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Consensus of Second-order Multi-agent Systems with Directed Networks Using Relative Position Measurements Only

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Abstract: This brief paper studies the consensus problem of second-order multi-agent systems when the agents' velocity measurements are unavailable. Firstly, two simple consensus protocols which do not need velocity measurements of the agents are derived to guarantee that the multi-agent systems achieve consensus in directed networks. Secondly, a key constant which is determined by the complex eigenvalue of the nonsymmetric Laplacian matrix and an explicit expression of the consensus state are respectively developed based on matrix theory. The obtained results show that all the agents can reach consensus if the feedback parameter is bigger than the key constant. Thirdly, the theoretical analysis shows that the followers can track the position and velocity of the leader provided that the leader has a directed path to all other followers and the feedback parameter is bigger enough. Finally, numerical simulations are given to illustrate the effectiveness of the proposed protocols.

Keywords: Consensus, directed networks, multi-agent system, velocity-free.

1. INTRODUCTION

As one of the most typical collective behaviors of multi-agent systems, the consensus of multi-agent systems has recently received considerable research attention due to its extensive applications in the cooperative control of autonomous mobile robots, the design of distributed sensor networks, unmanned aerial vehicles (UAVs), congestion control in communication networks, and in many other areas [1–9]. For a multi-agent system, leaderless consensus means that each agent updates its state based on the local information of its neighbors such that the agents eventually reach an agreement on a common value, while leader-following consensus means that there exists a virtual leader which specifies an objective for all the other agents to follow.

It is noted that the velocity measurements of the agents are usually unavailable in practical applications [10–21]. Without velocity measurements, the protocols proposed in [9] are no longer applicable, and it is thus necessary to design new protocols for a multi-agent system to reach consensus. By applying the linear matrix inequality technique and the common Lyapunov function approach, sufficient conditions for velocity-free consensus of multiple second-order agents were given in [10]. By introducing an auxiliary system, the consensus protocol and convergence conditions for multi-agent systems without velocity measurements were derived in [11]. A measurement output feedback controller with a dynamic observer for the unmeasurable states was proposed in [12], and the velocity consensus was achieved when only the position information was available for the feedback based on the proposed controller. By considering measurement noises, a measurement-based distributed protocol was designed and the convergence properties of the protocol were analyzed by using the stochastic analysis and algebraic graph theory in [13]. In [14], the sufficient conditions that characterized the relationship of formability, connectivity topology, formation properties and agent dynamics were obtained by using the matrix analysis and algebraic graph theory. Various control algorithms have also been proposed for velocity-free consensus of multi-agent systems [22–25], but the control methods applying only to the undirected networks.

In order to optimize convergence time, finite-time control techniques were presented in [26–28]. By computing the value of the Lyapunov function at the initial point, the finite settling time was theoretically estimated for the second-order multi-agent systems [27]. A novel finite-time discontinuous observer was proposed for the

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leader-following multi-agent systems [28]. Furthermore, the method given in [28] has also been used to analyze the consensus of networked mobile systems and linear multi-agent systems [29–34], networked Euler-Lagrange systems [35–38], and so on.

Most of the above works investigated the consensus problems of multi-agent systems, or networked mechanical systems in undirected networks. There is little research in the velocity-free consensus protocols and convergence conditions for multi-agent systems in directed networks. It is a common phenomenon that the information exchange topology between agents is directed in practical applications. For example, some agents may have transceivers, while the other less capable team members only have receivers in heterogeneous teams. It was shown that the second-order multi-agent systems might not achieve consensus even if the directed graph has a directed spanning tree [38]. Although some sufficient conditions for velocity-free consensus of multi-agent systems have been derived in undirected networks [11, 12], it is still necessary to derive the corresponding conditions in directed networks.

1.1. Motivations of this studies

Motivated by the recent work [11] and [12], we study the velocity-free consensus for multi-agent systems in directed networks. Firstly, in some specific environments, agents' velocity measurements are unavailable. For example, in order to save cost, space and weight, some agents are not equipped with velocity sensors or other agents may miss velocity measurements [39]. In other cases, the velocity may be unmeasurable or may not be precisely measured because of technology limitations or environment disturbances. Secondly, it is common that agents don't receive velocity measurements in directed networks. The lack of velocity measurements and the directed network topology render the design of consensus protocol and the analysis of consensus problems challenging. Hence, this paper focuses on the stationary consensus problem of multi-agent systems in directed networks and presents the protocol which only using agents' relative position measurements.

