

# UIO-based Fault Estimation and Accommodation for Nonlinear Switched Systems

Haoshuang Chen, Dongsheng Du\*, Dewen Zhu, and Yan Yang

**Abstract:** This paper investigates the problems of fault estimation and fault accommodation for a class of nonlinear switched systems. First, an augmented switched system is constructed by forming an augmented state vector composed of the state vector and the fault vectors. Then, an unknown input observer (UIO) is designed for the augmented switched system to estimate the augmented state vector. With the assist of the average dwell-time (ADT) method and the switched Lyapunov function technique, sufficient conditions are obtained to guarantee that the error system is globally uniformly asymptotically stable (GUAS) with a prescribed  $H_\infty$  performance index. An algorithm is provided to show the procedures on how to design the UIO. Moreover, the results are extended to the measurement disturbances case. Based on signal compensation principle, a dynamic output feedback controller is designed to ensure the asymptotical stability of the closed-loop system. Finally, simulation results are presented to demonstrate the proposed technique.

**Keywords:** Dynamic output feedback controller, fault estimation, nonlinear switched systems, UIO.

## 1. INTRODUCTION

With the quick development of modern techniques, the industrial systems are becoming more and more complex and expensive. Therefore, the demands of high reliability and security for such systems are increasing, which motivate the researches on fault diagnosis (FD) and fault tolerant control (FTC) techniques. FD technique can be divided into three parts: fault detection, fault isolation and fault estimation [1]. Fault estimation is the final step of FD, which is mainly to estimate the magnitude of the faults.

Back on the past literature, observer-based fault estimation is one of the most popular techniques. Until now, various observer-based approaches have been proposed. However, UIO is an effective observer to not only estimate the sensor but also estimate actuator faults, which can guarantee that is robust to unknown inputs and sensitive to faults. The problem of state estimation for nonlinear or linear system is solved by using UIO method in [2,3]. The authors in [4] considered the problem of robust design by combining the UIO and fault detection filter.

system is a kind of hybrid system, which is composed of several subsystems and a switching signal. ADT, one of the constrained switching signals, only needs to en-

sure that the average dwell time of the whole switched system is a constant, which is more general and less conservative than DT [5]. In [6], fault detection for switched linear parameter-varying systems was proposed by using an ADT approach. In [7], this paper dealt with the problem of simultaneous finite-time control and fault detection for linear switched systems with state delay and parameter uncertainties. However, it should be noted that there still have no references to report the issues of fault estimation and accommodation for nonlinear switched systems with actuator and sensor faults, and the estimated fault is generally a slope fault, which motivates us to try our best to investigate this issue.

The main contributions can be summarized as follows: (i) UIO-based robust fault estimation observer design is proposed for a class of nonlinear switched systems with the same term of actuator and sensor faults, which is capable of estimating all kinds of faults, i.e.,  $f^{(q)} = 0, q \in \mathbb{N} = \{1, 2, 3, \dots\}$ ; (ii) the outside disturbances are considered as being partially decoupled, and the results are popularized into the case with measurement disturbances; (iii) a dynamic output feedback controller is designed to compensate the effect of the faults and ensure the asymptotical stability of the closed-loop system.

The reminder of this paper is organized as follows: the

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model description and preliminaries are given in Section 2. An UIO-based fault estimation observer design is presented in Section 3. Section 4 is devoted to extend the theory to a nonlinear switched systems with measurement disturbances. In Section 5, a dynamic output feedback controller is given to ensure the asymptotical stability of the closed-loop system. Simulation results are shown in Section 6. Finally, the conclusion is provided in Section 7.

## 2. MODEL DESCRIPTION AND PRELIMINARIES

Consider the following nonlinear switched system:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^N \sigma_i(t)(A_i x(t) + B_i u(t) + D_{1i} d(t) \\ \quad + D_{2i} w(t) + F_{1i} f(t) + \Phi_i(x(t), u(t))), \\ y(t) = \sum_{i=1}^N \sigma_i(t)(C_i x(t) + F_{2i} f(t)), \end{cases} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^u$ ,  $y(t) \in \mathbb{R}^m$  and  $f(t) \in \mathbb{R}^l$  are the state vector, the control input vector, the measured output vector and the fault vector, respectively.  $d(t) \in \mathbb{R}^d$  is the unknown input, and  $w(t) \in \mathbb{R}^w$  is the independent Gaussian white noise disturbance, which can be easily decoupled.  $A_i, B_i, C_i, D_{1i}, D_{2i}, F_{1i}$ , and  $F_{2i}$  are known constant real matrices with appropriate dimensions. The piecewise constant function  $\sigma_i(t) : [0, \infty) \rightarrow \{0, 1\}$  ( $i \in \mathbb{N}$ ) is the switching signal satisfying  $\sum_{i=1}^N \sigma_i(t) = 1$ , which specifies which subsystem is activated at the switching instant. While  $\Phi_i(x(t), u(t)) \in \mathbb{R}^n$  are real nonlinear vector functions with Lipschitz constant  $\theta$ , namely,

$$\begin{aligned} & \| \Phi_i(\hat{x}(t), u(t)) - \Phi_i(x(t), u(t)) \| \leq \theta \| \hat{x}(t) - x(t) \|, \\ & \forall \{(\hat{x}(t), u(t)), (x(t), u(t))\} \in \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^u, \end{aligned} \quad (2)$$

where  $\hat{x}(t)$  is the state estimation of  $x(t)$ .

For the paper, the following preliminaries are given:

**Assumption 1:** The matrices  $D_{1i}$  are full column rank.

**Assumption 2:** The dimension of the measured output is no smaller than the dimension of the unknown input, i.e.,  $m \geq d$ .

**Assumption 3:**  $\text{rank}(C_i, D_{1i}) = \text{rank}(D_{1i})$ .

**Definition 1 [5]:** For any switching signal  $\sigma_i(t)$  and any  $t_2 > t_1 > 0$ , let  $N_{\sigma_i(t_1, t_2)}$  denote the number of switchings  $\sigma_i(t)$  on an interval  $(t_1, t_2)$ . If

$$N_{\sigma_i(t_1, t_2)} \leq N_0 + \frac{t_1 - t_2}{\tau_a}$$

holds for given  $N_0 \geq 0$  and  $\tau_a > 0$ , then the constant  $\tau_a$  is called the average dwell-time and  $N_0$  is the chattering bound.

