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Observer-based Controller Design for A T-S Fuzzy System with Unknown Premise Variables

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Abstract: For the stabilization problem of T-S fuzzy system, a new observer-based controller design approach is proposed when premise variables are not accessible. With a fuzzy observer, the estimated states error system is described as two parts: unknown premise variable caused terms and observer error terms. Consider the property that the norm of the unknown premise variable caused terms are under a Lipschitz condition constraint of observer error, an observer and controller errors augmented system is obtained. Then based on the Lyapunov function method, a series of linear matrix inequality conditions are proposed to asymptotically stabilize the system, the observer gain matrices are used to overcome the uncertainties caused by UPVs. Finally a simulation example is used to illustrate the effectiveness of the proposed method, comparisons with traditional method shows the conservatism reduction effects.

Keywords: Controller, observer, T-S fuzzy system, unknown premise variables.

1. INTRODUCTION

For the complicated nonlinear system control problems, abundant methods are proposed to achieve a satisfied control performance such as sliding mode, backstepping [1–3], adaptive fuzzy control [4] and some other approaches [5–7]. Since the T-S fuzzy modelling method is proposed in [8], abundant linear system control theories are adopted in nonliner system control for its systematic design process as in [9–15].

For the control problem with unmeasurable system states, observer-controller method is an effective approach [16–22]. The stability conditions of T-S control systems can be described as a series of linear matrix inequalities (LMIs) [23], and thus the control gain matrices can be obtained by solving these condition through a convex optimization process. But for the observer-controller approach, the stability conditions are often given as bilinear matrix inequalities (BMIs), and the BMIs could only be solved by recursively calculation process which is called two-step methods as in [24, 25]. In recent years, one-step methods are proposed to further reduce the conservatism caused by two-step methods: by using a similar transformation process, the BMI conditions are changed into LMI conditions with quadratic Lyapunov function in [26] and fuzzy Lyapunov function in [27]. For the T-S fuzzy system without external disturbance, the separation principle is satisfied: an individual observer and controller can be combined together to form a stable control structure [19, 28].

The T-S fuzzy observer-controller design problem can be considered as two cases according to the property of premise variables [29]: the first case refers to known premise variables, one can see [9]; the second case is that the premise variables are unknown and the membership functions also depends on the estimated premise variables [30]. In real engineering applications, the assumption that the premise variables are measurable is very restrictive, so the unknown premise variables (UPVs) case is widely studied: if the UPVs are partly unknown, the membership function estimated errors can be transformed into linear functions of observer error, then the observer-controller errors augmented system can be stabilized as in [31, 32]. In recent years, to reduce the conservatism for partly UPVs case, the known premise variables are sufficiently used in the observer-controller synthesis approach in [33, 34]. If the premise variables are all unaccessible, the control problem becomes more complicated, some results give a robust control approach to handle the uncertainties caused by UPVs as in [35]. In [36], the Lipschitz

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conditions with observer error are used to treat the uncertainties caused by UPVs; the observer and controller stability LMI conditions are designed separately based on the Lipschitz hypothesis in [37], for further results of the Lipschitz condition method, Guerra et.al proposed a differential mean value method to describe the UPVs caused uncertain terms in [38]. For other research results of the UPVs case, one can find H_∞ methods in [39,40], the UPVs caused terms are treated as coupled terms in error augmented systems, in the meanwhile, system disturbance is attenuated based on the robust approach. To further reduce the conservatism, a kind of polynomial control design approach based on SOS method is proposed for the UPVs case [41]. Although the current researches have achieved great results in this field, there is still certain conservatism in stability analysis.

In this paper, a new observer-based controller design approach is proposed when the premise variables are all unaccessible. Compared with the design methods in the existing literature, the predefined constants and the system conservatism can be reduced. With uncommon system output matrices, classical observer and controller are designed. Assuming that the UPVs caused uncertain terms are satisfied under a Lipschitz condition with the norm of observer error, then a new one-step method is designed in stability analysis process. It is shown that the observer gain matrices will make the main contribution of overcoming the membership functions uncertainties affections caused by UPVs, and during the increasement of membership functions partial derivatives with respect to premise variables, a high gain observer will guarantee the system stability. Finally, a simulation example is given to show the effectiveness of proposed method, also, comparisons with existing method are made to illustrate the conservatism reduction effects.

