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Mean-square Stability in Lagrange Sense for Stochastic Memristive Neural Networks with Leakage Delay

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Abstract: In this paper, a class of stochastic memristive neural networks with leakage delay, discrete and distribute transmission delays is investigated. Based on the theory of Filippov's solution, by using Lyapunov-Krasovskii functionals, the free-weighting matrix method and stochastic analysis technique, a sufficient criterion in terms of linear matrix inequalities (LMIs) is given to ascertain the network to be exponentially Lagrange stable in mean square sense, which can be easily checked via MATLAB. Meanwhile the estimation of globally attractive set is given. Finally, numerical simulations are carried out to illustrate the feasibility of theoretical results.

Keywords: Lagrange stability, leakage delay, memristive neutral networks, stochastic perturbations.

1. INTRODUCTION

The fourth passive circuit element: memristor was postulated by Chua in 1971 [1]. In 2008, Hewlett-Packard Laboratory fabricated the first memristor [2]. It possesses many properties of resistor and shares the same unit of measurement. The variable resistance of memristor is called memristance. The memristance depends on how much charge has passed through the memristor in a particular direction. It is a function of electric charge q given as follows:

$$M(q) = \frac{\mathrm{d}\varphi}{\mathrm{d}q},$$

where φ denotes the magnetic flux. Its current-voltage characteristic is shown as a pinched hysteretic line in Fig. 1. This characteristic has been demonstrated by experiments from the scientists at the Hewlett-Packard Laboratory (see Fig. 3d in [2]). When the voltage applied to the memristor is turned off, the memristor remembers its most recent value of memristance until it is turned on next time [3]. This feature makes memristor an attractive candidate for the next generation memory technology, especially as the synapses in artificial neural networks. Memristive neural networks can be implemented by replacing the resistors with memristors in VLSI circuits of conventional neural networks. This class of neural networks is a new model to emulate the human brain [4]. It can be applied in a lot of

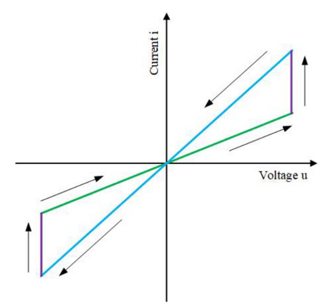

Fig. 1. Typical current-voltage characteristics of a memristor.

engineering problems. In [5], authors demonstrated pattern classification using a single-layer perceptron network implemented with a memrisitive crossbar circuit. In [6], the formation of associative memory in a simple neural network consisting of three electronic neurons connected by two memristor-emulator synapses was demonstrated by experiments. In these applications, memristive neural networks have advantages in computing speed, energy consumption and integration level. Recently, great attention has been paid to the applications of memristive neural networks. In [7], Hu and Wang proposed a simplified mathematical model to characterize the pinched hysteretic feature of the memristor. A memristive neural networks model was given in this paper and its uniformly asymp-

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totic stability was analysed. Soon afterwards many scholars were dedicated to studying dynamical properties of the memristive neural networks, such as stability, passivity, dissipativity and so on [8–14]. Among these properties, stability is one of the most important one. It is the prerequisite for memristive neural networks to be used in many applications. Up to now, many researches on stability of memristive neural networks have been published [12–16]. Most of the researches are about Lyapunov stability of monostable neural networks with a unique equilibrium attracting all trajectories. However, monostable neural networks have been found computationally restrictive in many applications. For example, the neural networks are required to have multistable equilibria when designed for associative memory or pattern recognition, so that they can get different results with diverse inputs (or initial values). In these applications, the neural networks are no longer globally stable in Lyapunov sense and it's meaningful to analyse their stability in Lagrange sense. Lagrange stability is concerned with the boundedness as well as the attractivity of systems. It has been proved that no equilibrium, chaos attractor or periodic state exists outside the global attractive set in a Lagrange stable neural network [18–20]. Moreover, the global stability in Lyapunov sense can be regarded as a special case of stability in Lagrange sense when the attractive set is an equilibrium. So far, some researches about Lagrange stability of memristive neural networks can be found in [15, 16] and references therein.

As well known, delays are inherent features in many practical networks, which can be caused by different reasons. Discrete transmission delays are often used to describe the delays caused by the finite switching speed of the neuron amplifiers. Distributed transmission delays are introduced to model the finite signal propagation speed and the presence of parallel pathways with different axon sizes or lengths. This two kinds of transmission delays have been considered in [15,16]. However, in [17], Gopalsamy proposed another kind of delays called leakage delay. Leakage delays are introduced to describe that the decay process of neurons is not instantaneous and time is required to isolate the static state. They always have a great impact on the dynamical behavior of neural networks. Thus, mixed delays should be considered in modelling memristive neural networks.

In real neural systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. Therefore, in [3, 21–30], authors have studied stochastic perturbations on systems. In [21], authors discussed almost sure exponential stability for a stochastic delay neural network. In [22], the exponentially stability of stochastic memristor-based recurrent neural networks with time-varying delays was investigated. In the model, only time-varying discrete transmission delays were considered. Some sufficient conditions in terms of inequalities were derived. Moreover, in [23], the stability of stochastic recurrent neural networks with unbounded time-varying discrete transmission delays was investigated. By constructing suitable Lyapunov functions and the semi-martingale convergence theorem, both pth moment exponential stability and almost sure exponential stability were obtained. M-matrix technique was used to make the results more applicable in this paper. In [24], both discrete and distributed transmission delays were considered in the model. A linear matrix inequality (LMI) approach was developed to establish sufficient conditions to ensure the global, robust asymptotic stability for the addressed system in the mean square. Some other results on stability of stochastic neural networks can be found in [25,26]. Also there are many researches about passivity, dissipativity and synchronization for stochastic neural networks, e.g., [27–29].

However, to the best of our knowledge, results on Lagrange stability of stochastic memristive neural networks have not been reported in the literature. Compared with traditional neural networks, the dynamical properties of memristive neural networks are more complex and difficult to analyse. To obtain more general and applicable results, mixed delays should be considered in the analysis. Meanwhile, the delays also complicate the dynamical behavior of memristive neural networks, especially the leakage delays.

Motivated by the discussions above, in this paper we analyse the Lagrange stability of stochastic memristive neural networks with leakage delay and time-varying discrete transmission delay as well as distribute transmission delay. The employing of memristors makes the neural networks state-dependent switching. These state-dependent switching neural networks are discontinuous on the right-hand side. To analyse this kind of stochastic differential equations, we turn to qualitative analysis of a relevant differential inclusion under the framework of Filippov's solution [11, 31]. Then by constructing suitable Lyapunov-Krasovskii functionals, using the free-weighting matrix method and stochastic analysis technique, a sufficient criterion in terms of LMIs is given to ascertain the network to be exponentially Lagrange stable in mean square sense. The criterion can be easily checked by Matlab LMI Toolbox. Meanwhile the estimation of globally attractive set is also given.

The main contribution of this paper is taking both leakage delay and stochastic perturbations into consideration in analysing the Lagrange stability of memristive neural networks. We not only obtain the criterion to ascertain the stability but also show the necessity of taking both of these two factors into consideration in modelling by numerical examples. At the same time, our results can extend some previous work.

The rest of the paper is organized as follows: The model description, some necessary definitions and lemmas are presented in Section 2. In Section 3, sufficient criteria are obtained respectively to ascertain the original network and two simplified networks to be exponentially Lagrange stable in mean square sense. And then, three numerical examples are given in Section 4 to demonstrate the feasibility of the theoretical results. Finally, we summarize this paper in Section 5.

Notations: $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. The superscripts A^T and A^{-1} stand for matrix transposition and matrix inverse of A, respectively. Throughout this paper, solutions of all the networks considered are in the Filippov's sense [31]. Let $C([-\eta,0],\mathbb{R}^n)$ be the Banach space of continuous functions $\psi: [-\eta,0]\to\mathbb{R}^n$ with the norm $\|\eta\|_c=\sup_{s\in[-\eta,0]}\|\psi(s)\|$. For a given constant $S>0$, $C_S=\{\psi\in C:\|\psi\|_c<S\}$. $\lambda_{\min}(\cdot)$ is the minimum eigenvalue of a certain matrix. $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. $D^+V(t)$ stands for the upper right Dini derivative of $V(t)$.