**Remark 1:** Definition 1 means that if there exists a positive number  $\tau_a$  such that a switching signal has the ADT property, the average time interval between consecutive switchings is at least  $\tau_a$ . This is a kind of slowly switching signals, which is less conservative than classical DT switching signal. For simplicity, we chose the chattering bound  $N_0$  is zero in this paper.

**Lemma 1 [8]:** For any matrices  $X \in \mathbb{R}^{s \times r}$ ,  $Y \in \mathbb{R}^{r \times s}$ , a time-varying matrix  $F(t) \in \mathbb{R}^{r \times r}$  with  $\|F(t)\| \leq 1$  and any scalar  $\varepsilon > 0$ , we have

$$XF(t)Y + Y^T F^T(t)X^T \leq \varepsilon^{-1}XX^T + \varepsilon Y^T Y.$$

**Lemma 2 (Schur complement) [9]:** Let  $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$  to be a symmetric matrix, then the LMI  $S < 0$  is equivalent to  $S_{22} < 0$  and  $S_{11} - S_{12}S_{22}^{-1}S_{21}^T < 0$ .

**Lemma 3 [5]:** Consider the continuous-time switched system  $\dot{x}(t) = f_{\sigma_i(t)}(x(t))$ , and let  $\alpha > 0$ ,  $\mu > 1$  be given constants. Suppose that there exist  $\mathcal{C}^1$  function  $V_{\sigma_i(t)} : \mathbb{R}^n \rightarrow \mathbb{R}$  and two class  $\mathcal{K}_\infty$  function  $\kappa_1$  and  $\kappa_2$  such that

$$\begin{aligned} \kappa_1(|x(t)|) &\leq V_{\sigma_i(t)}(x(t)) \leq \kappa_2(|x(t)|), \\ \dot{V}_{\sigma_i(t)}(x(t)) &\leq -\alpha V_{\sigma_i(t)}(x(t)), \end{aligned}$$

and  $\forall (\sigma_i(t_i) = i, \sigma_i(t_i^-) = j) \in \mathbb{N} \times \mathbb{N}, i \neq j$ ,

$$V_i(x(t_i)) \leq \mu V_j(x(t_i)),$$

then the switched system  $\dot{x}(t) = f_{\sigma_i(t)}(x(t))$  is GUAS for any switching signal with ADT

$$\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha}. \quad (3)$$

**Lemma 4:** For a given positive constant  $\beta$  and positive definite matrices  $R_1, Q_1$ , if there exist matrices  $R_2, Q_2$  and symmetric matrices  $R_3, Q_3$  satisfying

$$\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} > 0, \quad \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix}^{-1} = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix},$$

if and only if

$$\begin{bmatrix} R_1 & I \\ I & Q_1 \end{bmatrix} \geq 0, \quad \text{rank} \left( \begin{bmatrix} R_1 & I \\ I & Q_1 \end{bmatrix} \right) \leq n + \min(n, \beta).$$

## 3. UNKNOWN INPUT OBSERVER DESIGN

In this subsection, we mainly implement the design of UIO to achieve fault estimation.

Firstly, we construct the augmented state vector as

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ f(t) \\ \dot{f}(t) \\ \vdots \\ f^{(q-1)}(t) \end{bmatrix}, \quad (4)$$

we assume  $f^{(q)} = 0, q \in \mathbb{N}$ .

Then the following augmented system can be constructed:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^N \sigma_i(t) (\bar{A}_i \bar{x}(t) + \bar{B}_i u(t) + \bar{D}_{1i} d(t) \\ \quad + \bar{D}_{2i} w(t) + \bar{\Phi}_i(x(t), u(t)) + \bar{I} f^{(q)}(t)), \\ y(t) = \sum_{i=1}^N \sigma_i(t) \bar{C}_i \bar{x}(t), \end{cases} \quad (5)$$

where

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} A_i & F_{1i} & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \\ \bar{C}_i &= [C_i \quad F_{2i} \quad 0 \quad 0 \quad \cdots \quad 0], \\ \bar{D}_{1i} &= \begin{bmatrix} D_{1i} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad \bar{D}_{2i} = \begin{bmatrix} D_{2i} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad \bar{I} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ I \end{bmatrix}, \\ \bar{\Phi}_i(x(t), u(t)) &= \begin{bmatrix} \Phi_i(x(t), u(t)) \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (6)$$

Clearly, the state vector  $x(t)$  and the fault vector  $f(t)$  can be simultaneously estimated if an observer for the augmented system (5) can be constructed.

In order to estimate  $\bar{x}(t)$ , the following UIO is expressed by:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^N \sigma_i(t) (N_i z(t) + G_i u(t) + L_i y(t) \\ \quad + T_i \bar{\Phi}_i(\hat{x}(t), u(t))), \\ \hat{x}(t) = z(t) + \sum_{i=1}^N \sigma_i(t) H_i y(t), \end{cases} \quad (7)$$

where  $z(t) \in \mathbb{R}^{\bar{n}}$  ( $\bar{n} = n + ql$ ) is the state of the observer,  $\hat{x}(t) \in \mathbb{R}^{\bar{n}}$  represents the state estimation of  $\bar{x}(t)$ . Matrices  $N_i, G_i, H_i$  and  $L_i$  are the gains of UIO to be designed later. Further,  $T_i = I_{\bar{n}} - H_i \bar{C}_i$ ,  $L_i = L_{1i} + L_{2i}$ ,  $L_{1i}, L_{2i} \in \mathbb{R}^{\bar{n} \times (\bar{n} - 1)}$ .

