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Finite Time Controller Design of Nonlinear Quantized Systems with Non-strict Feedback Form

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Abstract: This article considers a finite-time control problem of nonlinear quantized systems in complex environments. The controlled system is in a non-strict feedback form. By applying a nonlinear decomposition of hysteretic quantizer, the quantization issue is tackled successfully. By employing a structural property of radial basis function (RBF) neural networks (NNs), the conventional backstepping method is extended to non-strict feedback nonlinear quantized systems. Based on the finite time stability criterion, a new adaptive neural control scheme is presented. The constructed neural controller can ensure the transient performance of nonlinear quantized systems.

Keywords: Adaptive control, finite-time control, neural network, quantized nonlinear systems.

1. INTRODUCTION

The growing number of applications in complex engineering environments, along with the increasing requirements for network, is posing new challenges for the controller design for the complex systems. A haptic identification approach was proposed for extreme learning machine(ELM) uncertain manipulator in [1]. Two neural control schemes were developed for bimanual robots and flexible joint robot in [2, 3]. By applying neural networks control technique, some control scheme were developed in [4–8]. At the same time, the quantized control problems have caught the imagination of many scholars. In [9], a quantized feedback stabilized scheme is first proposed for a kind of discrete linear systems. Serval stabilized strategies were presented continuous quantized systems in [10–15]. Furthermore, the quantized control problems were considered for uncertain nonlinear systems in [16–18]. Compared with the results in [9–15], the system models don't require to be exactly known for designer. In [18], by establishing a hysteretic quantizer, chattering is eliminated in [9–15].Considering that the quantized control scheme is hard to realize, by proposing a novel decomposition of quantizer, a new adaptive quantized control strategy was presented in [20] and [21]. The presented adaptive quantized control method in [20] and [21] provides an effective solution to nonlinear systems with quantized input. It must be said that the aforementioned researches are concerned with infinite-time stability problems, transient responses of systems can not be be assured in theory.However, it is significant that the practical control system possesses a transient performance.

Different from asymptotic stability control, finite-time control strategy can get a quick transient performance. Therefore, finite-time stability problem has attracted a lot of attention in recent years. By establishing a terminal sliding manifold, two sliding controllers were constructed [22] and [23]. To overcome chattering phenomena caused by discontinuous controller in [22] and [23], an important finite-time Lyapunov stability theory was first built in [24, 25]. After that, by utilizing Lyapunov stability theory in [24,25], the subsequent finite-time control problems were coped with for nonlinear systems in [25–57]. However, signal quantization is neglected in [25–57], the finite time control schemes in [26–57] cannot apply to nonlinear systems with quantized characteristic. In addition, the controlled systems in [26–57] are in strict feedback form. For more complex system, the finite time control approaches in [26–57] are unavailable.

For the above-mentioned description, this article considers a finite-time quantized feedback control problem of nonlinear systems with non-strict feedback form. In comparison with the available researches, this paper contributes in the following.

1) In the most studies on finite time control in [26–55], the data transmission is assumed to accurate transmission,

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the quantized error is ignored in media transmission. Although [58] has taken the quantized error into finite time controller design, the control strategy in [58] can be only applied to nonlinearly parameterized systems. Instead, not only the quantized input is considered in this manuscript, but also the nonlinearities and their bounding functions are unknown completely, which need not satisfy the nonlinearly parameterized form. In addition, the plant in this paper is in non-strict feedback form, which contains strict feedback form in [58] as a special case.

2) Compared with available studies on quantized nonlinear systems in [9–21], a new finite-time control strategy is presented. Under the constructed controller, transient performance of quantized system is quickly achieved. Theoretically, an effective finite time solution is provided for quantized control of nonlinear systems.

In this note, we work on develop a new finite time solution for quantized nonlinear system with non-strict feedback form, which can assure transient performance quickly.

2. PREREQUISITES AND PROBLEM FORMULATION

2.1. Prerequisites

Definition 1 [57]**:** The system $\dot{\varpi} = f(\varpi,u)$ is semi-global practical finite-time stable (SGPFS) if for $\varpi(t_0) = \varpi_0$, there are $\varepsilon > 0$ and a settling time $T(\varepsilon,\varpi_0) < \infty$ such that $\|\varpi(t)\| < \varepsilon$, for all $t \geq t_0 + T$.

Lemma 1 [61]**:** Let $z_j \in R, j = 1,\ldots,n, 0 < p \leq 1$, then inequality (1) holds:

$$\Big(\sum_{j=1}^{n}|z_j|\Big)^{p} \leq \sum_{j=1}^{n}|z_j|^{p} \leq n^{1-p}\Big(\sum_{j=1}^{n}|z_j|\Big)^{p}. \tag{1}$$

Lemma 2 [15]**:** For $\dot{\hat{\chi}}(t) = -\gamma\hat{\chi}(t) + \omega\varepsilon(t)$, if $\hat{\chi}(t_0) \geq 0$, then $\hat{\chi}(t) \geq 0$ for $\forall t \geq t_0$, where constants $\gamma > 0$, $\omega > 0$, and function $\varepsilon(t)$ is positive.

Lemma 3 [62]**:** For real variables π, ϑ and positive constants τ, λ, ι, the inequality (2) holds:

$$|\pi|^{\tau}|\vartheta|^{\lambda} \leq \frac{\tau}{\tau+\lambda}\iota|\pi|^{\tau+\lambda} + \frac{\lambda}{\tau+\lambda}\iota^{\frac{-\tau}{\lambda}}|\vartheta|^{\tau+\lambda}. \tag{2}$$

Lemma 4 [57]**:** For system $\dot{\varpi} = f(\varpi,u)$ and smooth positive-definite function $V(\varpi)$, if there are constants $c > 0$, $0 < \sigma < 1$ and $d > 0$ such that

$$\dot{V}(\varpi) \leq -cV^{\sigma}(\varpi) + d, t \geq 0, \tag{3}$$

then system $\dot{\varpi} = f(\varpi,u)$ is SGPFS.

Remark 1: For the sake of the following finite-time stability analysis, Lemma 2, Lemma 3, Lemma 1 will be used to deal with the inequalities (17), (25) and (27), respectively. To show that the system is semi-global practical finite-time stable under the condition (27), Lemma 4 will be adopted.

