Interval Observer-based Output Feedback Control for a Class of Interconnected Systems with Uncertain Interconnections

Zhi-Hui Zhang, Shujiang Li*💿 , and Hua Yan

Abstract: This paper studies the output feedback controller design problem for a class of interconnected systems with uncertain interconnections. First, the interval observers are built to guarantee the interval property between the system states and their estimations. Then, the state feedback controller is designed for each observer system other than output feedback controller for original system. A cyclic-small-gain condition is used to deal with the bound functions of the uncertain interconnections. The original closed-loop system is robust stable since that the interval properties are satisfied. The non-convex conditions caused by traditional observer-based output feedback are avoided by transforming the output to state feedback problem. Finally, the main results are demonstrated by numerical simulations.

Keywords: Cyclic-small-gain condition, interconnected systems, interval observer, output feedback control.

1. INTRODUCTION

As a practical matter, many systems appear in the form of several interacting subsystems, for instance, power networks and process control systems [1-5]. Hence, many researchers have devoted themselves to the control problem of such interconnected systems [6–9]. In [10], a controller was designed for large-scale systems subject to power constraint. In [11], an event-triggered controller was presented for interconnected linear systems. However, most of the results focused on state feedback control for interconnected systems. Unfortunately, in most cases, states are unmeasurable. Output feedback control is more suitable for such actual situation. Recently, many output feedback control schemes have been proposed for interconnected systems. For example, in [12], a static outputfeedback method has been proposed for large-scale T-S fuzzy systems. Furthermore, the robust H_{∞} dynamic output feedback control problem has been studied in [13]. But only the linear interconnections were considered in the above paper. It is well known that the linear ones are hard to describe the practical complex cases. Therefore, studies on the output feedback controller for interconnected systems with nonlinear interconnections are still insufficient.

Observer-based output feedback is an effective control method when only the output of the system is measurable [14–16]. Many observer design algorithms have been pro-

posed to estimate the system states, for instance, interval observers. In [17], the exponentially stable linear interval observer was built for linear systems. Furthermore, the interval observer with Luenberger observer structure was developed in [18]. Besides, coordinate transformation has proved to be a significant way to guarantee the interval property [19-22]. At the same time, [23] and [24] studied the output stabilization problem for nonlinear systems subject to parametric and signal uncertainties and extended the method to linear time-varying and linearparameter-varying systems. Furthermore, [25, 26] presented the interval observer-based output feedback controllers for time-varying input delay systems and switched systems, respectively. However, the above methods are proposed without consideration for the external disturbances. Therefore, interval observer-based output feedback control for a class of interconnected systems with uncertain interconnections and external disturbances is still an open problem.

In this paper, the output feedback controller for interconnected system with uncertain interconnections and external disturbances is designed. For each subsystem, an interval observer is built firstly by considering the upper and lower bounds of the interconnections and disturbances. Then, the output feedback controllers are built and the design conditions are derived in the formulation of linear matrix inequalities (LMIs). It is shown that the original closed-loop system can be proved to be robust sta-

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ble since that the interval properties are satisfied. The main contributions are as follows: 1) The state feedback controller is designed for observer system other than output feedback controller for original system. The non-convex conditions caused by traditional observer-based output feedback are avoided by transforming the output to state feedback problem. 2) A cyclic-small-gain condition is introduced to address the bound functions of the uncertain interconnections.

Note that the nonlinear uncertain interconnections are considered in this paper. Many methods have been proposed to control the system with nonlinear interconnections. For example, in [27], a sliding mode observer-based output feedback controller was presented for linear subsystems with nonlinear interconnections. In [28] and [29], the adaptive control techniques were developed for largescale nonlinear systems. However, there results were obtained based on the assumption that the interconnection matching condition is satisfied, which makes its application too inflexible. In addition, the neural networks have been introduced to deal with the nonlinear interconnections. In [30–32], the adaptive neural network outputfeedback control problem has been solved by backstepping technique. Furthermore, by a combination of the neural network and graph theory, the fault-tolerant control scheme has been developed in [33]. But the neural networks may considerably increase the complexity of the controller design. Different from the above mentioned results, the interconnection matching condition is removed and the proposed control scheme without employing the neural network simplifies the design process.

The structure of this paper is as follows: The system description is presented in Section 2 and the main results are proposed in Section 3. An example is given in Section 4 and conclusions are drawn in Section 5.

Notations: For vectors $\alpha = [\alpha_i]_{m \times 1}$, $\beta = [\beta_i]_{m \times 1}$, we define $\alpha \leq \beta$ ($\alpha \geq \beta$) by $\alpha_i \leq \beta_i$ ($\alpha_i \geq \beta_i$), $\forall 1 \leq i \leq m$. $\Omega > 0$ ($\Omega < 0$) means the matrix Ω is positive (negative) definite. For given matrix $\Gamma \in \mathbb{R}^{p \times q}$, define $\Gamma^+ = \max\{0, \Gamma\}$ and $\Gamma^- = \Gamma^+ - \Gamma$.

2. SYSTEM DESCRIPTION

Consider a system consisting of *N* interconnected subsystems S_i , $i = 1, 2, \dots, N$. Each of which is described by the following dynamic model:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \Psi_i(x(t), y(t), \delta(t)) + D_i d_i(t),$$

$$y_i(t) = C_i x_i(t),$$
(1)

where $u_i(t) \in \mathbb{R}^{s_i}$ is the control input vector, $x_i(t) \in \mathbb{R}^{n_i}$ and $y_i(t) \in \mathbb{R}^{q_i}$ are the state and output vectors of the *i*th subsystem. $x(t) = \begin{bmatrix} x_1^T(t), & x_2^T(t), & \cdots, & x_N^T(t) \end{bmatrix}^T \in \mathbb{R}^n$ and $y(t) = \begin{bmatrix} y_1^T(t), & y_2^T(t), & \cdots, & y_N^T(t) \end{bmatrix}^T \in \mathbb{R}^q$ are the state and output vectors of the overall system. The nonlinear function $\Psi_i(x(t), y(t), \delta(t)) : \mathbb{R}^{n+q+r} \to \mathbb{R}^{m_i}$ represents the interconnection between the *i*th subsystem and other subsystems. $\delta(t) \in \Delta \subset \mathbb{R}^r$ is the uncertain possibly time-varying parameter vector, the set Δ is assumed to be given, $\Psi_i(0,0,\delta(t)) = 0$ for any $\delta(t) \in \Delta$. $d_i(t) \in \mathbb{R}^{p_i}$ is the external disturbance vector. $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times s_i}$, $D_i \in \mathbb{R}^{n_i \times p_i}$, $C_i \in \mathbb{R}^{q_i \times n_i}$ are known constant matrices. $\sum_{i=1}^N n_i = n$, $\sum_{i=1}^N q_i = q$. Moreover, it is assumed that (A_i, C_i) is observable and (A_i, B_i) is controllable.