The rest of the paper is organized as follows: Section 2 introduces the T-S fuzzy system and control problem. Section 3 gives the observer-controller design procedure. Section 4 presents the main results in stability analysis. Section 5 illustrates the simulation example. Some closing remarks are presented in conclusion.

Notations: Throughout the paper, R^n denotes the n-dimensional Euclidean space. I is an identity matrix, and 0 denotes a zero matrix or a zero scalar without confusion. For a matrix X, X^T and X^{-1} denote its transpose and inverse matrices, respectively, the Hermitian section of X is denoted by $\mathrm{He}\{X\} = X + X^T$, $||*|| = \sqrt{*^T*}$ is the Euclidean-norm of a vector, while $||X|| = \{X : \int_o^\infty ||X(t)||^2 dt < \infty\}$ is an $\mathbf{L}_2$ norm. $\frac{\partial *}{\partial *}$ denotes the partial derivative. Without leading to confusion, f will be used to denote a function or a variable $f(*)$. $\star$ denotes the transpose of the corresponding block matrix.

2. PROBLEM STATEMENT

Consider a class of T-S fuzzy system:

$$\begin{aligned}&\textbf{Rule}\, i: \ \text{IF } z_1(t) \text{ is } M_{i1} \ \ldots \text{ and } z_p(t) \text{ is } M_{ip} \text{ THEN}\\ &\dot{x}(t) = A_i x(t) + B_i u(t),\\ &y(t) = C_i x(t),\end{aligned} \tag{1}$$

where M_{ij} are fuzzy sets; $i = 1, 2, \ldots, l$ with l denoting the number of fuzzy rules; $j = 1, 2, \ldots, p$ with p denoting the number of premise variables; $z(t) = [z_1(t) \ldots z_p(t)]^T \in \mathrm{R}^p$ is the premise variable; $x(t) \in \mathrm{R}^n$ is the system state; $y(t) \in \mathrm{R}^o$ is the measured output; $u(t) \in \mathrm{R}^m$ is the control input; $A_i \in \mathrm{R}^{n\times n}$, $B_i \in \mathrm{R}^{n\times m}$ and $C_i \in \mathrm{R}^{o\times n}$ are system parameter matrices, where $i = 1, 2, \ldots, l$. The system dynamics can be defined as:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{l} h_i(z(t))[A_i x(t) + B_i u(t)],\\ y(t) &= \textstyle\sum_{i=1}^{l} h_i(z(t)) C_i x(t),\end{aligned} \tag{2}$$

where

$$\begin{aligned}&\sum_{i=1}^{l} h_i(z(t)) = 1,\\ &h_i(z(t)) = \frac{\prod_{j=1}^{p} M_{ij}(z_j(t))}{\sum_{i=1}^{l}\prod_{j=1}^{p} M_{ij}(z_j(t))} \geq 0,\end{aligned}$$

for all i, $h_i(z(t))$ is the normalized grade of membership, and $M_{ij}(z_j(t))$ represents the grade of membership of $z_j(t)$ corresponding to the fuzzy set M_{ij}, all the elements in premise variable z are considered as unmeasurable. For simplicity, $h_i = h_i(z(t))$ is used in the rest of the paper.

For the system (2), an observer-based controller for stabilization control task will be designed in the consequent sections.

3. OBSERVER-BASED CONTROLLER

3.1. Observer design

An observer for system (2) with UPVs is designed as:

$$\begin{aligned}\dot{\hat{x}} &= \sum_{i=1}^{l} h_i(\hat{z})[A_i\hat{x} + B_i u + L_i(y - \hat{y})],\\ \hat{y} &= \sum_{i=1}^{l} h_i(\hat{z}) C_i \hat{x},\end{aligned} \tag{3}$$