2.2. Problem description

Let's consider the following quantized nonlinear systems:

$$\begin{cases} \dot{x} = F(x) + Gx + Bq(u(t)), \\ y = Cx, \end{cases} \tag{4}$$

where

$$\begin{aligned}
&F(x) = \begin{bmatrix} f_1(x) & \ldots & f_n(x) \end{bmatrix}^T, \\
&G = \begin{bmatrix} O^T & \Lambda \\ 0 & O \end{bmatrix}, \quad O = [0 \ \ldots \ 0] \in R^{n-1}, \\
&\Lambda = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & 1 \end{bmatrix}, \\
&B = \begin{bmatrix} 0 & 0 & \ldots & 1 \end{bmatrix}^T, \\
&C = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}^T.
\end{aligned}$$

In the aforementioned representations, $x = [x_1, x_2, \ldots, x_n]^T$ is a state vector, $y \in R$ represents system output, unknown nonlinear function $f_i(x)$ is continuously differentiable. $u(t) \in R$ is a controller to be established, system input $q(u(t))$ is affected by quantization action which is defined in the following [18, 19]:

$$q(u(t)) = \begin{cases} u_i sgn(u), & \frac{u_i}{1+\delta} < |u| \leq u_i, \dot{u} < 0, \text{ or} \\ & u_i < |u| \leq \frac{u_i}{1-\delta}, \dot{u} > 0 \\ u_i(1+\delta)sgn(u), & u_i < |u| \leq \frac{u_i}{1-\delta}, \dot{u} < 0, \text{ or} \\ & \frac{u_i}{1-\delta} < |u| \leq \frac{u_i(1+\delta)}{1-\delta}, \dot{u} > 0 \\ 0, & 0 \leq |u| < \frac{u_{min}}{1+\delta}, \dot{u} < 0, \text{ or} \\ & \frac{u_{min}}{1+\delta} \leq |u| \leq u_{min}, \dot{u} > 0, \\ q(u(t^-)), & \text{other cases,} \end{cases} \tag{5}$$

where $u_i = \rho^{1-i}u_{min} (i = 1, 2, \ldots)$ and $\delta = \frac{1-\rho}{1+\rho}$ with parameters $u_{min} > 0$ and $0 < \rho < 1$. $q(u(t))$ is in the set $U = \{0, \pm u_i, \pm u_i(1+\delta), i = 1, 2, \ldots\}$, the range of the dead-zone for $q(u(t))$ is determined by the positive parameter u_{min}, and the positive parameter ρ can be viewed as a measure of quantization density.

The aim of this article is to construct a neural controller for (1), such that y can follows a given signal y_r in finite time.

Let $\bar{y}_r^{(i)} = [y_r, y_r^{(1)}, \ldots, y_r^{(i)}]^T$, $1 \leq i \leq n$, where $y_r^{(i)}$ is the ith time derivative of y_r. It is assumed that y_r and $y_r^{(i)}$ are continuous and bounded in the following design.

Remark 2: Compared with finite-time works in [22–55], controlled system (1) is in a quantized feedback form.

Since the quantization of controller becomes closed-loop system a hybrid system, the traditional finite-time control schemes are not applicable to such quantized nonlinear system (1). In addition, the quantized feedback control scheme in [20] is based on infinite-time stability theory. Theoretically, the finite-time performance of system cannot be guaranteed under such a control strategy.

Remark 3: Different from nonlinear systems in finite time researches, function $f_i(x)$ contains all the state variables $x_1,..,x_n$, and so the system (4) is a non-strict feedback system. To cope with non-strict feedback form, the subsequent Lemma 6 will be provided. By using a useful characteristic of RBF NN, the backstepping design approach may be generalized to the system (4).

Lemma 5 [20]**:** The quantized signal $q(u(t))$ can be decomposed as follows:

$$q(u(t)) = D(u)u(t) + G(t), \tag{6}$$

where $D(u)$ and $G(t)$ satisfy

$$1-\delta \leq D(u) \leq 1+\delta, |G(t)| \leq u_{\min}. \tag{7}$$

2.3. RBF neural networks

In the following discussion, the radial basis functions (RBF) NN will be applied to approximate some unknown functions $f(\zeta)$ defined on some compact set Ξ. As stated in [63], for sufficiently large nodes number κ, the RBF neural networks $\Phi^{*T}\xi(\varpi)$ can approximate any continuous function $f(\varpi)$ over a compact set $\Xi \subset R^q$ to arbitrary accuracy $\varepsilon > 0$ in the following

$$f(\varpi) = \Phi^{*T}\xi(\varpi) + \varepsilon(\varpi), \forall \varpi \in \Xi \subset R^q,$$

where approximation error $\varepsilon(\varpi)$ satisfies $\mid \varepsilon(\varpi) \mid \leq \varepsilon$, $\xi(\varpi) = [\xi_1(\varpi), \xi_2(\varpi), ..., \xi_\kappa(\varpi)]^T$ is a basis function vector, $\Phi^* = [\phi_1, \phi_2, ..., \phi_\kappa]^T \in R^\kappa$ is

$$\Phi^* := \arg\min_{\Phi \in R^\kappa}\{\sup_{\varpi \in \Xi}|f(\varpi) - \Phi^T\xi(\varpi)|\},$$

where $\Phi \in R^\kappa$ is a weight vector. In this article, we will adopt Gaussian basis function $\xi_i(\varpi)$ as follows:

$$\xi_i(\varpi) = \exp[-\frac{(\varpi-\iota_i)^T(\varpi-\iota_i)}{\omega_i^2}], i = 1,2,...,\kappa, \tag{8}$$

where $\iota_i = [\iota_{i1}, \iota_{i2}, ..., \iota_{iq}]^T$ is a center of a receptive field and ω_i is a width of the Gaussian function.

Lemma 6 [64]**:** If $\xi(\bar{x}_q) = [\xi_1(\bar{x}_q), \cdots, \xi_l(\bar{x}_q)]^T$ with $\bar{x}_q = [x_1, \cdots, x_q]^T$ is basis function vector of a RBF NN, for $\forall k \leq q$, the following relation is satisfied:

$$||\xi(\bar{x}_q)||^2 \leq ||\xi(\bar{x}_k)||^2.$$

3. FINITE TIME CONTROL DESIGN

To facilitate the subsequent controller design, define unknown constant $\eta_i = ||\Phi_i^*||^2, i = 1\ldots,n$, where Φ_i^* is a weight vector of RBF neural networks which will be designed in the next analysis. Let $\hat{\eta}_i$ as an estimate of η_i, estimation error is $\tilde{\eta}_i = \eta_i - \hat{\eta}_i$. For $1 \leq i \leq n$, design the adaptive law as

$$\dot{\hat{\eta}}_i = \frac{p_i}{2a_i^2}\varpi_i^2\xi_i^T(X_i)\xi_i(X_i) - \rho_i\hat{\eta}_i, \hat{\eta}_i(t_0) \geq 0, \tag{9}$$

where $\xi_i(X_i)$ is a basis function vector of RBF neural networks with $X_i = [\bar{x}_i^T, \bar{y}_r^T, \bar{\hat{\eta}}_i^T]^T$, $\bar{\hat{\eta}}_i = [\hat{\eta}_1, ..., \hat{\eta}_i]^T$, p_i, a_i, ρ_i are positive design parameters, and ϖ_i is defined as follows:

$$\varpi_1 = y - y_r, \quad \varpi_i = x_i - \nu_{i-1}, \quad i = 2, \cdots, n. \tag{10}$$