The main idea of this paper is to design the interval observer-based output feedback controller, the following assumptions are considered firstly.

Assumption 1: There exists a matrix L_i such that $A_i - L_iC_i$ is Metzler.

Assumption 2: There exist the known bound functions $\underline{d}_i(t) \in \mathbb{R}^{p_i}$ and $\overline{d}_i(t) \in \mathbb{R}^{p_i}$ belong to $L_2[0,\infty)$ such that

$$\underline{d}_i(t) \preceq d_i(t) \preceq \overline{d}_i(t). \tag{2}$$

Assumption 3: There are known functions $\overline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)), \underline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) : \mathbb{R}^{2n+q} \to \mathbb{R}^{m_i}$ such that

$$\underline{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t)) \leq \Psi_{i}(x(t),y(t),\delta(t))$$
$$\leq \overline{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t))$$
(3)

is satisfied if $\underline{x}(t) \preceq x(t) \preceq \overline{x}(t)$ and $\delta(t) \in \Delta$.

Assumption 4: There are constants $a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}, a_{i6}$ such that for all $\overline{x}(t), \underline{x}(t) \in \mathbb{R}^n, y(t) \in \mathbb{R}^q$, the following inequalities hold

$$\begin{aligned} \|\underline{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t))\| &\leq a_{i1} \|\underline{x}(t)\| + a_{i2} \|\overline{x}(t)\| \\ &+ a_{i3} \|y(t)\| \\ \|\overline{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t))\| &\leq a_{i4} \|\underline{x}(t)\| + a_{i5} \|\overline{x}(t)\| \\ &+ a_{i6} \|y(t)\| \end{aligned}$$
(4)

Remark 1: Assumption 1 is common in [18–22]. It can be relaxed by coordinate transformations and will be discussed in Subsection 3.3. Assumption 2 means that the external disturbances are bounded with the known boundary functions. Assumptions 3 and 4 characterize the class of interconnections being considered.

3. MAIN RESULTS

3.1. Interval observer design

First, the following interval observer is designed for the *i*th subsystem (1).

$$\begin{aligned} \underline{\dot{x}}_{i}(t) = & (A_{i} - L_{i}C_{i})\underline{x}_{i}(t) + L_{i}y_{i}(t) + B_{i}u_{i}(t) \\ & + \underline{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t)) + D_{i}^{+}\underline{d}_{i}(t) - D_{i}^{-}\overline{d}_{i}(t), \\ \overline{\dot{x}}_{i}(t) = & (A_{i} - L_{i}C_{i})\overline{x}_{i}(t) + L_{i}y_{i}(t) + B_{i}u_{i}(t) \\ & + \overline{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t)) + D_{i}^{+}\overline{d}_{i}(t) - D_{i}^{-}\underline{d}_{i}(t), \end{aligned}$$

$$(5)$$

where L_i is the observer gain matrix to be determined. The interval property between the states of the system (1) and (5) is presented in the following lemma.

Lemma 1: Under Assumptions 1, 2 and 3, if the initial condition $\underline{x}_i(0) \preceq x_i(0) \preceq \overline{x}_i(0)$ is satisfied, then the relation $\underline{x}_i(t) \preceq x_i(t) \preceq \overline{x}_i(t)$ holds for any control law.

Proof: Let $\underline{e}_i(t) = x_i(t) - \underline{x}_i(t)$ and $\overline{e}_i(t) = \overline{x}_i(t) - x_i(t)$, from the system and interval observer dynamics (1) and (5), it yields the following *i*th estimation error dynamics,

$$\underline{\dot{e}}_{i}(t) = (A_{i}-L_{i}C_{i})\underline{e}_{i}(t) + D_{i}d_{i}(t) - [D_{i}^{+}\underline{d}_{i}(t) - D_{i}^{-}\overline{d}_{i}(t)]
+ \Psi_{i}(x(t), y(t), \delta(t)) - \underline{\Psi}_{i}(\overline{x}(t), \underline{x}(t), y(t)),
\dot{\overline{e}}_{i}(t) = (A_{i}-L_{i}C_{i})\overline{e}_{i}(t) + [D_{i}^{+}\overline{d}_{i}(t) - D_{i}^{-}\underline{d}_{i}(t)] - D_{i}d_{i}(t)
+ \overline{\Psi}_{i}(\overline{x}(t), \underline{x}(t), y(t)) - \Psi_{i}(x(t), y(t), \delta(t)).$$
(6)

From Assumptions 2 and 3, one gets

$$\begin{split} D_i d_i(t) &- [D_i^+ \underline{d}_i(t) - D_i^- d_i(t)] \succeq 0, \\ [D_i^+ \overline{d}_i(t) - D_i^- \underline{d}_i(t)] - D_i d_i(t) \succeq 0, \\ \Psi_i(x(t), y(t), \delta(t)) - \underline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) \succeq 0, \\ \overline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) - \Psi_i(x(t), y(t), \delta(t)) \succeq 0. \end{split}$$

In view of Assumption 1 and the cooperative system theory [22, 23], one gets

$$\underline{e}_i(t) \succeq 0, \ \overline{e}_i(t) \succeq 0$$

hold for all $i = 1, 2, \dots, N$. Furthermore, from

$$\underline{e}_i(t) = x_i(t) - \underline{x}_i(t) \succeq 0, \ \overline{e}_i(t) = \overline{x}_i(t) - x_i(t) \succeq 0,$$

and the initial condition $\underline{x}_i(0) \leq x_i(0) \leq \overline{x}_i(0)$, one gets

$$\underline{x}_i(t) \preceq x_i(t) \preceq \overline{x}_i(t)$$

holds for any control law.

