International Journal of Control, Automation and Systems 16(6) (2018) 2772-2780
http://dx.doi.org/10.1007/s12555-018-0266-8
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Stability Analysis for Time-delay Systems with Nonlinear Disturbances via New Generalized Integral Inequalities

Bin Wu, Chang-Long Wang*, Yong-Jiang Hu, and Xiao-Lin Ma

Abstract: This paper represents a novel less conservative stability criterion for time-delay systems with nonlinear disturbances. The main purpose is to obtain larger upper bound of the time-varying delay. A suitable Lyapunov-Krasovskii functional (LKF) with triple integral terms is constructed. Then, two new generalized double integral (GDI) inequalities are proposed which encompass Wirtinger-based double inequality as a special case. A simple case of the proposed GDI inequality is utilized to estimate double integral terms in the time derivative of the constructed LKF. Further, an improved delay-dependent stability criterion is derived in the form of linear matrix inequalities (LMIs). Finally, some numerical examples are given to illustrate the improvement of the proposed criteria.

Keywords: Generalized integral inequalities, Lyapunov-Krasovskii functional (LKF) method, nonlinear disturbances, stability analysis, time-delay systems.

1. INTRODUCTION

In the past few decades, time-delay systems have attracted much attention because of their wide applications in various practical systems such as attitude stabilization [1], neural networks [2, 3], infectious diseases and epidemics system [4] and network control systems [5]. However, time delay may lead to undesirable dynamic behaviour, poor performance and even instability of a real system. So, stability analysis of time-delay systems has become a hot issue [6–10]. Generally, delay-dependent stability criteria are less conservative than delay-independent ones. As is known to all, Lyapunov-Krasovskii functional (LKF) method is an effective way to obtain delay-dependent criteria. Nevertheless, stability criteria obtained based on LKF method are only sufficient [11–13]. Therefore, the main objective is to get the delay range as large as possible.

Various useful techniques have been put forward and many criteria with less conservatism have been derived, such as augmented LKF method [14], reciprocally convex combination approach [15, 16], delay-partitioning approach [17, 18], free-weighting-matrix (FWM) method [19] and zero equality approach [20]. Recently, a new insight to reduce conservatism is proposed in [21] by removing the restriants that all the matrices in the constructed LKF must be positive, instead, relaxed LMIs are added. In [22, 23], delay-product-type Lyapunov functional is established to obtain less conservative criteria.

Among existing results, integral inequalities play an important role to reduce the conservatism. Jensen inequality has usually been adopted in many literatures [24–26]. But, it has some undesirable conservatism. Hence, many efforts have been paid to derive improved inequalities. The most well-known inequality is Wirtinger-based integral (WBI) inequality. Then, some meaningful stability criteria are proposed based on single-integral or double-integral WBI inequality [27–29]. In [30], a new double integral inequality is proposed and applied to derive stability criterion of systems with state and distributed delays. Recently, a FWM inequality is proposed in [31]. The less conservatism of FWM inequality owes to some free matrices. However, it is pointed out in [32] that not all free matrices in the FWM inequality make contributions to reduce conservatism. Then, a generalized free-weighting-matrix (GFWM) inequality is proposed which encompasses FWM inequality as a special case in [32]. Very recently, second-order Bessel-Legendre inequality is used for stability analysis of time-varying delay systems in [33]. [34] proposes a relaxed integral inequality to deal with the sum of two single integral terms which are dependent on time-varying delay. However, the less conservatism inequalities in [31–34] can only handle single integral terms. In [35], a relaxed double integral inequality approach is proposed, but it still has unexpected conservatism.

Motivated by the above discussion, generalized double integral (GDI) inequalities are put forward with the help

Manuscript received April 24, 2018; revised June 27, 2018; accepted July 23, 2018. Recommended by Editor Jessie (Ju H.) Park. This work was supported by the National Natural Science Foundation of China (61501493).

Bin Wu, Chang-Long Wang, Yong-Jiang Hu and Xiao-Lin Ma are with the School of UAV Engineering, Army Engineering University, Shijiazhuang, 050003, China (e-mails: wubin435@163.com, wang-oec@126.com, huyongjiang_jxxy@163.com, mxl_linda@163.com).
* Corresponding author.

of Schur complement lemma. The proposed GDI inequalities are very flexible due to the free vectors and free matrices in them. It is theoretically proved that Wirtinger-based double integral inequality is a special case of the proposed GDI inequality. Then, an appropriate LKF with some triple terms is constructed. Two special cases of the proposed GDI inequalities are employed to estimate double integral terms in the derivative of the established LKF. Further, a less conservative stability criterion is derived in the form of LMIs. The effectiveness of the proposed criterion is shown by numerical examples in comparison with some recent results.

The remaining parts of the paper are organized as follows: Section 2 illustrates the problem description and some useful lemmas. Section 3 gives the proposed generalized double integral inequalities. Then, based on GDI inequality, an improved stability criterion for linear time-delay systems with nonlinear disturbances are derived. In Section 4, numerical examples are used to show the improvement of the proposed stability criterion. Conclusions are demonstrated in Section 5.

Notations: Throughout this paper, R^n denotes the n-dimensional Euclidean space. means that P is a symmetric and positive definite matrix. The symmetric terms in a symmetric matrix is donated by $*$. I donates a properly dimensioned identity matrix. In addition, diag$\{...\}$ represents block diagonal matrix, and $sym\{X\} = X + X^T$.

2. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider the following linear time-delay system with nonlinear disturbances:

$$\dot{x}(t) = Ax(t) + A_d x_d(t) + Bf(x(t)) + B_d f(x_d(t)), \quad (1)$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), ..., x_n(\cdot)]^T \in R^n$ is state vector and $x_d(t) = x(t-d(t))$. A, A_d, B, B_d are known system matrices with appropriate dimensions. $d(t)$ is time-varying state delay and $f(x(t))$ and $f(x_d(t))$ are nonlinear disturbances satisfying

$$0 \le d(t) \le d_U, \dot{d}(t) \le d_D, \quad (2)$$

$$\begin{cases} f(x(\cdot)) = [f_1(x_1(\cdot)), f_2(x_2(\cdot)), \dots, f_n(x_n(\cdot))]^T \in R^n, \\ 0 \le \dfrac{f_i(s_2) - f_i(s_1)}{s_2 - s_1} \le h_i, \ s_1 \ne s_2, i = 1, 2, \dots, n, \\ f_i(0) = 0, i = 1, 2, \dots, n, \end{cases} \quad (3)$$

where d_U, d_D, h_i are known constants, and $h_i > 0$.

Remark 1: The nonlinear disturbance $f(x(t))$ contains uncertain parts of system matrices A, A_d and unmodel dynamics of some practical systems.

Some helpful lemmas will be introduced, which are important to obtain the stability criteria.

