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Iterative Learning Control for Leader-following Consensus of Nonlinear Multi-agent Systems with Packet Dropout

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Abstract: In this paper, the consensus problem of leader-following nonlinear multi-agent systems with packet dropout is addressed. The iterative learning control method is applied to design the control protocol. Then, a distributed control protocol is presented, and a sufficient condition is derived. In addition, the Bernoulli distribution process is introduced to model the packet dropout case, where the dropout rate is converted into a stochastic parameter. The convergence of proposed control protocol is analyzed by norm theory. It is proved that, when there exists the packet dropout, the output of all the following agents can track the trajectory of leader under the proposed control protocol. Finally, two examples are provided to illustrate the validity of the theoretical analysis.

Keywords: Consensus control, iterative learning control, multi-agent systems, packet dropout.

1. INTRODUCTION

In the past decades, the cooperation control problems of multi-agent systems have been paid outstanding attention in many fields, such as robotics control systems, unmanned aerial vehicle engineering, autonomous vehicle systems [1–5], and so forth. The key problem of cooperative control is consensus, which is to design an appropriate control law such that the states output of a group of agents eventually reach agreement.

Currently, consensus problems of multi-agent systems have been increasing concerned by many researchers. The consensus problems of first-, second- and high-order linear multi-agent systems have been studied in [6–9]. Compared with the linear multi-agent systems, many literatures have focused on the consensus problems of multi-agent systems with nonlinear dynamics. In [10], the tracking problem of nonlinear multi-agent systems was concerned. The consensus problems of high-order nonlinear multi-agent systems were considered in [11, 12], and the formation control problems of nonlinear multi-agent systems were dealt with in [13, 14]. However, it should be noted that all of the above works considered being an ideal condition, that is, the communication between the agents is fully connected.

In practical systems, due to the actuator suspension, bandwidth constraint and external disturbance, the phenomenon of data packet may be occurred. Therefore, it is worth concerning that the packet dropout problem caused by the unreliable wireless communication. Some research results have been seen in the existing papers. In [15, 16], the stabilization problems of networked control systems with packet dropout were analyzed, and [17] designed the stabilizing controller for the network-induced delay and random packet dropout problems. Nevertheless, due to the packet transmission failure, the case of packet dropout may be existed among agents. The packet dropout problem of a class of linear multi-agent systems with communication link failure over the network was developed in [18]. In [19], the consensus problem of leader-following multi-agent systems with packet dropout was addressed. In [20], the consensus control of heterogeneous multi-agent systems with packet dropout was studied. Moreover, the sampled-data consensus problem of linear multi-agent systems with packet losses was investigated in [21]. However, the dynamics of multi-agent systems considered in [18–21] are linear. Hence, the first motivation of this work is to address the consensus problem of nonlinear multi-agent systems with packet dropout.

As we known, iterative learning control is based on the idea that the performance of a system that performs the same task repeatedly can be improved by learning from previous iterations [22]. It is first proposed by Arimoto *et al*. in 1984 [23]. Due to its simplicity and the ability for the problems with nonlinear and uncertainties, iterative learning control has been attracted considerable interest

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since it was proposed. In the early works, it was used to deal with the trajectory tracking problem [24], the data dropout problem [25], and the satellite attitude tracking control problem [26]. By now, the method has been extended to achieve the consensus of multi-agent systems. In [27, 28], the iterative learning control protocols for the consensus problem of multi-agent systems with nonlinear dynamics were proposed; the formation control of multi-agent systems with iterative learning control was addressed in [29]; and the consensus tracking control problems of high-order nonlinear multi-agent systems with iterative learning control were studied in [30, 31].

It should be pointed out that, the packet dropout problem of networked control systems has been paid some attention, but the consensus problem of multi-agent systems with packet dropout has gained little concerned. For an actual multi-agent system, the data dropout may occur due to the external disturbances or oversaturated network links. In view of the characteristics of iterative learning control, the consensus problem of multi-agent systems with packet dropout is discussed by applying the iterative learning control approach, which is the second motivation of this work.

Inspired by the above discussions, in this work, we divert our attention to address the iterative learning control for the consensus problem of leader-following nonlinear multi-agent systems with packet dropout. The main contributions are outlined as follows: (i) compared with [18–21], the consensus problem of a class of leader-following nonlinear multi-agent systems with packet dropout is investigated in this paper. It is supposed that only a portion of following agents can receive the leader's information. In addition, the nonlinear terms of agents are considered to satisfy the globally Lipschitz condition; (ii) the Bernoulli distribution process is introduced to describe the packet dropout process, in which the dropout rate is converted into a stochastic parameter. Therefore, the consensus problem of multi-agent systems can be thought as an equivalent asymptotic stability problem; (iii) the iterative learning control method is applied to design the control protocol. A distributed iterative learning control protocol is proposed, and a sufficient condition is derived. Then, the convergence of the proposed control protocol is analyzed by norm theory. It is proved that, when there exists the packet dropout, the consensus problem can be solved under the presented control protocol.

The rest of this paper is organized as follows: In Section 2, graph theory, some useful definitions and lemmas are introduced. In Section 3, the problem formulation on the leader-following nonlinear multi-agent systems is described. Then, the control protocol design and convergence analysis are discussed in Section 4, and the simulations analysis is provided in Section 5. Finally, some conclusions and future works are briefly drawn in Section 6.

2. PRELIMINARIES

2.1. Graph theory

A multi-agent system consists of n agents, the exchange information among agents may be modeled as an interaction directed graph with n vertices. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \boldsymbol{\mathcal{A}})$ denote the directed graph, where the set of vertices is $\mathcal{V} = \{v_1, \cdots, v_n\}$ and the set of edges is $\mathcal{E} = \{(i, j), i, j \in \mathcal{V}, \text{ and } i \neq j\}$. The weighted adjacency matrix of the graph $\mathcal{G}$ is defined as $\boldsymbol{\mathcal{A}} = [a_{ij}] \in R^{n\times n}$, where $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. In addition, it is assumed that $a_{ii} = 0$. The vertex j calls the neighbor of i, if i receives the information from j. The set of neighbors of i is defined as $\mathcal{N}_i = \{v_j : (v_j, v_i) \in \mathcal{E}\}$. The Laplacian matrix is $\boldsymbol{\mathcal{L}} = \boldsymbol{\mathcal{D}} - \boldsymbol{\mathcal{A}}$, where $\boldsymbol{\mathcal{D}} = \mathrm{diag}\{d_1, \cdots, d_n\}$ with $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. The graph $\mathcal{G}$ is connected if there exists a path between any two vertices.

