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Robust Observer Based Fault-tolerant Control for One-sided Lipschitz Markovian Jump Systems with General Uncertain Transition Rates

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Abstract: This paper presents an integrated design of adaptive sliding mode observer and fault-tolerant control for a class of one-sided Lipschitz Markovian jump systems with general uncertain transition rates. In the design process, an adaptive sliding mode observer is first constructed to estimate the states of the original system without knowing any information of the unknown input. Then a fault-tolerant control strategy is therefore proposed to stabilize the closed-loop system against the unknown input. Sufficient conditions of the existences of the designed observer and controller are deduced in the forms of linear matrix inequalities. In the end, several examples are given to illustrate the effectiveness and make some comparisons with other results.

Keywords: Fault-tolerant control, general uncertain transition rates, Markovian jump system, one-sided Lipschitz, observer design.

1. INTRODUCTION

In the past decades, the issues on observer design for nonlinear systems, especially for Lipschitz nonlinear systems, have attracted widespread attentions. Normally, many nonlinearities existed in the systems can be described as Lipschitz nonlinearities. However, when the Lipschitz constant becomes lager, this condition may turn out to be invalid [1], which is the main limitation of the existing results for Lipschitz nonlinear systems. In order to overcome this short-coming, the one-sided Lipschitz condition is introduced as a strategy to deal with the case when the Lipschitz constant becomes very large. It is proved that one-sided Lipschitz nonlinear systems, which include the traditional Lipschitz nonlinear systems, can represent larger class of nonlinear systems in a more general sense. In recent years, many excellent works were reported on the one-side Lipschitz systems [2–11]. For instance, Hu in [2] first proposed the concept of one-sided Lipschitz nonlinear systems and developed asymptotically stable conditions of the corresponding error dynamics. Following Hu's work, [3] and [4] proposed the improved results, which were less conservative than [2]. Later on, [5] proposed the existing conditions for designing observers for one-sided Lipschitz systems, which were given in the forms of nonlinear matrix inequalities. Moreover, [6] and [7] investigated the problems of observer design for a class of discrete-time nonlinear systems subjected to one-side Lipschitz nonlinear terms. [8] proposed full-order and reduced-order observers for one-sided Lipschitz nonlinear systems by solving the Riccati equations. Recently, [9] considered the exponential observer design problem for one-sided Lipschitz nonlinear systems, in which the one-sided Lipschitz constant didn't need to be considered in the design process.

In addition, Markovian jump system (MJS) has been paid much attention in the control field. As MJSs can be used to describe systems that subjected to random abrupt and environmental changes, they are able to describe a wide range of practical systems, including power systems, aerospace systems, manufacturing systems and network control systems. Due to this fact, a large number of results discussed the Markovian jump systems were proposed in recent years [12–20]. For instance, [12] addressed the robust stabilization problems for a class of MJSs, which were supposed to own uncertain switching probabilities. [15] studied the output tracking controller design for continuous-time MJSs, where adaptive laws were constructed in novel structures and the fault can therefore be estimated efficiently. [16] proposed an adaptive sliding mode controller for a class of nonlinear Markovian jump systems with partly unknown transition probabilities, where detailed information of the nonlinear term was assumed to be unknown. [20] developed a fault-

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tolerant control strategy for MJSs with general uncertain transition rates, in which the actuator fault were adjusted by the designed adaptive law.

As a critical factor, the transition rates (TRs) determine the system performance of MJS in the jump process. Until now, many works were developed under the assumptions that TRs were completely known [21–23]. In practice, it is, however, difficult for designers to estimate TRs precisely and completely. Therefore, some works on MJSs with partially unknown transition rates (PUTRs) [24–28] or bounded uncertain transition rates (BUTRs) [29] were widely studied. However, another description for uncertain TRs, named as general uncertain TRs (GUTRs), was presented in [30], including the BUTR and PUTR as special cases. In this description, each TR can be completely unknown or only the estimated value can be known, showing that the GUTR is more suitable for describing the TRs in practice [31].

To the best knowledge of the authors, few researches focus on the robust observer and fault-tolerant control designs for MJSs with GUTRs [20, 30–33], especially, when subjecting to the one-side Lipschitz nonlinearities. Although some effective results on observer and controller design were reported for Markovian jump systems, there still remain some problems to be solved. For example, [28] considered the problem of fault-tolerant control for the one-sided Lipschitz MJSs against unknown input, however it assumed that the transition rate was partially unknown. [34] and [35] proposed fault-tolerant control strategies for MJSs with PUTRs, but no reference to the one-side Lipschitz nonlinearities, and the bounds of the unknown inputs needed to be known in advance. The aforementioned results make the problem discussed in this paper more challenging and deserve further development. The main contributions of the proposed method are summarized as follows:

1) We suppose that the considered Markovian jump system is subjected to one-sided Lipschitz nonlinearity, which brings less conservatism than those involved Lipschitz nonlinearities [2, 5].

2) The designed observer and controller can be directly applied to the cases with BUTRs or PUTRs, implying that the proposed method is more universal than [27, 36].

3) Two novel stochastic adaptive laws are proposed, so as to estimate some parameters to eliminate the influences, brought by the unknown inputs, without knowing the upper bounds of the unknown inputs in advance. This shows some superiorities over the existing results that need the bounds of the unknown inputs being known in advance [34, 35].

Notations: R represents the field of real numbers. R_n denotes the n-dimensional Euclidean space. $\langle\cdot,\cdot\rangle$ is the inner product in R^n, given $x,y \in R^n$, then $\langle x,y\rangle = x^T y$, where x^T denotes the transpose of the column vector x. $R_{m\times n}$ refers to the real matrix of m row and n column. $\|x\| = \sqrt{x^T x}$ represents the 2-norm of vector x, defined as $\|x\| = \sqrt{x^T x}$. $'*'$ stands for the symmetric part of a matrix. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of the matrix A. I represents an identity matrix with appropriate dimension. $E\{\}$ represents the mathematical expectation.

2. SYSTEM DESCRIPTION

Consider a class of one-sided Lipschitz Markovian jump systems, which is subjected to unknown input as follows:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B(r(t))u(t) + D(r(t))\eta(t) \\ \qquad + f(x(t), r(t)), \\ y(t) = C(r(t))x(t), \end{cases} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ are the system state, control input, and $y(t) \in R^p$ is control output vector. $f(x(t), r(t)) \in R^s$ and $\eta(t) \in R^q$ are the one-sided Lipschitz nonlinear function and unknown input. $A(r(t))$, $B(r(t))$, $C(r(t))$ and $D(r(t))$ are coefficient matrices in appropriate dimensions that depend on $r(t)$. For convenience, the matrices $\Lambda(r(t))$ are denoted by $\Lambda_i = \Lambda(r(t) = i)$, $i \in S$ in the following. The mode jumping process $\{r(t), t \geq 0\}$ is a right-continuous Markov chain on the probability space, and takes values in a finite state space $S = \{1, 2, ..., \bar{s}\}$ with the mode transition probabilities

$$P_r\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \pi_{ij}\Delta + o(\Delta), & i = j, \end{cases}$$

where $\Delta > 0$, $\lim\limits_{\Delta\to 0} \frac{o(\Delta)}{\Delta} = 0$, and for $i \neq j$, $\pi_{ij} \geq 0$ is the transition rate from mode i at time t to mode j at time $t+\Delta$. Furthermore, the transition rate π_{ij} satisfies $\pi_{ii} = -\sum\limits_{j=1, i\neq j}^{\bar{s}} \pi_{ij}$.