Lemma 1: For given matrix $R > 0$ and differentiable function $x(t)$, the following inequality holds

$$\begin{aligned} &\int_a^b \dot{x}^T(s) R \dot{x}(s) ds \\ &\ge \frac{1}{b-a} \left(\chi_1^T R \chi_1 + 3\chi_2^T R \chi_2 + 5\chi_3^T R \chi_3 \right), \end{aligned} \quad (4)$$

where

$$\begin{aligned} &\chi_1 = x(b) - x(a), \ \ \chi_2 = x(b) + x(a) - \tfrac{2}{b-a} \int_a^b x(s) ds, \\ &\chi_3 = x(b) - x(a) + \frac{6}{b-a} \int_a^b x(s) ds \\ &\quad - \frac{12}{(b-a)^2} \int_a^b \int_u^b x(s) ds du. \end{aligned}$$

Lemma 2: For given matrix $R > 0$ and any continuously differentiable function $x(t)$, the following inequalities hold

$$\int_a^b \int_u^b \dot{x}^T(s) R_1 \dot{x}(s) ds du \ge 2\chi_4^T R_1 \chi_4 + 4\chi_5^T R_1 \chi_5, \quad (5)$$

$$\int_a^b \int_a^u \dot{x}^T(s) R_2 \dot{x}(s) ds du \ge 2\chi_6^T R_2 \chi_6 + 4\chi_7^T R_2 \chi_7, \quad (6)$$

where

$$\begin{aligned} &\chi_4 = x(b) - \frac{1}{b-a} \int_a^b x(s) ds, \\ &\chi_5 = x(b) + \frac{2}{b-a} \int_a^b x(s) ds \\ &\quad - \frac{6}{(b-a)^2} \int_a^b \int_u^b x(s) ds du, \\ &\chi_6 = x(a) - \frac{1}{b-a} \int_a^b x(s) ds, \\ &\chi_7 = x(a) - \frac{4}{b-a} \int_a^b x(s) ds \\ &\quad + \frac{6}{(b-a)^2} \int_a^b \int_u^b x(s) ds du. \end{aligned}$$

Lemma 3: For any vectors ϑ_1, ϑ_2, matrices S, $W_i > 0 (i = 1, 2, 3, 4)$ with proper dimensions satisfying

$$\begin{bmatrix} W_1 + W_3 & S \\ * & W_2 + W_4 \end{bmatrix} > 0, \quad (7)$$

and real scalar $0 < \rho < 1$, the following inequality holds

$$\begin{aligned} &\vartheta_1^T \left(\frac{1}{\rho} W_1 + \frac{1-\rho}{\rho} W_3 \right) \vartheta_1 \\ &\quad + \vartheta_2^T \left(\frac{1}{1-\rho} W_2 + \frac{\rho}{1-\rho} W_4 \right) \vartheta_2 \\ &\ge \begin{bmatrix} \vartheta_1 \\ \vartheta_2 \end{bmatrix}^T \begin{bmatrix} W_1 & S \\ * & W_2 \end{bmatrix} \begin{bmatrix} \vartheta_1 \\ \vartheta_2 \end{bmatrix}. \end{aligned} \quad (8)$$

Lemma 4: Suppose $\alpha_1 \leq \alpha(t) \leq \alpha_2$ satisfying $\alpha(\cdot): R^+ \to R^+$. For any known matrices Ξ, Ξ_1, Ξ_2 with appropriate dimensions, the following inequality holds

$$\Xi + (\alpha(t) - \alpha_1)\Xi_1 + (\alpha_2 - \alpha(t))\Xi_2 < 0, \tag{9}$$

if and only if

$$\begin{cases} \Xi + (\alpha_2 - \alpha_1)\Xi_1 < 0, \\ \Xi + (\alpha_2 - \alpha_1)\Xi_2 < 0. \end{cases} \tag{10}$$

3. MAIN RESULTS

In this section, two generalized double integral(GDI) inequalities are proposed. Then, a less conservative stability criterion is derived based on the proposed GDI inequalities.

3.1. The generalized double integral inequalities

Lemma 5: For positive definite symmetric matrix R_1, R_2 and any vector $x: [a,b] \to R^n$, two sets of non-zero scalar functions $p_{1i}(s), p_{2i}(s) (i = 1, 2, ..., n)$ satisfying

$$\int_a^b \int_u^b p_{1i}(s)p_{1k}(s)dsdu = 0, (i \neq k), \tag{11}$$

$$\int_a^b \int_a^u p_{2i}(s)p_{2k}(s)dsdu = 0, (i \neq k), \tag{12}$$

then the following inequalities hold

$$\begin{aligned} &-\int_a^b \int_u^b x^T(s)R_1x(s)dsdu \\ &\leq \chi_0^T \sum_{i=1}^n (q_{1i}S_iR_1^{-1}S_i^T)\chi_0 + 2\chi_0^T \sum_{i=1}^n S_i\Omega_{1i}, \end{aligned} \tag{13}$$

$$\begin{aligned} &-\int_a^b \int_a^u x^T(s)R_2x(s)dsdu \\ &\leq \chi_0^T \sum_{i=1}^n (q_{2i}S_iR_2^{-1}S_i^T)\chi_0 + 2\chi_0^T \sum_{i=1}^n S_i\Omega_{2i}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} q_{1i} &= \int_a^b \int_u^b p_{1i}^2(s)dsdu, \Omega_{1i} \\ &= \int_a^b \int_u^b p_{1i}(s)x(s)dsdu, \end{aligned} \tag{15}$$

$$\begin{aligned} q_{2i} &= \int_a^b \int_a^u p_{2i}^2(s)dsdu, \Omega_{2i} \\ &= \int_a^b \int_a^u p_{2i}(s)x(s)dsdu, \end{aligned} \tag{16}$$

and χ_0, S_i $(i = 1\ 2, \cdots, n)$ are any vector and matrices with proper dimensions.