where $\hat{x} \in \mathrm{R}^n$ is the estimated system state, $\hat{y} \in \mathrm{R}^o$ is the estimated system output, $L_i \in \mathrm{R}^{n\times o}$ are observer gain matrices. Define observer error $e(t) = x(t) - \hat{x}(t)$, and based on system (2) and observer (3), h, $\hat{h}$ are used to denote $h(z)$, $h(\hat{z})$ respectively in the consequent sections, the observer error dynamics can be described as:

$$\dot{e} = \sum_{i=1}^{l} h_i[A_i x + B_i u] - \sum_{i=1}^{l} \hat{h}_i[A_i\hat{x} + B_i u + L_i(y - \hat{y})]$$

$$=\sum_{i=1}^{l}(h_i-\hat{h}_i)(A_ix+B_iu)+\sum_{i=1}^{l}\sum_{j=1}^{l}\hat{h}_i\hat{h}_j(A_i-L_iC_j)e$$
$$+\sum_{i=1}^{l}\hat{h}_iL_i\sum_{j=1}^{l}(\hat{h}_j-h_j)C_jx. \quad (4)$$

Proposition 1: For system (2) whose premise variables are unknown, there exists a constant $\mu>0$, with which the following Lipschitz condition holds:

$$||\Delta||\le\mu||e||,$$

where

$$\Delta=\sum_{i=1}^{l}(h_i-\hat{h}_i)(A_ix+B_iu)+\sum_{i=1}^{l}\hat{h}_iL_i\sum_{j=1}^{l}(\hat{h}_j-h_j)C_jx.$$

Proof: Based on Theorem 4.5 in [31], the UPVs caused term which is related to system state and control input can be constrained as:

$$||\sum_{i=1}^{l}(h_i-\hat{h}_i)(A_ix+B_iu)||\le\mu_1||e||,$$

where constant scalar $\mu_1>0$. Similar with the above results, the system output uncertain term is satisfied with the following condition:

$$||\sum_{j=1}^{l}(\hat{h}_j-h_j)C_jx||\le\mu_2||e||,$$

where constant scalar $\mu_2>0$. Then the norm of uncertain term Δ is:

$$||\Delta||\le\mu_1||e||+||\sum_{i=1}^{l}\hat{h}_iL_i\sum_{j=1}^{l}(\hat{h}_j-h_j)C_jx||$$
$$\le\mu_1||e||+\sum_{i=1}^{l}||\hat{h}_iL_i||\cdot||\sum_{j=1}^{l}(\hat{h}_j-h_j)C_jx||$$
$$\le\mu_1||e||+\mu_2\sum_{i=1}^{l}||\hat{h}_iL_i||\cdot||e||. \quad (5)$$

Given the fact that the membership functions and observer gain matrices are both upper limited, so there exists $\mu_3>0$ making $\sum_{i=1}^{l}||\hat{h}_iL_i||\le\mu_3$ holds, and thus the above inequality is transformed into:

$$||\Delta||\le(\mu_1+\mu_2\mu_3)||e||=\mu||e||, \quad (6)$$

where $\mu=\mu_1+\mu_2\mu_3>0$. This completes the proof. □

Remark 1: With the conclusion in [31], as long as the membership functions are smooth and the variables are defined on a compact set, the constant μ can be obtained from:

$$\mu=||\partial\Delta/\partial e||_{\max}. \quad (7)$$

Then the observer error system can be transformed into:

$$\dot{e}=\Delta+\sum_{i=1}^{l}\sum_{j=1}^{l}\hat{h}_i\hat{h}_j(A_i-L_iC_j)e. \quad (8)$$

3.2. Estimated states feedback controller design

The controller is designed as:

$$u=\sum_{i=1}^{l}\hat{h}_iK_i\hat{x}, \quad (9)$$

where $K_i\in\mathrm{R}^{m\times n}$ are control gain matrices, substituting u into system (2), one has

$$\dot{x}=\sum_{i=1}^{l}\sum_{j=1}^{l}h_i\hat{h}_j(A_ix+B_iK_j\hat{x})$$
$$=\sum_{i=1}^{l}\sum_{j=1}^{l}h_i\hat{h}_j(A_i+B_iK_j)x-\sum_{i=1}^{l}\sum_{j=1}^{l}h_i\hat{h}_jB_iK_je. \quad (10)$$

