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Fractional-disturbance-observer-based Sliding Mode Control for Fractional Order System with Matched and Mismatched Disturbances

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Abstract: This paper addresses the sliding mode control for a class of fractional order systems with matched and mismatched disturbances. Firstly, fractional disturbance observer is presented to estimate both the matched and mismatched disturbances, and the boundedness of the estimation error can be guaranteed. Secondly, sliding mode surface is constructed based on the output of the observer. The bounded stability of the closed-loop system under the designed controller is revealed by theoretical analysis. Finally, simulation results show that the proposed control strategy can effectively suppress the effect of the matched and mismatched disturbances on the system.

Keywords: Fractional disturbance observer, fractional order system, matched and mismatched disturbances, sliding mode control.

1. INTRODUCTION

In the past decades, fractional order system, which can depict the properties of various real world process more suitable compared with integer order system, has become an important research issue and aroused considerable attention of scholars. For example, fractional order model can accurately describe the dynamic process of magnetic levitation system [1], lithium ion batteries [2], single-link lightweight flexible manipulator [3] and so on. Various researches have shown the interests in stability analysis and control synthesis for fractional order system [1, 3–6]. In fact, practical systems are unavoidably affected by uncertainties and disturbances. These disturbances will degrade the performance of the system, and even destabilizing the system. For a class of systems, the uncertainties and disturbances satisfy the matched condition, that is, the uncertainties and disturbances affect the system via the same passage with the control input. For instance, the dynamic output feedback control problem is discussed in [7] for systems with time delays and matched disturbances. In [8], disturbance attenuation is considered for stochastic Markovian jump system. But for another class of systems, such as magnetic leviation (MAGLEV) suspension system [1], continuous casting mold oscillatory system [10], and communication network model [9], the uncertainties and disturbances dissatisfy the matched condition, which are called mismatched disturbances. How to control fractional order system with matched and mismatched disturbances has become a challenging and open problem to be addressed.

Sliding mode control (SMC) has gained increasing attention recently due to its simple structure, strong robustness and has been widely adopted to control numerous fractional order systems, such as uncertain economic system [11] and chaotic system [12, 13]. However, the traditional sliding mode control can only ensure invariance to matched disturbance, but can not effectively suppress the mismatched disturbance, so the well-known property of invariance does not hold any longer when there exist mismatched disturbance in the system. Recently, several control strategies combination with sliding mode control were proposed for fractional order system with mismatched disturbance. Based on linear matrix inequality (LMI) technology, integral sliding mode control method was proposed for a class of fractional order systems in [14–16]. In [14–16], in order to attenuate the disturbance, the upper bound of disturbance must be known in advance, and higher controller gains were used, which could lead to chattering phenomenon. For the purpose of easing the chattering problem, [17, 18] designed adaptive law to adjust the disturbance. However, the mismatched disturbance considered in [14–18] must have bounded H_2 norms, which is an unreasonable assumption, for exam-

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ple, the mismatched disturbance in MAGLEV suspension system do not satisfy this assumption [1].

In addition to those methods, as an alternative way, disturbance observer can commendably estimate the disturbance and attenuate their effects on system. Recently, the studies of disturbance observer combination with sliding mode control for fractional order system have received much consideration. In [19], sliding mode observer was designed to evaluate the disturbance in nonlinear fractional order system. Nonlinear fractional disturbance observer was explored to handle disturbance, and then adaptive sliding mode control law was proposed for chaotic system in [20]. Fractional disturbance observer-based integral-type sliding mode control for fractional order system was presented in [21]. However, these methods given in [19–21] are not available for system with mismatched disturbance. For integer order system with mismatched disturbance, a novel disturbance observer-based sliding mode control method was developed in [22]. And the control strategy of [22] has been expanded to control fractional order system with mismatched disturbance by using Lyapunov stability theory in [1]. However, the mismatched disturbance in [1] was supposed to be constant in the stationary state and an integer order disturbance observer was proposed to evaluate the disturbance in fractional order system, especially. It would be better to design fractional disturbance observer to compensate fractional order system with mismatched disturbance. And to our best knowledge, there is no research on designing sliding mode control based on fractional disturbance observer for fractional order system with matched and mismatched disturbances.