Proof: According to Schur complement lemma, the following inequality is true with $R_1 > 0$ and any matrices S_i $(i = 1, 2, \ldots, n)$:

$$\Omega = \begin{bmatrix} S_1R_1^{-1}S_1^T & S_1R_1^{-1}S_2^T & \cdots & S_1R_1^{-1}S_n^T & S_1 \\ * & S_2R_1^{-1}S_2^T & \cdots & S_2R_1^{-1}S_n^T & S_2 \\ * & * & \ddots & \vdots & \vdots \\ * & * & * & S_nR_1^{-1}S_n^T & S_n \\ * & * & * & * & R_1 \end{bmatrix} \geq 0, \tag{17}$$

So, for any vector χ_0 and a set of scalar functions $p_{1i}(i = 1, 2, \ldots, n)$, it is obvious that

$$\int_a^b \int_u^b \begin{bmatrix} p_{11}(s)\chi_0 \\ \vdots \\ p_{1n}(s)\chi_0 \\ x(s) \end{bmatrix}^T \Omega \begin{bmatrix} p_{11}(s)\chi_0 \\ \vdots \\ p_{1n}(s)\chi_0 \\ x(s) \end{bmatrix} dsdu \geq 0, \tag{18}$$

The above inequality (18) is equivalent to the following inequality (19)

$$\begin{aligned} 0 \leq &\int_a^b \int_u^b x^T(s)R_1x(s)dsdu \\ &+ 2\chi_0^T \sum_{i=1}^n \left(S_i \int_a^b \int_u^b p_{1i}(s)x(s)dsdu\right) \\ &+ \chi_0^T \sum_{i=1}^n \left(S_iR_1^{-1}S_i^T \int_a^b \int_u^b p_{1i}^2(s)dsdu\right)\chi_0 \\ &+ 2\chi_0^T \sum_{i=1}^n \sum_{k=i+1}^n \left(S_iR_1^{-1}S_k^T \int_a^b \int_u^b p_{1i}(s)p_{1k}(s)dsdu\right)\chi_0, \end{aligned} \tag{19}$$

From (11) and (15), inequality (19) is equivalent to (13). Following a similar proof process, inequality (14) can be obtained. This is the end of proof. □

Remark 2: To the extent of the authors' knowledge, the GDI inequalities in (13) and (14) have not proposed in any existing literatures. Benefit from Schmidt orthogonalization approach [38], a set of orthogonal polynomials can be obtained conveniently. So, the inequalities in (13) and (14) can be implemented conveniently. Further, GDI inequalities are quite flexible and more general than some existing double integral inequalities because of free vector χ_0 and matrices $S_i (i = 1, 2, \ldots, n)$. Specifically,

1) By taking $n = 3$ and $p_{11} = 1$, $p_{12} = s - \frac{2b+a}{3}$, $p_{13} = s^2 - \frac{2(3b+2a)s}{5} + \frac{3b^2+a^2+6ab}{5}$ and $\chi_0^T S_i = -\frac{R_1\Omega_{1i}}{q_{1i}}$, the GDI inequality in (13) reduces to the one in Lemma 2.3 of [31].
2) By taking $n = 2$ and $p_{21} = 1$, $p_{22} = s - \frac{b+2a}{3}$, and $\chi_0^T S_i = -\frac{R_2\Omega_{2i}}{q_{2i}}$, the GDI inequality in (14) reduces to inequality (6) in Lemma 2.

Therefore, the GDI inequalities are more general than Wirtinger-based double integral inequality, and auxiliary function-based double integral inequalities [31].

From Lemma 5, the following lemma can be obtained easily which will be employed to derive stability criteria in the following parts.

Lemma 6: For a given symmetric matrix $R > 0$, any vector χ_0 and matrices S_1, S_2, S_3, S_4 with proper dimensions, the following inequalities hold for any continuously differentiable function $x : [a,b] \to R^n$

$$\begin{aligned}
&-\int_a^b \int_u^b \dot{x}^T(s) R_1 \dot{x}(s) ds du \\
&\le 2\chi_0^T (2S_1\gamma_1 + 4S_2\gamma_2) \\
&\quad + \chi_0^T \left(2S_1R_1^{-1}S_1^T + 4S_2R_1^{-1}S_2^T\right)\chi_0, \qquad (20) \\
&-\int_a^b \int_a^u \dot{x}^T(s) R_2 \dot{x}(s) ds du \\
&\le 2\chi_0^T (2S_3\gamma_3 + 4S_4\gamma_4) \\
&\quad + \chi_0^T \left(2S_3R_2^{-1}S_3^T + 4S_4R_2^{-1}S_4^T\right)\chi_0, \qquad (21)
\end{aligned}$$

where

$$\gamma_1 = \chi_4,\ \ \gamma_2 = \chi_5,\ \ \gamma_3 = -\chi_6,\ \ \gamma_4 = \chi_7,$$

and χ_4, χ_5, χ_6, χ_7 are defined in Lemma 2.

Proof: The inequality (20) can be easily from lemma 5 by choosing $n = 2$ and $p_{11} = \frac{2}{b-a}$, $p_{12} = \frac{12s-4(2b+a)}{(b-a)^2}$ in (13). Also, selecting $p_{21} = \frac{2}{b-a}$ and $p_{22} = \frac{12s-4(b+2a)}{(b-a)^2}$ in (14), (21) can be obtained. The detailed proof is omitted. □

3.2. Stability analysis

In this section, an less conservative stability criterion is derived based on the generalized double integral inequalities in lemma 6. Some notations are given to simplify the representation of following parts:

$$\begin{aligned}
&x_d(t) = x(t-d(t)),\ \ x_u(t) = x(t-d_U), \\
&d_2(t) = d_U - d(t),\ \ \varphi^T(t) = \left[x^T(t), \int_{t-d_U}^t x^T(s)ds\right], \\
&\varphi_1^T(t,s) = \left[x^T(t), x^T(s)\right],\ \ \varphi_2^T(s) = \left[x^T(s), f^T(x(s))\right], \\
&\xi_1(t) = \frac{1}{d(t)}\int_{t-d(t)}^t x(s)ds, \\
&\xi_2(t) = \frac{1}{d_2(t)}\int_{t-d_U}^{t-d(t)} x(s)ds, \\
&\xi_3(t) = \frac{1}{(d(t))^2}\int_{t-d(t)}^t \int_u^t x(s)dsdu, \\
&\xi_4(t) = \frac{1}{(d_2(t))^2}\int_{t-d_U}^{t-d(t)} \int_u^{t-d(t)} x(s)dsdu, \\
&\xi^T(t) = \left[x^T(t), x_d^T(t), x_u^T(t), \dot{x}^T(t), \xi_1^T(t), \xi_2^T(t), \xi_3^T(t),\right. \\
&\qquad\quad \left.\xi_4^T(t), f^T(x(t)), f^T(x_d(t)), f^T(x_u(t))\right].
\end{aligned}$$