In this paper, the transition rate matrix $\Pi = (\pi_{ij})$ is considered to be generally uncertain. For example, system (1) with $\bar{s}$ operation modes may have the following the transition rate matrix

$$\begin{bmatrix} \hat{\pi}_{11}+\Delta\pi_{11} & ? & \hat{\pi}_{13}+\Delta\pi_{13} & \cdots & ? \\ & \hat{\pi}_{22}+\Delta\pi_{22} & ? & \cdots & \hat{\pi}_{s2}+\Delta\pi_{s2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{\pi}_{s1}+\Delta\pi_{s1} & ? & \hat{\pi}_{s3}+\Delta\pi_{s3} & \cdots & ? \end{bmatrix}, \tag{2}$$

where $\hat{\pi}_{ij}$ and $\Delta\pi_{ij} \in [-\varepsilon_{ij}, \varepsilon_{ij}]$, $\varepsilon_{ij} \geq 0$, $i, j \in S$ represent the known estimated value and estimation error of π_{ij}, respectively. ε_{ij} represents the boundary of $\Delta\pi_{ij}$. The symbol '?' represents the complete unknown transition rate, which means that both the estimated value and the boundary of the estimated error are unknown. Without loss of generality, $\pi_{ij}-\varepsilon_{ij}\geq 0(\forall i,j\in S,j\neq i)$, $\pi_{ij}+\varepsilon_{ij}\leq 0(\forall i,j\in S,j=i)$, $\hat{\pi}_{ii}=-\sum_{j=1,j\neq i}^{\bar{s}}\hat{\pi}_{ij}$ and $\varepsilon_{ii}=-\sum_{j=1,j\neq i}^{\bar{s}}\varepsilon_{ij}$.

In order to distinguish symbols, for all $i\in S$, we denote

$$U_k^i=\{j: \text{The estimated value of } \pi_{ij} \text{ is known for } j\in S.\}$$

$$U_{uk}^i=\{j: \text{The estimated value of } \pi_{ij} \text{ is unknown for } j\in S.\}$$

Moreover, if $U_k^i\neq\emptyset$, it is further described as $U_k^i=\{l_i^1,l_i^2,\cdots,l_i^m\}$, where l_i^m represents the mth known element in the ith row of matrix Π.

Remark 1: The above descriptions of uncertain transition rates generalize those of the bounded uncertain transition rates and partially unknown transition rates. To show this clearly, we put the two uncertain rates as follows:
(i) Partially unknown transition rate matrix [24]

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \cdots & ? \\ ? & \pi_{22} & ? & \cdots & \pi_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \pi_{s1} & ? & \pi_{s3} & \cdots & \pi_{ss} \end{bmatrix}. \tag{3}$$

(ii) Bounded uncertain transition rate matrix [29]

$$\begin{bmatrix} \hat{\pi}_{11}+\Delta\pi_{11} & \hat{\pi}_{12}+\Delta\pi_{12} & \cdots & \hat{\pi}_{1s1}+\Delta\pi_{1s} \\ \hat{\pi}_{21}+\Delta\pi_{21} & \hat{\pi}_{22}+\Delta\pi_{22} & \cdots & \hat{\pi}_{2s}+\Delta\pi_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\pi}_{s1}+\Delta\pi_{s1} & \hat{\pi}_{s2}+\Delta\pi_{s2} & \cdots & \hat{\pi}_{ss}+\Delta\pi_{ss} \end{bmatrix}. \tag{4}$$

We can obviously see that if $\varepsilon_{ij}=0$, $\forall j\in U_k^i$, $\forall i\in S$, then the uncertain transition rate matrix (2) will come to a partial unknown transition rate matrix described in (3); If $U_{uk}^i=\emptyset$, $\forall i\in S$, the uncertain transition rate matrix (2) will come to a bounded uncertain transition rate matrix described in (4). Therefore, it is applicable to implement the proposed method in the cases of MJSs with BUTRs and PUTRs.

Remark 2: For the considered system, a typical example can be found in NCS (networked control system), where Markov chain can well describe the randomness of the time-delay and packet dropouts with all the transition rates being completely accessible. However, in fact, it may be impossible to obtain the exact knowledge of the transition rates in most real systems. Only estimated values and the boundaries of the estimation errors can be available. In summary, Markovian jump systems with general uncertain transition rates do exist in practical applications.

Definition 1 [5]**:** If the nonlinear term $f_i(x(t))$ is continuous with $x(t)$ and conforms to the one-sided Lipschitz condition in the region $\bar{D}_i$, then there exists a one-sided Lipschitz constant $\delta_i\in R$ such that $\forall x_1,x_2\in\bar{D}_i$.

$$\langle f_i(x(t))-f_i(\hat{x}(t)),x(t)-\hat{x}(t)\rangle\leq\delta_i\|x(t)-\hat{x}(t)\|^2, \tag{5}$$

where $\bar{D}_i$ is a compact region and contains the origin and the condition holds when the system operates in the ith mode.

Definition 2 [5]**:** The nonlinear term $f_i(x(t))$ is called quadratic inner-boundedness in the region $\tilde{D}_i$, if there exist constants $\sigma_i,\gamma_i\in R$ such that $\forall x_1,x_2\in\tilde{D}_i$

$$\begin{aligned}&(f_i(x(t))-f_i(\hat{x}(t)))^T(f_i(x(t))-f_i(\hat{x}(t)))\\&\leq\sigma_i\|x(t)-\hat{x}(t)\|^2\\&\quad+\gamma_i\langle x(t)-\hat{x}(t),f_i(x(t))-f_i(\hat{x}(t))\rangle,\end{aligned} \tag{6}$$

where $\tilde{D}_i$ is a compact region and contains the origin and the condition holds when the system operates in the ith mode.

Remark 3: For each $i\in S$, the one-sided Lipschitz constants δ_i, σ_i and γ_i can be positive, negative or zero, but the traditional Lipschitz constant must be positive. This shows some superiorities over the traditional Lipschitz condition.

Remark 4: The one-sided Lipschitz nonlinearities under consideration cover a broad family on practical nonlinear systems and includes the classic Lipschitz conditions as special cases [2]. It is proved that the one-sided Lipschitz constants are significantly smaller than the classical Lipschitz constants, so as to much conservatism can be reduced while solving the LMIs [5].

Definition 3 [37]**:** The Markovian jump system (1) with $u(t)\equiv 0$ is said to be stochastically stable, if for every initial condition $x_0\in R^n$ and initial mode $r_0\in S$,

$$E\left\{\int_0^\infty\|x(t)\|^2dt|x_0,r_0\right\}<\infty. \tag{7}$$

Assumption 1: We assume that the unknown input $\eta(t)$ is bounded that is $\|\eta(t)\|\leq\rho_\eta$, where ρ_η is unknown.

Assumption 2: There exist symmetric positive definite matrices $P_i\in R^{n\times n}$ and $W_i\in R^{n\times n}$, matrices H_i and G_i with appropriate dimensions such that

$$D_i^TP_i=H_iC_i, \tag{8}$$

and

$$D_i^TW_i=G_iC_i. \tag{9}$$

3. ADAPTIVE SLIDING MODE OBSERVER DESIGN

In this section, we construct an adaptive sliding mode observer which is then used to estimate the states of system (1).

$$\begin{cases}\dot{\hat{x}}(t)=A_i\hat{x}(t)+B_iu(t)+f_i(\hat{x}(t))+D_iv_{ei}(t)\\ \qquad\quad+L_i(y(t)-\hat{y}(t)),\\ \hat{y}(t)=\hat{C}_ix(t),\end{cases} \tag{10}$$

where $\hat{x}(t) \in R^n$ is the estimated state of $x(t)$. $L_i \in R^{n\times p}$ is the observer gain matrix to be designed. $v_{ei}(t)$ is a sliding mode control law which will be designed in the next part.

The estimation error is defined as $e(t) = x(t) - \hat{x}(t)$, then the error equation is governed by

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t). \tag{11}$$

Subtract (10) from (1), one can have access to the following error equation

$$\begin{aligned}\dot{e}(t) =& \dot{x}(t) - \dot{\hat{x}}(t)\\ =& (A_i - L_iC_i)e(t) + \tilde{f}_i(x(t),\hat{x}(t)) + D_i\eta(t)\\ &- D_iv_{ei}(t),\end{aligned} \tag{12}$$

where $\tilde{f}_i(x(t),\hat{x}(t)) = f_i(x(t)) - f_i(\hat{x}(t))$.