Considering (8) and (10), the augmented system of observer and controller errors can be written in contact form:

$$\dot{\bar{x}}=\sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{k=1}^{l}h_i\hat{h}_j\hat{h}_k(\bar{A}_{ijk}\bar{x}+D), \quad (11)$$

where

$$\bar{A}_{ijk}=\begin{bmatrix}A_i+B_iK_j & -B_iK_j\\ 0 & A_j-L_jC_k\end{bmatrix}\in\mathrm{R}^{2n\times2n},$$
$$D=\begin{bmatrix}0\\ \Delta\end{bmatrix}\in\mathrm{R}^{2n},\quad \bar{x}=\begin{bmatrix}x\\ e\end{bmatrix}\in\mathrm{R}^{2n}.$$

4. STABILITY ANALYSIS

Before illustrating the main results for stability analysis, some useful Lemmas are given below:

Lemma 1: For matrices X and Y with appropriate dimensions, a positive constant $\varepsilon>0$, and a matrix F which satisfies $F^TF\le I$, the following inequality holds [42]:

$$X^TFY+Y^TFX\le\varepsilon X^TX+\varepsilon^{-1}Y^TY. \quad (12)$$

Lemma 2: For a matrix $\Omega<0$ and a matrix X with appropriate dimension will make the $X^T\Omega X\le0$ satisfied, and there will exist a scalar α which makes the following inequality holds [43]:

$$X^T\Omega X\le-\alpha(X^T+X)-\alpha^2\Omega^{-1}. \quad (13)$$

With the above Lemmas, the main results are given in Theorem 1:

Theorem 1: For $\forall\ i,j,k=1,2,...,l,\ \alpha>0$ is predefined scalar, if there exists $\varepsilon\in\mathrm{R}$, $N_1=N_1^T>0\in\mathrm{R}^{n\times n}$, $X_2=X_2^T>0\in\mathrm{R}^{n\times n}$, $Y_i\in\mathrm{R}^{m\times n}$, $Z_i\in\mathrm{R}^{m\times o}$ which make the linear matrix inequalities (14) hold, system (11) is asymptotically stable.

$$\begin{bmatrix} \mathrm{He}\{A_iN_1+B_iY_j\} & -B_iY_j & N_1 & 0 & 0 & 0 & 0 \\ \star & \left\{\begin{matrix} \varepsilon\mu^2 I+ \\ \mathrm{He}\{P_2A_j-Z_jC_k\} \end{matrix}\right\} & 0 & P_2 & \alpha I & 0 & 0 \\ \star & \star & -\varepsilon I & 0 & 0 & \alpha I & 0 \\ \star & \star & \star & -\varepsilon I & 0 & 0 & \alpha I \\ \star & \star & \star & \star & -2\alpha N_1 & 0 & 0 \\ \star & \star & \star & \star & \star & -2\alpha N_1 & 0 \\ \star & \star & \star & \star & \star & \star & -2\alpha N_1 \end{bmatrix} < 0. \quad (14)$$

Proof: Given $P = diag[P_1\ P_2] > 0$, then define a Lyapunov function $V = \bar{x}^T P\bar{x} \geq 0$, based on Lemma 1 and Proposition 1, the derivative of V is:

$$\begin{aligned} \dot{V} =& \sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{k=1}^{l} h_i\hat{h}_j\hat{h}_k[\bar{x}^T(\bar{A}_{ijk}^T P + P\bar{A}_{ijk})\bar{x} \\ &+ D^T P\bar{x} + \bar{x}^T PD] \\ \leq& \sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{k=1}^{l} h_i\hat{h}_j\hat{h}_k[\bar{x}^T(\bar{A}_{ijk}^T P + P\bar{A}_{ijk})\bar{x} \\ &+ \varepsilon D^T D + \varepsilon^{-1}\bar{x}^T P^2\bar{x}] \\ \leq& \sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{k=1}^{l} h_i\hat{h}_j\hat{h}_k[\bar{x}^T(\bar{A}_{ijk}^T P + P\bar{A}_{ijk})\bar{x} \\ &+ \varepsilon\mu^2 e^T e + \varepsilon^{-1}\bar{x}^T P^2\bar{x}] \\ \leq& \sum_{i=1}^{l}\sum_{j=1}^{l}\sum_{k=1}^{l} h_i\hat{h}_j\hat{h}_k[\bar{x}^T(\bar{A}_{ijk}^T P + P\bar{A}_{ijk} \\ &+ \varepsilon\mu^2\mathrm{T} + \varepsilon^{-1}P^2)\bar{x}], \end{aligned}$$