The virtual controller ν_i and controller u are designed as follows:

$$\nu_i = -c_i\varpi_i^{2\sigma-1} - \frac{1}{2a_i^2}\hat{\eta}_i\varpi_i\xi_i^T(X_i)\xi_i(X_i), \tag{11}$$

$$u = -\frac{c_n\varpi_n^{2\sigma-1}}{1-\delta} - \frac{\varpi_n\hat{\eta}_n\xi_n^T(X_n)\xi_n(X_n)}{2a_n^2(1-\delta)}. \tag{12}$$

In the above definition, $c_i > 0$ and $\sigma = \frac{2l-1}{2l+1}(l > 2, l \in N)$ are design constants.

Remark 4: Having said that, in (11) and (12), the number of network nodes in $\Phi_i^T\xi_i(X_i)$ will increase exponentially as the system dimension increases (see the simulation section for details). Thus, huge dimension of nonlinear systems often has a high demand on memory in the process of calculation.

Theorem 1: For quantized nonlinear system (1), if the controller (12), the virtual controller (11) and parameter adaptive law (9) are adopted, the system output locates in a small neighborhood of the given signal in finite time.

Proof: Following by the typical backstepping technique in [59, 60], the proof procedure is given in this section.

Step 1: Choose Lyapunov function as $V_1 = \frac{\varpi_1^2}{2} + \frac{\tilde{\eta}_1^2}{2p_1}$, the time derivative of V_1 is

$$\dot{V}_1 = \varpi_1\Big(f_1(x) + \varpi_2 + \nu_1 - \dot{y}_d\Big) - \frac{1}{p_1}\tilde{\eta}_1\dot{\hat{\eta}}_1. \tag{13}$$

Applying Young's inequality, the following inequality is obtained.

$$\varpi_1\varpi_2 \leq \frac{\varpi_1^2}{2} + \frac{\varpi_2^2}{2}. \tag{14}$$

Define the following unknown packaged function

$$\bar{f}_1(Z_1) = \qquad f_1 - \dot{y}_d + \varpi_1, \tag{15}$$

where $Z_1 = [x^T, y_r, \dot{y}_r]^T$.
Then, we adopt a neural network $\Phi_1^* \xi_1(Z_1)$ to approximate $\bar{f}_1(Z_1)$, for a given $\varepsilon_1 > 0$,

$$\bar{f}_1 = \Phi_1^{*T} \xi_1(Z_1) + \delta_1(Z_1)$$

with δ_1 being the approximation error and satisfying $|\delta_1| \le \varepsilon_1$. Then, applying Lemma 6 and Young's inequality to the term $z_1 \bar{f}_1$ gives

$$\begin{aligned}\varpi_1 \bar{f}_1 &= \varpi_1(\Phi_1^{*T}\xi_1(Z_1) + \delta_1(Z_1)) \\ &\le |\varpi_1|(||\Phi_1^*||||\xi_1(Z_1)|| + \varepsilon_1) \\ &\le \frac{\varpi_1^2}{2a_1^2}\eta \xi_1^T(X_1)\xi_1(X_1) + \frac{a_1^2}{2} + \frac{\varpi_1^2}{2} + \frac{\varepsilon_1^2}{2}, \end{aligned} \quad (16)$$

where $X_1 = [x_1, y_r, \dot{y}_r]^T$. Substituting (9)-(11) and (14)-(16) into (13), one has

$$\dot{V}_1 \le -c_1 \varpi_1^{2\sigma} - \frac{\rho_1}{p_1}\tilde{\eta}_1 \hat{\eta}_1 + \frac{a_1^2 + \varepsilon_1^2}{2}. \quad (17)$$

Step i ($2 \le i \le n-1$)**:** Choose the Lyapunov function as $V_i = V_{i-1} + \frac{\varpi_i^2}{2} + \frac{\tilde{\eta}_i^2}{2p_i}$, then we have

$$\dot{V}_i = \dot{V}_{i-1} + \varpi_i \Big(f_i(x) + \varpi_{i+1} + \nu_i - \dot{\nu}_{i-1} \Big), \quad (18)$$

where

$$\begin{aligned}\dot{\nu}_{i-1} =& \sum_{j=1}^{i-1} \frac{\partial \nu_{i-1}}{\partial x_j} \Big[f_j(x) + x_{j+1} \Big] \\ &+ \sum_{j=1}^{i-1} \frac{\partial \nu_{i-1}}{\partial \hat{\eta}_j} \dot{\hat{\eta}}_j + \sum_{j=0}^{i-1} \frac{\partial \nu_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)}. \end{aligned} \quad (19)$$

Applying Young's inequality to $\varpi_i \varpi_{i+1}$, one has

$$\varpi_i \varpi_{i+1} \le \frac{\varpi_i^2}{2} + \frac{\varpi_{i+1}^2}{2}. \quad (20)$$

Define an unknown function $\bar{f}_i(Z_i)$ as

$$\bar{f}_i(Z_i) = f_i(x) - \dot{\nu}_{i-1} + \frac{3}{2}\varpi_i, \quad (21)$$

where $Z_i = [x^T, \bar{y}_r^T, \bar{\hat{\eta}}_i^T]^T$, $\bar{y}_r = [y_r, y_r^{(i)}]^T$.
Considering that $\bar{f}_i(Z_i)$ is unknown, we adopt a neural network $\Phi_i^* \xi_i(Z_i)$ to approximate it, for given $\varepsilon_i > 0$,

$$\bar{f}_i(Z_i) = \Phi_i^{*T} \xi_i(Z_i) + \delta_i(Z_i) \quad (22)$$

with δ_i being the approximation error and satisfying $|\delta_i| \le \varepsilon_i$. Then, applying Lemma 6 and Young's inequality to the term $z_i \bar{f}_i$ gives

$$\begin{aligned}\varpi_i \bar{f}_i &= \varpi_i(\Phi_i^{*T}\xi_i(Z_i) + \delta_i(Z_i)) \\ &\le |\varpi_i|(||\Phi_i^*||||\xi_i(Z_i)|| + \varepsilon_i) \\ &\le \frac{\varpi_i^2}{2a_i^2}\eta \xi_i^T(X_i)\xi_i(X_i) + \frac{a_i^2}{2} + \frac{\varpi_i^2}{2} + \frac{\varepsilon_i^2}{2}, \end{aligned} \quad (23)$$

where $X_i = [\bar{x}_i^T, \bar{y}_r^T, \bar{\hat{\eta}}_i^T]^T$.
Substituting (9)-(11) and (20)-(23) into (18), one has

$$\dot{V}_i \le -\sum_{j=1}^{n-1} c_j \varpi_j^{2\sigma} - \sum_{j=1}^{n-1} \frac{\rho_j}{p_j} \tilde{\eta}_j \hat{\eta}_j + \sum_{j=1}^{n-1} \frac{a_j^2 + \varepsilon_j^2}{2}. \quad (24)$$