Define the following sliding mode control laws:

$$v_{xi}(t) = \beta_{xi}(t)\mathrm{sgn}(S_{v_{xi}}(t)), \tag{13}$$

and

$$v_{ei}(t) = \beta_{ei}(t)\mathrm{sgn}(S_{v_{ei}}(t)), \tag{14}$$

which are used to eliminate the effect of unknown input $\eta(t)$, where $S_{v_{xi}}(t) = H_iC_ix(t) = D_i^TP_ix(t)$, $S_{v_{ei}}(t) = G_iC_ie(t) = D_i^TW_ie(t)$, H_i and G_i are denoted in Assumption 2 and $\beta_{xi}(t)$ and $\beta_{ei}(t)$ are unknown adaptive gains, and adjusted by adaption algorithms.

$$\begin{cases}\dot{\beta}_{xi}(t) = \alpha_{xi}\|S_{v_{xi}}(t)\|\,\mathrm{sgn}(\|S_{v_{xi}}(t)\| - \tau_{xi}) \\ \qquad\qquad\qquad\qquad \text{for } \beta_{xi}(t) > \mu_{xi}, \\ \dot{\beta}_{xi}(t) = \mu_{xi} \qquad\qquad \text{for } \beta_{xi}(t) \le \mu_{xi},\end{cases} \tag{15}$$

and

$$\begin{cases}\dot{\beta}_{ei}(t) = \alpha_{ei}\|S_{v_{ei}}(t)\|\,\mathrm{sgn}(\|S_{v_{ei}}(t)\| - \tau_{ei}) \\ \qquad\qquad\qquad\qquad \text{for } \beta_{ei}(t) > \mu_{ei}, \\ \dot{\beta}_{ei}(t) = \mu_{ei} \qquad\qquad \text{for } \beta_{ei}(t) \le \mu_{ei},\end{cases} \tag{16}$$

with $\alpha_{xi} < 2$, $\alpha_{ei} < 2$, $\beta_e(0) > 0$, $\beta_x(0) > 0$, $0 < \tau_{xi} < 1$, $0 < \tau_{ei} < 1$, and $\mu_{xi} > 0$, and $\mu_{ei} > 0$ being small enough.

Remark 5: In (15) and (16), the parameters $\mu_{xi} > 0$ and $\mu_{ei} > 0$ are introduced to guarantee that $\beta_{xi}(t)$ and $\beta_{ei}(t)$ are always positive scalars.

Remark 6: According to the structures of adaptive laws (15) and (16), we can see that the control gains $\beta_{xi}(t)$ and $\beta_{ei}(t)$ can be always adjusted to appropriate values online, avoiding being too small or large, so as to guarantee good estimations and control performances.

Lemma 1: With the sliding mode terms (13) and (14), the adaptive gains $\beta_{xi}(t)$ and $\beta_{ei}(t)$, defined in (15) and (16), have upper bounds $\beta_{xi}{}^*$ and $\beta_{ei}{}^*$, for all $t \ge 0$ with $\beta_{xi}{}^* \ge \rho_x$ and $\beta_{ei}{}^* \ge \rho_e$.

Proof: We omit the proved process, which is similar to [38].

4. OBSERVER-BASED FAULT-TOLERANT CONTROL DESIGN

In this section, we propose a fault-tolerant controller for the system (1). The fault-tolerant controller can be constructed as $u(t) = K(r(t))x(t) - B_i^\dagger D_iv_{xi}(t)$, where $x(t)$ is the real system state, $v_{xi}(t)$ is defined in (13) and $B_i^\dagger$ is by an inverse of B_i.

Assumption 3:

$$rank(\begin{bmatrix} B_i & D_i \end{bmatrix}) = rank(B_i).$$

Lemma 2: For any Penrose-Moore inverse $B_i^\dagger$ of the matrix B_i and any F_i that satisfies Assumption 3, we have $B_iB_i^\dagger D_i = D_i$. Proof: Assumption 3 implies that there exist some matrices $\sum_i$ such that $B_i^\dagger\sum_i = D_i$. Thus, $B_iB_i^\dagger D_i = B_iB_i^\dagger B_i\sum_i$. Since $B_i^\dagger$ is a Penrose-Moore inverse of B_i, we have $B_iB_i^\dagger = I$. Therefore, $B_iB_i^\dagger B_i\sum_i = B_i\sum_i = D_i$. This concludes the proof.

Using Lemma 2, the closed-loop system is given by

$$\dot{x}(t) = (A_i + B_iK_i)x(t) - D_iv_{xi}(t) + D_i\eta(t) + f_i(x(t),t). \tag{17}$$

The sufficient conditions of the existence of proposed controller are given in Theorem 1.

Theorem 1: If there exist scalar $\psi_i > 0$, symmetric positive definite matrices $P_i \in R^{n\times n}$, $W_i \in R^{n\times n}$, $Q_i \in R^{n\times n}$, $M_{ij} \in R^{n\times n}$, $N_{ij} \in R^{n\times n}$, $O_{ij} \in R^{n\times n}$, and some positive constants υ_{1i}, υ_{2i}, κ_{1i}, κ_{2i} and φ_i, such that for any $i \in S$

$$\begin{bmatrix} \Upsilon_{1i} & P_i + \frac{\gamma_i\upsilon_{2i} - \upsilon_{1i}}{2}I & 0 & 0 \\ * & -\upsilon_{2i}I & 0 & 0 \\ * & * & \Upsilon_{2i} & W_i + \frac{\gamma_i\kappa_{2i} - \kappa_{1i}}{2}I \\ * & * & * & -\kappa_{2i}I \end{bmatrix} < 0, \tag{18}$$

where $\Upsilon_{1i} = P_iA_i + A_i{}^TP_i + R_i + R_i{}^T + \sum_{j\in S_k^i}\hat{\pi}_{ij}(P_j - Q_i) + \sum_{j\in S_k^i} 2\varepsilon_{ij}M_{ij} - \sum_{j\in S_k^i}\varepsilon_{ij}(P_j - Q_i) + \upsilon_{1i}\delta_i + \upsilon_{2i}\sigma_i - \varphi_iI$, $R_i = P_iB_iK_i$ and $\Upsilon_{2i} = W_iA_i + A_i^TW_i - Y_iC_i - C_i{}^TY_i^T + \sum_{j\in S_k^i}\hat{\pi}_{ij}(W_j - O_i) + \sum_{j\in S_k^i} 2\varepsilon_{ij}N_{ij} - \sum_{j\in S_k^i}\varepsilon_{ij}(W_j - O_i) + \kappa_{1i}\delta_i + \kappa_{2i}\sigma_i - \varphi_iI$, $Y_i = W_iL_i$.

$$P_j - Q_i \ge 0, \quad j \in S_{uk}^i, \quad j = i, \tag{19}$$
$$P_j - Q_i \le 0, \quad j \in S_{uk}^i, \quad j \ne i, \tag{20}$$
$$P_j - Q_i - M_{ij} \le 0, \quad j \in S_k^i, \tag{21}$$
$$W_j - O_i \ge 0, \quad j \in S_{uk}^i, \quad j = i, \tag{22}$$
$$W_j - O_i \le 0, \quad j \in S_{uk}^i, \quad j \ne i, \tag{23}$$
$$W_j - O_i - N_{ij} \le 0, \quad j \in S_k^i, \tag{24}$$

$$\min\ \psi_i > 0$$

$$\begin{bmatrix} -\psi_iI & (D_i^TP_i - H_iC_i)^T \\ * & -I \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} -\psi_i I & (D_i^T W_i - G_i C_i)^T \\ * & -I \end{bmatrix} < 0. \tag{26}$$

then the error system (12) and the closed-loop system (17) are stochastically stable. The controller gains and observer gains can be obtained by $K_i = (B_i^T B_i)^{-1} B_i^T P_i^{-1} R_i$ and $L_i = W_i^{-1} Y_i$.

