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# Robust Asymptotic and Finite-time Tracking for Second-order Nonlinear Multi-agent Autonomous Systems

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**Abstract:** This paper investigates consensus based distributed robust asymptotic and finite-time tracking control strategy for second-order multi-agent autonomous systems. The protocol design uses states of the neighboring agents with directed communication topology in the presence of uncertainty associated with the autonomous agents. Robust adaptive learning algorithm uses with the protocol design for each follower agent to learn and adapt bounded uncertainty associated with nonlinear dynamics of the follower agents. Adaptive learning protocol also integrates with the follower agents protocol to learn and adapt bounded input of the leader. Lyapunov method with Graph, classical sliding mode, and terminal sliding mode theory use to guarantee that the proposed distributed control design can reach an agreement and follow the states of the leader in both finite-time and asymptotic sense. Analysis shows that consensus based protocol can force the states of the followers sliding surface to track the states of the leader sliding surface in finite-time and remain there. The proposed distributed consensus protocol does not require the exact bound of the uncertainty associated with the follower agents. Also, the proposed protocol does not require the exact bound of the leader input as opposed to other distributed cooperative control designs. Evaluation results with comparison are presented to demonstrate the validity of the theoretical argument for the real-time applications.

**Keywords:** Asymptotic and finite-time consensus control, linear sliding mode control, multi-agent autonomous systems, nonlinear terminal sliding mode control, robust adaptive control.

## 1. INTRODUCTION

Recently, research on the development of distributed cooperative control for multi-agent systems have gained significant attention by many researchers. The motivation behind distributed cooperative control for multi-agent systems is because of its wide variety of civilian and military applications such as satellite formation flying [1], swarming of aerial vehicles for radiation mapping [2], automated highway scheduling system [3], air traffic control [4], sensor network and cooperative surveillance [5–8]. This work emphasizes on the development of the consensus based distributed control algorithm design for multi-agent systems. The main objective in consensus design is to develop distributed cooperative protocol with the neighboring states information such that the states of all the agents can reach an agreement to perform desired task. Most recent results on this area for the first and second order multi-agent systems can be found in [9–13], and others.

Authors in [9, 10], provided framework to analysis consensus problem for the first-order multi-agent systems. Authors showed that agents can reach an agreement only if the topology of the interaction digraph is strongly connected. In [11], authors studied consensus problem for the second-order multi-agent systems examining the connection between communication topologies and controllability of the systems.

Authors in [12] also analyzed convergence condition of the states for multi-agent systems based on using simple linear protocols. Various consensus tracking protocols were presented for both first-order and second-order multi-agent systems in [13].

In view of the design and analysis, it can be seen that the existing algorithms can ensure asymptotic consensus property of the states of the multi-agent systems by using the dynamical model of the agents. However, in many practical applications, the convergence of the states consensus may require to achieve faster and in finite-time. To solve the finite-time consensus problem, various consensus based distributed cooperative protocols have been proposed for both first and second order multi-agent systems in the literature. Jiang and Wang in [14] showed that nonlinear interaction can be used to achieve finite-time consensus for first-order agents under both fixed and switch-

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ing topology. In [15], authors proposed protocol for single integrator to achieve finite-time consensus with connected interaction topology. Wang and Xiao developed framework for designing finite-time consensus laws for first-order multi-agent system. Many of these above mentioned results achieved finite-time consensus by assuming that the interaction topology among agents is undirected graph. Authors in [16–21], developed finite-time consensus protocols for second-order multi-agent systems. Authors showed in these designs that their methods can ensure fine-time stability based on using the bounds of the uncertainty associated with the nonlinear dynamic of the follower agents. Recently, distributed finite-time containment control design introduced for double integrator multi-agent systems in [22, 23]. However, there have been no results reported in the literature where both robust asymptotic and finite-time consensus tracking control designs were studied for second-order multi-agent systems with the presence of dynamical uncertainty.

In this work, both robust asymptotic and finite-time tracking problem is investigated for second-order nonlinear multi-agent autonomous system. The proposed protocol design for asymptotic and finite-time design only requires the states of the neighboring agents with directed communication topology. Robust adaptive learning protocol is employed for each follower agent to learn and adapt uncertainty associated with dynamical model of the follower agents. The protocol design also uses adaptive learning term to learn and adapt with the bound of the leader input. Using Graph, Lyapunov and sliding mode control theory, asymptotic and finite-time consensus convergence is given for the closed loop system formulated by the leader-followers multi-agent autonomous systems. The convergence analysis shows that consensus protocol can force the states of the follower agents to track the state of the leader in both finite-time and asymptotic sense. It is shown in convergence analysis that the distributed consensus can be ensured on the sliding mode surface if the directed communication topology has a directed spanning tree. The proposed protocol design does not require the exact bound of the leader input. Also, the protocol design does not need the exact bound of the uncertainty associated with the follower agent dynamics. Evaluation results with comparative studies are given to demonstrate the validity of the theoretical development for the real-time applications.

The rest of the paper is organized as follows: Section 2 provides the Graph theory, assumptions, lemmas and dynamical model related to design and analysis of the proposed method. Section 3 presents protocol design and convergence analysis for distributed asymptotic consensus tracking for second-order multi-agent autonomous systems. Section 3 also gives protocol design and convergence analysis for the distributed finite-time consensus tracking for second-order multi-agent autonomous systems. Evaluation results are presented in Section 4. Finally, the paper is concluded with future works in Section 5.

## 2. PRELIMINARIES, LEMMAS, ASSUMPTIONS AND DYNAMICAL MODEL

Graph theory uses to model the information exchange between leader and follower agents to solve the coordination problems in networked based autonomous agents [10]. The information exchanged topology between multiple agents, $N$, can be modeled by weighted graph as $\mathcal{G} = \{V, E, A\}$. $V = \{v_i, i \in \Omega\}$ is the set of nodes with agents belong to a finite set $\Omega = \{1, \ldots, N\}$. $E \subseteq V \times V$ is the set of edge, $A$ is the weighted adjacent matrix of the graph $\mathcal{G}$ with nonnegative elements $a_{ij}$. If there is a direct edge from vertex $j$ to vertex $i$ as $(v_j, v_i) \in E$, then agent $i$ can receive information from agent $j$ as $a_{ij} > 0$, otherwise $a_{ij} = 0$. It is assumed that $a_{ii} = 0$ for all $i \in \Omega$. The set of neighbors of agent $i$ is defined by $\mathcal{Y}_i = \{j \mid (v_i, v_j) \in E\}$. Then, the Laplacian matrix of directed graph $\mathcal{G}$ can be defined as