2. MODEL DESCRIPTION AND PRELIMINARIES

Based on the studies about stochastic perturbations on neural networks [3,21,22,24,26,27] and the models given in [6,7,32], we consider the following stochastic memristive neural networks model:

$$\begin{aligned}
\mathrm{d}x_i(t)=&\Bigg[-d_ix_i(t-\delta)+\sum_{j=1}^{n}a_{ij}(x_i)f_j(x_j(t))\\
&+\sum_{j=1}^{n}b_{ij}(x_i)f_j(x_j(t-\tau(t)))\\
&+\sum_{j=1}^{n}c_{ij}(x_i)\int_{t-\rho(t)}^{t}f_j(x_j(s))\mathrm{d}s+u_i(t)\Bigg]\mathrm{d}t\\
&+\sigma_i\left(t,x_i(t),x_i(t-\tau(t)),u_i(t)\right)\mathrm{d}\omega_i(t),\\
i=&1,2,\cdots,n,
\end{aligned}\tag{1}$$

where $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T\in\mathbb{R}^n$ is the state vector of the network at time t; $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\cdots,f_n(x_n(t)))^T$ denotes the neuron activation at time t. $U(t)=(u_1(t),u_2(t),\cdots,u_n(t))^T\in\mathbb{R}^n$ is a continuous external input, satisfying $|u_i(t)|\le u_i^*,(u_i^*=\max_{t\ge0}|u_i(t)|)$. Denote $U=(u_1^*,u_2^*,\cdots,u_n^*)^T$. $\sigma=(\sigma_1,\sigma_2,\cdots,\sigma_n)^T$ stands for the corresponding density of stochastic effects. $\omega(t)=(\omega_1(t),\omega_2(t),\cdots,\omega_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge0},P)$ with a filtration $\{\mathcal{F}_t\}_{t\ge0}$ satisfying the usual conditions. δ denotes the leakage delay; $\tau(t)$ and $\rho(t)$ are the discrete and distributed transmission delays, respectively. δ, $\tau(t)$ and $\rho(t)$ satisfy $\delta>0,0\le\tau(t)\le\tau,0\le\rho(t)\le\rho$, in which τ,ρ are positive constants. $D=\mathrm{diag}(d_1,d_2,\cdots,d_n)>0$ describes the rate that each neuron reset its potential to the resting state when disconnected from the networks and external inputs. Denote $d_{\max}=\max_{1\le i\le n}\{d_i\}$. $A=(a_{ij}(x_i(t)))_{n\times n}$, $B=(b_{ij}(x_i(t)))_{n\times n}$ and $C=(c_{ij}(x_i(t)))_{n\times n}$ are the connection weight matrices. According to the feature of the memristor and the current-voltage characteristic, $a_{ij}(x_i(t)),b_{ij}(x_i(t)),c_{ij}(x_i(t))$ satisfy

$$a_{ij}(x_i(t))=\begin{cases}a_{ij}^*, & |x_i(t)|\le\chi_i,\\ a_{ij}^{**}, & |x_i(t)|>\chi_i,\end{cases}$$

$$b_{ij}(x_i(t))=\begin{cases}b_{ij}^*, & |x_i(t)|\le\chi_i,\\ b_{ij}^{**}, & |x_i(t)|>\chi_i,\end{cases}$$

$$c_{ij}(x_i(t))=\begin{cases}c_{ij}^*, & |x_i(t)|\le\chi_i,\\ c_{ij}^{**}, & |x_i(t)|>\chi_i,\end{cases}$$

in which $\chi_i>0$, a_{ij}^*, a_{ij}^{**}, b_{ij}^*, b_{ij}^{**}, c_{ij}^*, c_{ij}^{**}, $(i,\ j=1,\ 2,\ \cdots,\ n)$ are constants. Obviously, for each i and j, $a_{ij}(x_i(t))$, $b_{ij}(x_i(t))$ and $c_{ij}(x_i(t))$ have two possible values. A certain state of x_i will determine the values of $3n$ parameters, thus the combination number of the possible form of A,B and C is 2^n. Order these 2^n cases in the following way:

$$(A_1,B_1,C_1),(A_2,B_2,C_2),\cdots,(A_{2^n},B_{2^n},C_{2^n}).$$

Then, at any fixed time $t\ge0$, the form of A,B and C must be one of the 2^n cases. For each case, we define the characteristic function as

$$\Psi_i(t)=\begin{cases}1, & A=A_i,\ \ B=B_i,\ \ C=C_i,\\ 0, & \text{otherwise},\end{cases}\tag{2}$$
$$i=1,2,\cdots,2^n.$$

We can easily conclude that $\sum_{i=1}^{2^n}\Psi_i(t)=1$ and

$$A=\sum_{i=1}^{2^n}\Psi_i(t)A_i,B=\sum_{i=1}^{2^n}\Psi_i(t)B_i,C=\sum_{i=1}^{2^n}\Psi_i(t)C_i.\tag{3}$$

Then, network (1) can be rewritten as the following vector form:

$$\begin{aligned}
\mathrm{d}x(t)=&\Bigg[-Dx(t-\delta)+\Big(\sum_{i=1}^{2^n}\Psi_i(t)A_i\Big)f(x(t))\\
&+\Big(\sum_{i=1}^{2^n}\Psi_i(t)B_i\Big)f(x(t-\tau(t)))\\
&+\Big(\sum_{i=1}^{2^n}\Psi_i(t)C_i\Big)\int_{t-\rho(t)}^{t}f(x(s))\mathrm{d}s+U(t)\Bigg]\mathrm{d}t\\
&+\sigma\left(t,x(t),x(t-\tau(t)),U(t)\right)\mathrm{d}\omega(t),\\
=&\sum_{i=1}^{2^n}\Psi_i(t)\Bigg[-Dx(t-\delta)+A_if(x(t))\\
&+B_if(x(t-\tau(t)))
\end{aligned}$$

$$+C_i\int_{t-\rho(t)}^{t} f(x(s))\mathrm{d}s+U(t)\Big]\mathrm{d}t$$
$$+\sigma(t,x(t),x(t-\tau(t)),U(t))\,\mathrm{d}\omega(t). \qquad (4)$$

The initial condition of network (1) is given as

$$x(t)=\psi(t),\ \ t\in[-\eta,0],\eta=\max\{\delta,\tau,\rho\}.$$

Throughout this paper, we always assume that

Assumption 1: For $i=1,2,\cdots,n$, $f_i(0)=0$ and there exist constants F_i^-,F_i^+ such that

$$F_i^-\le\frac{f_i(x_1)-f_i(x_2)}{x_1-x_2}\le F_i^+,$$

for all $x_1\ne x_2$. Denote

$$F_1=\mathrm{diag}\left(F_1^-F_1^+,F_2^-F_2^+,\cdots,F_n^-F_n^+\right),$$
$$F_2=\mathrm{diag}\left(\frac{F_1^-+F_1^+}{2},\frac{F_2^-+F_2^+}{2},\cdots,\frac{F_n^-+F_n^+}{2}\right).$$

Remark 1: The constants $F_i^-,F_i^+(i=1,2,\cdots,n)$ are allowed to be positive, negative or zero. Hence, this assumption is weaker than the assumptions in [15,20].

Assumption 2: There exist positive constant matrices R_1,R_2 and R_3 of appropriate dimensions such that

$$\mathrm{tr}\left(\sigma^{\mathrm{T}}(t,x,y,u)\,\sigma(t,x,y,u)\right)$$
$$\le x^{\mathrm{T}}R_1x+y^{\mathrm{T}}R_2y+u^TR_3u$$

holds for all $(t,x,y,u)\in\mathbb{R}^+\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n$.

For any initial condition $\psi\in C([-\eta,0],\mathbb{R}^n)$, the solution of network (1) that starts from the initial condition ψ will be denoted by $x(t,\psi)$. If there is no need to emphasize the initial condition, any solution of network (1) will also simply be denoted by $x(t)$. Let $C^{2,1}(\mathbb{R}^n\times\mathbb{R}_+,\mathbb{R}_+)$ denote the family of all nonnegative functions $V(x,t)$ on $\mathbb{R}^n\times\mathbb{R}_+$ which are twice differentiable in x and differentiable in t. For each such $V(x,t)$, we define an operator $\mathcal{L}$ associated with (1) as

$$\mathcal{L}V(x,t)=V_t(x,t)+V_x(x,t)\Big[-Dx(t-\delta)+Af(x(t))$$
$$+Bf(x(t-\tau(t)))+C\int_{t-\rho(t)}^{t} f(x(s))\mathrm{d}s+U(t)\Big]$$
$$+\frac{1}{2}\mathrm{tr}[\sigma^TV_{xx}(x,t)\sigma],$$

where $V_t(x,t)=\frac{\partial V(x,t)}{\partial t},V_x(x,t)=\left(\frac{\partial V(x,t)}{\partial x_1},\cdots,\frac{\partial V(x,t)}{\partial x_n}\right)$, $V_{xx}(x,t)=\left(\frac{\partial^2 V(x,t)}{\partial x_i\partial x_j}\right)_{n\times n}$.

Definition 1: Network (1) is said to be uniformly bounded in mean square sense, if for any $S>0$, there exists a constant $\kappa=\kappa(S)>0$ such that $\mathbf{E}\|x(t,\psi)\|^2<\kappa$ for all $\psi\in C_S$ and $t\ge0$.

Definition 2: If there exist a radially unbounded and positive definite function $V(x(t))$, a functional $\kappa\in C$, positive constants C_-,C^+,ℓ,α, such that for any solution $x(t)=x(t,\psi)$ of network (1), $t\ge0$ implies

$$C_-\|x(t)\|^2\le V(x(t),t)\le C^+\|x(t)\|^2,$$

and

$$\mathbf{E}V(x(t),t)-\ell\le\kappa(\psi)\exp(-\alpha t),$$

then network (1) is said to be globally exponentially attractive in the mean square sense, and the compact set $\Omega:=\{x\in\mathbb{R}^n,V(x,t)<\ell\}$ is called a globally exponentially attractive set of network (1) in mean square sense.

Definition 3: Network (1) is called exponentially Lagrange stable in mean square sense, if it is both uniformly bounded in mean square sense and globally exponentially attractive in mean square sense.