Theorem 1: For given scalars $d_U > 0$, d_D, and positive diagonal matrix $H_U = diag\{h_1, h_2, \ldots, h_n\}$, the system (1) is asymptotically stable, if there exist symmetric matrices P, $R_2 > 0$, $Q_i > 0\ (i = 1, 2, 3, 4)$, $Z_i > 0\ (i = 1, 2)$, positive diagonal matrices $\Lambda_i = diag\{\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{in}\}, (i = 1, 2)$, $H_i = diag\{h_{i1}, h_{i2}, \ldots, h_{in}\}$, $(i = 1, 2, 3, 4, 5)$ and any matrices S_i, T_i $(i = 1, 2, 3)$, Y_i $(i = 1, \cdots, 8)$ with appropriate dimensions, such that the following LMIs in (22)-(23) hold:

$$\begin{bmatrix} (2i-1)(R_2+Z_1) & S_i \\ * & (2i-1)(R_2+Z_2) \end{bmatrix} < 0, \quad (i = 1,2,3), \qquad (22)$$

$$\begin{bmatrix} \Phi + d_U\Pi_i & Y \\ * & -Z \end{bmatrix} < 0,\ \ (i = 1,2), \qquad (23)$$

where

$$\begin{aligned}
&\Phi = \textstyle\sum_{i=1}^5 \Phi_i + \Xi_1 + \Xi_2 + \Xi_3 - \Theta_1 - \Theta_2 - \Theta_S, \\
&P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix},\ \ Q_1 = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{13} \end{bmatrix}, \\
&\Phi_1 = sym\left\{e_1^T P_{11} e_4 + e_1^T P_{12} e_1 - e_1^T P_{12} e_3\right\}, \\
&\Phi_2 = \begin{bmatrix} e_1 \\ e_1 \end{bmatrix}^T (Q_1+Q_2) \begin{bmatrix} e_1 \\ e_1 \end{bmatrix} \\
&\qquad - \begin{bmatrix} e_1 \\ e_3 \end{bmatrix}^T Q_1 \begin{bmatrix} e_1 \\ e_3 \end{bmatrix} + sym\left\{d_U e_1^T Q_{11} e_4\right\} \\
&\qquad - (1-d_D)\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^T Q_2 \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \\
&\Phi_3 = \begin{bmatrix} e_1 \\ e_9 \end{bmatrix}^T (Q_3+Q_4) \begin{bmatrix} e_1 \\ e_9 \end{bmatrix} \\
&\qquad - \begin{bmatrix} e_3 \\ e_{11} \end{bmatrix}^T Q_3 \begin{bmatrix} e_3 \\ e_{11} \end{bmatrix} \\
&\qquad - (1-d_D)\begin{bmatrix} e_2 \\ e_{10} \end{bmatrix}^T Q_4 \begin{bmatrix} e_2 \\ e_{10} \end{bmatrix}, \\
&\Phi_4 = sym\left\{e_4^T \Lambda_1 e_9 + e_1^T H_U \Lambda_2 e_4 - e_4^T \Lambda_2 e_9\right\}, \\
&\Phi_5 = e_4^T \left(d_U^2 R_2 + \tfrac{1}{2} d_U^2 (Z_1+Z_2)\right) e_4 \\
&\qquad + sym\left\{\sum_{i=1}^8 \left(3+(-1)^i\right) Y_i \upsilon_i\right\}, \\
&\Theta_1 = \gamma_1^T R_2 \gamma_1 + 3\gamma_2^T R_2 \gamma_2 + 5\gamma_3^T R_2 \gamma_3, \\
&\Theta_2 = \gamma_4^T R_2 \gamma_4 + 3\gamma_5^T R_2 \gamma_5 + 5\gamma_6^T R_2 \gamma_6, \\
&\Theta_S = sym\left\{\gamma_1^T S_1 \gamma_4 + \gamma_2^T S_2 \gamma_5 + \gamma_3^T S_3 \gamma_6\right\}, \\
&\Xi_1 = sym\left\{e_9^T H_1 (H_U e_1 - e_9) + e_{10}^T H_2 (H_U e_2 - e_{10})\right. \\
&\qquad \left. + e_{11}^T H_3 (H_U e_3 - e_{11})\right\}, \\
&\Xi_2 = sym\left\{(e_9 - e_{10})^T H_4 (H_U e_1 - H_U e_2 - e_9 + e_{10})\right. \\
&\qquad \left. + (e_{10} - e_{11})^T H_5 (H_U e_2 - H_U e_3 - e_{10} + e_{11})\right\}, \\
&\Xi_3 = sym\left\{(e_1^T T_1 + e_2^T T_2 + e_4^T T_3)(A e_1\right. \\
&\qquad \left. + A_d e_2 + B e_9 + B_d e_{10} - e_4)\right\}, \\
&\Pi_1 = sym\left\{e_1^T Q_{21} e_4 + e_1^T P_{22} e_5 - e_3^T P_{22} e_5\right. \\
&\qquad \left. + e_4^T (P_{12} + Q_{12} + Q_{22}) e_5\right\},
\end{aligned}$$

$$\Pi_2 = sym\left\{e_1^T P_{22} e_6 - e_3^T P_{22} e_6 + e_4^T (P_{12}+Q_{12}) e_6\right\},$$
$$Y = 2\left[Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8\right],$$
$$Z = diag\left\{2Z_1, Z_1, 2Z_1, Z_1, 2Z_2, Z_2, 2Z_2, Z_2\right\},$$
$$\gamma_1 = e_1 - e_2,\ \ \gamma_2 = e_1 + e_2 - 2e_5,$$
$$\gamma_3 = e_1 - e_2 + 6e_5 - 12e_7,\ \ \gamma_4 = e_2 - e_3,$$
$$\gamma_5 = e_2 + e_3 - 2e_6,\ \ \gamma_6 = e_2 - e_3 + 6e_6 - 12e_8,$$
$$\upsilon_1 = e_1 - e_5,\ \ \upsilon_2 = e_1 + 2e_5 - 6e_7,\ \ \upsilon_3 = e_2 - e_6,$$
$$\upsilon_4 = e_2 + 2e_6 - 6e_8,\ \ \upsilon_5 = e_5 - e_2,$$
$$\upsilon_6 = e_2 - 4e_5 + 6e_7,$$
$$\upsilon_7 = e_6 - e_3,\ \ \upsilon_8 = e_3 - 4e_6 + 6e_8,$$
$$e_i = \left[0_{n\times(i-1)n}, I_{n\times n}, 0_{n\times(11-i)n}\right] (i = 1, 2, \ldots, 11).$$

Proof: Consider a LKF for system (1) as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t), \tag{24}$$

where

$$V_1(t) = \varphi^T(t) P \varphi(t),$$
$$V_2(t) = \int_{t-d_U}^{t} \varphi_1^T(t,s) Q_1 \varphi_1(t,s) ds + \int_{t-d(t)}^{t} \varphi_1^T(t,s) Q_2 \varphi_1(t,s) ds,$$
$$V_3(t) = \int_{t-d_U}^{t} \varphi_2^T(s) Q_3 \varphi_2(s) ds + \int_{t-d(t)}^{t} \varphi_2^T(s) Q_4 \varphi_2(s) ds,$$
$$V_4(t) = 2\sum_{i=1}^{n}\left(\lambda_{1i}\int_0^{x_i(t)} f_i(s) ds + \lambda_{2i}\int_0^{x_i(t)} (h_i s - f_i(s)) ds\right),$$
$$V_5(t) = d_U \int_{t-d_U}^{t}\int_u^t \dot{x}^T(s) R_2 \dot{x}(s) ds du,$$
$$V_6(t) = \int_{-d_U}^{0}\int_v^0\int_{t+u}^{t} \dot{x}^T(s) Z_1 \dot{x}(s) ds du dv + \int_{-d_U}^{0}\int_{-d_U}^{v}\int_{t+u}^{t} \dot{x}^T(s) Z_2 \dot{x}(s) ds du dv.$$

