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Leader-following Consensus of Nonlinear Multi-agent Systems via Reliable Control with Time-varying Communication Delay

K. Subramanian, P. Muthukumar, and Young Hoon Joo*

Abstract: This paper investigates the consensus problem of continuous-time leader-following nonlinear multi-agent systems with time-varying communication delay via reliable control. The parameter uncertainty is assumed to be bounded in given compact sets. With certain assumptions on the dynamic nonlinearity and underlying topology, the sufficient conditions are derived in terms of linear matrix inequality (LMI) by using a suitable Lyapunov-Krasovskii functional (LKF). It is ensure that the leader-following consensus can be achieved under the proposed reliable control scheme. Finally, numerical simulation results are presented to demonstrate the theoretical results.

Keywords: Communication delay, leader-following consensus, linear matrix inequality, multi-agent systems, reliable control.

1. INTRODUCTION

In recent years, consensus of multi-agent systems (MAS) has become a rapidly emerging topic due to its broad applications in various fields such as cooperative target tracking in sensor networks, vehicle formation and mobile robot formation [1–3]. In general, according to whether the final consensus values are predetermined, the consensus problem can be further classified into leaderless consensus [4] and leader-follower consensus [5] as two categories. In the leader-following consensus problem, the leader agent acts as a command generator, which generates the desired reference trajectory and ignores information from the follower agents. The leader's information is directly access by a subset of the followers. In addition, a consensus protocol is proposed to let all the follower agents track the leader's trajectory (see more details [5–8] and references therein).

Since nonlinear systems are ubiquitous in practice, research on the distributed control of multiple nonlinear systems has emerged and developed rapidly. Note that the results proposed in [5, 8, 9] only valid for MAS with linear dynamics. However, in practice, intelligent agents are more likely to be governed by complicated intrinsic nonlinear dynamics. Indeed, there are only few results available in the existing literature to investigate the consensus problem of MAS with nonlinear dynamics [6, 7, 10].

In practical applications, dynamical systems are often subjected to various perturbations, such as communication delays, uncertainty and etc. The delay is unavoidable for networked MAS due to the process of communicating information [11]. Therefore, the occurrence of communication delay is essential to study the consensus problem of nonlinear MAS. The authors in [12] derived a sufficient condition for all the second order agents reaching consensus with a constant communication delay. As for the time-varying delays, the consensus problem of MAS is discussed in [9, 13].

In addition, uncertainty coming from modeling, measurement and external disturbance is also inevitable in practical applications and must be taken into account in the design phase [14–16]. In the practical implementation of MAS, the values of system parameters are subject to uncertainties. Hence, it is important to ensure the consensus of MAS against such a parameter deviation. Since these deviations are lies in specified intervals called interval uncertainty [17]. In recent years, research on more general MAS, such as those with nonlinear and uncertain dynamics, has attracted more attention. Also, few results have been developed for consensus of nonlinear MAS with uncertainty [10, 18, 19].

In the real world control systems, the actuator may be subject to failures which may affect the system satisfactory performance and even lead to instability of the con-

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K. Subramanian and P. Muthukumar are with Department of Mathematics, The Gandhigram Rural Institute (Deemed to be University), Gandhigram - 624 302, Tamilnadu, India (e-mails: subramaniangri@gmail.com, pmuthukumargri@gmail.com). Young Hoon Joo is with the School of IT Information and Control Engineering, Kunsan National University, Korea (e-mail: yhjoo@kunsan.ac.kr).
* Corresponding author.

trolled system. Thus, due to the safety demand of real-time systems and to improve system reliability, it is important to design a reliable controller or fault-tolerant controller in the presence of unexpected actuator faults [20–22]. For example, the authors established in [21], the guaranteed cost consensus problem of MAS with actuator faults for both cases leaderless and leader-following consensus. However, to the best of authors knowledge, the reliable control design of nonlinear MAS in the presence of interval uncertainty and time-varying communication delay has not been considered in the existing literature.

Motivated by the above discussions, this paper proposes a more general MAS with the nonlinear and uncertain dynamics. In this paper, Theorem 1 proposes the leader-following consensus of nonlinear MAS by designing a reliable control with the case of actuator failures and time-varying communication delay. Unlike previous studies, the proposed approach in Theorem 1 does not require the healthy actuator and it may be subject to loss of effectiveness fault. In Theorem 2, the interval uncertainty is taken into the model considered in Theorem 1, and then we investigate the leader-following consensus of uncertain nonlinear multi-agent systems by designing a reliable control with time-varying communication delay and without actuator failures. The motivation of this work is to generalize the conventional control design for the leader-following MAS subject to nonlinearity and time-varying communication delay under directed topologies.

2. PRELIMINARIES AND PROBLEM FORMULATION

In MAS, the communication topology of a network of agents is represented using a digraph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{W})$. $\mathcal{V}=\{1,2,\ldots,N\}$ is the set of nodes and if node j can communicates with node i then there exists an edge $e_{ij}\in\mathcal{E}$, where $\mathcal{E}=\{(i,j):i,j\in\mathcal{V}\}\subset\mathcal{V}\times\mathcal{V}$ denotes edges of a graph. $\mathcal{W}=[w_{ij}]_{N\times N}$ is the weighted adjacency matrix of a graph $\mathcal{G}$ with nonnegative elements and it is defined as follows: For non-adjacent nodes i and j $(i\neq j)$, the (i,j)-entry is zero, and for adjacent nodes i and j, the (i,j)-entry is 1. The (i,i)-entry is zero for $i=1,2,\ldots,N$. The set of all the neighbor nodes of node i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}:(i,j)\in\mathcal{E},j\neq i\}$. In order to study a leader-following problem, a graph $\bar{\mathcal{G}}$ is employed to denote the communication topology between the N followers and the leader (labeled 0). Meanwhile, the connection weight between agent i and the leader is denoted by b_i, where if the $i-$th follower is connected to the leader, then $b_i=1$, otherwise $b_i=0$.

Consider the following MAS with N agents, each described as

$$\dot{x}_i(t)=Ax_i(t)+Bu_i^F(t)+f(t,x_i(t)),\ \ t\geq 0, \tag{1}$$

and dynamics of the leader node labeled as 0, is described as follows:

$$\begin{cases}\dot{x}_0(t)=Ax_0(t)+f(t,x_0(t)),\ \ t\geq 0,\\ x_0(0)=x_0^0,\end{cases} \tag{2}$$

$x_0(t)$ and $x_i(t)$ $(i=1,\ 2,\ \ldots,\ N)\in\mathbb{R}^n$ are, respectively, the state of the leader and agent i; $A=(a_{pq})_{n\times n}$ and $B=(b_{pq})_{n\times n}$ are known real constant matrices; $f(t,x_i(t))$ and $f(t,x_0(t))\in\mathbb{R}^n$ are nonlinear functions of agent and leader, respectively.