Proof: Consider the Lyapunov function candidate of

$$\begin{aligned} &V_\zeta(\zeta(t), r(t)) \\ &= \zeta(t)^T \tilde{P}(r(t))\zeta(t) + \frac{1}{2}(\beta_{xi}(t) - \beta_{xi}^*)^2 \\ &\quad + \frac{1}{2}(\beta_{ei}(t) - \beta_{ei}^*)^2 \\ &= x^T(t)P(r(t))x(t) + e^T(t)W(r(t))e(t) \\ &\quad + \frac{1}{2}(\beta_{xi}(t) - \beta_{xi}^*)^2 + \frac{1}{2}(\beta_{ei}(t) - \beta_{ei}^*)^2, \end{aligned} \tag{27}$$

where $\tilde{P}(r(t)) = diag\{P(r(t)), W(r(t))\}$ and $\zeta(t) = \begin{bmatrix} x(t)^T & e(t)^T \end{bmatrix}^T$. If at time t, $r(t) = i$, the weak infinitesimal operator acting on $V(\bullet)$ at time t is given by

$$\begin{aligned} &\ell V_\zeta(\zeta(t), i) \\ &= x^T(t)\left[P_i(A_i + B_i K_i) + (A_i + B_i K_i)^T P_i\right] x(t) \\ &\quad + x^T(t)\sum_{j=1}^{\bar{s}} \pi_{ij} P_j x(t) + 2x^T(t) P_i f_i(x(t)) \\ &\quad + 2x^T(t) P_i D_i \eta(t) - 2x^T(t) P_i D_i v_{xi}(t) \\ &\quad + \dot{\beta}_{xi}(t)(\beta_{xi}(t) - \beta_{xi}{}^*) \\ &\quad + e^T(t)\left[W_i(A_i - L_i C_i) + (A_i - L_i C_i)^T W_i\right] e(t) \\ &\quad + e^T(t)\sum_{j=1}^{\bar{s}} \pi_{ij} W_j e(t) + 2e^T(t) W_i \tilde{f}_i(x(t), \hat{x}(t)) \\ &\quad + 2e^T(t) W_i D_i \eta(t) \\ &\quad - 2e^T(t) W_i D_i v_{ei}(t) + \dot{\beta}_{ei}(t)(\beta_{ei}(t) - \beta_{ei}{}^*). \end{aligned} \tag{28}$$

In fact, for $Q_i > 0$ and $O_i > 0$, there exist $\sum_{j=1}^{\bar{s}} \pi_{ij} x^T(t) Q_i x(t) = 0$ and $\sum_{j=1}^{\bar{s}} \pi_{ij} e^T(t) O_i e(t) = 0$ since we have $\sum_{j=1}^{\bar{s}} \pi_{ij} = 0$.

Then, we have

$$\begin{aligned} &e^T(t)\sum_{j=1}^{\bar{s}} \pi_{ij} W_j e(t) - e^T(t)\sum_{j=1}^{\bar{s}} \pi_{ij} O_i e(t) \\ &= e^T(t)\sum_{j\in S_k^i} \pi_{ij} W_j e(t) + e^T(t)\sum_{j\in S_{uk}^i} \pi_{ij} W_j e(t) \\ &\quad - e^T(t)\sum_{j\in S_k^i} \pi_{ij} O_i e(t) - e^T(t)\sum_{j\in S_{uk}^i} \pi_{ij} O_i e(t) \\ &= e^T(t)\sum_{j\in S_k^i} \pi_{ij}(W_j - O_i) e(t) \\ &\quad + e^T(t)\sum_{j\in S_{uk}^i} \pi_{ij}(W_j - O_i) e(t). \end{aligned} \tag{29}$$

For the term $e^T(t)\sum_{j\in S_{uk}^i} \pi_{ij}(W_j - O_i)e(t)$, there are two scenarios for discussion.

(i) If $i = j$, then $\pi_{ij} \le 0$, and from (24) we can have

$$e^T(\mathrm{t})\sum_{j\in S_{uk}^i} \pi_{ij}(W_j - O_i) e(\mathrm{t}) < 0.$$

(ii) If $i \ne j$, then $\pi_{ij} \ge 0$, and from (25) we can have

$$e^T(t)\sum_{j\in S_{uk}^i} \pi_{ij}(W_j - O_i) e(t) < 0.$$

Thus, from (i) and (ii), we can conclude that

$$e^T(t)\sum_{j\in S_{uk}^i} \pi_{ij}(W_j - O_i) e(t) < 0. \tag{30}$$

So similarly, it's easy to derive

$$x^T(t)\sum_{j\in S_{uk}^i} \pi_{ij}(P_j - Q_i) x(t) < 0. \tag{31}$$

Substitute (30) and (31) into (28), we have

$$\begin{aligned} &\ell V_\zeta(\zeta(t), i) \\ &\le x^T(t)\left[P_i(A_i + B_i K_i) + (A_i + B_i K_i)^T P_i\right] x(t) \\ &\quad + 2x^T(t) P_i D_i \eta(t) - 2x^T(t) P_i D_i v_{xi}(t) \\ &\quad + \dot{\beta}_{xi}(t)(\beta_{xi}(t) - \beta_{xi}{}^*) \\ &\quad + 2x^T(t) P_i f_i(x(t)) + 2e^T(t) W_i \tilde{f}_i(x(t), \hat{x}(t)) \\ &\quad + e^T(t)\left[W_i(A_i - L_i C_i) + (A_i - L_i C_i)^T W_i\right] e(t) \\ &\quad + 2e^T(t) W_i D_i \eta(t) - 2e^T(t) W_i D_i v_{ei}(t) \\ &\quad + \dot{\beta}_{ei}(t)(\beta_{ei}(t) - \beta_{ei}{}^*) \\ &\quad + x^T(t)\sum_{j\in S_k^i} \pi_{ij}(P_j - Q_i) x(t) \\ &\quad + e^T(t)\sum_{j\in S_k^i} \pi_{ij}(W_j - O_i) e(t). \end{aligned} \tag{32}$$

Furthermore, notice that $\pi_{ij} = \hat{\pi}_{ij} + \Delta\pi_{ij}, |\Delta\pi_{ij}| \le \varepsilon_{ij}$ and (21), we have

$$\begin{aligned} &\sum_{j\in S_k^i} \pi_{ij}(P_i - Q_i) \\ &= \sum_{j\in S_k^i} (\hat{\pi}_{ij} + \Delta\pi_{ij})(P_j - Q_i) \\ &= \sum_{j\in S_k^i} \Big[\hat{\pi}_{ij}(P_j - Q_i) + (\Delta\pi_{ij} + \varepsilon_{ij})(P_j - Q_i) \\ &\quad - \varepsilon_{ij}(P_j - Q_i)\Big] \\ &\le \sum_{j\in S_k^i} \hat{\pi}_{ij}(P_j - Q_i) + \sum_{j\in S_k^i} 2\varepsilon_{ij} M_{ij} - \sum_{j\in S_k^i} \varepsilon_{ij}(P_j - Q_i). \end{aligned} \tag{33}$$

In the same way, we can derive from (24) that

$$\sum_{j\in S_k^i} \pi_{ij}(W_i - O_i)$$

$$\leq \sum_{j\in S_k^i}\hat{\pi}_{ij}(W_j-O_i)+\sum_{j\in S_k^i}2\varepsilon_{ij}N_{ij}-\sum_{j\in S_k^i}\varepsilon_{ij}(W_j-O_i). \tag{34}$$

