# Interval Observer-based Output Feedback Control for a Class of Interconnected Systems with Uncertain Interconnections

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Abstract: This paper studies the output feedback controller design problem for a class of interconnected systems with uncertain interconnections. First, the interval observers are built to guarantee the interval property between the system states and their estimations. Then, the state feedback controller is designed for each observer system other than output feedback controller for original system. A cyclic-small-gain condition is used to deal with the bound functions of the uncertain interconnections. The original closed-loop system is robust stable since that the interval properties are satisfied. The non-convex conditions caused by traditional observer-based output feedback are avoided by transforming the output to state feedback problem. Finally, the main results are demonstrated by numerical simulations.

Keywords: Cyclic-small-gain condition, interconnected systems, interval observer, output feedback control.

## 1. INTRODUCTION

<span id="page-0-0"></span>As a practical matter, many systems appear in the form of several interacting subsystems, for instance, power networks and process control systems  $[1-5]$  $[1-5]$ . Hence, many researchers have devoted themselves to the control problem of such interconnected systems [[6–](#page-6-2)[9\]](#page-6-3). In [[10\]](#page-6-4), a controller was designed for large-scale systems subject to power constraint. In [\[11](#page-6-5)], an event-triggered controller was presented for interconnected linear systems. However, most of the results focused on state feedback control for interconnected systems. Unfortunately, in most cases, states are unmeasurable. Output feedback control is more suitable for such actual situation. Recently, many output feedback control schemes have been proposed for interconnected systems. For example, in [\[12](#page-6-6)], a static outputfeedback method has been proposed for large-scale T-S fuzzy systems. Furthermore, the robust  $H_{\infty}$  dynamic output feedback control problem has been studied in [\[13](#page-6-7)]. But only the linear interconnections were considered in the above paper. It is well known that the linear ones are hard to describe the practical complex cases. Therefore, studies on the output feedback controller for interconnected systems with nonlinear interconnections are still insufficient.

Observer-based output feedback is an effective control method when only the output of the system is measurable [\[14](#page-6-8)[–16](#page-6-9)]. Many observer design algorithms have been pro-

posed to estimate the system states, for instance, interval observers. In [[17\]](#page-6-10), the exponentially stable linear interval observer was built for linear systems. Furthermore, the interval observer with Luenberger observer structure was developed in [\[18](#page-6-11)]. Besides, coordinate transformation has proved to be a significant way to guarantee the interval property [\[19](#page-6-12)[–22\]](#page-6-13). At the same time, [\[23](#page-6-14)] and [[24\]](#page-6-15) studied the output stabilization problem for nonlinear systems subject to parametric and signal uncertainties and extended the method to linear time-varying and linearparameter-varying systems. Furthermore, [\[25](#page-6-16), [26](#page-7-0)] presented the interval observer-based output feedback controllers for time-varying input delay systems and switched systems, respectively. However, the above methods are proposed without consideration for the external disturbances. Therefore, interval observer-based output feedback control for a class of interconnected systems with uncertain interconnections and external disturbances is still an open problem.

In this paper, the output feedback controller for interconnected system with uncertain interconnections and external disturbances is designed. For each subsystem, an interval observer is built firstly by considering the upper and lower bounds of the interconnections and disturbances. Then, the output feedback controllers are built and the design conditions are derived in the formulation of linear matrix inequalities (LMIs). It is shown that the original closed-loop system can be proved to be robust sta-

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ble since that the interval properties are satisfied. The main contributions are as follows: 1) The state feedback controller is designed for observer system other than output feedback controller for original system. The nonconvex conditions caused by traditional observer-based output feedback are avoided by transforming the output to state feedback problem. 2) A cyclic-small-gain condition is introduced to address the bound functions of the uncertain interconnections.

Note that the nonlinear uncertain interconnections are considered in this paper. Many methods have been proposed to control the system with nonlinear interconnections. For example, in [\[27](#page-7-1)], a sliding mode observer-based output feedback controller was presented for linear sub-systems with nonlinear interconnections. In [[28\]](#page-7-2) and [\[29](#page-7-3)], the adaptive control techniques were developed for largescale nonlinear systems. However, there results were obtained based on the assumption that the interconnection matching condition is satisfied, which makes its application too inflexible. In addition, the neural networks have been introduced to deal with the nonlinear interconnections. In  $[30-32]$  $[30-32]$ , the adaptive neural network outputfeedback control problem has been solved by backstepping technique. Furthermore, by a combination of the neural network and graph theory, the fault-tolerant control scheme has been developed in [\[33](#page-7-6)]. But the neural networks may considerably increase the complexity of the controller design. Different from the above mentioned results, the interconnection matching condition is removed and the proposed control scheme without employing the neural network simplifies the design process.

The structure of this paper is as follows: The system description is presented in Section [2](#page-1-0) and the main results are proposed in Section [3](#page-1-1). An example is given in Section [4](#page-5-0) and conclusions are drawn in Section [5.](#page-5-1)

**Notations:** For vectors  $\alpha = [\alpha_i]_{m \times 1}$ ,  $\beta = [\beta_i]_{m \times 1}$ , we define  $\alpha \leq \beta$   $(\alpha \geq \beta)$  by  $\alpha_i \leq \beta_i$   $(\alpha_i \geq \beta_i)$ ,  $\forall 1 \leq i \leq m$ .  $Ω > 0$  ( $Ω < 0$ ) means the matrix  $Ω$  is positive (negative) definite. For given matrix  $\Gamma \in \mathbb{R}^{p \times q}$ , define  $\Gamma^+$  =  $max\{0, \Gamma\}$  and  $\Gamma^- = \Gamma^+ - \Gamma$ .