The time derivatives of $V_i(t) (i = 1, \ldots, 6)$ along the trajectory of (1) are

$$\dot{V}_1(t) = \xi^T(t)\left(\Phi_1 + d(t)\Pi_{11} + d_2(t)\Pi_{21}\right)\xi(t), \tag{25}$$
$$\dot{V}_2(t) \le \xi^T(t)(\Phi_2 + d(t)(\Pi_{12} + \Pi_{13}) + d_2(t)\Pi_{22})\xi(t), \tag{26}$$
$$\dot{V}_3(t) \le \xi^T(t)\Phi_3\xi(t), \tag{27}$$
$$\dot{V}_4(t) = \xi^T(t)\Phi_4\xi(t), \tag{28}$$
$$\dot{V}_5(t) = d_U^2 \dot{x}^T(t) R_2 \dot{x}(t) + X_1 + X_2, \tag{29}$$
$$\dot{V}_6(t) = \tfrac{d_U^2}{2}\dot{x}^T(t)(Z_1 + Z_2)\dot{x}(t) + \textstyle\sum_{i=3}^{8} X_i, \tag{30}$$

where

$$\Pi_{11} = sym\left\{e_1^T P_{22} e_5 - e_3^T P_{22} e_5 + e_4^T P_{12} e_5\right\},$$
$$\Pi_{21} = sym\left\{e_1^T P_{22} e_6 - e_3^T P_{22} e_6 + e_4^T P_{12} e_6\right\},$$
$$\Pi_{12} = sym\left\{e_4^T Q_{12} e_5\right\}, \Pi_{22} = sym\left\{e_4^T Q_{12} e_6\right\},$$
$$\Pi_{13} = sym\left\{e_1^T Q_{21} e_4 + e_4^T Q_{22} e_5\right\},$$
$$X_1 = -d_U \int_{t-d(t)}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds,$$
$$X_2 = -d_U \int_{t-d_U}^{t-d(t)} \dot{x}^T(s) R_2 \dot{x}(s) ds,$$
$$X_3 = -d_2(t) \int_{t-d(t)}^{t} \dot{x}^T(s) Z_1 \dot{x}(s) ds,$$
$$X_4 = -d(t) \int_{t-d_U}^{t-d(t)} \dot{x}^T(s) Z_2 \dot{x}(s) ds,$$
$$X_5 = -\int_{t-d(t)}^{t}\int_u^t \dot{x}^T(s) Z_1 \dot{x}(s) ds du,$$
$$X_6 = -\int_{t-d_U}^{t-d(t)}\int_u^{t-d(t)} \dot{x}^T(s) Z_1 \dot{x}(s) ds du,$$
$$X_7 = -\int_{t-d(t)}^{t}\int_{t-d(t)}^{u} \dot{x}^T(s) Z_2 \dot{x}(s) ds du,$$
$$X_8 = -\int_{t-d_U}^{t-d(t)}\int_{t-d_U}^{u} \dot{x}^T(s) Z_2 \dot{x}(s) ds du.$$

According to Lemma 1 and Lemma 3 and inequality (22), for any matrices $S_i (i = 1, 2, 3)$ with proper dimensions, one can obtain

$$\begin{aligned}\textstyle\sum_{i=1}^{4} X_i &\le -\xi^T(t)\left[\frac{d_U}{d(t)}\Theta_1 + \frac{d_U}{d_2(t)}\Theta_2 + \frac{d_2(t)}{d(t)}\Theta_3 + \frac{d(t)}{d_2(t)}\Theta_4\right]\xi(t) \\ &\le -\xi^T(t)(\Theta_1 + \Theta_2 + \Theta_S)\xi(t),\end{aligned} \tag{31}$$

From Lemma 6, for any matrices $Y_i (i = 1, \ldots, 8)$ with proper dimensions and letting $\chi_0 = \xi(t)$ in (20)-(21), we have

$$\textstyle\sum_{i=5}^{8} X_i = \xi^T(t)\left[\Phi_Z + sym\left\{\sum_{i=1}^{8}(3 + (-1)^i)(Y_i\upsilon_i)\right\}\right]\xi(t), \tag{32}$$

where

$$\Phi_Z = \sum_{i=1}^{4}\left(3 + (-1)^i\right) Y_i Z_1^{-1} Y_i^T + \sum_{i=5}^{8}\left(3 + (-1)^i\right) Y_i Z_2^{-1} Y_i^T.$$

In addition, it follows from (3) that for any positive diagonal matrices $H_i = diag\{h_{i1}, h_{i2}, \ldots, h_{in}\} (i = 1, \ldots, 5)$, the following inequalities hold:

$$0 \le 2\sum_{i=1}^{n} f_i(x_i(t)) h_{1i}\left[h_i x_i(t) - f_i(x_i(t))\right]$$

$$
\begin{aligned}
&+2\sum_{i=1}^{n} f_i(x_i(t-d_U))h_{3i}\big[h_i x_i(t-d_U)\\
&-f_i(x_i(t-d_U))\big]\\
&+2\sum_{i=1}^{n} f_i(x_i(t-d(t)))h_{2i}big[h_i x_i(t-d(t))\\
&-f_i(x_i(t-d(t)))\big]\\
=&\xi^T(t)\Xi_1\xi(t),
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
0\le&2\sum_{i=1}^{n}[f_i(x_i(t))-f_i(x_i(t-d(t)))]h_{4i}[h_i x_i(t)\\
&-h_i x_i(t-d(t))-f_i(x_i(t))+f_i(x_i(t-d(t)))]\\
&+2\sum_{i=1}^{n}[f_i(x_i(t-d(t)))-f_i(x_i(t-d_U))]h_{5i}\\
&\times\big[h_i x_i(t-d(t))-h_i x_i(t-d_U)-f_i(x_i(t-d(t)))\big].
\end{aligned} \tag{34}
$$

Furthermore, from (1), the following zero equality is always true for any matrices $T_i (i=1,2,3)$ with proper dimensions,

$$
\begin{aligned}
0=&2\left(x^T(t)T_1+x_d^T(t)T_2+\dot{x}^T(t)T_3\right)(Ax(t)\\
&+A_d x_d(t)+Bf(x(t))+B_d f(x_d(t))-\dot{x}(t))\\
=&\xi^T(t)\Xi_3\xi(t).
\end{aligned} \tag{35}
$$

Hence, from (24)-(35), we can get

$$
\dot{V}(t)\le\xi^T(t)\left(\Phi+\Phi_Z+d(t)\Pi_1+d_2(t)\Pi_2\right)\xi(t). \tag{36}
$$

So, if the following inequality (37) holds, then the system (1) is asymptotically stable.

$$
\Phi+\Phi_Z+d(t)\Pi_1+d_2(t)\Pi_2<0. \tag{37}
$$

By Lemma 4, (37) is equivalent to (38),

$$
\Phi+\Phi_Z+d_U\Pi_i<0,(i=1,2). \tag{38}
$$

By Schur complement lemma, (38) is equivalent to (23). This is the end of proof. □

Remark 3: The proposed GDI inequalities are utilized to estimate double integral terms X_5, X_6, X_7, X_8 in (30). In order to show the improvement of the proposed GDI inequalities, Wirtinger-based double integral inequality is employed to estimate the same terms X_5, X_6, X_7, X_8. Then, the following criterion can be obtained.