Define the state error estimation  $e(t) = \bar{x}(t) - \hat{x}(t)$ , one has

$$\begin{aligned} e(t) &= \bar{x}(t) - z(t) - \sum_{i=1}^N \sigma_i(t) H_i y(t) \\ &= \sum_{i=1}^N \sigma_i(t) T_i \bar{x}(t) - z(t). \end{aligned} \quad (8)$$

From (5), (7), and (8), the derivative of the state error estimation can be calculated as

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^N \sigma_i(t) T_i \dot{\bar{x}}(t) - \dot{z}(t) \\ &= \sum_{i=1}^N \sigma_i(t) \left( (T_i \bar{A}_i - L_{1i} \bar{C}_i) e(t) \right. \\ &\quad + ((T_i \bar{A}_i - L_{1i} \bar{C}_i) H_i - L_{2i}) y(t) \\ &\quad + (T_i \bar{A}_i - N_i - L_{1i} \bar{C}_i) z(t) \\ &\quad + (T_i \bar{B}_i - G_i) u(t) + T_i \bar{D}_{1i} d(t) \\ &\quad \left. + T_i \bar{D}_{2i} w(t) + T_i \bar{\Phi} \right), \end{aligned} \quad (9)$$

where  $\bar{\Phi} = \bar{\Phi}(x(t), u(t)) - \bar{\Phi}(\hat{x}(t), u(t))$ .

If the following relationships hold:

$$(T_i \bar{A}_i - L_{1i} \bar{C}_i) H_i - L_{2i} = 0, \quad (10a)$$

$$T_i \bar{A}_i - N_i - L_{1i} \bar{C}_i = 0, \quad (10b)$$

$$T_i \bar{B}_i - G_i = 0, \quad (10c)$$

$$T_i \bar{D}_{1i} = 0. \quad (10d)$$

Then the state error estimation (9) reduces to

$$\dot{e}(t) = \sum_{i=1}^N \sigma_i(t) (N_i e(t) + T_i \bar{D}_{2i} w(t) + T_i \bar{\Phi}). \quad (11)$$

**Remark 2:** From (10d), the matrices  $H_i (i \in \mathbb{N})$  can be calculated by

$$H_i = \bar{D}_{1i} [(\bar{C}_i \bar{D}_{1i})^T (\bar{C}_i \bar{D}_{1i})]^{-1} (\bar{C}_i \bar{D}_{1i})^T, \quad (12)$$

and the matrices  $G_i$  can be solved by (10c) and (12).

**Remark 3:** If we have  $\text{rank}(C_i, D_{1i}) = \text{rank}(D_{1i})$ , one can get that  $\text{rank}(\bar{C}_i, \bar{D}_{1i}) = \text{rank}(\bar{D}_{1i})$ .

Next, based on the state error system (11), the following theorem provides how to design the UIO (7).

**Theorem 1:** For given scalars  $\alpha > 0, \mu > 1, \varepsilon > 0$  and  $\gamma > 0$ , if the conditions (10a)-(10d) hold and there exist positive definite matrices  $P_i$  and  $P_j$ , matrices  $Q_i, \forall i, j \in \mathbb{N}$ , which satisfy

$$P_i \leq \mu P_j, \quad (13)$$

$$\begin{bmatrix} \Psi_{11i} & P_i \hat{D}_{2i} & P_i T_i \\ * & -\gamma^2 I_{\times d} & 0 \\ * & * & -\varepsilon I_{\bar{n}} \end{bmatrix} < 0, \quad (14)$$

where  $\Psi_{11i} = \hat{A}_i^T P_i + P_i \hat{A}_i - \bar{C}_i^T Q_i^T - Q_i \bar{C}_i + \alpha P_i + (\varepsilon \theta^2 + 1)I$ ,  $\hat{A}_i^T = T_i \bar{A}_i$ ,  $\hat{D}_{2i} = T_i \bar{D}_{2i}$ , and  $L_{1i} = P_i^{-1} Q_i$ , then, for the switching signal  $\sigma_i(t)$  with ADT constraint (3), there exists an UIO (7) for the augmented switched system (5) such that the error system (11) is GUAS with  $H_\infty$  performance index  $\gamma$ .

**Proof:** Firstly, we establish the stability of system (11). If we assume  $w(t) = 0$ , consider the following switched Lyapunov function for the error dynamics (11) as

$$V_i(t, e(t)) = e^T(t) \left( \sum_{i=1}^N \sigma_i(t) P_i \right) e(t). \quad (15)$$

One has

$$\begin{aligned} & \dot{V}_i(t, e(t)) + \alpha V_i(t, e(t)) \\ &= \sum_{i=1}^N \sigma_i(t) (e^T(t) (N_i^T P_i + P_i N_i + \alpha P_i) e(t) \\ & \quad + \tilde{\Phi}^T T_i^T P_i e(t) + e^T(t) P_i T_i \tilde{\Phi}). \end{aligned} \quad (16)$$

By using Lemma 1 and (2), one can get

$$\begin{aligned} & \dot{V}_i(t, e(t)) + \alpha V_i(t, e(t)) \\ & \leq \sum_{i=1}^N \sigma_i(t) (e^T(t) (N_i^T P_i + P_i N_i + \alpha P_i \\ & \quad + \varepsilon^{-1} P_i T_i T_i^T P_i + \varepsilon \theta^2 I_{\bar{n}}) e(t)). \end{aligned} \quad (17)$$

Lemma 2 and (14) imply

$$\Psi_{11i} + \varepsilon^{-1} P_i T_i (P_i T_i)^T < 0, \quad (18)$$

which leads to

$$N_i^T P_i + P_i N_i + \alpha P_i + \varepsilon^{-1} P_i T_i T_i^T P_i + \varepsilon \theta^2 I_{\bar{n}} < 0. \quad (19)$$

Apparently, we have  $\dot{V}_i(t, e(t)) < -\alpha V_i(t, e(t))$ . According to Lemma 3 and (13), one can get that the system (11) is GUAS.

