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Sampled-data Robust H_∞ Control for T-S Fuzzy Time-delay Systems with State Quantization

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Abstract: This paper investigates the sampled-data robust H_∞ control for T-S fuzzy time-delay systems with state quantization. Based on a modified Lyapunov-Krasovskii function(LKF), which is fully considered the characteristics of sample-data and state quantization, a sample-data and state quantized controller is designed. By introducing the free weighting matrices, some integral techniques and modified inequalities, the results in this paper are less conservative than other existing results. At the end of the paper, two examples are given to show the effectiveness and superiority of the proposed methods.

Keywords: Sampled-data H_∞ control, state quantization, T-S fuzzy systems, time-delay systems.

1. INTRODUCTION

The Takagi-Sugeno (T-S) fuzzy model is an effective modeling method for nonlinear systems [1]. It is described by a set of fuzzy IF-THEN rules with fuzzy sets in the antecedents and linear time-invariant dynamic systems in the consequent, which works for many typical nonlinear systems, and a lot of stability conditions are obtained [2–6].

Network and information technology are gradually applied in many fields, more and more people are devoted to the study of T-S fuzzy systems combined with sampled-data [7–9]. The information data obtained at discrete time intervals and it can be the digital data or simulated data. The control signals of sampled-data systems will be held constant between any two consecutive sampling instants and only be changed at each sampling instant [10, 11]. The sampling period is an important issue as we solve the stability problem of the system with a sampled-data controller. It is clear that a longer sampling period will be detrimental to the stability of the system. So, it is very important to consider the control design problem with a longer sampling period. In [12], the input delay approach is first proposed, and it has the advantage that $t_{k+1} - t_k$ does not need to be a fixed value. It combines with other technologies and is widely used in many papers [13–16].

It should be noted that the previous research results only assumed that all data transmissions can be performed with infinite precision, while the data quantization is not take into account in their papers. The first paper that mentions quantification is [17]. Before the transmission process, the sampling signal usually needs to be quantized to solve the problem of limited network transmission capacity due to the rapid development of the internet. The concept of quantizer is first proposed to solve the channel finite problem of the network control system [18]. Shannon proposed the concept of distortion rate [19], which arouses the scholars' interest in quantizer. Quantification is mainly applied to the transformation from continuous signal to digital signal, as shown in Fig. 1. According to the structure, quantizer can be divided into linear quantizer and logarithmic quantizer. According to different design techniques, quantizer can be divided into static quantizer and dynamic quantizer [20]. [21] proved that the optimal quantizer in the discrete linear time invariant system with single input and single output is logarithmic quantizer, because of the small density. [22] uses the upward bound method to confirm that the quantitative quantizer is not only applicable to the system of single input single output but also for multi-input and multi-output systems. It is the first paper to study the stability of sampled data systems with quantization. Inspired by [22], [23] studies the quantitative stability of discrete time invariant systems, the concept of state quantification is first proposed and the sector bound approach for sampled-data systems with a logarithmic quantizer is derived. Even if the stability of the system controller is good, some random behav-

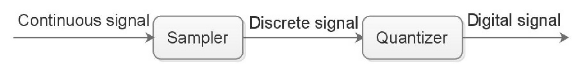

Fig. 1. The process of signal transmission.

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iors may affect the stability of the system in the process of quantification. So, the study of sampled-data systems with the quantization has became an active research topic [24–26]. Inspired by these works and [27–29], this paper will develop the sampled-data robust H_∞ control for T-S fuzzy time-delay systems with state quantization.

Then, the problem of the sampled-data robust H_∞ control for T-S fuzzy time-delay systems with state quantization is considered in this paper. First, we construct a improved LKF, which considered all available information about the actual sampling and state quantization pattern, some new stability conditions are obtained. Second, based on these stability conditions, the sample-data and state quantized controller is designed for T-S fuzzy time-delay systems. Particularly, by introducing the free weighting matrices, some integral techniques and modified inequalities, the results obtained from this paper have a larger sampling period and are less conservative than other existing results. The improved inequality is inspired by $X^TAX \geq 0$. In the end, a numerical example and a comparison example are given to show the good effect of the results we obtained.

Notation: In this paper, $*$ denotes the elements below the main diagonal of a symmetric block matrix. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $diag\{\cdots\}$ denotes the block-diagonal matrix, and $diag\left(\underbrace{X,\ldots,X}_{4}\right)$ denotes $diag(X,X,X,X)$. For symmetric matrices A and B , the notation $A > B$ means that the matrix $A-B$ is positive definite.

2. PREAMBLE FORMULATION

2.1. Fuzzy plant model

The i th rule of the system is expressed in the following IF-THEN form, consider the following T-S fuzzy time-delay systems:

Rule i : IF $\theta_1(t)$ is ϖ_1^i and ... and $\theta_n(t)$ is ϖ_n^i, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-\tau) + B_i u(t) + B_{wi}\omega(t). \\ y(t) = C_{1i}x(t) + C_{2i}x(t-\tau) + D_{1i}u(t) + D_{2i}\omega(t), \\ x(t) = \varphi(t), t \in [-\max(\tau,h), 0], \quad i = 1,2,\ldots,r, \end{cases} \tag{1}$$

where $\theta_1(t),\ldots,\theta_n(t)$ are the premise variables, and $\varpi_j^i(i = 1,2,\ldots,r, j = 1,2,\ldots,r)$ is the fuzzy set, denote $\theta(t) = [\theta_1(t), \ \ldots, \theta_{1n}(t)]^T$ and assume that $\theta(t)$ is already given or a function of $x(t)$ but doesn't depend on r, scalar r is the number of IF-THEN rules; $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the sampled-data input vector, $\omega(t) \in \ell_2[0,\infty)$ denotes exogenous input disturbance signal, $y(t) \in \mathbb{R}^p$ denotes system output, $\phi(t)$ is the initial condition of this system state; τ is constant state delay; $\mathrm{A_i}, \mathrm{A_{di}}, B_i, B_{\mathrm{wi}}, \mathrm{C_{1i}}, \mathrm{C_{2i}}, D_{1i}, D_{2i}$ are constant real matrices with appropriate dimensions;

By using a center-average defuzzifier, product inference and singleton fuzzifier, the global dynamics of the T-S fuzzy system (1) can be inferred as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \eta_i(\theta(t)) [A_i x(t) + A_{di} x(t-\tau) + B_i u(t) \\ \qquad + B_{wi}\omega(t)] \\ y(t) = \sum_{i=1}^{r} \eta_i(\theta(t)) [C_{1i} x(t) + C_{2i} x(t-\tau) + D_{1i} u(t) \\ \qquad + D_{2i}\omega(t)] \\ x(t) = \varphi(t), t \in [-\max(\tau,h)], i = 1,2,\ldots,r, \end{cases} \tag{2}$$

where $\eta_i = \eta_i(\theta(t))$ denotes the normalized membership function satisfying:

$$\eta_i(\theta(t)) = \frac{\omega_i(\theta(t))}{\sum_{i=1}^{r}\omega_i(\theta(t))}, \omega_i(\theta(t)) = \prod_{j=1}^{r} M_{ij}(\theta_j(t)), \tag{3}$$

where $M_{ij}(\theta_j(t))$ is the membership value of $\theta_j(t)$ in M_{ij}. It can be seen that $\forall i \in \{1,2,\ldots,r\}$, $\eta_i(\theta(t))$ has the following condition:

$$\eta_i(\theta(t)) \geq 0, \sum_{i=1}^{r} \eta_i(\theta(t)) = 1. \tag{4}$$

2.2. Sampled-data and state quantized controller

The parallel distributed compensation (PDC) is used to design the controller and make the T-S fuzzy system (2) stabilization. The sampled-data input $u(t) = u_j(t)$, $t_k \leq t < t_{k+1}$ with sampling instants $t_k(k = 0,1,2,\ldots)$ satisfying

$$0 = t_0 < t_1 < \ldots < \lim_{k\to+\infty} t_k = +\infty, \tag{5}$$

and

$$0 < t_{k+1} - t_k = h, \forall k \geq 0, \tag{6}$$

where $h \geq 0$ is the upper bound of sampled time.