1.2. Contributions of this paper

Based on network control theory, the velocity-free consensus protocols are presented for the multi-agent systems with directed networks. A key constant depending on the complex eigenvalues of the nonsymmetric Laplacian matrix, and the exact expression of the final consensus state, is analytically developed based on matrix theory. The key constant indicates that the real and imaginary parts of all eigenvalue of Laplacian matrix play an important role in achieving consensus. The convergence conditions for multi-agent systems to reach consensus in directed networks are also derived.

It is worth pointing out that different from [9] and [11], here the network topology is directed, and thus, the method used in [11] is not suitable to the consensus analysis. Also, unlike the existing results in [39], where partial agents' velocity measurements are needed, the obtained protocols do not need any velocity information.

1.3. Notations and organization

Some standard mathematical notations will be used in this paper. Let R define a set of real numbers; R^m be the m-dimensional real vector; $R^{n\times n}$ be the set of $n\times n$ real matrices; $\bar{n}=\{1,2,\cdots,n\}$ be an index set; and $\|x\|$ be the Euclidean norm of vector x. Let I_n and 0_n denote the identity and zero matrix, respectively.

The rest of the paper is organized as follows: Section 2 presents the model of multi-agent systems and some fundamental math knowledge. Section 3 gives the main consensus results of multi-agent systems including the leadless and leader-follower consensus in directed networks. Furthermore, two consensus protocols which need relative position measurements only, and the convergence conditions for multi-agents systems to achieve consensus, are respectively presented. Two simulation examples are given to show the effectiveness of the proposed consensus protocols in Section 4. Finally, Section 5 presents a brief conclusion to this paper.

2. PROBLEM FORMULATIONS

The dynamics of multi-agents systems composed of n second-order agents is described by

$$\begin{cases}\dot{x}_i=v_i,\\ \dot{v}_i=u_i,\end{cases}\quad i\in\bar{n},\tag{1}$$

where $x_i=(x_{i1},\cdots,x_{im})^\top\in R^m$, $v_i=(v_{i1},\cdots,v_{im})^\top\in R^m$ are the position and velocity states of the agent i, respectively. $u_i\in R^m$ is a control input for the agent i. Moreover, x_{il} and v_{il}, $l=1,2,\cdots,m$, denote the l-th component of the vector x_i and v_i, respectively. The main aim of the paper is to determine u_i for all agents in (1) to achieve stationary consensus, which means $\lim_{t\to\infty}\|x_i-x_j\|=0$, $i,j\in\bar{n}$, and the velocity of all agents in (1) converge to 0^m, namely, $\lim_{t\to\infty}\|v_i\|=0$, $i\in\bar{n}$.

In the analysis of the convergence conditions for multi-agent systems, the directed graph of order n, $\mathcal{G}=\{\mathcal{V},\mathcal{E},C\}$ with the node set $\mathcal{V}=\{1,2,\cdots,n\}$ will be used to model the interaction among n agents. A set of directed edges $\mathcal{E}=\{(i,j)\in\mathcal{V}\times\mathcal{V}|i\sim j\}$ containing the pairs of nodes represents communication connections. A weighted adjacency matrix $C=(c_{ij})\in R^{n\times n}$ is defined such that $c_{ij}>0$ if $e_{ij}\in\mathcal{E}$, which means agent i can receive information from agent j, while $c_{ij}=0$ if $e_{ij}\notin\mathcal{E}$, which means there is no information exchange from agents i to j. Moreover,

c_{ii} is defined as $c_{ii} = -\sum_{j=1, j\neq i}^{n} c_{ij}$, $i = 1, 2, \cdots, n$. A directed path of $\mathcal{G}$ is a sequence of edges of the form (i_1, i_2), (i_2, i_3), $\cdots$, where $i_j \in \mathcal{V}$. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all the other nodes.

The following lemmas will be needed.

Lemma 1 [40]**:** Let $\mathcal{L}$ be the nonsymmetric Laplacian matrix associated with $\mathcal{G}$. Then $\mathcal{L}$ has a simple zero eigenvalue and all the other eigenvalues have positive real parts if and only if $\mathcal{G}$ has a directed spanning tree.