where $\mathrm{T} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$. If the following condition holds, $\dot{V} < 0$ will be satisfied:

$$\bar{A}_{ijk}^T P + P\bar{A}_{ijk} + \varepsilon\mu^2\mathrm{T} + \varepsilon^{-1}P^2 < 0, \quad (15)$$

the above inequalities are in the form of nonlinear matrix inequalities, and with schur complement property, the equivalent inequalities are obtained as:

$$\begin{bmatrix} \bar{A}_{ijk}^T P + P\bar{A}_{ijk} + \varepsilon\mu^2\mathrm{T} & P \\ \star & -\varepsilon I \end{bmatrix} < 0, \quad (16)$$

and condition (16) can be expanded as:

$$\begin{bmatrix} \mathrm{He}\{P_1A_i+P_1B_iK_j\} & -P_1B_iK_j & P_1 & 0 \\ \star & \left\{\begin{matrix} \varepsilon\mu^2 I+ \\ \mathrm{He}\{P_2A_j-P_2L_jC_k\} \end{matrix}\right\} & 0 & P_2 \\ \star & \star & -\varepsilon I & 0 \\ \star & \star & \star & -\varepsilon I \end{bmatrix} < 0. \quad (17)$$

Define $Z_j = P_2L_j$, $N_1 = P_1^{-1}$ and $Y_j = K_jN_1$, pre- and post-multiply matrix $diag[N_1\ N_1\ N_1\ N_1]$ with the above inequalities, we get:

$$\begin{bmatrix} \mathrm{He}\{A_iN_1+B_iY_j\} & -B_iY_j & N_1 & 0 \\ \star & \left\{\begin{matrix} \varepsilon\mu^2 N_1N_1+ \\ \mathrm{He}\{N_1(P_2A_j-Z_jC_k)N_1\} \end{matrix}\right\} & 0 & N_1P_2N_1 \\ \star & \star & -\varepsilon N_1N_1 & 0 \\ \star & \star & \star & -\varepsilon N_1N_1 \end{bmatrix} < 0, \quad (18)$$

based on Lemma 2, the right-bottom block matrix in (18) is rewritten as inequalities:

$$\begin{aligned} &\begin{bmatrix} \left\{\begin{matrix} \varepsilon\mu^2 N_1N_1+ \\ \mathrm{He}\{N_1(P_2A_j-Z_jC_k)N_1\} \end{matrix}\right\} & 0 & N_1P_2N_1 \\ \star & -\varepsilon N_1N_1 & 0 \\ \star & \star & -\varepsilon N_1N_1 \end{bmatrix} \\ &= F\begin{bmatrix} \left\{\begin{matrix} \varepsilon\mu^2 I+ \\ \mathrm{He}\{P_2A_j-Z_jC_k\} \end{matrix}\right\} & 0 & P_2 \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{bmatrix} F \\ &\leq -2\alpha F - \alpha^2\begin{bmatrix} \left\{\begin{matrix} \varepsilon\mu^2 I+ \\ \mathrm{He}\{P_2A_j \\ -Z_jC_k\} \end{matrix}\right\} & 0 & P_2 \\ \star & -\varepsilon I & 0 \\ \star & \star & -\varepsilon I \end{bmatrix}^{-1}, \end{aligned} \quad (19)$$

where $F = diag[N_1\ N_1\ N_1]$. With schur complement property, it is easy to obtain condition (14), and this completes the proof. □

Remark 2: If one wants to reduce the conservatism caused by the quadratic fuzzy Lyapunov function $V = \bar{x}^T P\bar{x}$, many membership functions dependent research results can be used in most cases, such as the method of bounding the time derivatives of the membership function in [40], and linear piecewise function approximation methods in [44, 45].