$$\mathcal{M} = \Big(K - A\Big) \tag{1}$$

with the degree matrix $K = diag\{K_1, \ldots, K_N\}$ and $K_i = \sum_{j \in \mathcal{Y}_i} a_{ij}$. For directed Graph, the interaction between nodes $v_i$ and $v_{ik}$ is the sequence of $\{v_{i1}, \ldots, v_{ik}\}$ as $(v_{i_l}, v_{i_{l+1}}) \in E$ with $l = 1, \ldots, k-1$. In this work, the digraph is assumed to be strongly connected by assuming that every vertex can be reachable from every other vertex. It also considers that the digraph has a directed spanning tree provided that there exist at least one node which has a directed path to all the other nodes through directed path. The leader-following networked of multi-agent autonomous systems is composed of $N+1$ agents as labeled as $0, \ldots, N$. The leader is labeled by 0 while followers agents are labeled from 1 to $N$. The interaction between $i$-th follower agent and the leader can be denoted by $c_i$. If there is an edge between the $i$-th follower and leader, then the connection weight $c_i$ is $c_i > 0$. Otherwise, $c_i = 0$. The followers interaction with the leader can be defined by a matrix $P = diag(c_1, \ldots, c_N)$. In this work, a class of second-order nonlinear multi-agent autonomous systems is considered that composed of a leader and follower agents. It is assumed that the agents can communicate by their states via local communication networks. The dynamics of the follower agents can be written as

$$\dot{x}_i = v_i, \dot{v}_i = \mathcal{F}_i(x_i, v_i, t) + u_i, \tag{2}$$

where $v_i \in \Re^m$, $x_i \in \Re^m$, $\mathcal{F}_i(x_i, v_i, t)$ and $u_i$ are the velocity, position, unknown bounded nonlinear function and control input for $i$-th follower agent, respectively. The dynamics of the leader can be modeled as

$$\dot{x}_o = v_o, \quad \dot{v}_o = u_o, \tag{3}$$

where $v_o \in \Re^m$, $x_o \in \Re^m$ and $u_o$ are the velocity, position and control input for the leader, respectively. It is assumed that the state of the leader is active and time varying. Also, the dynamic behavior of the leader is independent of the follower agents. Then, the consensus error function can be defined as

$$e_{1i} = \sum_{j=1}^{N} a_{ij}(x_i - x_j) + (x_i - x_o),$$
$$e_{2i} = \sum_{j=1}^{N} a_{ij}(v_i - v_j) + (v_i - v_o) \quad (4)$$

with $i \in \Omega$. Now, define new variable $E_p = [e_{11}, ..., e_{1_N}]^T$ and $E_v = [e_{21}, ..., e_{2_N}]^T$. Then, the consensus error function can be rewritten in the following form

$$E_p = \Big(\mathcal{M} + P\Big) \otimes I_m \tilde{\mathcal{X}}, E_v = \Big(\mathcal{M} + P\Big) \otimes I_m \tilde{\mathcal{V}} \quad (5)$$

with $\tilde{x}_i = (x_i - 1_N \otimes x_o)$, $\tilde{v}_i = (v_i - 1_N \otimes v_o)$, $\tilde{\mathcal{X}} = (\tilde{x}_1, ..., \tilde{x}_N)^T$, $\tilde{\mathcal{V}} = (\tilde{v}_1, ..., \tilde{v}_N)^T$ and $1_N = [1, ..., 1]^T$. The time derivative of the above consensus error (5) be expressed as

$$\dot{E}_p = E_v, \quad (6)$$
$$\dot{E}_v = \Big(\mathcal{M} + P\Big) \otimes I_m \Big[F(x, v, t) - 1_N \otimes u_o + u\Big] \quad (7)$$

with $F(x, v, t) = [\mathcal{F}(x_1, v_1, t), ..., \mathcal{F}(x_N, v_N, t)]^T$. Before presenting the design and convergence analysis, the following assumptions are considered.

**Assumption 1:** The time varying input of the leader $u_o$ is continuous and bounded with $\pi_o > 0$ as $\|u_o\| \le \pi_o$.

**Assumption 2:** The position and velocity states of the leader dynamic is assumed to be bounded.

**Assumption 3:** The nonlinear dynamical functions $\mathcal{F}(x_i, v_i, t)$ for the follower agents are assumed to be continuous and bounded satisfies the following inequality

$$\|\mathcal{F}(x, v, t)\| \le \eta_1 \|\tilde{\mathcal{X}}\| + \eta_2 \|\tilde{\mathcal{V}}\| + \eta_o, \quad (8)$$

where $\eta_o = (\eta_{o1} + \eta_{02})$ with $\eta_{o1} = \eta_1 \alpha_o$ , $\eta_{o2} = \eta_2 \beta_o$, $\alpha_o \le \|x_o\|$, $\beta_o \le \|v_o\|$, $\eta_o > 0$, $\eta_1 > 0$, $\eta_2 > 0$, $\alpha_o > 0$ and $\beta_o > 0$.

The proposed design and stability analysis is based on the following Lemmas.

**Lemma 1** [24]**:** Consider the nonlinear function $\dot{g} = g(x)$ with $g(0) = 0$. Then, there exist continuous differentiable function $V_g(x)$ defined closed to the neighborhood of the origin such that $V_g(x(t))$ is positive definite and $\dot{V}_g(x(t)) \le -\gamma_g V_g^{\alpha_g}(x(t))$ with $\gamma_g > 0$ and $0 < \alpha_g < 1$. Then, the origin is finite-time stable and the function $V_g(x(t))$ reaches to zero in finite-time $t_f \le \frac{V_g(x(0))^{(1-\alpha_g)}}{\gamma_g(1-\alpha_g)}$ for all $t \ge t_f$.

**Lemma 2** [19, 20]**:** The digraph $\mathcal{G} = \{V, E, A\}$ is assumed to have a directed spanning tree provided that $\{V, E, A\}$ has at least one root-node agent which has access to the leader trajectory. When the graph has a directed spanning tree, then the matrix $(\mathcal{M} + P)$ is invertible.

**Lemma 3** [20, references there in]**:** If $z_1, ..., z_N \ge 0$ and $0 < a \le 1$, then $\Big(\sum_{i=1}^{N} z_i\Big)^a \| \le \sum_{i=1}^{N} z_i^a$.

**Remark 1:** Note that Assumption 1 is flexible and realistic as the input signals are usually bounded in real-time applications. Assumptions 2 and 3 are also realistic as the states of the nonlinear functions $\mathcal{F}(x_i, v_i, t)$ and the leader states are assumed to be continuous and bounded over the given compact sets.