Lemma 2 [41]**:** Consider the cubic polynomial $F(\lambda) = \lambda^3 + c\lambda^2 + (a_1 + b_1\mathbf{i})\lambda + (a_2 + b_2\mathbf{i})$, where $\mathbf{i} = \sqrt{-1}$, and a_1, a_2, c, b_1 and b_2 are real. Then all three roots of $F(\lambda) = 0$ have negative real parts if and only if $c > 0$, $c^2a_1 - ca_2 - b_1^2 > 0$ and $a_2(a_2 - ca_1)^2 - b_1^2(a_1a_2 + b_1b_2) + cb_1b_2(ca_1 - 3a_2) - c^3b_2^2 > 0$.

3. CONSENSUS OF MULTI-AGENT SYSTEMS

In this section, the leaderless and leader-follower velocity-free consensus of multi-agent systems in directed networks will be investigated, respectively. Two consensus protocols, which only need relative position measurements of the agents, will be given.

3.1. Leaderless velocity-free consensus of multi-agent systems

The control inputs and the convergence conditions for system (1) to achieve consensus are respectively derived based on matrix theory.

The protocol u_i in (1) is designed as

$$\begin{cases} u_i = \sum_{j=1}^{n} c_{ij}(x_j - x_i) + a(y_i - x_i), \\ \dot{y}_i = x_i - y_i, \end{cases} \quad i \in \bar{n}, \tag{2}$$

where $a > 0$ is the feedback parameter, $y_i = (y_{i1}, \cdots, y_{im})^\top \in R^m$ is the auxiliary system. Let $\xi = (\xi_1, \cdots, \xi_n)^\top \in R^n$ be the left eigenvectors of $C = (c_{ij}) \in R^{n\times n}$ associated with eigenvalue zero.

Theorem 1: Consider system (1) with the protocol (2) and assume that $r = \alpha_i + \beta_i\mathbf{i}$, where $\mathbf{i} = \sqrt{-1}$, is an eigenvalue of C. Suppose that the following conditions hold.

1) The graph $\mathcal{G}$ has a directed spanning tree.

2) Feedback parameter a satisfies

$$a > \theta = \frac{(\underline{\alpha}+1)\beta^2 - |\beta(\underline{\alpha}-1)|\sqrt{\beta^2 - 4\underline{\alpha}}}{2\bar{\alpha}},$$

where $\beta = \max_{i\in\bar{n}}\{|\beta_i|\}$, $\bar{\alpha} = \max_{i\in\bar{n}}\{\alpha_i\}$ and $\underline{\alpha} = \min_{i\in\bar{n}}\{\alpha_i\}$. Then, $x_i \to \sum_{i=1}^{n} \xi_i\big(v_i(0) + ay_i(0)\big)$ and $v_i \to 0$ as $t \to \infty$, where $\sum_{i=1}^{n} \xi_i = 1/a$, $i \in \bar{n}$.

Proof: The process of the proof is divided into the following three steps. In step 1, the multi-agent system will be written in matrix form, and the left and right eigenvector of system matrix, which will be defined later, associated with eigenvalue zero will be given, respectively. In step 2, the conditions that all nonzero eigenvalues of system matrix have negative real parts are derived. Finally, an explicit expression of the consensus state is developed based on matrix theory.

Step 1: Let $X = (x_1^\top, \cdots, x_n^\top)^\top \in R^{nm}$, $V = (v_1^\top, \cdots, v_n^\top)^\top \in R^{nm}$, $Y = (y_1^\top, \cdots, y_n^\top)^\top \in R^{nm}$ and $H = C - aI$. Then, system (1) with protocol (2) can be written as

$$\begin{bmatrix} \dot{X} \\ \dot{V} \\ \dot{Y} \end{bmatrix} = D \otimes I_m \begin{bmatrix} X \\ V \\ Y \end{bmatrix}, \tag{3}$$

where $D = \begin{bmatrix} 0_n & I_n & 0_n \\ H & 0_n & aI \\ I & 0_n & -I \end{bmatrix}$ is the system matrix.