To prove our results, the following lemmas are necessary.

Lemma 1 [33]**:** Let $a,b\in\mathbb{R}^n$ and Q be a positive definite matrix, then $2a^Tb\le a^TQ^{-1}a+b^TQb$.

Lemma 2 [34]**:** Let h be a positive constant, and $Q\in\mathbb{R}^{n\times n}$ be a positive definite constant matrix, then

$$\left(\int_{t-h}^{t}x(s)\mathrm{d}s\right)^TQ\int_{t-h}^{t}x(s)\mathrm{d}s\le h\int_{t-h}^{t}x^T(s)Qx(s)\mathrm{d}s,$$

for $t\ge0$ and any vector function $x(s)\in\mathbb{R}^n$.

Lemma 3 [28]**:** The LMI $Y=\begin{pmatrix}y_{11}&y_{12}\\y_{12}^T&y_{22}\end{pmatrix}<0$ with $y_{11}^T=y_{11},y_{22}^T=y_{22}$ is equivalent the following condition:

$$y_{22}<0,\ \ y_{11}-y_{12}y_{22}^{-1}y_{12}^T<0.$$

Lemma 4 [35]**:** Let $V(x(t)):\mathbb{R}^n\to\mathbb{R}_+$ be a positive definite and radially unbounded function, and suppose there exist two positive constants ϖ,π such that

$$D^+\mathbf{E}V(x(t))\le-\varpi\mathbf{E}V(x(t))+\pi,\ \ t\ge t_0,$$

then,

$$\mathbf{E}V(x(t))-\frac{\pi}{\varpi}\le\left(\mathbf{E}V(x(t_0))-\frac{\pi}{\varpi}\right)e^{-\varpi(t-t_0)}.$$

3. MAIN RESULTS

Theorem 1: Under Assumptions 1 and 2, if there exist positive definite matrices P_1, P_2, P_3, P_4, P_5, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8, positive definite diagonal matrices S_1, S_2, matrices K_1, K_2, K_3, K_4 and positive constants $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ such that the following LMIs hold:

$$P_1<\lambda_1I,\ Q_2<\lambda_2I,\ Q_4<\lambda_3I,\ P_4<\lambda_4I, \qquad (5)$$

$$\Pi_i=\begin{bmatrix}\Xi_i&\Theta\\\Theta^T&\Upsilon\end{bmatrix}<0,\ \ i=1,2,\cdots,2^n, \qquad (6)$$

where

$$\Theta = \begin{bmatrix} P_1 & 0 & 0 & 0 & K_3 & K_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_4 & K_4 \\ 0 & 0 & 0 & K_2 & 0 & 0 & 0 & 0 \\ 0 & P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon = \text{diag}\bigg(-Q_5, -Q_6, -Q_7, -Q_8, -\frac{Q_1}{\delta}, -Q_2, -\frac{Q_3}{\tau}, -Q_4 \bigg),$$

$$\Xi_i = \begin{bmatrix} \Sigma_{11} & K_4 & K_3 & P_1D-P_1 & P_1A_i+F_2S_1 & P_1B_i & P_1C_i & P_4 \\ * & \Sigma_{22} & 0 & 0 & 0 & F_2S_2 & 0 & 0 \\ * & * & \Sigma_{33} & 0 & K_2A_i & K_2B_i & K_2C_i & -K_1D-K_2 \\ * & * & * & P_1-\frac{e^{-\delta}}{\delta}P_2 & -P_1A_i & -P_1B_i & -P_1C_i & 0 \\ * & * & * & * & \rho P_3-S_1 & 0 & 0 & K_1A_i \\ * & * & * & * & * & -S_2 & 0 & K_1B_i \\ * & * & * & * & * & * & -\frac{e^{-\rho}}{\rho}P_3 & K_1C_i \\ * & * & * & * & * & * & * & \Sigma_{88} \end{bmatrix},$$

in which

$$\begin{aligned} \Sigma_{11} &= P_1+P_4+(1+e^{\delta}\delta)P_5-P_1D-D^TP_1^T \\ &\quad -K_3-K_3^T-F_1S_1+\delta D^TP_2D \\ &\quad +\Big(\lambda_1+e^{\delta}\delta\lambda_2+e^{\tau}\tau\lambda_3+\lambda_4\Big)R_1, \\ \Sigma_{22} &= -K_4-K_4^T-F_1S_2 \\ &\quad +\Big(\lambda_1+e^{\delta}\delta\lambda_2+e^{\tau}\tau\lambda_3+\lambda_4\Big)R_2, \\ \Sigma_{33} &= -P_5-K_2D-D^TK_2^T, \\ \Sigma_{88} &= -K_1-{K_1}^T+e^{\delta}\delta Q_1+e^{\tau}\tau Q_3, \end{aligned}$$

then network (1) is exponentially Lagrange stable in mean square sense, and

$$\Omega = \left\{ x(t)\in\mathbb{R}^n, \|x(t)\| \le \sqrt{\frac{W}{\lambda_{\min}(P_1)}}e^{\delta d_{\max}} \right\} \cap \left\{ x(t)\in\mathbb{R}^n, \|x(t)\| \le \sqrt{\frac{W}{\lambda_{\min}(P_4)}} \right\},$$

is a globally exponentially attractive set of network (1) in mean square sense, in which $W = U^T\left[Q_5+Q_6+Q_7+Q_8+\left(\lambda_1+e^{\delta}\delta\lambda_2+e^{\tau}\tau\lambda_3+\lambda_4\right)R_3\right]U$.

Proof: Let

$$\begin{aligned} y(t) &= -Dx(t-\delta)+Af(x(t))+Bf(x(t-\tau(t))) \\ &\quad +C\int_{t-\rho(t)}^{t} f(x(s))ds+U(t), \\ \alpha(t) &= \sigma\left(t,x(t),x(t-\tau(t)),U(t)\right), \end{aligned}$$

then network (1) can be rewritten as

$$\mathrm{d}x(t) = y(t)\mathrm{d}t+\alpha(t)\mathrm{d}\omega(t). \tag{7}$$

Define the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(x(t),t) =& V_1(x(t),t)+V_2(x(t),t)+V_3(x(t),t) \\ &+V_4(x(t),t)+V_5(x(t),t), \end{aligned} \tag{8}$$

where

$$\begin{aligned} V_1(x(t),t) &= \left(x(t)-D\int_{t-\delta}^{t}x(s)\mathrm{d}s\right)^T P_1\left(x(t)-D\int_{t-\delta}^{t}x(s)\mathrm{d}s\right), \\ V_2(x(t),t) &= \int_{-\delta}^{0}\int_{t+\theta}^{t} e^{s-t}\Big\{x(s)^{\mathrm{T}}D^{\mathrm{T}}P_2Dx(s) \\ &\quad +e^{\delta}x(s)^{\mathrm{T}}P_5x(s)+e^{\delta}y^T(s)Q_1y(s) \\ &\quad +e^{\delta}\mathrm{tr}\left[\alpha^T(s)Q_2\alpha(s)\right]\Big\}\mathrm{d}s\mathrm{d}\theta, \\ V_3(x(t),t) &= \int_{-\rho}^{0}\int_{t+\theta}^{t} e^{s-t}f^T(x(s))P_3f(x(s))\mathrm{d}s\mathrm{d}\theta, \\ V_4(x(t),t) &= \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{s-t}\{e^{\tau}y^T(s)Q_3y(s) \\ &\quad +e^{\tau}\mathrm{tr}\left[\alpha^T(s)Q_4\alpha(s)\right]\}\mathrm{d}s\mathrm{d}\theta, \\ V_5(x(t),t) &= x^T(t)P_4x(t)+\int_{t-\delta}^{t}x^T(s)P_5x(s)\mathrm{d}s. \end{aligned}$$

Applying the Itô differential formula to $V_i(x(t),t)$ yields

$$\begin{aligned} \mathcal{L}V_1(x(t),t) &= 2\left(x(t)-D\int_{t-\delta}^{t}x(s)\mathrm{d}s\right)^{\mathrm{T}}P_1\Big[-Dx(t) \\ &\quad +Af(x(t))+Bf(x(t-\tau(t))) \\ &\quad +C\int_{t-\rho(t)}^{t}f(x(s))\mathrm{d}s+U(t)\Big] \\ &\quad +\mathrm{tr}\left[\alpha^{\mathrm{T}}(t)P_1\alpha(t)\right], \\ \mathcal{L}V_2(x(t),t) &= -V_2((x(t),t)+\delta x^T(t)D^TP_2Dx(t) \\ &\quad +e^{\delta}\delta x(t)^{\mathrm{T}}P_5x(t) \\ &\quad +e^{\delta}\delta y^T(t)Q_1y(t)+e^{\delta}\delta\mathrm{tr}\left[\alpha^T(t)Q_2\alpha(t)\right] \\ &\quad -\int_{t-\delta}^{t}e^{s-t}x^T(s)D^TP_2Dx(s)\mathrm{d}s \\ &\quad -\int_{t-\delta}^{t}e^{s-t+\delta}x^T(s)P_5x(s)\mathrm{d}s \end{aligned}$$