In this paper, a new fractional sliding mode control combination with fractional disturbance observer is presented to attenuate matched and mismatched disturbances in fractional order system. The main highlights of this paper are listed as follows:

1) fractional disturbance observer is designed to compensate both matched and mismatched disturbances;

2) the novel sliding mode control method presented in [22] is extended for nonlinear fractional order system in presence of matched and mismatched disturbances;

3) a more extensive type of mismatched disturbance is considered.

The paper is organized as follows: Section 2 gives the system to be considered and some preliminaries. Fractional disturbance observer and sliding mode control for the system are proposed in Section 3. Section 4 gives the numerical simulation results. And, conclusion is given in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

In this paper, we adopt the following Caputo derivative:

Definition 1: The Caputo fractional derivative of order α of a continuous function $h(t)$ is defined as follows:

$$D^{\alpha}h(t) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_0^t \dfrac{h^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}}d\tau, \\ \qquad\qquad\qquad\qquad m-1<\alpha<m \\ \dfrac{d^m}{dt^m}h(t), \qquad \alpha = m \end{cases} \tag{1}$$

where Γ is the Gamma function, which is defined as $\Gamma(z) = \int_0^{\infty} e^{-u}u^{z-1}du$.

Consider a kind of fractional order systems in presence of matched and mismatched disturbances, described by

$$\begin{cases} D^{\alpha}x_1 = x_2 + d_1(t), \\ D^{\alpha}x_2 = a(x) + b(x)u + d_2(t), \end{cases} \tag{2}$$

where $\alpha \in (0,1)$ is the fractional order of the system, $x(t) = [x_1, x_2]^T$ is the system pseudostate, $d_1(t)$ and $d_2(t)$ are the mismatched and matched disturbances, u is the control input, $a(x)$ and $b(x) \neq 0$ represent smooth known functions.

Assumption 1: The matched and mismatched disturbances and their fractional order derivatives are assumed to be bounded, that is, $|d_i(t)| \leq \tau_i$ and $|D^{\alpha}d_i(t)| \leq \rho_i$, $i = 1, 2$, where τ_i and ρ_i are unknown positive constants.

Remark 1: In this paper, the fractional derivatives of disturbances are only required to be bounded and the upper bounds do not need to be known in advance, which is different from the assumption on disturbance in [1].

In system (2), $d_2(t)$ affect the system via the same passage with the control input, which is called matched disturbance, yet $d_1(t)$ is called as mismatched disturbance. The aim of this paper is to design a sliding mode controller based on fractional disturbance observer to stabilize system (2) with matched and mismatched disturbances.

Lemma 1 [23]**:** Let $x(t) \in \mathbf{R}^n$ be a differentiable and continuous function. Then, for $\forall t \geq t_0$ and $\forall \alpha \in (0,1)$

$$\frac{1}{2}D^{\alpha}(x^T(t)x(t)) \leq x^T(t)D^{\alpha}x(t). \tag{3}$$

Lemma 2 [23]**:** For the fractional order system

$$D^{\alpha}x(t) = f(x(t)), \tag{4}$$

where $\alpha \in (0,1)$, $x = 0$ is the equilibrium point and $x(t) \in R^n$, if the following condition is satisfied

$$x(t)f(x(t)) \leq 0, \tag{5}$$

then the origin of the system (4) is stable. And if

$$x(t)f(x(t)) < 0, \quad \forall x \neq 0, \tag{6}$$

then the origin of the system (4) is asymptotically stable.

Lemma 3 [24, 25]**:** Consider the following system described in the state-space form

$$D^{\alpha}x = Ax + Bu. \tag{7}$$

Let us suppose that $u(t)$ is bounded, that is, $\|u(t)\| \le M$ for $M > 0$, and that all eigenvalues of A satisfy $|arg(eig(A))| > \frac{\alpha\pi}{2}$, then the state of the system with arbitrary initial condition x_0 is bounded input-bounded output stable.

Property 1 [26]**:** The Caputo fractional derivative satisfies the following linear characteristic:

$$D^{\alpha}(a_1 f(t) + a_2 g(t)) = a_1 D^{\alpha} f(t) + a_2 D^{\alpha} g(t), \tag{8}$$

where $f(t), g(t)$ are functions and a_1, a_2 are constants.