Step n: Choose the Lyapunov function as $V_n = V_{n-1} + \frac{\varpi_n^2}{2} + \frac{\tilde{\eta}_n^2}{2p_n}$ and apply (6), then we have

$$\dot{V}_n = \dot{V}_{n-1} + \varpi_i \Big(f_n(x) + D(u)u(t) + G(t) - \dot{\nu}_{n-1} \Big), \quad (25)$$

where $\dot{\nu}_{n-1}$ is defined by (19).
As (9) satisfies the condition of Lemma 2, $\hat{\eta}_n(t) \ge 0$ for $\forall t > t_0$. Therefore, from (9) and (12), one has:

$$\varpi_n D(u)u(t) \le -c_n \varpi_n^{2\sigma} - \frac{1}{2a_n^2}\varpi_n^2 \hat{\eta}_n \xi_n^T \xi_n. \quad (26)$$

Applying Young's inequality to $\varpi_n G(t)$, one has

$$\varpi_n G(t) \le \frac{1}{2}\varpi_n^2 + \frac{1}{2}u_{\min}^2. \quad (27)$$

Define an unknown function $\bar{f}_n(Z_n)$ as

$$\bar{f}_n(Z_n) = f_n(x) - \dot{\nu}_{n-1} + \frac{3}{2}\varpi_n, \quad (28)$$

where $Z_n = [x^T, \bar{y}_r^T, \bar{\hat{\eta}}_n^T]^T$, $\bar{y}_r = [y_r, y_r^{(n)}]^T$. Similar to the processing in (13), and substituting (9)-(12), (26)-(27) into (25), one has

$$\dot{V}_n \le -\sum_{i=1}^{n} c_i \varpi_i^{2\sigma} - \sum_{i=1}^{n} \frac{\rho_i}{p_i} \tilde{\eta}_i \hat{\eta}_i + \sum_{i=1}^{n} \frac{a_i^2 + \varepsilon_i^2}{2} + \frac{1}{2}u_{\min}^2. \quad (29)$$

At the same time, from the definition of $\tilde{\eta}_i$, we have

$$\tilde{\eta}_i \hat{\eta}_i = \tilde{\eta}_i(\eta_i - \tilde{\eta}_i) \le -\frac{1}{2}\tilde{\eta}_i^2 + \frac{1}{2}\eta_i^2. \quad (30)$$

Based on (30), (29) can be rewritten as

$$\dot{V}_n \le -\sum_{i=1}^{n} c_i \varpi_i^{2\sigma} - \sum_{i=1}^{n} \frac{\rho_i}{2p_i} \tilde{\eta}_i^2 + \gamma, \quad (31)$$

where $\gamma = \sum_{i=1}^{n} \frac{\rho_i}{2p_i}\eta_i^2 + \sum_{i=1}^{n} \frac{a_i^2 + \varepsilon_i^2}{2} + \frac{1}{2}u_{\min}^2$.
Then, applying Lemma 3, let $\pi = 1$, $\vartheta = \frac{1}{2p_i}\tilde{\eta}_i^2$, and $\tau = \sigma, \lambda = 1 - \sigma, \iota = (1-\sigma)^{\frac{1-\sigma}{\sigma}}$, one has

$$\Big(\frac{1}{2p_i}\tilde{\eta}_i^2\Big)^{\sigma} \le \sigma\iota + \frac{1}{2p_i}\tilde{\eta}_i^2. \quad (32)$$

Combing (34) and (35), one has

$$\dot{V}_n \le -\sum_{i=1}^{n} c_i \varpi_i^{2\sigma} - \sum_{i=1}^{n} \rho_i \Big(\frac{1}{2p_i}\tilde{\eta}_i^2\Big)^{\sigma} + d$$

$$\leq -c\sum_{i=1}^{n}\varpi_i^{2\sigma} - c\sum_{i=1}^{n}\left(\frac{1}{2p_i}\tilde{\eta}_i^2\right)^{\sigma} + d, \tag{33}$$

where $c = \min\{c_i, \rho_i, i = 1, 2, \ldots, n\}$ and $d = \gamma + \sum_{i=1}^{n}\rho_i\sigma\iota$.

Furthermore, applying Lemma 1, we have

$$\dot{V}_n \leq -c\Big(\sum_{i=1}^{n}\Big(\frac{\varpi_i^2}{2} + \frac{1}{2p_i}\tilde{\eta}_i^2\Big)\Big)^{\sigma} + d$$
$$\leq -cV_n^{\sigma} + d. \tag{34}$$

Let $T^* = \frac{1}{(1-\sigma)\varphi c}\left[V_n^{1-\sigma}(\varpi(0),) - \left(\frac{d}{(1-\varphi)c}\right)^{(1-\sigma)/\sigma}\right]$ with $(\varpi(0)) = (\varpi_1(0), \ldots, \varpi_n(0), \eta_1(0), \ldots, \eta_n(0))^T$, then from Lemma 4, for $\forall t \geq T^*$, $V_n^{\sigma}(\varpi) \leq \frac{d}{(1-\varphi)c}$, that is, all the signals in the closed-loop system are SGPFS. Furthermore, from the definition of V, for $\forall t \geq T^*$, we have

$$|y - y_r| \leq 2\Big(\frac{d}{(1-\varphi)c}\Big)^{\frac{1}{2\sigma}}. \tag{35}$$

That is, system output locates in a small neighborhood of the reference signal in finite time.

4. SIMULATION EXAMPLE

Example 1: Here, the following example is presented to show the effectiveness of the proposed scheme.

$$\dot{x}_1 = x_2 + f_1(x),$$
$$\dot{x}_2 = x_3 + f_2(x),$$
$$\dot{x}_3 = q(u) + f_3(x),$$
$$y = x_1, \tag{36}$$

where $q(u)$ defined in (9), and $\delta = 0.25$, $\mu_{min} = 0.2$ are set. The reference signal is chosen as $y_r = \sin(t)$.