## 3. MAIN RESULTS AND ANALYSIS

In this section, we develop distributed consensus protocols for achieving asymptotic and finite-time consensus tracking for the networked of nonlinear leader-follower multi-agent autonomous systems with classical and terminal sliding model mechanism. The contribution of this work can divided into three parts. First, the paper presents both asymptotic and finite-time consensus tracking algorithm with comparative studies for the second-order nonlinear leader-follower multi-agent autonomous agents by using classical and nonlinear terminal sliding mode mechanism. Second, the proposed consensus tracking protocol design does not require the exact bound of the uncertainty associated with the follower agents dynamic. Third, protocol design for the follower agents does not require the exact bound of the input of the leader.

### 3.1. Algorithm design and convergence analysis for asymptotic consensus tracking

This section develops consensus protocol for achieving asymptotic consensus tracking property for the networked of nonlinear multi-agent autonomous systems by using linear sliding mode dynamics. The protocol needs to ensure asymptotic consensus property as defined as $\lim_{t\to\infty} \|x_i(t) - x_o(t)\| = 0, \lim_{t\to\infty}$ $\|v_i(t) - v_o(t)\| = 0$. Then, present the following Theorem 1 based on classical sliding mode dynamics.

**Theorem 1 :** Consider Assumptions 1 to 3 holds. Then, if the interaction graph is directed and have a directed spanning tree, then there exist sliding mode vectors $S = E_p + \alpha E_v$ with $\alpha > 0$ and protocol $u = (u_l + u_u + u_p)$

$$u_p = -\alpha^{-1} (\mathcal{M} + P)^{-1} \otimes I_m \Big( \big(b \otimes \hat{\Theta}_o\big) Sgn(S) \Big),$$
$$u_u = -\alpha^{-1} (\mathcal{M} + P)^{-1} \otimes I_m \left( Y_F \hat{\Theta}_F \right),$$
$$u_l = -\alpha^{-1} \Big( (\mathcal{M} + P)^{-1} \otimes I_m \Big) (E_v + k_s S),$$
$$\dot{\hat{\Theta}}_F = -\Gamma_F Y_F^T S, \dot{\hat{\Theta}}_o = -\Gamma_o Sgn^T(S) b^T S \quad (9)$$

with $b = P 1_N$, $k_s > 0$, symmetric positive-definite matrix $\Gamma_{F_i} \in \Re^{3\times 3}$ and $\Gamma_{oi} > 0$ such that the network of the leader-follower multi-agent systems (2)-(3) achieve consensus tracking asymptotically.

**Proof:** The proof can be divided into three parts. In first part, it proves the upper bounded of the parameter estimates. Second, it shows that each sliding mode vector can reach the sliding surface in finite-time. Finally, analysis prove that the states of the multi-agent autonomous systems can achieve consensus tracking in finite time. First, define the following linear classical sliding mode variable as

$$s_i = e_{1i} + \alpha e_{2i}. \tag{10}$$

The sliding dynamics can be written in compact form as

$$S = E_p + \alpha E_v \tag{11}$$

with $S = (s_1, ..., s_n)^T$. Applying (6)-(7), the derivative of $\dot{S}$ has the following form

$$\dot{S} = E_v + \mathcal{A}\left[F(x,v,t) - 1_n \otimes u_o + u\right] \tag{12}$$

with $\mathcal{A} = \alpha\Big((\mathcal{M}+P)\otimes I_m\Big)$. To prove the first part of Theorem 1, the following Lyapunov function candidate is considered

$$V = \frac{S^T S}{2} + \frac{\tilde{\Theta}_F^T \Gamma_F^{-1} \tilde{\Theta}_F}{2} + \frac{\Gamma_o^{-1} \tilde{\Theta}_o^T \tilde{\Theta}_o}{2}, \tag{13}$$

where $\tilde{\Theta}_F = \big(\Theta_F - \hat{\Theta}_F\big)$ and $\tilde{\Theta}_o = \big(\Theta_o - \hat{\Theta}_o\big)$. Take the derivative along the trajectory of (12). Then, using Assumptions 1 and 2, $\dot{V}$ can derived as

$$\begin{aligned}\dot{V} =& S^T\Bigg[E_v + \alpha\Big((\mathcal{M}+P)\otimes I_m\Big)\Big(Y_F\Theta_F + u \\ & - 1_n \otimes \Theta_o\Big)\Bigg] + \tilde{\Theta}_F^T \Gamma_F^{-1} \dot{\tilde{\Theta}}_F + \Gamma_o^{-1} \tilde{\Theta}_o^T \dot{\tilde{\Theta}}_o \end{aligned} \tag{14}$$

with $Y_F = [Sgn(S),\, \tilde{\mathcal{X}},\, \tilde{\mathcal{V}}]$, $Sgn(S) = [Sgn(s_1), \ldots, Sgn(s_n)]^T$, $\Theta_{Fi} = [\eta_o,\, \eta_1,\, \eta_2]^T$, $\Theta_{oi} = \pi_o$ and $u = (u_l + u_u + u_p)$. In view of (9), $\dot{V}$ can be simplified as $\dot{V} = -k_s \sum_{i=1}^{n} \|s_i\|^2$. Then, it is possible to conclude from the Lyapunov stability theorem that all signals in the closed-loop system are bounded. The parameter estimates are bounded such that there exists positive constants $\Theta_F^*$ and $\Theta_o^*$ such that $\hat{\Theta}_F \le \Theta_F^*$ and $\hat{\Theta}_o \le \Theta_o^*$ for $\forall t \ge 0$. This ensures the upper boundedness of the parameter estimates and estimation errors. Second step is to prove that the sliding mode motion occurs within a finite-time. To show that, the following Lyapunov function candidate is considered

$$V = \frac{1}{2} S^T S + \frac{\tilde{\Theta}_F^T \gamma_{F1}^{-1} \tilde{\Theta}_F}{2} + \frac{\gamma_{o1}^{-1} \tilde{\Theta}_o^T \tilde{\Theta}_o}{2}, \tag{15}$$

where $\gamma_{F1i}^{-1} \in \Re^{3\times 3}$ is a symmetric positive-definite matrix and $\gamma_{o1i}^{-1} > 0$. Using (9), $\dot{V}$ can be simplified as

$$\begin{aligned}\dot{V} =& \sqrt{2}\mathcal{D}\frac{\|S\|}{\sqrt{2}} - \sqrt{2\gamma_{F1}}\Big(\varepsilon_1 - \|S\|Y_F\Big)\frac{\big(\|\Theta_F^* - \hat{\Theta}_F\|\big)}{\sqrt{2\gamma_{F1}}} \\ & - \sqrt{2\gamma_{o1}}\Big(\varepsilon_2 - \|S\|b\Big)\frac{\big(\|\Theta_o^* - \hat{\Theta}_o\|\big)}{\sqrt{2\gamma_{o1}}}\end{aligned} \tag{16}$$

