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Adaptive Controller Design Based on Predicted Time-delay for Teleoperation Systems Using Lambert W Function

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Abstract: This study develops an approach of controller design, on the basis of Lambert W function structure for Internet-based bilateral teleoperation systems. Actually, time-delay terms in bilateral teleoperation systems lead to an infinite number of characteristic equation roots making difficulty in analysis of systems by classical methods. As delay differential equations (DDEs) have infinite eigenspectrums, all closed-loop eigenvalues are not feasible to locate in desired positions by using classical control methods. Therefore, this study suggests a new feedback controller for assignment of eigenvalues, in compliance with Lambert W function. In this regard, an adaptive controller is accurately employed in order to provide the controller with updated predicted time-delay and robust the system against the time-delay. This novel control approach causes the rightmost eigenvalues to locate exactly in desired positions in the stable left hand of the imaginary axis. The simulation results show strong and robust closed-loop performance and better tracking in constant and time-varying delay.

Keywords: Adaptive controller, delay differential equations(DDEs), eigenvalue assignment, Lambert W function, teleoperation systems, time-delay.

1. INTRODUCTION

Up to now, various types of teleoperation systems have been presented to enable human being to accomplish a work in distant or perilous sites, with a diversity of application cases, ranging from underwater to space, nuclear activities, etc. [1]. Teleoperation systems highly suffer from the delays in communication channel which are usually time-varying delay leading to the system instability. The customary wave variable technique used for constant time delays could not provide satisfactory outcomes to stabilize the system under time-varying delay [1,2]. Another problem is observed in the complex structure and uncertain dynamics of nonlinear bilateral teleoperation systems resulting in the poor performance of the system. Therefore, designing controller for the teleoperation systems is very necessary and highly recommended to guarantee the system stability and strong performance of system. However, a large number of studies are conducted in this regard to resolve the problems and meet the appropriate conditions [1,2].

As regards teleoperation systems strongly suffer from an uncertain dynamics and kinematics, Zhao *et al.* suggested new nonlinear adaptive controllers for which no thorough knowledge should be gained in terms of kinematics of the master-slave as well as dynamics of the master-slave-operator-environment [2]. Lee *et al.* addressed the adaptive control used actuator saturation for teleoperation systems. In order to direct the survey of passive external forces, actuator saturation, asymmetric time-varying delays, and unknown parameters in the similar structure, a new switched control method is introduced, where an especial switched filter is scrutinized [3]. Peñaloza-Mejía *et al.* exploited actuator faults to synchronize time-delay problem of bilateral teleoperation systems. They modeled error dynamic system with actuator faults as a time-varying delay system [4]. Mellah *et al.* proposed an adaptive PID controller in order to improve the transparency in the bilateral teleoperation systems [5].

Xie *et al.* designed an observer-based H_∞ sliding mode controller for a type of nonlinear uncertain neutral Markovian switching systems (NUNMSSs) with general uncertain transition rates [6]. In this condition, each transition rate can be completely unknown or only its estimate value is known. Jiang *et al.* addressed the problem of robust passivity-based sliding mode control (SMC) for uncertain

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singular systems with semi-Markov switching and actuator failures [7]. They emphasized on designing a common sliding surface to reduce the jumping effect and developing a sliding mode controller to accommodate to actuator faults for pacification of the singular semi-Markovian jump system. Du *et al.* focused on the interval type-2 (IT2) fuzzy sampled-data tracking control problem of nonlinear systems subject to uncertain parameters [8]. The controlled plant employed the Takagi–Sugeno (T–S) fuzzy structure, where the parameter uncertainties are captured efficiently by the upper and lower membership functions.

This study is a serious attempt at providing a description of an innovative structure and adaptive controller on the basis of Lambert W function and predicted time-varying delay turning the Internet-oriented bilateral teleoperation systems as robust ones when are dealing with time-varying delay. By applying the Lambert W function, it is possible to locate a critical subset of the eigenvalues for desired positions in the left hand of the imaginary axis, which leads to stability of the system. This new approach involves eigenvalue assignment for the purpose of avoiding undesirable effects of time-delay and making improvements in position, force tracking, and transparency. The other way round, predicted time-varying delay turns controller into an intelligent type, being of great importance to stable the teleoperation systems. This predicted time-delay is utilized by lookup table to apply the correct controller designed by Lambert W function for each zone.

This study is developed in the following sections: in Section 2, bilateral teleoperation systems are described and Lambert W function is introduced in Section 3. Section 4 presents the new adaptive control architecture based on the Lambert W function and predicted time-delay as well as the stability of new methods is represented is Section 5. After designing the controllers, simulation results for different types of time-delay are demonstrated and the validity of these schemes vividly is established in the Section 6.

2. TELEOPERATION SYSTEMS

By and large, the bilateral teleoperation systems are combined of a local site, where a hand-controller named master is handled by a human operator, a distant site, where a manipulator named slave tracks the master motion and interacts with the environment in order to perform a given work, and a communication channel which connects both sites [9]. All in all, when a human operator controls one or some manipulators in the remote site, the system is so-called teleoperation system.