The sampled-data and state quantized controller and the i th rule of the controller is expressed in the following IF-THEN form:

Rule j : IF $\theta_1(t_k)$ is ϖ_1^j and ... and $\theta_n(t_k)$ is ϖ_n^j, THEN

$$u(t) = K_j q(x(t_k)), t_k \leq t < t_{k+1}, \tag{7}$$

where $u_j(t)$ is the input vector of rule j, K_j is the gain matrix of the state feedback with appropriate dimension, $x(t_k)$ is the state vector of subsystem at the instant t_k, which is a piecewise constant function by using a zero-order-holder(ZOH); The logarithmic quantizer is described as:

$$q(\cdot) = [q_1(\cdot), q_2(\cdot), ..., q_n(\cdot)]^T \tag{8}$$

with the d th subquantizer $q(\cdot)$, which is symmetric:

$$q_d(x_d(t_k)) = -q_d(-x_d(t_k)), \tag{9}$$

and the set of its quantized levels is described by:

$$\begin{aligned} &\left\{\pm\phi_d^{(i)} \middle| \phi_d^{(i)} = (\rho_d)^i \phi_d^{(0)}, d = 0, \pm1, \pm2, ...\right\} \cup \{0\}, \\ &0 < \rho_i < 1, \ \ \phi_d^{(0)} > 0, \end{aligned} \tag{10}$$

where ρ_d and $\phi_d^{(0)}$ denote the quantizer density and initial quantization respectively. The definition of $q_d(x_d(t_k))$ is expressed as:

$$q_d(x_d(t_k)) = \begin{cases} \phi_d^{(i)}, \text{ if } \dfrac{\phi_d^{(i)}}{1+l_d} < x_d(t_k) \leqslant \dfrac{\phi_d^{(i)}}{1-l_d}, \\ 0, \quad \text{if } x_d(t_k) = 0, \\ -q_d(-x_d(t_k)), \text{ if } x_d(t_k) < 0, \end{cases} \tag{11}$$

where $l_d = \frac{1-\rho_d}{1+\rho_d} (d = 1, 2, ..., n)$ is the quantizer parameters. Then, we have the following inequality:

$$\begin{cases} (1-l_m)x_d(t_k) \leqslant \phi_d^{(i)} \leqslant (1+l_d)x_d(t_k), \\ \qquad\qquad\qquad\qquad \text{for } x_d(t_k) \geqslant 0, \\ (1+l_m)x_d(t_k) \leqslant \phi_d^{(i)} \leqslant (1-l_d)x_d(t_k), \\ \qquad\qquad\qquad\qquad \text{for } x_d(t_k) < 0. \end{cases} \tag{12}$$

Then, we can get the quantizer as follows:

$$q(x(t_k)) = x(t_k) + f(x(t_k)), \tag{13}$$

where

$$f(x(t_k)) = [f_1(x_1(t_k)), f_2(x_2(t_k)), ..., f_n(x_n(t_k))]^T, \tag{14}$$

with

$$-l_d[x_d(t_k)]^2 \leq x_d(t_k) f_d(x_d(t_k)) \leq l_d[x_d(t_k)]^2. \tag{15}$$

The defuzzified output of controller (7) is expressed as:

$$u(t_k) = \sum_{j=1}^{r} \eta_j(\theta(t_k)) K_j [x(t_k) + f(x(t_k))]. \tag{16}$$

In this paper, the sampling distance does not need to be a fixed value and between any two consecutive sampling instants belongs to an interval and $t_{k+1} - t_k = h_k \leq h$ for all $k \geq 0$, where $h > 0$.

Substituting (16) into (2), the T-S fuzzy system with sampled-data and state quantization is formulated as:

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\theta(t))\eta_j(\theta(t_k))[A_i x(t) \\ \qquad +A_{di}x(t-\tau) + B_i K_j(x(t_k) + f(x(t_k))) \\ \qquad +B_{wi}\omega(t)], \\ y(t) = \displaystyle\sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\theta(t))\eta_j(\theta(t_k))[C_{1i}x(t) \\ \qquad +C_{2i}x(t-\tau) + D_{1i}K_j(x(t_k) + f(x(t_k))) \\ \qquad +D_{2i}\omega(t)] \\ x(t) = \varphi(t), t \in [-\max(\tau, h)], i = 1, 2, ..., r, \end{cases} \tag{17}$$

Remark 1: The T-S fuzzy system with sampled-data has been investigated in [10, 11, 13, 14, 16], but the quantification is ignored in the study. In [17–22], they only investigated the system with quantization, and few people consider the combination of sampled-data and quantization. Although the problem of sampled-data control with quantization has been considered in [27], which cannot directly be employed to solving the H_∞ control problems of the system with time-delay, so, there are still some room to be improved. So, the problem of the sampled-data robust H_∞ control for T-S fuzzy time-delay systems with state quantization is considered in this paper.

The purpose of this paper is to design a robust H_∞ controller (16) for the T-S fuzzy system (1). In the following, we give some definition and lemmas which are useful in deriving the stability criteria.

Definition 1 [28]**:** For all admissible uncertainties, the system (17) is said to be robustly asymptotically stable with an norm H_∞ bound γ, if the following hold:

1) With $\omega(t) \equiv 0$, the trivial solution (equilibrium point) is asymptotically stable if $\lim\limits_{t\to\infty} |x(t)| = 0$ holds.
2) Under the assumption of zero initial condition $x(t) = 0$, $\forall t \in [-\max(\tau_d, h), 0]$, the controlled output $y(t)$ satisfies $\|y(t)\|_2 \leq \gamma \|\omega(t)\|_2$ for any nonzero $\omega(t) \in \ell_2[0, \infty)$, where γ is a attenuation index.

Lemma 1 (Jensen's inequality [30])**:** For any constant matrix $S = S^T > 0$ and vector $\eta > 0$, function $\omega : [0, \eta] \to \Re^n$ such that the integrations concerned are well defined, then

$$\begin{aligned} &-\eta \int_0^\eta \dot{\omega}^T(s) S \dot{\omega}(s) ds \\ &\leq -\left[\int_0^\eta \dot{\omega}(s) ds\right]^T S \left[\int_0^\eta \dot{\omega}(s) ds\right]. \end{aligned}$$

Lemma 2 (Wirtinger's inequality [31])**:** Consider a given matrix $R > 0$. Then, for all continuous function ω

in $[a,b] \to R^n$, the following inequality holds:

$$\int_a^b \dot{x}^T(s) R \dot{x}(s)\, ds \leqslant \frac{1}{b-a}[x(b)-x(a)]^T R [x(b)-x(a)] + \frac{3}{b-a}\Pi^T R \Pi,$$

where $\Pi = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)\, ds$.

Lemma 3 (Linear convex combination [28])**:** For any matrices $W_i (i = 1,2,3)$ with appropriate dimensions, and $0 \le t - t_k < h$, the condition of

$$W_1 + (t-t_k) W_2 + (h - (t - t_k)) W_2 < 0$$

holds if and only if

$$\begin{cases} W_1 + hW_2 < 0, \\ W_1 + hW_3 < 0. \end{cases}$$

3. MAIN RESULTS

This section aims to develop some stability criteria for the system described by (17), and the asymptotic stability and stabilization conditions of the T-S fuzzy with sampled-data and state quantization are derived in terms of linear matrix inequalities (LMIs).