Assumption 1: The nonlinear function f is Lipschitz continuous with the Lipschitz constant being $\alpha\geq 0$, that is, $\|f(x)-f(y)\|\leq\alpha\|x-y\|$ is satisfied for $\forall x,y\in\mathbb{R}^n$.

By the continuation, $u_i^F(t)=[u_{i1}^F(t),u_{i2}^F(t),\ldots,u_{in}^F(t)]^T\in\mathbb{R}^n$ denotes the control input of actuator failure and it is described as $u_i^F(t)=(I-\Lambda_i(t))u_i(t)$, where $\Lambda_i(t)=diag\{\Lambda_{i,1}(t),\ \Lambda_{i,2}(t),\ \ldots,\Lambda_{i,n}(t)\}$; $u_i(t)$ is the control input; $u_{ik}^F(t)=(1-\Lambda_{i,k}(t))u_{ik}(t)$ and $0\leq\Lambda_{i,k}(t)\leq\bar{\Lambda}_k<1$ for $i=1,2,\ldots,N$ and $k=1,2,\ldots,n$. At the same time, consensus protocol $u_i(t)$ with time-varying communication delay is defined as follows:

$$\begin{aligned}u_i(t)=&K\Big\{\sum_{j\in\mathcal{N}_i}w_{ij}[x_i(t-\tau(t))-x_j(t-\tau(t))]\\&\qquad+b_i[x_i(t-\tau(t))-x_0(t-\tau(t))]\Big\},\end{aligned} \tag{3}$$

where $\tau(t)$ is a time-varying communication delay which satisfying $0\leq\tau(t)\leq\tau$ and $\dot{\tau}(t)\leq\mu$; K is the state feedback gain to be designed. Therefore, the actuator fault control input (3) is consider into (1), we get

$$\begin{cases}\dot{x}_i(t)\\ \quad=Ax_i(t)+B(I-\Lambda_i(t))K\Big\{\sum_{j\in\mathcal{N}_i}w_{ij}[x_i(t-\tau(t))\\ \qquad-x_j(t-\tau(t))]+b_i[x_i(t-\tau(t))-x_0(t-\tau(t))]\Big\}\\ \qquad+f(t,x_i(t)),\ \ t\geq 0,\\ x_i(t)=x_i^0,\ \ t\in[-\tau,0],\ \ i=1,2,\ldots,N,\end{cases} \tag{4}$$

where $0\leq\Lambda_i(t)\leq\Lambda\leq I$ with $\Lambda=diag\{\bar{\Lambda}_1,\ \bar{\Lambda}_2,\ \ldots,\bar{\Lambda}_n\}$. Define the tracking error signal as $e_i(t)=x_i(t)-x_0(t)$ and based on the above analysis, we have the following closed loop system

$$\begin{cases}\dot{e}_i(t)=Ae_i(t)+B(I-\Lambda_i(t))K\\ \qquad\quad\times\Big\{\sum_{j\in\mathcal{N}_i}w_{ij}[x_i(t-\tau(t))-x_j(t-\tau(t))]\\ \qquad\quad+b_i[x_i(t-\tau(t))-x_0(t-\tau(t))]\Big\}\\ \qquad\quad+g(t,e_i(t)),\ \ t\geq 0,\\ e_i(t)=x_i^0-x_0^0,\ \ t\in[-\tau,0],\ \ i=1,2,\ldots,N,\end{cases} \tag{5}$$

where $g(t,e_i(t)) = f(t,x_i(t)) - f(t,x_0(t))$. Define $D = \text{diag}\{b_1, b_2, \dots, b_N\}$. Then the Laplacian matrix $L = [L_{ij}]_{N\times N}$ of the weighted directed graph $\mathcal{G}$ is defined as $L_{ii} = \sum_{j=1, j\neq i}^{N} w_{ij}$ and $L_{ij} = -w_{ij}$ for $i \neq j$. The compact form of error system can be written as

$$\begin{aligned}\dot{e}(t) =&(I\otimes A)e(t) + (L\otimes BK)e(t-\tau(t)) + (D\otimes BK)\\&\times e(t-\tau(t)) - (L\otimes B)\Lambda(t)(I\otimes K)e(t-\tau(t))\\&-(D\otimes B)\Lambda(t)(I\otimes K)e(t-\tau(t)) + G(t,e(t)),\end{aligned} \tag{6}$$

where $e(t) = [e_1(t), e_2(t), \dots, e_N(t)]^T$ denotes the error state vector. $G(t,e(t)) = [g(t,e_1(t)), g(t,e_2(t)), \dots, g(t,e_N(t))]^T$ is the nonlinear function and $\Lambda(t) = diag\{\Lambda_1(t), \Lambda_2(t), \dots, \Lambda_N(t)\}$.

Definition 1 [8]**:** For the error system (6) is said to be asymptotically solve the consensus problem if and only if for any initial conditions we have

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\|^2 = 0 \text{ or } \lim_{t\to\infty} \|e_i(t)\|^2 = 0.$$

3. MAIN RESULTS

3.1. Consensus of MAS without uncertainty

In this section, we study the leader-following consensus of MAS by designing a reliable control with time-varying communication delay.

Theorem 1: For given positive constants τ, μ, the consensus of system (6) can be achieved asymptotically if there exist positive definite matrices $\hat{P}_j$ $(j = 1, 2, \dots, 6)$, X, Q_0, Q_1 and any matrix Z such that the following inequality holds

$$\bar{\Omega} = \begin{bmatrix} \bar{\Xi} & \bar{\Upsilon} \\ * & \bar{\Theta} \end{bmatrix} < 0, \tag{7}$$

where $\bar{\Xi}$, $\bar{\Upsilon}$ and $\bar{\Theta}$ are defined in Appendix A. Moreover the control gain matrix is defined as $K = ZX^{-1}$.

Proof: Consider the LKF as follows: $V(t) = \sum_{i=1}^{4} V_i(t)$, where

$$\begin{aligned}V_1(t) =& e^T(t)(I\otimes P_1)e(t),\\ V_2(t) =& \int_{t-\tau(t)}^{t} e^T(s)(I\otimes P_2)e(s)ds\\&+\int_{t-\tau}^{t} e^T(s)(I\otimes P_3)e(s)ds,\\ V_3(t) =& \tau\int_{t-\tau}^{t}\int_{\theta}^{t} \dot{e}^T(s)(I\otimes P_4)\dot{e}(s)dsd\theta\\&+\frac{\tau^2}{2}\int_{t-\tau}^{t}\int_{\gamma}^{t}\int_{\theta}^{t} \dot{e}^T(s)(I\otimes P_5)\dot{e}(s)dsd\theta d\gamma,\\ V_4(t) =& \frac{\tau^3}{6}\int_{t-\tau}^{t}\int_{\delta}^{t}\int_{\gamma}^{t}\int_{\theta}^{t} \dot{e}^T(s)(I\otimes P_6)\dot{e}(s)dsd\theta d\gamma d\delta.\end{aligned}$$