with $\mathcal{D} = \Big(Y_F\left(\Theta_F^* - \Theta_F\right) + b\left(\Theta_o^* - \Theta_o\right)\Big)$, $\varepsilon_1 = \gamma_{F1}^{-1}\Gamma_F Y_F^T \|S\|$ and $\varepsilon_2 = \gamma_{o1}^{-1}\Gamma_o b^T \|S\|$. Now, define $\Pi_1 = \Big(Y_F\left(\Theta_F^* - \Theta_F\right) + b\left(\Theta_o^* - \Theta_o\right)\Big)$, $\Pi_2 = \sqrt{\gamma_{F1}}\Big(\varepsilon_1 - \|S\|Y_F\Big)$, $\Pi_3 = \sqrt{\gamma_{o1}}\Big(\varepsilon_2 - \|S\|b\Big)$. Using $\Pi = \min\{\Pi_1, \Pi_2, \Pi_3\}$, $\dot{V}$ can be written as $\dot{V} \le -\sqrt{2}\Pi\mathcal{K}_L$ with $\mathcal{K}_L = \Big(\frac{\|S\|}{\sqrt{2}} + \frac{(\|\Theta_F^* - \hat{\Theta}_F\|)}{\sqrt{2\gamma_{F1}}} - \frac{(\|\Theta_o^* - \hat{\Theta}_o\|)}{\sqrt{2\gamma_{o1}}}\Big)$. Now, it is possible to select $\Theta_o^* > \Theta_o$, $\Theta_F^* > \Theta_F$, $\gamma_{o1}^{-1} < \Gamma_o^{-1}$ and $\|\gamma_{F1}^{-1}\| < \|\Gamma_F^{-1}\|$ so that $\Pi > 0$, $\Pi_1 > 0$, $\Pi_2 > 0$ and $\Pi_3 > 0$. Then, applying Lemma 3, $\dot{V}$ can be written as $\dot{V} = -\sqrt{2}\Pi V^{\frac{1}{2}}$. Using $\Pi > 0$ and Lemma 1, it can be stated that the states of the closed systems with the proposed consensus protocol can reach the sliding surface $S = 0$ in finite-time depending on the initial state of the sliding surface $S(0)$ and remain on this sliding surface for all time with bounded dynamical uncertainty. Third step is show that the state of the followers can reach consensus and track the states of the leader on the sliding surface in the presence of uncertainty. To do that, the following Lyapunov function candidate is chosen as $V_o = \frac{E_p E_p}{2}$. On $S = 0$, we have $E_v = -\alpha E_p$. Then, the time derivative of $V_o$ can be written as $\dot{V}_o = -\alpha E_p^T E_p$. In view of $V_o$, $\dot{V}_o$ can be simplified as $\dot{V}_o \le -2\alpha V_o$. Then, applying Lemma 2, $E_p \to 0$ as $t \to \infty$. Also, on the sliding surface $S = 0$, $E_v \to 0$ as $t \to \infty$ as $E_v = -\alpha E_p$. This implies that the leader-follower multi-agent autonomous systems with the proposed protocol can reach an agreement and achieve asymptotic consensus tracking property. □

### 3.2. Algorithm design and convergence analysis for finite-time consensus tracking

In this section, distributed consensus protocol is developed for achieving finite-time tracking for the networked of nonlinear leader-follower multi-agent autonomous systems by using nonsingular terminal sliding model control theory. This means that, for any initial conditions, there exist a constant $t^*$ such that the finite-time consensus tracking can be achieved as $\lim_{t\to t^*} \|x_i(t) - x_o(t)\| = 0$, $\lim_{t\to t^*} \|v_i(t) - v_o(t)\| = 0$. Then, the nonlinear terminal sliding mode based finite-time consensus tracking is presented in Theorem 2.

**Theorem 2:** Consider the uncertain nonlinear leader-follower multi-agent autonomous systems (2)-(3) under

Assumptions 1 to 3. If the interaction graph is directed and have a directed spanning tree, then there exists nonsingular terminal sliding vectors $S = E_p + k_1 E_v^{\varepsilon} Sgn(E_v)$ and $u = (u_p + u_u + u_l + u_s)$ with

$$\begin{aligned}
u_p &= \left(k_1\varepsilon\right)^{-1}\left(\mathcal{M}+P\right)^{-1}\otimes I_m\left(\left(b\otimes\hat{\Theta}_o\right)Sgn\left(\alpha_c\right)\right),\\
u_u &= -\left(k_1\varepsilon\right)^{-1}\left(\mathcal{M}+P\right)^{-1}\otimes I_m\left(Y_F\hat{\Theta}_F\right),\\
u_l &= -\left(k_1\varepsilon\right)^{-1}\left(\mathcal{M}+P\right)^{-1}\otimes I_m\left(E_v^{2-\varepsilon}\right),\\
u_s &= -k_s\left(k_1\varepsilon\right)^{-1}\left(\mathcal{M}+P\right)^{-1}\otimes I_m\left(Sgn(\alpha_c)\right),\\
\dot{\hat{\Theta}}_F &= -\Gamma_F Y_F^T\alpha_c, \dot{\hat{\Theta}}_o = -\Gamma_o Sgn^T\left(\alpha_c\right)b^T\alpha_c \qquad (17)
\end{aligned}$$

with $k_1 > 0$ and $1 < \varepsilon < 2$, $\alpha_c = diag\left(E_v^{\varepsilon-1}\right)S$, $b = P1_N$, symmetric positive-definite matrix $\Gamma_{F_i} \in \Re^{3\times 3}$ and $\Gamma_{oi} > 0$ such that parameters estimate are bounded as $\hat{\Theta}_F \leq \Theta_F^*$ and $\hat{\Theta}_o \leq \Theta_o^*$ and all the states of the leader-follower multi-agent autonomous systems reach to the sliding mode surface in finite-time and remain there achieving leader-follower finite-time tracking consensus.