$$
\begin{aligned}
&-\int_{t-\delta}^{t} e^{s-t+\delta} y^T(s) Q_1 y(s)\mathrm{d}s \\
&-\int_{t-\delta}^{t} e^{s-t+\delta} \mathrm{tr}\left[\alpha^T(s) Q_2 \alpha(s)\right]\mathrm{d}s, \\
\mathcal{L}V_3(x(t),t) = &-V_3(x(t),t) + \rho f^T(x(t)) P_3 f(x(t)) \\
&-\int_{t-\rho}^{t} e^{s-t} f^T(x(s)) P_3 f(x(s))\mathrm{d}s, \\
\mathcal{L}V_4(x(t),t) = &-V_4(t) + e^{\tau}\tau y^T(t) Q_3 y(t) \\
&+ e^{\tau}\tau \mathrm{tr}\left[\alpha^T(t) Q_4 \alpha(t)\right] \\
&-\int_{t-\tau}^{t} e^{s-t+\tau} y^T(s) Q_3 y(s)\mathrm{d}s, \\
&-\int_{t-\tau}^{t} e^{s-t+\tau} \mathrm{tr}\left[\alpha^T(s) Q_4 \alpha(s)\right]\mathrm{d}s, \\
\mathcal{L}V_5(x(t),t) = &\,\mathrm{tr}(\alpha^{\mathrm{T}}(t) P_4 \alpha(t)) + 2x^T(t) P_4 y(t) \\
&+ x^T(t) P_5 x(t) - x^T(t-\delta) P_5 x(t-\delta).
\end{aligned} \tag{9}
$$

From Assumption 2 and inequalities in (5), we have

$$
\begin{aligned}
&\mathrm{tr}\left[\alpha^{\mathrm{T}}(t) P_1 \alpha(t)\right] \\
&\le \lambda_1\left[x^{\mathrm{T}}(t) R_1 x(t) + x^{\mathrm{T}}(t-\tau(t)) R_2 x(t-\tau(t)) \right.\\
&\quad \left. + U^T(t) R_3 U(t)\right], \\
&\mathrm{tr}\left[\alpha^T(s) Q_2 \alpha(s)\right] \\
&\le \lambda_2\left[x^{\mathrm{T}}(t) R_1 x(t) + x^{\mathrm{T}}(t-\tau(t)) R_2 x(t-\tau(t)) \right.\\
&\quad \left. + U^T(t) R_3 U(t)\right], \\
&\mathrm{tr}\left[\alpha^T(s) Q_4 \alpha(s)\right] \\
&\le \lambda_3\left[x^{\mathrm{T}}(t) R_1 x(t) + x^{\mathrm{T}}(t-\tau(t)) R_2 x(t-\tau(t)) \right.\\
&\quad \left. + U^T(t) R_3 U(t)\right], \\
&\mathrm{tr}\left[\alpha^{\mathrm{T}}(t) P_4 \alpha(t)\right] \\
&\le \lambda_4\left[x^{\mathrm{T}}(t) R_1 x(t) + x^{\mathrm{T}}(t-\tau(t)) R_2 x(t-\tau(t)) \right.\\
&\quad \left. + U^T(t) R_3 U(t)\right].
\end{aligned} \tag{10}
$$

From the definition of $y(t)$, we have

$$
\begin{aligned}
0 = &\left(2y^T(t) K_1 + 2x^T(t-\delta) K_2\right)\Big[-y(t) \\
&- Dx(t-\delta) + Af(x(t)) + Bf(x(t-\tau(t))) \\
&+ C\int_{t-\rho(t)}^{t} f(x(s))\mathrm{d}s + U(t)\Big].
\end{aligned} \tag{11}
$$

Integrating both sides of (7) from $t-\delta$ to t, we obtain

$$
x(t) - x(t-\delta) - \int_{t-\delta}^{t} y(s)\mathrm{d}s - \int_{t-\delta}^{t} \alpha(s)\mathrm{d}\omega(s) = 0.
$$

By Lemmas 1 and 2, we get

$$
\begin{aligned}
0 = &-2x^T(t) K_3\Big(x(t) - x(t-\delta) - \int_{t-\delta}^{t} y(s)\mathrm{d}s \\
&- \int_{t-\delta}^{t} \alpha(s)\mathrm{d}\omega(s)\Big) \\
\le &\, x^T(t)\left(-2K_3 + \delta K_3 Q_1^{-1} K_3^T + K_3 Q_2^{-1} K_3^T\right) x(t) \\
&+ 2x^T(t) K_3 x(t-\delta) + \int_{t-\delta}^{t} y^T(s) Q_1 y(s)\mathrm{d}s \\
&+ \left(\int_{t-\delta}^{t} \alpha(s)\mathrm{d}\omega(s)\right)^T Q_2 \left(\int_{t-\delta}^{t} \alpha(s)\mathrm{d}\omega(s)\right).
\end{aligned} \tag{12}
$$

Integrating both sides of (7) from $t-\tau(t)$ to t, we have

$$
x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} y(s)\mathrm{d}s - \int_{t-\tau(t)}^{t} \alpha(s)\mathrm{d}\omega(s) = 0.
$$

By Lemmas 1 and 2 and noting that $0 \le \tau(t) \le \tau$, we get

$$
\begin{aligned}
0 = &\, 2x^T(t-\tau(t)) K_4\Big(x(t) - x(t-\tau(t)) \\
&- \int_{t-\tau(t)}^{t} y(s)\mathrm{d}s - \int_{t-\tau(t)}^{t} \alpha(s)\mathrm{d}\omega(s)\Big) \\
\le &\, x^T(t-\tau(t))\big(-2K_4 + \tau K_4 Q_3^{-1} K_4^T \\
&+ K_4 Q_4^{-1} K_4^T\big) x(t-\tau(t)) + \int_{t-\tau}^{t} y^T(s) Q_3 y(s)\mathrm{d}s \\
&+ \left(\int_{t-\tau}^{t} \alpha(s)\mathrm{d}\omega(s)\right)^T Q_4 \left(\int_{t-\tau}^{t} \alpha(s)\mathrm{d}\omega(s)\right) \\
&+ 2x^T(t-\tau(t)) K_4 x(t).
\end{aligned} \tag{13}
$$

Obviously,

$$
\begin{aligned}
&-\int_{t-\delta}^{t} e^{s-t+\delta} x^T(s) P_5 x(s)\mathrm{d}s \le -\int_{t-\delta}^{t} x^T(s) P_5 x(s)\mathrm{d}s, \\
&-\int_{t-\delta}^{t} e^{s-t+\delta} y^T(s) Q_1 y(s)\mathrm{d}s \le -\int_{t-\delta}^{t} y^T(s) Q_1 y(s)\mathrm{d}s, \\
&-\int_{t-\tau}^{t} e^{s-t+\tau} y^T(s) Q_3 y(s)\mathrm{d}s \le -\int_{t-\tau}^{t} y^T(s) Q_3 y(s)\mathrm{d}s.
\end{aligned} \tag{14}
$$

By using Lemma 2 and noting that $0 \le \rho(t) \le \rho$, we have

$$
\begin{aligned}
&-\int_{t-\rho}^{t} e^{s-t} f^T(x(s)) P_3 f(x(s))\mathrm{d}s \\
&\le -\frac{e^{-\rho}}{\rho}\int_{t-\rho(t)}^{t} f^T(x(s))\mathrm{d}s P_3 \int_{t-\rho(t)}^{t} f(x(s))\mathrm{d}s, \\
&-\int_{t-\delta}^{t} e^{s-t} x^T(s) D^T P_2 D x(s)\mathrm{d}s \\
&\le -\frac{e^{-\delta}}{\delta}\int_{t-\delta}^{t} x^T(s)\mathrm{d}s D^T P_2 D \int_{t-\delta}^{t} x(s)\mathrm{d}s.
\end{aligned} \tag{15}
$$

By Lemma 1 we have

$$
\begin{aligned}
&2x^{\mathrm{T}}(t) P_1 U(t) - 2\int_{t-\delta}^{t} x^{\mathrm{T}}(s)\mathrm{d}s D^{\mathrm{T}} P_1 U(t) \\
&\quad + \left(2y^T(t) K_1 + 2x^T(t-\delta) K_2\right) U(t) \\
&\le -\xi^T(t)\tilde{\Theta}\tilde{\Upsilon}^{-1}\tilde{\Theta}^T\xi(t) \\
&\quad + U^T(t)\left(Q_5 + Q_6 + Q_7 + Q_8\right) U(t),
\end{aligned} \tag{16}
$$

where

$$\xi(t)=\Big[x^T(t),x^T(t-\tau(t)),x^T(t-\delta),\int_{t-\delta}^{t}x^T(s)dsD^T,\\ f^T(x(t)),f^T(x(t-\tau(t))),\\ \int_{t-\rho(t)}^{t}f^T(x(s))ds,y^T(t)\Big]^T,$$

$$\tilde{\Upsilon}=\mathrm{diag}\left(-Q_5,-Q_6,-Q_7,-Q_8\right),$$

$$\tilde{\Theta}=\begin{bmatrix} P_1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & K_2\\ 0 & P_1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & K_1 & 0\end{bmatrix}.$$