Calculating the time derivatives of $V_i(t)$, $(i = 1, 2, 3, 4)$ along the trajectory of MAS (6) yields,

$$\dot{V}_1(t) = e^T(t)(I\otimes P_1)\dot{e}(t) + \dot{e}^T(t)(I\otimes P_1)e(t), \tag{8}$$

$$\begin{aligned}\dot{V}_2(t) \leq& e^T(t)[(I\otimes P_2) + (I\otimes P_3)]e(t) - (1-\mu)\\&\times e^T(t-\tau(t))(I\otimes P_2)e(t-\tau(t))\\&- e^T(t-\tau)(I\otimes P_3)e(t-\tau),\end{aligned} \tag{9}$$

$$\begin{aligned}\dot{V}_3(t) =& \dot{e}^T(t)\left[\tau^2(I\otimes P_4) + \frac{\tau^4}{4}(I\otimes P_5)\right]\dot{e}(t)\\&-\tau\int_{t-\tau}^{t} \dot{e}^T(s)(I\otimes P_4)\dot{e}(s)ds\\&-\frac{\tau^2}{2}\int_{t-\tau}^{t}\int_{\theta}^{t} \dot{e}^T(s)(I\otimes P_5)\dot{e}(s)dsd\theta,\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{V}_4(t) =& \frac{\tau^6}{36}\dot{e}^T(t)(I\otimes P_6)\dot{e}(t)\\&-\frac{\tau^3}{6}\int_{t-\tau}^{t}\int_{\gamma}^{t}\int_{\theta}^{t} \dot{e}^T(s)(I\otimes P_6)\dot{e}(s)dsd\theta d\gamma.\end{aligned} \tag{11}$$

According to Jensen's inequality Lemma [23], we have

$$\begin{aligned}&-\tau\int_{t-\tau}^{t} \dot{e}^T(s)(I\otimes P_4)\dot{e}(s)ds\\&\leq -[e^T(t-\tau(t)) - e^T(t-\tau)](I\otimes P_4)[e(t-\tau(t))\\&\quad - e(t-\tau)] - [e^T(t) - e^T(t-\tau(t))](I\otimes P_4)\\&\quad\times[e(t) - e(t-\tau(t))].\end{aligned} \tag{12}$$

By using the lemma as in [23] for double integral inequalities, it gives

$$\begin{aligned}&-\frac{\tau^2}{2}\int_{t-\tau}^{t}\int_{\theta}^{t} \dot{e}^T(s)(I\otimes P_5)\dot{e}(s)dsd\theta\\&\leq -\left[\tau e(t) - \int_{t-\tau}^{t} e(s)ds\right]^T(I\otimes P_5)\left[\tau e(t) - \int_{t-\tau}^{t} e(s)ds\right].\end{aligned} \tag{13}$$

Using lemma as in [23] for triple integral terms, one can obtain that from (11) as follows:

$$-\frac{\tau^3}{6}\int_{t-\tau}^{t}\int_{\gamma}^{t}\int_{\theta}^{t} \dot{e}^T(s)(I\otimes P_6)\dot{e}(s)dsd\theta d\gamma$$

$$\leq \left[\frac{\tau^2}{2}e(t)-\int_{t-\tau}^{t}\int_{\theta}^{t}e(s)dsd\theta\right]^T (I\otimes P_6) \times \left[\frac{\tau^2}{2}e(t)-\int_{t-\tau}^{t}\int_{\theta}^{t}e(s)dsd\theta\right]. \quad (14)$$

Moreover, from Assumption 1, one can seen that,

$$e^T(t)(I\otimes\Gamma^T\Gamma)e(t)-G^T(t,e(t))G(t,e(t))\geq 0, \quad (15)$$

where $\Gamma = diag\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$. In addition, for any appropriately dimensioned matrix $(I\otimes Y)>0$, the following equality holds.

$$\begin{aligned}&2(\dot{e}^T(t)+e^T(t))(I\otimes Y)[-\dot{e}(t)+(I\otimes A)e(t)\\&\quad+((L+D)\otimes BK)e(t-\tau(t))-((L+D)\otimes B)\Lambda(t)\\&\quad\times(I\otimes K)e(t-\tau(t))+G(t,e(t))]=0. \quad (16)\end{aligned}$$

Applying matrix Cauchy inequality [24] in (16) for any symmetric positive definite matrices $(I\otimes Q_0)$ and $(I\otimes Q_1)$, it follows that

$$\begin{aligned}&-2\dot{e}^T(t)((L+D)\otimes YB)\Lambda(t)(I\otimes K)e(t-\tau(t))\\&\leq \dot{e}^T(t)((L+D)\otimes YB)(I\otimes Q_0)((L+D)\otimes YB)^T\dot{e}(t)\\&\quad+e^T(t-\tau(t))(I\otimes K)^T\Lambda(I\otimes Q_0)^{-1}\\&\quad\times\Lambda(I\otimes K)e(t-\tau(t)), \quad (17)\\&\quad-2e^T(t)((L+D)\otimes YB)\Lambda(t)(I\otimes K)e(t-\tau(t))\\&\leq e^T(t)((L+D)\otimes YB)(I\otimes Q_1)((L+D)\otimes YB)^Te(t)\\&\quad+e^T(t-\tau(t))(I\otimes K)^T\Lambda(I\otimes Q_1)^{-1}\\&\quad\times\Lambda(I\otimes K)e(t-\tau(t)). \quad (18)\end{aligned}$$

Using the relationship (8) – (18), and Schur complement lemma, we have the new upper bound for derivative of LKF $\dot{V}(t)$ as follows:

$$\dot{V}(t)\leq\zeta^T(t)\Omega\zeta(t), \quad (19)$$

where $\zeta^T(t)=\left[e^T(t)\ e^T(t-\tau(t))\ e^T(t-\tau)\ \int_{t-\tau}^{t}e^T(s)ds\ \int_{t-\tau}^{t}\int_{\theta}^{t}e^T(s)dsd\theta\quad \dot{e}^T(t)\quad G^T(t,e(t))\right]$, and the matrix Ω is defined in Appendix B. Due to the control gain matrix K, the matrix Ω is not an LMI. In order to obtain the LMI based constraint, let us define $K=ZX^{-1}$, and $X=Y^{-1}$. After that pre and post multiplying the matrix Ω by $diag\{\underbrace{(I\otimes X),\ldots(I\otimes X)}_{6times},\underbrace{(I\otimes I),\ldots,(I\otimes I)}_{6times}\}$ and utilizing the relationships $\hat{P}_i=XP_iX(i=1,2,\ldots,6)$, we have the left hand side of (7). From (7), it is observed that if $\bar{\Omega}<0$ then $\dot{V}(t)<0$. Hence, it concludes that the error system (6) is asymptotically stable. Also, according to Definition 1, the leader-following consensus is achieved via reliable control with time-varying communication delay and without uncertainty. This completes the proof of theorem. □