3. MAIN RESULTS

3.1. Fractional disturbance observer design

Fractional disturbance observer will be designed to evaluate the matched and mismatched disturbances in system (2), which can be rewritten as

$$D^{\alpha}x = f(x) + g(x)u + d(t), \tag{9}$$

where $f(x) = [x_2, a(x)]^T$, $g(x) = [0, b(x)]^T$ and disturbance $d(t) = [d_1(t), d_2(t)]^T$.

The fractional disturbance observer, introduced by [20], can be described as following

$$\begin{cases} D^{\alpha}p = -lp - l[lx + f(x) + g(x)u], \\ \hat{d}(t) = p + lx, \end{cases} \tag{10}$$

where $\hat{d}(t) = \left[\hat{d}_1(t), \hat{d}_2(t)\right]^T$ is the estimation vector of $d(t)$, p represents the auxiliary variable of the fractional disturbance observer and $l = diag(l_1, l_2)$ is the gain matrix of disturbance observer to be established.

Consider (9) and (10), one can obtain that

$$\begin{aligned} D^{\alpha}p &= -l\,[p + lx + f(x) + g(x)u] \\ &= -l\left[\hat{d}(t) + D^{\alpha}x - d(t)\right], \end{aligned}$$

from which one has

$$D^{\alpha}\hat{d}_i(t) = -l_i\left[\hat{d}_i(t) - d_i(t)\right], \quad i = 1, 2. \tag{11}$$

So the Caputo derivative of the disturbance estimation errors $e_{di}(t) = d_i(t) - \hat{d}_i(t)$ can be deduced as

$$D^{\alpha}e_{di}(t) = -l_i e_{di}(t) + D^{\alpha}d_i(t), \quad i = 1, 2. \tag{12}$$

According to Lemma 3 and (12), one can find that the estimation errors $e_{di}(t)$, $i = 1, 2$ are both bounded.

3.2. Sliding mode controller design and stability analysis

In this paper, we consider the following sliding mode surface for system (2)

$$\sigma = cx_1 + x_2 + \hat{d}_1, \tag{13}$$

where c is a positive constant to be designed, and $\hat{d}_1$ is the estimated value of mismatched disturbance given by disturbance observer (10).

The sliding mode control law in this paper is designed as following:

$$\begin{aligned} u = &- b^{-1}(x)\big(a(x) + \hat{d}_2 + c(x_2 + \hat{d}_1) + \xi_1\sigma \\ &+ \xi_2 sgn(\sigma)\big). \end{aligned} \tag{14}$$

Theorem 1: Considering the system (2) in presence of matched and mismatched disturbances, under the controller (14), the closed loop system is bounded stable in case of the observer gains $l_i, i = 1, 2$ are selected as positive constants and the switching gain is chosen so that $\xi_2 > \sup|(c + l_1)e_{d1} + e_{d2}|$.

Proof: By taking the fractional derivative of σ defined as (13), gives

$$\begin{aligned} D^{\alpha}\sigma &= D^{\alpha}\left(cx_1 + x_2 + \hat{d}_1\right) \\ &= c\,(x_2 + d_1) + a(x) + b(x)u + d_2 + D^{\alpha}\hat{d}_1. \end{aligned} \tag{15}$$

Substituting the controller (14) and (11) into (15), yields

$$\begin{aligned} D^{\alpha}\sigma = &- \xi_1\sigma - \xi_2 sgn(\sigma) + d_2 - \hat{d}_2 \\ &+ (c + l_1)\left(d_1 - \hat{d}_1\right) \\ = &- \xi_1\sigma - \xi_2 sgn(\sigma) + (c + l_1)e_{d1} + e_{d2}. \end{aligned} \tag{16}$$

Consider a Lyapunov function candidate as

$$V = \frac{1}{2}\sigma^2. \tag{17}$$

On the basis of Lemma 1, (16) and (17), the fractional derivative of V can be obtained as

$$\begin{aligned} D^{\alpha}V &\le \sigma D^{\alpha}\sigma \\ &= \sigma\left(-\xi_1\sigma - \xi_2 sgn(\sigma) + (c + l_1)e_{d1} + e_{d2}\right) \\ &\le -\xi_1\sigma^2 - |\sigma|\left(\xi_2 - (c + l_1)e_{d1} - e_{d2}\right). \end{aligned} \tag{18}$$