It is assumed that the feedback gain matrices K_j have been well designed for ensuring the asymptotically stable of the system (17) with an H_∞ norm bound γ. The corresponding results are summarized in the following theorem.

For the sake of simplicity of matrix and vector representation, the notations are defined as

$$\xi^T(t) = \Big[x^T(t), x^T(t_k), x^T(t-\tau), x^T(t-h), f^T(x(t_k)), \dot{x}^T(t), \frac{1}{\tau}\int_{t-\tau}^t x^T(s)\, ds, \frac{1}{t-t_k}\int_{t_k}^t x^T(s)\, ds, \frac{1}{h}\int_{t-h}^t x^T(s)\, ds, \omega^T(t)\Big],$$

$$\Phi_1 = [I, -I, 0, 0, 0, 0, 0, 0, 0, 0],$$
$$\Phi_2 = [I, -I, 0, 0, 0, 0, 0, -2I, 0, 0].$$

3.1. Stability analysis with an H_∞ norm bound γ

This subsection provide some stability conditions to guarantee that the system (17) is robustly asymptotically stable with an H_∞ norm bound γ.

Theorem 1: For some given positive constants τ, λ and h, diagonal matrix L, and gain matrices K_j of the subsystem controllers (7), the system (17) is robustly asymptotically stable with an H_∞ norm bound γ, if there exist symmetric positive matrices $P = \begin{bmatrix} P_1 & P_2 & P_3 \\ * & P_4 & P_5 \\ * & * & P_6 \end{bmatrix}, M = \begin{bmatrix} M_1 & M_2 & M_3 \\ * & M_4 & M_5 \\ * & * & M_6 \end{bmatrix}, R_1, R_2, N_1, N_2, Q_1, Q_2$, positive diagonal matrix G, and any matrices $Y_{1ij} = [Y_{11ij}, Y_{12ij}, \ldots, Y_{110ij}]$, $Y_{2ij} = [Y_{21ij}, Y_{22ij}, \ldots, Y_{210ij}]$ with appropriate dimensions, such that the following inequalities hold for $i, j = 1, 2, \ldots, r$:

$$\begin{bmatrix} \Omega^1 + h\Omega^2 & \Gamma_1 & hY_{1ij}^T & 3hY_{2ij}^T \\ * & -I & 0 & 0 \\ * & * & -hM^{-1} & 0 \\ * & * & * & -3hM^{-1} \end{bmatrix} < 0, \quad (18)$$

$$\begin{bmatrix} \Omega^1 + h\Omega^3 & \Gamma_1 \\ * & -I \end{bmatrix} < 0, \quad (19)$$

where

$$\Omega^1 = \begin{bmatrix} \Omega_{11}^1 & \Omega_{12}^1 & \Omega_{13}^1 & \ldots & \Omega_{110}^1 \\ * & \Omega_{22}^1 & \Omega_{23}^1 & \ldots & \Omega_{210}^1 \\ * & * & \Omega_{33}^1 & \ldots & \Omega_{310}^1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \ldots & \Omega_{1010}^1 \end{bmatrix},$$

$$\Omega^2 = \begin{bmatrix} \Omega_{11}^2 & \Omega_{12}^2 & \Omega_{13}^2 & \ldots & \Omega_{110}^2 \\ * & \Omega_{22}^2 & \Omega_{23}^2 & \ldots & \Omega_{210}^2 \\ * & * & \Omega_{33}^2 & \ldots & \Omega_{310}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \ldots & \Omega_{1010}^2 \end{bmatrix},$$

$$\Omega^3 = \begin{bmatrix} \Omega_{11}^3 & \Omega_{12}^3 & \Omega_{13}^3 & \ldots & \Omega_{110}^3 \\ * & \Omega_{22}^3 & \Omega_{23}^3 & \ldots & \Omega_{210}^3 \\ * & * & \Omega_{33}^3 & \ldots & \Omega_{310}^3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \ldots & \Omega_{1010}^3 \end{bmatrix},$$

$\Omega_{11}^1 = P_2 + P_2^T + P_3 + P_3^T + R_1 + R_2 - 4Q_1 - 4Q_2 + Z_1 A_i + {A_i}^T {Z_1}^T - Y_{11ij}^T - Y_{11ij} - 3Y_{21ij}^T - 3Y_{21ij}$, $\Omega_{12}^1 = -M_2^T + Z_1 B_i K_j + Y_{11ij}^T - Y_{12ij} - 3Y_{21ij}^T - 3Y_{22ij}$, $\Omega_{13}^1 = -P_2 - 2Q_1 + Z_1 A_{di} - Y_{13ij} - 3Y_{23ij}$, $\Omega_{14}^1 = -2Q_1 - Y_{14ij} - 3Y_{24ij}$, $\Omega_{15}^1 = -M_3^T + Z_1 B_i K_j - Y_{15ij} - 3Y_{25ij}$, $\Omega_{16}^1 = P_1 - Z_1 + A_i^T Z_2^T - Y_{16ij} - 3Y_{26ij}$, $\Omega_{17}^1 = \tau P_4 + \tau P_5^T + 6Q_1 - Y_{17ij} - 3Y_{27ij}$, $\Omega_{18}^1 = 6Y_{21ij}^T - Y_{18ij} - 3Y_{28ij}$, $\Omega_{19}^1 = 6Q_2 - Y_{19ij} - 3Y_{29ij}$, $\Omega_{110}^1 = Z_1 B_{\omega i} - Y_{110ij} - 3Y_{210ij}$, $\Omega_{22}^1 = M_2 + M_2^T + 2L^T GL + Y_{12ij}^T + Y_{12ij} - 3Y_{22ij}^T - 3Y_{22ij}$, $\Omega_{23}^1 = Y_{13ij} - 3Y_{23ij}$, $\Omega_{24}^1 = Y_{14ij} - 3Y_{24ij}$, $\Omega_{25}^1 = M_3^T - 2LG + Y_{15ij} - 3Y_{25ij}$, $\Omega_{26}^1 = K_j^T B_i^T Z_2^T + Y_{16ij} - 3Y_{26ij}$, $\Omega_{27}^1 = Y_{17ij} - 3Y_{27ij}$, $\Omega_{28}^1 = 6Y_{22ij}^T + Y_{18ij} - 3Y_{28ij}$, $\Omega_{29}^1 = Y_{19ij} - 3Y_{29ij}$, $\Omega_{210}^1 = Y_{110ij} - 3Y_{210ij}$, $\Omega_{33}^1 = -R_1 - 4Q_1$, $\Omega_{36}^1 = A_{di}^T Z_2^T$, $\Omega_{37}^1 = -\tau P_4 + 6Q_1$, $\Omega_{38}^1 = 6Y_{23ij}^T$, $\Omega_{44}^1 = -R_2 - 4Q_2$,