3.2. Consensus of MAS with interval uncertainty

Next, we consider the consensus problem of leader-following nonlinear MAS with interval uncertainty and time-varying communication delay. It is important to study the leader-following consensus problem for nonlinear MAS against such parameter deviations. Since these deviations are bounded in practice, the quantities may be intervalised as follows: $A=(a_{pq})_{n\times n}$, $B=(b_{pq})_{n\times n}$, $\bar{A}=(\bar{a}_{pq})_{n\times n}$, $\underline{A}=(\underline{a}_{pq})_{n\times n}$, $\bar{B}=(\bar{b}_{pq})_{n\times n}$, $\underline{B}=(\underline{b}_{pq})_{n\times n}$,

$$\begin{aligned}N[\bar{A},\underline{A}]=&\{A=(a_{pq})_{n\times n}\in\mathbb{R}^{n\times n}|\underline{A}<A<\bar{A},\\&\text{i.e., }\underline{a}_{pq}<a_{pq}<\bar{a}_{pq},\ p,q=1,2,\ldots,n.\},\\N[\bar{B},\underline{B}]=&\{B=(b_{pq})_{n\times n}\in\mathbb{R}^{n\times n}|\underline{B}<B<\bar{B},\\&\text{i.e., }\underline{b}_{pq}<b_{pq}<\bar{b}_{pq},\ p,q=1,2,\ldots,n.\}.\end{aligned}$$

For notational convenience, let $A_0=\frac{1}{2}(\underline{A}+\bar{A})$, $B_0=\frac{1}{2}(\underline{B}+\bar{B})$, $H_A=\frac{1}{2}(\bar{A}-\underline{A})=(\eta_{pq})_{n\times n}$, $H_B=\frac{1}{2}(\bar{B}-\underline{B})=(\beta_{pq})_{n\times n}$. Note that, every element in the matrices H_A and H_B is non negative. So we can define

$$\begin{aligned}E_A&=[\sqrt{\eta_{11}}e_1,\cdots,\sqrt{\eta_{1n}}e_1,\cdots\sqrt{\eta_{n1}}e_n,\cdots,\sqrt{\eta_{nn}}e_n],\\F_A&=[\sqrt{\eta_{11}}e_1,\cdots,\sqrt{\eta_{1n}}e_n,\cdots\sqrt{\eta_{n1}}e_1,\cdots,\sqrt{\eta_{nn}}e_n]^T,\\E_B&=[\sqrt{\beta_{11}}e_1,\cdots,\sqrt{\beta_{1n}}e_1,\cdots\sqrt{\beta_{n1}}e_n,\cdots,\sqrt{\beta_{nn}}e_n],\\F_B&=[\sqrt{\beta_{11}}e_1,\cdots,\sqrt{\beta_{1n}}e_n,\cdots\sqrt{\beta_{n1}}e_1,\cdots,\sqrt{\beta_{nn}}e_n]^T,\end{aligned}$$

where $E_A\in\mathbb{R}^{n\times n^2}$, $F_A\in\mathbb{R}^{n^2\times n}$, $E_B\in\mathbb{R}^{n\times n^2}$, $F_B\in\mathbb{R}^{n^2\times n}$ and e_k denotes the $k-th$ standard basis of $\mathbb{R}^{n\times 1}$. Furthermore, we denote

$$\begin{aligned}\Sigma^*=&\{\Sigma\in\mathbb{R}^{n^2\times n^2}|\Sigma=diag\{\varepsilon_{11},\cdots,\varepsilon_{1n},\cdots,\varepsilon_{n1},\\&\cdots,\varepsilon_{nn}\},\ |\varepsilon_{pq}|\leq 1,\ p,\ q=1,2,\ldots,n\}.\end{aligned}$$

Based on Lemma 1 in [17] and substituting $\Lambda_{i,j}(t)=0$ in (4), then the system (4) without actuator fault and (2) can be rewritten as follows:
Agent system:

$$\begin{aligned}\dot{x}_i(t)=&(A_0+E_A\Sigma_AF_A)x_i(t)+(B_0+E_B\Sigma_BF_B)K\\&\times\Big\{\sum_{j\in\mathcal{N}_i}w_{ij}[x_i(t-\tau(t))-x_j(t-\tau(t))]\\&+b_i[x_i(t-\tau(t))-x_0(t-\tau(t))]\Big\}+f(t,x_i(t)),\end{aligned} \quad (20)$$

Leader system:

$$\dot{x}_0(t)=(A_0+E_A\Sigma_AF_A)x_0(t)+f(t,x_0(t)), \quad (21)$$

Error system:

$$\begin{aligned}\dot{e}(t)=&(I\otimes(A_0+E_A\Sigma_AF_A))e(t)\\&+(L\otimes(B_0+E_B\Sigma_BF_B)K)e(t-\tau(t))\\&+(D\otimes(B_0+E_B\Sigma_BF_B)K)e(t-\tau(t))\\&+G(t,e(t)). \quad (22)\end{aligned}$$

For systems (20)-(22), we have the following result.

Theorem 2: For given positive constants τ and μ, the leader-following consensus of system (22) without actuator fault can be achieved asymptotically if there exist positive definite matrices $\hat{P}_j$ $(j = 1,2,\ldots,6)$, X, any matrix Z and positive scalars ε_l $(l = 1,2,\ldots,6)$ such that the following inequality holds

$$\bar{\Pi} = \begin{bmatrix} \tilde{\Xi} & \tilde{\Upsilon} \\ * & \tilde{\Theta} \end{bmatrix} < 0, \tag{23}$$

where $\tilde{\Xi}$, $\tilde{\Upsilon}$ and $\tilde{\Theta}$ are defined in Appendix C. Moreover the control gain matrix is defined as $K = ZX^{-1}$.

Proof: Consider the same LKF as in Theorem 1. Then taking time derivative of LKF and the following zero equation is hold for any appropriate dimension matrix Y.

$$\begin{aligned} &2[\dot{e}^T(t)+e^T(t)](I\otimes Y)[-\dot{e}(t)+(I\otimes(A_0+E_A\Sigma_A F_A)) \\ &\quad \times e(t)+(L\otimes(B_0+E_B\Sigma_B F_B)K)e(t-\tau(t)) \\ &\quad +(D\otimes(B_0+E_B\Sigma_B F_B)K)e(t-\tau(t))+G(t,e(t))] \\ &=0. \end{aligned} \tag{24}$$