For positive diagonal matrices S_1,S_2, it follows from Assumption 1 and the proof of Theorem 1 in [36] that

$$0\le\begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix}^T\begin{bmatrix} -F_1S_1 & F_2S_1\\ F_2S_1 & -S_1\end{bmatrix}\begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix},$$
$$0\le\begin{bmatrix} x(t-\tau(t))\\ f(x(t-\tau(t)))\end{bmatrix}^T\begin{bmatrix} -F_1S_2 & F_2S_2\\ F_2S_2 & -S_2\end{bmatrix}\times\begin{bmatrix} x(t-\tau(t))\\ f(x(t-\tau(t)))\end{bmatrix}. \tag{17}$$

From the proof of Theorem 1 in [37], we can get that

$$\mathbf{E}\left\{\left(\int_{t-\tau}^{t}\alpha(s)\mathrm{d}\omega(s)\right)^T Q_2\left(\int_{t-\tau}^{t}\alpha(s)\mathrm{d}\omega(s)\right)\right\}\\ =\mathbf{E}\left\{\int_{t-\delta}^{t}\mathrm{tr}\left[\alpha^T(s)Q_2\alpha(s)\right]\mathrm{d}s\right\},\\ \mathbf{E}\left\{\left(\int_{t-\tau}^{t}\alpha(s)\mathrm{d}\omega(s)\right)^T Q_4\left(\int_{t-\tau}^{t}\alpha(s)\mathrm{d}\omega(s)\right)\right\}\\ =\mathbf{E}\left\{\int_{t-\tau}^{t}\mathrm{tr}\left[\alpha^T(s)Q_4\alpha(s)\right]\mathrm{d}s\right\}. \tag{18}$$

From the definition of Dini-derivative and the generalized Itô formula (see [38]), we have $D^+\mathbf{E}V(x(t),t)=\mathbf{E}\mathcal{L}V(x(t),t)$, then it follows from (8) to (18) that

$$D^+\mathbf{E}V(x(t),t)\\ \le-\mathbf{E}V(x(t),t)+\xi^T(t)\Xi\xi(t)-\xi^T(t)\Theta\Upsilon^{-1}\Theta^T\xi(t)\\ +U^T(t)\left(Q_5+Q_6+Q_7+Q_8\right)U(t)\\ +\left(\lambda_1+e^{\delta}\delta\lambda_2+e^{\tau}\tau\lambda_3+\lambda_4\right)U^T(t)R_3U(t), \tag{19}$$

in which

$$\Xi=\begin{bmatrix} \Sigma_{11} & K_4 & K_3 & P_1D-P_1 & P_1A+F_2S_1 & P_1B & P_1C & P_4\\ * & \Sigma_{22} & 0 & 0 & 0 & F_2S_2 & 0 & 0\\ * & * & \Sigma_{33} & 0 & K_2A & K_2B & K_2C & -K_1D-K_2\\ * & * & * & P_1-\frac{e^{-\delta}}{\delta}P_2 & -P_1A & -P_1B & -P_1C & 0\\ * & * & * & * & \rho P_3-S_1 & 0 & 0 & K_1A\\ * & * & * & * & * & -S_2 & 0 & K_1B\\ * & * & * & * & * & * & -\frac{e^{-\rho}}{\rho}P_3 & K_1C\\ * & * & * & * & * & * & * & \Sigma_{88}\end{bmatrix}.$$

We can derive from (3) and (6) that

$$\sum_{i=1}^{2^n}\Psi_i(t)\Pi_i<0.$$

According to Lemma 3, we have

$$\Xi-\Theta\Upsilon^{-1}\Theta^T<0. \tag{20}$$

Substituting (20) into (19) yields

$$D^+\mathbf{E}V(x(t),t)\le-\mathbf{E}V(x(t),t)+W.$$

Then, by Lemma 4, we know that

$$\mathbf{E}V(x(t),t)-W\le\left(\mathbf{E}V(x(0),0)-W\right)e^{-t},$$

which implies network (1) is globally exponentially attractive in the mean square sense. Also, we can get from

$$\lambda_{\min}(P_4)\mathbf{E}\|x(t)\|^2\le\mathbf{E}V(x(t),t)\le\mathbf{E}V(x(0),0)+W,$$

that

$$\mathbf{E}\|x(t)\|^2\le\frac{\mathbf{E}V(x(0),0)+W}{\lambda_{\min}(P_4)},$$

thus network (1) is uniformly bounded in mean square sense. Then (1) is exponentially Lagrange stable in mean square sense.
Solving the inequality

$$\lambda_{\min}(P_1)\left\|x(t)-D\int_{t-\delta}^{t}x(s)ds\right\|^2\le V_1(x(t),t)\le W$$

with Gronwall inequality, we obtain

$$\|x(t)\|\le\sqrt{\frac{W}{\lambda_{\min}(P_1)}}e^{\delta d_{\max}}.$$

Solving the inequality

$$\lambda_{\min}(P_4)\|x(t)\|^2 \le V_5(x(t),t) \le W,$$

we have

$$\|x(t)\| \le \sqrt{\frac{W}{\lambda_{\min}(P_4)}}.$$

So that

$$\Omega = \left\{x(t) \in \mathbb{R}^n, \|x(t)\| \le \sqrt{\frac{W}{\lambda_{\min}(P_1)}} e^{\delta d_{\max}}\right\} \cap \left\{x(t) \in \mathbb{R}^n, \|x(t)\| \le \sqrt{\frac{W}{\lambda_{\min}(P_4)}}\right\}$$

is a globally exponentially attractive set of network (1) in mean square sense. This completes the proof.

Remark 2: There are some researches about switched stochastic systems in [39–41]. Switches of the systems in these papers are time-dependent, while the switches in memristive neural networks are state-dependent. The ways to deal with the switches are different. However, in this paper, we essentially analysed the stability of each subsystem. So when analysing the subsystems of switched stochastic systems, our method can be used for reference.

Remark 3: The perturbations we considered in this paper are Gaussian noises. There are some other kinds of noises in practical applications, such as non-Gaussian Lévy noise and Poisson white noise. Some researches about non-Gaussian noises in systems can be found in [42–45]. However, the stability of neural networks with non-Gaussian noises is still an open problem and needs further study.

Remark 4: Obviously, Theorem 1 also works for traditional neural networks with perturbations and mixed delays. Moreover, the model in this paper is more general than the models in [16, 21, 22, 27, 28], so our result can extend these work.

If there is no leakage delay, network (1) reduces to the following neural network

$$\begin{aligned} \mathrm{d}x(t) = \Big[& -Dx(t) + Af(x(t)) + Bf(x(t-\tau(t))) \\ & + C\int_{t-\rho(t)}^{t} f(x(s))\mathrm{d}s + U(t)\Big]\mathrm{d}t \\ & + \alpha(t)\mathrm{d}\omega(t). \end{aligned} \tag{21}$$

When there is no stochastic effects, network (1) turns to

$$\begin{aligned} \frac{\mathrm{d}x(t)}{\mathrm{d}t} = & -Dx(t-\delta) + Af(x(t)) \\ & + Bf(x(t-\tau(t))) + C\int_{t-\rho(t)}^{t} f(x(s))\mathrm{d}s \\ & + U(t). \end{aligned} \tag{22}$$

From the proof of Theorem 1, we can obtain the following corollaries for network (21) and (22).

Corollary 1: For network (21), under assumptions **(A1)** and **(A2)**, if there exist positive definite matrices $P_1, P_2, Q_1, Q_2, Q_3, Q_4$, positive definite diagonal matrices S_1, S_2, matrices K_1, K_2 and positive constants λ_1, λ_2 such that the following LMIs hold:

$$\begin{aligned} & P_1 < \lambda_1 I, Q_2 < \lambda_2 I, \\ & \Pi_i = \begin{bmatrix} \Xi_i & \Theta \\ \Theta^T & \Upsilon \end{bmatrix} < 0, \quad i = 1,2,\cdots,2^n, \end{aligned} \tag{23}$$

where

$$\Theta = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ 0 & 0 & K_2 & K_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & K_1 & 0 & 0 \end{bmatrix},$$

$$\Upsilon = \mathrm{diag}\left(-Q_3, -Q_4, -\frac{Q_1}{\tau}, -Q_2\right),$$

$$\Xi_i = \begin{bmatrix} \Sigma_{11} & K_2 & P_1A_i + F_2S_1 & P_1B_i & P_1C_i & -K_1D \\ * & \Sigma_{22} & 0 & F_2S_2 & 0 & 0 \\ * & * & \rho P_2 - S_1 & 0 & 0 & K_1A_i \\ * & * & * & -S_2 & 0 & K_1B_i \\ * & * & * & * & -\frac{e^{-\rho}}{\rho}P_2 & K_1C_i \\ * & * & * & * & * & \Sigma_{66} \end{bmatrix},$$

in which

$$\begin{aligned} \Sigma_{11} &= P_1 - P_1D - D^TP_1^T - F_1S_1 + (\lambda_1 + e^{\tau}\tau\lambda_2)R_1, \\ \Sigma_{22} &= -K_2 - K_2^T - F_1S_2 + (\lambda_1 + e^{\tau}\tau\lambda_2)R_2, \\ \Sigma_{66} &= -K_1 - K_1{}^T + e^{\tau}\tau Q_1. \end{aligned}$$

then network (21) is exponentially Lagrange stable in mean square sense, and

$$\Omega = \left\{x(t) \in \mathbb{R}^n, \|x(t)\| \le \sqrt{\frac{W}{\lambda_{\min}(P_1)}}\right\}$$

is a globally exponentially attractive set of network (1) in mean square sense, in which $W = U^T[Q_3 + Q_4 + (\lambda_1 + e^{\tau}\tau\lambda_2)R_3]U$.