Theorem 2: For given scalars $d_U>0$, d_D, and positive diagonal matrix $H_U=diag\{h_1,h_2,\ldots,h_n\}$, the system (1) is asymptotically stable, if there exist symmetric matrices P, $R_2>0$, $Q_i>0$ $(i=1,2,3,4)$, $Z_i>0$ $(i=1,2)$, positive diagonal matrices $\Lambda_i=diag\{\lambda_{i1},\lambda_{i2},\ldots,\lambda_{in}\},(i=1,2)$, $H_i=diag\{h_{i1},h_{i2},\ldots,h_{in}\}$, $(i=1,2,3,4,5)$ and any matrices S_i, T_i $(i=1,2,3)$ with appropriate dimensions, such that the following LMIs in (39)-(40) hold:

$$
\begin{bmatrix}(2i-1)(R_2+Z_1) & S_i\\ * & (2i-1)(R_2+Z_2)\end{bmatrix}<0, \quad (i=1,2,3), \tag{39}
$$

$$
\bar{\Phi}+d_U\Pi_i<0, \quad (i=1,2), \tag{40}
$$

where

$$
\begin{aligned}
\bar{\Phi}=&\textstyle\sum_{i=1}^{4}\Phi_i+\bar{\Phi}_5+\bar{\Phi}_Z+\Xi_1+\Xi_2+\Xi_3\\
&-\Theta_1-\Theta_2-\Theta_S,\\
\bar{\Phi}_Z=&-\sum_{i=1}^{4}\left(3+(-1)^i\right)\upsilon_i^T Z_1\upsilon_i\\
&-\sum_{i=5}^{8}\left(3+(-1)^i\right)\upsilon_i^T Z_2\upsilon_i,\\
\bar{\Phi}_5=&e_4^T\left(d_U^2R_2+\frac{1}{2}d_U^2(Z_1+Z_2)\right)e_4,
\end{aligned}
$$

and $\Phi_i,\Xi_1,\Xi_2,\Xi_3,\Theta_1,\Theta_2,\Theta_S,e_4,\upsilon_k$ $(i=1,2,3,4;\ k=1,\ldots,8)$ are defined in Theorem 1.

Proof: Selecting the same LKF as (24). In a similar way to the proof of Theorem 1, Theorem 2 can be obtained easily. So, it is omitted. □

Remark 4: For some special cases of system (1), for instance, time derivative of the delay $\dot{d}(t)$ is unknown or $f(x(t))\equiv 0$, stability criteria can be derived easily by removing relevant terms in the constructed LKF in (24).

4. NUMERICAL EXAMPLES

In this section, two well-known numerical examples will be given to illustrate the improvement of the proposed criteria in this paper.

Example 1: For system (1) with the following system matrices:

$$
\begin{aligned}
&A=diag\{-1.2769,-0.6231,-0.9230,-0.4480\},\\
&A_d=diag\{0,0,0,0\},\\
&B=\begin{bmatrix}-0.0373 & 0.4852 & -0.3351 & 0.2336\\ -1.6033 & 0.5988 & -0.3224 & 1.2352\\ 0.3394 & -0.0860 & -0.3824 & -0.5785\\ -0.1311 & 0.3253 & -0.9534 & -0.5015\end{bmatrix},\\
&B_d=\begin{bmatrix}0.8674 & -1.2405 & -0.5325 & 0.0220\\ 0.0474 & -0.9164 & 0.0360 & 0.9816\\ 1.8495 & 2.6117 & -0.3788 & 0.8428\\ -2.0413 & 0.5179 & 1.1734 & -0.2775\end{bmatrix},\\
&h_1=0.1137,\ h_2=0.1279,\ h_3=0.7994,\ h_4=0.2368.
\end{aligned}
$$

The maximum upper bounds of delay(MUBD) that guarantee the asymptotical stability of system (1) with different d_D are listed in Table 1. For comparison, the results obtained in [3, 33, 35] are also given in Table 1. From Table 1, the MUBD derived by the proposed criteria in Theorems 1 are larger than some exiting results. Therefore, our method is less conservative.

Table 1. MUBD of d_U with various d_D in Example 1.

Methods	$d_D = 0.1$	$d_D = 0.5$	$d_D = 0.9$	NoDV
[33]	4.2778	3.2152	2.9361	$73n^2 + 13n$
[35]	4.370	3.187	2.907	$34.5n^2 + 20.5n$
[3]	4.4530	3.4929	3.0726	$119n^2 + 21n$
Theorem 2	5.101	3.822	3.210	$17.5n^2 + 13.5n$
Theorem 1	5.135	3.836	3.228	$81.5n^2 + 13.5n$

* NoDV represents the number of the decision variables.

Table 2. MUBD of d_U with various d_D in Example 2.

Methods	$d_D = 0.4$	$d_D = 0.45$	$d_D = 0.5$	$d_D = 0.55$
[33]	8.3489	7.3817	7.0219	6.8156
[35]	9.211	7.187	6.807	6.498
[3]	10.4317	9.1910	8.6957	8.3806
Theorem 2	11.618	10.473	9.932	9.102
Theorem 1	11.986	10.825	10.213	9.515

When $d_D \leq 0.5$, setting $d(t) = 3.336 + 0.5\sin(t) \leq 3.836$, $x(0) = [1, -1, -1, 1]^T$, and $f(x(t)) = [0.1137\tanh(x_1(t)), 0.1279\tanh(x_2(t)), 0.7994\tanh(x_3(t)), 0.2368\tanh(x_4(t))]^T$, the state responses of system (1) in Example 1 are shown in Fig. 1.

Example 2: Consider the time-delay system (1) with matrix parameters:

$$A = \begin{bmatrix} -1.5 & 0 \\ 0 & -0.7 \end{bmatrix}, \ A_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.0505 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix}, \ B_d = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix},$$
$$h_1 = 0.3, \ \ h_2 = 0.8.$$

The allowable maximum upper bounds of delay(MUBD) d_U for various d_D are given in Table 2. It is clear that Theorem 1 gets better results than some recent results [3, 33, 35], which shows the improvement of the proposed GDI inequality in reducing conservatism.

