter α is usually selected in the interval $[0.5, 0.9]$.

Generally, it is a challenge to avoid Zeno behavior. The following Theorem proves that the time interval $t^i_{k_i+1} - t^i_{k_i}$ has a lower bound so that the Zeno triggering does not exist.

Theorem 2: Consider multi-agent system (1) with the event-triggered control algorithm (5) and triggering function (6). Suppose that Assumption 1 holds for all agents. Then, for any time $t > 0$, each agent i will exclude Zeno behavior before consensus is achieved.

Proof: At $t = t^i_{k_i}$, the controller of agent i updates its control input, hence the measurement error is set to zero, i.e., $\left\|e_{ir}\left(t^i_{k_i}\right)\right\| = 0$ and $\left\|e_{iv}\left(t^i_{k_i}\right)\right\| = 0$. During the time interval $\left[t^i_{k_i}, t^i_{k_i+1}\right)$, we have

$$\begin{aligned}
&\frac{d}{dt}\left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\| \le \left\|\frac{d}{dt}\left(\beta e_{ir}(t) + \gamma e_{iv}(t)\right)\right\| \\
&= \left\|\beta v_i(t) + \gamma \dot{v}_i(t)\right\| \\
&= \left\|\beta v_i + \gamma\left(\Phi_i + d_i + f - (C_1 + C_2)\, sgn\left(S_i\left(t^i_{k_i}\right)\right)\right)\right\| \\
&\le \left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2 + C_1 + C_2\right).
\end{aligned} \tag{21}$$

Before the system trajectory reaches the sliding manifold, we have

$$\begin{aligned}
&\frac{d}{dt}\left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\| \\
&\le \left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2 + C_1 + C_2\right),
\end{aligned} \tag{22}$$

$$\begin{aligned}
&\left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\| \\
&= \int_{t^i_{k_i}}^{t} \frac{d}{dt}\left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\| ds \\
&\le \int_{t^i_{k_i}}^{t} \left(\left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2 + C_1 + C_2\right)\right) ds \\
&= \left(\left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2 + C_1 + C_2\right)\right)\left(t - t^i_{k_i}\right).
\end{aligned} \tag{23}$$

When the system trajectory reaches the sliding manifold, we have

$$\begin{aligned}
&\frac{d}{dt}\left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\| \\
&\le \left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2\right),
\end{aligned} \tag{24}$$

$$\begin{aligned}
&\left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\| \\
&= \int_{t^i_{k_i}}^{t} \frac{d}{dt}\left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\| ds \\
&\le \int_{t^i_{k_i}}^{t} \left(\left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2\right)\right) ds \\
&= \left(\left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2\right)\right)\left(t - t^i_{k_i}\right).
\end{aligned} \tag{25}$$

When the event is triggered, we have

$$\left\|\beta e_{ir}(t) + \gamma e_{iv}(t)\right\| > \frac{\xi}{\|L\|}\left\|\beta p_i\left(t^i_{k_i}\right) + \gamma q_i\left(t^i_{k_i}\right)\right\|. \tag{26}$$

Before consensus is achieved, it follows from Theorem 1 that $\left\|\beta p_i\left(t^i_{k_i}\right) + \gamma q_i\left(t^i_{k_i}\right)\right\| > 0$. Before the system trajectory reaches the sliding manifold, we have

$$\begin{aligned}
&\left(t^i_{k_i+1} - t^i_{k_i}\right) \\
&> \frac{\xi\left\|\beta p_i\left(t^i_{k_i}\right) + \gamma q_i\left(t^i_{k_i}\right)\right\|}{\|L\|\left(\left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2 + C_1 + C_2\right)\right)} > 0.
\end{aligned} \tag{27}$$

When the system trajectory reaches the sliding manifold, we have

$$\left(t^i_{k_i+1} - t^i_{k_i}\right) > \frac{\xi\left\|\beta p_i\left(t^i_{k_i}\right) + \gamma q_i\left(t^i_{k_i}\right)\right\|}{\|L\|\left(\left\|\beta v_i\right\| + \gamma\left(\left\|\Phi_i\right\| + \rho_1 + \rho_2\right)\right)} > 0. \tag{28}$$

We can conclude that $t^i_{k_i+1} - t^i_{k_i} > 0$ before consensus is achieved. In turns, $t^i_{k_i+2} - t^i_{k_i+1} > 0$. We can conclude that Zeno behavior is excluded for agent i. This is the end of proof. □

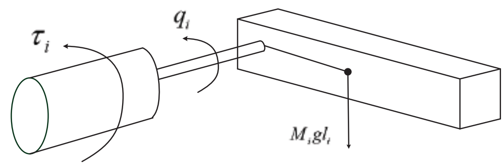

Fig. 2. Single-link robot arm.