In this paper, the augmented graph $\bar{\mathcal{G}}$ with n following agents whose information topology graph is $\mathcal{G}$ and one leader agent is considered. The interconnection weighted matrix between following agents and the leader is defined as $\boldsymbol{\mathcal{B}} = \mathrm{diag}\{b_1, \cdots, b_n\}$, where $b_i > 0$ if the following agent i receives the information from the leader, and $b_i = 0$ otherwise. Then, it can be obtained that $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{B}}$ is a matrix associated with $\bar{\mathcal{G}}$.

2.2. Some definitions and lemmas

Definition 1: A Bernoulli distribution as a sequence X_1, $X_2, \cdots$ of independent Bernoulli random variables X_i with

$$\begin{cases} \Pr(X_i = 1) = \mathrm{E}(X_i = 1) = \rho, \\ \Pr(X_i = 0) = \mathrm{E}(X_i = 0) = 1 - \rho, \end{cases}$$

where $\Pr(\cdot)$ is the probability of event; $\mathrm{E}(\cdot)$ is the numerical expectation and $\rho \in [0, 1]$.

Definition 2: The consensus of multi-agent systems is said to be achieved if, for any initial condition, there satisfies $\lim\limits_{k\to\infty} x_i^k(t) = x_0(t)$, as $t \in \{0, 1, \cdots, T\}$.

Lemma 1: For the matrix $A \in R^{n\times n}$, if there exist n eigenvalues are respectively $\lambda_1, \lambda_2, \cdots, \lambda_n$, then the spectral radius of A is defined as

$$\rho(A) = \max_{i\in\{1,\cdots,n\}} (\lambda_i).$$

Lemma 2: The matrix $A \in R^{n\times n}$ is a convergence matrix, then have $\lim_{k\to\infty} A^k \to \boldsymbol{O}$ for the necessary and sufficient condition $\rho(A) < 1$, where $\rho(A)$ represents the spectral radius of A.

3. PROBLEM FORMULATION

Consider a class of leader-following nonlinear multi-agent systems with n following agents, the dynamics of following agent i at the kth iteration are described as

$$\begin{cases} x_i^k(t+1) = f\left(x_i^k(t),t\right) + b\left(x_i^k(t),t\right)u_i^k(t), \\ y_i^k(t) = Cx_i^k(t), \end{cases} \tag{1}$$

where $x_i^k(t) = \left[x_{i1}^k(t), x_{i2}^k(t)\right]^{\mathrm{T}} \in R^2$, $u_i^k(t) \in R$ and $y_i^k(t) \in R^2$ are the state, control input and output of the following agent i at the kth iteration, respectively; $f(\cdot,\cdot): R^2 \times [0,T] \to R^2$ and $b(\cdot,\cdot): R^2 \times [0,T] \to R^2$ are nonlinear function vectors; t is the discrete time index and $t \in \{0, 1, 2, \cdots, T\}$; $C = I_2$ and I_2 is the unit matrix with 2 dimensions and $i = 1, \cdots, n$.

The matrix form of (1) is expressed as

$$\begin{cases} \boldsymbol{x}^k(t+1) = \boldsymbol{f}\left(\boldsymbol{x}^k(t),t\right) + \boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\boldsymbol{u}^k(t), \\ \boldsymbol{y}^k(t) = \bar{\boldsymbol{C}}\boldsymbol{x}^k(t), \end{cases} \tag{2}$$

where $\boldsymbol{x}^k(t) = [(x_1^k(t))^{\mathrm{T}}, \cdots, (x_n^k(t))^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{u}^k(t) = [u_1^k(t), \cdots, u_n^k(t)]^{\mathrm{T}}$, $\boldsymbol{y}^k(t) = [(y_1^k(t))^{\mathrm{T}}, \cdots, (y_n^k(t))^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{f}(\boldsymbol{x}^k(t),t) = [(f(x_1^k(t),t))^{\mathrm{T}}, \cdots, (f(x_n^k(t),t))^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{B}(\boldsymbol{x}^k(t),t) = \mathrm{diag}\{b(x_1^k(t),t), \cdots, b(x_n^k(t),t)\}$ and $\bar{\boldsymbol{C}} = C \otimes \boldsymbol{I}_n$, where $\otimes$ represents the Kronecker product.

The dynamics of leader agent are given as

$$\begin{cases} x_0(t+1) = f\left(x_0(t),t\right) + b\left(x_0(t),t\right)u_0(t), \\ y_0(t) = Cx_0(t), \end{cases} \tag{3}$$

where $x_0(t) = [x_{01}(t), x_{02}(t)]^{\mathrm{T}} \in R^2$, $u_0(t) \in R$ and $y_0(t) \in R^2$ are the state, given input and output of the leader agent, respectively; $f(x_0(t),t)$ and $b(x_0(t),t)$ are nonlinear function vectors.

Remark 1: In this paper, it is considered that the model of the multi-agent systems is only affected by the nonlinear dynamics. Actually, due to the physical factors of the sensors themselves and the external environment, the time delays and external disturbances often occur in a system. Therefore, when characterizing the actual multi-agent systems, the model proposed in this paper has certain defects. In the follow-up work, the more general multi-agent systems will be considered and studied.

To facilitate the below discussions, the following assumptions are made.

Assumption 1: The nonlinear functions $\boldsymbol{f}(\cdot,\cdot)$ and $\boldsymbol{B}(\cdot,\cdot)$ are uniformly globally Lipschitz in $\boldsymbol{x}$, that is, there exist constants $\tau_{\boldsymbol{f}}$ and $\tau_{\boldsymbol{B}}$, such that

$$\begin{aligned} &\left\|\boldsymbol{f}\left(\boldsymbol{x}_1(t),t\right) - \boldsymbol{f}\left(\boldsymbol{x}_2(t),t\right)\right\| \le \tau_{\boldsymbol{f}}\left\|\boldsymbol{x}_1(t) - \boldsymbol{x}_2(t)\right\|, \\ &\left\|\boldsymbol{B}\left(\boldsymbol{x}_1(t),t\right) - \boldsymbol{B}\left(\boldsymbol{x}_2(t),t\right)\right\| \le \tau_{\boldsymbol{B}}\left\|\boldsymbol{x}_1(t) - \boldsymbol{x}_2(t)\right\|. \end{aligned}$$

Assumption 2: The given input $u_0(t)$ is bounded, that is, there exists

$$b_{u0} = \max_{t\in[0,T]} \left\|\boldsymbol{u}_0(t)\right\|,$$

where $\boldsymbol{u}_0(t) = \boldsymbol{1}_n \otimes u_0(t)$ and $\boldsymbol{1}_n = [1, \cdots, 1]^{\mathrm{T}}$.

Assumption 3: The resetting condition is satisfied for each iteration, that is, there satisfies

$$\boldsymbol{x}^k(0) = \boldsymbol{x}^0(0), \quad k = 1, 2, \cdots, \infty,$$

where $\boldsymbol{x}^0(0)$ is the initial state of all the following agents.