Next, we give the proof of  $H_\infty$  performance  $\gamma$  as follows: Let

$$\Gamma = \int_0^T (e^T(t) e(t) - \gamma^2 w^T(t) w(t) + \alpha V_i(t, e(t))). \quad (20)$$

One has

$$\begin{aligned} \Gamma &= \int_0^T (e^T(t) e(t) - \gamma^2 w^T(t) w(t) + \alpha V_i(t, e(t)) \\ & \quad + \dot{V}_i(t, e(t))) dt - \int_0^T \dot{V}_i(t, e(t)) dt \\ &= \int_0^T \sum_{i=1}^N \sigma_i(t) \begin{bmatrix} e^T(t) & w^T(t) \end{bmatrix} \Omega \begin{bmatrix} e(t) \\ w(t) \end{bmatrix} dt \\ & \quad - \int_0^T \dot{V}_i(t, e(t)) dt, \end{aligned} \quad (21)$$

where

$$\Omega = \begin{bmatrix} \Psi_{11i} + \varepsilon^{-1} P_i T_i T_i^T P_i & P_i \hat{D}_{2i} \\ * & -\gamma^2 I_l \end{bmatrix} < 0. \quad (22)$$

Under the initial condition  $e(0) = 0$ , one has

$$\int_0^T \dot{V}_i(t, e(t)) dt > 0. \quad (23)$$

By (22) and (23), we get

$$e^T(t) e(t) - \gamma^2 w^T(t) w(t) + \dot{V}_i(t, e(t)) < -\alpha V_i(t, e(t)). \quad (24)$$

Then, one can conclude that the error system (11) has  $H_\infty$  performance  $\gamma$ . The proof is completed.  $\square$

In the following algorithm, the procedure on how to design the UIO is given.

**Algorithm 1:**

**Step 1:** Construct the augmented system (5);

**Step 2:** Calculate the UIO gain  $H_i$  in (12), then  $G_i$  can be solved by (10c);

**Step 3:** Solve the matrices  $P_i$  and  $Q_i$  by Theorem 1, and calculate the gain  $L_{1i} = P_i^{-1} Q_i$ ;

**Step 4:** Calculate the other gains  $L_{2i}$  and  $N_i$  by (10a) and (10b);

**Step 5:** Implement the robust UIO (7) to obtain the augmented state estimation  $\hat{x}(t)$ , and the state estimation  $\hat{x}(t)$  and fault estimation  $\hat{f}(t)$  can be got by the following equations:

$$\begin{aligned} \hat{x}(t) &= \begin{bmatrix} I_n & \underbrace{0 \cdots 0} \end{bmatrix} \hat{\hat{x}}(t), \\ & \quad (q-1) 0_{n \times 2l} \text{ matrices} \\ \hat{f}(t) &= \begin{bmatrix} 0_{n \times (n+l)} & I_l & \underbrace{0 \cdots 0} \end{bmatrix} \hat{\hat{x}}(t). \end{aligned} \quad (25)$$

(q-2) 0\_{l \times l} matrices

#### 4. UIO DESIGN FOR MEASUREMENT DISTURBANCES

In this section, a more general case is taken into consideration, i.e., the nonlinear switched system (1) is not only affected by process disturbances, but also corrupted by measurement disturbances, which is described by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^N \sigma_i(t) (A_i x(t) + B_i u(t) + D_{1i} d(t) \\ \quad + D_{2i} w(t) + F_{1i} f(t) + \Phi_i(x(t), u(t))), \\ y(t) = \sum_{i=1}^N \sigma_i(t) (C_i x(t) + F_{2i} f(t) + E_i d_s(t)), \end{cases} \quad (26)$$

where  $d_s(t) \in \mathbb{R}^s$  stands for the distribution matrix of the measurement noise,  $E_i$  are constant matrices with appropriate dimensions, and the other symbols are the same as defined in the system (1).

Firstly, an augmented state vector is defined as in (4), then one can get the augmented switched system as

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^N \sigma_i(t) (\bar{A}_i \bar{x}(t) + \bar{B}_i u(t) + \bar{D}_i d(t) \\ \quad + \bar{D}_{2i} w(t) + \bar{\Phi}_i(x(t), u(t)) + \bar{I} f^{(q)}(t)), \\ y(t) = \sum_{i=1}^N \sigma_i(t) (\bar{C}_i \bar{x}(t) + E_i d_s(t)), \end{cases} \quad (27)$$

where the coefficient matrices are defined as in (6) except for  $E_i$  and  $d_s(t)$ . Next, an UIO is constructed as same form as in (7). Similarly, the error estimation  $\bar{e}(t) = \bar{x}(t) - \hat{x}(t)$  can be got.

By (7) and (27), and under the same conditions (10a)-(10d), the state error estimation can be calculated as

$$\dot{\bar{e}}(t) = \sum_{i=1}^N \sigma_i(t)(N_i \bar{e}(t) + M_i v(t) + T_i \bar{\Phi}), \quad (28)$$

where  $M_i = [T_i \bar{D}_{2i} \quad -L_{1i} E_i \quad -H_i E_i]$  and  $v(t) = [w^T(t) \quad d_s^T(t) \quad \bar{d}_s^T(t)]^T$ .

**Remark 4:** Similarly, from the condition (10d), the gain matrices  $H_i (i \in \mathbb{N})$  are the same as before.

The following theorem is given to design the parameters of observer (7).

**Theorem 2:** For given scalars  $\alpha > 0$ ,  $\mu > 1$ ,  $\varepsilon > 0$  and  $\gamma > 0$ , if the conditions (10a)-(10d) hold and there exist positive definite matrices  $P_i$  and  $P_j$ , matrices  $Q_i, \forall i, j \in \mathbb{N}$ , which satisfy

$$P_i \leq \mu P_j, \quad (29)$$

$$\begin{bmatrix} \Psi_{11i} & P_i \bar{D}_{2i} & P_i T_i & -Q_i E_i & -P_i \hat{E}_i \\ * & -\gamma^2 I_{l \times d} & 0 & 0 & 0 \\ * & * & -\varepsilon I_{\bar{n}} & 0 & 0 \\ * & * & * & -\gamma^2 I_s & 0 \\ * & * & * & * & -\gamma^2 I_s \end{bmatrix} < 0, \quad (30)$$

where  $\Psi_{11i} = \hat{A}_i^T P_i + P_i \hat{A}_i - \bar{C}_i^T Q_i^T - Q_i \bar{C}_i + \alpha P_i + (\varepsilon \theta^2 + 1)I$ ,  $\hat{A}_i^T = T_i \bar{A}_i$ ,  $\hat{D}_{2i} = T_i \bar{D}_{2i}$ ,  $\hat{E}_{2i} = H_i E_i$  and  $L_{1i} = P_i^{-1} Q(i)$ , then, for the switching signal  $\sigma_i(t)$  with ADT constraint (3), there exists an UIO (7) for the augmented switched system (27) such that the error system (28) is GUAS with  $H_\infty$  performance index  $\gamma$ .