According to Lemma 2 in [25], one can see

$$\begin{aligned} &2\dot{e}^T(t)(I\otimes Y)(I\otimes(E_A\Sigma_A F_A))e(t) \\ &\le \varepsilon_1\dot{e}^T(t)(I\otimes YE_AE_A^TY^T)\dot{e}(t)+\varepsilon_1^{-1}e^T(t) \\ &\quad \times(I\otimes F_A^TF_A)e(t), \end{aligned} \tag{25}$$

$$\begin{aligned} &2\dot{e}^T(t)(I\otimes Y)(L\otimes(E_B\Sigma_B F_BK))e(t-\tau(t)) \\ &\le \varepsilon_2\dot{e}^T(t)(LL^T\otimes YE_BE_B^TY^T)\dot{e}(t)+\varepsilon_2^{-1}e^T(t-\tau(t)) \\ &\quad \times(I\otimes K^TF_B^TF_BK)e(t-\tau(t)), \end{aligned} \tag{26}$$

$$\begin{aligned} &2\dot{e}^T(t)(I\otimes Y)(D\otimes(E_B\Sigma_B F_BK))e(t-\tau(t)) \\ &\le \varepsilon_3\dot{e}^T(t)(DD^T\otimes YE_BE_B^TY^T)\dot{e}(t)+\varepsilon_3^{-1}e^T(t-\tau(t)) \\ &\quad \times(I\otimes K^TF_B^TF_BK)e(t-\tau(t)), \end{aligned} \tag{27}$$

$$\begin{aligned} &2e^T(t)(I\otimes Y)(L\otimes(E_B\Sigma_B F_BK))e(t-\tau(t)) \\ &\le \varepsilon_4e^T(t)(LL^T\otimes YE_BE_B^TY^T)e(t)+\varepsilon_4^{-1}e^T(t-\tau(t)) \\ &\quad \times(I\otimes K^TF_B^TF_BK)e(t-\tau(t)), \end{aligned} \tag{28}$$

$$\begin{aligned} &2e^T(t)(I\otimes Y)(D\otimes(E_B\Sigma_B F_BK))e(t-\tau(t)) \\ &\le \varepsilon_5e^T(t)(DD^T\otimes YE_BE_B^TY^T)e(t)+\varepsilon_5^{-1}e^T(t-\tau(t)) \\ &\quad \times(I\otimes K^TF_B^TF_BK)e(t-\tau(t)), \end{aligned} \tag{29}$$

$$\begin{aligned} &2e^T(t)(I\otimes Y)(I\otimes(E_A\Sigma_A F_A))e(t) \\ &\le \varepsilon_6^{-1}e^T(t)(I\otimes F_A^TF_A)e(t) \\ &\quad +\varepsilon_6e^T(t)(I\otimes YE_AE_A^TY^T)e(t). \end{aligned} \tag{30}$$

Then, by utilizing the above inequalities (25)-(30), zero equation (24) and with the help of Schur complement lemma, the derivative of LKF can be written as follows:

$$\dot{V}(t) \le \zeta^T(t)\Pi\zeta(t), \tag{31}$$

where $\zeta(t)$ is defined in Theorem 1 and the entries of $\Pi = (\pi_{i,j})_{20\times 20}$ is given in Appendix D. Setting $K = ZX^{-1}$, $X = Y^{-1}$ and $\hat{P}_j = XP_jX$ $(j = 1,\ 2,\ \ldots,\ 6)$. Then, pre and post multiplying both the sides of Π by $\mathrm{diag}\{\underbrace{(I\otimes X),\ldots(I\otimes X)}_{6times},\underbrace{(I\otimes I),\ldots,(I\otimes I)}_{14times}\}$, we have the matrix $\bar{\Pi}$. From (23) and (31), it is easily to obtain that $\dot{V}(t) < 0$. Therefore, we conclude that the error system is asymptotically stable with interval uncertainty and time-varying communication delay. The proof of Theorem 2 is completed. □

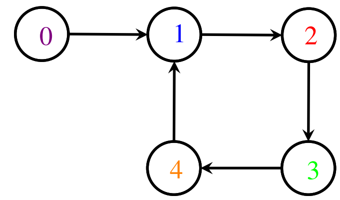

Fig. 1. Communication topology of MAS.

4. NUMERICAL EXAMPLES

In this section, the simulation examples are presented to validate the effectiveness of the developed theoretical results for considered nonlinear MAS with and without uncertainty.

Example 1: Consider the MAS (2) and (4) consisting of a leader and four followers indexed by 0 and 1,2,3,4, respectively. The system topology is given as in Fig. 1 and the system matrices are

$$A = \begin{bmatrix} -1.0 & 0.3 \\ 0.1 & -1.9 \end{bmatrix},\ B = \begin{bmatrix} 0.5 & -1.6 \\ -0.1 & 0.17 \end{bmatrix},$$
$$D = \begin{bmatrix} 1&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix},\ L = \begin{bmatrix} 1&0&0&-1 \\ -1&1&0&0 \\ 0&-1&1&0 \\ 0&0&-1&1 \end{bmatrix}.$$

Let the time-varying communication delay as $\tau(t) = 0.1\sin t + 0.1$ with $\tau = 0.2$ and $\mu = 0.1$. The nonlinear function is assumed to be $f(t,x_i(t)) = \tanh(x_i(t))$. The fault model is adopted as $\Lambda_i(t) = 0.3|\sin(t)|I_2$ and it satisfying $\Lambda = 0.3I_2$, where I_2 represents the 2×2 identity matrix. By using these values to solve the LMI (7) with the help of MATLAB LMI control toolbox, the state feedback gain K for reliable control protocol is estimated as follows:

$$K = \begin{bmatrix} 0.2160 & 0.1872 \\ 2.0225 & 0.3962 \end{bmatrix}. \tag{32}$$

In this example, we choose the initial conditions of leader and follower agents as $x_0(0) = [1.5\ 0.2]^T$ and $x_1(0) = [-1.5\ 1.6]^T$, $x_2(0) = [1.5\ \ -0.9]^T$, $x_3(0) = [0.5\ 1.0]^T$, $x_4(0) = [-1.0\ 2.6]^T$, respectively. Based on the reliable control law, the state responses of error system is shown

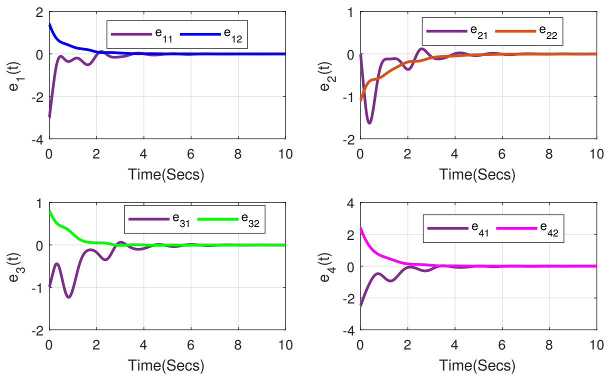

Fig. 2. The error between states of all follower and a leader in (6).

in Fig. 2 and it indicates that the error system (6) is asymptotically stable. It is shown that the leader-following consensus can be achieved through the proposed control scheme.