A general framework of the teleoperation systems is displayed in Fig. 1. The framework is composed of five major sections: operator, master, control and communication, slave, and environment. Actually, human operator applies a force Fm to move master manipulator with

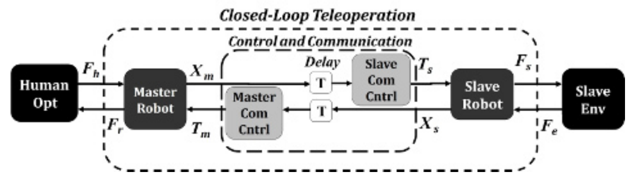

Fig. 1. Framework of closed-loop teleoperation systems.

$X_m = [\, x_m \quad \dot{x}_m \,]$ transferred through the communication channel to the slave manipulator. Then, the slave manipulator is derived by a local control T_s on the slave side. Since the manipulator of slave has an undeniable interaction with the environment, the remote force F_s is sent back from the slave side via force F_e which pushes the slave with $X_s = [\, x_s \quad \dot{x}_s \,]$ which is transmitted back to the master side by way of the communication channel as control signal T_m. Therefore, by receiving this reflected force F_r, human operator finds a virtual sense of the remote environment.

2.1. Dynamics of teleoperation systems

The general form of dynamic equation for a couple of n-DOF nonlinear robot manipulator in the lack of friction or other disturbances is organized as [10]:

$$M(p)\ddot{p} + C(p,\dot{p}) + G(p) = T, \tag{1}$$

where $M(p)$, $C(p,\dot{p}) \in R^{n\times n}$ are positive-definite inertia matrices and the Coriolis/Centripetal vector, respectively. Moreover, $\dot{M}(p) - C(p,\dot{p})$ is skew-symmetric, $G(p)$ shows the gravity vector and T stands as the torque vector. In this study, the degree of freedom (DOF) for the master and the slave manipulator is considered as one. The dynamic model of a 1-DOF manipulator is:

$$J\ddot{\theta}(t) + b\dot{\theta}(t) + \frac{1}{2}mgl\sin\theta(t) = u(t), \tag{2}$$

where J is the inertia, m and l are the mass and length of manipulator, respectively, g is the gravity acceleration, $\theta(t)$ is the angle of the rotate, $u(t)$ is the control signal and b is the viscous friction coefficient; however, proof is given in [11]. The simplified linear dynamic model is:

$$J\ddot{\theta}(t) + b\dot{\theta}(t) = u(t). \tag{3}$$

If position and velocity states are considered as $(x_1(t) = \theta(t))$, $(x_2(t) = \dot{\theta}(t))$, respectively, the master-slave manipulators can be symbolized in state-space description as:

$$\begin{bmatrix}\dot{x}_{m1}(t)\\ \dot{x}_{m2}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{b_m}{J_m}\end{bmatrix}\begin{bmatrix}x_{m1}(t)\\ x_{m2}(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J_m}\end{bmatrix}u_m(t). \tag{4}$$

$$y_m(t) = [\,1 \quad 0\,]\begin{bmatrix}x_{m1}(t)\\ x_{m2}(t)\end{bmatrix}, \tag{5}$$

$$\begin{bmatrix}\dot{x}_{s1}(t)\\ \dot{x}_{s2}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{b_s}{J_s}\end{bmatrix}\begin{bmatrix}x_{s1}(t)\\ x_{s2}(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J_s}\end{bmatrix}u_s(t), \tag{6}$$

$$y_s(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{s1}(t) \\ x_{s2}(t) \end{bmatrix}. \tag{7}$$

2.2. Environment

There is no doubt that environment force, produced due to the interaction of slave and remote environment, plays a key role in teleoperation systems. As a result, it is of great importance to introduce a mathematical model in order to calculate this reaction force. In this study, the Kelvin model simplification of the environment is employed [12]. This reaction force which performs opposite the slave is:

$$f_s(t) = k_e \theta_s(t) + b_e \dot{\theta}_s(t), \tag{8}$$

where k_e displays stiffness and b_e shows viscous friction. To regard the slave-master force feedback, the R_m vector is described as:

$$R_m = \begin{bmatrix} r_{m1} & r_{m2} \end{bmatrix} = \begin{bmatrix} k_f k_e & k_f b_e \end{bmatrix} \tag{9}$$

where k_f is the force feedback gain.