**Proof:** The proof of Theorem 2 can be shown along the line of Theorem 1. First, the upper bounded property of the parameter estimates is proven. To do that, nonsingular terminal sliding variable for the $i$-th agent is defined as

$$s_i = e_{1i} + k_1 e_{2i}^{\xi} Sgn(e_{2i}). \qquad (18)$$

The sliding dynamics can be written in compact form as

$$S = E_p + k_1 E_v^{\varepsilon} Sgn(E_v). \qquad (19)$$

Applying (6)-(7), the time derivative of (19) can be derived as

$$\begin{aligned}
\dot{S} =& E_v + k_1\varepsilon diag\left(E_v^{\varepsilon-1}\right)\left(\mathcal{M}+P\right)\otimes I_m\Big[F(x,v,t)\\
&-1_N\otimes u_o + u\Big]. \qquad (20)
\end{aligned}$$

Now, the following Lyapunov function candidate is chosen to show the boundedness of the parameter estimates.

$$V = \frac{S^T S}{2} + \frac{\tilde{\Theta}_F^T\Gamma_F^{-1}\tilde{\Theta}_F}{2} + \frac{\Gamma_o^{-1}\tilde{\Theta}_o^T\tilde{\Theta}_o}{2}. \qquad (21)$$

Now, first take the time derivative of above Lyapunov function. Then use assumption 3 and then use assumption 1 to simplify the time derivative of Lyapunov function as

$$\begin{aligned}
=& S^T\left(E_v + k_1\varepsilon diag\left(E_v^{\varepsilon-1}\right)\left((\mathcal{M}+P)\otimes I_m\right)\Big[Y_F\Theta_F\right.\\
&\left.+u - 1_N\otimes\pi_o\Big]\right) + \tilde{\Theta}_F^T\Gamma_F^{-1}\dot{\tilde{\Theta}}_F + \Gamma_o^{-1}\tilde{\Theta}_o^T\dot{\tilde{\Theta}}_o \qquad (22)
\end{aligned}$$

with $Y_F = [sgn(S), \tilde{\mathcal{X}}, \tilde{\mathcal{V}}]$, $\Theta_{Fi} = [\eta_o, \eta_1, \eta_2]^T$, $\Theta_{oi} = \pi_o$. Now, using (17), $\dot{V}$ can be simplified as $\dot{V} = -\Pi\left(\|S\|^2\right)^{\frac{1}{2}} \leq 0$ for $S \neq 0$ and $\Pi = min\left\{k_s e_{21}^{(\varepsilon-1)}, \ldots, k_s e_{2N}^{(\varepsilon-1)}\right\}$. In view of $\dot{V}$, it is possible to conclude from the Lyapunov theorem that all the signals in leader-follower closed loop system and the parameter estimates are bounded. This means that there exist positive constants $\Theta_F^*$ and $\Theta_o^*$ such that $\hat{\Theta}_F \leq \Theta_F^*$ and $\hat{\Theta}_o \leq \Theta_o^*$ for $\forall t \geq 0$. To prove that the sliding mode motion occurs within a finite-time, the following Lyapunov function candidate is selected.

$$V = \frac{1}{2}S^T S + \frac{\tilde{\Theta}_F^T\gamma_{F1}^{-1}\tilde{\Theta}_F}{2} + \frac{\gamma_{o1}^{-1}\tilde{\Theta}_o^T\tilde{\Theta}_o}{2}. \qquad (23)$$

Now, using protocol (17), the time derivative of $\dot{V}$ can be simplified as

$$\begin{aligned}
\dot{V} =& -\sqrt{2}\left(\xi_1 Y_F\left(\Theta_F^* - \Theta_F\right) + b\xi_1\left(\Theta_o^* - \Theta_o\right)\right)\frac{\|S\|}{\sqrt{2}}\\
&-\sqrt{2\gamma_{F1}}\left(a_F - \|S\|\xi_1 Y_F\right)\frac{\left(\|\Theta_F^* - \hat{\Theta}_F\|\right)}{\sqrt{2\gamma_{F1}}}\\
&-\sqrt{2\gamma_{o1}}\left(a_o - \|S\|\xi_1 b\right)\frac{\left(\|\Theta_o^* - \hat{\Theta}_o\|\right)}{\sqrt{2\gamma_{o1}}} \qquad (24)
\end{aligned}$$

with $\xi_1 = diag\left(E_v^{\varepsilon-1}\right)$, $a_F = \gamma_{F1}^{-1}\Gamma_F Y_F^T\alpha_c$ and $a_o = \gamma_{o1}^{-1}\Gamma_o b^T\alpha_c$. By defining $\Pi_1 = -\sqrt{2}\left(Y_F\left(\Theta_F^* - \Theta_F\right) + b\left(\Theta_o^* - \Theta_o\right)\right)$, $\Pi_2 = \sqrt{2\gamma_{F1}}\left(a_F - \|S\|\xi_1 Y_F\right)$, $\Pi_3 = \sqrt{2\gamma_{o1}}\left(a_o - \|S\|\xi_1 b\right)$ and using $\Pi = \min\left\{\Pi_1, \Pi_2, \Pi_3\right\}$, $\dot{V}$ can be simplified as

$$\dot{V} \leq -\Pi\left(\frac{\|S\|}{\sqrt{2}} + \frac{\left(\|\Theta_F^* - \hat{\Theta}_F\|\right)}{\sqrt{2\gamma_{F1}}} - \frac{\left(\|\Theta_o^* - \hat{\Theta}_o\|\right)}{\sqrt{2\gamma_{o1}}}\right). \qquad (25)$$

Now, by choosing $\Theta_o^* > \Theta_o$, $\Theta_F^* > \Theta_F$, $\gamma_{o1} < \Gamma_o$ and $\|\gamma_{F1}\| < \|\Gamma_F\|$, one has $\Pi_1 > 0$, $\Pi_2 > 0$, $\Pi_3 > 0$ and $\Pi > 0$. Then, applying Lemma 3, (25) has the following compact form $\dot{V} \leq= -\Pi V^{\frac{1}{2}}$. This implies that the bound on $\|diag\left(E_v^{\varepsilon-1}\right)\| > 0$ exists when $E_v \neq 0$. This arguments together with Lemma 1 and $\Pi > 0$, one can conclude that the states of the leader-follower multi-agent autonomous systems can reach the sliding surface $S = 0$ in finite-time depending on the initial state $S(0)$ and then remain on the surface for all time in the presence of uncertainty. The finite-time can be calculated as $t^* = \frac{V(0)^{\frac{1}{2}}}{\Pi}$. Finally, one needs to show that the state of the followers can reach consensus and track the states of the leader in finite-time on the sliding surface in the presence of uncertainty. To do that, the Lyapunov function candidate is chosen as $V_o = \frac{E_p^T E_p}{2}$. On the sliding surface