$\Omega_{48}^1=6Y_{24ij}^T$, $\Omega_{55}^1=-2G$, $\Omega_{56}^1=K_j^TB_i^TZ_2^T$, $\Omega_{58}^1=6Y_{25ij}^T$, $\Omega_{66}^1=-Z_2^T-Z_2+\tau^2Q_1+h^2Q_2$, $\Omega_{67}^1=\tau P_2^T$, $\Omega_{68}^1=6Y_{26ij}^T$, $\Omega_{610}^1=Z_2B_{\omega i}$, $\Omega_{77}^1=-12Q_1$, $\Omega_{78}^1=6Y_{27ij}^T$, $\Omega_{88}^1=6Y_{28ij}^T+6Y_{28ij}$, $\Omega_{89}^1=6Y_{29ij}$, $\Omega_{810}^1=6Y_{210ij}$, $\Omega_{99}^1=-12Q_2$, $\Omega_{1010}^1=-\gamma^2I$, $\Omega_{18}^2=P_5+P_6$, $\Omega_{22}^2=-M_4-N_1$, $\Omega_{25}^2=-M_5$, $\Omega_{38}^2=-P_5$, $\Omega_{55}^2=-M_6-N_2$, $\Omega_{68}^2=P_3$, $\Omega_{22}^3=M_4+N_1$, $\Omega_{25}^3=M_5$, $\Omega_{26}^3=M_2^T$, $\Omega_{55}^3=M_6+N_2$, $\Omega_{56}^3=M_3^T$, $\Omega_{66}^3=M_1$, $\Gamma_1^T=\left(C_{1i}^T,K_j^TD_{1i}^T,C_{2i}^T,0,K_j^TD_{1i}^T,0,0,0,0,D_{2i}^T\right)$, $\Gamma_2=\left(A_i,B_iK_j,A_{di}X,0,B_iK_j,-X,0,0,0,B_{\omega i}X\right)$.

Proof: Construct a LKF candidate as

$$V(t)=\sum_{i=1}^{5}V_i(t),t\in[t_k,t_{k+1}), \tag{20}$$

where

$$\begin{aligned}
V_1(t)&=\begin{bmatrix} x(t)\\ \int_{t-\tau}^t x(s)ds\\ \int_{t_k}^t x(s)ds\end{bmatrix}^T P\begin{bmatrix} x(t)\\ \int_{t-\tau}^t x(s)ds\\ \int_{t_k}^t x(s)ds\end{bmatrix},\\
V_2(t)&=\int_{t-\tau}^t x^T(s)R_1x(s)ds+\int_{t-h}^t x^T(s)R_2x(s)ds,\\
V_3(t)&=(h-(t-t_k))\\
&\quad\times\int_{t_k}^t\begin{bmatrix}\dot x(s)\\ x(t_k)\\ f(x(t_k))\end{bmatrix}^T M\begin{bmatrix}\dot x(s)\\ x(t_k)\\ f(x(t_k))\end{bmatrix}ds,\\
V_4(t)&=(t_{k+1}-t)(t-t_k)\left[x^T(t_k)N_1x(t_k)\right.\\
&\quad\left.+f^T(x(t_k))N_2f(x(t_k))\right],\\
V_5(t)&=\tau\int_{t-\tau}^t\int_\theta^t \dot x^T(s)Q_1\dot x(s)dsd\theta\\
&\quad+h\int_{t-h}^t\int_\theta^t \dot x^T(s)Q_2\dot x(s)dsd\theta.
\end{aligned}$$

Taking the time derivative of $V(t)$ for the system (17), we have

$$\begin{aligned}
\dot V_1(t)=&2\begin{bmatrix} x(t)\\ \int_{t-\tau}^t x(s)ds\\ \int_{t_k}^t x(s)ds\end{bmatrix}^T P\begin{bmatrix}\dot x(t)\\ x(t)-x(t-\tau)\\ x(t)\end{bmatrix}\\
=&2x^T(t)P_1\dot x(t)+2x^T(t)P_2[x(t)-x(t-\tau)]\\
&+2x^T(t)P_3x(t)+2\int_{t-\tau}^t x^T(s)P_2^T\dot x(t)ds\\
&+2\int_{t-\tau}^t x^T(s)P_4[x(t)-x(t-\tau)]ds\\
&+2\int_{t-\tau}^t x^T(s)P_5x(t)ds+2\int_{t_k}^t x^T(s)P_3^T\dot x(t)ds\\
&+2\int_{t_k}^t x^T(s)P_4^T[x(t)-x(t-\tau)]ds\\
&+2\int_{t_k}^t x^T(s)P_6x(t)ds,
\end{aligned}\tag{21}$$

$$\begin{aligned}
\dot V_2(t)=&x^T(t)R_1x(t)-x^T(t-\tau)R_1x(t-\tau)\\
&+x^T(t)R_2x(t)-x^T(t-h)R_2x(t-h),
\end{aligned}\tag{22}$$

$$\begin{aligned}
\dot V_3(t)=&-\int_{t_k}^t\begin{bmatrix}\dot x(s)\\ x(t_k)\\ f(x(t_k))\end{bmatrix}^T M\begin{bmatrix}\dot x(s)\\ x(t_k)\\ f(x(t_k))\end{bmatrix}ds\\
&+(h-(t-t_k))\begin{bmatrix}\dot x(t)\\ x(t_k)\\ f(x(t_k))\end{bmatrix}^T M\begin{bmatrix}\dot x(t)\\ x(t_k)\\ f(x(t_k))\end{bmatrix}\\
=&-\int_{t_k}^t\dot x^T(s)M_1\dot x(s)ds-2[x(t)-x(t-\tau)]^T\\
&\times M_2x(t_k)-2[x(t)-x(t-\tau)]^TM_3f(x(t_k))\\
&+(t-t_k)\left[x^T(t_k)M_4x(t_k)+2x^T(t_k)M_5f(x(t_k))\right.\\
&\left.+f^T(x(t_k))M_6f(x(t_k))\right]+(h-(t-t_k))\left[\dot x^T(t)\right.\\
&\times M_1\dot x(t)+\dot x^T(t_k)M_2x(t_k)+\dot x^T(t_k)M_3f(x(t_k))\\
&+x^T(t_k)M_4x(t_k)+x^T(t_k)M_5f(x(t_k))\\
&\left.+f^T(x(t_k))M_6f(x(t_k))\right],
\end{aligned}\tag{23}$$

$$\begin{aligned}
\dot V_4(t)=&(t_{k+1}-t)[x^T(t_k)N_1x(t_k)\\
&+f^T(x(t_k))N_2f(x(t_k))]\\
&-(t-t_k)[x^T(t_k)N_1x(t_k)\\
&+f^T(x(t_k))N_2f(x(t_k))]\\
=&(h_k-(t-t_k))[x^T(t_k)N_1x(t_k)\\
&+f^T(x(t_k))N_2f(x(t_k))]\\
&-(t-t_k)[x^T(t_k)N_1x(t_k)\\
&+f^T(x(t_k))N_2f(x(t_k))],
\end{aligned}\tag{24}$$

$$\begin{aligned}
\dot V_5(t)=&\tau^2\dot x^T(t)Q_1\dot x(t)-\tau\int_{t-\tau}^t\dot x^T(s)Q_1\dot x(s)ds\\
&+h^2\dot x^T(t)Q_1\dot x(t)-h\int_{t-h}^t\dot x^T(s)Q_2\dot x(s)ds.
\end{aligned}\tag{25}$$

Using Lemma 1 to deal with the integral term in (23),

$$\begin{aligned}
&-\int_{t_k}^t\dot x^T(s)M_1x(t)ds\leqslant\\
&-\frac{1}{t-t_k}\xi^T(t)\left[\Phi_1^TM_1\Phi_1+3\Phi_2^TM_1\Phi_2\right]\xi(t).
\end{aligned}\tag{26}$$