3. LAMBERT W FUNCTION APPROACH

It is possible to use delay differential equations (DDE) in order to describe time-delay systems. In fact, an infinite spectrum of frequencies is created due to delay term and this phenomenon makes analysis of systems by classical methods difficult especially as regard verifying stability and designing parameters of controllers. For overcoming this difficulty, approximations such as Pade can be used indirectly. Controllers are also designed by employing the Lyapunov method (e.g., algebraic Riccati equations (AREs) or linear matrix inequalities (LMIs)) [13,14]. The mentioned approaches need complicated equations and can lead to redundant terms. The estimation of this infinite frequency spectrum demands corresponding determination roots of the infinite dimensional characteristic equations, which is not possible for time-delay systems. Generally, we can gain understanding of it by applying standard methods developed for systems of linear ordinary differential equations (ODE). Thereby, in place of closed-form solutions, DDE is often satisfied by using asymptotic solutions, numerical techniques, and graphical methods principally including analysis of stability as well as controller design.

In this section, the researcher extends this technique to achieve a complete response for DDE system established on Lambert W function [15]. As the response presents an analytical structure in terms of DDE parameter, it is possible to define how parameters are included in the response and how each term affects each eigenvalue as well as the solution. Furthermore, each eigenvalue relates to a specific “branch” of the Lambert W function. In this technique, the format of response is similar to the general solution format of ODEs, and notion of state transition matrix in ODEs can be developed with regard to DDEs by employing the Lambert W function. This reveals that some methods for control and analysis of ODE system, in accordance with notion of state transition matrix, can extend to DDE System [16]. Actually, the Lambert W function is introduced as function which fulfills:

$$W_k(H_k) e^{W_k(H_k)} = H_k. \tag{10}$$

The Lambert W function [15] is a function with a complex argument and contains an infinite number of W_k branches, where $k = (-\infty, ..., -1, 0, 1, ..., \infty)$ [17]. The Lambert W function method is exploited to find the roots of matrix transcendental characteristic equation like linear matrix differential equation containing delayed argument. This characteristic equation has infinite root matrices. Asymptotic stability of matrix differential equation solutions with delayed argument has the most influence in root matrices corresponding to the values of the Lambert W function in its principal and neighboring branch. In time-delay systems (TDS) with real coefficients, the maximum real part of the characteristic equation roots correlates to one real root or one pair of complex conjugate roots. In fact, such a root or a pair of conjugate roots will be so-called the rightmost root. An equilibrium point of TDS is asymptotically stable if and only if the maximum real part of the characteristic equation roots (eigenvalues) is negative and this means that all of the eigenvalues have negative real parts. It is clear that the stability of the system is guaranteed if all of eigenvalues have negative real parts. In [18], a linear time-invariant (LTI) of DDEs, with a single constant time-delay, T, is given as:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + A_d x(t-T) + Bu(t), \quad t > 0, \\ &x(t) = g(t), \quad t \in [-T, 0), \\ &x(t) = x_0, \quad t = 0, \end{aligned} \tag{11}$$

where A and A_d are $n \times n$ matrices, $x(t)$ is a $n \times 1$ state vector, B is a $n \times r$ matrix, $u(t)$ is a $r \times 1$ vector indicating the external stimulation, $g(t)$ and $x(t)$ are a defined reshape function and an initial value respectively being specified in the Banach space of continuous mappings. In (11), the reshape function $g(t)$ and $x(t)$ are equal on the interval $t \in [-T, 0)$. The solution for (11), in term of the matrix Lambert W function is [19]:

$$x(t) = \sum_{k=-\infty}^{\infty} e^{S_k t} C_k^I + \int_0^t \sum_{k=-\infty}^{\infty} e^{S_k(t-\xi)} C_k^N Bu(\xi) d\xi, \tag{12}$$

where S_K is the solution matrix described as:

$$S_k = \frac{1}{T} W_k(A_d T Q_k) + A. \tag{13}$$

Both types of the coefficient C in (12) are consisted of A, A_d, T and computing approaches of them are proposed in

[14]. In what follows, the solution for the unknown matrix Q_k is represented [18]:

$$W_k(A_d T Q_k) e^{W_k(A_d T Q_k)+AT} = A_d T. \tag{14}$$

According to the aforementioned equation, the principal ($k=0$) and other ($k \neq 0$) branches of the Lambert W function can be computed analytically [15]. On the basis of the supposition that any pair of characteristic equation roots of (11) has a minimum separation distance, it is feasible to indicate that the unbounded series in (12), always converge when all the roots have negative real parts or during any limited distance [20]. The solution form in (12), asserts that the eigenvalues of the matrix S_k, and the matrix e^{S_k} define the stability condition for the system of (11). A time-delay system specified by (11) is asymptotically stable if and only if [18]:

All the Eigenvalues of S_k, $k = (-\infty, ..., -1, 0, 1, ..., \infty)$, have negative real parts or, equivalently, in the sense of Lyapunov:

All the Eigenvalues of e^{S_k}, $k = (-\infty, ..., -1, 0, 1, ..., \infty)$, are located in the unit circle.

However, calculating matrices S_k or e^{S_k} for a great deal of branches, $k = (-\infty, ..., -1, 0, 1, ..., \infty)$ sounds intricate. On the other hand, all eigenvalue branches of the Lambert W function technique $k = (-\infty, ..., -1, 0, 1, ..., \infty)$ are distinguishable. Actually, the obtained eigenvalues of the principal branch ($k=0$) are in nearest distance to the imaginary axis; hence, define the system stability [21].