Remark 3: From the stability conditions (14), one can clearly find that a positive constant matrix $\varepsilon\mu^2 I$, μ is a

Table 1. Number of predefined constants comparison.

	Number of predefined constants
Theorem 1 from [37]	3
Theorem 1 from [37]	5
Theorem 1 of this paper	2

predefined constant, when the value of μ increases, the term $\text{He}\{P_2A_j - Z_jC_k\}$ must be a negative definite matrix with negative eigenvalues so as to overcome the impacts of $\varepsilon\mu^2 I$. In other words, the eigenvalues of $\text{He}\{Z_jC_k\}$ will drastically decrease as μ increasing, so a high gain observer could be obtained when μ is large.

Remark 4: Compared with the results in [37], Theorem 1 in this paper has some improvements. As observed from Theorem 1 and 2 in [37], the LMI conditions contain many constants which must be predefined, and may lead to considerable conservativeness. In Theorem 1 of this paper, however, there are only two predefined constants, that may give less conservatism, Table 1 is given below to show the comparisons.

Remark 5: The Theorem 2 of [37] restricts the control input u in order to limit the items caused by UPVs, however, the limit of the control input will make the upper bound of x_1 smaller, and the stability region is also restricted, so it will lead to a certain conservatism. In the following simulation examples, by calculating the upper bound of x_1, the conservatism comparisons between [37] and this paper for upper bound of x_1 are presented.

5. SIMULATION EXAMPLE

The simulation example in [37] is adopted here, the parameter matrices of the simulation model are:

$$A_1 = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} 2.5 & 0 \\ -2.3 & -1 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T,\ C_1 = C_2 = \begin{bmatrix} 10 & 2 \end{bmatrix},$$

and membership functions are given as follows:

$$\begin{cases} h_1(x_2) = 0.5 + \arctan(x_2)/\pi, \\ h_2(x_2) = 1 - h_1(x_2), \end{cases}$$

the system has common output matrix C and common control input matrix B, so the UPVs caused uncertain term is described as:

$$\sum_{i=1}^{2}(h_i - \hat{h}_i)A_i x = \frac{1}{\pi}(\arctan(x_2) - \arctan(\hat{x}_2))\Delta A x \leq \mu||e||, \tag{20}$$

where $\Delta A = \begin{bmatrix} -1.5 & 0 \\ 1.3 & 0 \end{bmatrix}$, and :

$$\mu = ||\frac{1}{\pi}\partial(\arctan(x_2) - \arctan(\hat{x}_2))\Delta A x/\partial e||_{\max}$$
$$= ||\frac{1}{\pi}\frac{\Delta A x}{1 + (x_2 - e(2))^2}||_{\max}$$
$$= \max\{\frac{3.94x_1^2}{\pi^2[1 + (x_2 - e(2))^2]}\}^{0.5}$$
$$= \max\{\frac{1.9849|x_{1\max}|}{\pi}\},$$

Table 2. Comparison of obtained $x_{1\max}$.

	$x_{1\max}$
Theorem 1 from [37]	1.139
Theorem 1 from [37]	1.0572×10^{-6}
Theorem 1 of this paper	3.1496×10^4

where $x_{1\max}$ denotes the upper bound of x_1. Using (20), $x_{1\max}$ in Theorem 1 of this paper and [37] can be obtained. For Theorem 2 in [37], by using feasible function of MATLAB R2016b, the norm upper limit of control law is $\eta = 0.55$, control gain matrices are $K_1 = K_2 = \begin{bmatrix} 255077 & -50916 \end{bmatrix}$, consider the state feedback control law as:

$$||u|| = ||\sum_{i=1}^{2} h_i K_i x|| = 5.2022 \times 10^5 |x_{1\max}| \leq \eta.$$