Example 2: Consider the MAS (21) and (20) with a leader and 4 follower agents which is depicted in the interaction graph Fig. 3. The parameters of given system as

$$\underline{A} = \begin{bmatrix} -0.3 & 0.02 \\ 0.01 & 0.05 \end{bmatrix}, \ \bar{A} = \begin{bmatrix} -0.1 & 0.04 \\ 0.03 & 0.07 \end{bmatrix},$$
$$\underline{B} = \begin{bmatrix} 2.1 & 0.2 \\ 0.3 & 2.1 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} 2.3 & 0.4 \\ 0.5 & 2.3 \end{bmatrix},$$

with nonlinear function $f(t,x_i(t)) = 0.01\tanh(x_i(t))$. The time-varying communication delay as $\tau(t) = 0.1\sin t + 0.2$ with $\tau = 0.3$ and $\mu = 0.1$. The Laplacian matrix L and the degree matrix D are given as

$$L = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \ D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

By solving the LMI in Theorem 2, the corresponding control gain matrix is calculated as

$$K = \begin{bmatrix} -0.0936 & 0.0137 \\ 0.0065 & -0.2240 \end{bmatrix}. \tag{33}$$

Choose an initial conditions of leader and follower agents are $x_0(0) = [10.7\ 5.01]^T$ and $x_1(0) = [11.2\ 10.9\ ]^T$, $x_2(0) = [-1.7\ 2.0]^T$, $x_3(0) = [4.0\ \ -2.9]^T$, $x_4(0) = [0.57\ 1.0]^T$, respectively. Based on the control gain matrix (33), the state trajectories of leader and follower agents are plotted in Fig. 4. It can be observed that all follower agents can reach the region around the leader asymptotically, which confirms the effectiveness of Theorem 2.

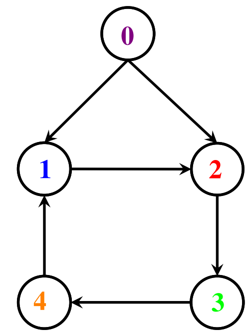

Fig. 3. Interaction graph for Leader and follower.

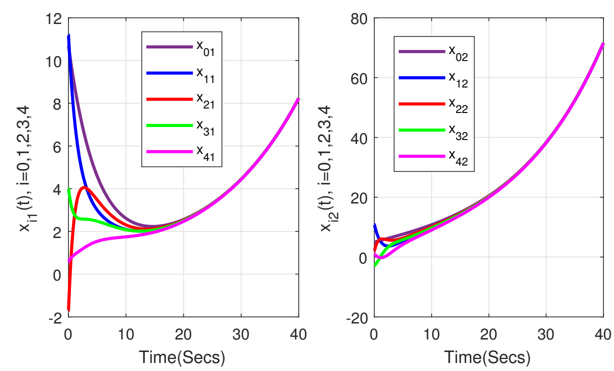

Fig. 4. State trajectories of $x_{i1}(t)$ and $x_{i2}(t)$, $i = 0, 1, 2, 3, 4$ in Example 2.

5. CONCLUSION

In this paper, the problem of leader-following consensus for a MAS with time-varying communication delay and interval uncertainty has been investigated. By using the Kronecker product properties and proposed protocols, the leader-following consensus problem has been transformed into stabilization problem of error system. On the basis of suitable LKF involving triple and quadruple integral terms, a sufficient condition assuring the consensus of MAS has been addressed in terms of LMIs. Finally, numerical examples have been provided to illustrate the effectiveness of the proposed theoretical results.

APPENDIX A

$\bar{\Xi} = \bar{\Xi}^T = (\bar{\Xi}_{p,q})_{6\times 6}$ with entries:

$$\begin{aligned}
\bar{\Xi}_{1,1} &= (I\otimes\hat{P}_2) + (I\otimes\hat{P}_3) - \tau^2(I\otimes\hat{P}_5) - \frac{\tau^4}{4}(I\otimes\hat{P}_6) \\
&\quad - (I\otimes\hat{P}_4) + (I\otimes AX) + (I\otimes AX)^T, \\
\bar{\Xi}_{1,2} &= (I\otimes\hat{P}_4) + ((L+D)\otimes BZ), \ \ \bar{\Xi}_{1,4} = \tau(I\otimes\hat{P}_5), \\
\bar{\Xi}_{1,5} &= \frac{\tau^2}{2}(I\otimes\hat{P}_6), \\
\bar{\Xi}_{1,6} &= (I\otimes\hat{P}_1) + (I\otimes AX)^T - (I\otimes X),
\end{aligned}$$

$$\bar{\Xi}_{2,2} = -(1-\mu)(I\otimes\hat{P}_2) - 2(I\otimes\hat{P}_4),$$
$$\bar{\Xi}_{2,3} = (I\otimes\hat{P}_4),\ \ \bar{\Xi}_{2,6} = ((L+D)\otimes BZ)^T,$$
$$\bar{\Xi}_{3,3} = -(I\otimes\hat{P}_3) - (I\otimes\hat{P}_4),$$
$$\bar{\Xi}_{4,4} = -(I\otimes\hat{P}_5),\ \ \bar{\Xi}_{5,5} = -(I\otimes\hat{P}_6),$$
$$\bar{\Xi}_{6,6} = \tau^2(I\otimes\hat{P}_4) + \frac{\tau^4}{4}(I\otimes\hat{P}_5) + \frac{\tau^6}{36}(I\otimes\hat{P}_6) - 2(I\otimes X).$$

$\bar{\Upsilon} = (\bar{\Upsilon}_{p,q})_{6\times 6}$ with entries:

$$\bar{\Upsilon}_{1,1} = (I\otimes I),\ \ \bar{\Upsilon}_{1,4} = (I\otimes \Gamma X),$$
$$\bar{\Upsilon}_{1,5} = ((L+D)\otimes BQ_1),\ \ \bar{\Upsilon}_{2,2} = (I\otimes Z)^T\Lambda,$$
$$\bar{\Upsilon}_{2,6} = (I\otimes Z)^T\Lambda,\ \ \bar{\Upsilon}_{6,1} = (I\otimes I),$$
$$\bar{\Upsilon}_{6,3} = ((L+D)\otimes BQ_0).$$

$\bar{\Theta} = diag\{\bar{\Theta}_{1,1}, \bar{\Theta}_{2,2}, \bar{\Theta}_{3,3}, \bar{\Theta}_{4,4}, \bar{\Theta}_{5,5}, \bar{\Theta}_{6,6}\}$ with entries:

$$\bar{\Theta}_{1,1} = -(I\otimes I),\ \ \bar{\Theta}_{2,2} = -(I\otimes Q_0),$$
$$\bar{\Theta}_{3,3} = -(I\otimes Q_0),\ \ \bar{\Theta}_{4,4} = -(I\otimes I),$$
$$\bar{\Theta}_{5,5} = -(I\otimes Q_1),\ \ \bar{\Theta}_{6,6} = -(I\otimes Q_1),$$

and the remaining terms are zero.