#### 2. SYSTEM DESCRIPTION

<span id="page-1-0"></span>Consider a system consisting of *N* interconnected subsystems  $S_i$ ,  $i = 1, 2, \dots, N$ . Each of which is described by the following dynamic model:

$$
\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \Psi_i(x(t), y(t), \delta(t)) + D_i d_i(t), \n y_i(t) = C_i x_i(t),
$$
\n(1)

where  $u_i(t) \in \mathbb{R}^{s_i}$  is the control input vector,  $x_i(t) \in \mathbb{R}^{n_i}$ and  $y_i(t) \in \mathbb{R}^{q_i}$  are the state and output vectors of the *i*th subsystem.  $x(t) = \begin{bmatrix} x_1^T(t), & x_2^T(t), & \cdots, & x_N^T(t) \end{bmatrix}^T \in \mathbb{R}^n$ and  $y(t) = [y_1^T(t), y_2^T(t), \dots, y_N^T(t)]^T \in \mathbb{R}^q$  are the state and output vectors of the overall system. The nonlinear function  $\Psi_i(x(t), y(t), \delta(t)) : \mathbb{R}^{n+q+r} \to \mathbb{R}^{m_i}$  represents the interconnection between the *i*th subsystem and other subsystems.  $\delta(t) \in \Delta \subset \mathbb{R}^r$  is the uncertain possibly time-varying parameter vector, the set  $\Delta$  is assumed to be given,  $\Psi_i(0,0,\delta(t)) = 0$  for any  $\delta(t) \in \Delta$ .  $d_i(t) \in \mathbb{R}^{p_i}$  is the external disturbance vector.  $A_i \in \mathbb{R}^{n_i \times n_i}$ ,  $B_i \in \mathbb{R}^{n_i \times s_i}$ ,  $D_i \in \mathbb{R}^{n_i \times p_i}$ ,  $C_i \in \mathbb{R}^{q_i \times n_i}$  are known constant matrices.  $\sum_{i=1}^{N} n_i = n$ ,  $\sum_{i=1}^{N} q_i = q$ . Moreover, it is assumed that  $(A_i, C_i)$  is observable and  $(A_i, B_i)$  is controllable.

The main idea of this paper is to design the interval observer-based output feedback controller, the following assumptions are considered firstly.

<span id="page-1-2"></span>Assumption 1: There exists a matrix  $L_i$  such that  $A_i$  – *LiC<sup>i</sup>* is Metzler.

<span id="page-1-3"></span>Assumption 2: There exist the known bound functions *d*<sub>*i*</sub>(*t*) ∈  $\mathbb{R}^{p_i}$  and  $\overline{d}_i(t)$  ∈  $\mathbb{R}^{p_i}$  belong to  $L_2[0, \infty)$  such that

$$
\underline{d}_i(t) \preceq d_i(t) \preceq \overline{d}_i(t). \tag{2}
$$

<span id="page-1-4"></span>**Assumption 3:** There are known functions  $\overline{\Psi}_i(\overline{x}(t))$ ,  $\underline{x}(t), y(t)), \underline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) : \mathbb{R}^{2n+q} \to \mathbb{R}^{m_i}$  such that

$$
\underline{\Psi}_{i}(\overline{x}(t), \underline{x}(t), y(t)) \leq \Psi_{i}(x(t), y(t), \delta(t))
$$
\n
$$
\leq \overline{\Psi}_{i}(\overline{x}(t), \underline{x}(t), y(t))
$$
\n(3)

is satisfied if  $x(t) \leq x(t) \leq \overline{x}(t)$  and  $\delta(t) \in \Delta$ .

<span id="page-1-5"></span>Assumption 4: There are constants  $a_{i1}$ ,  $a_{i2}$ ,  $a_{i3}$ ,  $a_{i4}$ ,  $a_{i5}$ ,  $a_{i6}$  such that for all  $\bar{x}(t), \underline{x}(t) \in \mathbb{R}^n, y(t) \in \mathbb{R}^q$ , the following inequalities hold

$$
\|\underline{\Psi}_{i}(\overline{x}(t), \underline{x}(t), y(t))\| \le a_{i1} \|\underline{x}(t)\| + a_{i2} \|\overline{x}(t)\| + a_{i3} \|y(t)\| \|\overline{\Psi}_{i}(\overline{x}(t), \underline{x}(t), y(t))\| \le a_{i4} \|\underline{x}(t)\| + a_{i5} \|\overline{x}(t)\| + a_{i6} \|y(t)\|
$$
\n(4)

Remark 1: Assumption [1](#page-1-2) is common in [\[18](#page-6-11)[–22](#page-6-13)]. It can be relaxed by coordinate transformations and will be discussed in Subsection [3.3.](#page-4-0) Assumption [2](#page-1-3) means that the external disturbances are bounded with the known boundary functions. Assumptions [3](#page-1-4) and [4](#page-1-5) characterize the class of interconnections being considered.