Assumption 4: The packet dropout only occurs among the following agents, that is to say, there is no packet dropout between the leader and the following agents.

Remark 2: In the existing literature, there are two main methods are applied to deal with the nonlinear dynamics. One is that the nonlinear dynamics satisfies the uniformly globally Lipschitz condition; and another is that the nonlinear dynamics are approximated by the neural network. Compared with the latter, although there is a little conservative, the former is easier to deal with nonlinear terms. After weighing, we finally provide the Assumption 1 to deal with the nonlinear dynamics of the agents.

Remark 3: For the Assumption 2, the desired input signal for an actual system should be not too large; and the bounded input is usually required and easily achieved. Hence, we assume the given input is bounded. The resetting condition considered in the Assumption 3, the purpose is to simplify the analysis of convergence. For other initial conditions, such as the alignment initial condition, can be also applied.

Remark 4: In the Assumption 4, it is supposed that the packet dropout only occurs among the following agents. Considering that the consensus problem of the leader-following multi-agent systems is addressed in this paper, the information of leader agent must be obtained by some following agents. Otherwise, the consensus may not be achieved due to the packet dropout between the following agents and the leader.

In this paper, the control objective is to find a control protocol $\boldsymbol{u}^k(t)$ for the leader-following nonlinear multi-agent systems with packet dropout such that the output of all the following agents can track the trajectory of leader as k tends to infinity, that is, $x_i^k(t) \to x_0(t)$ as $t \in \{0, 1, \cdots, T\}$ for $i = 1, \cdots, n$.

4. CONTROL PROTOCOL DESIGN AND CONVERGENCE ANALYSIS

4.1. Control protocol design

During the information transmission among agents, the packet dropout is inevitably occurred, which will cause the instability and poor performance of a multi-agent system. Consider the Definition 1 and Assumption 4, the Bernoulli process for the transmitted data among following agents is described as

$$x_i^k(t) = \theta x_i^k(t) + (1-\theta)x_i^k(t-1), \tag{4}$$

where θ is a stochastic parameter satisfying Bernoulli distribution, that is, $\theta \in \{0, 1\}$. Here, it is assumed that the data packet is received as $\theta = 1$ and $\theta = 0$ is lost.

According to the multi-agent systems (1) and (3), the consensus tracking error for the ith following agent at the kth iteration is defined as

$$\varepsilon_i^k(t) = \sum_{j\in\mathcal{N}_i} a_{ij}\left(y_j^k(t)-y_i^k(t)\right)+b_i\left(y_0(t)-y_i^k(t)\right), \tag{5}$$

where $i\neq j\in\{1,\cdots,n\}$.
Substituting (4) into (5), the consensus error with stochastic parameter is expressed as

$$\begin{aligned}\varepsilon_i^k(t) &= \theta\Bigg(\sum_{j\in\mathcal{N}_i} a_{ij}C\left(x_j^k(t)-x_i^k(t)\right)+b_iC\left(x_0(t)-x_i^k(t)\right)\Bigg)\\ &\quad+(1-\theta)\Bigg(\sum_{j\in\mathcal{N}_i} a_{ij}C\left(x_j^k(t-1)-x_i^k(t-1)\right)\\ &\quad+b_iC\left(x_0(t)-x_i^k(t-1)\right)\Bigg),\end{aligned} \tag{6}$$

and the vector form of (6) is

$$\begin{aligned}\boldsymbol{\varepsilon}^k(t) =&\theta(\mathcal{H}\otimes I_2)\left(\boldsymbol{y}_0(t)-\boldsymbol{y}^k(t)\right)\\ &+(1-\theta)(\mathcal{H}\otimes I_2)\left(\boldsymbol{y}_0(t)-\boldsymbol{y}^k(t-1)\right)\\ =&\theta\bar{\mathcal{H}}\boldsymbol{e}^k(t)+(1-\theta)\bar{\mathcal{H}}\boldsymbol{e}^k(t-1),\end{aligned} \tag{7}$$

where $\boldsymbol{y}_0(t)=\mathbf{1}_n\otimes y_0(t)$, $\mathcal{H}=\mathcal{L}+\mathcal{B}$, $\bar{\mathcal{H}}=\mathcal{H}\otimes I_2$ and $\boldsymbol{e}^k(t)=\boldsymbol{y}_0(t)-\boldsymbol{y}^k(t)$.

Hence, the distributed iterative learning control protocol for the multi-agent systems (1) and (3) is designed as

$$\boldsymbol{u}^{k+1}(t)=\boldsymbol{u}^k(t)+\boldsymbol{\Gamma}\boldsymbol{\varepsilon}^k(t+1) \tag{8}$$

where $\boldsymbol{\Gamma}$ is the learning gain matrix with appropriate dimension.

4.2. Convegence analysis

In what follows, the main results of this paper are shown in Theorem 1.

Theorem 1: Consider the leader-following nonlinear multi-agent systems with packet dropout (1) and (3), and suppose that the Assumptions 1-4 are held, and the control protocol (8) is applied, if there exists $\left\|\boldsymbol{I}-\rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right\|<1$ for all t and k, then the output of all the following agents can track the trajectory of leader, that is, $\lim\limits_{k\to\infty}x_i^k(t)=x_0(t)$ as $t=0,1,\cdots,T$ for $i=1,\cdots,n$.

Proof: Substituting (7) into (8), we have

$$\begin{aligned}\boldsymbol{u}^{k+1}(t) =&\boldsymbol{u}^k(t)+\boldsymbol{\Gamma}\boldsymbol{\varepsilon}^k(t+1)\\ =&\boldsymbol{u}^k(t)+\boldsymbol{\Gamma}\left[\theta\bar{\mathcal{H}}\boldsymbol{e}^k(t+1)+(1-\theta)\bar{\mathcal{H}}\boldsymbol{e}^k(t)\right]\\ =&\boldsymbol{u}^k(t)+\theta\boldsymbol{\Gamma}\bar{\mathcal{H}}\boldsymbol{e}^k(t+1)+(1-\theta)\boldsymbol{\Gamma}\bar{\mathcal{H}}\boldsymbol{e}^k(t).\end{aligned} \tag{9}$$