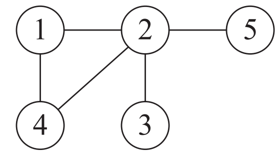

Fig. 3. Topology of the interaction graph.

To consider communication distance or other reasons, we introduce switching topologies $G = \{G_c \,|c = 1,2,\cdots,v\}$, $v \in N^+$. $\sigma(t) : [0,+\infty) \to k$ denotes switching signal. $G_{\sigma(t)}$ is the switching topology at time t. Without loss of generality, if the topology changes, the topology switches at the triggering time. In the following, we present Theorem 3 on switching topologies.

Theorem 3: Consider switching topologies $G = \{G_c \mid c = 1,2,\cdots,v\}$ and each topology G_c is connected. With the event-triggered control algorithm (5) and the triggering function (6), the finite-time consensus problem can be solved if $C_1 > \rho_1$, $C_2 > \rho_2$, $a\gamma^2 > \beta$ and $\xi < \min\left\{\frac{a\gamma^2-\beta}{3a\gamma^2-\beta}, 1\Big/\sqrt{(mn)^{1-\alpha}}\right\}$, irrespective of the nonlinear dynamics and disturbances.

Where $a = \min\limits_{1\le c\le v}\left\{\lambda_2\left(L_{\sigma(t)}\right)\right\}$.

Proof: The proof is similar to Theorem 1 and is hence omitted here. This is the end of proof. □

4. SIMULATION RESULTS

To verify the validity of finite-time event-triggered consensus algorithm, we consider cooperative control problem of single-link robotic arms [41]. Each single-link robotic arm consists of a rigid link coupled through a gear train to a DC motor, as shown in Fig. 2. The multi-agent system consists of five single-link robotic arms, and each agent is modeled as the Lagrangian dynamics with external disturbance.

$$J_i\ddot{q}_i + B_i\dot{q}_i + M_i g l_i \sin(q_i) = \tau_i + d_i, \tag{29}$$

where the states q_i and $\dot{q}_i$ are angle and angular velocity of the ith link, J_i is the total rotational inertia of the link and the motor, B_i is the damping coefficient, M_i denotes the total mass of the link, g is the gravitational acceleration, l_i is the distance from the joint axis to the link center of mass for ith agent, and d_i is the external disturbance with upper bound.

To achieve finite-time consensus, the control input for each agent is designed as

$$\tau_i = J_i u_i. \tag{30}$$

With the control input (30), the dynamics can be rewritten as

$$\ddot{q}_i = u_i + J_i^{-1} d_i - J_i^{-1}\left(B_i\dot{q}_i + M_i g l_i \sin(q_i)\right). \tag{31}$$

Let $r_i = q_i$ and $v_i = \dot{q}_i$, the dynamics of single-link robot arms become (1). The single-link robot arms are linked as shown in Fig. 3.

Next, the event-triggered controller (5) and triggering function (6) are used to control the single-link robot arms. For the simulation, the parameters are chosen as $J = [6.86, 7.57, 8.31, 9.66, 10.81]^T$, $B = [5.08, 5.08, 5.08, 5.08, 5.08]^T$, $g = 9.8$, $l = [1.02, 1.15, 1.27, 1.36, 1.45]^T$, $d_i = 0.5\sin(t)$. The initial conditions are chosen as $\dot{q}_i(0) = 0$, $q(0) = \left[5\pi/11, \pi/6, 2\pi/7, \pi/12, \pi/3\right]^T$. Based on Theorem 1 and Remark 3, we design the controller u_i with $\alpha = 0.5$, $\beta = 2.9$, $\gamma = 3$, $C_1 = 0.8$, $C_2 = 10$, $\xi = 0.24$. The simulation duration is set to 10s, and the simulation step size is 0.01s.

With the proposed event-triggered control algorithm, the trajectories of the robotic arms angle and angular velocity are shown in Fig. 4 and Fig. 5, from which one can see that the angle and angular velocity of the five robot arms converge rapidly from their respective initial states to the same angle and angular velocity. As a result, consensus of single-link robotic arms system with disturbance can be achieved in finite time using the proposed control algorithm.