Therefore, noticing (33) and (34), then (32) can result in

$$\begin{aligned}
&\ell V_\zeta(\zeta(t),i)\\
&\leq x^T(t)\Big[P_i(A_i+B_iK_i)+(A_i+B_iK_i)^TP_i\\
&\quad+\sum_{j\in S_k^i}\hat{\pi}_{ij}(P_j-Q_i)+\sum_{j\in S_k^i}2\varepsilon_{ij}M_{ij}\\
&\quad-\sum_{j\in S_k^i}\varepsilon_{ij}(P_j-Q_i)\Big]x(t)+2x^T(t)P_if_i(x(t))\\
&\quad+2x^T(t)P_iD_i\eta(t)-2x^T(t)P_iD_iv_{xi}(t)\\
&\quad+\dot{\beta}_{xi}(t)(\beta_{xi}(t)-\beta_{xi}{}^*)+e^T(t)\Big[W_i(A_i+B_iK_i)\\
&\quad+(A_i+B_iK_i)^TW_i+\sum_{j\in S_k^i}\hat{\pi}_{ij}(W_j-Q_i)\\
&\quad+\sum_{j\in S_k^i}2\varepsilon_{ij}M_{ij}-\sum_{j\in S_k^i}\varepsilon_{ij}(W_j-O_i)\Big]e(t)\\
&\quad+2e^T(t)W_i\tilde{f}_i(x(t),\hat{x}(t))+2e^T(t)W_iD_i\eta(t)\\
&\quad-2e^T(t)W_iD_iv_{ei}(t)+\dot{\beta}_{ei}(t)(\beta_{ei}(t)-\beta_{ei}{}^*).
\end{aligned}\tag{35}$$

Based on (14) and (16), we have

$$\begin{aligned}
&2e^T(t)W_iD_i\eta(t)-2e^T(t)W_iD_iv_{ei}(t)\\
&\quad+\dot{\beta}_{ei}(t)(\beta_{ei}(t)-\beta_{ei}{}^*)\\
&=2S_{v_{ei}}^T(t)\eta(t)-2\beta_{ei}(t)\left\|S_{v_{ei}}(t)\right\|\\
&\quad+\dot{\beta}_{ei}(t)(\beta_{ei}(t)-\beta_{ei}{}^*)\\
&\leq 2\left\|S_{v_{ei}}(t)\right\|\rho_e-2\beta_{ei}(t)\left\|S_{v_{ei}}(t)\right\|\\
&\quad+2\beta_{ei}{}^*\left\|S_{v_{ei}}(t)\right\|-2\beta_{ei}{}^*(t)\left\|S_{v_{ei}}(t)\right\|\\
&\quad+\alpha_{ei}\left\|S_{v_{ei}}(t)\right\|\operatorname{sgn}(\left\|S_{v_{ei}}(t)\right\|-\tau_{ei})(\beta_{ei}(t)-\beta_{ei}{}^*)\\
&\leq 2(\rho_e-\beta_{ei}{}^*)\left\|S_{v_{ei}}(t)\right\|-2(\beta_{ei}(t)-\beta_{ei}{}^*)\left\|S_{v_{ei}}(t)\right\|\\
&\quad+\alpha_{ei}\left\|S_{v_{ei}}(t)\right\|\operatorname{sgn}(\left\|S_{v_{ei}}(t)\right\|-\tau_{ei})(\beta_{ei}(t)-\beta_{ei}{}^*).
\end{aligned}\tag{36}$$

In the same way, we can derive that

$$\begin{aligned}
&2x^T(t)P_iD_i\eta(t)-2x^T(t)P_iD_iv_{xi}(t)\\
&\quad+\dot{\beta}_{xi}(t)(\beta_{xi}(t)-\beta_{xi}{}^*)\\
&\leq 2(\rho_x-\beta_{xi}{}^*)\left\|S_{v_{xi}}(t)\right\|-2(\beta_{xi}(t)-\beta_{xi}{}^*)\left\|S_{v_{xi}}(t)\right\|\\
&\quad+\alpha_{xi}\left\|S_{v_{xi}}(t)\right\|\operatorname{sgn}(\left\|S_{v_{xi}}(t)\right\|-\tau_{xi})(\beta_{xi}(t)-\beta_{xi}{}^*).
\end{aligned}\tag{37}$$

For the sign function in (36) and (37), we handle it in different ways in two cases.

Case 1: Suppose that $\|S_{v_{xi}}(t)\|\geq\tau_{xi}$ and $\|S_{v_{ei}}(t)\|\geq\tau_{ei}$, then with the aid of Lemma 1, (36) and (37) can be written as

$$\begin{aligned}
&2x^T(t)P_iD_i\eta(t)-2x^T(t)P_iD_iv_{xi}(t)\\
&\quad+\dot{\beta}_{xi}(t)(\beta_{xi}(t)-\beta_{xi}{}^*)\\
&\leq 2(\rho_x-\beta_{xi}{}^*)\left\|S_{v_{xi}}(t)\right\|\\
&\quad+(\alpha_{xi}-2)(\beta_{xi}(t)-\beta_{xi}{}^*(t))\left\|S_{v_{xi}}(t)\right\|\\
&<0,
\end{aligned}$$

and

$$\begin{aligned}
&2e^T(t)P_iD_i\eta(t)-2e^T(t)P_iD_iv_{ei}(t)\\
&\quad+\dot{\beta}_{ei}(t)(\beta_{ei}(t)-\beta_{ei}{}^*)\\
&\leq 2(\rho_e-\beta_{ei}{}^*)\left\|S_{v_{ei}}(t)\right\|\\
&\quad+(\alpha_{ei}-2)(\beta_{ei}(t)-\beta_{ei}{}^*)\left\|S_{v_{ei}}(t)\right\|\\
&<0.
\end{aligned}$$

From Lemma 1 and the definitions of α_{xi} and α_{ei}, (35) becomes

$$\begin{aligned}
&\ell V_\zeta(\zeta(t),i)\\
&\leq x^T(t)\Big[P_i(A_i+B_iK_i)+(A_i+B_iK_i)^TP_i\\
&\quad+\sum_{j\in S_k^i}\hat{\pi}_{ij}(P_j-Q_i)+\sum_{j\in S_k^i}2\varepsilon_{ij}M_{ij}\\
&\quad-\sum_{j\in S_k^i}\varepsilon_{ij}(P_j-Q_i)\Big]x(t)+2x^T(t)P_if_i(x(t))\\
&\quad+e^T(t)\Big[W_i(A_i+B_iK_i)+(A_i+B_iK_i)^TW_i\\
&\quad+\sum_{j\in S_k^i}\hat{\pi}_{ij}(W_j-O_i)+\sum_{j\in S_k^i}2\varepsilon_{ij}N_{ij}\\
&\quad-\sum_{j\in S_k^i}\varepsilon_{ij}(W_j-O_i)\Big]e(t)+2e^T(t)W_i\tilde{f}_i(x(t),\hat{x}(t)).
\end{aligned}\tag{38}$$

On the other hand, based on Definition 1 and 2, we can get

$$\begin{aligned}
&\upsilon_{1i}\delta_ix^T(t)x(t)-\frac{\upsilon_{1i}}{2}x^T(t)f_i(x(t))-\frac{\upsilon_{1i}}{2}f_i^T(x(t))x(t)\\
&\geq 0,
\end{aligned}\tag{39}$$

$$\begin{aligned}
&\upsilon_{2i}\sigma_ix^T(t)x(t)+\frac{\upsilon_{2i}}{2}\gamma_ix^T(t)f_i(x(t))\\
&\quad+\frac{\upsilon_{2i}}{2}\gamma_if_i^T(x(t))x(t)-\upsilon_{2i}f_i^T(x(t))f_i(x(t))\\
&\geq 0,
\end{aligned}\tag{40}$$

$$\begin{aligned}
&\kappa_{i1}\delta_ie^T(t)e(t)-\frac{\kappa_{i1}}{2}e^T(t)\tilde{f}_i(x(t),\hat{x}(t))\\
&\quad-\frac{\kappa_{i1}}{2}\tilde{f}_i^T(x(t),\hat{x}(t))e(t)\\
&\geq 0,
\end{aligned}\tag{41}$$

$$\begin{aligned}
&\kappa_{i2}\sigma_ie^T(t)e(t)-\kappa_{i2}\tilde{f}_i^T(x(t),\hat{x}(t))\tilde{f}_i(x(t),\hat{x}(t))\\
&\quad+\frac{\kappa_{i2}}{2}\gamma_ie^T(t)\tilde{f}_i(x(t),\hat{x}(t))\\
&\quad+\frac{\kappa_{i2}}{2}\gamma_i\tilde{f}_i^T(x(t),\hat{x}(t))e(t)\\
&\geq 0,
\end{aligned}\tag{42}$$

$$\ell V_\zeta(\zeta(t),i) \le \begin{bmatrix} x(t) \\ f_i(x(t)) \\ e(t) \\ \tilde{f}_i(x(t),\hat{x}(t)) \end{bmatrix}^T \begin{bmatrix} \Upsilon_{1i}-\varphi_i I & P_i+\frac{\gamma_i \upsilon_{2i}-\upsilon_{1i}}{2}I & 0 & 0 \\ * & -\upsilon_{2i}I & 0 & 0 \\ * & * & \Upsilon_{2i}-\varphi_i I & W_i+\frac{\gamma_i \kappa_{2i}-\kappa_{1i}}{2}I \\ * & * & * & -\kappa_{2i}I \end{bmatrix} \begin{bmatrix} x(t) \\ f_i(x(t)) \\ e(t) \\ \tilde{f}_i(x(t),\hat{x}(t)) \end{bmatrix}. \quad (43)$$

where υ_{1i}, υ_{2i}, κ_{1i} and κ_{2i} are some positive constants.