Noting,

$$\begin{aligned}&\boldsymbol{e}^k(t+1)\\ &=\boldsymbol{y}_0(t+1)-\boldsymbol{y}^k(t+1)\\ &=\bar{\boldsymbol{C}}\left(\boldsymbol{x}_0(t+1)-\boldsymbol{x}^k(t+1)\right)\\ &=\bar{\boldsymbol{C}}\Big(\boldsymbol{f}\left(\boldsymbol{x}_0(t),t\right)+\boldsymbol{B}\left(\boldsymbol{x}_0(t),t\right)\boldsymbol{u}_0(t)\\ &\quad-\boldsymbol{f}\left(\boldsymbol{x}^k(t),t\right)-\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\boldsymbol{u}^k(t)\Big)\\ &=\bar{\boldsymbol{C}}\left(\boldsymbol{f}\left(\boldsymbol{x}_0(t),t\right)-\boldsymbol{f}\left(\boldsymbol{x}^k(t),t\right)\right)\\ &\quad+\bar{\boldsymbol{C}}\left(\boldsymbol{B}\left(\boldsymbol{x}_0(t),t\right)-\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right)\boldsymbol{u}_0(t)\\ &\quad+\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\Delta\boldsymbol{u}^k(t),\end{aligned} \tag{10}$$

where $\boldsymbol{f}\left(\boldsymbol{x}_0(t),t\right)=\mathbf{1}_n\otimes f\left(x_0(t),t\right)$, $\boldsymbol{B}\left(\boldsymbol{x}_0(t),t\right)=\mathrm{diag}\left\{b(x_0(t),t),\cdots,b(x_0(t),t)\right\}$, $\boldsymbol{u}_0(t)=\mathbf{1}_n\otimes u_0(t)$, and $\Delta\boldsymbol{u}^k(t)=\boldsymbol{u}_0(t)-\boldsymbol{u}^k(t)$.

Substituting $\boldsymbol{e}^k(t)$ and $\boldsymbol{e}^k(t+1)$ into (9) yields

$$\begin{aligned}\Delta\boldsymbol{u}^{k+1}(t) =&\Delta\boldsymbol{u}^k(t)-\theta\boldsymbol{\Gamma}\bar{\mathcal{H}}\boldsymbol{e}^k(t+1)-(1-\theta)\boldsymbol{\Gamma}\bar{\mathcal{H}}\boldsymbol{e}^k(t)\\ =&\Delta\boldsymbol{u}^k(t)-\theta\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\Big(\boldsymbol{f}\left(\boldsymbol{x}_0(t),t\right)-\boldsymbol{f}\left(\boldsymbol{x}^k(t),t\right.\\ &+\left(\boldsymbol{B}\left(\boldsymbol{x}_0(t),t\right)-\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right)\boldsymbol{u}_0(t)\\ &+\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\Delta\boldsymbol{u}^k(t)\Big)\\ &-(1-\theta)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\Big(\boldsymbol{f}\left(\boldsymbol{x}_0(t-1),t-1\right.\\ &-\boldsymbol{f}\left(\boldsymbol{x}^k(t-1),t-1\right)+\Big(\boldsymbol{B}\left(\boldsymbol{x}_0(t-1),t-1\right)\\ &-\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\Big)\boldsymbol{u}_0(t-1)\\ &+\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\Delta\boldsymbol{u}^k(t-1)\Big)\\ =&\left(\boldsymbol{I}-\theta\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right)\Delta\boldsymbol{u}^k(t)\\ &-(1-\theta)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\Delta\boldsymbol{u}^k(t-1)\\ &-\theta\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\Big(\left(\boldsymbol{f}\left(\boldsymbol{x}_0(t),t\right)-\boldsymbol{f}\left(\boldsymbol{x}^k(t),t\right)\right)\\ &+\left(\boldsymbol{B}\left(\boldsymbol{x}_0(t),t\right)-\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right)\boldsymbol{u}_0(t)\Big)\\ &-(1-\theta)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\Big(\Big(\boldsymbol{f}\left(\boldsymbol{x}_0(t-1),t-1\right)\\ &-\boldsymbol{f}\left(\boldsymbol{x}^k(t-1),t-1\right)\Big)+\Big(\boldsymbol{B}\left(\boldsymbol{x}_0(t-1),t-1\right)\\ &-\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\Big)\boldsymbol{u}_0(t-1)\Big).\end{aligned} \tag{11}$$

Taking norm on both sides of (11), and applying the Assumptions 1 and 2, one gets

$$\begin{aligned}&\left\|\Delta\boldsymbol{u}^{k+1}(t)\right\|\\ &\leq\left\|\boldsymbol{I}-\theta\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right\|\left\|\Delta\boldsymbol{u}^k(t)\right\|\\ &\quad+\left\|(1-\theta)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\right\|\left\|\Delta\boldsymbol{u}^k(t-1)\right\|\\ &\quad+(\tau_{\boldsymbol{f}}+\tau_{\boldsymbol{B}}b_{\boldsymbol{u}0})\left\|\theta\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\right\|\left\|\Delta\boldsymbol{x}^k(t)\right\|\\ &\quad+(\tau_{\boldsymbol{f}}+\tau_{\boldsymbol{B}}b_{\boldsymbol{u}0})\left\|(1-\theta)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\right\|\left\|\Delta\boldsymbol{x}^k(t-1)\right\|,\end{aligned} \tag{12}$$

where $\Delta\boldsymbol{x}^k(t)=\boldsymbol{x}_0(t)-\boldsymbol{x}^k(t)$.
Due to θ is independent of $\boldsymbol{x}^k(t)$, and taking expectation on both sides of (12), then we obtain

$$\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^{k+1}(t)\right\|\right\}$$

$$
\begin{aligned}
&\le \left\|\boldsymbol{I}-\rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right\| \mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t)\right\|\right\} \\
&\quad+\left\|(1-\rho)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\right\| \mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t-1)\right\|\right\} \\
&\quad+\left\|\rho\left(k_{\boldsymbol{f}}+k_{\boldsymbol{B}}b_{u0}\right)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\right\| \mathrm{E}\left\{\left\|\Delta\boldsymbol{x}^k(t)\right\|\right\} \\
&\quad+\left\|(1-\rho)\left(k_{\boldsymbol{f}}+k_{\boldsymbol{B}}b_{u0}\right)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\right\| \mathrm{E}\left\{\left\|\Delta\boldsymbol{x}^k(t-1)\right\|\right\} \\
&\le \mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t)\right\|\right\}+\mu_2\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t-1)\right\|\right\} \\
&\quad+\mu_3\mathrm{E}\left\{\left\|\Delta\boldsymbol{x}^k(t)\right\|\right\}+\mu_4\mathrm{E}\left\{\left\|\Delta\boldsymbol{x}^k(t-1)\right\|\right\}, \qquad (13)
\end{aligned}
$$

where ρ is called recovery rate and satisfies $\Pr(\theta=1)=\mathrm{E}(\theta=1)=\rho=1-\Pr(\theta=0)$; and

$$
\begin{aligned}
\mu_1&=\left\|\boldsymbol{I}-\rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right\|, \\
\mu_2&=\left\|(1-\rho)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\right\|, \\
\mu_3&=\left\|\rho\left(k_{\boldsymbol{f}}+k_{\boldsymbol{B}}b_{u0}\right)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\right\|, \\
\mu_4&=\left\|(1-\rho)\left(k_{\boldsymbol{f}}+k_{\boldsymbol{B}}b_{u0}\right)\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\right\|.
\end{aligned}
$$