3.1. Assumption

$$\text{Max [Re\{eigenvalues of } S_0\} \geq \text{Re\{other eigenvalues of } S_k\}.$$

It has been proven that the roots obtained with the principal branch ($k=0$) of the Lambert W function can define the stability of the DDE systems [22]. Yi *et al.* formulated the above Assumption as the foundation not only to reveal DDE system stability, but also to locate a subset of the eigenvalues in favorite places. On the one hand, delayed system, such as teleoperation systems shown by (11), suffer from countless solutions for matrices S_k, $k = (-\infty, ..., -1, 0, 1, ..., \infty)$, and on the other hand, the number of control parameters used in feedback controller is limited; therefore, it is not practical to design all of them at once [23].

4. LAMBERT CONTROLLER DESIGN METHOD

In order to locate the rightmost eigenvalues to the desired places in the left hand of imaginary axis, a Lambert W function controller is designed in this part. In controllable ODE systems, the noticeable point is that assigning all the closed-loop eigenvalues by designing the gains is

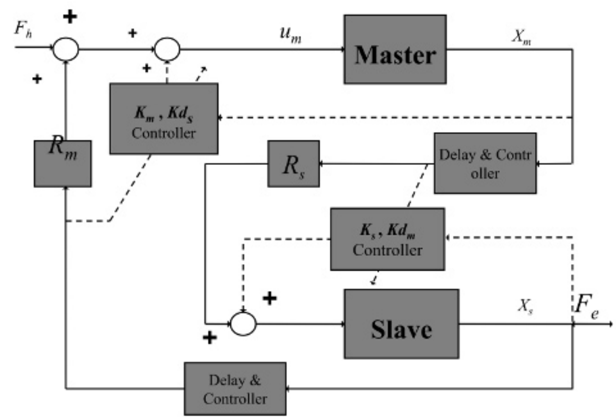

Fig. 2. Position-position structure of 1-DOF linear bilateral teleoperation systems.

applicable. Moreover, as mentioned before, DDEs suffer from numberless eigenvalues, and it is not practical to assign all of them in the desired locations by using classical methods. Hence, for controllable DDE systems, the researcher employs the Lambert W function method to determine the first matrix, S_0, correlating to the major branch, $k=0$, as it is significant for the solution form of (11), by designing a feedback controller and selecting the feedback gains. Fig. 2 shows a position-position structure of 1-DOF linear bilateral teleoperation systems with time-delay in the communication channel exploiting a Lambert W function controller. In what follows, all the possible terms of the operator-master-slave-environment model are considered:

F_h, F_e: respective the operator and environment force;

u_m, u_s: respective the master and slave control input;

X_m, X_s: respective position and velocity vectors for the master and slave;

K_m, Kd_s: Lambert Controller vectors in master;

K_s, Kd_m: Lambert Controller vectors in slave;

R_m: slave-master interaction which causes to the force reflection to the master;

R_s: master-slave interaction;

The block of "*Delay & Controller*" is depictured in Fig. 3. This figure clearly shows that an adaptive predictor controller is utilized to predict the time-varying delay and choose adaptively the appropriate amounts for "K_s, Kd_m" taking advantage of a lookup table. Also it is of great importance to say that this takes exactly happen for "K_m, Kd_s" in return channel and this demonstrates that delay is predicted from master to slave and slave to master.

The delay block demonstrates time-delay, T, in communication channel. It is of great importance to consider that the master and slave are developed by nth-order linear differential equations, and form of the matrices in the system can be represented in the following:

$$K_m = \begin{bmatrix} k_{m1} & k_{m2} & \dots & k_{mn} \end{bmatrix}, \quad K_s = \begin{bmatrix} k_{s1} & k_{s2} & \dots & k_{sn} \end{bmatrix},$$

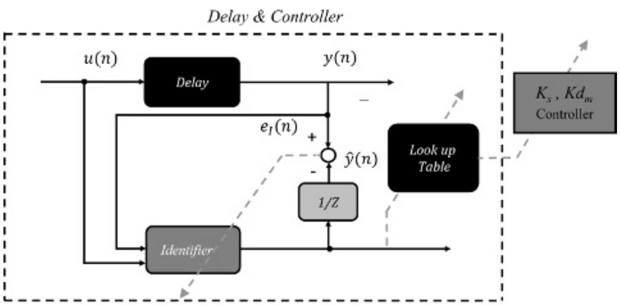

Fig. 3. Delay & controller block in detail.