Let $\eta_r = [x_r^\top, v_r^\top, y_r^\top]^\top$ be the right eigenvector of matrix D associated with eigenvalue zero. Then,

$$\begin{bmatrix} 0_n & I_n & 0_n \\ H & 0_n & aI \\ I & 0_n & -I \end{bmatrix} \begin{bmatrix} x_r \\ v_r \\ y_r \end{bmatrix} = \begin{bmatrix} 0^{n\times 1} \\ 0^{n\times 1} \\ 0^{n\times 1} \end{bmatrix}. \tag{4}$$

It follows from (4) that

$$\begin{cases} v_r = 0^{n\times 1}, \\ Cx_r = 0^{n\times 1}, \\ y_r = x_r. \end{cases} \tag{5}$$

It can be obtained from (5) that $\eta_r = [1, \cdots, 1, 0, \cdots, 0, 1, \cdots, 1]^\top$. Similarly, let $\eta_l = [x_l^\top, v_l^\top, y_l^\top]^\top$ be the left eigenvector of matrix D associated with eigenvalue zero. Then,

$$[x_l^\top, v_l^\top, y_l^\top] \begin{bmatrix} 0_n & I_n & 0_n \\ H & 0_n & aI \\ I & 0_n & -I \end{bmatrix} = [0^{1\times n}, 0^{1\times n}, 0^{1\times n}]. \tag{6}$$

Expansion of (6) results in

$$\begin{cases} v_l^\top C = 0^{1\times n}, \\ x_l^\top = 0^{1\times n}, \\ y_l^\top = av_l^\top. \end{cases} \tag{7}$$

Analysis of the above equation (7) indicates that $\eta_l = [0, \cdots, 0, \xi_1, \cdots, \xi_n, a\xi_1, \cdots, a\xi_n]^\top$.

Step 2: Let λ, $r = \alpha + \beta\,\mathbf{i}$ be an eigenvalue of D and C, respectively. Then,

$$\begin{bmatrix} 0_n & I_n & 0_n \\ H & 0_n & aI \\ I & 0_n & -I \end{bmatrix} \begin{bmatrix} x \\ v \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ v \\ y \end{bmatrix}. \tag{8}$$

It follows from (8) that

$$\begin{cases} v = \lambda x, \\ Cx = ax - ay + \lambda v, \\ x = y + \lambda y. \end{cases} \tag{9}$$

Then, it gives that $\lambda^3 + \lambda^2 + (a-r)\lambda - r = 0$ from (9).

If $r = 0$, we obtain that $\lambda_1 = 0$, $\lambda_2 = \frac{-1-\sqrt{1-4a}}{2}$ and $\lambda_3 = \frac{-1+\sqrt{1-4a}}{2}$.

If $r = \alpha + \beta\,\mathbf{i}$, then all nonzero eigenvalues of matrix D have negative real parts when $a > \frac{(\alpha+1)\beta^2 - |\beta(\alpha-1)|\sqrt{\beta^2-4\alpha}}{2\alpha}$ from Lemma 2.

Step 3: From the Jordan decomposition theorem [40], the matrix D can be written in Jordan canonical form as $D = PJP^{-1} = P\begin{bmatrix} 0 & 0_{1\times(3n-1)} \\ 0_{(3n-1)\times 1} & \hat{J} \end{bmatrix}P^{-1} = [\mu_1, \cdots, \mu_{3n}]$
$\begin{bmatrix} 0 & 0_{1\times(3n-1)} \\ 0_{(3n-1)\times 1} & \hat{J} \end{bmatrix}\begin{bmatrix} \phi_1^\top \\ \vdots \\ \phi_{3n}^\top \end{bmatrix}$, where $\mu_i \in R^{3n}$ and $\phi_i \in R^{3n}$, $i = 1, 2, \cdots, 3n$, can be chosen to be the right and left eigenvectors or generalized eigenvectors of D, respectively, and $\hat{J}$ is the Jordan upper diagonal block matrix corresponding to eigenvalues λ_i, $i = 2, 3, \cdots, 3n$. Given that $P^{-1}P = I$, μ_i and ϕ_i must satisfy that $\phi_i^\top \mu_i = 1$ and $\phi_i^\top \mu_k = 0$, where $i \neq k$. It is easy to note that $\mu_1 = \eta_r$, $\phi_1 = \eta_l$.