Meanwhile, note that

$$
\begin{aligned}
\Delta\boldsymbol{x}^k(t)=&\boldsymbol{f}\left(\boldsymbol{x}_0(t-1),t-1\right)-\boldsymbol{f}\left(\boldsymbol{x}^k(t-1),t-1\right) \\
&+\Big(\boldsymbol{B}\left(\boldsymbol{x}_0(t-1),t-1\right) \\
&-\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\Big)\boldsymbol{u}_0(t-1) \\
&+\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\Delta\boldsymbol{u}^k(t-1). \qquad (14)
\end{aligned}
$$

Taking norm on both sides of (14), and considering Assumption 1, one has

$$
\left\|\Delta\boldsymbol{x}^k(t)\right\| \le \mu_5\left\|\Delta\boldsymbol{u}^k(t-1)\right\|+\mu_6\left\|\Delta\boldsymbol{x}^k(t-1)\right\|, \quad (15)
$$

where

$$
\begin{aligned}
\mu_5&=\left\|\boldsymbol{B}\left(\boldsymbol{x}^k(t-1),t-1\right)\right\|, \\
\mu_6&=k_{\boldsymbol{f}}+k_{\boldsymbol{B}}b_{u0}.
\end{aligned}
$$

Recursive operation for (15), one gets

$$
\left\|\Delta\boldsymbol{x}^k(t)\right\| \le \mu_5\sum_{n=1}^{t}(\mu_6)^{t-n}\left\|\Delta\boldsymbol{u}^k(n-1)\right\|, \qquad (16)
$$

where Assumption 3 is considered.

Accordingly, taking expectation on both sides of (16), we have

$$
\mathrm{E}\left\{\left\|\Delta\boldsymbol{x}^k(t)\right\|\right\} \le \mu_5\sum_{n=1}^{t}(\mu_6)^{t-n}\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(n-1)\right\|\right\}. \qquad (17)
$$

Substituting (17) into (13), it is held that

$$
\begin{aligned}
&\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^{k+1}(t)\right\|\right\} \\
&\le \mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t)\right\|\right\}+\mu_2\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t-1)\right\|\right\} \\
&\quad+\mu_3\mu_5\sum_{n=1}^{t}(\mu_6)^{t-n}\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(n-1)\right\|\right\} \\
&\quad+\mu_4\mu_5\sum_{n=1}^{t-1}(\mu_6)^{t-n-1}\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(n-1)\right\|\right\} \\
&=\mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t)\right\|\right\}+\mu_2\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t-1)\right\|\right\} \\
&\quad+\sum_{n=0}^{t-1}\eta_1(n)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(n)\right\|\right\}+\sum_{n=0}^{t-2}\eta_2(n)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(n)\right\|\right\}, \qquad (18)
\end{aligned}
$$

where

$$
\begin{aligned}
\eta_1(n)&=\mu_3\mu_5(\mu_6)^{t-n-1}, \\
\eta_2(n)&=\mu_4\mu_5(\mu_6)^{t-n-2}.
\end{aligned}
$$

Hence, the expansion expression of (18) from $t=0$ to $t=T$ is expressed as

$$
\begin{aligned}
&\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^{k+1}(0)\right\|\right\} \le \mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\}, \\
&\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^{k+1}(1)\right\|\right\} \\
&\le \mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(1)\right\|\right\}+\mu_2\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\} \\
&\quad+\eta_1(0)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\} \\
&=\left(\eta_1(0)+\mu_2\right)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\} \\
&\quad+\mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(1)\right\|\right\}, \\
&\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^{k+1}(2)\right\|\right\} \\
&\le \mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(2)\right\|\right\}+\mu_2\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(1)\right\|\right\} \\
&\quad+\left(\eta_1(0)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\}+\eta_1(1)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(1)\right\|\right\}\right) \\
&\quad+\eta_2(0)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\} \\
&=\left(\eta_1(0)+\eta_2(0)\right)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\} \\
&\quad+\left(\eta_1(1)+\mu_2\right)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(1)\right\|\right\} \\
&\quad+\mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(2)\right\|\right\}, \\
&\quad\vdots \\
&\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^{k+1}(T)\right\|\right\} \\
&\le \mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(T)\right\|\right\}+\mu_2\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(T-1)\right\|\right\} \\
&\quad+\sum_{n=0}^{T-1}\eta_1(n)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(n)\right\|\right\} \\
&\quad+\sum_{n=0}^{T-2}\eta_2(n)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(n)\right\|\right\} \\
&=\left(\eta_1(0)+\eta_2(0)\right)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\} \\
&\quad+\left(\eta_1(1)+\eta_2(1)\right)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(1)\right\|\right\} \\
&\quad+\cdots+\left(\eta_1(T-2)+\eta_2(T-2)\right)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(T-2)\right\|\right\} \\
&\quad+\left(\eta_1(T-1)+\mu_2\right)\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(T-1)\right\|\right\} \\
&\quad+\mu_1\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(T)\right\|\right\}. \qquad (19)
\end{aligned}
$$

Consequently, the matrix form of (19) is reorganized as

$$
\boldsymbol{\Phi}^{k+1} \le \boldsymbol{\Omega}\Phi^k, \qquad (20)
$$

where

$$
\begin{aligned}
\boldsymbol{\Phi}^k=&\Big[\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(0)\right\|\right\},\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(1)\right\|\right\}, \\
&\cdots,\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(T)\right\|\right\}\Big]^{\mathrm{T}},
\end{aligned}
$$

$$\boldsymbol{\Omega}=\begin{bmatrix} \mu_1 & 0 & 0 & \cdots & 0 \\ \eta_1(0)+\mu_2 & \mu_1 & 0 & \cdots & 0 \\ \eta_1(0)+\eta_2(0) & \eta_1(1)+\mu_2 & \mu_1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 \\ \eta_1(0)+\eta_2(0) & \cdots & \eta_1(T-2)+\mu_2 & \mu_1 & 0 \\ \eta_1(0)+\eta_2(0) & \eta_1(1)+\eta_2(1) & \cdots & \eta_1(T-1)+\mu_2 & \mu_1 \end{bmatrix}.$$

Since $\boldsymbol{\Omega}$ is a lower triangular matrix, all its eigenvalues at the kth iteration are $\lambda_1(\boldsymbol{\Omega})=\cdots=\lambda_{n+1}(\boldsymbol{\Omega})=\mu_1=\left\|\boldsymbol{I}-\rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right\|>0$ for $t=0,\,1,\,2,\,\cdots,\,T$.
Hence, implementing recursive operation on (20), we have

$$\boldsymbol{\Phi}^k \le \boldsymbol{\Omega}^k\boldsymbol{\Phi}^0, \tag{21}$$

where

$$\boldsymbol{\Phi}^0=\left[\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^0(0)\right\|\right\},\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^0(1)\right\|\right\},\right.$$
$$\left.\cdots,\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^0(T)\right\|\right\}\right]^{\mathrm{T}}.$$