Remark 4: It is clear that system (36) is in a non-strict feedback system because $f_i(x)$ include all the state variables x_i, which is more general than the linearly parameterized quantized systems in [58]. In addition, the nonlinearities $f_i(x)$ are unknown to designer. Therefore, the only existing finite-time quantized control strategy in [58] is unavailable.

In the simulation, $f_1(x) = 0.5x_2^2\sin(x_3) + x_2\sin^2(x_2)$, $f_2(x) = \sin(x_1)\cos(x_1)x_2^2 + \frac{2x_3+x_3^3}{1+x_2^2+x_3^2}$, $f_3(x) = \sin(x_1 - x_2)x_3^2$. To apply the design strategy in Theorem 1, the centers of the basis functions are set evenly in the interval $[-2, 2]$, and the width of the basis functions are selected as 2. In particular, five nodes for each input dimension of $\Phi_1^T\xi_1(X_1)$, $\Phi_2^T\xi_2(X_2)$ and $\Phi_3^T\xi_3(X_3)$ are adopted. Thus, $\Phi_1^T\xi_1(X_1)$ includes 5^3 nodes with centers spaced evenly in $[-2, 2]^3$; $\Phi_2^T\xi_2(X_2)$ includes 5^6 nodes with centers spaced evenly in $[-2, 2]^6$; $\Phi_3^T\xi_3(X_3)$ includes 5^9 nodes with centers spaced evenly in $[-2, 2]^9$. In simulation, the design parameters are chosen as $\sigma = \frac{103}{107}$, $c_1 = c_2 = c_3 = 10$, $a_1 = a_2 = a_3 = 1$, $r_1 = r_2 = r_3 = 7.5$, $\rho_1 = 1.6$, $\rho_2 = \rho_3 = 0.8$; the initial conditions is given as $[x_1(0), x_2(0), x_3(0), \hat{\eta}_1(0), \hat{\eta}_2(0), \hat{\eta}_3(0)]^T = [0.25, 0, 0, 0.2, 0.2, 2]^T$. The corresponding simulation results are shown by Figs. 1-6. From the simulation results, we can see that output y can follow the given signal to a bounded set, in addition, all the closed-loop system signals are bounded in finite time.

To show that the effect of the main design parameters on the system performances, two group parameters are considered.

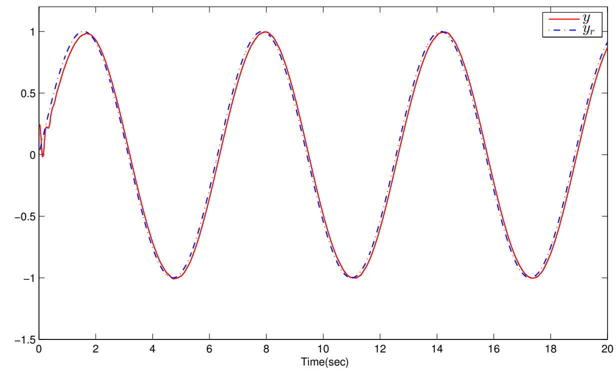


Fig. 1. y and y_r.

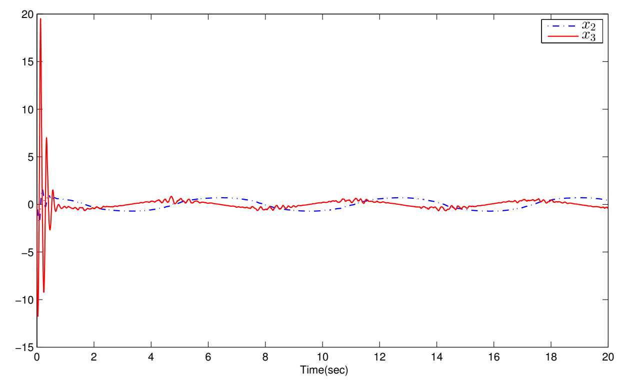


Fig. 2. x_2 and x_3.

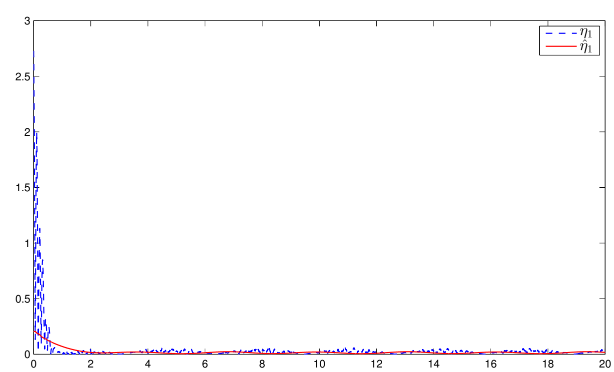


Fig. 3. η_1 and $\hat{\eta}_1$.

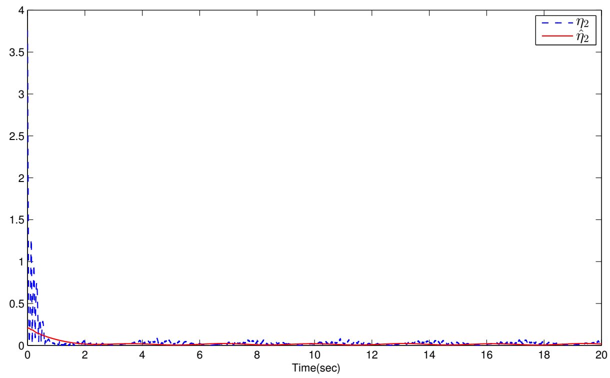


Fig. 4. η_2 and $\hat{\eta}_2$.

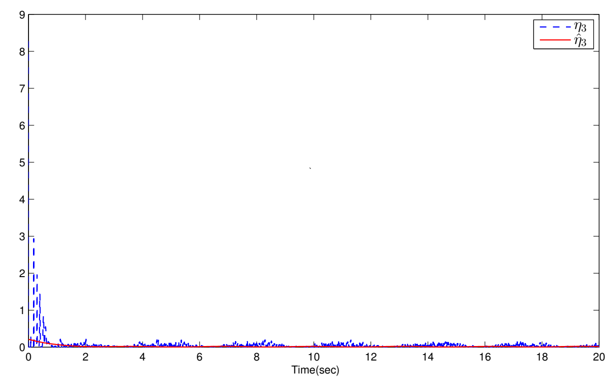


Fig. 5. η_3 and $\hat{\eta}_3$.