With the matrices Y_{1ij} and Y_{2ij}, we can see that

$$\begin{aligned}
&\frac{1}{(t-t_k)}[M_1\Phi_1-(t-t_k)Y_{1ij}]^TM_1^{-1}\\
&\quad\times[M_1\Phi_1-(t-t_k)Y_{1ij}],\\
&\frac{1}{(t-t_k)}[M_1\Phi_2-(t-t_k)Y_{2ij}]^TM_1^{-1}\\
&\times[M_1\Phi_2-(t-t_k)Y_{2ij}]
\end{aligned}$$

are nonnegative. So, the following inequalities holds:

$$\begin{aligned}
&-\frac{1}{(t-t_k)}\Phi_1^TM_1\Phi_1\\
&\leqslant-Y_{1ij}^T\Phi_1-\Phi_1^TY_{1ij}+(t-t_k)Y_{1ij}^TM_1^{-1}Y_{1ij},
\end{aligned}\tag{27}$$

$$-\frac{1}{(t-t_k)}\Phi_2^TM_1\Phi_2$$

$$\leqslant -Y_{2ij}^T\Phi_2 - \Phi_2^T Y_{2ij} + (t-t_k) Y_{2ij}^T M_1^{-1} Y_{2ij}. \quad (28)$$

Using Lemma 2 to deal with the integral terms in (25),

$$\begin{aligned}&-\tau\int_{t-\tau}^{t} \dot{x}^T(s) Q_1 \dot{x}(s)\, ds\\ &\leqslant -[x(t)-x(t-\tau)]^T R[x(t)-x(t-\tau)], \quad (29)\\ &-h\int_{t-h}^{t} \dot{x}^T(s) Q_1 \dot{x}(s)\, ds\\ &\leqslant -[x(t)-x(t-h)]^T R[x(t)-x(t-h)]. \quad (30)\end{aligned}$$

Based on the system (17), it is easy to know that

$$\begin{aligned}&2\left[x^T(t) Z_1 + \dot{x}^T(t) Z_2\right] \times \{-\dot{x}(t)\\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\theta(t))\eta_j(\theta(t_k))[A_i x(t) + A_{di}x(t-\tau)\\ &\quad + B_i K_j(x(t_k) + f(x(t_k))) + B_{wi}\omega(t)]\} = 0, \quad (31)\end{aligned}$$

that is

$$2\left[x^T(t) Z_1 + \dot{x}^T(t) Z_2\right] \times \Gamma_2 \xi(t) = 0. \quad (32)$$

From (15), for a diagonal matrix $G > 0$, we can obtain

$$-2[f(x(t_k)) + Lx(t_k)]^T G[f(x(t_k)) - Lx(t_k)] \geq 0. \quad (33)$$

From (21)-(35), we have

$$\begin{aligned}&\dot{V}(t) + y^T(t) y(t) - \gamma^2\omega^T(t)\omega(t)\\ &\leqslant \sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\theta(t))\eta_j(\theta(t_k))\xi^T(t)\Omega\xi(t), \quad (34)\end{aligned}$$

where $\Omega = \left(\Omega^1 - \Gamma^T I\Gamma\right) + (t-t_k)\left(\Omega^2 + Y_{1ij}^T M_1^{-1} Y_{1ij} + 3Y_{2ij}^T M_1^{-1} Y_{2ij}\right) + (h-(t-t_k))\Omega^3$.

Using Lemma 3, we know $\Omega < 0$ if and only if

$$\begin{aligned}&\left(\Omega^1 - \Gamma^T I\Gamma\right) + h\left(\Omega^2 + Y_{1ij}^T M_1^{-1} Y_{1ij} + 3Y_{2ij}^T M_1^{-1} Y_{2ij}\right)\\ &< 0, \quad (35)\\ &\left(\Omega^1 - \Gamma^T I\Gamma\right) + h\Omega^3 < 0. \quad (36)\end{aligned}$$

Based on the Schur Complement [32], (37) is equivalent to (18) and (38) is equivalent to (19). Therefore, it follows from (36) that

$$\dot{V}(t) \leq -y^T(t) y(t) + \gamma^2\omega^T(t)\omega(t). \quad (37)$$

Under zero initial condition, integrating both sides of (39) from t_0 to $t \to \infty$, we have

$$\int_{t_0}^{\infty} y^T(s) y(s)\, ds \leq \int_{t_0}^{\infty} \gamma^2\omega^T(s)\omega(s)\, ds, \quad (38)$$

which implies that $\|y(t)\|_2 \leq \gamma\|\omega(t)\|_2$.

Next, when $\omega(t) \equiv 0$, we can get $\dot{V}(t) < 0$, obviously. According to Definition 1, the result is established. This completes the proof. □

Remark 2: In constructed LKF (20), the characteristic of sampling and state quantization is fully considered. Particularly, the item of $V_3(t)$ and $V_4(t)$ in (20) plays a key role to prove the stability of the system. The items $(h-(t-t_k))$ in $V_3(t)$ and the items $(t_{k+1}-t)(t-t_k)$ in $V_4(t)$, which can fully use the linear convex combination (Lemma 3) in the derivation to replace h with $(t-t_k)$ and $(h-(t-t_k))$. It is more easy to obtain the stability conditions of the system.

Remark 3: There are some constructed inequalities and some lemmas to deal with the integral items. By using the combination of the constructed inequality (27-30) and lemma 2, less delay-dependent integral items are arbitrarily ignored. So, the results obtained form this paper are less conservative than other paper.

3.2. Sampled-data and state quantized controller design

In this subsection, the sampled-data and state quantized controller is design for the T-S fuzzy system (17).

Theorem 2: For given some positive constants h, ε, and a diagonal matrix L, the system (17) is robustly asymptotically stable with an H_∞ norm bound γ and feedback gain matrices $K_j = J_j X^{-T}$ if there exist symmetric positive matrices $\tilde{P} = \begin{bmatrix} \tilde{P}_1 & \tilde{P}_2 & \tilde{P}_3\\ * & \tilde{P}_4 & \tilde{P}_5\\ * & * & \tilde{P}_6\end{bmatrix}$, $\tilde{M} = \begin{bmatrix} \tilde{M}_1 & \tilde{M}_2 & \tilde{M}_3\\ * & \tilde{M}_4 & \tilde{M}_5\\ * & * & \tilde{M}_6\end{bmatrix}$, $\tilde{R}_1, \tilde{R}_2, \tilde{N}_1, \tilde{N}_2, \tilde{Q}_1, \tilde{Q}_2$, positive diagonal matrix $\tilde{G}$, and any matrices $\tilde{Y}_{1ij}, \tilde{Y}_{2ij}, \tilde{Z}_1, \tilde{Z}_2$ with appropriate dimensions,such that the following LMIs hold for $i, j = 1, 2, \ldots, r$:

$$\begin{bmatrix} \tilde{\Omega}^1 + h\tilde{\Omega}^2 & \tilde{\Gamma}_1 & h\tilde{Y}_{1ij}^T & 3h\tilde{Y}_{2ij}^T\\ * & -I & 0 & 0\\ * & * & -h\tilde{M}^{-1} & 0\\ * & * & * & -3h\tilde{M}^{-1}\end{bmatrix} < 0, \quad (39)$$

$$\begin{bmatrix} \tilde{\Omega}^1 + h\tilde{\Omega}^3 & \tilde{\Gamma}_1\\ * & -I\end{bmatrix} < 0, \quad (40)$$

where

$$\Omega^1 = \begin{bmatrix} \tilde{\Omega}_{11}^1 & \tilde{\Omega}_{12}^1 & \tilde{\Omega}_{13}^1 & \ldots & \tilde{\Omega}_{110}^1\\ * & \tilde{\Omega}_{22}^1 & \tilde{\Omega}_{23}^1 & \ldots & \tilde{\Omega}_{210}^1\\ * & * & \tilde{\Omega}_{33}^1 & \ldots & \tilde{\Omega}_{310}^1\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & \ldots & \tilde{\Omega}_{1010}^1\end{bmatrix},$$

$$\Omega^2 = \begin{bmatrix} \tilde{\Omega}_{11}^2 & \tilde{\Omega}_{12}^2 & \tilde{\Omega}_{13}^2 & \ldots & \tilde{\Omega}_{110}^2\\ * & \tilde{\Omega}_{22}^2 & \tilde{\Omega}_{23}^2 & \ldots & \tilde{\Omega}_{210}^2\\ * & * & \tilde{\Omega}_{33}^2 & \ldots & \tilde{\Omega}_{310}^2\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & \ldots & \tilde{\Omega}_{1010}^2\end{bmatrix},$$

$$\Omega^3 = \begin{bmatrix} \tilde{\Omega}^3_{11} & \tilde{\Omega}^3_{12} & \tilde{\Omega}^3_{13} & \cdots & \tilde{\Omega}^3_{110} \\ * & \tilde{\Omega}^3_{22} & \tilde{\Omega}^3_{23} & \cdots & \tilde{\Omega}^3_{210} \\ * & * & \tilde{\Omega}^3_{33} & \cdots & \tilde{\Omega}^3_{310} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & \tilde{\Omega}^3_{1010} \end{bmatrix},$$