$S = 0$, one has $E_v = -k_1^{\varepsilon} E_p^{\frac{1}{\varepsilon}}$. Then, the time derivative of $V_o$ can be written as $\dot{V}_o = -k_1^{\varepsilon} E_p^T E_p^{\frac{1}{\varepsilon}}$. Now, using $E_p^T E_p^{\frac{1}{\varepsilon}} = \sum_{i=1}^{N} \sum_{j=1}^{m} (e_{1ij})^{2^{\frac{(\varepsilon+1)}{2\varepsilon}}}$, $V_o$ can be written as $\dot{V}_o \le -k_1^{\varepsilon} 2^{\frac{(\varepsilon+1)}{2\varepsilon}} V_o^{\frac{(\varepsilon+1)}{2\varepsilon}}$. Then, using $1 < \varepsilon < 2$ and Lemma 1, one can conclude that there exist a finite-time $t_1$ depending on the initial state $E_p(t_o)$ with $t_o > t^*$ such that the error $E_p$ will converge to zero. Since $E_p$ converge to zero, then, in view of $\dot{V}_o$, the states $E_v$ also converge to zero in finite-time. Also, Lemma 2 ensures that the matrix $\mathcal{M} + P$ is full rank. Then, $\tilde{x}_i = 0$ and $\tilde{v}_i = 0$ can be obtained as the error function is zero as $e_{1i} = 0$ and $e_{2i} = 0$. □

**Remark 2:** The design uses $sgn(.)$ function which may generate chattering phenomenon. To avoid this problem, one can estimate $sgn(.)$ by using $sat(.)$ function. $sat(.)$ is a bounded saturation function that satisfies

$$sat(s_i) = \begin{cases} -1, & s_i < -\vartheta, \\ \dfrac{s_i}{\vartheta}, & |s_i| \le \vartheta \\ 1, & s_i > \vartheta. \end{cases}$$

By using $sat(.)$, one can also ensure the same results that presented in Theorems 1 and 2. To show that, one can follow the same steps as used in Theorem 2 and Theorem 1. Then, using $sat(.)$ in Theorem 2, $\dot{V}$ can be simplified as $\dot{V} = -\Pi \sum_{i=1}^{N} s_i sat(s_i)$. This means that, for $s_i \neq 0$, $\dot{V} < 0$ as $s_i sat(s_i) > 0$. This implies that the function $V$ continue to decrease until $s_i \le \vartheta$. Now, applying $sat(s_i) = \frac{s_i}{\vartheta}$, $\dot{V}$ can be simplified as $\dot{V} \le -\frac{\Pi}{\vartheta} \sum_{i=1}^{N} \|s_i\|^2 \le 0$. Then, using Lyapunov theory, one can conclude that all the sliding modes converge to zero. Thus all signals in leader-follower closed-loop multi-agent autonomous systems are bounded provided that the parameter estimates are bounded. This means that it is possible to replace $sgn(.)$ by $sat(.)$ function so that the finite-time convergence can be ensured.

## 4. EVALUATION RESULTS

This section implements and evaluates the designs to show the effectiveness of the proposed distributed finite-time and asymptotic consensus algorithms. In evaluation, second-order multi-agent autonomous systems are considered with one leader and six follower agents. The interaction topologies with autonomous agents are depicted in Fig. 1.

For the given topology, the Laplacian matrix $\mathcal{M}$ can be written as $\mathcal{M} = [0\ 0\ 0\ 0\ 0\ 0; -1\ 1\ 0\ 0\ 0\ 0; -1\ 0\ 1\ 0\ 0\ 0; 0\ -1\ 0\ 1\ 0\ 0; 0\ 0\ 0\ -1\ 2\ -1; 0\ 0\ -1\ -1\ 0\ 2]$. The interaction matrix $P$ between leader and followers can be defined as $P = [1\ 0\ 0\ 0\ 0\ 0; 0\ 1\ 0\ 0\ 0\ 0; 0\ 0\ 0\ 0\ 0\ 0; 0\ 0\ 0\ 0\ 0\ 0; 0\ 0\ 0\ 0\ 0\ 0; 0\ 0\ 0\ 0\ 0\ 0]$.

The initial states of the leader are chosen as $x_o(0) = 1$, $v_o(0) = 0.5$. To examine the consensus behavior, four sets of time varying and state dependents inputs are selected for the leader as $u_{o1} = 0.01\sin(0.1t)$, $u_{o2} = 0.5\cos(0.01t)$, $u_{o3} = 5 + \sin(0.2x_o) + \cos(0.5v_o)$, $u_{o4} = 3 + \sin(0.6x_o) + \sin(.5v_o)$. $u_{o1}$ and $u_{o3}$ applies from 0 sec. to 50 sec. while $u_{o2}$ and $u_{o4}$ uses from 50 sec. to 120 sec. The input profile for the leader input is depicted in Fig. 2.

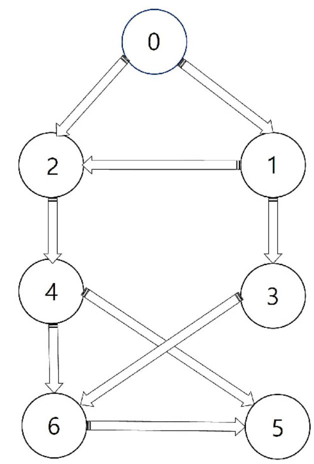


Fig. 1. The interaction topology between autonomous agents and leader for the second-order nonlinear multi-agent autonomous systems [20].

The initial states of the follower agents are chosen as $x_{0i}(0) = [4,\ 2,\ 3,\ -0.5,\ -1,\ -2]^T$, $v_{0i}(0) = [-1,\ 0.8,\ -1,\ -0.5,\ -0.5,\ 2]^T$ with $i \in \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ [20]. The nonlinear dynamics for the follower agents are defined as $\mathcal{F}_1(x_1, v_1, t) = \sin(.2x_1) + 0.5v_1 + 2$, $\mathcal{F}_2(x_2, v_2, t) = \sin(.2x_2) + 0.5v_2 + .08$, $\mathcal{F}_3(x_3, v_3, t) = \sin(x_3) + 0.5v_3 + 1$, $\mathcal{F}_4(x_4, v_4, t) = \sin(0.5x_4) + v_4 + 0.5$, $\mathcal{F}_5(x_5, v_5, t) = \sin(0.1x_5) + 0.05v_5 + 0.7$ and $\mathcal{F}_6(x_6, v_6, t) = \cos(0.5x_6) + 2v_6 + 0.3$.

The control design parameters for the finite-time consensus design introduced in Theorem 2 are chosen as

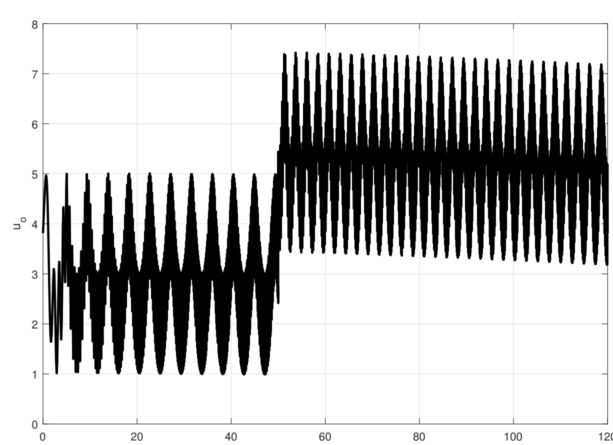


Fig. 2. The time history of the input for the leader $u_o$.