The state measurement errors and event triggering instants for all robotic arms are shown in Fig. 6 and Fig. 7, where each vertical line represents a triggering event. It indicates that the generation of triggering event is asynchronous and the time interval $t^i_{k_i+1} - t^i_{k_i}$ has a lower bound. According to Fig. 6 and Fig. 7, we can see that the measurement errors of all robotic arms are 0 at the initial moment. Taking the first robotic arm as an example, its controller continuously measures its own state to calculate the measurement error $\|L\|\,\|\beta e_{1r}(t) + \gamma e_{1v}(t)\|$. When the measurement error is greater than the threshold $\xi\left\|\beta p_1\left(t^1_{k_1}\right) + \gamma q_1\left(t^1_{k_1}\right)\right\|$, its controller generates an event while the controller updates and broadcasts current state, then the measurement errors are set to 0. Therefore, for a robotic arm, communication and controller update are performed when it generates triggering events and its neighbor agents generate triggering events. Compared with continuous communication, the proposed algorithm avoids continuous communication in controller update and event triggering, saving a lot of communication resources.

In order to better show the performance of the pro-

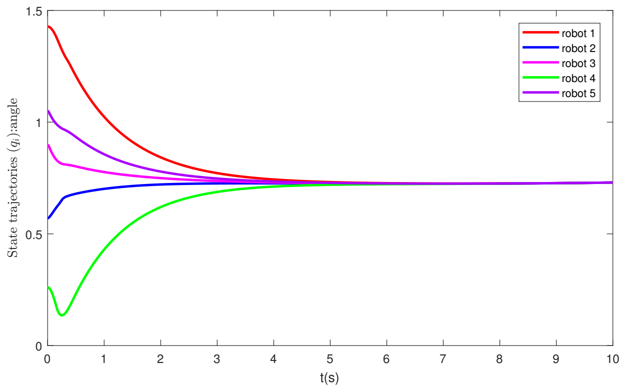

Fig. 4. The trajectories of angle (q_i).

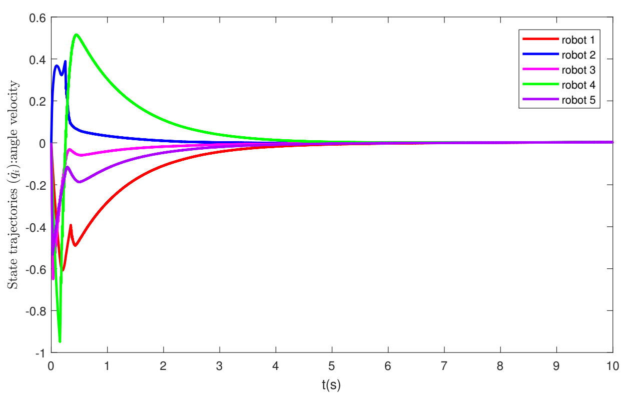

Fig. 5. The trajectories of angular velocity $(\dot{q}_i)$.

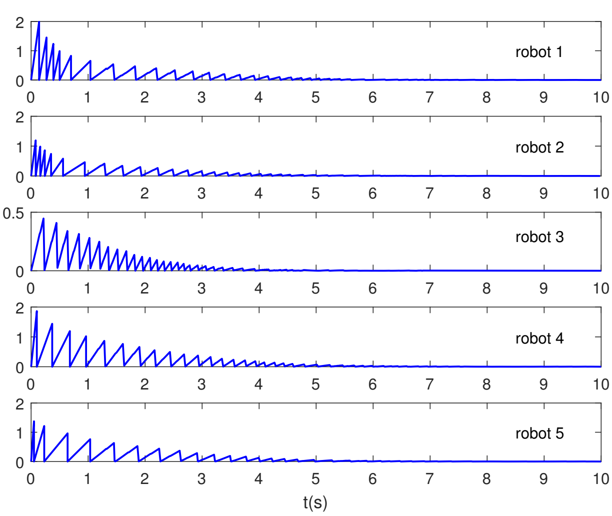

Fig. 6. The state measurement errors of robots.

posed event-triggered finite-time algorithm (5), the following continuous communication finite-time consensus algorithm is constructed.

$$u_i(t) = \Phi_i(t) - (C_1 + C_2)\, sgn(S_i(t)), \tag{32}$$

where $\Phi_i(t) = -\left(\beta \sum_{j=1}^{n} a_{ij}(r_i - r_j) + \gamma \sum_{j=1}^{n} a_{ij}(v_i - v_j)\right) - sig\left(\beta \sum_{j=1}^{n} a_{ij}(r_i - r_j) + \gamma \sum_{j=1}^{n} a_{ij}(v_i - v_j)\right)^{\alpha}$, $S_i(t) = v_i(t) - v_i(0) - \int_0^t \Phi_i(t)\, d\tau$, $\beta > 0$, $\gamma > 0$, $\alpha \in (0,1)$, $C_1 > 0$, $C_1 > 0$. The proof of stability is similar to Theorem 1, and is omitted here.