**Proof:** By using Lemma 1, 2 and 3 and following the same proof lines as in theorem 1, the condition (30) can be easily deduced. The detailed proof is thus omitted for the brevity  $\square$

**Remark 5:** The procedures on how to design the UIO is similar to Algorithm 1.

## 5. DYNAMIC OUTPUT FEEDBACK CONTROLLER DESIGN

In this section, to guarantee the asymptotical stability of the closed-loop system, a dynamic output feedback controller is designed to compensating the faulty signals. The following dynamic output feedback controller is constructed:

$$\begin{cases} \dot{x}_c(t) = \sum_{i=1}^N \sigma_i(t)(A_{ci} x_c(t) + B_{ci} y_c(t)), \\ u(t) = \sum_{i=1}^N \sigma_i(t)(C_{ci} x_c(t) - B_i^\dagger F_{1i} \hat{f}(t)), \end{cases} \quad (31)$$

where  $A_{ci}$ ,  $B_{ci}$  and  $C_{ci}$  are controller gains to be calculated later,  $B_i^\dagger$  is the generalized inverse of matrices  $B_i$ . On the basis of the estimation for the augmented state vector  $\hat{x}(t)$ , the considered faults can be constructed as

$$\hat{f}(t) = [0 \quad I_l \quad 0 \quad \cdots \quad 0] \hat{x}(t) = J \hat{x}(t), \quad (32)$$

where  $J = [0 \quad I_l \quad 0 \quad \cdots \quad 0]$ .

Then, the measurement output can be compensated as follows:

$$y_c = y(t) - F_{2i} \hat{f}(t) = y(t) - F_{2i} J \hat{x}(t). \quad (33)$$

Define  $\tilde{x}(t) = [x^T(t) \quad x_c^T(t)]^T$ , and substitute (31) to (1), one has

$$\dot{\tilde{x}}(t) = \sum_{i=1}^N \sigma_i(t)(\tilde{A}_i \tilde{x}(t) + \tilde{D}_i \psi(t) + \tilde{I} \Xi_i), \quad (34)$$

where

$$\begin{aligned} \tilde{A}_i &= \begin{bmatrix} A_i & B_i C_{ci} \\ B_{ci} C_i & A_{ci} \end{bmatrix}, \quad \Xi_i = \begin{bmatrix} \Phi_i(x(t), u(t)) \\ 0 \end{bmatrix}, \\ \psi(t) &= \begin{bmatrix} d(t) \\ w(t) \\ \tilde{f}(t) \end{bmatrix}, \quad \tilde{D}_i = \begin{bmatrix} D_{1i} & D_{2i} & F_{1i} \\ 0 & 0 & B_{ci} F_{2i} \end{bmatrix}, \\ \tilde{I} &= [I_{n \times n} \quad 0], \quad \tilde{f}(t) = f(t) - J \hat{x}(t). \end{aligned} \quad (35)$$

In the following theorem, the design of the dynamic output feedback controller (31) for the descriptor switched system (1) is given.

**Theorem 3:** For some given positive constants  $\gamma$ ,  $\tilde{\alpha}$ , and  $\tilde{\mu} > 1$ , if there exist positive definite matrices  $P_{11i} \in \mathbb{N}^{n \times n}$ ,  $P_{22i} \in \mathbb{N}^{n \times n}$ ,  $P_{11j} \in \mathbb{N}^{n \times n}$ ,  $P_{22j} \in \mathbb{N}^{n \times n}$ ,  $Q_{11i} \in \mathbb{N}^{n \times n}$ , matrices  $P_{12i} \in \mathbb{N}^{n \times n}$ ,  $M_i \in \mathbb{N}^{n \times n}$ ,  $N_i \in \mathbb{N}^{n \times m}$ , and  $W_i \in \mathbb{N}^{l \times n}$ , for  $\forall i, j \in \mathbb{N}$ , which satisfy

$$\begin{bmatrix} P_{11i} & I_n \\ I_n & Q_{11i} \end{bmatrix} > 0, \quad (36)$$

$$\begin{bmatrix} P_{11i} & P_{12i} \\ P_{12i}^T & P_{22i} \end{bmatrix} \leq \tilde{\mu} \begin{bmatrix} P_{11j} & P_{12j} \\ P_{12j}^T & P_{22j} \end{bmatrix}, \quad (37)$$

$$\begin{bmatrix} \Upsilon_{11i} & \Upsilon_{12i} & \Upsilon_{13i} & \Upsilon_{14i} \\ * & -\gamma I_{d+2l} & 0 & 0 \\ * & * & -I_n & 0 \\ * & * & * & \Upsilon_{14i} \end{bmatrix} < 0, \quad (38)$$

where

$$\begin{aligned} \Upsilon_{11i} &= \begin{bmatrix} \mathcal{P} & A_i + M_i^T \\ A_i^T + M_i & \mathcal{Q} \end{bmatrix}, \\ \Upsilon_{12i} &= \begin{bmatrix} D_{1i} & D_{2i} & F_{1i} & 0 \\ P_{11i} D_{1i} & P_{11i} D_{2i} & P_{11i} F_{1i} & N_i F_{2i} \end{bmatrix}, \\ \Upsilon_{13i} &= \begin{bmatrix} Q_{11i} \\ I_n \end{bmatrix}, \quad \Upsilon_{14i} = \begin{bmatrix} \tilde{\alpha} I_n & 0 & \tilde{I} \\ \tilde{\alpha} P_{11i} & \tilde{\alpha} P_{12i} & 0 \end{bmatrix}, \\ \Upsilon_{44i} &= \begin{bmatrix} -\tilde{\alpha} P_{11i} & -\tilde{\alpha} P_{12i} & 0 \\ -\tilde{\alpha} P_{12i}^T & -\tilde{\alpha} P_{22i} & 0 \end{bmatrix}, \end{aligned} \quad (39)$$

where  $\mathcal{P} = A_i Q_{11i} + Q_{11i} A_i^T + B_i W_i + W_i^T B_i^T$ ,  $\mathcal{Q} = P_{11i} A_i + A_i^T P_{11i} + N_i C_i + C_i^T N_i^T$ .