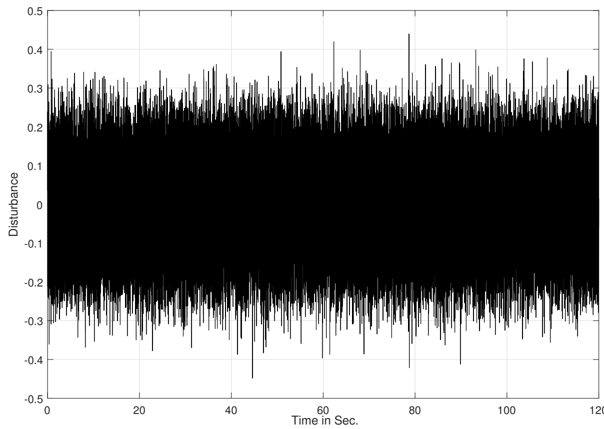


Fig. 3. The external disturbance for the follower agents.

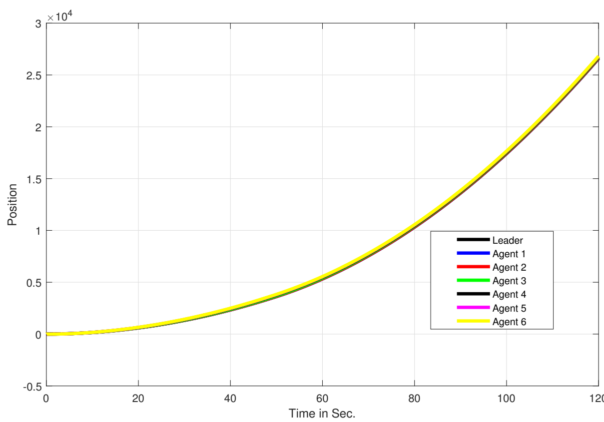


Fig. 4. The time history of the position states for the leader and follower agents for Theorem 2.

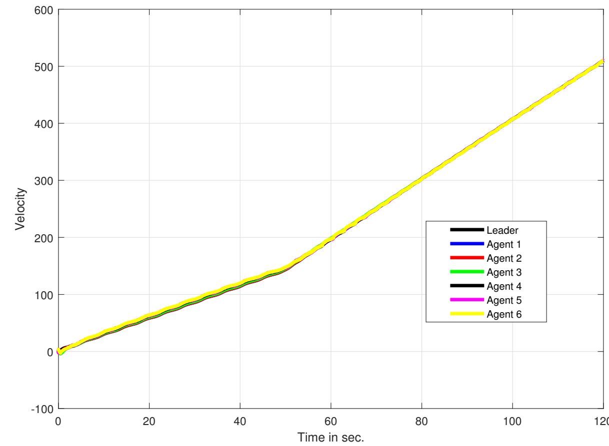


Fig. 5. The time history of the velocity states for the leader and follower agents for Theorem 2.

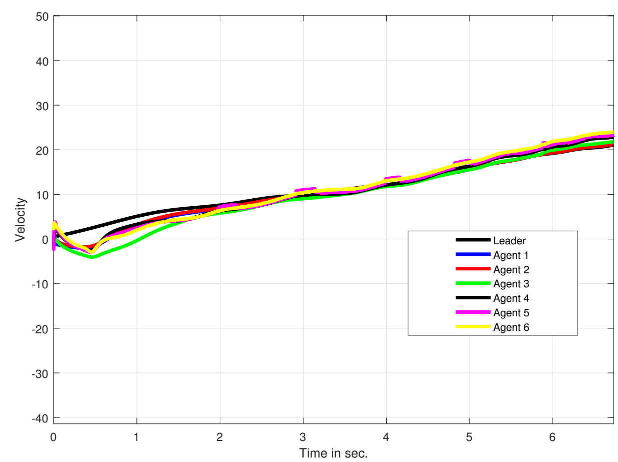


Fig. 6. The time history of the zoomed velocity states for the leader and follower agents for Theorem 2.

$k_1 = 4.50$, $\varepsilon = 0.05$, $k_s = .5$, $\Gamma_{F_i} = 10^{-3} I_{3\times 3}$, $\Gamma_{o_i} = 10^{-3}$. All the followers dynamics are corrupted by external disturbances as depicted in Fig. 3.

Then, apply finite-time consensus tracking based control algorithm developed in Theorem 2 for the given leader-follower multi-agent autonomous systems. The sampling time is chosen as 0.001 sec. The evaluation results are shown in Figs. 4 to 6. Fig. 4 presents the evaluation results of the position states of the leader and followers. The velocity and zoom velocity states of the leader and followers are shown in Figs. 5 to 6. From these figs., we see that the consensus based distributed controller design can ensure consensus tracking property with the presence of nonlinear dynamical model uncertainty and external disturbance uncertainty.

The proposed design in Theorem 2 is now compare with robust finite-time consensus algorithm designed by using terminal sliding mode control mechanism [20]. The control design parameters are remained similar to our previous evaluation. For fair comparison, all other design parameters and scenarios are also kept same as to our previous test. Then, apply robust finite-time distributed consensus control designed by Theorem 1 of [20]. Figs. 7 to 8 depict the evaluation results of the position and velocity states for the leader and follower autonomous agents. It can be seen from these results that the systems explodes approximately at 54.5 sec. due to very large modeling error and external disturbance uncertainty.

The goal is now to compare proposed asymptotic consensus protocol with the finite-time consensus protocol on the same second-order nonlinear multi-agent autonomous systems as used in previous evaluation. The control design parameters for asymptotic consensus design are chosen as $\alpha = 4$, $k_s = 50$, $\Gamma_{F_i} = .005 I_{3\times 3}$, $\Gamma_{o_i} = 0.001$. To examine the consensus convergence property, all other design parameters and test scenarios are kept same as previous evaluation. Then, apply robust consensus tracking control proposed in Theorem 1 for the given leader-followers multi-agent autonomous systems.