$x_{1\max}$ obtained from Theorem 1 of this paper and Theorems 1-2 in [37] are listed in Table 2. It is easy to see that $x_{1\max}$ indicates the region of stability, which also shows the conservatism. In the simulation process, $\alpha = 1$ is chosen. As shown in (20), the value of μ is dependent on system state x_1, so the greater value of μ is obtained from the LMI condition in Theorem 1, the larger stability area of x_1 can be obtained, and it is obvious that there is no limitation for state x_2. $\mu_{\max} = 1.99 \times 10^4$ can be got by a recursively calculation procedure of the conditions in Theorem 1, the corresponding $x_{1\max} = 3.1496 \times 10^4$. The conditions (14) in Theorem 1 can be solved by linear matrix inequalities toolbox in MATLAB R2016b, then the matrix variables N_1, P_2, Y_i, Z_i can be obtained. Therefore, the observer and controller gains can be calculated as:

$$L_1 = \begin{bmatrix} 3234.1 \\ -2172.0 \end{bmatrix},\ L_2 = \begin{bmatrix} 3311.4 \\ 22224.0 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -3.739 & 0.119 \end{bmatrix},\ K_2 = \begin{bmatrix} -3.550 & 0.161 \end{bmatrix}.$$

Obviously, the observer gain matrices are considerable high so as to overcome the effects of term $\varepsilon\mu^2 I$. The system initial state is set as $x_0 = \begin{bmatrix} 3.1496 \times 10^4 & 1 \times 10^5 \end{bmatrix}^T$, where $x_1 = 3.1496 \times 10^4$ is $x_{1\max}$, $x_2 = 1 \times 10^5$ is a great enough arbitrary chosen value, and these are used as initial values in simulation as a tough case to test the proposed method. From Fig.1, high gain observer and controller could make the system stable. But for most cases, the operating area of x_1 will only be limited in a small zone, for example $|x_1| \leq 100$, and the corresponding $\mu = 63.183$, this will greatly release the requirement of observer gain, the calculated observer and controller matrices are given

Table 3. Comparison of obtained $x_{1\,\max}$.

	$x_{1\,\max}$
The quadratic Lyapunov function method	3.1496×10^4
The fuzzy Lyapunov function method	2.6925×10^4

Table 4. Comparison of convergent time t for 5% error relative tolerance.

	$t(s)$
Theorem 1 from [37]	x_1: 3.2, x_2: 4.5
Theorem 2 from [37]	x_1: 2.6, x_2: 4.3
Theorem 2 from [43]	x_1: 2.9, x_2: 4.7
Theorem 1 of this paper	x_1: 2.5, x_2: 4.1

as follows:

$$L_1=\begin{bmatrix}0.535\\-0.274\end{bmatrix},\ L_2=\begin{bmatrix}0.687\\-0.397\end{bmatrix}$$
$$K_1=\begin{bmatrix}-4.924 & 0.740\end{bmatrix},\ K_2=\begin{bmatrix}-5.001 & 0.611\end{bmatrix}.$$

In order to compare with the simulation in [37], in this paper, the simulation initial condition is set as $x=\begin{bmatrix}100 & 100\end{bmatrix}^T$. From Fig.2, it is shown that the low gain observer-based controller can accomplish the stabilization control task when the value of μ is relatively small, so one can adjust the value of μ according to the specific system properties and control task.

Remark 6: Aim at reducing the conservatism caused by the quadratic fuzzy Lyapunov function $V=\bar{x}^TP\bar{x}$, the fuzzy Lyapunov function $V=\bar{x}^T\sum_{i=1}^{l}h_iP_i\bar{x}$ according to reference [46] is used in Theorem 1. However, with the same simulation example in this paper, the results of the fuzzy Lyapunov function method have increased the conservatism to some extent. Because the boundary values of the membership function derivatives $|\dot{h}_i(\hat{z}(t))|=|\frac{\partial h_i}{\partial\hat{z}}\frac{\partial\hat{z}}{\partial\hat{x}}\dot{\hat{x}}(t)|\le|\phi|$ involve certain conservatism. Table 3 is given to show the comparisons, therefore the quadratic Lyapunov function is chosen to be used in Theorem 1 of this paper.

Remark 7: Fig. 2 clearly show that the state variables under control law (9) rapidly converge to zero with the initial conditions $x=\begin{bmatrix}100 & 100\end{bmatrix}^T$. In addition, with the proposed method, the state variables converge faster than that in [37, 43]. Table 4 is given below to show the comparisons.