So, by choosing the switching gain as $\xi_2 > \sup|(c + l_1)e_{d1} + e_{d2}|$, from (18), one can obtain

$$D^{\alpha}V \le -\xi_1\sigma^2 \le 0. \tag{19}$$

Consequently, according to Lemma 2, the system states will convergence to the sliding mode surface $\sigma = 0$. When $\sigma = 0$, from (2) and (13), one can get

$$D^{\alpha}x_1 = -cx_1 + e_{d1}. \tag{20}$$

Combining (20) with (12), the closed-loop system can be expressed as

$$\begin{cases} D^{\alpha}x_1 = -cx_1 + e_{d1}, \\ D^{\alpha}e_d = -le_d + D^{\alpha}d, \end{cases} \tag{21}$$

which can also be rewritten as

$$\begin{aligned} \begin{bmatrix} D^{\alpha}x_1 \\ D^{\alpha}e_d \end{bmatrix} &= \begin{bmatrix} -c & G \\ 0 & -l \end{bmatrix} \begin{bmatrix} x_1 \\ e_d \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} h \\ &= A \begin{bmatrix} x_1 \\ e_d \end{bmatrix} + Bh, \end{aligned} \tag{22}$$

where $e_d = \begin{bmatrix} e_{d1} \\ e_{d2} \end{bmatrix}$, $A = \begin{bmatrix} -c & G \\ 0 & -l \end{bmatrix}$, $G = [1 \;\; 0]$, $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$, $h = D^{\alpha}d$ and I denotes the identity matrix.

Due to the observer gain $l_i, i = 1, 2$ and the design parameter in sliding mode surface c are chosen as positive constants, so all the eigenvalues of matrix A are located in the left half plain, that is, $|arg(eig(A))| > \frac{\alpha\pi}{2}$ for $0 < \alpha < 1$ is always holds. On account of Assumption 1, h is bounded. According to Lemma 3, it can be derived that the closed-loop system is bounded input-bounded output stable.

Remark 2: The switching gain in Theorem 1 is selected as $\xi_2 > \sup|(c + l_1)e_{d1} + e_{d2}|$ in order to ensure the bounded stability of the closed loop system. In virtue of the matched and mismatched disturbances can be precise evaluated by the fractional disturbance observer, the switching gain can be chosen as a very small value, which is beneficial to reduce chattering and such excellent characteristic will be displayed in simulation section.

4. SIMULATION EXAMPLE

In order to evaluate the availability of the presented control method for fractional order systems in presence of matched and mismatched disturbances, a numerical example given in [27] is investigated

$$\begin{cases} D^{\alpha}x_1 = 2x_2 + \dfrac{1}{6}\sqrt{x_1^2 + x_2^2}, \\ D^{\alpha}x_2 = x_1 + u + \dfrac{1}{6}\sin(x_1)\cos(u). \end{cases} \tag{23}$$

Assume system (23) is also influenced by external disturbances, so we can rewrite (23) as system (2) with matched and mismatched disturbances. Where the nonlinear term and partial linear term in the first equation of (23) are regarded as mismatched disturbance $d_1(t) = x_2 + \frac{1}{6}\sqrt{x_1^2 + x_2^2} + 0.05\sin(2t) + 3(1 - e^{-2t})$, and the nonlinear term in the second equation of (23) is regarded as the matched disturbance $d_2(t) = 0.5\sin(2t) + 1.5 - e^{-2t} + \frac{1}{6}\sin(x_1)\cos(u)$. For system (23), the initial states are chosen as $x_{10} = -1.5$, $x_{20} = 1$.