Remark 2: In Lemma 1, the matrix L_i is designed such that $(A_i - L_iC_i)$ is Metlzer, but the stability of the observer states $\underline{x}_i(t)$ and $\overline{x}_i(t)$ can not be guaranteed. The output feedback control designed next will achieve the desired control objective.

3.2. Interval observer-based output feedback control

In this paper, the output feedback control problem for the system (1) will be transformed into designing the state feedback controller for the interval observer systems (5) based on Lemma 1. The system (1) is robust stable if the upper and lower estimate dynamics (5) are robust stable. The block diagram of the control scheme is given in Fig. 1.

Toward this control objective, the following interval observer-based controller is designed,

$$u_i(t) = \underline{K}_i \underline{x}_i(t) + \overline{K}_i \overline{x}_i(t), \tag{7}$$



Fig. 1. Architecture of the interval observer-based outputfeedback control.

where \underline{K}_i and \overline{K}_i are the controller gains for the *i*th interval observer system.

Denoting $\zeta_i(t) = \begin{bmatrix} \underline{x}_i(t) \\ \overline{x}_i(t) \end{bmatrix}$, $\tilde{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) = \begin{bmatrix} \underline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) \\ \overline{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) \end{bmatrix}$, $\tilde{d}_i(t) = \begin{bmatrix} \underline{d}_i(t) \\ \overline{d}_i(t) \end{bmatrix}$ and applying the controller (7) to the observer system, the resulting *i*th closed-loop subsystem is given by

$$\begin{aligned} \dot{\zeta}_i(t) = &\tilde{A}_i \zeta_i(t) + \tilde{L}_i y_i(t) + \tilde{\Psi}_i(\overline{x}(t), \underline{x}(t), y(t)) \\ &+ \tilde{D}_i \tilde{d}_i(t), \end{aligned}$$
(8)

where $\tilde{L}_i = \begin{bmatrix} L_i \\ L_i \end{bmatrix}$, $\tilde{D}_i = \begin{bmatrix} D_i^+ & -D_i^- \\ -D_i^- & D_i^+ \end{bmatrix}$, $\tilde{A}_i = \bar{A}_i + \bar{B}_i K_i$ with $\bar{A}_i = \begin{bmatrix} A_i - L_i C_i & 0 \\ 0 & A_i - L_i C_i \end{bmatrix}$, $\bar{B}_i = \begin{bmatrix} B_i \\ B_i \end{bmatrix}$, $K_i = \begin{bmatrix} \underline{K}_i & \overline{K}_i \end{bmatrix}$.

The main goal is to design the control law such that for prescribed scalars $\kappa_i > 0$, $\tilde{\gamma} > 0$, the following L_{∞} performance

$$\sup_{t\in[0,\infty)} \left(\sum_{i=1}^{N} \kappa_i \zeta_i^T(t) \zeta_i(t)\right) \le \tilde{\gamma}^2 \sup_{t\in[0,\infty)} \left(\sum_{i=1}^{N} \kappa_i \tilde{d}_i^T(t) \tilde{d}_i(t)\right)$$
(9)

is satisfied under zero initial conditions for all nonzero vector $\tilde{d}_i(t) \in L_2[0,\infty)$.

The main theorem is given as follows.

Theorem 1: For given positive scalars α and γ , the closed-loop system (8) is robust stable with disturbance attenuation performance (9) if there exists matrices $Q_i > 0$, X_i , positive scalars ρ_i , a_i , ε_i and β_i satisfying the cyclic-small-gain condition

$$\sum_{j=1}^{N-1} j \sum_{1 \le i_1 \le i_2 \le \dots \le i_{j+1} \le j+1} \beta_{i_1} \beta_{i_2} \cdots \beta_{i_{j+1}} < 1,$$
(10)

such that the following LMI holds

$$\begin{bmatrix} \bar{A}_{i}Q_{i} + Q_{i}\bar{A}_{i}^{T} + \bar{B}_{i}X_{i} + X_{i}^{T}\bar{B}_{i}^{T} + \rho_{i}^{-1}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T} + a_{i}^{2}I + \alpha Q_{i} \\ & * \\ & * \\ \tilde{D}_{i} & Q_{i} \\ -\gamma I & 0 \\ * & -(\beta_{i}^{-1} + 1 + \beta_{i}^{-1}\varepsilon_{i} + \rho_{i})^{-1}I \end{bmatrix} < 0.$$
(11)

Then the controller gains are given as $K_i = X_i Q_i^{-1}$.

Proof: Consider the closed-loop observer system (8), the following Lyapunov function is chosen,

$$V_i(\zeta_i(t)) = \zeta_i^T(t) P_i \zeta_i(t).$$

Along the solutions of (8), one gets

$$\dot{V}_{i}(\zeta_{i}(t)) = \zeta_{i}^{T}(P_{i}\tilde{A}_{i} + \tilde{A}_{i}^{T}P_{i})\zeta_{i} + 2\zeta_{i}^{T}P_{i}\tilde{L}_{i}y_{i} + 2\zeta_{i}^{T}P_{i}\tilde{\Psi}_{i}(\bar{x},\underline{x},y).$$
(12)

From Assumption 4 and $||y(t)|| \le \sqrt{2} ||\bar{C}|| ||\zeta(t)||$ with $\bar{C} = diag\{C_1, C_2, \cdots, C_N\}$, one gets

$$\begin{split} \|\tilde{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t))\| \leq & \|\underline{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t))\| \\ & + \|\overline{\Psi}_{i}(\overline{x}(t),\underline{x}(t),y(t))\| \\ \leq & a_{i}\|\zeta(t)\| \end{split}$$
(13)

where $a_i = \sqrt{2} \max\{(a_{i1} + a_{i4}), (a_{i2} + a_{i5})\} + \sqrt{2}(a_{i3} + a_{i6}) \|\bar{C}\|.$