Then, for the switching signal  $\sigma_i(t)$  with ADT constraint (3), there exists a dynamic output feedback controller (31) such that the system (1) is GUAS with  $H_\infty$  performance index  $\gamma$ . Under this case, the gains of the output feedback controller (31) are given by

$$A_{ci} = P_{12i}^{-1} (M_i - P_{11i} A_i Q_{11i} - N_i C_i Q_{11i} - P_{11i} B_i W_i) Q_{12i}^{-T}, \quad (40)$$

$$B_{ci} = P_{12i}^{-1} N_i, \quad (41)$$

$$C_{ci} = W_i Q_{12i}^{-T}, \quad (42)$$

where the matrices  $P_{12i}$  and  $Q_{12i}$  satisfy  $P_{11i} Q_{11i} + P_{12i} Q_{12i}^T = I_n$ .

**Proof:** The following switched Lyapunov function is constructed

$$V_i(t, \tilde{x}(t)) = \tilde{x}^T(t) \left( \sum_{i=1}^N \sigma_i(t) \tilde{P}_i \right) \tilde{x}(t), \quad (43)$$

where  $\tilde{P}_i = \begin{bmatrix} P_{11i} & P_{12i} \\ P_{12i}^T & P_{22i} \end{bmatrix} > 0$ . By using (34), one can get its derivative is

$$\begin{aligned} \dot{V}_i(t, \tilde{x}(t)) &= \sum_{i=1}^N \sigma_i(t) (\tilde{x}^T(t) (\tilde{A}_i^T \tilde{P}_i + \tilde{P}_i \tilde{A}_i) \tilde{x}(t) \\ &\quad + 2\tilde{x}^T(t) \tilde{P}_i \tilde{D}_i \psi(t) + \Xi^T \tilde{I}^T \tilde{P}_i \tilde{x}(t) \\ &\quad + \tilde{x}^T(t) \tilde{P}_i \tilde{I} \Xi). \end{aligned} \quad (44)$$

Define

$$K = \int_0^T (\tilde{x}^T(t) \tilde{x}(t) - \gamma^2 \psi^T(t) \psi(t) + \alpha V_i(t, \tilde{x}(t)) + \dot{V}_i(t, \tilde{x}(t))) dt. \quad (45)$$

By submitting (43) and (44) to (45), one has

$$K = \sum_{i=1}^N \sigma_i(t) \int_0^T \xi^T(t) \Lambda_i \xi(t) dt, \quad (46)$$

where

$$\begin{aligned} \Lambda_i &= \begin{bmatrix} \tilde{A}_i^T \tilde{P}_i + \tilde{P}_i \tilde{A}_i + \tilde{I}^T \tilde{I} + \tilde{\alpha} \tilde{P}_i & \tilde{P}_i \tilde{D}_i & \tilde{P}_i \tilde{I} \\ & \tilde{D}_i^T \tilde{P}_i & 0 \\ & \tilde{I}^T \tilde{P}_i & 0 \end{bmatrix}, \\ \xi(t) &= \begin{bmatrix} \tilde{x}(t) \\ \psi(t) \\ \Xi \end{bmatrix}, \quad \tilde{I} = \begin{bmatrix} I & 0 \end{bmatrix}. \end{aligned} \quad (47)$$

If  $\Lambda_i < 0$ , based Lemma 2, which is equivalent to

$$\begin{bmatrix} \tilde{A}_i^T \tilde{P}_i + \tilde{P}_i \tilde{A}_i & \tilde{P}_i \tilde{D}_i & \tilde{I}^T & \tilde{\alpha} \tilde{P}_i & \tilde{P}_i \tilde{I} \\ * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\tilde{\alpha} \tilde{P}_i & 0 \\ * & * & * & * & 0 \end{bmatrix}. \quad (48)$$

Let

$$\tilde{P}_i = \begin{bmatrix} P_{11i} & P_{12i} \\ P_{12i}^T & P_{22i} \end{bmatrix}, \quad \tilde{P}_i^{-1} = \begin{bmatrix} Q_{11i} & Q_{12i} \\ Q_{12i}^T & Q_{22i} \end{bmatrix}, \quad (49)$$

which follows

$$\tilde{P}_i \begin{bmatrix} Q_{11i} \\ Q_{12i}^T \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}. \quad (50)$$

Define

$$\Pi_i = \begin{bmatrix} Q_{11i} & I \\ Q_{12i}^T & 0 \end{bmatrix}, \quad (51)$$

and pre-multiply and post-multiply  $\text{diag}\{\Pi_i^T, I, I, I, I\}$  and  $\text{diag}\{\Pi_i, I, I, I, I\}$  to both sides of (48), which is equivalent to (38) with the definitions in (40)-(42). By combining the condition (37), it obtains that the augmented system (34) is asymptotically stable with  $H_\infty$  performance  $\|x(t)\|_2 \leq \|g(t)\|_2$ . By Lemma 4, the existence of matrices  $\tilde{P}_i$  and  $\tilde{P}_i^{-1}$  in the condition (36) is guaranteed. Then the proof is completed.  $\square$

Then, the following algorithm on how to design the dynamic output feedback controller is given.

#### Algorithm 2:

**Step 1:** Substitute dynamic output feedback controller (31) into system (1), one can get the augmented systems (34);

**Step 2:** Calculate the matrices  $P_{11i}$ ,  $P_{12i}$ ,  $Q_{11i}$ ,  $M_i$ ,  $N_i$ ,  $W_i$  by using LMI toolbox to solving the condition (37) and (38) with the ADT constraint (3);

**Step 3:** According to the matrices  $P_{11i}$ ,  $Q_{11i}$ ,  $P_{12i}$ , the matrices  $Q_{12i}$  are calculated by using the equation  $P_{12i}$  and  $Q_{12i}$  satisfy  $P_{11i} Q_{11i} + P_{12i} Q_{12i}^T = I_n$ ;

**Step 4:** Substitute the matrices  $P_{11i}$ ,  $P_{12i}$ ,  $Q_{11i}$ ,  $Q_{12i}$ ,  $M_i$ ,  $N_i$  and  $W_i$  into (40)-(42) to calculate the controller gains  $A_{ci}$ ,  $B_{ci}$  and  $C_{ci}$ .