By adding (39)-(42) into the right-hand side of (38), (43) can be readily obtained, which is at the top of this page.

According to Theorem 1, we obtain that

$$\begin{bmatrix} \Upsilon_{1i} & P_i+\frac{\gamma_i \upsilon_{2i}-\upsilon_{1i}}{2}I & 0 & 0 \\ * & -\upsilon_{2i}I & 0 & 0 \\ * & * & \Upsilon_{2i} & W_i+\frac{\gamma_i \kappa_{2i}-\kappa_{1i}}{2}I \\ * & * & * & -\kappa_{2i}I \end{bmatrix} < 0. \quad (44)$$

Therefore, (43) combining with (44) implies that

$$\begin{aligned} \ell V_\zeta(\zeta(t),i) &\le \vartheta^T(t) \begin{bmatrix} -\varphi_i I & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & -\varphi_i I & 0 \\ * & * & * & 0 \end{bmatrix} \vartheta(t) \\ &= -\varphi_i \zeta^T(t)\zeta(t), \end{aligned} \quad (45)$$

where $\vartheta(t) = \begin{bmatrix} x(t) & f_i(x(t)) & e(t) & \tilde{f}_i(x(t),\hat{x}(t)) \end{bmatrix}^T$. Denote that $\varphi = \min_{i\in S}\{\varphi_i\}$, then

$$\ell V_\zeta(\zeta(t),i) \le -\varphi_i \zeta^T(t)\zeta(t) \le -\varphi \zeta^T(t)\zeta(t). \quad (46)$$

Therefore, by using Dynkin's formula [39], we obtain

$$\begin{aligned} &E\left\{\int_0^t \ell V_\zeta(\zeta(s),r(s))d\zeta\right\} \\ &= E\left\{V_\zeta(\zeta(t),r(t))\right\} - E\left\{V_{\zeta 0}\right\}, \end{aligned} \quad (47)$$

where $V_{\zeta 0}$ is the initial value of $V_\zeta(\zeta(t),r(t))$.

Employing (46) and (47), we can get

$$\begin{aligned} &E\left\{V_\zeta(\zeta(t),r(t))\right\} - E\left\{V_{\zeta 0}\right\} \\ &\le -\varphi E\left\{\int_0^t V_\zeta(\zeta(s),r(s))ds\right\}. \end{aligned} \quad (48)$$

Dividing both sides of (50) by $-\varphi$, we can get

$$\begin{aligned} &E\left\{\int_0^t V_\zeta(\zeta(s),r(s))ds\right\} \\ &\le \frac{1}{-\varphi}\left[E\left\{V_\zeta(\zeta(t),r(t))\right\} - E\left\{V_{\zeta 0}\right\}\right] \\ &\le \frac{E\left\{V_{\zeta 0}\right\}}{\varphi}, \end{aligned} \quad (49)$$

which implies that

$$E\left\{\int_0^\infty \|\zeta(t)\|^2 dt|\zeta_0, r_0\right\} < \infty \quad (50)$$

by noticing $\varphi > 0$. It means that the error dynamics (12) and the augmented system (17) are stochastically stable according to Definition 3.

Case 2: Suppose now $\|S_{v_{xi}}(t)\| \le \tau_{xi}$ and $\|S_{v_{ei}}(t)\| \le \tau_{ei}$, then $\ell V_\zeta(\zeta(t),i) \le -\varphi\zeta^T(t)\zeta(t)$ will sign indefinite and this will lead to instability of the closed-loop system temporarily. The instability may cause that $\|S_{v_{xi}}(t)\|$ increases over τ_{xi} and $\|S_{v_{ei}}(t)\|$ increases over τ_{ei}. As soon as $\|S_{v_{xi}}(t)\|$ becomes greater than τ_{xi} and $\|S_{v_{ei}}(t)\|$ becomes greater than τ_{ei}, $\ell V_\zeta(\zeta(t),i) \le -\varphi\zeta^T(t)\zeta(t)$ will be satisfied as discussed in case 1.

In addition, the conditions in (8) and (9) can be converted into a minimization problem based on some LMI constraints. The linear equality condition (8) is in equivalent to

$$tr((D_i^T P_i - H_i C_i)^T (D_i^T P_i - H_i C_i)) = 0.$$

Introduce the conditions

$$(D_i^T P_i - H_i C_i)^T (D_i^T P_i - H_i C_i) < \psi_i I,$$

where ψ_i is a positive scalar, and by Schurs complement it is equivalent to

$$\begin{bmatrix} -\psi_i I & (D_i^T P_i - H_i C_i)^T \\ * & -I \end{bmatrix} < 0.$$

In the same way, according to the linear equality condition (9), we can derive that

$$\begin{bmatrix} -\psi_i I & (D_i^T W_i - G_i C_i)^T \\ * & -I \end{bmatrix} < 0.$$

Therefore, the problems of solving the existence conditions of the designed observer and controller are now converted into the problems of finding the global minimization problems:

$$\min \ \psi_i \ \text{subject to } (8), (9) \text{ and Theorem 1.} \quad (53)$$

Remark 7: From observing the derivations (36) and (37), we add the terms $\frac{1}{2}(\beta_{xi}(t)-\beta_{xi}^*)^2$ and $\frac{1}{2}(\beta_{ei}(t)-\beta_{ei}^*)^2$

into the Lyapunov function in order to cope with the unknown input terms $2x^T(t)P_iD_i\eta(t)$ and $2e^T(t)W_iD_i\eta(t)$ existing in the error dynamics (12) and closed-loop system (17).

Remark 8: It is worth noting that the auxiliary matrices Q_i and O_i are important since they can provide more freedom for the inequality constraints (19)-(26).