The solution of equation (3) is given by

$$\begin{bmatrix} X \\ V \\ Y \end{bmatrix} = (Pe^{Jt}P^{-1}) \otimes I_m \begin{bmatrix} X(0) \\ V(0) \\ Y(0) \end{bmatrix}.$$

Using the result in step 2 that all nonzero eigenvalues of D have negative real parts when $a > \frac{(\alpha+1)\beta^2 - |\beta(\alpha-1)|\sqrt{\beta^2-4\alpha}}{2\alpha}$, we have that $\lim_{t\to\infty} e^{\hat{J}t} = 0_{(3n-1)\times(3n-1)}$ and

$$\lim_{t\to\infty}\begin{bmatrix} X \\ V \\ Y \end{bmatrix} = [\eta_r, \cdots, \mu_{3n}]M \begin{bmatrix} \eta_l^\top \\ \vdots \\ \phi_{3n}^\top \end{bmatrix} \otimes I_m \begin{bmatrix} x_1(0) \\ \vdots \\ y_n(0) \end{bmatrix}.$$

where $M = \begin{bmatrix} 1 & 0_{1\times(3n-1)} \\ 0_{(3n-1)\times 1} & 0_{(3n-1)\times(3n-1)} \end{bmatrix}$.

So, it gives that

$$x_i \to \sum_{i=1}^{n} \xi_i \big(v_i(0) + a y_i(0)\big), \quad v_i \to 0, \quad i \in \bar{n},$$

as $t \to \infty$, $x_i(0)$, $v_i(0)$ and $y_i(0)$ are the initial states. This completes the proof. □

Remark 1: Different from previous works, Theorem 1 presents the key constant θ, which is determined by the real and imaginary parts of all complex eigenvalue of the nonsymmetric Laplacian matrix and affects the velocity-free consensus process of multi-agent systems. Moreover, the final consensus state where all agents converge is derived.

Theorem 1 shows that system (1) with protocol (2) can achieve consensus even if all agents do not measure velocity measurements.

3.2. Leader-follower velocity-free consensus of multi-agent systems

In this section, the case when the network of n agents is regulated to a leader's state is considered. The velocity of the leader which is labeled as 0 is unavailable and satisfies the following equation

$$\begin{cases} \dot{x}_0 = v_0, \\ \dot{v}_0 = -b(x_0 - \psi_0), \end{cases} \tag{10}$$

where b is strictly positive scalar gain. The vectors ψ_0 is given by $\dot{\psi}_0 = x_0 - \psi_0 \in R^m$. In order to guarantee that all agents in (1) converge to the leader's state, the protocol u_i in (1) is designed as

$$\begin{cases} u_i = \sum_{j=1}^{n} c_{ij}(x_j - x_i) + b(\psi_i - x_i) + c_{i0}(x_0 - x_i), \\ \dot{\psi}_i = x_i - \psi_i, \quad i \in \bar{n}, \end{cases} \tag{11}$$

where ψ_i is the auxiliary system. Here, c_{i0} is the communication connection between the agent i and the leader, and $c_{i0} > 0$ denotes that agent i knows the position x_0 directly; otherwise, $c_{i0} = 0$.

Corollary 1: Consider system (1) with the protocol (11) and assume that $\alpha_i^* + \beta_i^*\,\mathbf{i}$ is an eigenvalue of $\bar{C}$. Suppose that the following conditions hold.

1) The leader has a directed path to all the other agents.
2) Feedback parameter b satisfies

$$b > \eta = \frac{(\underline{\alpha^*}+1)(\beta^*)^2 - |\beta^*(\underline{\alpha^*}-1)|\sqrt{(\beta^*)^2 - 4\underline{\alpha^*}}}{2\bar{\alpha^*}},$$

where $\beta^* = \max_{i\in\bar{n}}\{|\beta_i^*|\}$, $\bar{\alpha^*} = \max_{i\in\bar{n}}\{\alpha_i^*\}$, $\underline{\alpha^*} = \min_{i\in\bar{n}}\{\alpha_i^*$.

Then, $x_i \to x_0$, $v_i \to v_0$ as $t \to \infty$, $i \in \bar{n}$.

Proof: Let $e_{i1} = x_i - x_0$, $e_{i2} = v_i - v_0$, $z_i = \psi_i - \psi_0$, $E_1 = (e_{11}^\top, \cdots, e_{n1}^\top)^\top \in R^{nm}$, $E_2 = (e_{12}^\top, \cdots, e_{n2}^\top)^\top \in R^{nm}$ and $Z = (z_1^\top, \cdots, z_n^\top)^\top \in R^{nm}$. Then, system (1) with protocol (11) can be written in matrix form as

$$\begin{bmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} 0_n & I_n & 0_n \\ \bar{H} & 0_n & bI \\ I & 0_n & -I \end{bmatrix} \otimes I_m \begin{bmatrix} E_1 \\ E_2 \\ Z \end{bmatrix} = \bar{D} \otimes I_m \begin{bmatrix} E_1 \\ E_2 \\ Z \end{bmatrix}.$$