#### <span id="page-1-7"></span>3. MAIN RESULTS

#### <span id="page-1-1"></span>3.1. Interval observer design

<span id="page-1-6"></span>First, the following interval observer is designed for the *i*th subsystem ([1\)](#page-1-6).

$$
\dot{x}_i(t) = (A_i - L_iC_i)\dot{x}_i(t) + L_i y_i(t) + B_i u_i(t) \n+ \underline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) + D_i^+ \underline{d}_i(t) - D_i^- \overline{d}_i(t), \n\dot{\overline{x}}_i(t) = (A_i - L_iC_i)\overline{x}_i(t) + L_i y_i(t) + B_i u_i(t) \n+ \overline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) + D_i^+ \overline{d}_i(t) - D_i^- \underline{d}_i(t),
$$
\n(5)

where  $L_i$  is the observer gain matrix to be determined. The interval property between the states of the system ([1\)](#page-1-6) and [\(5](#page-1-7)) is presented in the following lemma.

<span id="page-2-0"></span>Lemma 1: Under Assumptions [1,](#page-1-2) [2](#page-1-3) and [3,](#page-1-4) if the initial condition  $\underline{x}_i(0) \leq x_i(0) \leq \overline{x}_i(0)$  is satisfied, then the relation  $\underline{x}_i(t) \leq x_i(t) \leq \overline{x}_i(t)$  holds for any control law.

**Proof:** Let  $\underline{e}_i(t) = x_i(t) - \underline{x}_i(t)$  and  $\overline{e}_i(t) = \overline{x}_i(t) - x_i(t)$ , from the system and interval observer dynamics ([1\)](#page-1-6) and [\(5](#page-1-7)), it yields the following *i*th estimation error dynamics,

$$
\dot{\mathbf{e}}_i(t) = (A_i - L_i C_i) \mathbf{e}_i(t) + D_i d_i(t) - [D_i^+ \underline{d}_i(t) - D_i^- \overline{d}_i(t)] \n+ \Psi_i(x(t), y(t), \delta(t)) - \underline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)), \n\dot{\overline{e}}_i(t) = (A_i - L_i C_i) \overline{e}_i(t) + [D_i^+ \overline{d}_i(t) - D_i^- \underline{d}_i(t)] - D_i d_i(t) \n+ \overline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) - \Psi_i(x(t), y(t), \delta(t)).
$$
\n(6)

From Assumptions [2](#page-1-3) and [3,](#page-1-4) one gets

$$
D_i d_i(t) - [D_i^+ \underline{d}_i(t) - D_i^- \overline{d}_i(t)] \succeq 0,
$$
  
\n
$$
[D_i^+ \overline{d}_i(t) - D_i^- \underline{d}_i(t)] - D_i d_i(t) \succeq 0,
$$
  
\n
$$
\Psi_i(x(t), y(t), \delta(t)) - \underline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) \succeq 0,
$$
  
\n
$$
\overline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) - \Psi_i(x(t), y(t), \delta(t)) \succeq 0.
$$

In view of Assumption [1](#page-1-2) and the cooperative system theory [\[22](#page-6-13), [23\]](#page-6-14), one gets

$$
\underline{e}_i(t) \succeq 0, \ \overline{e}_i(t) \succeq 0
$$

hold for all  $i = 1, 2, \cdots, N$ . Furthermore, from

$$
\underline{e}_i(t) = x_i(t) - \underline{x}_i(t) \succeq 0, \ \overline{e}_i(t) = \overline{x}_i(t) - x_i(t) \succeq 0,
$$

and the initial condition  $\underline{x}_i(0) \leq x_i(0) \leq \overline{x}_i(0)$ , one gets

$$
\underline{x}_i(t) \preceq x_i(t) \preceq \overline{x}_i(t)
$$

holds for any control law. □

**Remark 2:** In Lemma [1,](#page-2-0) the matrix  $L_i$  is designed such that  $(A_i - L_i C_i)$  is Metlzer, but the stability of the observer states  $x_i(t)$  and  $\bar{x}_i(t)$  can not be guaranteed. The output feedback control designed next will achieve the desired control objective.

#### 3.2. Interval observer-based output feedback control

In this paper, the output feedback control problem for the system ([1\)](#page-1-6) will be transformed into designing the state feedback controller for the interval observer systems ([5\)](#page-1-7) based on Lemma [1](#page-2-0). The system ([1\)](#page-1-6) is robust stable if the upper and lower estimate dynamics ([5\)](#page-1-7) are robust stable. The block diagram of the control scheme is given in Fig. [1.](#page-2-1)

Toward this control objective, the following interval observer-based controller is designed,

$$
u_i(t) = \underline{K}_i \underline{x}_i(t) + \overline{K}_i \overline{x}_i(t), \qquad (7)
$$

<span id="page-2-1"></span>

Fig. 1. Architecture of the interval observer-based outputfeedback control.

where  $\underline{K}_i$  and  $K_i$  are the controller gains for the *i*th interval observer system.