$\tilde{\Omega}^1_{11} = \tilde{P}_2 + \tilde{P}_2^T + \tilde{P}_3 + \tilde{P}_3^T + \tilde{R}_1 + \tilde{R}_2 - 4\tilde{Q}_1 - 4\tilde{Q}_2 + A_iX^T + XA_i{}^T - \tilde{Y}^T_{11ij} - \tilde{Y}_{11ij} - 3\tilde{Y}^T_{21ij} - 3\tilde{Y}_{21ij}$, $\tilde{\Omega}^1_{12} = -\tilde{M}_2^T + B_iJ_j + \tilde{Y}^T_{11ij} - \tilde{Y}_{12ij} - 3\tilde{Y}^T_{21ij} - 3\tilde{Y}_{22ij}$, $\Omega^1_{13} = -\tilde{P}_2 - 2\tilde{Q}_1 + A_{di}X^T - \tilde{Y}_{13ij} - 3\tilde{Y}_{23ij}$, $\tilde{\Omega}^1_{14} = -2\tilde{Q}_1 - \tilde{Y}_{14ij} - 3\tilde{Y}_{24ij}$, $\Omega^1_{15} = -\tilde{M}_3^T + B_iJ_j - \tilde{Y}_{15ij} - 3\tilde{Y}_{25ij}$, $\tilde{\Omega}^1_{16} = \tilde{P}_1 - X^T + \varepsilon XA_i^T - \tilde{Y}_{16ij} - 3\tilde{Y}_{26ij}$, $\tilde{\Omega}^1_{17} = \tau\tilde{P}_4 + \tau\tilde{P}_5^T + 6\tilde{Q}_1 - \tilde{Y}_{17ij} - 3\tilde{Y}_{27ij}$, $\tilde{\Omega}^1_{18} = 6\tilde{Y}^T_{21ij} - \tilde{Y}_{18ij} - 3\tilde{Y}_{28ij}$, $\tilde{\Omega}^1_{19} = 6\tilde{Q}_2 - \tilde{Y}_{19ij} - 3\tilde{Y}_{29ij}$, $\tilde{\Omega}^1_{110} = B_{\omega i} - \tilde{Y}_{110ij} - 3\tilde{Y}_{210ij}$, $\tilde{\Omega}^1_{22} = \tilde{M}_2 + \tilde{M}_2^T + 2L^T\tilde{G}L + \tilde{Y}^T_{12ij} + \tilde{Y}_{12ij} - 3\tilde{Y}^T_{22ij} - 3\tilde{Y}_{22ij}$, $\tilde{\Omega}^1_{23} = \tilde{Y}_{13ij} - 3\tilde{Y}_{23ij}$, $\tilde{\Omega}^1_{24} = \tilde{Y}_{14ij} - 3\tilde{Y}_{24ij}$, $\tilde{\Omega}^1_{25} = \tilde{M}_3^T - 2L\tilde{G} + \tilde{Y}_{15ij} - 3\tilde{Y}_{25ij}$, $\tilde{\Omega}^1_{26} = \varepsilon J_j^T B_i^T + \tilde{Y}_{16ij} - 3\tilde{Y}_{26ij}$, $\tilde{\Omega}^1_{27} = \tilde{Y}_{17ij} - 3\tilde{Y}_{27ij}$, $\tilde{\Omega}^1_{28} = 6\tilde{Y}^T_{22ij} + \tilde{Y}_{18ij} - 3\tilde{Y}_{28ij}$, $\tilde{\Omega}^1_{29} = \tilde{Y}_{19ij} - 3\tilde{Y}_{29ij}$, $\tilde{\Omega}^1_{210} = \tilde{Y}_{110ij} - 3\tilde{Y}_{210ij}$, $\tilde{\Omega}^1_{33} = -\tilde{R}_1 - 4\tilde{Q}_1$, $\tilde{\Omega}^1_{36} = \varepsilon XA^T_{di}$, $\tilde{\Omega}^1_{37} = -\tau\tilde{P}_4 + 6\tilde{Q}_1$, $\tilde{\Omega}^1_{38} = 6\tilde{Y}^T_{23ij}$, $\tilde{\Omega}^1_{44} = -\tilde{R}_2 - 4\tilde{Q}_2$, $\tilde{\Omega}^1_{48} = 6\tilde{Y}^T_{24ij}$, $\tilde{\Omega}^1_{55} = -2\tilde{G}$, $\tilde{\Omega}^1_{56} = \varepsilon J_j^T B_i^T$, $\tilde{\Omega}^1_{58} = 6\tilde{Y}^T_{25ij}$, $\tilde{\Omega}^1_{66} = -\tilde{Z}_2^T - \tilde{Z}_2 + \tau^2\tilde{Q}_1 + h^2\tilde{Q}_2$, $\tilde{\Omega}^1_{67} = \tau\tilde{P}_2^T$, $\tilde{\Omega}^1_{68} = 6\tilde{Y}^T_{26ij}$, $\tilde{\Omega}^1_{610} = \varepsilon B_{\omega i}$, $\tilde{\Omega}^1_{77} = -12\tilde{Q}_1$, $\tilde{\Omega}^1_{78} = 6\tilde{Y}^T_{27ij}$, $\tilde{\Omega}^1_{88} = 6\tilde{Y}^T_{28ij} + 6\tilde{Y}_{28ij}$, $\tilde{\Omega}^1_{89} = 6\tilde{Y}_{29ij}$, $\tilde{\Omega}^1_{89} = 6\tilde{Y}_{29ij}$, $\Omega^1_{99} = -12\tilde{Q}_2$, $\Omega^1_{1010} = -\gamma^2 I$, $\tilde{\Omega}^2_{18} = \tilde{P}_5 + \tilde{P}_6$, $\tilde{\Omega}^2_{22} = -\tilde{M}_4 - \tilde{N}_1$, $\tilde{\Omega}^2_{25} = -\tilde{M}_5$, $\tilde{\Omega}^2_{38} = -\tilde{P}_5$, $\tilde{\Omega}^2_{55} = -\tilde{M}_6 - \tilde{N}_2$, $\tilde{\Omega}^2_{55} = -\tilde{M}_6 - \tilde{N}_2$, $\tilde{\Omega}^3_{22} = \tilde{M}_4 + \tilde{N}_1$, $\tilde{\Omega}^3_{25} = \tilde{M}_5$, $\tilde{\Omega}^3_{26} = \tilde{M}_2^T$, $\tilde{\Omega}^3_{55} = \tilde{M}_6 + \tilde{N}_2$, $\tilde{\Omega}^3_{56} = \tilde{M}_3^T$, $\tilde{\Omega}^3_{66} = \tilde{M}_1$, $\tilde{\Gamma}_1^T = \left(XC^T_{1i}, J_j^T D^T_{1i}, XC^T_{2i}, 0, J_j^T D^T_{1i}, 0,0,0,0, XD^T_{2i}\right)$, $\tilde{\Gamma}_2 = \left(A_iX^T, B_iJ_j, A_{di}X^T, 0, B_iJ_j, -X^T, 0,0,0, B_{\omega i}X^T\right)$.