Matrix $\bar{H} = \bar{C} - bI$, and $\bar{C} = (\bar{c}_{ij}) \in R^{n\times n}$ is defined as

$$\bar{c}_{ij} = \begin{cases} c_{ij}, & i \neq j, \\ -\sum\limits_{j=1, j\neq i}^{n} c_{ij} - c_{i0}, & i = j. \end{cases}$$

Let $r = \alpha^* + \beta^* \mathbf{i}$ be an eigenvalue of $\bar{C}$. Then, all eigenvalues of $\bar{C}$ have negative real parts when the leader has a directed path to all the other agents. It can be obtained from Theorem 1 that all eigenvalues of $\bar{D}$ have negative real parts if $b > \frac{(\alpha^*+1)(\beta^*)^2 - |\beta^*(\alpha^*-1)|\sqrt{(\beta^*)^2 - 4\alpha^*}}{2\alpha^*}$. Thus, by using a similar analysis presented in Theorem 1, we conclude that $\lim\limits_{t\to\infty} \|E_1\| = 0$ and $\lim\limits_{t\to\infty} \|E_2\| = 0$. So, we have $x_i \to x_0$, $v_i \to v_0$ as $t \to \infty$. This completes the proof. □

Remark 2: Corollary 1 indicates that all follower agents can converge to the leader's state under the condition that the feedback parameter b is bigger enough.

By the calculation, we get that $x_i \to \psi_0(0) + \frac{v_0(0)}{b}$, $v_i \to 0$, as $t \to \infty$, where $v_0(0)$ and $\psi(0)$ are the initial states of v_0 and ψ, respectively.

4. NUMERICAL SIMULATIONS

In this section, numerical simulations on a system consisting of six second-order agents are performed. The consensus process of system (1) with protocol (2) and (11) will be given, respectively, to show the effectiveness of the proposed protocols.

The initial positions and velocities of six agents are chosen as $(x_1(0),\ v_1(0),\ y_1(0)) = (-0.5,\ -0.3,\ -0.2)$, $(x_2(0),\ v_2(0),\ y_2(0)) = (0.1,\ -0.6,\ 0.3)$, $(x_3(0),\ v_3(0),\ y_3(0)) = (0.4, -0.6, 0.5)$, $(x_4(0),\ v_4(0),\ y_4(0)) = (-0.8,\ 0.9,\ 0.4)$, $(x_5(0),\ v_5(0),\ y_5(0)) = (-0.5,\ -0.1,\ 0.8)$ and $(x_6(0), v_6(0), y_6(0)) = (0.7, 0.4, 0.9)$. Let the parameters $c_{ij} = 1$, or 0, depending on network topology. The units of the position and velocity are m and m/s, respectively.

4.1. Leaderless consensus

Consider system (1) composed of $n = 6$ agents with protocol (2) and network topology shown in Fig. 1.

Example 1: It is calculated that the eigenvalues of matrix C associated with network topology shown in Fig. 1 are $\lambda_1 = 0$, $\lambda_{2,3} = -1$, $\lambda_4 = -3.32$, $\lambda_{5,6} = -1.34 \pm 0.56\mathbf{i}$, the left eigenvector of matrix C associated eigenvalue zero is $\frac{1}{7a}(2, 2, 1, 1, 1, 0)^\top$. By applying the results of Theorem 1, the positions of the agents in (1) can reach the state $\sum\limits_{i=1}^{6} \xi_i \big(v_i(0) + a y_i(0)\big)$ when $a > 4.85$.

Figs. 3-4 show the change process of positions and velocities of the agents in (1) with $a = 1.5$ and $a = 5$, respectively. Noted that consensus is achieved after a long period of time in Fig. 3. It is observed in Fig. 4 that the positions and velocities of six agents achieve consensus within a short period of time. It can be concluded from Figs. 3-4 that the value of parameter a heavily affects the velocity-free consensus of multi-agent systems.

Fig. 5 shows the evolution of the agents in system (1)

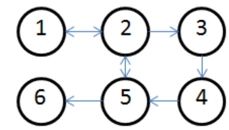

Fig. 1. Network of six interacting agents.

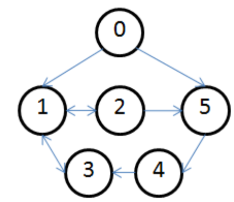

Fig. 2. Network of five follower agents and one leader.