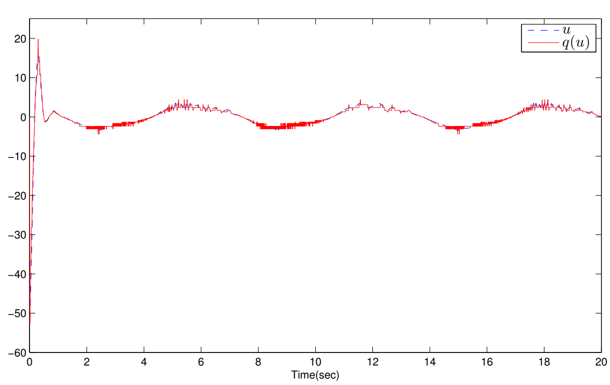


Fig. 6. u and $q(u)$.

Case 1: $a_1 = a_2 = a_3 = 1, p_1 = p_2 = p_3 = 7.5, \rho_1 = 1.6, \rho_2 = \rho_3 = 0.8, \sigma = \frac{103}{107}, c_1 = c_2 = c_3 = 10.$

Case 2: $a_1 = a_2 = a_3 = 1.2, p_1 = p_2 = p_3 = 6, \rho_1 = 1.2, \rho_2 = \rho_3 = 0.6$, and the selection of σ, c_i is similar to Case 1.

Under Case 1 and Case 2, define the tracking error index as $\sum_{k=1}^{M}[y(k) - y_r(k)]^2$, define the estimating error indexes as $\sum_{k=1}^{M}[\eta_1 - \hat{\eta}_1]^2$, $\sum_{k=1}^{M}[\eta_2 - \hat{\eta}_2]^2$ and $\sum_{k=1}^{M}[\eta_3 - \hat{\eta}_1]^3$, define the control gain index as , $\sum_{k=1}^{M}[u(k)]^2$, where M represents the sampling number. The above performance indexes are calculated within $0 \sim 20s$ and the sampling period is set as 0.01 s. The performance indexes comparisons under two cases are shown by Table I. From Table 1, it is clear that the smaller a_i, and the larger p_i and ρ_i, the smaller the the tracking error and the estimating errors. However, the control energy becomes larger when a_i are smaller and p_i, ρ_i are larger. Thus, the design parameters should be chosen on the basis of the system requirements.

Table 1. Performance index comparisons under Case 1 and Case 2.

Performance comparisons	Case 1	Case 2
$\sum_{k=1}^{M}[y(k) - y_r(k)]^2$	0.481	0.668
$\sum_{k=1}^{M}[\eta_1 - \hat{\eta}_1]^2$	0.3890	0.576
$\sum_{k=1}^{M}[\eta_2 - \hat{\eta}_2]^2$	0.6430	0.9163
$\sum_{k=1}^{M}[\eta_3 - \hat{\eta}_3]^2$	0.8430	1.2163
$\sum_{k=1}^{M}[u(k)]^2$	12.5897	18.8097

5. CONCLUSION

This article discusses a finite-time control issue of quantized nonlinear systems with non-strict feedback form. Based on the backstepping technique, by applying a nonlinear decomposition of hysteretic quantizer and structural property of radial basis function neural networks, an adaptive quantized control scheme is put foreword. The designed neural controller can ensure the transient performance of nonlinear quantized systems. Finally, we put Theorem 1 to the simulation to testify the effectiveness of main result. It should be pointed out that the proposed controller in this paper is based on the known state. More general case, if the state variables are partially measurable, then it would be better to synthesize the controller based on the measurement output [65–67]. Thus, as our future works mainly focus on the finite-time output feedback control design. On the other hand, as stated in [68–71], for the underlying systems suffering from actuator faults, the reliable control is an interesting issue. How to extend the current results to the finite time control of nonlinear systems actuator faults, is also our future research direction.

REFERENCES

[1] C. Yang, Y. Jiang, Z. Li, W. He, and C. Su, "Neural control of bimanual robots with guaranteed global stability and motion precision," *IEEE Transactions on Industrial Informatics,* vol. 13, no. 3, pp. 1162-1171, 2017.

[2] W. He, Z. J. Li, and C. L. Philip Chen, "A survey of human-centered intelligent robots: Issues and challenges," *IEEE/CAA Jouirnal of Automatica Sinica*, vol. 4, no. 4, pp. 602-609, Oct. 2017.

[3] W. He, Z. C. Yan, and C. Y. Sun, "Adaptive neural network control of a flapping wing micro aerial vehicle with distur-

bance observer," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3452-3465, 2017.

[4] C. Yang, X. Wang, Z. Li, Y. Li, and C. Y. Su, "Teleoperation control based on combination of wave variable and neural networks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 8, pp. 2125-2136, 2017.

[5] W. He, Z. C. Yan, Y. K. Sun, Y. S. Qu, and C. Y. Sun, "Neural-learning-based control for a constrained robotic manipulator with flexible joints," *IEEE Trans Neural Netw Learn Syst.*, vol. 29, no. 12, pp. 5993-6003, 2018. .

[6] Y. L. Wei, J. H. Park, H. R. Karimi, Y. C. Tian, and H. Jung, "Improved stability and stabilization results for stochastic synchronization of continuous-time semi-Markovian jump neural networks with time-varying delay," *IEEE Trans Neural Netw Learn Syst.*, vol. 29, no. 6, pp. 2488-2501, 2018.

[7] C. Y. Sun, W. He, and W. L. Ge, "Adaptive neural network control of biped robots," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 47, no. 2, pp. 315-326, 2017.

[8] H. Gao, W. He, C. Zhou, and C. Sun, "Neural network control of a two-link flexible robotic manipulator using assumed mode method," *IEEE Trans. Ind. Inform.*, pp. 1-1, March 2018. DOI: 10.1109/TII.2018.2818120

[9] N. Elia and S. Mitter, "Stabilization of linear systems with limited information," *IEEE Trans. Autom. Control,* vol. 46, no. 9, pp. 1384-1400, Sep. 2001.

[10] S. Tatikonda and S. Mitter, "Control under communication constraints," *IEEE Trans. Autom. Control,* vol. 49, no. 7, pp. 1056-1068, Jul. 2004.

[11] H. Ma, Q. Zhou, L. Bai, and H. Liang, "Observer-based adaptive fuzzy fault-tolerant control for stochastic nonstrict-feedback nonlinear systems with input quantization," *IEEE Transactions on Systems, Man and Cybernetics: Systems*, pp. 1-12, June 2018. DOI: 10.1109/TSMC.2018.2833872

[12] C. Persis and A. Isidori, "Stabilizability by state feedback implies stabilizability by encoded state feedback," *Syst. Control Lett.,* vol. 53, pp. 249-258, 2004.

[13] D. Liberzon and J. Hespanha, "Stabilization of nonlinear systems with limited information feedback," *IEEE Trans. Autom. Control,* vol. 50, no. 6, pp. 910-915, Jun. 2005.

[14] H. Gao and T. Chen, "A new approach to quantized feedback control systems," *Automatica*, vol. 44, no. 2, pp. 534-542, 2008.