In addition,

$$x_i^T(t)x_i(t) \leq \overline{x}_i^T(t)\overline{x}_i(t) + \underline{x}_i^T(t)\underline{x}_i(t) \leq \zeta_i^T(t)\zeta_i(t),$$

we have

$$\begin{split} \dot{V}_{i}(\zeta_{i}(t)) + \alpha\zeta_{i}(t)P_{i}\zeta_{i}(t) &- \gamma \tilde{d}_{i}^{T}(t)\tilde{d}_{i}(t) \\ &= \zeta_{i}^{T}(t)(P_{i}\tilde{A}_{i} + \tilde{A}_{i}^{T}P_{i})\zeta_{i}(t) + 2\zeta_{i}^{T}(t)P_{i}\tilde{L}_{i}y_{i}(t) \\ &+ 2\zeta_{i}^{T}(t)P_{i}\tilde{\Psi}_{i}(\bar{x}(t),\underline{x}(t),y(t)) + 2\zeta_{i}^{T}(t)P_{i}\tilde{D}_{i}\tilde{d}_{i}(t) \\ &+ \alpha\zeta_{i}(t)P_{i}\zeta_{i}(t) - \gamma \tilde{d}_{i}^{T}(t)\tilde{d}_{i}(t) \\ &\leq \zeta_{i}^{T}(t)(P_{i}\tilde{A}_{i} \\ &+ \tilde{A}_{i}^{T}P_{i})\zeta_{i}(t) + \rho_{i}^{-1}\zeta_{i}^{T}(t)P_{i}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T}P_{i}\zeta_{i}(t) \\ &+ \rho_{i}x_{i}^{T}(t)x_{i}(t) + a_{i}^{2}\zeta_{i}^{T}(t)P_{i}^{2}\zeta_{i}(t) + \zeta^{T}(t)\zeta(t) \\ &+ 2\zeta_{i}^{T}(t)P_{i}\tilde{D}_{i}\tilde{d}_{i}(t) + \alpha\zeta_{i}^{T}(t)P_{i}\zeta_{i}(t) - \gamma \tilde{d}_{i}^{T}(t)\tilde{d}_{i}(t) \\ &\leq \zeta_{i}^{T}(t)[P_{i}\tilde{A}_{i} + \tilde{A}_{i}^{T}P_{i} + \rho_{i}^{-1}P_{i}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T}P_{i} + a_{i}^{2}P_{i}^{2} + \rho_{i}I \\ &+ \alpha P_{i}]\zeta_{i}(t) + \zeta^{T}(t)\zeta(t) + 2\zeta_{i}^{T}(t)P_{i}\tilde{D}_{i}\tilde{d}_{i}(t) \\ &- \gamma \tilde{d}_{i}^{T}(t)\tilde{d}_{i}(t) \\ &\leq \left[\zeta_{i}^{T}(t) \quad \tilde{d}_{i}^{T}(t)\right] \\ &\times \left[P_{i}\tilde{A}_{i} + \tilde{A}_{i}^{T}P_{i} + \rho_{i}^{-1}P_{i}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T}P_{i} + a_{i}^{2}P_{i}^{2} + \rho_{i}I + \alpha P_{i}\right] \\ &\times \left[P_{i}\tilde{A}_{i} + \tilde{A}_{i}^{T}P_{i} + \rho_{i}^{-1}P_{i}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T}P_{i} + a_{i}^{2}P_{i}^{2} + \rho_{i}I + \alpha P_{i}\right] \\ & \times \left[P_{i}\tilde{D}_{i} - \gamma I\right] \left[\zeta_{i}(t) \\ &= \zeta_{i}(t)\right] \\ &+ \zeta_{i}(t)\right] \\ & \times \left[P_{i}\tilde{D}_{i} - \gamma I\right] \left[\zeta_{i}(t)\right] \\ &+ \zeta_{i}(t)\right] \\ &+ \zeta_{i}(t)\zeta(t), \\ & (14) \end{array}$$

if the following inequality

$$\begin{bmatrix} P_{i}\tilde{A}_{i}+\tilde{A}_{i}^{T}P_{i}+\rho_{i}^{-1}P_{i}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T}P_{i}+a_{i}^{2}P_{i}^{2}+\alpha P_{i} \\ +(\beta_{i}^{-1}+1+\beta_{i}^{-1}\varepsilon_{i}+\rho_{i})I \\ * \\ P_{i}\tilde{D}_{i} \\ -\gamma I \end{bmatrix} < 0$$
(15)

holds, it follows that

$$\begin{split} \dot{V}_{i}(\zeta_{i}(t)) &+ \alpha \zeta_{i}(t) P_{i} \zeta_{i}(t) - \gamma d_{i}^{T}(t) d_{i}(t) \\ &\leq -(\beta_{i}^{-1} + 1 + \beta_{i}^{-1} \varepsilon_{il}) \zeta_{i}^{T}(t) \zeta_{i}(t) + \zeta^{T}(t) \zeta(t) \\ &\leq -\beta_{i}^{-1} \varepsilon_{il} \zeta_{i}^{T}(t) \zeta_{i}(t) - \beta_{i}^{-1} \zeta_{i}^{T}(t) \zeta_{i}(t) \\ &+ \sum_{j \neq i, j=1}^{N} \zeta_{j}^{T}(t) \zeta_{j}(t). \end{split}$$
(16)