The trajectories of system (1) in Example 2 are shown in Fig. 2 by setting $d(t) = 8.965 + 0.55\sin(t) \leq 9.515$ with $d_D \leq 0.55$, $f(x(t)) = [0.3\tanh(x_1(t)), 0.8\tanh(x_2(t))]^T$ and $x(0) = [1, -1]^T$.

5. CONCLUSION

In this paper, stability analysis for linear time-delay systems with nonlinear disturbances has been investigated. In order to decrease the conservatism of stability criteria, two improved generalized double integral inequalities are proposed to reduce estimation gap of the time derivative of the constructed LKF . It has been proved that the proposed GDI inequalities encompass well-known Wirtinger-based double integral inequality. Stability criteria with the framework of LMIs are obtained with the proposed GDI inequalities and Wirtinger-based double integral inequality, respectively. Finally, two well-known numerical examples are given to demonstrate the less conservatism of stability criteria derived from the proposed GDI inequalities.

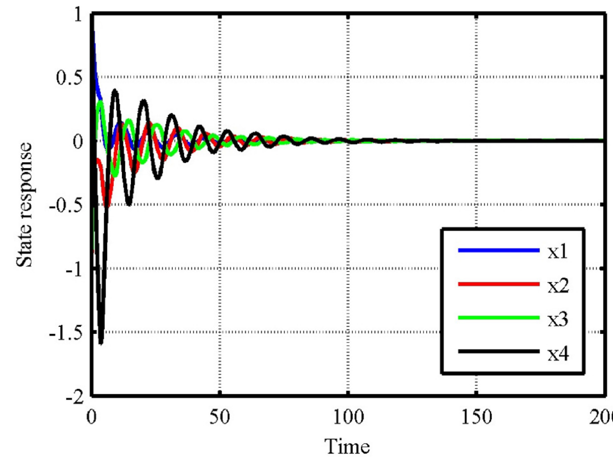


Fig. 1. State trajectories of the system in Example 1.

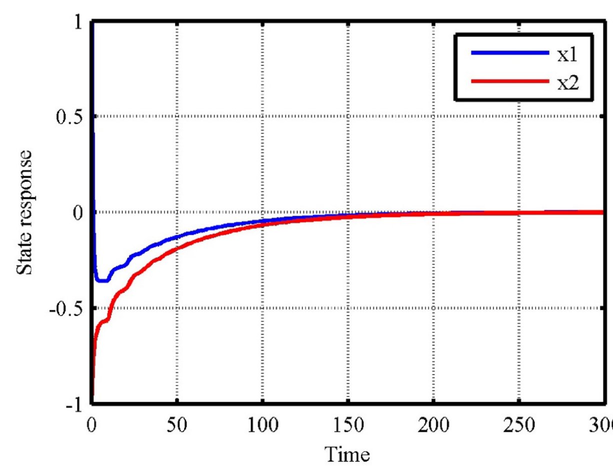


Fig. 2. State trajectories of the system in Example 2.

REFERENCES

[1] E. Samiei, E. A. Butcher, A. K. Sanyal, and R. Paz, "Attitude stabilization of rigid spacecraft with minimal attitude coordinates and unknown time-varying delay," *Aerospace Science and Technology*, vol. 46, no. 4, pp. 412-421, 2015.

[2] Q. H. Shan, H. G. Zhang, Z. S. Wang, and J. Y. Wang, "Adjustable delay interval method based stochastic robust stability analysis of delayed neural networks," *Neurocomputing*, vol. 219, no. 3, pp. 389-395, 2017.

[3] B. Yang, J. Wang, and J. Wang, "Stability analysis of delayed neural networks via a new integral inequality," *Neural Networks*, vol. 88, no. 1, pp. 49-57, 2017.

[4] T. H. Lee, H. M. Trinh, and J. H. Park, "Stability analysis of neural networks with time-varying delay by con-

structing novel Lyapunov functionals," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 9, pp. 4238-4247, 2018.

[5] Y. Muroya, T. Kuniya, and J. Wang, "Stability analysis of a delayed multi-group SIS epidemic model with nonlinear incidence rates and patch structure," *Journal of Mathematical Analysis and Applications*, vol. 425, no. 1, pp. 415-439, 2015.

[6] T. Wang, H. Gao, and J. Qiu, "A combined fault-tolerant and predictive control for network-based industrial processes," *IEEE Transactions on Industrial Electronics*, vol.63, no. 4, pp. 2529-2536, 2016.

[7] G. Rajchakit, R. Saravanakumar, C. K. Ahn, and H. R. Karimi, "Improved exponential convergence results for generalized neural networks including interval time-varying delayed signal," *Neural Networks*, vol. 86, no. 1, pp. 10-17, 2017.

[8] G. L. Chen, J. W. Xia, G. M. Zhuang, and J. S. Zhao, "Improved delay-dependent stabilization for a class of networked control systems with nonlinear perturbations and two delay components," *Applied Mathematics and Computation*, vol. 316, no. 1, pp. 1-17, 2018.

[9] L. S. Zhang, L. He, and Y. D. Song, "New results on stability analysis of delayed systems derived from extended Wirtinger's integral inequality," *Neurocomputing*, vol. 283, no. 1, pp. 98-106, 2018.

[10] W. Qian, M. M. Yuan, L. Wang, Y. G. Chen, and J. Q. Yang, "Robust stability criteria for uncertain systems with interval time-varying delay based on multi-integral functional approach," *Journal of the Franklin Institute*, vol. 355, no. 2, pp. 849-861, 2018.

[11] R. Saravanakumar, M. S. Ali, H. Huang, J. Cao, and Y. H. Joo, "Robust $H\infty$ state-feedback control for nonlinear uncertain systems with mixed time-varying delays," *International Journal of Control, Automation and Systems*, vol. 16, no. 1, pp. 225-233, 2018.

[12] W. I. Lee, S. Y. Lee, and P. G. Park, "Improved criteria on robust stability and $H\infty$ performance for linear systems with interval time-varying delays via new triple integral functionals," *Applied Mathematics and Computation*, vol. 243, no. 3, pp. 570-577, 2014.

[13] T. H. Lee, J. H. Park, and S. Y. Xu, "Relaxed conditions for stability of time-varying delay systems," *Automatica*, vol. 75, no. 1, pp. 11-15, 2017.

[14] T. H. Lee and J. H. Park, "A novel Lyapunov functional for stability of time-varying delay systems via matrix-refined-function," *Automatica*, vol. 80, no. 4, pp. 239-242, 2017.