$$Kd_m = \left[\, kd_{m1} \ \ kd_{m2} \ \ \ldots \ \ kd_{mn} \,\right],$$
$$Kd_s = \left[\, kd_{s1} \ \ kd_{s2} \ \ \ldots \ \ kd_{sn} \,\right],$$
$$R_m = \left[\, r_{m1} \ \ r_{m2} \ \ \ldots \ \ r_{mn} \,\right], \ \ R_s = \left[\, r_{s1} \ \ r_{s2} \ \ \ldots \ \ r_{sn} \,\right],$$

where 'm' and 's' are indications of *master system* and *slave system* respectively which can take the following form:

$$\dot{X}_m(t) = A_m X_m(t) + B_m u_m(t), \tag{15}$$
$$Y_m(t) = C_m X_m(t), \tag{16}$$
$$\dot{X}_s(t) = A_s X_s(t) + B_s u_s(t), \tag{17}$$
$$Y_s(t) = C_s X_s(t). \tag{18}$$

In this structure of assigning unstable pole to the desirable locations, three control modes take place:

$$\text{Mode 1:} \ \ u(t) = KX(t), \tag{19}$$
$$\text{Mode 2:} \ \ u(t) = K_d X(t-T), \tag{20}$$
$$\text{Mode 3:} \ \ u(t) = KX(t) + K_d X(t-T). \tag{21}$$

It is worth noting that we can choose either similar or different types of controller for master system and slave system. According to the selected control mode in the master and slave systems, the control signal, $u(t)$, is constructed and utilized in dynamic of time-delay system shown in (11). By combining (11) and (21), we can determine the closed-loop bilateral teleoperation systems:

$$\dot{X}(t) = (A + BK)X(t) + (A_d + BK_d)X(t-T), \tag{22}$$

where,

$$\dot{X}(t) = \left[\, \dot{X}_m(t) \ \ \dot{X}_s(t) \,\right]^T, \tag{23}$$
$$X(t) = \left[\, X_m(t) \ \ X_s(t) \,\right]^T, \tag{24}$$
$$X(t-T) = \left[\, X_m(t-T) \ \ X_s(t-T) \,\right]^T. \tag{25}$$

The major contribution of eigenvalue assignment in Lambert controller with the new coefficients, $AA = A + BK$ and $AA_d = A_d + BK_d$, in (22) is that we can arrive at a solution for the matrix S_k, by using (12), and by locating the rightmost eigenvalues to the favorite places.

$$\lambda_i(S_0) = \lambda_{i,desired} \ \ \text{for} \ \ i = 1, 2, \ldots, n. \tag{26}$$

Control signal of the master, $u_m(t)$, and the slave, $u_s(t)$, as shown in Fig. 2, can be given as follows:

$$u_m(t) = F_h - R_m X_s(t-T) + K_m X_m(t) + Kd_s X_s(t-T), \tag{27}$$
$$u_s(t) = R_s X_m(t-T) + K_s X_s(t) + Kd_m X_m(t-T). \tag{28}$$

By substituting (27) & (28) in (15) & (17), it can be deduced:

$$\dot{X}_m(t) = (A_m + B_m K_m)X_m(t) + B_m(Kd_s - R_m)X_s(t-T) + B_m F_h, \tag{29}$$
$$\dot{X}_s(t) = (A_s + B_s K_s)X_s(t) + B_s(Kd_m + R_s)X_m(t-T). \tag{30}$$

Bilateral teleoperation systems with respect to control signals sent from master to slave through the communication channel can be expressed as follows:

$$\begin{bmatrix} \dot{X}_m(t) \\ \dot{X}_s(t) \end{bmatrix} = \begin{bmatrix} A_m + B_m K_m & 0 \\ 0 & A_s + B_s K_s \end{bmatrix} \begin{bmatrix} X_m(t) \\ X_s(t) \end{bmatrix} + \begin{bmatrix} 0 & B_m(Kd_s - R_m) \\ B_s(Kd_m + R_s) & 0 \end{bmatrix} \begin{bmatrix} X_m(t-T) \\ X_s(t-T) \end{bmatrix} + \begin{bmatrix} B_m & 0 \\ 0 & B_s \end{bmatrix} \begin{bmatrix} F_h \\ 0 \end{bmatrix}, \tag{31}$$

where A_m and As are 2×2 matrices, B_m and B_s are 2×1 vectors, K_m and Kd_s are 1×2 Lambert controller gain vectors in master system, K_s and Kd_m are 1×2 Lambert controller gain vectors used in slave system, X_m and X_s are 2×1 state vectors in master and slave, respectively. Success in feedback controller design depends on the control gain matrices, K and K_d, for master and slave systems in order to guarantee the stability in closed-loop system of (22).

The control parameters in the teleoperation systems (see Fig. 2) are: $K_m = [\, k_{m1} \ \ k_{m2} \,]$, $K_s = [\, k_{s1} \ \ k_{s2} \,]$, $Kd_m = [\, kd_{m1} \ \ kd_{m2} \,]$, and $Kd_s = [\, kd_{s1} \ \ kd_{s2} \,]$. In other words, there are eight control parameters for assigning rightmost eigenvalues to desired locations in the left hand of imaginary axis. Four steps should be taken for the gains, K and K_d:

Step 1: Choose the desired eigenvalues, $\lambda_{i,desired}$ for (i = 1, 2, ..., n) in (26), and set an equation so that the selected eigenvalues are viewed as eigenvalues of the matrix S_0. Note that S_0 is the solution matrix resulting from the principal branch ($k = 0$) and $\lambda_i(S_0)$ which are the corresponding eigenvalues.