Similar to [34] and [35], the cyclic-small-gain condition (10) is introduced, then there exist constants $c_i > 0$ for all $1 \le i \le N$ such that

$$\begin{split} \sum_{i=1}^{N} c_{i}\beta_{i}(\dot{V}_{i}(\zeta_{i}(t)) + \alpha\zeta_{i}(t)P_{i}\zeta_{i}(t) - \gamma\tilde{d}_{i}^{T}(t)\tilde{d}_{i}(t)) \\ \leq & -\sum_{i=1}^{N} c_{i}\varepsilon_{il}\zeta_{i}^{T}(t)\zeta_{i}(t) + \sum_{i=1}^{N} c_{i}(-\zeta_{i}^{T}(t)\zeta_{i}(t) \\ & +\beta_{i}\sum_{j=1,j\neq i}^{N} \zeta_{j}^{T}(t)\zeta_{j}(t)) \\ = & -\sum_{i=1}^{N} c_{i}\varepsilon_{il}\zeta_{i}^{T}(t)\zeta_{i}(t) \\ & + \begin{bmatrix} \|\zeta_{1}(t)\|^{2} \\ \|\zeta_{2}(t)\|^{2} \\ \|\zeta_{3}(t)|^{2} \\ \vdots \\ \|\zeta_{N}(t)\|^{2} \end{bmatrix}^{T} \begin{bmatrix} -1 & \beta_{2} & \beta_{3} & \cdots & \beta_{N} \\ \beta_{1} & -1 & \beta_{3} & \cdots & \beta_{N} \\ \beta_{1} & \beta_{2} & -1 & \cdots & \beta_{N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_{1} & \beta_{2} & \beta_{3} & \cdots & -1 \end{bmatrix} \begin{bmatrix} c_{1} \\ c_{2} \\ c_{3} \\ \vdots \\ c_{N} \end{bmatrix} \\ = & -\sum_{i=1}^{N} c_{i}\varepsilon_{il}\zeta_{i}^{T}(t)\zeta_{i}(t) - \zeta^{T}(t)\zeta(t). \end{split}$$
(17)

Furthermore, we have

$$\dot{\bar{V}}(\bar{\zeta}(t)) \le -\alpha \bar{V}(\bar{\zeta}(t)) + \gamma \bar{\omega}^T(t) \bar{\omega}(t),$$
(18)

where $\bar{V}(\bar{\zeta}(t)) = \bar{\zeta}^T(t)\bar{P}\bar{\zeta}(t)$ with $\bar{P} = diag\{P_1, P_2, \cdots, P_N\},$ $\bar{\omega}(t) = \left[(c_1\beta_1)^{\frac{1}{2}}\bar{d}_1^T(t) \ (c_2\beta_2)^{\frac{1}{2}}\bar{d}_2^T(t) \ \cdots \ (c_N\beta_N)^{\frac{1}{2}}\bar{d}_N^T(t)\right]^T,$ $\bar{\zeta}(t) = \left[(c_1\beta_1)^{\frac{1}{2}}\zeta_1^T(t) \ (c_2\beta_2)^{\frac{1}{2}}\zeta_2^T(t) \ \cdots \ (c_N\beta_N)^{\frac{1}{2}}\zeta_N^T(t)\right]^T.$ From the solution of differential inequality (18), one obtains

$$\bar{V}(\bar{\zeta}(t)) \leq e^{-\alpha t} \bar{V}(\bar{\zeta}(0)) + \gamma \int_{0}^{t} e^{-\alpha \tau} \bar{\omega}^{T}(t-\tau) \bar{\omega}(t-\tau) d\tau$$

$$\leq \sup_{\tau \in [0,t]} \left\{ e^{-\alpha t} \bar{V}(\bar{\zeta}(0)) + \frac{\gamma}{\alpha} \bar{\omega}^{T}(t-\tau) \bar{\omega}(t-\tau)(1-e^{-\alpha t}) \right\}, \quad (19)$$

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it follows that

$$\sup_{t \in [0,\infty)} \lambda_{\min}(\bar{P})\bar{\zeta}^{T}(t)\bar{\zeta}(t)
\leq \sup_{t \in [0,\infty)} \sup_{\tau \in [0,t]} \left\{ e^{-\alpha t}\bar{V}(\bar{\zeta}(0))
+ \frac{\gamma}{\alpha}\bar{\omega}^{T}(t-\tau)\bar{\omega}(t-\tau)(1-e^{-\alpha t}) \right\}
\leq \bar{V}(\bar{\zeta}(0)) + \frac{\gamma}{\alpha} \sup_{t \in [0,\infty)} \bar{\omega}^{T}(t)\bar{\omega}(t).$$
(20)

The above inequality implies that

$$\sup_{t \in [0,\infty)} \sum_{i=1}^{N} c_i \beta_i \zeta_i^T(t) \zeta_i(t)
\leq \frac{\lambda_{\max}(\bar{P})}{\lambda_{\min}(\bar{P})} \sum_{i=1}^{N} c_i \beta_i \zeta_i^T(0) \zeta_i(0)
+ \frac{\gamma}{\alpha \lambda_{\min}(\bar{P})} \sup_{t \in [0,\infty)} \sum_{i=1}^{N} c_i \beta_i \tilde{d}_i^T(t) \tilde{d}_i(t),$$
(21)

which implies that the system (8) satisfies the L_{∞} performance (9).

Left-multiplying and right-multiplying $diag\{P_i^{-1}, I\}$ to the inequality (15) yields

$$\begin{bmatrix} \tilde{A}_{i}P_{i}^{-1} + P_{i}^{-1}\tilde{A}_{i}^{T} + \rho_{i}^{-1}\tilde{L}_{i}C_{i}C_{i}^{T}\tilde{L}_{i}^{T} + a_{i}^{2}I + \alpha P_{i}^{-1} \\ & * \\ & * \\ \tilde{D}_{i} & P_{i}^{-1} \\ -\gamma I & 0 \\ * & -(\beta_{i}^{-1} + 1 + \beta_{i}^{-1}\varepsilon_{i} + \rho_{i})^{-1}I \end{bmatrix} < 0 \quad (22)$$

denote $P_i^{-1} = Q_i$ and $X_{il} = K_{il}Q_i$, the above inequality can be rewritten as (11).

Remark 3: The design condition in Theorem 1 is LMI, it avoids the non-convex problems which arise in the design of dynamic output feedback controller.