Denoting  $\zeta_i(t) = \begin{bmatrix} \frac{x_i(t)}{\bar{x}_i(t)} \end{bmatrix}$  $\bar{x}_i(t)$  $\int$ ,  $\tilde{\Psi}_i(\bar{x}(t), \underline{x}(t), y(t)) =$  $\sqrt{2}$  $\frac{\Psi_i(\overline{x}(t),\underline{x}(t),y(t))}{\overline{\Psi}_i(\overline{x}(t),\underline{x}(t),y(t))}$ ,  $\tilde{d}_i(t) = \begin{bmatrix} \frac{d_i(t)}{d_i(t)} \end{bmatrix}$  $d_i(t)$ ] and applying the controller [\(7](#page-2-2)) to the observer system, the resulting *i*th closed-loop subsystem is given by

<span id="page-2-3"></span>
$$
\dot{\zeta}_i(t) = \tilde{A}_i \zeta_i(t) + \tilde{L}_i y_i(t) + \tilde{\Psi}_i(\bar{x}(t), \underline{x}(t), y(t)) + \tilde{D}_i \tilde{d}_i(t),
$$
\n(8)

where  $\tilde{L}_i =$  $\left[ \begin{array}{c} L_i \end{array} \right]$ *Li*  $\left| \right|$ ,  $\tilde{D}_i =$  $\left[ \begin{array}{cc} D_i^+ & -D_i^- \\ -D_i^- & D_i^+ \end{array} \right]$  $\left| \begin{array}{c} \tilde{A}_i = \bar{A}_i + \end{array} \right|$  $\bar{B}_i K_i$  with  $\bar{A}_i =$  $\begin{bmatrix} A_i - L_i C_i & 0 \end{bmatrix}$ 0  $A_i - L_iC_i$  $\Big]$ ,  $\bar{B}_i =$  $\left[ \begin{array}{c} B_i \end{array} \right]$ *Bi* ] ,  $K_i = \begin{bmatrix} \underline{K}_i & \overline{K}_i \end{bmatrix}$ .

The main goal is to design the control law such that for prescribed scalars  $\kappa_i > 0$ ,  $\tilde{\gamma} > 0$ , the following  $L_{\infty}$  performance

<span id="page-2-4"></span>
$$
\sup_{t\in[0,\infty)}(\sum_{i=1}^N\kappa_i\zeta_i^T(t)\zeta_i(t))\leq\tilde{\gamma}^2\sup_{t\in[0,\infty)}(\sum_{i=1}^N\kappa_i\tilde{d}_i^T(t)\tilde{d}_i(t))
$$
\n(9)

is satisfied under zero initial conditions for all nonzero vector  $\tilde{d}_i(t) \in L_2[0, \infty)$ .

The main theorem is given as follows.

<span id="page-2-6"></span>**Theorem 1:** For given positive scalars  $\alpha$  and  $\gamma$ , the closed-loop system ([8\)](#page-2-3) is robust stable with disturbance at-tenuation performance ([9](#page-2-4)) if there exists matrices  $Q_i > 0$ ,  $X_i$ , positive scalars  $\rho_i$ ,  $a_i$ ,  $\varepsilon_i$  and  $\beta_i$  satisfying the cyclicsmall-gain condition

<span id="page-2-5"></span><span id="page-2-2"></span>
$$
\sum_{j=1}^{N-1} j \sum_{1 \le i_1 \le i_2 \le \dots \le i_{j+1} \le j+1} \beta_{i_1} \beta_{i_2} \dots \beta_{i_{j+1}} < 1,\tag{10}
$$

such that the following LMI holds

$$
\begin{bmatrix}\n\bar{A}_{i}Q_{i} + Q_{i}\bar{A}_{i}^{T} + \bar{B}_{i}X_{i} + X_{i}^{T}\bar{B}_{i}^{T} + \rho_{i}^{-1}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T} + a_{i}^{2}I + \alpha Q_{i} \\
* \\
\delta_{i} & * \\
\tilde{D}_{i} & Q_{i} \\
-\gamma I & 0 \\
* -(\beta_{i}^{-1} + 1 + \beta_{i}^{-1}\varepsilon_{i} + \rho_{i})^{-1}I\n\end{bmatrix} < 0.
$$
\n(11)

Then the controller gains are given as  $K_i = X_i Q_i^{-1}$ .

Proof: Consider the closed-loop observer system [\(8](#page-2-3)), the following Lyapunov function is chosen,

$$
V_i(\zeta_i(t)) = \zeta_i^T(t) P_i \zeta_i(t).
$$

Along the solutions of ([8\)](#page-2-3), one gets

$$
\dot{V}_i(\zeta_i(t)) = \zeta_i^T (P_i \tilde{A}_i + \tilde{A}_i^T P_i) \zeta_i + 2 \zeta_i^T P_i \tilde{L}_i y_i + 2 \zeta_i^T P_i \tilde{\Psi}_i(\bar{x}, \underline{x}, y).
$$
\n(12)

From Assumption [4](#page-1-5) and  $||y(t)|| \le \sqrt{2} ||\bar{C}|| ||\zeta(t)||$  with  $\bar{C} =$  $diag{C_1, C_2, \cdots, C_N}$ , one gets

$$
\|\tilde{\Psi}_i(\bar{x}(t), \underline{x}(t), y(t))\| \le \|\underline{\Psi}_i(\bar{x}(t), \underline{x}(t), y(t))\| + \|\overline{\Psi}_i(\bar{x}(t), \underline{x}(t), y(t))\|
$$
  

$$
\le a_i \|\zeta(t)\| \tag{13}
$$

where  $a_i = \sqrt{2} \max\{(a_{i1} + a_{i4}), (a_{i2} + a_{i5})\} + \sqrt{2}(a_{i3} + a_{i4})$  $a_{i6}$ ) $||\bar{C}||$ .