Proof: First, pre-multiply and post-multiply both sides of (18) with $diag\left\{\underbrace{X,\ldots,X}_{9}, I, I, X, X\right\}$ and its transpose, and (19) with $diag\left\{\underbrace{X,\ldots,X}_{9}, I\right\}$ and its transpose, respectively, and define $Z_1 = X^{-1}$, $Z_2 = \varepsilon X^{-1}$, $J_j = K_jX^T$, $\tilde{P} = diag\{X,X,X\}Pdiag\{X^T,X^T,X^T\}$, $\tilde{M} = diag\{X,X,X\}Mdiag\{X^T,X^T,X^T\}$, $\tilde{G} = XGX^T$, $\tilde{R}_1 = XR_1X^T$, $\tilde{R}_2 = XR_2X^T$, $\tilde{N}_1 = XN_1X^T$, $\tilde{N}_2 = XN_2X^T$, $\tilde{Q}_1 = XQ_1X^T$, $\tilde{Q}_2 = XQ_2X^T$, $\tilde{Y}_{1ij} = XY_{1ij}diag\left\{\underbrace{X,\ldots,X}_{9}, I\right\}$, $\tilde{Y}_{2ij} = XY_{2ij}diag\left\{\underbrace{X,\ldots,X}_{9}, I\right\}$, $\tilde{Y}_{3ij} = XY_{3ij}diag\left\{\underbrace{X,\ldots,X}_{9}, I\right\}$. Then, we can obtain (41) and (42). This completes the proof. □

Remark 4: Unlike the proposed method in [10, 11, 13, 14, 16], we attempt to analyze quantization effect of sampled-data fuzzy systems. The introduction of quantizer can effectively reduce the burden of communication channel in the signal transmission of network control systems, and also reduce the influence of network fixed bandwidth on the system. The results obtained from this paper have a larger sampling period h and a smaller attenuation interference index γ than other existing results, this will be demonstrated later through Example 1 and Example 2. So, the sampled-data and state quantized controller we obtained is more conducive to the stability of the system.

4. EXAMPLES

This section provides two examples to verify the validity and superiority of the proposed methods. To prepare for the appropriate examples, we also study T-S fuzzy systems with sampled-data or state quantized controller investigated in the recent literature [33–38].

Example 1: Consider the following fuzzy system. The T-S fuzzy model of this fuzzy system is the following form:

$$\begin{aligned} \dot{x}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} \eta_i(\theta(t))\eta_j(\theta(t))[A_ix(t) + A_{di}x(t-\tau) \\ &+ B_iK_j(x(t_k) + f(x(t_k))) + B_{wi}\omega(t)], \end{aligned}$$

where

$$A_1 = \begin{bmatrix} -2 & 2 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & -2 \end{bmatrix},$$

$$A_{d1} = A_{d2} = \begin{bmatrix} -0.5 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -0.5 \end{bmatrix},$$

$$B_{\omega 1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad B_{\omega 2} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -1 \\ -2 \\ 0.5 \end{bmatrix}, C_{11} = \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \quad C_{12} = \begin{bmatrix} -0.1 & 0.1 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.1 \end{bmatrix},$$

$$C_{21} = C_{22} = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.3 & 0 \\ 0 & 0 & -0.1 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0.1 \\ 0.5 \\ 1 \end{bmatrix},$$

$$D_{21} = D_{22} = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}.$$

Consider the quantizer densities

$$\rho_1 = \frac{1}{2}, \ \rho_2 = \frac{1}{3}, \ \rho_3 = \frac{1}{6},$$

Table 1. Upper bounds sampling period h and controller feedback gains for $\gamma = 2$ and different values of state delay τ.

τ	0.1	0.3	0.5
h	0.88	0.74	0.65

so we have the quantizer parameters are

$$l_1 = \frac{1-\rho_1}{1+\rho_1} = \frac{1}{3}, l_2 = \frac{1-\rho_2}{1+\rho_2} = \frac{1}{2}, \quad l_3 = \frac{1-\rho_3}{1+\rho_3} = \frac{5}{7},$$

then we have content matrices

$$L = \begin{bmatrix} l_1 & 0 & 0 \\ * & l_2 & 0 \\ * & * & l_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ * & \frac{1}{2} & 0 \\ * & * & \frac{5}{7} \end{bmatrix}.$$

Employ the following membership function:

$$\begin{cases} \eta_1 (x_i (t)) = \dfrac{1}{1+\exp(0.5 (x_i (t)+1))}, \\ \eta_2 (x_i (t)) = 1 - f_1 (x_i (t)), \end{cases}$$

and the disturbance input is assumed to be $\omega(t) = \sin(-3t)\exp(-t)$, and given $\varepsilon = 0.9$.

Then, by using Theorem 2 with LMIs, we obtain the results of some feasible solutions K_1, K_2 and the upper bounds of sampling period h under different levels of state constant delay τ, which can guarantee the asymptotic stability of the system (17) with H_∞ performance level $\gamma = 2$. The corresponding results is shown in Table 1.

From Table 1, we can seen that the upper bounds of sampling period is $h = 0.88$ when $\tau = 0.1$. The similar conclusions can be obtained with different τ.
With τ=0.1, h=0.88, $K_1 = [-0.0252\ -0.0831\ -0.0051]$, $K_2 = [-0.0365\ -0.0773\ -0.0014]$, the state responses $x(t)$ of the system (17) is shown in Fig. 2 and the control input $u(t)$ is shown in Fig. 3 under an initial condition $x(0) = [0,0,0]^T$.

Remark 5: By using the results of Theorem 2 and combining with the method of linear matrix inequalities, we obtained many sets of data. However, due to space constraints, this paper only shows three sets of better data. In the process of calculation, we find that the larger γ, the smaller h, and they are inversely proportional to each other. After repeated calculation and comparison, we find that $\gamma = 2$ can get better results. From Table 1 , it can be seen that the method of this paper can obtain a larger sampling period $h = 0.88$ when $\tau = 0.1$.

Remark 6: From Fig. 1 and Fig. 2, it can be seen that the state responses of the system state and controller input tend to be stable from 4s. This proved the feasibility of the method proposed in this paper.

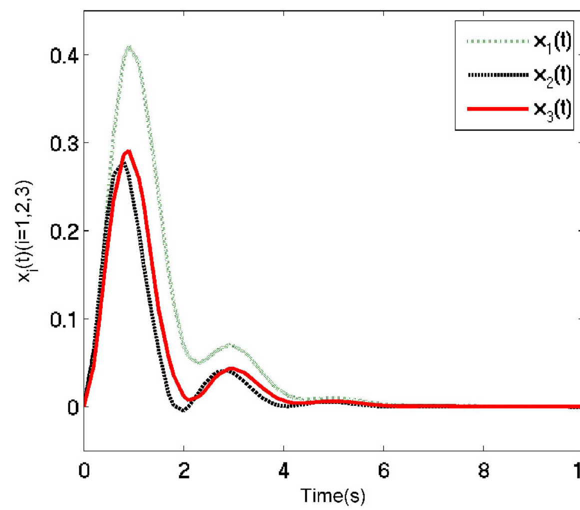

Fig. 2. The state responses of the system (17).