3.3. Coordinate transformation

Assumption 1 can be relaxed by finding a transformation of coordinates $x_i(t) = T_i z_i(t)$, the system (1) can be rewritten as follows,

$$\dot{z}_{i}(t) = T_{i}^{-1}(A_{i} - L_{i}C_{i})T_{i}z_{i}(t) + T_{i}^{-1}L_{i}y_{i}(t) + T_{i}^{-1}B_{i}u_{i}(t) + T_{i}^{-1}\Psi_{i}(Tz(t), y(t), \delta(t)) + T_{i}^{-1}D_{i}d_{i}(t), y_{i}(t) = C_{i}T_{i}z_{i}(t),$$
(23)

where $T = diag\{T_1, T_1, \dots, T_N\}$. Denote $T_i^{-1}(A_i - L_iC_i)T_i = \mathcal{A}_i, T_i^{-1}L_i = \mathcal{L}_i, T_i^{-1}B_i = \mathcal{B}_i,$ $T_i^{-1}D_i = \mathcal{D}_i, T_i^{-1} = R_i$, then (23) can be rewritten as

$$\dot{z}_i(t) = \mathcal{A}_i z_i(t) + \mathcal{L}_i y_i(t) + \mathcal{B}_i u_i(t) + \mathcal{D}_i d_i(t) + R_i \Psi_i(Tz(t), y(t), \delta(t))$$

$$y_i(t) = C_i T_i z_i(t). \tag{24}$$

From

$$T_i^+ \underline{z}_i(t) - T_i^- \overline{z}_i(t) \preceq T_i z_i(t) \preceq T_i^+ \overline{z}_i(t) - T_i^- \underline{z}_i(t),$$

$$T^+ z(t) - T^- \overline{z}(t) \preceq T z(t) \preceq T^+ \overline{z}(t) - T^- z(t),$$

it is clear that

$$\underline{\Phi}_{i}(\overline{z}(t),\underline{z}(t),y(t)) \leq R_{i}\Psi_{i}(Tz(t),y(t),\delta(t))$$
$$\leq \overline{\Phi}_{i}(\overline{z}(t),\underline{z}(t),y(t))$$

where

$$\begin{split} \overline{\Phi}_i(\overline{z},\underline{z},y) &= R_i^+ \overline{\Psi}_i(T^+ \overline{z} - T^- \underline{z},T^+ \underline{z} - T^- \overline{z},y) \\ &\quad -R_i^- \underline{\Psi}_i(T^+ \overline{z} - T^- \underline{z},T^+ \underline{z} - T^- \overline{z},y), \\ \underline{\Phi}_i(\overline{z},\underline{z},y) &= R_i^+ \underline{\Psi}_i(T^+ \overline{z} - T^- \underline{z},T^+ \underline{z} - T^- \overline{z},y) \\ &\quad -R_i^- \overline{\Psi}_i(T^+ \overline{z} - T^- \underline{z},T^+ \underline{z} - T^- \overline{z},y). \end{split}$$

Then, the following interval observer is designed for the system (24),

$$\underline{\dot{z}}_{i}(t) = \mathcal{A}_{i}\underline{z}_{i}(t) + \mathcal{L}_{i}y_{i}(t) + \mathcal{B}_{i}u_{i}(t) + \underline{\Phi}_{i}(\overline{z}(t),\underline{z}(t),y(t)) \\
+ \mathcal{D}_{i}^{+}\underline{d}_{i}(t) - \mathcal{D}_{i}^{-}\overline{d}_{i}(t,) \\
\underline{\dot{z}}_{i}(t) = \mathcal{A}_{i}\overline{z}_{i}(t) + \mathcal{L}_{i}y_{i}(t) + \mathcal{B}_{i}u_{i}(t) + \overline{\Phi}_{i}(\overline{z}(t),\underline{z}(t),y(t)) \\
+ \mathcal{D}_{i}^{+}\overline{d}_{i}(t) - \mathcal{D}_{i}^{-}\underline{d}_{i}(t).$$
(25)

Furthermore, the following controller is designed for the transformed interval observer system (25)

$$u_i(t) = \underline{\mathcal{K}}_i \underline{z}_i(t) + \overline{\mathcal{K}}_i \overline{z}_i(t).$$
(26)

In the new coordinates, the interval observer (25) is similar to (5), therefore, a similar conclusion can be obtained.

Based on the above analysis and design, an algorithm is given to design the interval observer-based output feedback controller.

Algorithm 1: Interval observer-based output feedback controller design

- **Step 1:** If there exists a matrix L_i such that $A_i L_iC_i$ is Metzler and Hurwitz, go to Step 3 with the selected interval observer gain L_i . Otherwise, go to Step 2.
- **Step 2:** Choose the free matrices S_i , Y_i such that the matrix S_i is Metzler and Hurwitz. Solve the Sylvester equation $S_iT_i^{-1} T_i^{-1}A_i + Y_iC_i = 0$ with respect to the unknown transformation matrix T_i , then $L_i = T_iY_i$. Transform the system (1) into (23) by using $x_i(t) = T_iz_i(t)$.
- **Step 3:** Design the interval observer with the obtained gain matrix L_i .
- **Step 4:** Solve the LMI conditions to get the observerbased controller gain K_i .

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4. SIMULATION

In this section, a simulation example is presented to demonstrate the effectiveness of the proposed method. Consider the following system,

$$\dot{x}_{i}(t) = A_{i}x_{i}(t) + B_{i}u_{i}(t) + \Psi_{i}(y(t), \delta(t)) + D_{i}d_{i}(t),$$

$$y_{i}(t) = C_{i}x_{i}(t),$$
(27)

where $A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ 9.81 & 0 \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$, $D_1 = D_2 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}$, $C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $\Psi_1(y(t), \delta(t)) = \Psi_2(y(t), \delta(t)) = \begin{bmatrix} 0 \\ \delta(t)(y_2 - y_1) \end{bmatrix}$. The uncertain parameter $\delta(t)$ satisfies the inequality $|\delta(t)| \le \overline{\delta}$, then the interconnection functions satisfy

$$\underline{\Psi}_{1}(y(t)) \leq \Psi_{1}(y(t), \ \delta(t)) \leq \Psi_{1}(y(t)),$$

$$\underline{\Psi}_{2}(y(t)) \leq \Psi_{2}(y(t), \ \delta(t)) \leq \overline{\Psi}_{2}(y(t)),$$

where $\underline{\Psi}_1(y(t)) = -\overline{\delta}(||y_1|| + ||y_2||)$ and $\overline{\Psi}_1(y(t)) = \overline{\delta}(||y_1|| + ||y_2||)$ satisfy the inequalities $||\underline{\Psi}_1(y(t))|| \le a_{13}||y(t)||$ and $||\overline{\Psi}_1(y(t))|| \le a_{16}||y(t)||$ with $a_{13} = a_{16} = \sqrt{2}\overline{\delta}$. For simulation, the value of the uncertain parameter has been chosen as $\delta(t) = 0.3535 \sin(t), d(t) = \cos(\pi t) + 0.5 \sin(\pi t), \underline{d}(t) = \cos(\pi t) - 0.5, \overline{d}(t) = \cos(\pi t) + 0.5$.