Step 2: Apply two new coefficient matrices, $AA = A + BK$ and $AA_d = A_d + BK_d$, in (22) to (14), and come up with a numerical solution to calculate the matrix Q_0 for the principal branch ($k = 0$). It should be noticed that K

and K_d are unknown matrices with unknown terms, and the matrix Q_0 is a function consisting of K and K_d.

Step 3: Calculate S_0 and its eigenvalues as the function of the unknown matrix K and K_d by substituting the matrix Q_0 in (14) with (13).

Step 4: Using numerical technique such as "fsolve" function in Matlab software to solve S_0 for the unknown K and K_d.

5. SCATTERING THEORY AND STABILITY

This section contains a theorem that describes an end-to-end model for the teleoperation systems based on the scattering matrix analysis. In accordance with scattering matrix, teleoperation systems is described as $b = S(s)a$, where $a = [\ a_1 \quad a_2\ ]^T$ and $b = [\ b_1 \quad b_2\ ]^T$ are input and output waves of the teleoperation systems, respectively.

5.1. Theorem

Necessary and sufficient conditions for robust stability of teleoperation systems are [24]:

(a) $S(s)$ includes no poles in the closed right half plane.

(b) If Δ is the structured perturbation of s:

$$Sup_{\omega}\left[\mu_{\Delta}(S(j\omega))\right] \leq 1,$$

where $\mu_{\Delta}(s)$ is the structured singular value of matrix S. A practical advantage of $\mu_{\Delta}(s)$ is $\mu_{\Delta}(s) \leq \bar{\alpha}(s)$, where $\bar{\alpha}(s)$ is the maximum singular value of Δ and $S(s)$ is the scattering matrix. As the structure in Fig. 2 shows, input-output relation can be expressed as follows:

$$\dot{X}_m = A_m X_m + B_m u_m, \tag{32}$$

$$u_m = K_m X_m + Kd_s X_s(t-T) + F_h - F_e(t-T). \tag{33}$$

If aforementioned control signal is substituted in the master state space description:

$$X_m = (sI - A_m - B_m K_m)^{-1} B_m [F_h + e^{-Ts} Kd_s X_s - e^{-Ts} F_e]. \tag{34}$$

In slave subsystem:

$$\dot{X}_s = A_s X_s + B_s u_s, \tag{35}$$

$$u_s = R_s X_m(t-T) + K_s X_s + Kd_m X_m(t-T). \tag{36}$$

By replacing the mentioned control signal in the slave state space description, it is obtained:

$$X_s = \psi(s) X_m, \tag{37}$$

$$\psi(s) = (sI - A_s - B_s K_s)^{-1} B_s (Kd_m + R_s) e^{-Ts}. \tag{38}$$

By inserting (37) into (34) and providing a concise summary, we will have:

$$X_m = a(s) F_h + b(s) F_e, \tag{39}$$

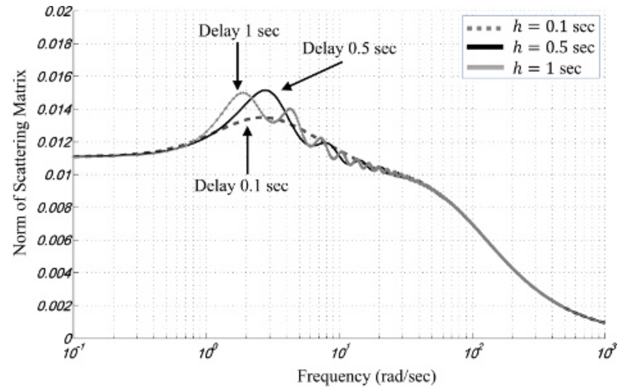

Fig. 4. $S(s)$ SVD for different time-delays.

where

$$a(s) = \beta_{(s)}{}^{-1} B_m, \tag{40}$$

$$b(s) = -\beta_{(s)}{}^{-1} e^{-Ts} B_m, \tag{41}$$

$$\beta_{(s)} = sI - A_m - B_m K_m - e^{-2Ts} B_m Kd_s (sI - A_s - B_s K_s)^{-1} B_s (Kd_m + R_s). \tag{42}$$

By inserting (39) into (37), we obtain:

$$X_s = c(s) F_h + d(s) F_e, \tag{43}$$

where

$$c(s) = \alpha(s) a(s), \tag{44}$$

$$d(s) = \alpha(s) b(s). \tag{45}$$

As the result, for establishing a structure for teleoperation systems by exploiting Lambert W function controller, it is required to estimate the scattering matrix as follows:

$$S(s) = \begin{bmatrix} a(s) & b(s) \\ c(s) & d(s) \end{bmatrix}. \tag{46}$$

Then, SVD (singular value decomposition) of $S(j\omega)$ for different time-delays is computed and plotted. Fig. 4 is an indication of scattering matrix norm for different time-delays. It is clear that $Sup_{\omega}[\mu_{\Delta}(S(j\omega))] \leq 1$ for different time-delays is acceptable; therefore teleoperation systems controlled by Lambert W function has stable structure and robust performance on different values of time-delay. This figure shows that maximum norm occurs in interval $[1, 10]$ frequency while minimum norm belongs to time-delay 0.1 sec and maximum is possessed by time-delay 0.5 sec.