Corollary 2: For network (22), under Assumption 1, if there exist positive definite matrices $P_1, P_2, P_3, P_4, P_5, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8$, positive definite diagonal matrices S_1, S_2, matrices K_1, K_2, K_3, K_4 such that the following LMIs hold:

$$\Pi_i = \begin{bmatrix} \Xi_i & \Theta \\ \Theta^T & \Upsilon \end{bmatrix} < 0, i = 1,2,\cdots,2^n \tag{24}$$

where

$$\Theta = \begin{bmatrix} P_1 & 0 & 0 & 0 & K_3 & K_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_4 & K_4 \\ 0 & 0 & 0 & K_2 & 0 & 0 & 0 & 0 \\ 0 & P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon = \mathrm{diag}\Big(-Q_5, -Q_6, -Q_7, -Q_8, -\frac{Q_1}{\delta}, -Q_2, -\frac{Q_3}{\tau}, -Q_4\Big),$$

$$\Xi_i = \begin{bmatrix} \Sigma_{11} & K_4 & K_3 & P_1D - P_1 & P_1A_i + F_2S_1 & P_1B_i & P_1C_i & P_4 \\ * & \Sigma_{22} & 0 & 0 & 0 & F_2S_2 & 0 & 0 \\ * & * & \Sigma_{33} & 0 & K_2A_i & K_2B_i & K_2C_i & -K_1D - K_2 \\ * & * & * & P_1 - \frac{e^{-\delta}}{\delta}P_2 & -P_1A_i & -P_1B_i & -P_1C_i & 0 \\ * & * & * & * & \rho P_3 - S_1 & 0 & 0 & K_1A_i \\ * & * & * & * & * & -S_2 & 0 & K_1B_i \\ * & * & * & * & * & * & -\frac{e^{-\rho}}{\rho}P_3 & K_1C_i \\ * & * & * & * & * & * & * & \Sigma_{88} \end{bmatrix},$$

in which

$$\Sigma_{11} = P_1 + P_4 + (1 + e^{\delta}\delta)P_5 - P_1D - D^TP_1^T - K_3 - K_3^T - F_1S_1 + \delta D^TP_2D,$$
$$\Sigma_{22} = -K_4 - K_4^T - F_1S_2,$$
$$\Sigma_{33} = -P_5 - K_2D - D^TK_2^T,$$
$$\Sigma_{88} = -K_1 - {K_1}^T + e^{\delta}\delta Q_1 + e^{\tau}\tau Q_3.$$

Then network (22) is exponentially Lagrange stable in mean square sense, and

$$\Omega = \left\{x(t) \in \mathbb{R}^n, \|x(t)\| \le \sqrt{\frac{W}{\lambda_{\min}(P_1)}}e^{\delta d_{\max}}\right\} \cap \left\{x(t) \in \mathbb{R}^n, \|x(t)\| \le \sqrt{\frac{W}{\lambda_{\min}(P_4)}}\right\},$$

is a globally exponentially attractive set of network (22) in mean square sense, in which $W = U^T(Q_5 + Q_6 + Q_7 + Q_8)U$.

4. NUMERICAL SIMULATIONS

In this section, three examples are provided to demonstrate the feasibility of the theoretical results.

Example 1 is provided to demonstrate the feasibility of Theorem 1. By simulating the state trajectories of the neural network with and without inputs, we show that the conditions can ensure the Lagrange stability of network (1). Then, we adjust the value of leakage delay to show its effect on destabilizing the system. Moreover, by comparing the state trajectories of the network in Example 1 with and without stochastic perturbations, we demonstrate the effect of stochastic perturbations on destabilizing the system. The comparisons also indicate the necessity of taking both leakage delay and stochastic perturbations into consideration in analysing the Lagrange stability of memristive neural.

Examples 2 and 3 are provided to demonstrate the feasibility of the Corollary 1 and 2 respectively. Example 3 also show that our results can also be applied to traditional neural networks.

To simulate the stochastic differential equations, in Examples 1 and 2, we used the algorithm approved in [46].

Example 1: Consider two-dimensional stochastic memristive neural network (1) with following parameters:

$$d_1 = 1.8,\ d_2 = 1.7,$$
$$a_{11}(x_1) = \begin{cases} -0.5, |x_1| \le 1, \\ 0.2, |x_1| > 1, \end{cases} a_{12}(x_1) = \begin{cases} 0.4, |x_1| \le 1, \\ 0.2, |x_1| > 1, \end{cases}$$
$$a_{21}(x_2) = \begin{cases} 0.3, |x_2| \le 1, \\ -0.7, |x_2| > 1, \end{cases} a_{22}(x_2) = \begin{cases} -0.5, |x_2| \le 1, \\ -0.4, |x_2| > 1, \end{cases}$$
$$b_{11}(x_1) = \begin{cases} 0.2, |x_1| \le 1, \\ 0.3, |x_1| > 1, \end{cases} b_{12}(x_1) = \begin{cases} 0.3, |x_1| \le 1, \\ 0.2, |x_1| > 1, \end{cases}$$
$$b_{21}(x_2) = \begin{cases} -0.2, |x_2| \le 1, \\ 0.3, |x_2| > 1, \end{cases} b_{22}(x_2) = \begin{cases} 0.2, |x_2| \le 1, \\ -0.4, |x_2| > 1, \end{cases}$$
$$c_{11}(x_1) = \begin{cases} 0.4, |x_1| \le 1, \\ 0.5, |x_1| > 1, \end{cases} c_{12}(x_1) = \begin{cases} 0.8, |x_1| \le 1, \\ 0.2, |x_1| > 1, \end{cases}$$
$$c_{21}(x_2) = \begin{cases} -0.4, |x_2| \le 1, \\ 0.8, |x_2| > 1, \end{cases} c_{22}(x_2) = \begin{cases} -0.9, |x_2| \le 1, \\ 0.3, |x_2| > 1. \end{cases}$$

Select the activation functions and density functions as $f_i(x) = \tanh x, \sigma_i(t, x_i(t), x_i(t-\tau(t)), u_i(t)) = 0.1x_i(t) + 0.1x_i(t-\tau(t)) + 0.1u_i(t), i = 1,2$, correspondingly, $F_1 = 0, F_2 = \mathrm{diag}(0.5, 0.5), R_1 = R_2 = R_3 = 0.01$. The delays and external input are chosen as $\delta = 0.05, \tau(t) = 0.15 + 0.15\sin(t), \rho(t) = 0.5|\cos(t)|, u_1(t) = \sin(t), u_2(t) = \cos(t)$, so that $\tau = 0.3, \rho = 0.5, U = (1,1)^T$. Solving the LMIs (5)-(6) in Theorem 1 by LMI tools in MATLAB, we

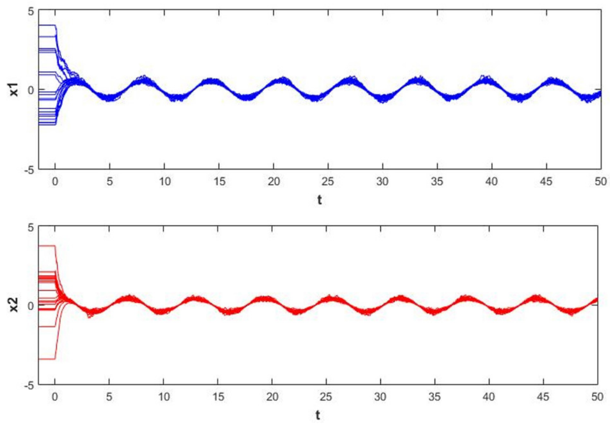

Fig. 2. The state trajectories of Example 1 with external inputs $U(t) = (\sin(t), \cos(t))^T$.