Based on the Algorithm 1, choosing the free matrices $S_i = \begin{bmatrix} -0.9 & 10 \\ 0 & -0.95 \end{bmatrix}$, $Q_i = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$ and solving the Sylvester equation $S_i T_i^{-1} - T_i^{-1} A_i + Q_i C_i = 0$, one gets

$$T_i = \begin{bmatrix} 1.7822 & 0.1356 \\ 1.6931 & 17.9439 \end{bmatrix}, \ L_i = \begin{bmatrix} 1.8500 \\ 10.6650 \end{bmatrix}.$$

Given the constants $\rho_i = 0.2, \alpha = 0.3, \varepsilon_i = 0.1, \beta_i = 0.0455$, solving the LMI condition, we get the controller gain matrices

$$\underline{K}_1 = \underline{K}_2 = \begin{bmatrix} -2993 & -2496.6 \end{bmatrix},$$

$$\overline{K}_1 = \overline{K}_2 = \begin{bmatrix} -3021.6 & -1183.4 \end{bmatrix}.$$

Simulation results are shown in Figs. 2-3, it can be seen that the interval relations hold, furthermore, both the system states and the interval observer states achieve the good steady performance, which further verifies that the proposed interval observer-based output feedback controller achieves desired control performance for interconnected systems.

5. CONCLUSION

In this paper, the interval observer-based output feedback control scheme for a class of interconnected systems with uncertain interconnections has been proposed.



Fig. 2. State responses of the subsystem S_1 .



Fig. 3. State responses of the subsystem S_2 .

For each subsystem, the output feedback controller design problem for original system is transformed to the state feedback problem for observer system. The cyclicsmall gain condition is introduced to formulate the problem to LMIs. The closed-loop system stability is guaranteed by coordinate transformation and interval property. The achieved method has been finally demonstrated via simulation results. Furthermore, by reference to the existing results [36–42], a direction for the future is to extend the proposed method to the stochastic interconnected systems.

REFERENCES

- P. Ioannou, "Decentralized adaptive control of interconnected systems," *IEEE Transactions on Automatic Control*, vol. 31, no. 4, pp. 291-298, April 1986.
- [2] D. D. Šljak, Decentralized Control of Complex Systems, Academic Press, San Diego, 1991.
- [3] W. J. Wang and W. W. Lin, "Decentralized PDC for largescale TS fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 6, pp. 779-786, December 2005.
- [4] Y. H. Chen, "Decentralized robust control system design for large-scale uncertain systems," *International Journal of Control*, vol. 47, no. 5, pp. 1195-1205, January 1988.
- [5] K. K. Shyu, W. J. Liu, and K. C. Hsu, "Design of largescale time-delayed systems with dead-zone input via variable structure control," *Automatica*, vol. 41, no. 7, pp. 1239-1246, July 2005.
- [6] T. N. Lee and U. L. Radovic, "Decentralized stabilization of linear continuous and discrete-time systems with delays in interconnections," *IEEE Transactions on Automatic Control*, vol. 33, no. 8, pp. 757-761, August 1988.
- [7] S. N. Huang, K. K. Tan, and T. H. Lee, "Decentralized control of a class of large-scale nonlinear systems using neural networks," *Automatica*, vol. 41, no. 9, pp. 1645-1649, September 2005.
- [8] H. M. Wang and G. H. Yang, "Decentralized state feedback control of uncertain affine fuzzy large-scale systems with unknown interconnections," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 5, pp. 1134-1146, October 2016.
- [9] Y. Zhu, F. Yang, C. Li, and Y. Zhang, "Simultaneous stability of large-scale systems via distributed control network with partial information exchange," *International Journal* of Control, Automation and Systems, vol. 16, no. 4, pp. 1502-1511, August 2018.
- [10] D. Zhang, P. Shi, and Q. G. Wang, "Energy-efficient distributed control of large-scale systems: A switched system approach," *International Journal of Robust and Nonlinear Control*, vol. 26, no. 14, pp. 3101-3117, September 2016.
- [11] M. Guinaldo, D. V. Dimarogonas, K. H. Johansson, J. Sánchez, and S. Dormido, "Distributed event-based control strategies for interconnected linear systems," *IET Control Theory & Applications*, vol. 7, no. 6, pp. 877-886, April 2013.