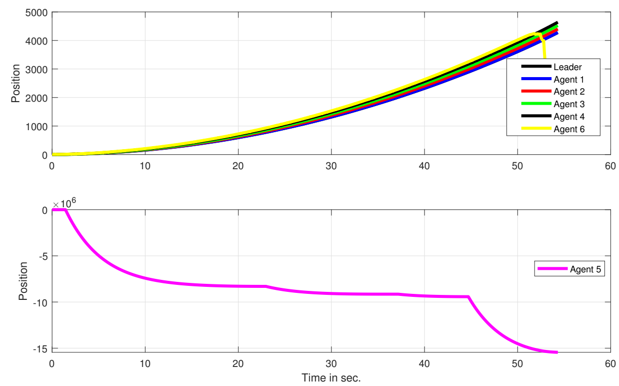


Fig. 7. The time history of the position states for the leader and follower agents for Theorem 1 [20].

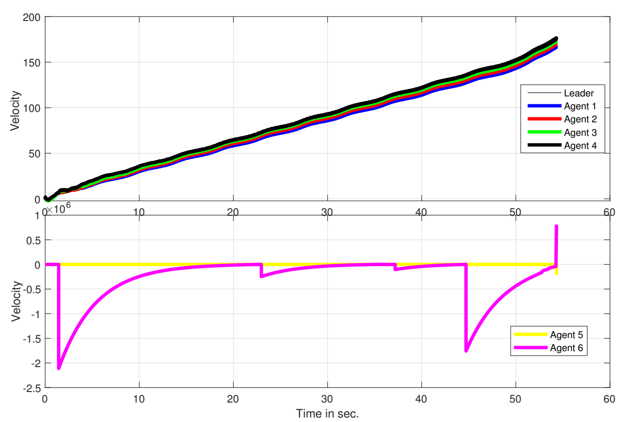


Fig. 8. The time history of the velocity states for the leader and follower agents for Theorem 1 [20].

The evaluation results are shown in Figs. 9 to 11. Fig. 9 shows the evaluation results of the position states of the leader and followers autonomous agents. The velocity and zoomed velocity states of the leader and followers agents are given in Figs. 10 to 11. From these results, it can be seen that the distributed controller can ensure consensus tracking with the presence of model uncertainty and disturbance. In view of these results, it can also be noticed that the consensus tracking performance under Theorems1 and 2 is almost same. The design parameters for asymptotic consensus design can be selected to ensure the same consensus tracking speed as finite-time consensus tracking design.

**Remark 3:** It should be noted that finite-time consensus design parameters are more sensitive than asymptotic consensus. Therefore, the designer can tune the parameters of the classical linear sliding mode based asymptotic consensus more easily than the nonlinear terminal sliding mode based finite-time design.

**Remark 4:** The adaptation law given in protocols may exhibit discontinuous property. To ensure that the parameters estimates remain bounded over the compact sets, we can introduce a projection mechanism to limit the parameter estimates [25].

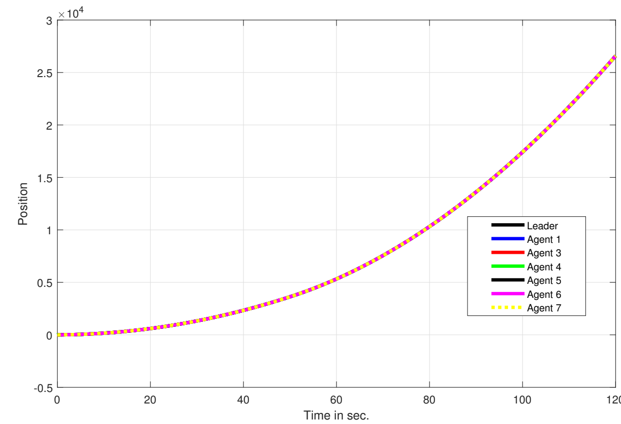


Fig. 9. The time history of the position states for the leader and follower agents for Theorem 1.

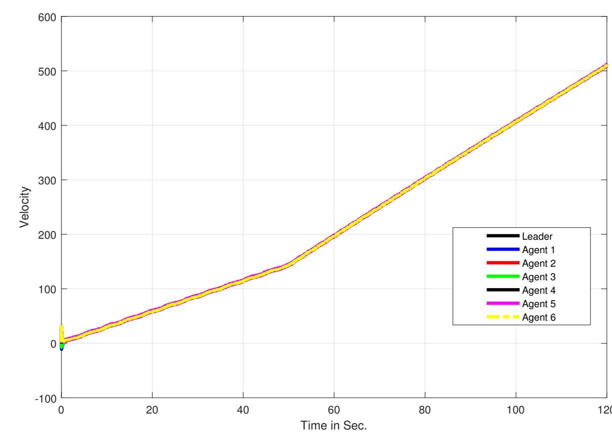


Fig. 10. The time history of the velocity states for the leader and follower agents for Theorem 1.

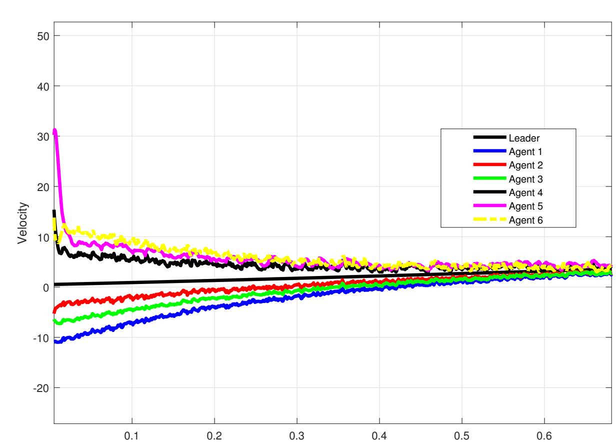


Fig. 11. The time history of the zoomed velocity states for the leader and follower agents for Theorem 1.

## 5. CONCLUSION AND FUTURE WORKS

In this paper, both robust asymptotic and finite-time consensus tracking algorithms have been studied for second-order nonlinear leader-following autonomous agents. The protocols have been designed by using the states of the neighboring agents with directed communication network topology in the presence of uncertainty. Lyapunov and Graph theory with linear classical sliding mode and nonlinear terminal sliding mode control theory has been used to prove robust asymptotic and finite-time consensus tracking of autonomous agents with the presence of uncertainty. The evaluation results with comparison have been presented to show the effectiveness of the proposed results for real-time applications. Both designs are simple and easy to implement as they do not use the bounds of the leader input and uncertainty associated with the follower agent systems. However, the evaluation shows that the asymptotic design is more easier to develop and implement than finite-time consensus design. The design parameters in asymptotic consensus can be chosen to achieve faster tracking convergence as the finite-time design. Moreover, the proposed design and analysis is assumed that the data transmission delay between agents is performed by using dedicated communication network. Therefore, the data transmission delay was assumed to be zero as autonomous agents are connected via local network. In future, the proposed design and development will be extended for open distributed communication network along the line of the results presented in [26–28].

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