6. CONCLUSION

The stabilization problem of observer-based controller design for a class of UPVs T-S fuzzy system has been addressed. The terms caused by the unknown premise variables in observer error equations are restricted by the

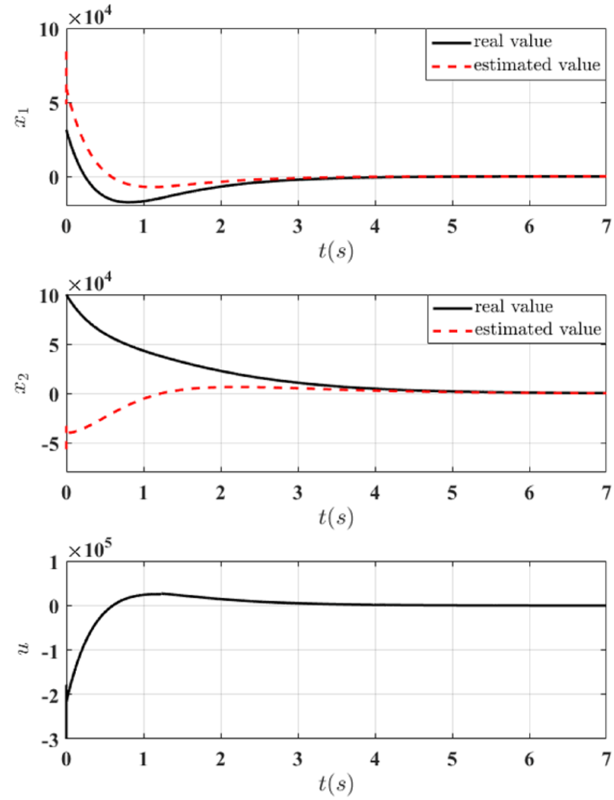

Fig. 1. Observer and controller performance and control input u with $x_{10}=3.1496\times10^4$, $x_{20}=1\times10^5$.

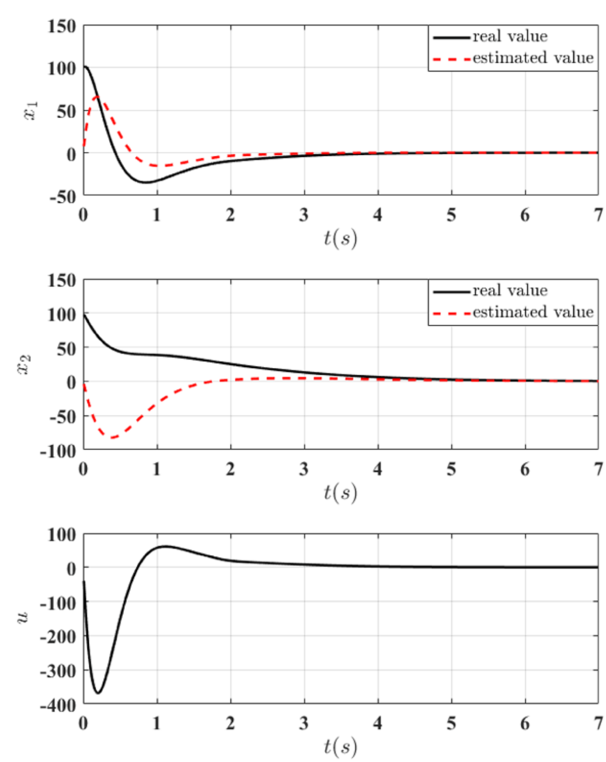

Fig. 2. Observer and controller performance and control input u with $x_{10}=100$, $x_{20}=100$.

Lipschitz conditions. Then a new method for transforming the BMI conditions into LMI form is proposed, the observer gain matrices mainly contributes to overcome the unknown premise variables caused uncertainties, and it has been proved that the observer-controller error system is asymptotically stable. A simulation example has been used to demonstrate the effectiveness of the designed method, the observer gain matrices can be calculated according to the specific control task or system properties, and compared with the design methods in the existing literature, the system conservatism can be reduced.

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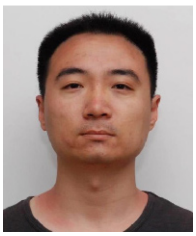

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