In addition,

$$
x_i^T(t)x_i(t) \leq \overline{x}_i^T(t)\overline{x}_i(t) + \underline{x}_i^T(t)\underline{x}_i(t) \leq \zeta_i^T(t)\zeta_i(t),
$$

we have

$$
\dot{V}_{i}(\zeta_{i}(t)) + \alpha \zeta_{i}(t) P_{i} \zeta_{i}(t) - \gamma \tilde{d}_{i}^{T}(t) \tilde{d}_{i}(t) \n= \zeta_{i}^{T}(t) (P_{i}\tilde{A}_{i} + \tilde{A}_{i}^{T} P_{i}) \zeta_{i}(t) + 2 \zeta_{i}^{T}(t) P_{i} \tilde{L}_{i} y_{i}(t) \n+ 2 \zeta_{i}^{T}(t) P_{i} \tilde{\Psi}_{i}(\bar{x}(t), \underline{x}(t), y(t)) + 2 \zeta_{i}^{T}(t) P_{i} \tilde{D}_{i} \tilde{d}_{i}(t) \n+ \alpha \zeta_{i}(t) P_{i} \zeta_{i}(t) - \gamma \tilde{d}_{i}^{T}(t) \tilde{d}_{i}(t) \n\leq \zeta_{i}^{T}(t) (P_{i} \tilde{A}_{i} \n+ \tilde{A}_{i}^{T} P_{i}) \zeta_{i}(t) + \rho_{i}^{-1} \zeta_{i}^{T}(t) P_{i} \tilde{L}_{i} C_{i} C_{i}^{T} \tilde{L}_{i}^{T} P_{i} \zeta_{i}(t) \n+ \rho_{i} x_{i}^{T}(t) x_{i}(t) + \alpha_{i}^{2} \zeta_{i}^{T}(t) P_{i} \tilde{L}_{i} C_{i} C_{i}^{T} \tilde{L}_{i}^{T} P_{i} \zeta_{i}(t) \n+ 2 \zeta_{i}^{T}(t) P_{i} \tilde{D}_{i} \tilde{d}_{i}(t) + \alpha \zeta_{i}^{T}(t) P_{i} \zeta_{i}(t) - \gamma \tilde{d}_{i}^{T}(t) \tilde{d}_{i}(t) \n\leq \zeta_{i}^{T}(t) [P_{i} \tilde{A}_{i} + \tilde{A}_{i}^{T} P_{i} + \rho_{i}^{-1} P_{i} \tilde{L}_{i} C_{i} C_{i}^{T} \tilde{L}_{i}^{T} P_{i} + a_{i}^{2} P_{i}^{2} + \rho_{i} I \n+ \alpha P_{i}] \zeta_{i}(t) + \zeta_{i}^{T}(t) \zeta(t) + 2 \zeta_{i}^{T}(t) P_{i}
$$

if the following inequality

<span id="page-3-1"></span>
$$
\begin{bmatrix}\nP_i\tilde{A}_i + \tilde{A}_i^T P_i + \rho_i^{-1} P_i \tilde{L}_i C_i C_i^T \tilde{L}_i^T P_i + a_i^2 P_i^2 + \alpha P_i \\
+ (\beta_i^{-1} + 1 + \beta_i^{-1} \varepsilon_i + \rho_i)I \\
* \\
P_i \tilde{D}_i \\
-\gamma I\n\end{bmatrix} < 0
$$
\n(15)

<span id="page-3-2"></span>holds, it follows that

$$
\dot{V}_{i}(\zeta_{i}(t)) + \alpha \zeta_{i}(t) P_{i} \zeta_{i}(t) - \gamma \tilde{d}_{i}^{T}(t) \tilde{d}_{i}(t) \n\leq -(\beta_{i}^{-1} + 1 + \beta_{i}^{-1} \varepsilon_{il}) \zeta_{i}^{T}(t) \zeta_{i}(t) + \zeta^{T}(t) \zeta(t) \n\leq -\beta_{i}^{-1} \varepsilon_{il} \zeta_{i}^{T}(t) \zeta_{i}(t) - \beta_{i}^{-1} \zeta_{i}^{T}(t) \zeta_{i}(t) \n+ \sum_{j \neq i,j=1}^{N} \zeta_{j}^{T}(t) \zeta_{j}(t).
$$
\n(16)

Similar to [[34](#page-7-7)] and [\[35](#page-7-8)], the cyclic-small-gain condition ([10\)](#page-2-5) is introduced, then there exist constants  $c_i > 0$  for all  $1 \leq i \leq N$  such that