In order to display the availability of the presented control strategy, denoted by SMC-FDO, the simulation results

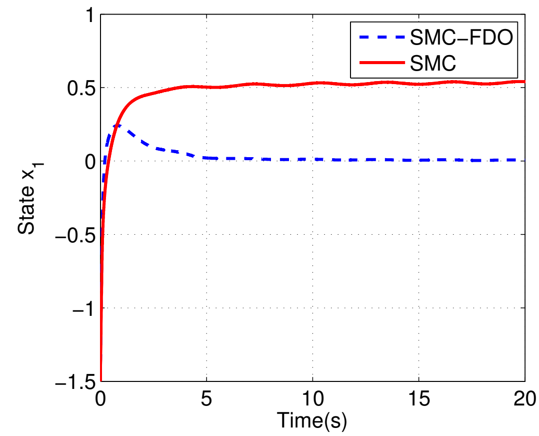

Fig. 1. Response curves of state x_1.

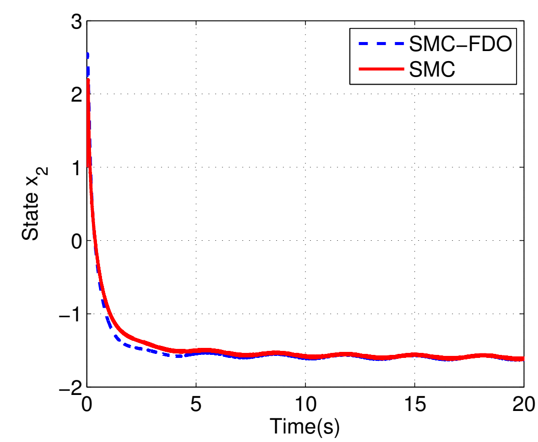

Fig. 2. Response curves of state x_2.

of SMC-FDO are compared with the results of the normal sliding mode control method, denoted by SMC. The control parameters of SMC-FDO are selected as $c = 3$, $\xi_1 = 2$, $\xi_2 = 0.5$, $l_1 = 5$ and $l_2 = 10$. The same parameters c and ξ_1 are chosen for SMC, but in order to ensure the stability of SMC, ξ_2 should be selected with a larger value like $\xi_2 = 10$. Figs. 1-8 show the simulation results.

From Figs. 1 and 2, one can see that even though there exists mismatched disturbance, state x_1 can still reach the equilibrium point under the proposed controller, that is the presented SMC-FDO has strong robustness. However, the robustness of SMC does not exist any more. Fig. 4 shows that the chattering phenomenon is evident in SMC, due to a larger switching gain is chosen in SMC. But in SMC-FDO, only a relatively small switching gain is used, so the chattering phenomenon is obviously suppressed, as shown in Fig. 3. It can be seen from Figs. 5 and 7 that the fractional disturbance observer designed in this paper can effectively estimate the matched and mismatched disturbances, and the estimation error are bounded, as shown in Figs. 6 and 8.

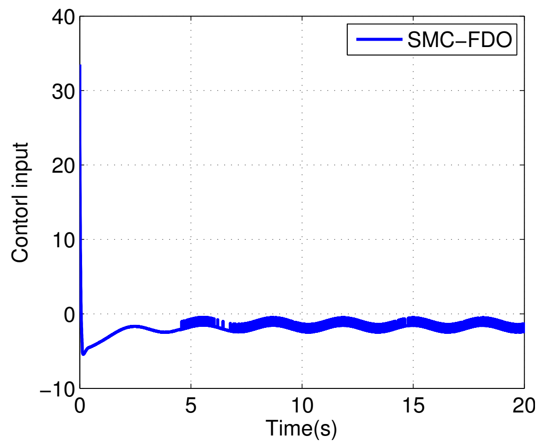

Fig. 3. Response curve of control input.

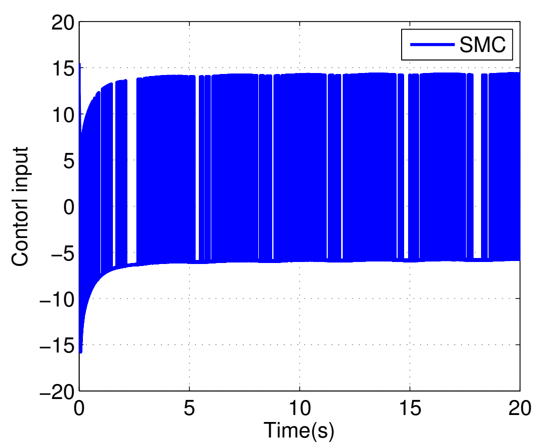

Fig. 4. Response curve of control input.