6. SIMULATION RESULTS

The most common and generic dynamic test for control scheme is step response, through which the tracking control performance with human operator is evaluated. For a better response, a local feedback is taken into consideration in order to place the closed-loop poles in master and slave systems at locations $[\,-2 \quad -4\,]$, and then we calculate Lambert adaptive controller gain matrix for locating unstable poles in system of (31), to the desired locations. For calculating these gains, the “fsolve” function in Matlab is exploited. Despite undesirable effect of time-delay, this adaptive composed controller causes the controlled system to show an appropriate performance and stable behavior in Internet-based bilateral teleoperation systems. Seen from other way round, Azorin *et al.* introduced a feedback controller for the bilateral teleoperation systems [25] and in order to validate the simulation results, they are compared with Azorin’s controller scheme results. Simulation parameters are as follows:

$$\text{Master: } \begin{cases} J_m = 1.5 \text{ kgm}^2, \\ b_m = 11 \dfrac{\text{Nm}}{\text{rad/s}}, \end{cases} \quad \text{Slave: } \begin{cases} J_s = 2 \text{ kgm}^2, \\ b_s = 15 \dfrac{\text{Nm}}{\text{rad/s}}, \end{cases}$$

$$\text{Environment: } \begin{cases} k_e = 100 \text{ Nm/rad}, \\ b_e = 1 \dfrac{\text{Nm}}{\text{rad/s}}, \\ R_s = [\,r_{s1} \ \ r_{s2}\,] = [\,1 \ \ 1\,],' \end{cases}$$

Force reflection gain: $k_f = 0.1$,

Desired poles: $\lambda_{desired} = -1.5 \pm 0.01i$.

Fig. 5 falls into three parts, in which we can see simulation results of constant time-delay, 500 msec, part (a) contains time-varying input with different amplitudes and frequencies. When this input is applied to the system and output response displays a strong performance and does good tracking, controller design is acceptable. Part (b) consists of position of master, slave of Lambert W function, and slave of Azorin’s controller [22] for time-varying input relating to part (a). This figure clearly exposes that the slave of Lambert W function controller predicts the performance of the master better and tracks master quicker rather than slave of previous controller (Azorin). This part strongly demonstrates the superiority of the proposed controller in comparison with the previous controller. Master and slave control signal of proposed (Lambert) and previous (Azorin) controller are illustrated in part (c). Although proposed controller reaches master faster, it needs lower control signal in compliance with previous controller and it is other advantage of this impressive controller. However, all of them have bounded and acceptable response with respect to input signal.

There are four parts in Fig. 6, representing controller parameters for 500 msec time-delay and time-varying input. Internet is utilized in teleoperation systems as communication channel; consequently, delay is undeniable and should be considered. Part (a) illustrates K_m having two parameters stabling master subsystem and placing its poles in the desired places. Part (b) explains Kd_s gains being designed to make telepresence at master joystick and keep system behavior in an acceptable manner. This graph also has 2 lines which the former is constant and the latter is variable and this shows that one variable parameter can satisfy the system needs. Located in the master side, K_m and Kd_s keep master poles in the left side of imaginary axis and this, in turn, stabilize the system; moreover, the goal of utilizing the Kd_s is to improve systems performance. Part (c) and (d) contain K_s and Kd_m controller gains in the slave side for 500 msec time-delay, respectively. Frankly speaking, K_s places slave poles at the desired place in the left of imaginary axis and Kd_m transfers delayed master data to slave. Each of them has 2 parameters which the first is constant and the second is variable. These parameters guarantee the system stability and improve system performance to obtain the appropriate results.

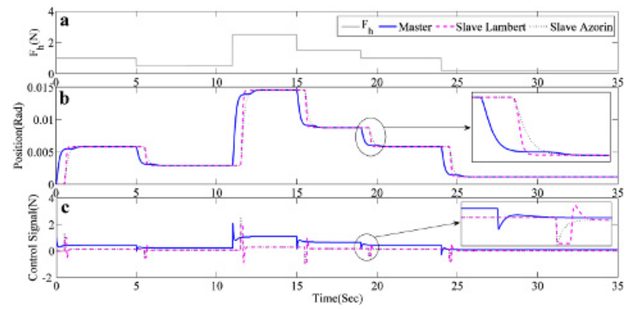

Fig. 5. Simulation results for 500 msec time-delay.