APPENDIX B

The matrix Ω is defined as follows: $\Omega = \begin{bmatrix} \Xi & \Upsilon \\ * & \Theta \end{bmatrix}$, where $\Xi = \Xi^T = (\Xi_{p,q})_{6\times 6}$, $\Upsilon = (\Upsilon_{p,q})_{6\times 6}$, $\Theta = \Theta^T = diag\{\Theta_{1,1}, \Theta_{2,2}, \Theta_{3,3}, \Theta_{4,4}, \Theta_{5,5}, \Theta_{6,6}\}$ with

$$\Xi_{1,1} = (I\otimes P_2) + (I\otimes P_3) - \tau^2(I\otimes P_5) - \frac{\tau^4}{4}(I\otimes P_6) - (I\otimes P_4) + (I\otimes YA) + (I\otimes YA)^T,$$
$$\Xi_{1,2} = (I\otimes P_4) + ((L+D)\otimes YBK),\ \ \Xi_{1,4} = \tau(I\otimes P_5),$$
$$\Xi_{1,5} = \frac{\tau^2}{2}(I\otimes P_6),\ \Xi_{1,6} = (I\otimes P_1) + (I\otimes YA)^T - (I\otimes Y),$$
$$\Xi_{2,2} = -(1-\mu)(I\otimes P_2) - 2(I\otimes P_4),$$
$$\Xi_{2,3} = (I\otimes P_4),\ \ \Xi_{2,6} = ((L+D)\otimes YBK)^T,$$
$$\Xi_{3,3} = -(I\otimes P_3) - (I\otimes P_4),\ \ \Xi_{4,4} = -(I\otimes P_5),$$
$$\Xi_{5,5} = -(I\otimes P_6),$$
$$\Xi_{6,6} = \tau^2(I\otimes P_4) + \frac{\tau^4}{4}(I\otimes P_5) + \frac{\tau^6}{36}(I\otimes P_6) - 2(I\otimes Y),$$
$$\Upsilon_{1,1} = (I\otimes Y),\ \ \Upsilon_{1,4} = (I\otimes\Gamma),$$
$$\Upsilon_{1,5} = ((L+D)\otimes YBQ_1),\ \ \Upsilon_{2,2} = (I\otimes K)^T\Lambda,$$
$$\Upsilon_{2,6} = (I\otimes K)^T\Lambda,\ \ \Upsilon_{6,1} = (I\otimes Y),$$
$$\Upsilon_{6,3} = ((L+D)\otimes YBQ_0),$$
$$\Theta_{1,1} = -(I\otimes I),\ \ \Theta_{2,2} = -(I\otimes Q_0),$$
$$\Theta_{3,3} = -(I\otimes Q_0),\ \ \Theta_{4,4} = -(I\otimes I),$$
$$\Theta_{5,5} = -(I\otimes Q_1),\ \ \Theta_{6,6} = -(I\otimes Q_1),$$

and the remaining terms are zero.

APPENDIX C

$$\tilde{\Xi} = \tilde{\Xi}^T = (\tilde{\Xi}_{p,q})_{10\times 10},\ \ \tilde{\Upsilon} = (\tilde{\Upsilon}_{p,q})_{10\times 10},$$
$$\tilde{\Theta} = (\tilde{\Theta}_{p,q})_{10\times 10}$$

with

$$\tilde{\Xi}_{1,1} = (I\otimes\hat{P}_2) + (I\otimes\hat{P}_3) - \tau^2(I\otimes\hat{P}_5) - \frac{\tau^4}{4}(I\otimes\hat{P}_6) - (I\otimes\hat{P}_4) + (I\otimes A_0X) + (I\otimes A_0X)^T,$$
$$\tilde{\Xi}_{1,2} = (I\otimes\hat{P}_4) + ((L+D)\otimes B_0Z),\ \ \tilde{\Xi}_{1,4} = \tau(I\otimes\hat{P}_5),$$
$$\tilde{\Xi}_{1,5} = \frac{\tau^2}{2}(I\otimes\hat{P}_6),\ \ \tilde{\Xi}_{1,6} = (I\otimes\hat{P}_1) + (I\otimes A_0X)^T,$$
$$\tilde{\Xi}_{1,8} = (I\otimes\Gamma X),\ \ \tilde{\Xi}_{1,9} = (I\otimes F_AX)^T,$$
$$\tilde{\Xi}_{2,2} = -(1-\mu)(I\otimes\hat{P}_2) - 2(I\otimes\hat{P}_4),$$
$$\tilde{\Xi}_{2,3} = (I\otimes\hat{P}_4),\ \ \tilde{\Xi}_{2,6} = ((L+D)\otimes B_0Z)^T,$$
$$\tilde{\Xi}_{2,10} = (I\otimes F_BZ)^T,\ \ \tilde{\Xi}_{3,3} = -(I\otimes\hat{P}_3) - (I\otimes\hat{P}_4),$$
$$\tilde{\Xi}_{4,4} = -(I\otimes\hat{P}_5),\ \ \tilde{\Xi}_{5,5} = -(I\otimes P_6),$$
$$\tilde{\Xi}_{6,6} = \tau^2(I\otimes\hat{P}_4) + \frac{\tau^4}{4}(I\otimes\hat{P}_5) + \frac{\tau^6}{36}(I\otimes\hat{P}_6) - 2(I\otimes X),$$
$$\tilde{\Xi}_{6,7} = (I\otimes I),\ \ \tilde{\Xi}_{7,7} = -(I\otimes I),\ \ \tilde{\Xi}_{8,8} = -(I\otimes I),$$
$$\tilde{\Xi}_{9,9} = -\varepsilon_1(I\otimes I),\ \ \tilde{\Xi}_{10,10} = -\varepsilon_2(I\otimes I),$$
$$\tilde{\Upsilon}_{1,5} = \varepsilon_4(LL^T\otimes E_B),\ \ \tilde{\Upsilon}_{1,6} = \varepsilon_5(DD^T\otimes E_B),$$
$$\tilde{\Upsilon}_{1,7} = \varepsilon_6(I\otimes E_A),\ \ \tilde{\Upsilon}_{1,8} = (I\otimes F_AX)^T,$$
$$\tilde{\Upsilon}_{2,1} = (I\otimes F_BZ)^T,\ \ \tilde{\Upsilon}_{2,9} = (I\otimes F_BZ)^T,$$
$$\tilde{\Upsilon}_{2,10} = (I\otimes F_BZ)^T,\ \ \tilde{\Upsilon}_{6,2} = \varepsilon_1(I\otimes E_A),$$
$$\tilde{\Upsilon}_{6,3} = \varepsilon_2(LL^T\otimes E_B),\ \ \tilde{\Upsilon}_{6,4} = \varepsilon_3(DD^T\otimes E_B),$$
$$\tilde{\Theta}_{1,1} = -\varepsilon_3(I\otimes I),\ \ \tilde{\Theta}_{2,2} = -\varepsilon_1(I\otimes I),$$
$$\tilde{\Theta}_{3,3} = -\varepsilon_2(I\otimes I),\ \ \tilde{\Theta}_{4,4} = -\varepsilon_3(I\otimes I),$$
$$\tilde{\Theta}_{5,5} = -\varepsilon_4(I\otimes I),\ \ \tilde{\Theta}_{6,6} = -\varepsilon_5(I\otimes I),$$
$$\tilde{\Theta}_{7,7} = -\varepsilon_6(I\otimes I),\ \ \tilde{\Theta}_{8,8} = -\varepsilon_6(I\otimes I),$$
$$\tilde{\Theta}_{9,9} = -\varepsilon_4(I\otimes I),\ \ \tilde{\Theta}_{10,10} = -\varepsilon_5(I\otimes I),$$

and the remaining terms are zero.