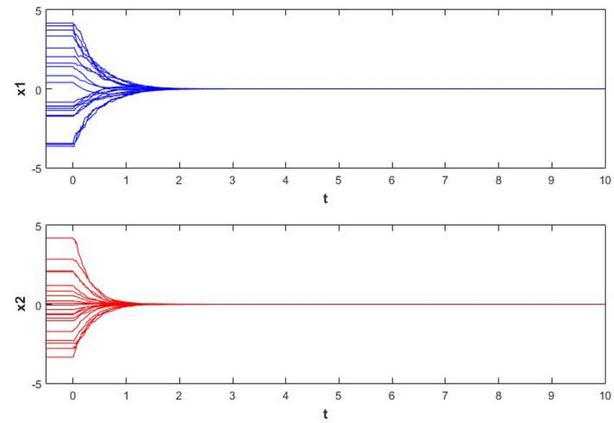

Fig. 3. The state trajectories of the network in Example 1 without external inputs.

can get a feasible solution:

$$P_1 = \begin{bmatrix} 191.43 & 15.15 \\ 15.15 & 160.49 \end{bmatrix}, P_2 = \begin{bmatrix} 68.62 & 8.95 \\ 8.95 & 53.89 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 214.51 & 52.57 \\ 52.57 & 224.97 \end{bmatrix}, P_4 = \begin{bmatrix} 4.96 & -3.34 \\ -3.33 & 8.53 \end{bmatrix},$$

$$P_5 = \begin{bmatrix} 105.49 & 1.20 \\ 1.20 & 97.78 \end{bmatrix}, Q_1 = \begin{bmatrix} 35.05 & 3.11 \\ 3.11 & 30.22 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 762.64 & 97.41 \\ 97.41 & 616.17 \end{bmatrix}, Q_3 = \begin{bmatrix} 48.00 & 3.62 \\ 3.62 & 37.96 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 617.83 & 57.25 \\ 57.25 & 512.51 \end{bmatrix}, Q_5 = 10^3 \times \begin{bmatrix} 2.06 & 0.45 \\ 0.45 & 1.15 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 532.73 & 23.20 \\ 23.20 & 483.54 \end{bmatrix}, Q_7 = \begin{bmatrix} 582.05 & 56.07 \\ 56.07 & 500.45 \end{bmatrix},$$

$$Q_8 = \begin{bmatrix} 434.45 & -1.20 \\ -1.20 & 430.26 \end{bmatrix}, S_1 = \begin{bmatrix} 366.26 & 0 \\ 0 & 310.31 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 95.42 & 0 \\ 0 & 106.00 \end{bmatrix}, K_1 = \begin{bmatrix} 20.70 & 1.64 \\ 1.64 & 18.20 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -5.34 & 1.05 \\ 1.05 & 0 \end{bmatrix}, K_3 = \begin{bmatrix} 63.71 & 5.33 \\ 5.33 & 62.74 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 52.78 & 0 \\ 0 & 50.63 \end{bmatrix},$$

$$\lambda_1 = 224.08, \ \lambda_2 = 1.01 \times 10^3, \ \lambda_3 = 695.40,$$
$$\lambda_4 = 45.50.$$

By Theorem 1, we know that the network in Example 1 is exponentially Lagrange stable in mean square sense, and $\Omega = \{x \in \mathbb{R}^2, \|x\| \le 7.49\}$ is a globally exponentially attractive set. We set the initial values of the network as $x(t) = ({x_0}^1, {x_0}^2)^T, -\eta \le t \le 0$. The simulation results with 20 random $({x_0}^1, {x_0}^2)^T$ are as Fig. 2.

Based on the parameters in Example 1, when there are no external inputs, the equilibrium $(0,0)$ is a globally exponentially attractive set by Theorem 1. Also we can say that the equilibrium $(0,0)$ is globally exponentially stable in mean square sense. Simulation results of the network in Example 1 without external inputs are as Fig. 3.

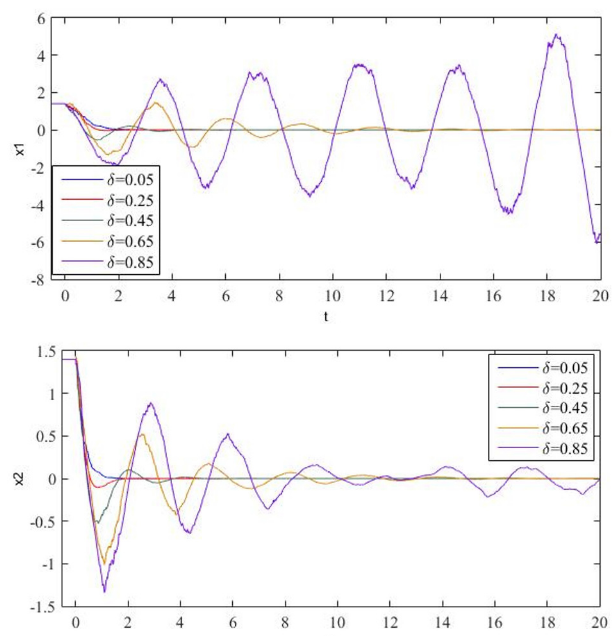

Fig. 4. The state trajectories of the network in Example 1 without external inputs when $\delta = 0.05$, 0.25, 0.45, 0.65, 0.85, respectively.

When we increase the value of δ from 0.05 to 0.85, the network becomes unstable, so the leakage delays do show a tendency to destabilize the system. Set $x_0 = (1.4, 1.4)^T$ and $\delta = 0.05, 0.25, 0.45, 0.65, 0.85$, respectively. The simulation results of the network in Example 1 are shown in Fig. 4.

Moreover, we find that the stochastic perturbations can amplify the destabilizing effects of the leakage delays. Set $x_0 = (0.6, 0.6)^T, U(t) = (0,0)^T$ and $\delta = 0.82$, the simu-

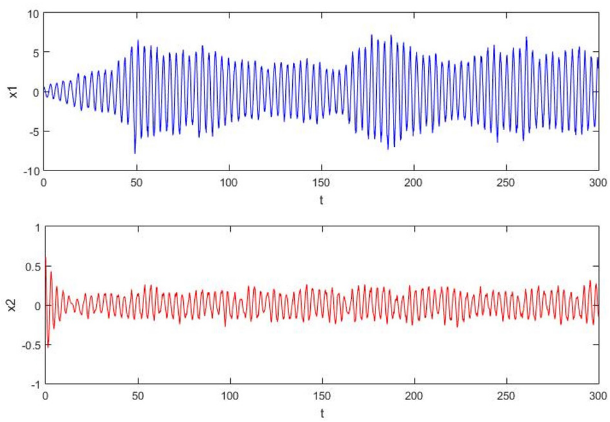

Fig. 5. The state trajectories of the network in Example 1 without external inputs when $\delta = 0.82$.

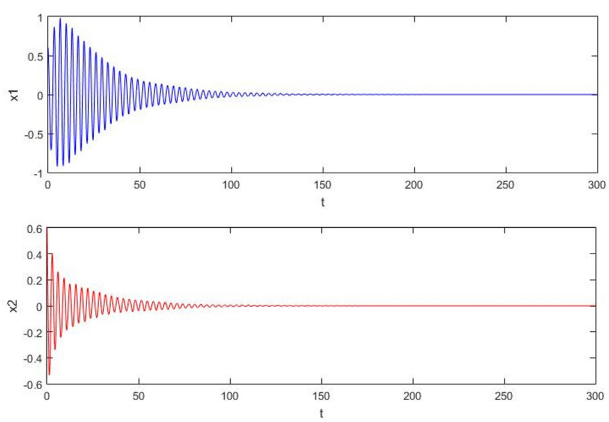

Fig. 6. The state trajectories of the network in Example 1 without external inputs when $\delta = 0.82$ and there are no stochastic perturbations.

lation results of the network in Example 1 are shown in Fig. 5. When there are no stochastic perturbations the simulation results of the network in Example 1 are shown in Fig. 6. We can see from the results that the state trajectories tend to 0 when there are no stochastic perturbations. When the stochastic perturbations exist, the network becomes unstable.

Example 2: Consider two-dimensional stochastic memristive neural network (21) with following parameters:

$$d_1 = 2.2, d_2 = 2.5,$$

$$a_{11}(x_1) = \begin{cases} 0.9, |x_1| \le 1, \\ 1, |x_1| > 1, \end{cases} a_{12}(x_1) = \begin{cases} 0.8, |x_1| \le 1, \\ 0.5, |x_1| > 1, \end{cases}$$

$$a_{21}(x_2) = \begin{cases} 0.3, |x_2| \le 1, \\ -0.3, |x_2| > 1, \end{cases} a_{22}(x_2) = \begin{cases} 0.4, |x_2| \le 1, \\ 0.6, |x_2| > 1, \end{cases}$$

$$b_{11}(x_1) = \begin{cases} 0.2, |x_1| \le 1, \\ -0.2, |x_1| > 1, \end{cases} b_{12}(x_1) = \begin{cases} 0.1, |x_1| \le 1, \\ -0.1, |x_1| > 1, \end{cases}$$

$$b_{21}(x_2) = \begin{cases} 0.4, |x_2| \le 1, \\ 0.2, |x_2| > 1, \end{cases} b_{22}(x_2) = \begin{cases} 0.7, |x_2| \le 1, \\ 0.8, |x_2| > 1, \end{cases}$$

$$c_{11}(x_1) = \begin{cases} -1, |x_1| \le 1, \\ -0.8, |x_1| > 1, \end{cases} c_{12}(x_1) = \begin{cases} 0.7, |x_1| \le 1, \\ -0.7, |x_1| > 1, \end{cases}$$

$$c_{21}(x_2) = \begin{cases} -0.4, |x_2| \le 1, \\ 0.4, |x_2| > 1, \end{cases} c_{22}(x_2) = \begin{cases} -0.6, |x_2| \le 1, \\ 0.6, |x_2| > 1. \end{cases}$$