Note that the condition $\left\|\boldsymbol{I}-\rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t),t\right)\right\|<1$, and the Lemmas 1 and 2, then we have $\lim\limits_{k\to\infty}\boldsymbol{\Phi}^k=\boldsymbol{O}$ and $\lim\limits_{k\to\infty}\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t)\right\|\right\}=0$ for all t.
In addition, according to (8), we have

$$\Delta\boldsymbol{u}^{k+1}(t)=\Delta\boldsymbol{u}^k(t)-\boldsymbol{\Gamma}\boldsymbol{\varepsilon}^k(t+1), \tag{22}$$

and then,

$$\begin{aligned}&\lim_{k\to\infty}\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^{k+1}(t)\right\|\right\}\\ &\le\lim_{k\to\infty}\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t)\right\|\right\}+\|\boldsymbol{\Gamma}\|\lim_{k\to\infty}\mathrm{E}\left\{\left\|\boldsymbol{\varepsilon}^k(t+1)\right\|\right\}.\end{aligned} \tag{23}$$

By means of $\lim\limits_{k\to\infty}\mathrm{E}\left\{\left\|\Delta\boldsymbol{u}^k(t)\right\|\right\}=0$, then there exists $\lim\limits_{k\to\infty}\mathrm{E}\left\{\left\|\boldsymbol{\varepsilon}^k(t+1)\right\|\right\}=0$ for all $t=0,\,1,\,2,\,\cdots,\,T-1$. Hence, it is obtained form (7) that $\lim\limits_{k\to\infty}\boldsymbol{y}^k(t)=\boldsymbol{y}_0(t)$. Consequently, we can get $\lim\limits_{k\to\infty}x_i^k(t)=x_0(t)$ as $t=0,\,1,\,\cdots,\,T$ for $i=1,\,\cdots,\,n$, which implies that the output of all the following agents track the trajectory of leader. The proof is completed. □

5. SIMULATION ANALYSIS

In this section, two examples will be provided to illustrate the effectiveness of the distributed iterative learning control protocol (8). Consider the leader-following nonlinear multi-agent systems with four following agents and one leader agent (labeled as 0), the directed communication topology is shown in Fig. 1.

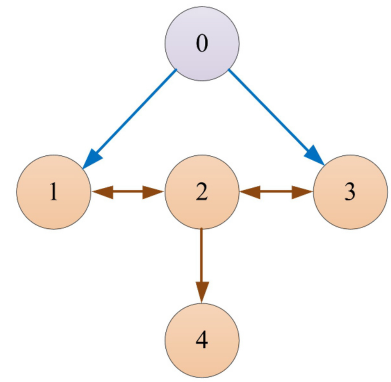

Fig. 1. Communication topology.

Form Fig. 1, the weighted adjacency matrices are

$$\mathcal{A}=\begin{bmatrix}1&0&0&0\\1&0&1&0\\0&1&0&0\\0&1&0&0\end{bmatrix},\quad \mathcal{B}=\mathrm{daig}\{1,0,1,0\}.$$

The dynamics of the ith following agent at the kth iteration are described as

$$\begin{cases} x_i^k(t+1)=\begin{bmatrix}x_{i2}^k(t)\\ \sin(x_{i1}^k(t)x_{i2}^k(t))\end{bmatrix}+\begin{bmatrix}0\\ \cos(x_{i2}^k(t))\end{bmatrix}u_i^k(t),\\ y_i^k(t)=\begin{bmatrix}x_{i1}^k(t)\\ x_{i2}^k(t)\end{bmatrix},\end{cases} \tag{24}$$

where $x_i^k(t)=[x_{i1}^k(t),\,x_{i2}^k(t)]^{\mathrm{T}}$, $i=1,\,2,\,3,\,4$ and k is the iteration number. The initial position and velocity of four following agents are $\boldsymbol{x}_1^0(0)=[-0.5,\,0.6,\,0.2,\,-0.1]^{\mathrm{T}}$ and $\boldsymbol{x}_2^0(0)=[0.5,\,-0.9,\,-0.4,\,0.1]^{\mathrm{T}}$, respectively.
The dynamics of leader agent are given as

$$\begin{cases} x_0(t+1)=\begin{bmatrix}x_{02}(t)\\ \sin(x_{01}(t)x_{02}(t))\end{bmatrix}+\begin{bmatrix}0\\ \cos(x_{02}(t))\end{bmatrix}u_0(t),\\ y_0(t)=\begin{bmatrix}x_{01}(t)\\ x_{02}(t)\end{bmatrix},\end{cases} \tag{25}$$

where $x_0(t)=[x_{01}(t),\,x_{02}(t)]^{\mathrm{T}}$. The initial position and velocity are $x_{01}(0)=0$ and $x_{02}(0)=0$, respectively; and the control input $u_0(t)=\sin(\pi t)$.
The rest parameters are given as: the time interval $t\in[0,10]$; the sampling time $\Delta T=0.01$; the sampling size at each iteration is $T=1000$ and the iteration number $k_{\max}=50$.

Example 1: Analysis with the proposed control protocol (8).

In this example, three cases for the given systems (1) and (3) are considered as:

Case 1: $\rho = 1$, that is, no packet dropout;
Case 2: $\rho = 0.9$, that is, 10% packet dropout;
Case 3: $\rho = 0.6$, that is, 40% packet dropout.

According to the iterative learning control protocol (8), let the learning gain matrix $\boldsymbol{\Gamma} = [0.5]_{4\times 8}$. Check the condition $\|\boldsymbol{I} - \rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t), t\right)\|$ in Theorem 1, we have

Case 1: $\max \left\|\boldsymbol{I} - \rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t), t\right)\right\| = 0.93 < 1;$
Case 2: $\max \left\|\boldsymbol{I} - \rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t), t\right)\right\| = 0.9365 < 1;$
Case 3: $\max \left\|\boldsymbol{I} - \rho\boldsymbol{\Gamma}\bar{\mathcal{H}}\bar{\boldsymbol{C}}\boldsymbol{B}\left(\boldsymbol{x}^k(t), t\right)\right\| = 0.9567 < 1.$

which imply that the condition of Theorem 1 is satisfied.

The simulation results for Case 1, Case 2 and Case 3 at the 50th iteration are shown in Figs. 2-7.