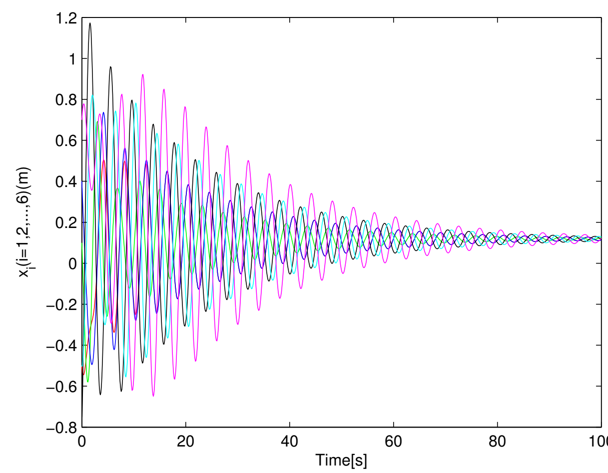

Fig. 3. Positions converge to 0.12 with $a = 1.5$.

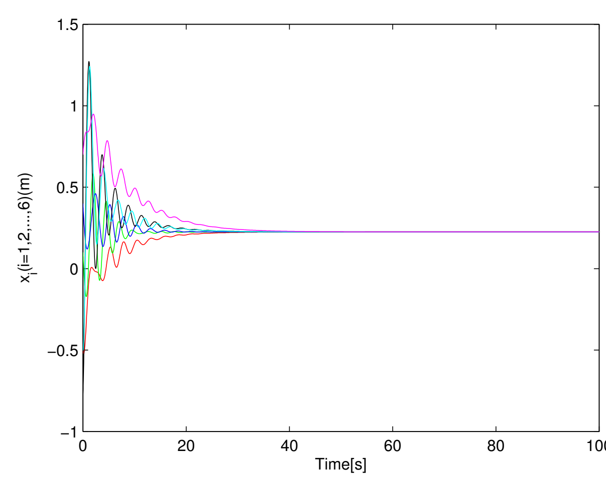

Fig. 4. Positions converge to 0.23 with $a = 5$.

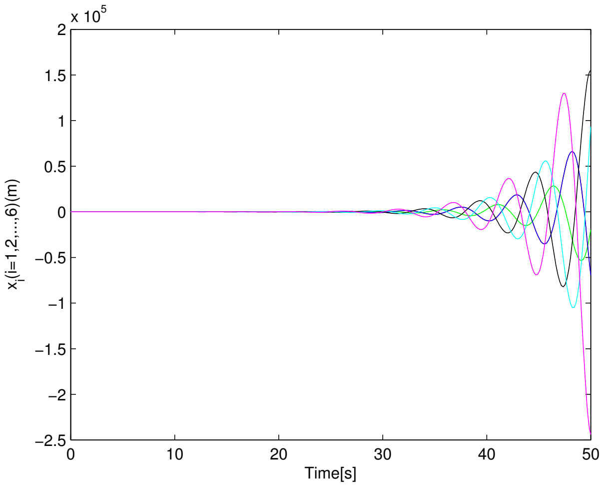

Fig. 5. Failure to reach consensus without velocity measurements.

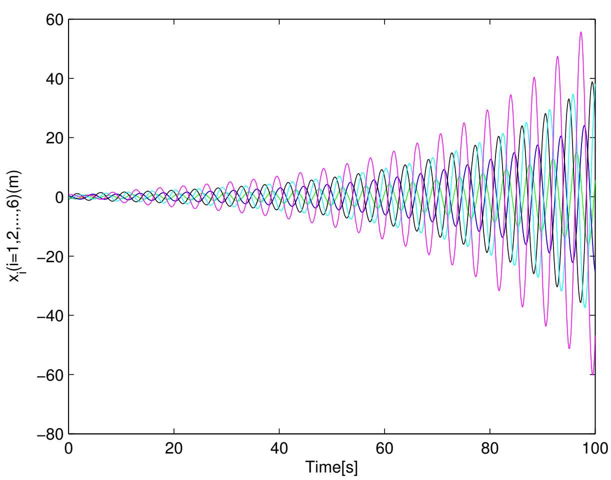

Fig. 6. Unstable consensus with a=1.

with the protocol presented in [9]. Noted that in the absence of velocity measurements, the agents in system (1) cannot achieve consensus. Fig. 6 illustrates that the agents in (1) cannot achieve consensus due to a smaller $a = 1$. The trajectories of all agents have tendency to go infinity as time goes.