$$
\sum_{i=1}^{N} c_{i} \beta_{i} (\dot{V}_{i}(\zeta_{i}(t)) + \alpha \zeta_{i}(t) P_{i} \zeta_{i}(t) - \gamma \bar{d}_{i}^{T}(t) \tilde{d}_{i}(t))
$$
\n
$$
\leq - \sum_{i=1}^{N} c_{i} \epsilon_{il} \zeta_{i}^{T}(t) \zeta_{i}(t) + \sum_{i=1}^{N} c_{i} (-\zeta_{i}^{T}(t) \zeta_{i}(t))
$$
\n
$$
+ \beta_{i} \sum_{j=1, j \neq i}^{N} \zeta_{j}^{T}(t) \zeta_{j}(t))
$$
\n
$$
= - \sum_{i=1}^{N} c_{i} \epsilon_{il} \zeta_{i}^{T}(t) \zeta_{i}(t)
$$
\n
$$
+ \begin{bmatrix} ||\zeta_{1}(t)||^{2} \\ ||\zeta_{2}(t)||^{2} \\ ||\zeta_{3}(t)||^{2} \\ \vdots \\ ||\zeta_{N}(t)||^{2} \end{bmatrix}^{T} \begin{bmatrix} -1 & \beta_{2} & \beta_{3} & \cdots & \beta_{N} \\ \beta_{1} & -1 & \beta_{3} & \cdots & \beta_{N} \\ \beta_{1} & \beta_{2} & -1 & \cdots & \beta_{N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_{1} & \beta_{2} & \beta_{3} & \cdots & -1 \end{bmatrix} \begin{bmatrix} c_{1} \\ c_{2} \\ c_{3} \\ \vdots \\ c_{N} \end{bmatrix}
$$
\n
$$
= - \sum_{i=1}^{N} c_{i} \epsilon_{il} \zeta_{i}^{T}(t) \zeta_{i}(t) - \zeta^{T}(t) \zeta(t).
$$
\n(17)

Furthermore, we have

<span id="page-3-0"></span>
$$
\dot{\nabla}(\bar{\zeta}(t)) \le -\alpha \bar{V}(\bar{\zeta}(t)) + \gamma \bar{\omega}^T(t)\bar{\omega}(t), \tag{18}
$$

where  $\bar{V}(\bar{\zeta}(t)) = \bar{\zeta}^T(t)\bar{P}\bar{\zeta}(t)$  with  $\bar{P} = diag\{P_1, P_2, \dots, P_N\},$  $\bar{\omega}(t) = \left[ (c_1\beta_1)^{\frac{1}{2}} \tilde{d}_1^T(t) (c_2\beta_2)^{\frac{1}{2}} \tilde{d}_2^T(t) \cdots (c_N\beta_N)^{\frac{1}{2}} \tilde{d}_N^T(t) \right]^T$  $\bar{\zeta}(t) = \left[ (c_1\beta_1)^{\frac{1}{2}} \zeta_1^T(t) (c_2\beta_2)^{\frac{1}{2}} \zeta_2^T(t) \cdots (c_N\beta_N)^{\frac{1}{2}} \zeta_N^T(t) \right]^T$ . From the solution of differential inequality [\(18](#page-3-0)), one  $\overrightarrow{ob}$ tains

$$
\bar{V}(\bar{\zeta}(t)) \leq e^{-\alpha t} \bar{V}(\bar{\zeta}(0)) + \gamma \int_0^t e^{-\alpha \tau} \bar{\omega}^T(t-\tau) \bar{\omega}(t-\tau) d\tau
$$
\n
$$
\leq \sup_{\tau \in [0,t]} \left\{ e^{-\alpha t} \bar{V}(\bar{\zeta}(0)) + \frac{\gamma}{\alpha} \bar{\omega}^T(t-\tau) \bar{\omega}(t-\tau) (1-e^{-\alpha t}) \right\}, \quad (19)
$$

Interval Observer-based Output Feedback Control for a Class of Interconnected Systems with Uncertain ... 961

it follows that

$$
\sup_{t \in [0,\infty)} \lambda_{\min}(\bar{P}) \bar{\zeta}^T(t) \bar{\zeta}(t)
$$
\n
$$
\leq \sup_{t \in [0,\infty)} \sup_{\tau \in [0,t]} \left\{ e^{-\alpha t} \bar{V}(\bar{\zeta}(0)) + \frac{\gamma}{\alpha} \bar{\omega}^T(t-\tau) \bar{\omega}(t-\tau) (1-e^{-\alpha t}) \right\}
$$
\n
$$
\leq \bar{V}(\bar{\zeta}(0)) + \frac{\gamma}{\alpha} \sup_{t \in [0,\infty)} \bar{\omega}^T(t) \bar{\omega}(t).
$$
\n(20)

The above inequality implies that

$$
\sup_{t \in [0,\infty)} \sum_{i=1}^{N} c_i \beta_i \zeta_i^T(t) \zeta_i(t)
$$
\n
$$
\leq \frac{\lambda_{\max}(\bar{P})}{\lambda_{\min}(\bar{P})} \sum_{i=1}^{N} c_i \beta_i \zeta_i^T(0) \zeta_i(0)
$$
\n
$$
+ \frac{\gamma}{\alpha \lambda_{\min}(\bar{P})} \sup_{t \in [0,\infty)} \sum_{i=1}^{N} c_i \beta_i \tilde{d}_i^T(t) \tilde{d}_i(t), \tag{21}
$$

which implies that the system [\(8](#page-2-3)) satisfies the  $L_{\infty}$  performance  $(9)$  $(9)$ .

Left-multiplying and right-multiplying  $diag\{P_i^{-1}, I\}$  to the inequality [\(15](#page-3-1)) yields

$$
\begin{bmatrix}\n\tilde{A}_{i}P_{i}^{-1} + P_{i}^{-1}\tilde{A}_{i}^{T} + \rho_{i}^{-1}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T} + a_{i}^{2}I + \alpha P_{i}^{-1} \\
* \\
* \\
\tilde{D}_{i} & F_{i}^{-1} \\
-\gamma I & 0 \\
* -(\beta_{i}^{-1} + 1 + \beta_{i}^{-1}\varepsilon_{i} + \rho_{i})^{-1}I\n\end{bmatrix} < 0 \quad (22)
$$

denote  $P_i^{-1} = Q_i$  and  $X_{il} = K_{il}Q_i$ , the above inequality can be rewritten as  $(11)$  $(11)$ .