[15] O. M. Kwon, M. J. Park, and J. H. Park, "Analysis on robust $H\infty$ performance and stability for linear systems with interval time-varying state delays via some new augmented Lyapunov-Krasovskii functional," *Applied Mathematics and Computation*, vol. 224, no. 1, pp. 108-122, 2013.

[16] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235-238, 2011.

[17] X. F. Liu, X. G. Liu, M. L. Tang, and F. X. Wang, "Improved exponential stability criterion for neural networks with time-varying delay," *Neurocomputing*, vol. 234, no. 2, pp. 154-163, 2017.

[18] J. J. Hui, X. Y. Kong, H. X. Zhang, and X. Zhou, "Delay-partitioning approach for systems with interval time-varying delay and nonlinear perturbations," *Journal of Computational and Applied Mathematics*, vol. 281, no. 1, pp. 74-81, 2015.

[19] L. M. Ding, Y. He, M. Wu, and Z. M. Zhang, "A novel delay partitioning methods for stability analysis of interval time-varying delay systems," *Journal of Franklin Institute*, vol. 354, no. 6, pp. 1209-1219, 2017.

[20] M. Wu, Y. He, J. H. She, and G. P. Liu, "Delay-dependent criteria for robust stability of time-varying delay systems," *Automatica*, vol. 40, no. 8, pp. 1435-1439, 2004.

[21] S. Y. Lee, W. I. Lee, and P. G. Park, "Improved stability criteria for linear systems with interval time-varying delays: generalized zero equalities approach," *Applied Mathematics and Computation*, vol. 292, no. 2, pp. 336-348, 2017.

[22] S. Y. Xu, J. Lam, B. Zhang, and Y. Zou, "New insight into delay-dependent stability of time-delay systems," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 7, pp. 961-970, 2015.

[23] H. T. Xu, C. K. Zhang, L. Jiang, and J. Smith, "Stability analysis of linear systems with two additive time-varying delays via delay-product-type Lyapunov functional," *Applied Mathematical Modelling*, vol. 45, no. 6, pp. 955-964, 2017.

[24] T. H. Lee and J. H. Park, "Improved stability conditions of time-varying delay systems based on new Lyapunov functionals," *Journal of the Franklin Institute*, vol. 355, no. 2, pp. 1176-1191, 2018.

[25] X. Ge, "Stability analysis for continuous system with additive time-varying delays: a less conservative result," *Applied Mathematics and Computation*, vol. 241, no. 1, pp. 42-46, 2014.

[26] J. Cheng, H. Zhu, S. M. Zhong, Y. P. Zhang, and Y. Zeng, "Improved delay-dependent stability criteria for continuous system with two additive time-varying delay components," *Communications of Nonlinear Sciences and Numerical Simulation*, vol. 19, no. 1, pp. 210-215, 2014.

[27] X. L. Zhu, Y. Y. Wang, and X. Du, "Stability criteria for continuous-time systems with additive time-varying delays," *Optimal Control Applications & Methods*, vol. 35, no. 2, pp. 166-178, 2014.

[28] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860-2866, 2013.

[29] O. M. Kwon, M. J. Park, J. H. Park, S. M. Lee, and E. J. Cha, "Improved results on stability of linear systems with time-varying delays via Wirtinger-based integral inequality," *Journal of the Franklin Institute*, vol. 351, pp. 5386-5398, 2014.

[30] E. Gyurkovics and T. Takacs, "Multiple integral inequalities and stability analysis of time delay systems," *System & Control Letters*, vol. 96, no. 1, pp. 72-80, 2016.

[31] N. Zhao, C. Lin, B. Chen, and Q. G. Wang, "A new double integral inequality and application to stability test for time-delay systems," *Applied Mathematics Letters*, vol. 65, no. 1, pp. 26-31, 2017.

[32] H. B. Zeng, Y. He, M. Wu, and J. H. She, "Free-matrix-based integral inequality for stability analysis of systems with time-varying delay," *IEEE Transactions on Automatic Control*, vol. 60, no. 10, pp. 2768-2772, 2015.

[33] C. K. Zhang, Y. He, L. Jiang, W. J. Lin, and M. Wu, "Delay-dependent stability analysis of neural networks with time-varying delay: A generalized free-weighting-matrix approach," *Applied Mathematics and Computation*, vol. 294, no. 1, pp. 102-120, 2017.

[34] K. Liu, A. Seuret, and Y. Q. Xia, "Stability analysis of systems with time-varying delays via the second-order Bessel-Legendre inequality," *Automatica*, vol. 76, no. 2, pp. 138-142, 2017.

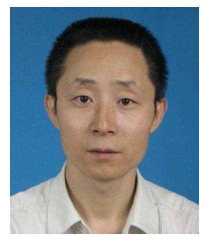
[35] W. J. Wen, Y. He, C. K. Zhang, and M. Wu, "Stability analysis of neural networks with time-varying delay: enhanced stability criteria and conservatism comparisons," *Communications of Nonlinear Sciences and Numerical Simulation*, vol. 54, no. 1, pp. 118-135, 2018.

[36] S. Y. Lee, W. I. Lee, and P. G. Park, "Orthogonal-polynomials-based integral inequality and its applications to systems with additive time-varying delays," *Journal of the Franklin Institute*, vol. 355, no. 1, pp. 421-435, 2018.

[37] Z. C. Li, Y. Bai, C. Z. Huang, and H. C. Yan, "Further results on stabilization for interval time-delay systems via new integral inequality approach," *ISA Transactions*, vol. 68, no. 1, pp. 170-180, 2017.

[38] C. Gong, X. Zhang, and L. G. Lu, "Multiple-integral inequalities to stability analysis of linear time-delay systems," *Journal of the Franklin Institute*, vol. 354, no. 2, pp. 1446-1463, 2017.

Bin Wu received his B.S. and M.S. degrees in Department of UAV Engineering from Ordnance Engineering College in 2012 and 2014, respectively. Now, he is a Ph.D. student in School of UAV Engineering at Army Engineering University Shijiazhuang Campus. His research interests include spacecraft control, and nonlinear and robust control.



Chang-Long Wang received his M.S. degree from Harbin Institute of Technology in 1992, and a Ph.D. degree from Xi'an Jiaotong University in 2006. Now, he is a professor in Army Engineering University Shijiazhuang Campus. His research interests include spacecraft control, and signal and information processing.

Yong-Jiang Hu received his M.S. degree from Shanghai Jiaotong University, and his Ph.D. degree in guidance, navigation and control from Ordnance Engineering College in 2007. He is currently an associate professor in Army Engineering University Shijiazhuang Campus. His mainly research interests are in and signal processing and spacecraft control.

Xiao-Lin Ma received her B.S. degree in electronics from Hebei University of Technology, and the M.S. degree in microwave from Communication University of China. She is currently an associate professor in Army Engineering University Shijiazhuang Campus. Her mainly research interests are in wireless communication and signal processing.