## 6. SIMULATION EXAMPLE

A liquid level control system, in this part, has been considered to test the effectiveness of the proposed UIO approach which is described in Fig. 1 [10]. As in [10], the system is composed of two tanks: one flow source, two outlet pipes, and one connecting pipe. The pipes contain valves that can be opened or closed by an external controller. Based on the status of each valve (open or closed), there exist eight different system modes, however, just three valve configurations are considered as follows.

Model 1: V-2 ON, V-1 and V-3 OFF

Model 2: V-1 and V-2 ON, V-3 OFF

Model 3: V-2 and V-3 ON, V-1 OFF

Consider the flow through the valves is laminar, which implies that the relation between the flow rate in the valves

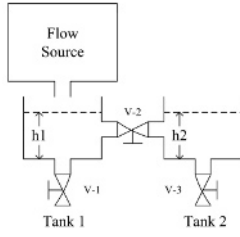


Fig. 1. A liquid lever control system.

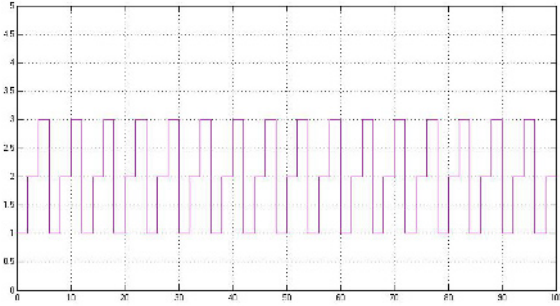


Fig. 2. The switching signal  $\sigma_i(t)$ .

and the height of the liquid is linear. Depending on the value of the tank capacity  $C_T$  and the pipe resistance  $V$  in each mode, the behavior of the system is governed by a specific differential equation. The state-space representation of the system is given by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^3 \sigma_i(t)(A_i x(t) + B_i u(t)), \\ y(t) = \sum_{i=1}^3 \sigma_i(t) C_i x(t), \end{cases} \quad (52)$$

where the state  $x(t) = [h_1^T(t) \ h_2^T(t)]^T$  contains the heights of liquid in the tanks, and  $u(t) = 2e^{-0.5t}(1 + \sin 3\pi t)$  is the input flow from the flow source to tank 1. The switching signal  $\sigma_i(t)$  in this example is a piecewise constant function with the set of images equal to  $\{1, 2, 3\}$  satisfies the constrained ADT condition, which is described in Fig. 2. The output  $y(t)$  is the liquid in each tank while the valve configuration can jump between the three given modes. It is assumed that the system (52) is affected by disturbances and system faults. The following values for the system parameters is considered:

$$\begin{aligned} C_{T1} &= 5 \text{ m}, \quad C_{T2} = 5 \text{ m}, \\ V-1 &= V-2 = 300 \text{ s/m}^2, \quad V-3 = 100 \text{ s/m}^2. \end{aligned}$$

One can obtain the following parameters:

$$A_1 = \begin{bmatrix} -0.0007 & 0.0007 \\ 0.0011 & -0.0011 \end{bmatrix}, B_i = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.0013 & 0.0007 \\ 0.0011 & -0.0011 \end{bmatrix}, D_{1i} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.0007 & 0.0007 \\ 0.0011 & -0.0044 \end{bmatrix}, D_{2i} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix},$$

$$C_i = \begin{bmatrix} 0.01 & 0 \\ 0.01 & 0 \end{bmatrix}, F_{1i} = F_{2i} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$\bar{\Phi}_i(x(t), u(t)) = [\Phi_i^T(x(t), u(t)) \ 0]^T, i = 1, 2, 3.$$

For given  $\mu = 1.002$ ,  $\alpha = 0.001$ ,  $\gamma = 0.1$ ,  $\varepsilon = 0.1$ , and Lipschitz constant  $\theta = 0.1$ , one has  $\tau_a^* = \frac{\ln \mu}{\alpha} = 1.998$ . We assume  $\bar{\Phi}_i(x(t), u(t)) = \sin(x(t))$ .

**Case 1:** constant faults. For the given switched system (1), if only constant faults are considered, i.e.  $\dot{f} = 0$ . Then the augmented state is  $\bar{x}(t) = [x^T(t) \ f^T(t)]^T$ , and the coefficient matrices in (6) are

$$\bar{A}_i = \begin{bmatrix} A_i & F_{1i} \\ 0 & 0 \end{bmatrix}, \bar{C}_i = [C_i \ F_{2i}], \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix},$$

$$\bar{D}_{1i} = \begin{bmatrix} D_{1i} \\ 0 \end{bmatrix}, \bar{D}_{2i} = \begin{bmatrix} D_{2i} \\ 0 \end{bmatrix}, \bar{I} = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

Then, the observer gain  $H_i (i = 1, 2, 3)$  can be calculated by Step 2 of Algorithm 1 as follows:

$$H_i = \bar{D}_{1i} [(\bar{C}_i \bar{D}_{1i})^T (\bar{C}_i \bar{D}_{1i})]^{-1} (\bar{C}_i \bar{D}_{1i})^T = \begin{bmatrix} 100 & 0 \\ 50 & 0 \\ 0 & 0 \end{bmatrix},$$

$$i = 1, 2, 3.$$

$P_i$  and  $Q_i (i=1,2,3)$  in Theorem 1 can be solved by using Matlab tool. Then, one can obtain the other gains of the UIO (7) according to Algorithm 1:

$$N_1 = 10^2 \times \begin{bmatrix} -1.4214 & 0.0000 & 0.1979 \\ -0.6916 & -0.1000 & 0.1000 \\ 0.0123 & 0.0000 & -0.3509 \end{bmatrix},$$

$$N_2 = 10^2 \times \begin{bmatrix} -1.4397 & 0.0000 & -0.1835 \\ -0.7049 & -0.1000 & -0.0734 \\ 0.1235 & 0.0000 & -0.3373 \end{bmatrix},$$

$$N_3 = 10^2 \times \begin{bmatrix} -1.2227 & 0.0000 & -0.4526 \\ -0.5834 & -0.3000 & -0.1585 \\ 0.1980 & 0.0000 & -0.3011 \end{bmatrix},$$