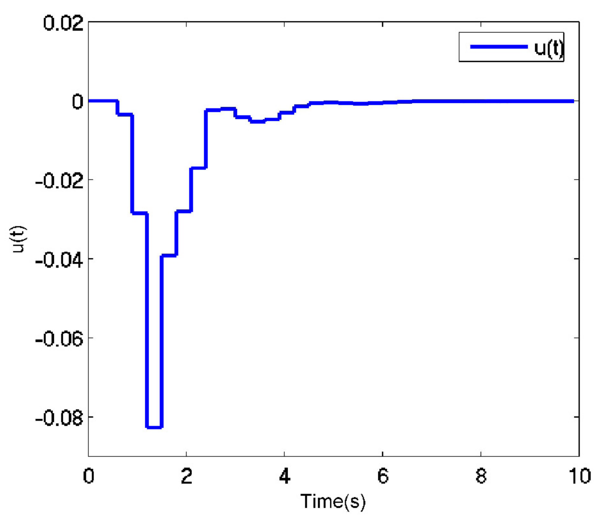

Fig. 3. The control input of the system (17).

Example 2: Consider the problem of balancing and swing-up of an inverted pendulum on a cart. The equations of the pendulum motion are given by [35]

$$\begin{cases} \dot{x}_1 (t) = x_2 (t), \\ \dot{x}_2 (t) \\ = \dfrac{g\sin(x_1 (t)) - amlx_2^2 (t) \frac{\sin(2x_1(t))}{2} - a\cos(x_1 (t)) u(t)}{\frac{4l}{3} - aml\cos^2 (x_1 (t))} \\ \quad + \omega(t), \\ z(t) = x_2 (t), \end{cases}$$

where $x_1 (t)$ is the angle (in radians) of the pendulum from the vertical, $x_2 (t)$ is the angular velocity, and $u(t)$ is the force applied to the cart (in newtons), and $\omega(t)$ is the disturbance. g is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length

Table 2. The upper bounds of sampling period h obtained by various methods.

Methods	[36]	[37]	[38]	[35]	Theorem 2
h	14ms	16ms	24ms	42ms	112ms

of the pendulum. As the same as in [35], the parameters are given by $m = 2.0kg, M = 8.0kg, 2l = 1.0m, g = 9.8m/s^2, a = 1/(m+M)$. The control objective here is to balance the inverted pendulum for the approximate range $x_1(t) \in (-\pi/2, \pi/2)$ and $x_2(t) \in (-\pi, \pi)$ via a sampled-data control approach.

The system (41) in this example can be represented by a two-rule T-S fuzzy model:
Rule 1: IF $x_1(t)$ is about 0, THEN

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_1 u(t) + B_{w1}\omega(t), \\ z(t) = C_{11} x(t). \end{cases}$$

Rule 2: IF $x_1(t)$ is about $\pm\frac{\pi}{2}$, THEN

$$\begin{cases} \dot{x}(t) = A_2 x(t) + B_2 u(t) + B_{w2}\omega(t), \\ z(t) = C_{12} x(t), \end{cases}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{4l/3 - aml} & 0 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ -\frac{a}{4l/3 - aml} \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix},$$
$$E_1 = E_2 = [0,1]^T, C_1 = C_2 = [0,1], \beta = \cos(88^\circ).$$

The membership function are defined as

$$\eta_1(x_1(t)) = \begin{cases} 1 - \dfrac{2}{\pi} x_1(t), if 0 \leqslant x_1(t) < \dfrac{\pi}{2}, \\ 1 + \dfrac{2}{\pi} x_1(t), if -\dfrac{\pi}{2} < x_1(t) < 0, \end{cases}$$

and $\eta_2(x_1(t)) = 1 - \eta_1(x_1(t))$.

By using Matlab LMI tool box to solve the LMIs obtained in Theorem 2, we can obtain the upper bounds of sampling period $h = 112ms$. In order to verify the superiority of our methods, the comparison results with other papers are presented in Table 2.

When $h = 30ms$, the minimal H_∞ performance index γ is obtained as 0.0133 by Theorem 2, and the corresponding controller gains are as

$$K_1 = [6.5666, -0.3832], K_2 = [10.2418, -0.2755].$$

By using the above results, the state responses $x(t)$ of the fuzzy system are shown in Fig. 4 and corresponding control input $u(t)$ is shown in Fig. 5 under the initial state $x(0) = [\pi/3, 0]^T$.

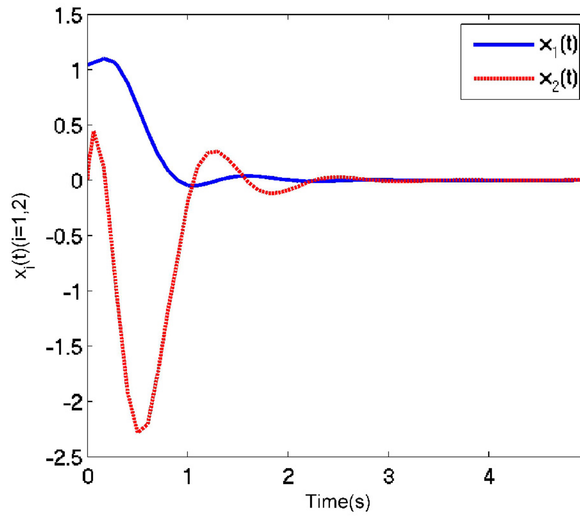

Fig. 4. The state responses of the system (41).

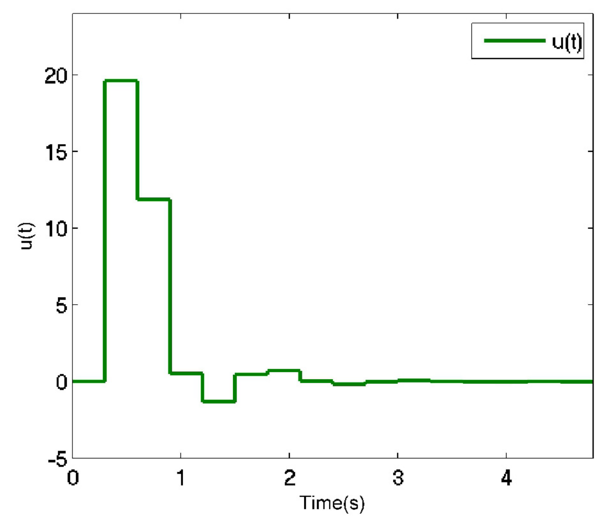

Fig. 5. The control input of the system (41).

Table 3. The comparison results of γ.

Methods	[35]	Theorem 2
γ	0.6062	0.0133

Remark 7: It can be seen from Table 2 that the upper bounds of sampling period h obtained by Theorem 2 is larger than others. Moreover, the comparison results of the minimum attenuation interference index γ is shown in Table 3. From Table 3, we know that the minimum attenuation interference index γ obtained by Theorem 2 is smaller than others. All these results prove the superiority of our method.

5. CONCLUSIONS

In this paper, the problem of sampled-data robust H_∞ control for the T-S fuzzy time-delay system with state quantization has been investigated. We constructed a improved LKF, which is fully considered the characteristics of sample-data and state quantization, and by introducing the free weighting matrices, some integral techniques and modified inequalities, the results obtained in this paper are less conservative than other existing results. Based on the obtained stability conditions, the sample-data and state quantized controller has been designed for the robustly asymptotically stable of the T-S fuzzy time-delay system. Finally, a larger sampling period h and a smaller attenuation interference index γ obtained in the two examples have been proved the effectiveness and superiority of the proposed methods.

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