The tracking results of four following agents without packet dropout at the 50th iteration are shown in Fig. 2, which indicate that the consensus problem of leader-

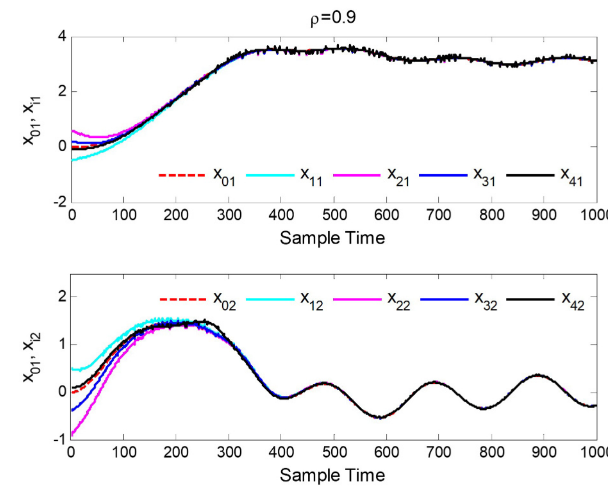

Fig. 4. Tracking results of four agents with Case 2.

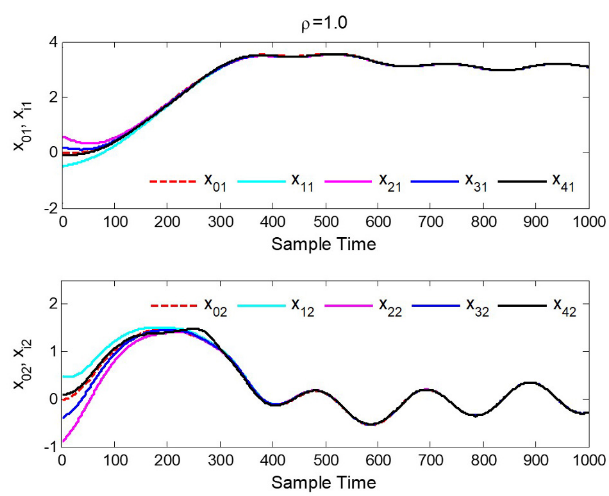

Fig. 2. Tracking results of four agents with Case 1.

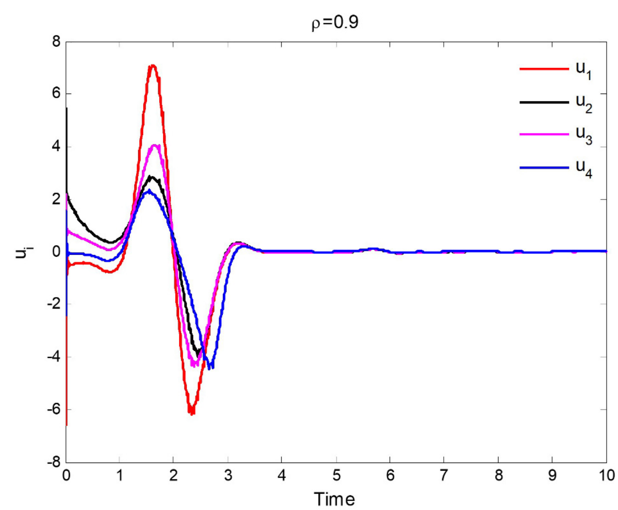

Fig. 5. Control output of four agents with Case 2.

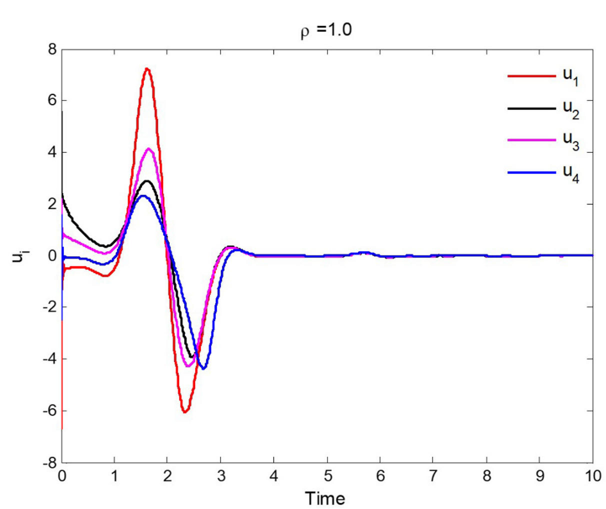

Fig. 3. Control output of four agents with Case 1.

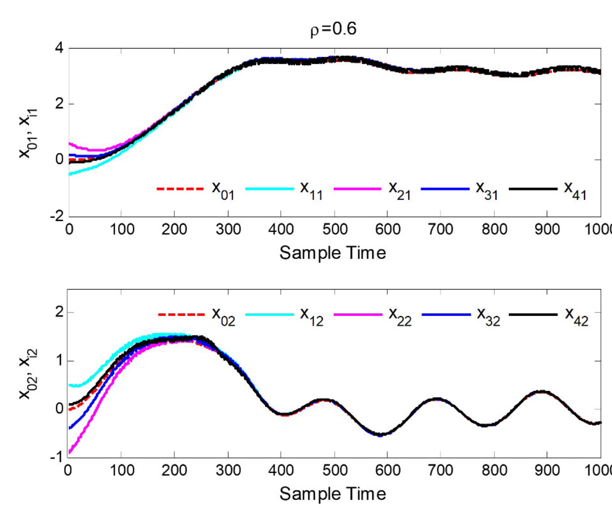

Fig. 6. Tracking results of four agents with Case 3.

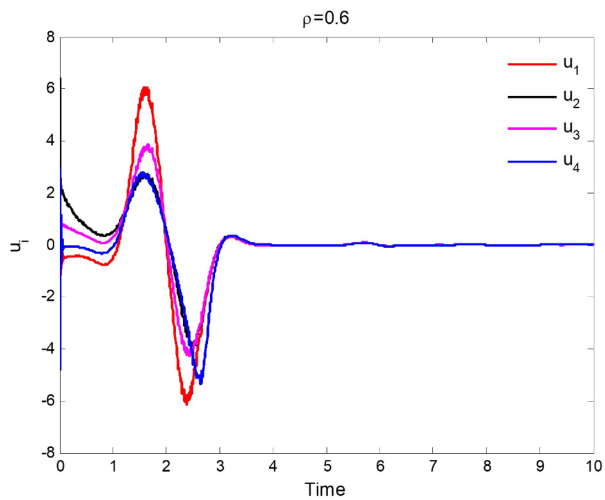

Fig. 7. Control output of four agents with Case 3.