- [12] G. B. Koo, J. B. Park, and Y. H. Joo, "Robust fuzzy controller for large-scale nonlinear systems using decentralized static output-feedback," *International Journal of Control, Automation and Systems*, vol. 9, no. 4, pp. 649-658, August 2011.
- [13] Y. F. Xie, W. H. Gui, Z. H. Jiang, and C. Huang, "Decentralized robust H_∞ output feedback control for value bounded uncertain large-scale interconnected systems," *International Journal of Control, Automation and Systems*, vol. 8, no. 1, pp. 16-28, February 2010.
- [14] D. Zhai, L. W. An, J. X. Dong, and Q. L. Zhang, "Output feedback adaptive sensor failure compensation for a class of parametric strict feedback systems," *Automatica*, vol. 97, pp. 48-57, November 2018.
- [15] D. Zhai, L. W. An, J. X. Dong, and Q. L. Zhang, "Switched adaptive fuzzy tracking control for a class of switched nonlinear systems under arbitrary switching," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 2, pp. 585-597, April 2018.
- [16] J. Huang, "Adaptive distributed observer and the cooperative control of multi-agent systems," *Journal of Control and Decision*, vol. 4, no. 1, pp. 1-11, November 2017.
- [17] F. Mazenc and O. Bernard, "Asymptotically stable interval observers for planar systems with complex poles," *IEEE Transactions on Automatic Control*, vol. 55, no. 2, pp. 523-527, February 2010.
- [18] F. Mazenc and O. Bernard, "Interval observers for linear time-invariant systems with disturbances," *Automatica*, vol. 47, no. 1, pp. 140-147, January 2011.
- [19] F. Mazenc, T. N. Dinh, and S. I. Niculescu, "Interval observers for discrete-time systems," *Proceedings of the 51st IEEE Conference on Decision and Control*, Maui, Hawaii, USA, pp. 6755-6760, 2012.
- [20] F. Mazenc, T. N. Dinh, and S. I. Niculescu, "Interval observers for discrete-time systems," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 17, pp. 2867-2890, November 2014.
- [21] D. Efimov, W. Perruquetti, T. Raissi, and A. Zolghadri, "Interval observers for time-varying discrete-time systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 12, pp. 3218-3224, December 2013.
- [22] S. Chebotarev, D. Efimov, T. Raissi, and A. Zolghadri, "Interval observers for continuous-time LPV systems with L_1/L_2 performance," *Automatica*, vol. 58, pp. 82-89, August 2015.
- [23] D. Efimov, T. Raissi, and A. Zolghadri, "Control of nonlinear and LPV systems: Interval observer-based framework," *IEEE Transactions on Automatic Control*, vol. 58, no. 3, pp. 773-778, March 2013.
- [24] X. Cai, G. Lv, and W. Zhan, "Stabilisation for a class of non-linear uncertain systems based on interval observers," *IET Control Theory & Applications*, vol. 6, no. 13, pp. 2057-2062, September 2012.
- [25] A. Polyakov, D. Efimov, W. Perruquetti, and J. P. Richard, "Output stabilization of time-varying input delay systems using interval observation technique," *Automatica*, vol. 49, no. 11, pp. 3402-3410, November 2013.

- [26] Z. W. He and W. Xie, "Control of non-linear switched systems with average dwell time: Interval observer-based framework," *IET Control Theory & Applications*, vol. 10, no. 1, pp. 10-16, January 2016.
- [27] K. Kalsi, J. Lian, and S. H. Zak, "Decentralized dynamic output feedback control of nonlinear interconnected systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 8, pp. 1964-1970, August 2010.
- [28] S. C. Tong, H. X. Li, and G. R. Chen, "Adaptive fuzzy decentralized control fora class of large-scale nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B Cybernetics*, vol. 34, no. 1, pp. 770-775, February 2004.
- [29] Y. S. Huang, D. Q. Zhou, and X. X. Chen, "Decentralized direct adaptive output feedback fuzzy H_∞ tracking design of large-scale nonaffine nonlinear systems," *Nonlinear Dynamics*, vol. 58, no. 1, pp. 153-167, October 2009.
- [30] J. Li, W. S. Chen, and J. M. Li, "Adaptive NN outputfeedback decentralized stabilization for a class of largescale stochastic nonlinear strict-feedback systems," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 4, pp. 452-472, March 2011.
- [31] S. C. Tong, Y. M. Li, and H. G. Zhang, "Adaptive neural network decentralized backstepping output-feedback control for nonlinear large-scale systems with time delays," *IEEE Transactions on Neural Networks*, vol. 22, no. 7, pp. 1073-1086, July 2011.
- [32] W. S. Chen and J. M. Li, "Decentralized output-feedback neural control for systems with unknown interconnections," *IEEE Transactions on Systems, Man, and Cybernetics, Part B Cybernetics*, vol. 38, no. 1, pp. 258-266, February 2008.
- [33] X. J. Li and G. H. Yang, "Neural-network-based adaptive decentralized fault-tolerant control for a class of interconnected nonlinear systems," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 1, pp. 144-155, January 2018.
- [34] Y. Jiang and Z. P. Jiang, "Robust adaptive dynamic programming for large-scale systems with an application to multimachine power systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 59, no. 10, pp. 693-697, October 2012.
- [35] H. M. Wang and G. H. Yang, "Decentralized dynamic output feedback control for affine fuzzy large-scale systems with measurement errors," *Fuzzy Sets and Systems*, vol. 314, pp. 116-134, May 2017.
- [36] H. Shen, F. Li, S. Y. Xu, and V. Sreeram, "Slow state variables feedback stabilization for semi-Markov jump systems with singular perturbations," *IEEE Transactions on Automatic Control*, vol. 63, no. 8, pp. 2709-2714, August 2018.

- [37] W. H. Qi, J. H. Park, J. Cheng, Y. G. Kao, and X. W. Gao, "Anti-windup design for stochastic Markovian switching systems with mode-dependent time-varying delays and saturation nonlinearity," *Nonlinear Analysis: Hybrid Systems*, vol. 26, pp. 201-211, November 2017.
- [38] W. H. Qi, G. D. Zong, and H. R. Karim, "Observerbased adaptive SMC for nonlinear uncertain singular semi-Markov jump systems with applications to DC motor," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 65, no. 9, pp. 2951-2960, September 2018.
- [39] W. H. Qi, G. D. Zong, and H. R. Karim, "ℒ_∞ control for positive delay systems with semi-Markov process and application to a communication network model," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 3, pp. 2081-2091, March 2019.
- [40] H. Shen, S. C. Huo, Cao, J. D. Cao, and T. W. Huang, "Generalized state estimation for Markovian coupled networks under round-robin protocol and redundant channels," *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2018.2799929, 2018 (in press).
- [41] L. W. Li and G. H. Yang, "Decentralized stabilization of Markovian jump interconnected systems with unknown interconnections and measurement errors," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 6, pp. 2495-2512, April 2018.
- [42] H. Shen, F. Li, H. C. Yan, H. R. Karimi, and H. K. Lam, "Finite-time event-triggered H_{∞} control for T-S fuzzy Markov jump systems," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 5, pp. 3122-3135, October 2018.



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