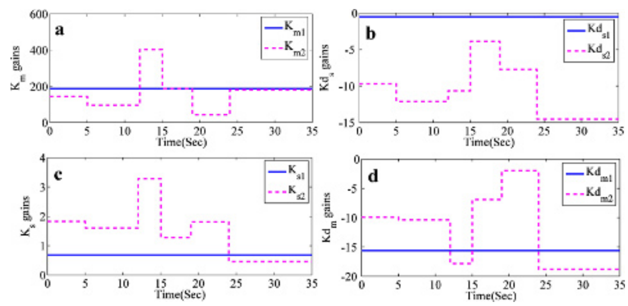

Fig. 6. Adaptive controller gains for 500 msec time-delay.

There are three parts in Fig. 7, indicating simulation results for time-varying delay. Internet is utilized in teleoperation systems as communication channel. As its time-delay types are variable, we can test controller of time-varying delay shown in part (a). Part (b) demonstrates master and Lambert-Azorin slave positions being obtained by applying Fig. 5(a) as time-varying input and part (a) of this figure as time-varying delay. This figure firmly validates the adequate performance of the Lambert-based controller. As a matter of fact, although the proposed controller is brightly robust over noise of time-varying delay, output of previous controller strongly is affected by fluctuations relating to time-varying delay and presents a poor

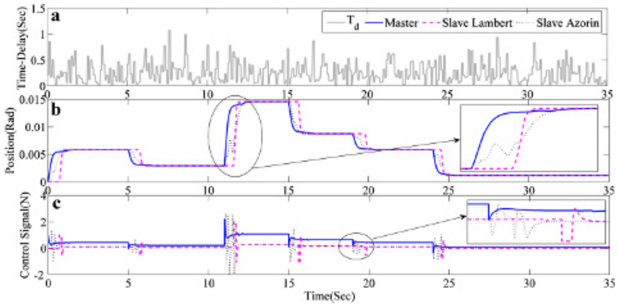

Fig. 7. Simulation results for time-varying delay.

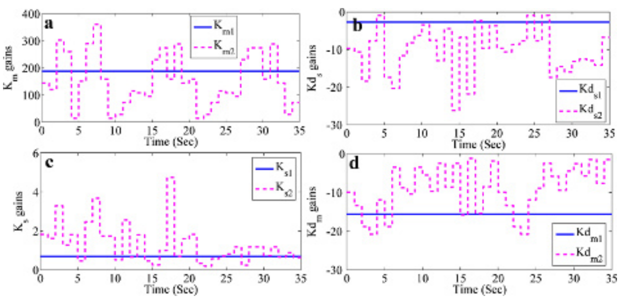

Fig. 8. Adaptive controller gains for time-varying delay.

and non-robust performance.

Part (c) contains master and Lambert-Azorin slave control signal for time-varying delay. This part, also, approves the excellence of the proposed controller with respect to the control signal. In fact, not only slave of previous controller suffers from a fluctuated output which negatively affects actuator, but also needs higher value of control signal which is harmful for actuator. Parts (b) and (c) in this figure completely demonstrate the unquestionable superiority of the proposed controller.

There are four parts in Fig. 8, representing controller parameters for time-varying delay and time-varying input. Internet is utilized in teleoperation systems as communication channel; consequently, delay is undeniable and should be considered. Part (a) demonstrates K_m gains for time-varying delay. Part (b) explains Kd_s gains being designed to make telepresence at master joystick and keep system behavior in an acceptable manner. This graph also has 2 lines which the former is constant and the latter is variable and this show that one variable parameter can satisfy the system needs. Located in the master side, K_m and Kd_s keep master poles in the left side of imaginary axis and this, in turn, stabilize the system; moreover, the goal of utilizing the Kd_s is to improve systems performance. Part (c) and (d) contain K_s and Kd_m controller gains in the slave side for time-varying delay, respectively. In fact, K_s places slave poles at the desired place in the left of imaginary axis and Kd_m transfers delayed master data to slave. Each of them has 2 parameters which the first is constant and the second is variable.

7. CONCLUSION

In this study, the researcher deployed a new controller for teleoperation systems based on Lambert W function method. Time-delays in communication channel lead to an infinite number of eigenvalues in teleoperation systems. They are calculated by delay differential equations and cause instability in these systems. Due to infinite number of eigenspectrums, the systems face difficult; however, by using this method a critical subset of these eigenspectrums is assigned to the desired possible locations in the complex plane. Moreover, in order to robust the system against the time-varying delay of communication channel, an adaptive controller is employed. Then, the researcher described a method for preserving stability in teleoperation systems in accordance with scattering theory. The findings demonstrated that the proposed controller of Lambert W function can improve performance of teleoperation systems and reduce unfavorable influence of time-delays in teleoperation systems. The simulation results demonstrated the effectiveness of this neoteric approach.

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