[15] H. Gao, X. Meng, and T. Chen, "Stabilization of networked control systems with a new delay characterization," *IEEE Trans. Autom. Control,* vol. 53, no. 9, pp. 2142-2148, 2008.

[16] T. Hayakawaa, H. Ishii, and K. Tsumurac, "Adaptive quantized control for linear uncertain discrete-time systems," *Automatica,* vol. 45, pp. 692-700, 2009.

[17] H. Sun, N. Hovakimyan, and T. Basar, "L_1 adaptive controller for systems with input quantization," *Proc. of Amer. Control Conf., Baltimore*, MD, USA, pp. 253-258. Jun, 2010.

[18] T. Hayakawaa, H. Ishii, and K. Tsumurac, "Adaptive quantized control for nonlinear uncertain systems," *Syst. Control Lett.,* vol. 58, pp. 625-632, 2009.

[19] J. Zhou, C. Wen, and G. Yang, "Adaptive backstepping stabilization of nonlinear uncertain systems with quantized input signal," *IEEE Trans. Autom. Control,* vol. 59, no. 2, pp. 460-464, Feb. 2014.

[20] Z. Liu, F. Wang, Y. Zhang, and C. L. Philip Chen, "Fuzzy adaptive quantized control for a class of stochastic nonlinear uncertain systems," *IEEE Trans. Cybern.,* vol. 46, no. 2, pp. 524-534, 2016.

[21] F. Wang, Z. Liu, Y. Zhang, and C. L. Philip Chen, "Adaptive quantized controller design via backstepping and stochastic small-gain approach," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 2, pp. 330-343, 2016.

[22] Y. Tang, "Terminal sliding mode control for rigid robots," *Automatica*, vol. 34, no. 1, pp. 51-56, 1998.

[23] C. Tan, X. Yu, and Z. Man, "Terminal sliding mode observers for a class of nonlinear systems," *Automatica*, vol. 46, no. 8, pp. 1401-1404, 2010.

[24] S. P. Bhat and D. S. Bernstein, "Continuous finite-time stabilization of the translational and rotational double integrators,"*IEEE Trans. Autom. Control,* vol. 43, no. 5, pp. 678-682, 1998.

[25] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751-766, 2000.

[26] X. Huang, W. Lin, and B. Yang, "Global finite-time stabilization of a class of uncertain nonlinear systems," *Automatica*, vol. 41, no. 5, pp. 881-888, 2005.

[27] Y. Hong, J. Wang, and D. Cheng, "Adaptive finite-time control of nonlinear systems with parametric uncertainty," *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 858-862, 2006.

[28] C. Yang, Y. Jiang, W. He, J. Na, Z. Li, and B. Xu, "Adaptive parameter estimation and control design for robot manipulators with finite-time convergence," *IEEE Trans. Ind. Electron.*, vol. 65, no. 10, pp. 8112-8123, Oct. 2018.

[29] F. Wang, B. Chen, Y. M. Sun, and C. Lin, "Finite time control of switched stochastic nonlinear systems," *Fuzzy Sets Syst.*, 2018. DOI: 10.1016/j.fss.2018.04.016

[30] Y. J. Cui, W. J. Ma, Q. Sun, and X. Su, "New uniqueness results for boundary value problem of fractional differential equation," *Nonlinear Analysis: Modelling and Control*, vol. 23, no. 1, pp. 31-39, 2018.

[31] S. Ding, S. Li, and W. X. Zheng, "Nonsmooth stabilization of a class of nonlinear cascaded systems," *Automatica*, vol. 48, no. 10, pp. 2597-2606, 2012.

[32] W. Lv and F. Wang, "Adaptive tracking control for a class of uncertain nonlinear systems with infinite number of actuator failures using neural networks," *Adv. Differ. Equ.,* vol. 2017, pp. 374-390, 2017.

[33] J. M. Wang, H. D. Cheng, Y. Li, and X. N. Zhang, "The geometrical analysis of a predator-prey model with multi-state dependent impulses," *J. Appl. Anal. Comput.*, vol. 8, no. 2, pp. 427-442, 2018.

[34] J. Wang, H. Cheng, H. Liu, and Y. Wang, "Periodic solution and control optimization of a prey-predator model with two types of harvesting," *Adv. Differ. Equ.,* vol. 2018, pp. 41-54, 2018.

[35] Y. Li, H. Cheng, J. Wang, and Y. Wang, "Dynamic analysis of unilateral diffusion Gompertz model with impulsive control strategy," *Adv. Differ. Equ.*, vol. 2018, pp. 32-45, 2018.

[36] F. Liu and H. Wu, "Regularity of discrete multisublinear fractional maximal functions," *Sci. China Math.*, vol. 60, no. 8, pp. 1461-1476, 2017.

[37] F. Liu, "Continuity and approximate differentiability of multisublinear fractional maximal functions," *Math. Inequal. Appl.,* vol. 21, no. 1, pp. 25-40, 2018.

[38] W. Lv, F. Wang, and Y. Li, "Adaptive finite-time tracking control for nonlinear systems with unmodeled dynamics using neural networks," *Adv. Differ. Equ.*, vol. 2018, pp. 159-175, 2018.

[39] Q. Y. Su and X. L. Jia, "Finite-time H_∞ control of cascade nonlinear switched systems under state-dependent switching," *Int. J. Control Autom. Syst.,* vol. 16, no. 1, pp. 120-128, 2018.

[40] Q. H. Meng, Z. Y. Sun, and Y. S. Li, "Finite-time controller design for four-wheel-steering of electric vehicle driven by four in-wheel motors," *Int. J. Control Autom. Syst.,* vol. 16, no. 4, pp. 1814-1823, 2018.

[41] R. C. Ma, B. Jiang, and Y. Liu, "Finite-time stabilization with output-constraints of a class of highorder nonlinear systems," *Int. J. Control Autom. Syst.,* vol. 16, no. 3, pp. 945-952, 2018.

[42] G. D. Zong, X. H. Wang, and H. J. Zhao, "Robust finite-time guaranteed cost control for impulsive switched systems with time-varying delay,"*Int. J. Control Autom. Syst.,* vol. 15, no. 1, pp. 113-121, 2017.

[43] H. H. Dong, T. T. Chen, L. F. Chen, and Y. Zhang, "A new integrable symplectic map and the lie point symmetry associated with nonlinear lattice equations," *J. Nonlinear Sci. Appl.,* vol. 9, pp. 5107-5118, 2016.

[44] F. Wang, X. Y. Zhang, B. Chen, C. Lin, X. Li, and J. Zhang, "Adaptive finite-time tracking control of switched nonlinear systems," *Information Sciences*, vol. 421, pp. 126-135, 2017.