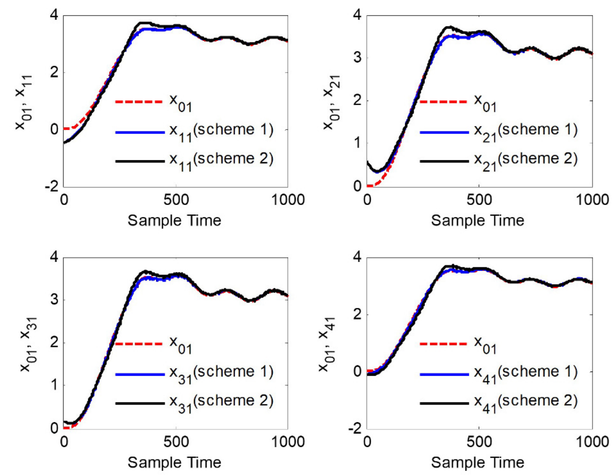

Fig. 8. Comparison results of position with $\rho = 0.9$.

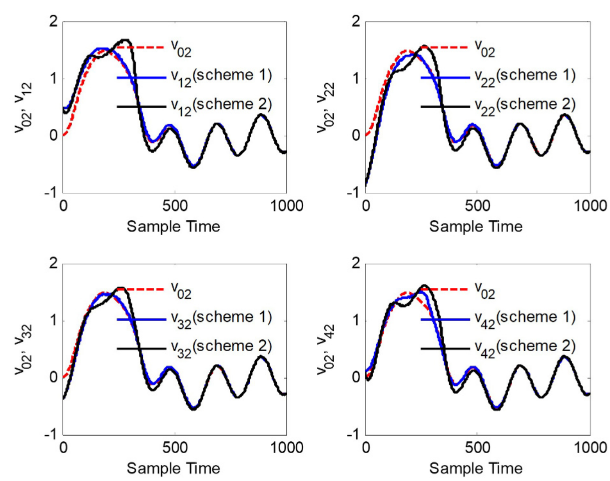

Fig. 9. Comparison results of velocity with $\rho = 0.9$.

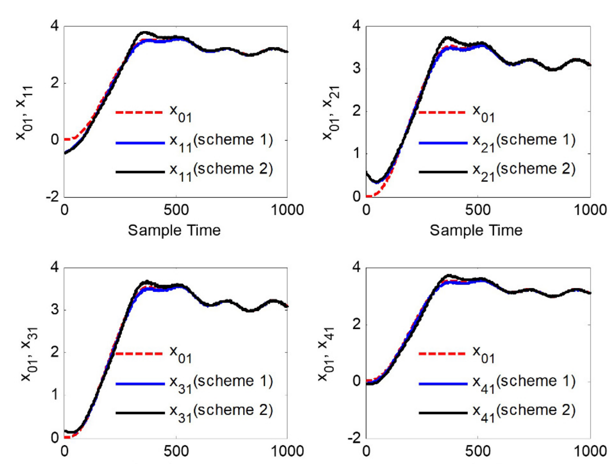

Fig. 10. Comparison results of position with $\rho = 0.6$.

following nonlinear multi-agent systems (1) and (3) without packet dropout can be achieved under the control protocol (8). Fig. 3 gives the control output curves.

The tracking results of four following results with different packet dropout rates at the 50th iteration are shown in Figs. 4 and 6, respectively. Although the packet dropout is considered, the consensus problem of multi-agent systems (1) and (3) with the proposed control protocol (8) can be obtained. It can be seen from Figs. 4 and 6 that the chattering is occurred during the tracking process due to some packet loss. Also, the larger the packet loss, the more obvious the chattering. Nevertheless, the better control effects can still be obtained by adopting the control protocol proposed in this paper. The control output curves with different packet dropout rates are shown in Figs. 5 and 7, respectively.

Example 2: Comparison with other control law.

To further analyze the effectiveness of the control protocol (8) (scheme 1) designed in this paper, the control law of [19] (scheme 2) is considered. Here, the $\rho = 0.9$ and $\rho = 0.6$, that is, 10% and 40% packet dropout are considered in this example, respectively. The parameters setting are same with Example 1. The simulation results are shown in Figs. 8-11.

The comparison results of position and velocity with 10% packet dropout are shown in Figs. 8 and 9, respectively. It can be seen that the consensus problem can be solved by using the control law of [19], but the control effects are worse than the results of the control protocol proposed in this paper. From Figs. 8 and 9, although the chattering occurs, the overshoot of position and velocity results using the proposed control protocol (8) are obviously less than the results using the control law of [19]. In addition, compared with the control law of [19], the convergence speed of position and velocity are faster by using

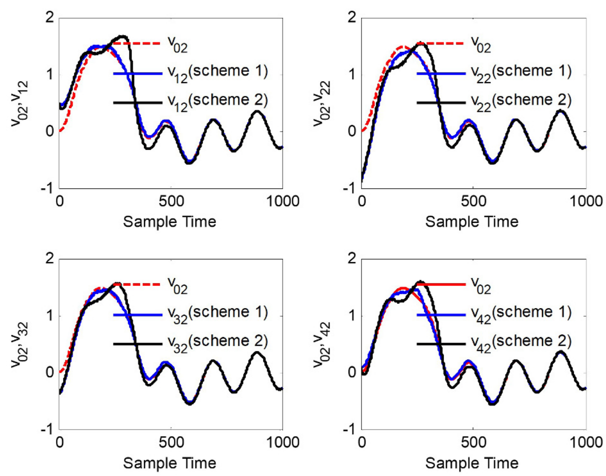

Fig. 11. Comparison results of velocity with $\rho = 0.6$.

the control protocol (8).

The comparison results of position and velocity with 40% packet dropout are shown in Figs. 10 and 11. The packet dropout rate is increased, but the similar results can still be obtained. Compared with the control law of [19], the overshoot of position and velocity is less and the convergence speed is faster by utilizing the control protocol (8), which also imply that the validity of the control protocol proposed in this paper from another viewpoint.

6. CONCLUSION

In this paper, we investigate the packet dropout problem of leader-following nonlinear multi-agent systems with iterative learning control approach. The Bernoulli distribution process is applied to model the packet dropout case, in which the packet dropout rate is converted into a stochastic parameter; and then the packet dropout problem is thought as an equivalent asymptotic stability problem. A distributed control protocol is designed to solve the consensus packet dropout problem. Also, a sufficient condition is given, which guarantees that the output of all the following agents track the trajectory of leader. The validity of the proposed control protocol is verified by simulation analysis. In addition, comparison with other control law, the better control effects can be obtained via adopting the control protocol designed in this paper.

In this work, we only consider the packet dropout problem of multi-agent systems with iterative learning control. Sometimes, time delays are associated with the packet dropout, and the external disturbances may be have a certain impact on a system. Meanwhile, the model of the multi-agents systems considered in this paper is homogeneous. Therefore, in our future work, the iterative learning control for the consensus problem of nonlinear multi-agent systems with time delay and packet dropout will be studied. For the heterogeneous multi-agent systems, we will future explore as well.

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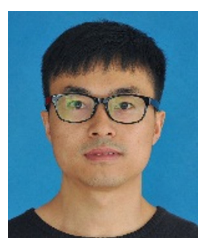

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