4.2. Leader-follower consensus

Consider system (1) composed of five follower agents and one leader with protocol (11) and network topology shown in Fig. 2. The protocol used in this simulation is

$$\begin{cases} u_i = \sum_{j=1}^{5} c_{ij}(x_j - x_i) + b(\psi_i - x_i) + c_{i0}(x_0 - x_i), \\ \dot{\psi}_i = x_i - \psi_i, \ \ i = 1, 2, \cdots, 5, \end{cases}$$

where $c_{10} = 2$, $c_{50} = 2$, $c_{i0} = 0$, $i = 2, 3, 4$, $x_0(0) = 0.5$, $v_0(0) = -0.5$ and $\psi_0(0) = -0.2$.

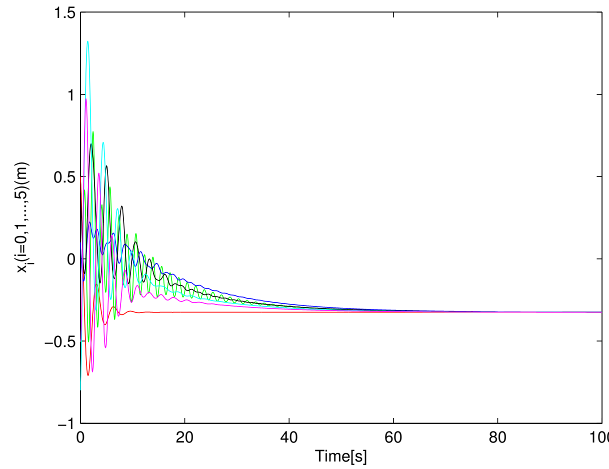

Fig. 7. Positions converge to -0.325.

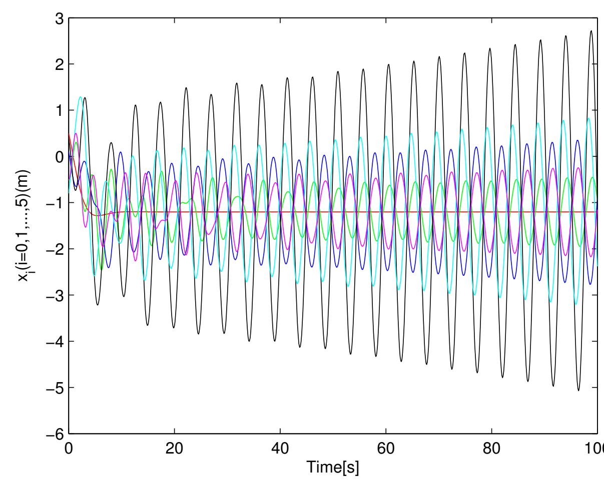

Fig. 8. Unstable leader-follower consensus with b=0.5.

Example 2: It is calculated that the eigenvalues of $\bar{C}$ associated with network topology shown in Fig. 2 are $\lambda_1 = -0.47$, $\lambda_2 = -3.1$, $\lambda_3 = -4.64$, $\lambda_{4,5} = -1.39 \pm 0.305\mathbf{i}$. From the result of Corollary 1, all positions and velocities of the agents in system (1) can converge to the leader's position when $b > 14.45$.

Fig. 7 shows the process of positions and velocities of all agents with $b = 4$, which indicates that the agents can achieve the leader's state -0.325 and the developed protocol (11) is effective. Fig. 7 also indicates that the conditions in Theorem 1 and Corollary 1 are sufficient. Fig. 8 illustrates that the agents in (1) cannot achieve the leader's state since the parameter $b = 0.5$ is small.

It can be concluded from Figs. 3-8 that the theoretical results are in good agreement with numerical simulations. Therefore, the feasibility of Theorem 1 and the effective of the developed protocols (2) and (11) are verified.

5. CONCLUSIONS

The consensus for second-order multi-agent systems, in which all agents do not obtain their velocity measurements, has been studied in directed networks. Two simple consensus protocols for multi-agent systems to reach consensus were derived. A key constant which is determined by the eigenvalue of the Laplacian matrix was analytically developed. Numerical simulations were used to illustrate the theoretical results. In future work, we will focus on the consensus problems of heterogeneous multi-agent systems without velocity measurements.

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