[45] F. F. Bian, W. C. Zhao, Y. Song, and R. Yue, "Dynamical analysis of a class of prey-predator model with Beddington-DeAngelis functional response, stochastic perturbation, and impulsive toxicant input," *Complexity*, vol. 2017, no. 3, pp. 1-18, 2017.

[46] C. D. Li, J. L. Gao, J. Q. Yi, and G. Q. Zhang, "Analysis and design of functionally weighted single-input-rule-modules connected fuzzy inference systems," *IEEE Trans. Fuzzy Syst.,* vol. 26, no. 1, pp. 56-71, 2018.

[47] C. D. Li, Z. X. Ding, D. B. Zhao, J. Yi, and G. Zhang, "Building energy consumption prediction: an extreme deep learning approach," *Energies*, vol. 10, no. 10, pp. 1-20, Oct. 2017.

[48] Y. J. Liang, R. Ma, M. Wang, and J. Fu, "Global finite-time stabilisation of a class of switched nonlinear systems," *Int. J. Syst. Sci.* vol. 46, no. 16, pp. 2897-2904, 2015.

[49] F. Wang, B. Chen, C. Lin, J. Zhang, and X. Z. Meng, "Adaptive neural network finite-time output feedback control of quantized nonlinear systems," *IEEE Trans. Cybern.,* vol. 48, no. 6, pp. 1839-1848, 2018.

[50] B. Niu, D. Wang, H. Li, X. Xie, N. D. Alotaibi, and F. E. Alsaadi, "A novel neural-network-based adaptive control scheme for output-constrained stochastic switched nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* pp. 1-15, 2017. DOI: 10.1109/TSMC.2017.2777472

[51] B. Niu, D. Wang, N. D. Alotaibi, and F. E. Alsaadi, "Adaptive neural state-feedback tracking control of stochastic nonlinear switched systems: an average dwell-time method," *IEEE Trans. Neural Netw.,* pp. 1-12, 2018, DOI:10.1109/TNNLS.2018.2860944

[52] J. Huang, C. Wen, W. Wang, and Y.-D. Song, "Design of adaptive finite-time controllers for nonlinear uncertain systems based on given transient specifications," *Automatica*, vol. 69, no. 1, pp. 395-404, 2016.

[53] X. P. Li, X. Y. Lin, and Y. Q. Lin, "Lyapunov-type conditions and stochastic differential equations driven by G-Brownian motion," *J. Math. Anal. Appl.*, vol. 439, no. 1, pp. 235-255, 2016.

[54] Y. Yang, C. Hua, and X. Guan, "Adaptive fuzzy finite-time coordination control for networked nonlinear bilateral teleoperation system," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 3, pp. 631-641, 2014.

[55] J. Wu, W. S. Chen, and J. Li, "Global finite-time adaptive stabilization for nonlinear systems with multiple unknown control directions," *Automatica*, vol. 69, no. 1, pp. 298-307, 2016.

[56] F. Wang, B. Chen, X. P. Liu, and C. Lin, "Finite-time adaptive fuzzy tracking control design for nonlinear systems," *IEEE Trans. Fuzzy Syst.,* vol. 26, no. 3, pp. 1207-1216, 2018.

[57] Z. Zhu, Y. Q. Xia, and M. Y. Fu, "Attitude stabilization of rigid spacecraft with finite-time convergence," *Int. J. Robust Nonlinear Control*, vol. 21, pp. 686-702, 2011.

[58] W. Liu, D. W. C Ho, and S. Xu, "Adaptive finite-time stabilization of a class of quantized nonlinearly parameterized systems," *Int. J. Robust Nonlinear Control*, vol. 27, no. 18, pp. 4554-4573, 2017.

[59] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Trans. Autom. control,* vol. 56, no. 7, pp. 1672-1678, 2011.

[60] J. P. Cai, C. Y. Wen, H. Y. Su, Z. Liu, and L. Xing, "Adaptive backstepping control for a class of nonlinear systems with non-triangular structural uncertainties," *IEEE Trans. Autom. control,* vol. 62, no. 10, pp. 5220-5226, 2017.

[61] G. H. Hardy, J. E. Littlewood, and G. Polya, *Inequalities*, Cambridge University Press, Cambridge, 1952.

[62] C. Qian and W. Lin, "Non-Lipschitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization," *Syst. Control Lett.,* vol. 42, no. 3, pp. 185-200, 2001.

[63] H. Q. Wang, P. Shi, H. Li, and Q. Zhou, "Adaptive neural tracking control for a class of nonlinear systems with unmodeled dynamics," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3075-3087, 2017.

[64] F. Wang, B. Chen, C. Lin, and X. H. Li, "Distributed adaptive neural control for stochastic nonlinear multiagent systems," *IEEE Trans. Cybern.*, vol. 47.no. 7, pp. 1795-1803, 2017.

[65] Y. Wei, J. H. Park, J. Qiu, L. Wu, and H. Y. Jung, "Sliding mode control for semi-Markovian jump systems via output feedback," *Automatica*, vol. 81, pp. 133-141, 2017.

[66] Y. L. Wei, J. B. Qiu, P. Shi, and M. ChaDli, "Fixed-order piecewise-affine output feedback controller for fuzzy-affine-model-based nonlinear systems with time-varying delay," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 64, no. 4, pp. 945-958, 2017.

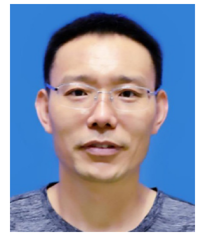
[67] Y. J. Liu, M. Z. Gong, S. C. Tong, C. L. P. Chen, and D. J. Li, "Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 2607-2617, Oct. 2018.

[68] L. Liu, Z. Wang, and H. Zhang,"Adaptive fault-tolerant tracking control for MIMO discrete-time systems via reinforcement learning algorithm with less learning parameters", *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 1, pp. 299-313, 2017.

[69] F. Wang and X. Y. Zhang, "Adaptive finite time control of nonlinear systems under time-varying actuator failures," *IEEE Trans. Syst., Man, Cybern., Syst.*, DOI:10.1109/TSMC.2018.2868329.

[70] J. B. Qiu, Y. L. Wei, H. R. Karimi, and H. Gao, "Reliable control of discrete-time piecewise-affine time-delay systems via output feedback," *IEEE Trans. Rel.*, vol. 67, no. 1, pp. 79-91, 2018.

[71] C. Wang, C. Wen, and L. Guo, "Decentralized output-feedback adaptive control for a class of interconnected nonlinear systems with unknown actuator failures," *Automatica*, vol. 71, pp. 187-196, 2016.



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