Remark 3: The design condition in Theorem [1](#page-2-6) is LMI, it avoids the non-convex problems which arise in the design of dynamic output feedback controller.

#### <span id="page-4-0"></span>3.3. Coordinate transformation

Assumption [1](#page-1-2) can be relaxed by finding a transformation of coordinates  $x_i(t) = T_i z_i(t)$ , the system ([1\)](#page-1-6) can be rewritten as follows,

$$
\dot{z}_i(t) = T_i^{-1} (A_i - L_i C_i) T_i z_i(t) + T_i^{-1} L_i y_i(t) + T_i^{-1} B_i u_i(t) \n+ T_i^{-1} \Psi_i (T z(t), y(t), \delta(t)) + T_i^{-1} D_i d_i(t), \ny_i(t) = C_i T_i z_i(t),
$$
\n(23)

where  $T = diag\{T_1, T_1, \cdots, T_N\}$ . Denote  $T_i^{-1}(A_i - L_iC_i)T_i = A_i, T_i^{-1}L_i = \mathcal{L}_i, T_i^{-1}B_i = \mathcal{B}_i,$  $T_i^{-1}D_i = D_i$ ,  $T_i^{-1} = R_i$ , then [\(23\)](#page-4-1) can be rewritten as

$$
\dot{z}_i(t) = \mathcal{A}_i z_i(t) + \mathcal{L}_i y_i(t) + \mathcal{B}_i u_i(t) + \mathcal{D}_i d_i(t) + R_i \Psi_i(Tz(t), y(t), \delta(t))
$$

$$
y_i(t) = C_i T_i z_i(t). \tag{24}
$$

From

$$
T_i^+ \underline{z}_i(t) - T_i^- \overline{z}_i(t) \preceq T_i z_i(t) \preceq T_i^+ \overline{z}_i(t) - T_i^- \underline{z}_i(t),
$$
  
\n
$$
T^+ \underline{z}(t) - T^- \overline{z}(t) \preceq T z(t) \preceq T^+ \overline{z}(t) - T^- \underline{z}(t),
$$

it is clear that

$$
\underline{\Phi}_i(\overline{z}(t),\underline{z}(t),y(t)) \leq R_i \Psi_i(Tz(t),y(t),\delta(t))
$$
  

$$
\leq \overline{\Phi}_i(\overline{z}(t),\underline{z}(t),y(t))
$$

where

$$
\overline{\Phi}_i(\overline{z},\underline{z},y) = R_i^+ \overline{\Psi}_i (T^+ \overline{z} - T^- \underline{z}, T^+ \underline{z} - T^- \overline{z},y) \n- R_i^- \underline{\Psi}_i (T^+ \overline{z} - T^- \underline{z}, T^+ \underline{z} - T^- \overline{z},y), \n\underline{\Phi}_i(\overline{z},\underline{z},y) = R_i^+ \underline{\Psi}_i (T^+ \overline{z} - T^- \underline{z}, T^+ \underline{z} - T^- \overline{z},y) \n- R_i^- \overline{\Psi}_i (T^+ \overline{z} - T^- \underline{z}, T^+ \underline{z} - T^- \overline{z},y).
$$

Then, the following interval observer is designed for the system ([24\)](#page-4-2),

<span id="page-4-3"></span>
$$
\dot{\underline{z}}_i(t) = \mathcal{A}_i \underline{z}_i(t) + \mathcal{L}_i y_i(t) + \mathcal{B}_i u_i(t) + \underline{\Phi}_i (\overline{z}(t), \underline{z}(t), y(t)) \n+ \mathcal{D}_i^+ \underline{d}_i(t) - \mathcal{D}_i^- \overline{d}_i(t, ) \n\dot{\overline{z}}_i(t) = \mathcal{A}_i \overline{z}_i(t) + \mathcal{L}_i y_i(t) + \mathcal{B}_i u_i(t) + \overline{\Phi}_i (\overline{z}(t), \underline{z}(t), y(t)) \n+ \mathcal{D}_i^+ \overline{d}_i(t) - \mathcal{D}_i^- \underline{d}_i(t).
$$
\n(25)

Furthermore, the following controller is designed for the transformed interval observer system ([25\)](#page-4-3)

$$
u_i(t) = \underline{\mathcal{K}}_{i\overline{\mathcal{Z}}_i}(t) + \overline{\mathcal{K}}_{i\overline{\mathcal{Z}}_i}(t). \tag{26}
$$

In the new coordinates, the interval observer  $(25)$  $(25)$  is similar to ([5\)](#page-1-7), therefore, a similar conclusion can be obtained.

Based on the above analysis and design, an algorithm is given to design the interval observer-based output feedback controller.