Remark 9: In this paper, the proposed method possesses generality as it can be directly applied to the Markovian jump systems with BUTRs and PUTRs. In particular, (18), (21) and (24)-(26) are the existing conditions for BUTR case; (18)-(20), (22)-(23) and (25)-(26) are for PUTR case by setting $\pi_{ij} = \hat{\pi}_{ij}$ and $\varepsilon_{ij} = 0$ for $i, j \in S$. □

5. SIMULATION STUDY

5.1. Practical example study

In this section, a practical example will be provided to demonstrate the effectiveness of the proposed to approach. Consider the linearized model of an F-404 aircraft engine system in [34]

$$A(t) = \begin{bmatrix} -1.46 & 0 & 2.428 \\ 0.1643+0.5\theta(t) & -0.4+\theta(t) & -0.3788 \\ 0.3107 & 0 & -2.23 \end{bmatrix}$$

with $\theta(t)$ being an uncertain model parameter. Let $\theta(t)$ be subjected to a Markov process $r(t)$ with $s = 3$, described as $\theta(t) = \begin{cases} 0, & r=1, \\ -1, & r=2, \\ -2, & r=3. \end{cases}$

The transition rate matrix is chosen as

$$\Pi = \begin{bmatrix} -1+\Delta\pi_{11} & ? & ? \\ 2 & -3+\Delta\pi_{22} & ? \\ 0.1+\Delta\pi_{31} & 0.1+\Delta\pi_{32} & -0.2+\Delta\pi_{33} \end{bmatrix}.$$

Other coefficient matrices are set as follows:

$$B_1 = \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.1 \\ -0.1 \\ -1 \end{bmatrix},\ B_3 = \begin{bmatrix} 0.3 \\ 0 \\ 1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.1 \\ -0.1 \\ -1 \end{bmatrix},\ D_3 = \begin{bmatrix} 0.3 \\ 0 \\ 1 \end{bmatrix},$$
$$C_1 = C_2 = C_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Assume that $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{31} = \varepsilon_{32} = \varepsilon_{33} = 1$ and $\kappa_{1i} = \upsilon_{1i} = 0.5$ and $\kappa_{2i} = \upsilon_{2i} = 0.6$ and $\varphi_i = 1$, $i = 1,2,3$.
We assume the $f(x,u,i) = 3.33sin(x)$, $i = 1,2,3$ and by the expression and Definition 1 and 2, we can prove that

$$f_i^T(x(t))x(t) \le 3.33\|x(t)\|^2,$$

$$f_i^T(x(t))f_i(x(t)) \le 11.2\|x(t)\|^2.$$

So we can define that $\delta_i = 3.33$, $\sigma_i = 11.2$, and $\gamma_i = 0$, $i = 1,2,3$.
Next, solving (18)-(26) yields

$$P_1 = \begin{bmatrix} 59.1650 & -1.0503 & 14.2392 \\ -1.0503 & 53.0335 & 15.6147 \\ 14.2392 & 15.6147 & 56.7398 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 9.9569 & -0.0510 & 0.0972 \\ -0.0510 & 9.8800 & 0.1586 \\ 0.0972 & 0.1586 & 9.8794 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 5.5216 & 0.0021 & -0.111 \\ 0.0021 & 5.4937 & 0.0361 \\ -0.0111 & 0.0361 & 5.5080 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 20.2979 & -0.0091 & 0.0187 \\ -0.0091 & 20.2831 & 0.0283 \\ 0.0187 & 0.0283 & 20.2821 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 14.4492 & 0.0042 & -0.0142 \\ 0.0042 & 14.4356 & 0.0073 \\ -0.0142 & 0.0073 & 14.4500 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 41.6633 & -0.0018 & 0.0027 \\ -0.0018 & 41.6639 & 0.0031 \\ 0.0027 & 0.0031 & 41.6614 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 26.3589 & -0.6638 & 6.3016 \\ -0.6638 & 24.5583 & 7.2155 \\ 6.3016 & 7.2155 & 25.4580 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 12.8143 & -0.3878 & 1.0631 \\ -0.3878 & 12.4159 & 1.3967 \\ 1.0631 & 1.3967 & 12.2690 \end{bmatrix},$$

$$W_3 = \begin{bmatrix} 8.4069 & -0.1517 & 0.4467 \\ -0.1517 & 8.2914 & 0.6101 \\ 0.4467 & 0.6101 & 8.1221 \end{bmatrix},$$

$$O_1 = \begin{bmatrix} 30.8718 & -0.0683 & 0.1907 \\ -0.0683 & 30.8085 & 0.2488 \\ 0.1907 & 0.2488 & 30.7720 \end{bmatrix},$$

$$O_2 = \begin{bmatrix} 26.3009 & -0.0544 & 0.1425 \\ -0.0544 & 26.2673 & 0.1847 \\ 0.1425 & 0.1847 & 26.2144 \end{bmatrix},$$

$$O_3 = \begin{bmatrix} 17.8571 & -0.0018 & 0.0025 \\ -0.0018 & 17.8565 & 0.0029 \\ 0.0025 & 0.0029 & 17.8552 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 397.3631 & 30.4289 & -110.2430 \\ 46.0398 & 364.5661 & -105.5184 \\ -121.7591 & -90.9029 & 412.9625 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 2194.7245 & 17.8486 & -65.7473 \\ 21.2350 & 1888.8452 & 55.4966 \\ -67.9463 & 58.8492 & 2087.3421 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 3.9119*10^3 & 8.7327 & -59.4770 \\ 8.7683 & 3.4744*10^3 & 112.0319 \\ -59.3017 & 113.5079 & 3.7446*10^3 \end{bmatrix},$$

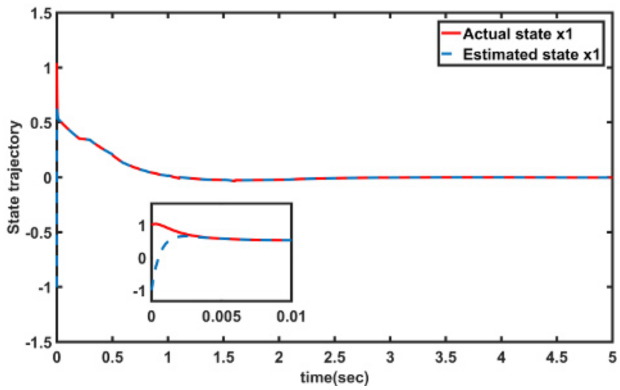

Fig. 1. State estimation of $x_1(t)$ with generally uncertain TRs.

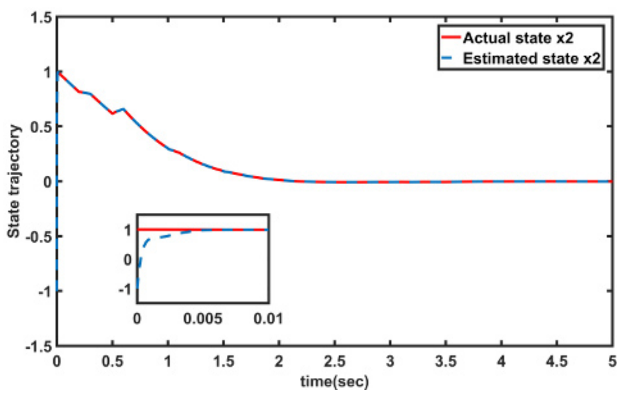

Fig. 2. State estimation of $x_2(t)$ with generally uncertain TRs.

$$K_1 = \begin{bmatrix} -193.54 & -648.14 & -269.81 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 106.79 & 200.11 & 509.40 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -267.74 & -118.24 & -462.59 \end{bmatrix}.$$

For this simulation, the unknown input is assumed to be $\eta(t) = 5\sin(t)$. Fig. 4 shows the switching signal $r(t)$. According to Figs. 1-3, we can see that the estimated values of states can track the real values quickly and precisely, which implies that the designed observer can be employed successfully. In addition, all the state trajectories shown in Figs. 1-3 converge to zero, which means that the stochastic stability of the closed-loop system can be guaranteed.

5.2. Compared study 1

In order to verify the generality of the proposed approach, we modify the transition rate matrix (2) to BUTR type as $\Pi = \begin{bmatrix} -1+\Delta\pi_{11} & 0.5+\Delta\pi_{12} & 0.5+\Delta\pi_{13} \\ 2+\Delta\pi_{21} & -3+\Delta\pi_{22} & 1+\Delta\pi_{23} \\ 0.1+\Delta\pi_{31} & 0.1+\Delta\pi_{32} & -0.2+\Delta\pi_{33} \end{bmatrix}$. In this case, the existence conditions in Theorem 1 become (18), (21) and (24)-(26). According to Figs. 5-7, we can see that the proposed method can be applied to MJSs with BUTRs successfully, as discussed in Remark 9.