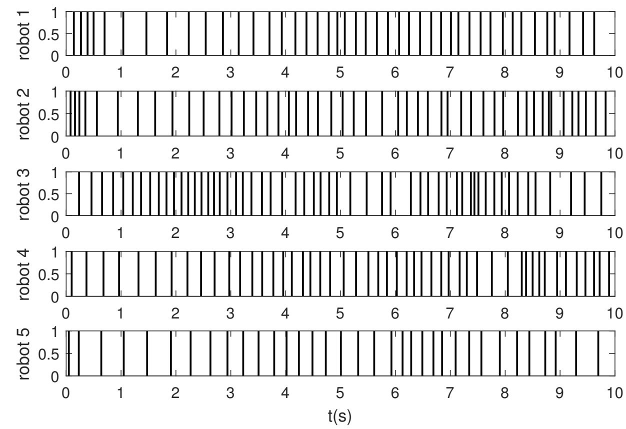

Fig. 7. Event triggering instants for all robotic arms.

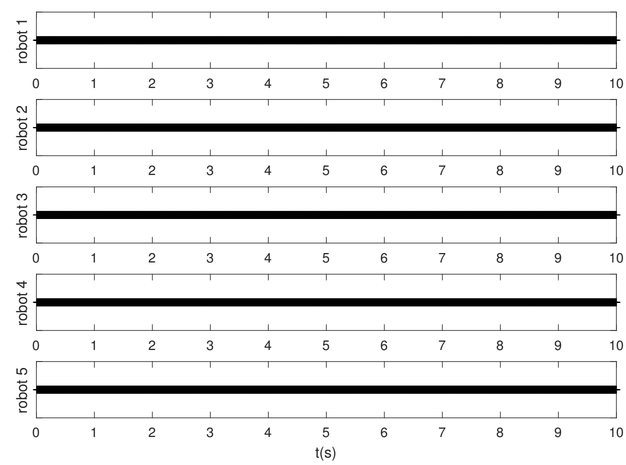

Fig. 8. The controller update using control algorithm (32).

The parameters are the same as the event-triggered controller. The controller update of each robotic arm using control algorithm (32) is given in Fig. 8, and the controller update using event-triggered control protocol (5) is given in Fig. 9. Fig. 8 shows that, using control algorithm (32), each robotic arms controller updates continuously. Fig. 9 shows that the controller only updates when its own triggering condition is satisfied and its neighbors broadcast states. Compared with the continuous control algorithm, the proposed event-triggered control algorithm can reduce the update frequency of the controller.

The simulation results verify that the proposed event-triggered algorithm solves finite-time consensus problem with continuous communication free, and the event-triggered algorithm avoids frequent updates of the controller.

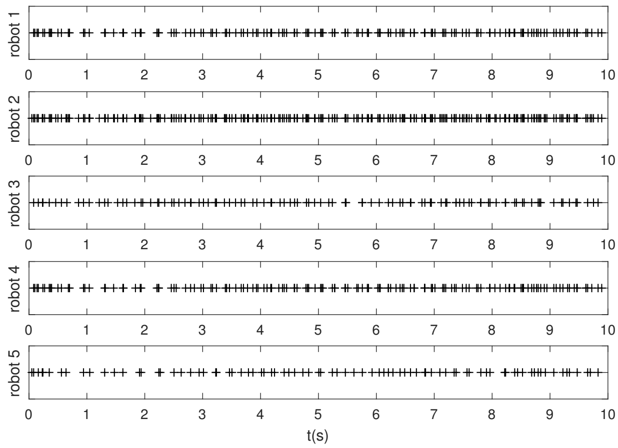

Fig. 9. The controller update using event-triggered control protocol (5).

5. CONCLUSION

This paper investigated finite-time consensus problems of second-order multi-agent systems with intrinsic nonlinear dynamics and external bounded disturbances. We proposed a novel event-triggered algorithm by utilizing integral sliding mode control strategy. With the constructed triggering function, the proposed algorithm solved the finite-time consensus problem with continuous communication free in controller update and event triggering. By applying Lyapunov stability theory and finite-time stability theory, we proved that the finite-time consensus was achieved under several conditions. Moreover, we proved that Zeno behavior was excluded. Finally, the simulation of single-link robotic arms proved that the proposed event-triggered algorithm can solve the finite-time consensus problem and save communication resources greatly.

Recently, as noted in [3, 5, 42], stochastic disturbances and time-delay have attracted an increasing interest. Stochastic disturbances and time-delay are frequently encountered in multi-agent systems and often have significant influences on the system performances. Based on [3, 5, 42], future work will focus on finite-time event-triggered consensus of stochastic multi-agent systems with time-delay.

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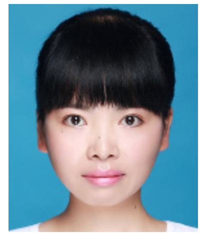

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