Algorithm 1: Interval observer-based output feedback controller design

- Step 1: If there exists a matrix  $L_i$  such that  $A_i L_i C_i$  is Metzler and Hurwitz, go to Step 3 with the selected interval observer gain *L<sup>i</sup>* . Otherwise, go to Step 2.
- <span id="page-4-1"></span>**Step 2:** Choose the free matrices  $S_i$ ,  $Y_i$  such that the matrix  $S_i$  is Metzler and Hurwitz. Solve the Sylvester equation  $S_i T_i^{-1} - T_i^{-1} A_i + Y_i C_i = 0$  with respect to the unknown transformation matrix  $T_i$ , then  $L_i = T_i Y_i$ . Transform the system [\(1](#page-1-6)) into [\(23](#page-4-1)) by using  $x_i(t) =$  $T_i z_i(t)$ .
- Step 3: Design the interval observer with the obtained gain matrix *L<sup>i</sup>* .
- Step 4: Solve the LMI conditions to get the observerbased controller gain *K<sup>i</sup>* .

<span id="page-4-2"></span>

### 4. SIMULATION

<span id="page-5-0"></span>In this section, a simulation example is presented to demonstrate the effectiveness of the proposed method. Consider the following system,

$$
\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \Psi_i(y(t), \delta(t)) + D_i d_i(t), \n y_i(t) = C_i x_i(t),
$$
\n(27)

where  $A_1 = A_2 =$  $\begin{bmatrix} 0 & 1 \\ 9.81 & 0 \end{bmatrix}$ ,  $B_1 = B_2 =$  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ 0*.*5 ]  $D_1 =$  $D_2 =$  $\begin{bmatrix} 0.1 \end{bmatrix}$ 0*.*5 ] ,  $C_1 = C_2 = [1 \ 0], \Psi_1(y(t), \delta(t)) =$  $\Psi_2(y(t),\delta(t)) = \begin{bmatrix} 0 \\ 0 \\ \delta(t) \end{bmatrix}$  $\delta(t)(y_2 - y_1)$ ] . The uncertain parameter  $\delta(t)$  satisfies the inequality  $|\delta(t)| < \overline{\delta}$ , then the interconnection functions satisfy

$$
\underline{\Psi}_1(y(t)) \leq \Psi_1(y(t), \delta(t)) \leq \overline{\Psi}_1(y(t)),
$$
  

$$
\underline{\Psi}_2(y(t)) \leq \Psi_2(y(t), \delta(t)) \leq \overline{\Psi}_2(y(t)),
$$

 $\frac{\Psi_1(y(t))}{= -\bar{\delta}(\|y_1\| + \|y_2\|)}$  and  $\overline{\Psi}_1(y(t)) =$  $\delta(\|y_1\| + \|y_2\|)$  satisfy the inequalities  $\|\Psi_1(y(t))\| \le$  $a_{13}$  ||*y*(*t*)|| and  $||\Psi_1(y(t))|| \le a_{16}$  ||*y*(*t*)|| with  $a_{13} = a_{16} =$  $\overline{2}\overline{\delta}$ . For simulation, the value of the uncertain parameter has been chosen as  $\delta(t) = 0.3535 \sin(t), d(t) = \cos(\pi t) +$  $0.5 \sin(\pi t)$ ,  $d(t) = \cos(\pi t) - 0.5$ ,  $\overline{d}(t) = \cos(\pi t) + 0.5$ .

Based on the Algorithm 1, choosing the free matrices  $S_i =$  $\sqrt{2}$ *−*0*.9* 10 *−*0*.95*, *Q<sub>i</sub>* =  $\begin{bmatrix} 1 \end{bmatrix}$ 0*.*5 ] and solving the Sylvester equation  $S_i T_i^{-1} - T_i^{-1} A_i + Q_i C_i = 0$ , one gets

$$
T_i = \left[ \begin{array}{cc} 1.7822 & 0.1356 \\ 1.6931 & 17.9439 \end{array} \right], L_i = \left[ \begin{array}{c} 1.8500 \\ 10.6650 \end{array} \right].
$$

Given the constants  $\rho_i = 0.2, \alpha = 0.3, \varepsilon_i = 0.1, \beta_i =$ 0*.*0455, solving the LMI condition, we get the controller gain matrices

$$
\underline{K}_1 = \underline{K}_2 = \begin{bmatrix} -2993 & -2496.6 \end{bmatrix},
$$
  
\n
$$
\overline{K}_1 = \overline{K}_2 = \begin{bmatrix} -3021.6 & -1183.4 \end{bmatrix}.
$$

Simulation results are shown in Figs. [2](#page-5-2)[-3](#page-5-3), it can be seen that the interval relations hold, furthermore, both the system states and the interval observer states achieve the good steady performance, which further verifies that the proposed interval observer-based output feedback controller achieves desired control performance for interconnected systems.

#### 5. CONCLUSION

<span id="page-5-1"></span>In this paper, the interval observer-based output feedback control scheme for a class of interconnected systems with uncertain interconnections has been proposed.

<span id="page-5-2"></span>

Fig. 2. State responses of the subsystem *S*1.

<span id="page-5-3"></span>

Fig. 3. State responses of the subsystem  $S_2$ .

For each subsystem, the output feedback controller design problem for original system is transformed to the state feedback problem for observer system. The cyclicsmall gain condition is introduced to formulate the problem to LMIs. The closed-loop system stability is guaranteed by coordinate transformation and interval property. The achieved method has been finally demonstrated via simulation results. Furthermore, by reference to the existing results  $[36-42]$  $[36-42]$ , a direction for the future is to extend the proposed method to the stochastic interconnected systems.

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