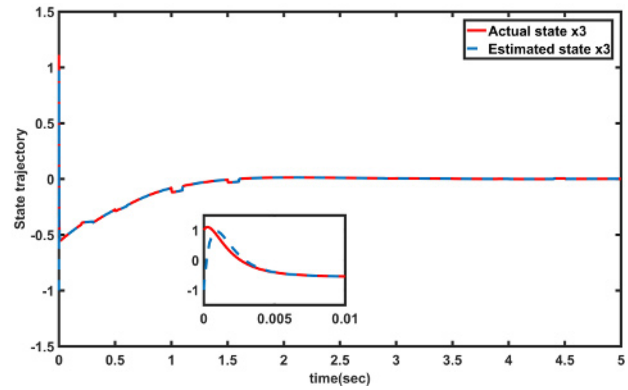

Fig. 3. State estimation of $x_3(t)$ with generally uncertain TRs.

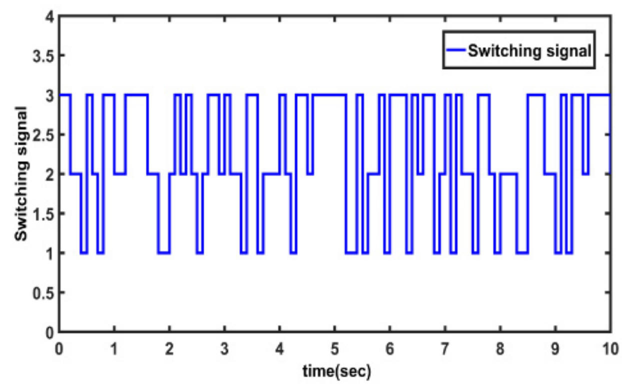

Fig. 4. Switching signal.

5.3. Compared study 2

We modify the transition rate matrix (2) to PUTR type as $\Pi = \begin{bmatrix} -1 & ? & ? \\ 2 & -3 & ? \\ 0.1 & 0.1 & -0.2 \end{bmatrix}$. In this case, the existence conditions in Theorem 1 become ((18)-(20), (22)-(23) and (25)-(26) with $\pi_{ij} = \hat{\pi}_{ij}$ and $\varepsilon_{ij} = 0$ for $i, j \in S$. According to Figs. 8-10, we can see that the estimation and control performances are both satisfactory.

6. CONCLUSION

In this paper, we investigate the problem of designing robust observer and fault-tolerant controller for one-sided Lipschitz Markovian jump systems with general uncertain transition rates against the existing unknown input. First, a robust observer involved sliding mode control terms is proposed to provide the estimations of the states. Then a fault-tolerant controller, by virtue of sliding mode technique, is proposed to stabilize the closed-loop system. Finally, the simulations validate the effectiveness of the proposed method. In future research, we will develop some other control methods, such as networked control [40], sliding mode control [41], sampled-data control

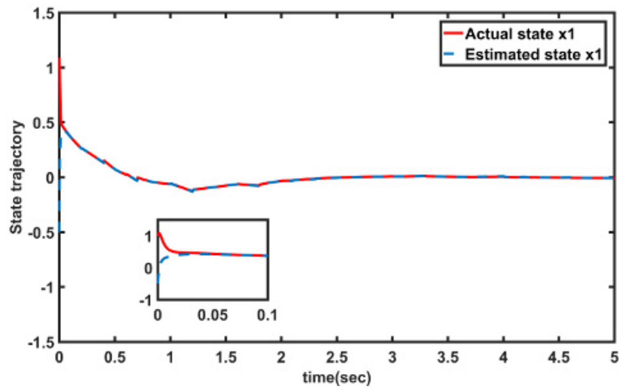

Fig. 5. State estimation of $x_1(t)$ with bounded uncertain TRs.

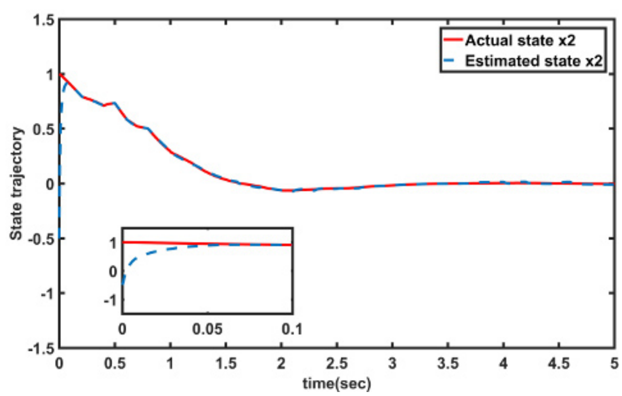

Fig. 6. State estimation of $x_2(t)$ with bounded uncertain TRs.

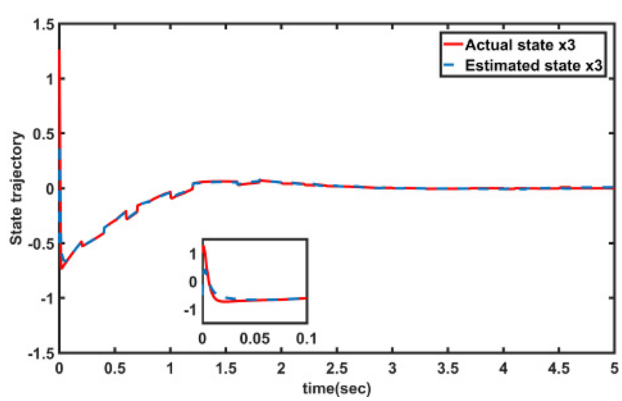

Fig. 7. State estimation of $x_3(t)$ with bounded uncertain TRs.

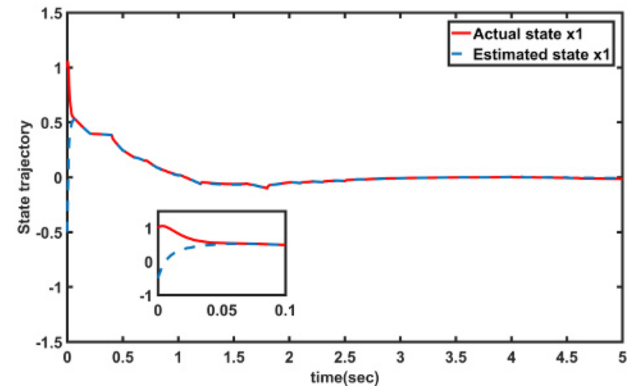

Fig. 8. State estimation of $x_1(t)$ with partially unknown TRs.

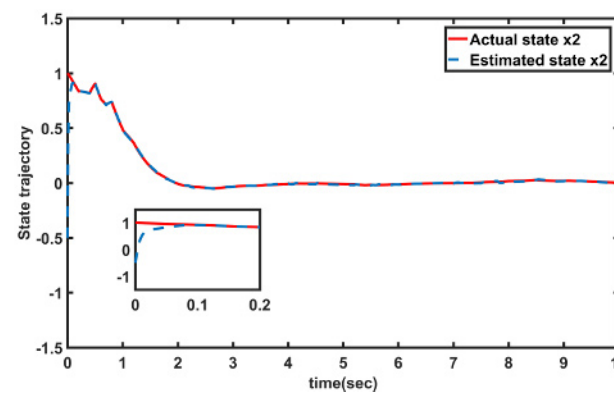

Fig. 9. State estimation of $x_2(t)$ with partially unknown TRs.

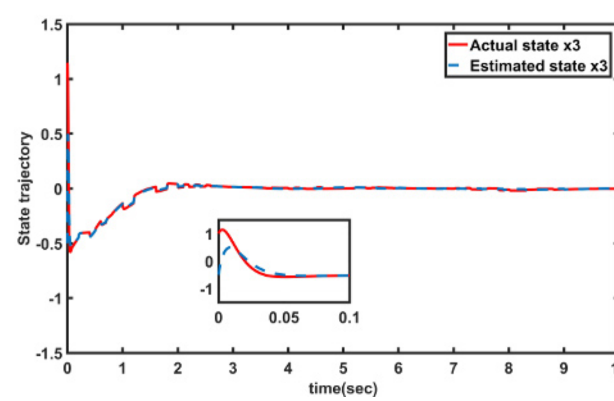

Fig. 10. State estimation of $x_3(t)$ with partially unknown TRs.

[42, 43] and event-based control [44], for the considered system.

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