Select the activation functions and density functions as $f_i(x) = \tanh x - x, \sigma_i(t, x_i(t), x_i(t - \tau(t)), u_i(t)) = 0.1x_i(t) + 0.1x_i(t - \tau(t)) + 0.1u_i(t), i = 1, 2$, correspondingly, $F_1 = 0, F_2 = \mathrm{diag}(-0.5, -0.5), R_1 = R_2 = R_3 = 0.01$. The delays and external input are chosen as $\tau(t) = \frac{e^t}{5e^t+5}, \rho(t) = \frac{e^t}{e^t+1}, u_1(t) = u_2(t) = 0$, so that $\tau = 0.2, \rho = 1, U = (0, 0)^T$. Solving the LMIs (23) in Corollary 1 by LMI tools in MATLAB, we can get a feasible solution:

$$P_1 = \begin{bmatrix} 53.92 & 4.85 \\ 4.85 & 101.90 \end{bmatrix}, P_2 = \begin{bmatrix} 114.55 & 27.48 \\ 27.48 & 197.12 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 11.58 & 2.48 \\ 2.48 & 18.71 \end{bmatrix}, Q_2 = \begin{bmatrix} 172.27 & 1.82 \\ 1.82 & 177.83 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 298.31 & -72.01 \\ -72.01 & 324.73 \end{bmatrix}, Q_4 = \begin{bmatrix} 143.60 & 2.30 \\ 2.30 & 146.39 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 174.29 & 0 \\ 0 & 354.04 \end{bmatrix}, S_2 = \begin{bmatrix} 33.02 & 0 \\ 0 & 118.29 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 4.06 & 1.38 \\ 1.38 & 7.02 \end{bmatrix}, K_2 = \begin{bmatrix} 18.44 & 9.30 \\ 9.30 & 39.61 \end{bmatrix},$$

$$\lambda_1 = 138.00, \quad \lambda_2 = 271.38.$$

By Corollary 1, the original point is globally exponentially stable in mean square sense. Simulation results of the network in Example 2 with 20 random initial values are shown in Fig. 7.

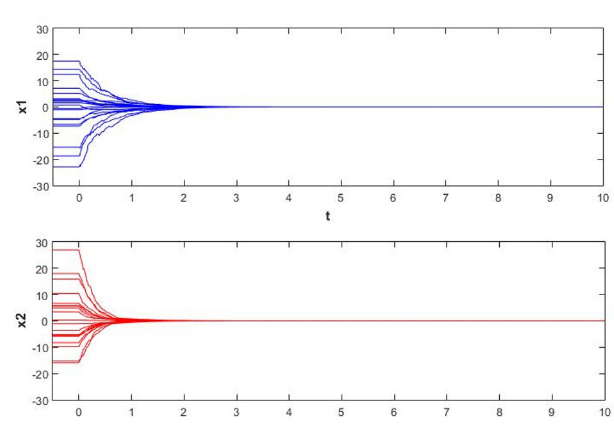

Fig. 7. The state trajectories of the network in Example 2 with 20 random initial values .

Example 3: Consider two-dimensional neural network (22) with following parameters:

$$\begin{aligned}
&d_1 = 1.9, \ \ d_2 = 2.2,\\
&a_{11} = -0.7, \ \ a_{12} = -0.1, \ \ a_{21} = 0.2, \ \ a_{22} = -0.9,\\
&b_{11} = 0.7, \ \ b_{12} = 0.9, \ \ b_{21} = -0.2, \ \ b_{22} = -0.5,\\
&c_{11} = 0.4, \ \ c_{12} = 0.6, \ \ c_{21} = -0.9, \ \ c_{22} = -1.
\end{aligned}$$

Select the activation functions and density functions as $f_i(x) = \tanh x$, correspondingly, $F_1 = 0, F_2 = \mathrm{diag}(0.5, 0.5)$. The delays and external input are chosen as $\delta = 0.1, \tau(t) = 0.2, \rho(t) = 0.4, u_1(t) = \sin(2t), u_2(t) = 0.5\cos(t)$, so that $\tau = 0.3, \rho = 0.5, U = (1, 0.5)^T$. Solving the LMIs in Corollary 2 by LMI tools in MATLAB, we can get a feasible solution:

$$P_1 = \begin{bmatrix} 1.04 & 0.48 \\ 0.48 & 2.31 \end{bmatrix}, P_2 = \begin{bmatrix} 0.53 & 0.26 \\ 0.26 & 1.40 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 1.96 & 0.96 \\ 0.96 & 3.36 \end{bmatrix}, P_4 = \begin{bmatrix} 0.11 & 0.06 \\ 0.06 & 0.42 \end{bmatrix},$$

$$P_5 = \begin{bmatrix} 0.87 & 0.66 \\ 0.66 & 2.80 \end{bmatrix}, Q_1 = \begin{bmatrix} 0.33 & 0.06 \\ 0.06 & 0.47 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 4.87 & 0.03 \\ 0.03 & 3.72 \end{bmatrix}, Q_3 = \begin{bmatrix} 0.41 & 0.20 \\ 0.20 & 0.75 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 6.09 & 0.81 \\ 0.81 & 5.08 \end{bmatrix}, Q_5 = \begin{bmatrix} 7.16 & 0.12 \\ 0.12 & 5.37 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 3.56 & 0.17 \\ 0.17 & 3.92 \end{bmatrix}, Q_7 = \begin{bmatrix} 3.82 & 0.36 \\ 0.36 & 3.82 \end{bmatrix},$$

$$Q_8 = \begin{bmatrix} 3.20 & 0.01 \\ 0.01 & 3.20 \end{bmatrix}, S_1 = \begin{bmatrix} 1.84 & 0 \\ 0 & 4.44 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 1.06 & 0 \\ 0 & 3.51 \end{bmatrix}, K_1 = \begin{bmatrix} 0.17 & 0.12 \\ 0.12 & 0.34 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.02 & 0.02 \\ 0.02 & -0.07 \end{bmatrix}, K_3 = \begin{bmatrix} 0.54 & 0.22 \\ 0.22 & 0.94 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 0.57 & 0.32 \\ 0.32 & 1.17 \end{bmatrix}.$$

By Corollary 2, we know that the network in Example 3 is exponentially Lagrange stable in mean square sense, and $\Omega = \{x \in \mathbb{R}^2, \|x\| \leq 6.30\}$ is a globally exponentially attractive set. Simulation results of the network in Example 3 with 20 random initial values are shown in Fig. 8.

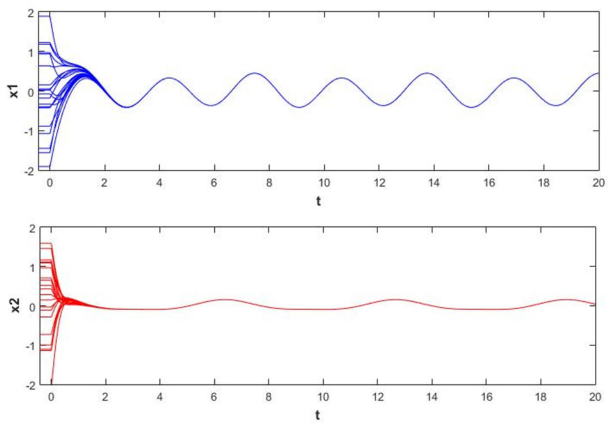

Fig. 8. The state trajectories of the network in Example 3 with 20 random initial values.

5. SUMMARY

In this paper, the Lagrange stability of stochastic memristive neural networks with leakage delay and mixed transmission delays was investigated. We turned to qualitative analysis of a relevant differential inclusion under the framework of Filippov's solution to solve the discontinuity caused by using memristors in the neural networks. Then, by using Lyapunov-Krasovskii functionals, the free-weighting matrix method and stochastic analysis technique, sufficient criteria in terms of linear matrix inequalities (LMIs) were given to ascertain the original network, the network without leakage delay and the network without stochastic perturbations to be exponentially Lagrange stable in mean square sense, respectively.

The main contribution of this paper is taking both leakage delay and stochastic perturbations into consideration in analysing the Lagrange stability of memristive neural networks. Usually, both of these two factors have quite negative influence to the stability of neural networks and generally exist in practical networks. The given criterion for checking the mean-square stability in Lagrange sense of network (1) is dependent on leakage delay and transmission delays, which implicates that the information on the sizes of these delays is sufficiently utilized. Meanwhile, the estimation of globally attractive set is associated with the density functions of stochastic perturbations. This indicates that stochastic perturbations may enlarge the range of globally attractive set, which is reasonable. Also, the assumptions in our paper are quite weak. In assumption **(A1)**, the constants $F_i^-, F_i^+ (i = 1, 2, \cdots, n)$ are allowed to be positive, negative or zero. Hence, this assumption is weaker than the assumptions in [15, 20]. And the assumptions on time-varying discrete delay and distribute delay are only bounded.

The numerical simulations in Section 4 illustrated that our theoretical results are feasible. Moreover, the simulation results in Fig. 4 showed the tendency of leakage delay to destabilize the network. Furthermore, comparing the results in Figs. 5 and 6, we see that when the leakage delay was 0.82, the state trajectories of the network in Example 1 tended to stable if there were no stochastic perturbations. But when there were stochastic perturbations in the network, the network became unstable. The stochastic perturbations amplified the destabilizing effect of the leakage delay, which indicates it's necessary to take both of these two factors into consideration in modelling.

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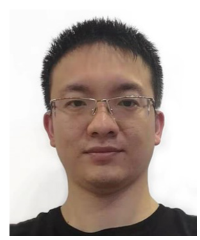

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