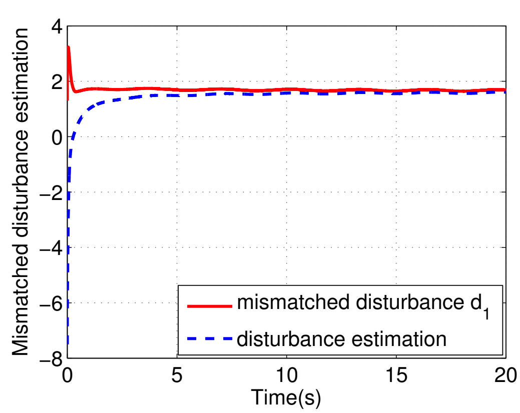

Fig. 5. Response curves of mismatched disturbance d_1.

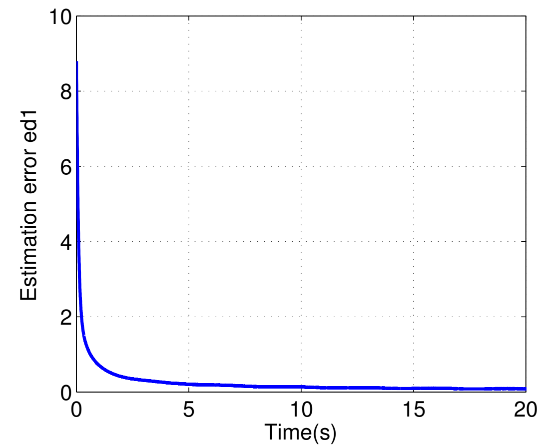

Fig. 6. Response curves of mismatched disturbance estimation error.

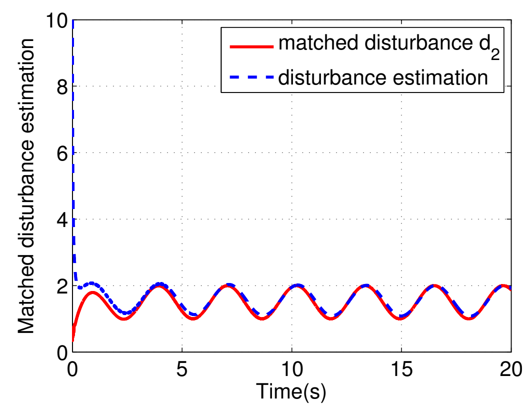

Fig. 7. Response curves of matched disturbance d_2.

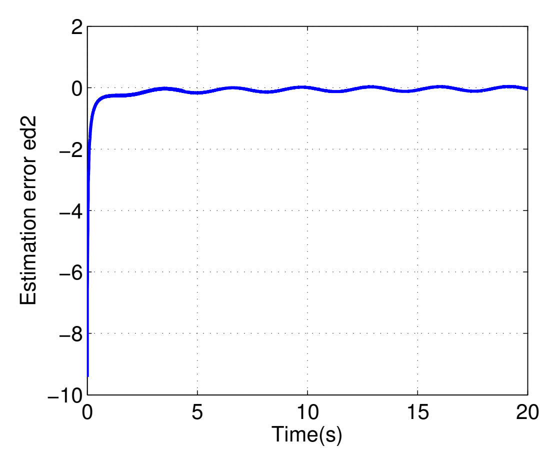

Fig. 8. Response curves of matched disturbance estimation error.

5. CONCLUSION

In order to suppress the influence of matched and mismatched disturbances on fractional order system, a novel sliding mode control method was established in this paper. Firstly, fractional disturbance observer was proposed to evaluate the matched and mismatched disturbances. The disturbance estimation were used to construct sliding mode surface and sliding mode control law. Under the designed controller, system states can effectively suppress the influence of the mismatched disturbance. Simulation results demonstrate that the presented control strategy has better robustness to mismatched disturbance.

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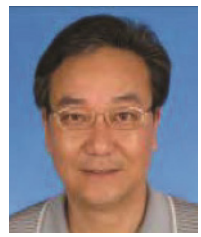

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