$$L_1 = 10^2 \times \begin{bmatrix} 0 & -0.1923 \\ 0 & -0.1003 \\ 0.0000 & 0.0040 \end{bmatrix},$$

$$L_2 = 10^2 \times \begin{bmatrix} -0.0000 & -0.1916 \\ -0.0023 & -0.0959 \\ 0 & 0.0188 \end{bmatrix},$$

$$L_3 = 10^2 \times \begin{bmatrix} 0 & -0.0185 \\ 0.0374 & -0.0859 \\ 0 & 0.0032 \end{bmatrix},$$

$$G_1 = G_2 = G_3 = \begin{bmatrix} 0 \\ 0.0050 \\ 0 \end{bmatrix}.$$

In the simulation, we consider the following piecewise constant fault:

$$f_1(t) = \begin{cases} 0, & 0s \leq t < 10s; \\ 2, & 10s \leq t < 50s; \\ -2, & 50s \leq t < 100s. \end{cases}$$

The fault estimation result is shown in Fig. 3.

**Case 2: slope fault.** For the given switched system (1), if slop faults are considered, i.e.  $\dot{f} = 0$ . Then the augmented state is  $\bar{x}(t) = [x^T(t) \ f^T(t) \ \dot{f}^T(t)]^T$ , and the coefficient matrices in (6) are

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} A_i & F_{1i} & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix}, \\ \bar{D}_{1i} &= \begin{bmatrix} D_{1i} \\ 0 \\ 0 \end{bmatrix}, \quad \bar{D}_{2i} = \begin{bmatrix} D_{2i} \\ 0 \\ 0 \end{bmatrix}, \quad \bar{I} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \\ \bar{C}_i &= [C_i \ F_{2i} \ 0], \end{aligned}$$

Then, by substituting the matrices  $\bar{C}_i$  and  $\bar{D}_{1i}$  into (12), the gains of  $H_i (i = 1, 2, 3)$  can be calculated as

$$H_i = \bar{D}_{1i} [(\bar{C}_i \bar{D}_{1i})^T (\bar{C}_i \bar{D}_{1i})]^{-1} (\bar{C}_i \ \bar{D}_{1i})^T = \begin{bmatrix} 100 & 0 \\ 50 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$i = 1, 2, 3.$$

Similarly, by solving the conditions in Theorem 1 and following the calculation procedures in Algorithm 1, the other gains  $N_i, L_i, G_i$  of the UIO (7) can be obtained. For the simulation, a piecewise slop fault is considered, which is defined as

$$f_2(t) = \begin{cases} t, & 0s \leq t < 10s; \\ 2 \times (t - 50), & 10s \leq t < 50s; \\ 10 - t, & 50s \leq t < 100s. \end{cases}$$

The fault estimation result is shown in Fig. 4.

**Case 3:  $f^{(q)} = 0, q \in \mathbb{N}$ .** Consider the case of  $f^{(4)} = 0$  in this part. As the same routine as in the Case 1 and Case 2, the gains  $H_i, L_i, N_i$ , are obtained.

To verify the accuracy of the UIO method, we define a piecewise fault signal as follows:

$$f_3(t) = \begin{cases} 2 \times (t + 60)^2, & 0s \leq t < 10s; \\ t^2 \times (10 - t), & 10s \leq t < 50s; \\ (t - 20) \times (t - 60) \times (t - 100), & 50s \leq t < 100s. \end{cases}$$

The fault estimation result is shown in Fig. 5.

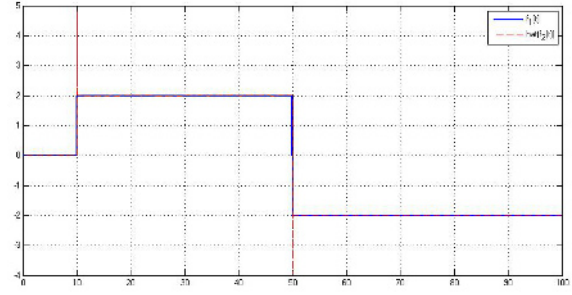


Fig. 3. The fault  $f_1(t)$  (solid line) and its estimation  $\hat{f}_1(t)$  (dotted line).

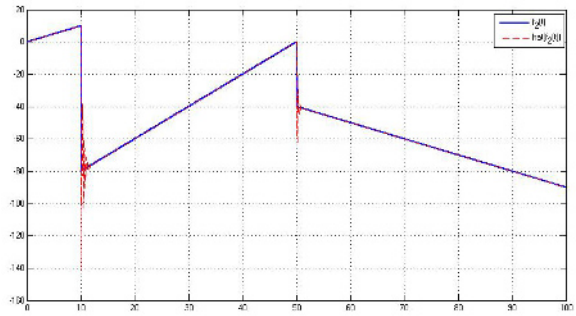


Fig. 4. The fault  $f_2(t)$  (solid line) and its estimation  $\hat{f}_2(t)$  (dotted line).

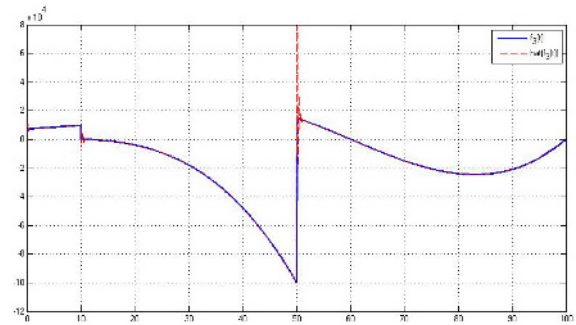


Fig. 5. The fault  $f_3(t)$  (solid line) and its estimation  $\hat{f}_3(t)$  (dotted line).

## 7. CONCLUSION

This paper deals with robust fault estimation and fault accommodation problems for nonlinear switched systems. By using the switched Lyapunov function method and the ADT technique, an UIO is designed to be as robust as possible to the disturbances and as sensitive as possible to the faults. Furthermore, a dynamic output feedback controller is given to ensure the asymptotical stability of the closed-loop system. The results are also extended to measurement disturbances case. Finally, a liquid control system is provided to show the effectiveness of the presented scheme.



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