APPENDIX D

The entries of $\Pi = (\pi_{p,q})_{20\times 20}$ as

$$\pi_{1,1} = (I\otimes P_2) + (I\otimes P_3) - \tau^2(I\otimes P_5) - \frac{\tau^4}{4}(I\otimes P_6) - (I\otimes P_4) + (I\otimes YA_0) + (I\otimes YA_0)^T,$$

$$\pi_{1,2} = (I \otimes P_2) + ((L+D) \otimes YB_0K),$$
$$\pi_{1,4} = \tau(I \otimes P_5), \ \ \pi_{1,5} = \frac{\tau^2}{2}(I \otimes P_6),$$
$$\pi_{1,6} = (I \otimes P_1) + (I \otimes YA_0)^T, \ \ \pi_{1,8} = (I \otimes \Gamma),$$
$$\pi_{1,9} = (I \otimes F_A)^T, \ \ \pi_{1,15} = \varepsilon_4(LL^T \otimes YE_B),$$
$$\pi_{1,16} = \varepsilon_5(DD^T \otimes YE_B), \ \ \pi_{1,17} = \varepsilon_6(I \otimes YE_A),$$
$$\pi_{1,18} = (I \otimes F_A)^T,$$
$$\pi_{2,2} = -(1-\mu)(I \otimes P_2) - 2(I \otimes P_4), \ \ \pi_{2,3} = (I \otimes P_4),$$
$$\pi_{2,6} = ((L+D) \otimes YB_0K)^T, \ \ \pi_{2,10} = (I \otimes K^T F_B^T),$$
$$\pi_{2,11} = (I \otimes K^T F_B^T), \ \ \pi_{2,19} = (I \otimes F_B K)^T,$$
$$\pi_{2,20} = (I \otimes F_B K)^T, \ \ \pi_{3,3} = -(I \otimes P_3) - (I \otimes P_4),$$
$$\pi_{4,4} = -(I \otimes P_5), \ \ \pi_{5,5} = -(I \otimes P_6),$$
$$\pi_{6,6} = \tau^2(I \otimes P_4) + \frac{\tau^4}{4}(I \otimes P_5) + \frac{\tau^6}{36}(I \otimes P_6) - 2(I \otimes Y),$$
$$\pi_{6,7} = (I \otimes Y), \ \ \pi_{6,12} = \varepsilon_1(I \otimes YE_A),$$
$$\pi_{6,13} = \varepsilon_2(LL^T \otimes YE_B), \ \ \pi_{6,14} = \varepsilon_3(DD^T \otimes YE_B),$$
$$\pi_{7,7} = -(I \otimes I), \ \ \pi_{8,8} = -(I \otimes I), \ \ \pi_{9,9} = -\varepsilon_1(I \otimes I),$$
$$\pi_{10,10} = -\varepsilon_2(I \otimes I), \ \ \pi_{11,11} = -\varepsilon_3(I \otimes I),$$
$$\pi_{12,12} = -\varepsilon_1(I \otimes I), \ \ \pi_{13,13} = -\varepsilon_2(I \otimes I),$$
$$\pi_{14,14} = -\varepsilon_3(I \otimes I), \ \ \pi_{15,15} = -\varepsilon_4(I \otimes I),$$
$$\pi_{16,16} = -\varepsilon_5(I \otimes I), \ \ \pi_{17,17} = -\varepsilon_6(I \otimes I),$$
$$\pi_{18,18} = -\varepsilon_6(I \otimes I), \ \ \pi_{19,19} = -\varepsilon_4(I \otimes I),$$
$$\pi_{20,20} = -\varepsilon_5(I \otimes I),$$

and the other terms are zero.

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K. Subramanian received the B.Sc and M.Sc degrees in mathematics from Sri Ramakrishna Mission Vidyalaya College of Arts and Science affiliated to Bharathiar University, Coimbatore, Tamilnadu, India, in 2011 and 2013, respectively. He received the Master of Philosophy from Department of Mathematics, Bharathiar University, Coimbatore, Tamilnadu, India, in 2014. He is currently pursuing the Ph.D. degree with the Department of Mathematics, The Gandhigram Rural Institute (Deemed to be University), Gandhigram, Tamilnadu, India. His current research interests include neural networks, multi-agent systems and robust control theory.

P. Muthukumar received the Ph.D. degree from the Department of Mathematics, Gandhigram Rural Institute, Gandhigram, India, in 2009. Since 2010, he has been an Assistant Professor with the Department of Mathematics, Gandhigram Rural Institute. He was a Visiting Faculty, University of Texas at Arlington (UTA) Research Institute, TX, USA for a year. He was selected as a Visiting Faculty from the Research Center for Wind Energy Systems, Kunsan National University, Gunsan, Korea for three months in 2018. His current research interests include control theory, stochastic differential systems, and nonlinear control and its applications. Dr. Muthukumar was a recipient of the IUSSTF Research Fellow in 2012 from the DST, Govt. of India and the UGC-SAP Project Fellow Award in 2005 and the CSIR-SRF Award in 2009 from the Indian Government.

Young Hoon Joo received the B.S., M.S., and Ph.D. degrees in Electrical Engineering from Yonsei University, Seoul, Korea, in 1982, 1984, and 1995, respectively. He worked with Samsung Electronics Company, Seoul, Korea, from 1986 to 1995, as a project manager. He was with the University of Houston, Houston, TX, from 1998 to 1999, as a visiting professor in the Department of Electrical and Computer Engineering. He is currently a professor in the Department of Control and Robotics Engineering, Kunsan National University, Korea. His major interest is mainly in the field of intelligent control, intelligent robot, human-robot interaction, wind-farm control, power system stabilization, and intelligent surveillance systems. He served as the President for Korea Institute of Intelligent Systems (KIIS) (2008-2009) and as the Editor-in-Chief for the International Journal of Control, Automation, and Systems (IJCAS) (2014-2017), and the Director of Research Center for Wind Energy Systems funded by Korean Government (2016-present) and the President of Korean Institute of Electrical Engineers (KIEE) (2019).