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Stability Analysis and Dynamic Output Feedback Control for Nonlinear T-S Fuzzy System with Multiple Subsystems and Normalized Membership Functions

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Abstract: This paper addresses the stability analysis and dynamic output-feedback control problems for a class of nonlinear Takagi-Sugeno (T-S) fuzzy systems with multiple subsystems and normalized membership functions. First, the switching control law of the membership function is proposed based on the membership function for the nonlinear T-S fuzzy subsystems. Secondly, the relaxation parameter is introduced into this switching control law to guarantee a minimal dwell time between two consecutive switching. Then, based on the proposed switching control law of the membership function and relaxation parameter, the dynamic output feedback controller with the estimate algorithm is designed to estimate the attraction domain. By introducing the new switched Lyapunov-Krasovskii functional, it can be seen that the solutions of the resultant closed-loop system converge to an adjustable bounded region. Compared with the previous works, the developed controller in this paper is flexible and smooth, which only uses the system output. And the results are further extended to the mobile robot case and the chemical process case. Finally, two simulation examples are performed to show the effectiveness of the theoretical results.

Keywords: Dynamic output-feedback, Lyapunov-Krasovskii functional, multiple subsystems, relaxation parameter, switching control law, T-S fuzzy system.

1. INTRODUCTION

The nonlinear control problem is a common phenomenon for many industrial process systems [1]. The stability analysis and intelligent control for the nonlinear dynamic systems have attracted considerable attentions [2]. The presence of such nonlinear uncertainties in the nonlinear systems may induce severe deterioration of the system performances [3, 4]. Many controller design strategies were proposed to deal with the nonlinear uncertainties and time-delays cases, see [5–7] and the references therein. In practice, it is generally known that the fuzzy control theory can be employed to control a class of nonlinear uncertain systems, which has become an important methodology for the nonlinear system design [8, 9].

On the other hand, it is well known that the fuzzy control theory provides a powerful method to solve the control design issues for the nonlinear systems [10]. The most advantage of T-S fuzzy control is that the dynamics of the concerned nonlinear system can be achieved by the smooth combination of linear system models [11]. Recently, a lot of important investigations on the class of T-S fuzzy systems in both the discrete-time and continuous-time cases have been discussed in the literatures based on the linear matrix inequalities (LMIs) methods. In [12], the intelligent fuzzy digital redesign approach was proposed for the fuzzy control systems based on state-matching error cost function. A new fuzzy filtering control scheme was proposed for the nonlinear system to approximate the local dynamics of the system in a certain region determined by a set of T-S rules [13]. Based on the fuzzy back-stepping dynamic surface control theory, an adaptive tracking controller was designed for the nonlinear dynamic MIMO system [14]. In [15], an adaptive fuzzy-decentralized robust output-feedback control strategy was proposed for a class of large-scale strict-feedback nonlinear systems with unmeasured states. Then, a new adaptive T-S fuzzy method was developed in [16] to improve the haptic feedback fidelity of the nonlinear affine system. It should be point out that in order to reduce the conservatism, a switched controller was proposed for the fuzzy systems [17, 18]. In addition, based on the values of

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membership functions, a new switched parallel distributed compensation controller was proposed in [19]. Recently, some adaptive control strategies were proposed for the switched nonlinear systems via the LMIs design technique [10, 15, 18]. In [10], the robust control approach based on the LMIs was proposed for a class of fuzzy dynamic systems with one-sided control constraint, without considering the switched Lyapunov-Krasovskii functional technique. In [15], the adaptive decentralized output feedback control approach was proposed for a class of stochastic fuzzy systems with nonlinear uncertainties, without considering the dynamic output feedback and the attraction domain issues. The membership functions are assumed to be non-differentiable and the state-feedback controller was constructed for the T-S fuzzy system [18]. However, the relaxation parameter is not considered such that the dwell time between two consecutive switching may not be minimal. In addition, since the average dwell-time method was employed in [20], the switching signals are not arbitrary and need to satisfy some restrictive conditions. However, for the switching law in [17] was based on the membership functions and has no minimal dwell time, the famous Metzler matrix method [19] and average dwell time method [21] cannot be applied to the fuzzy systems with switching control. Therefore, dealing with the trade-off between a less conservative condition and increased design complexity remains an important problem in system control design.

Recently, the adaptive fuzzy control methods have been studied extensively in the nonlinear control problems and application problems, see [22–26] and the references therein. In [24], an output-based adaptive backstepping control method was developed for a class of nonlinear nonstrict-feedback systems against actuator fault. In [25], an adaptive fuzzy backstepping method was proposed for a class of single-input single-output nonlinear systems with network-induced delay and data loss. Moreover, an observer-based adaptive fuzzy backstepping control method was proposed for a class of nonlinear nonstrict-feedback stochastic systems with input saturation and prescribed performance [26]. In [27], a fuzzy-model-based static output-feedback method was proposed and the reliable distributed fuzzy controller was constructed. In order to deal with the unknown nonlinear uncertainties, some adaptive fuzzy and robust output-feedback decentralized control strategies [28–30] have been developed. Based on the above reasons, the neural network fuzzy-logic control systems and output-feedback control are often considered as universal effective controllers. In [31], the dynamic output-feedback control theory was considered for a class of nonlinear industrial system with unknown disturbances, and the network-based fuzzy design methodology was presented. Recently, the LMIs-based design conditions were employed for the system stabilization via the fuzzy and output feedback theories in [32–34]. In [35], the dynamic output feedback control method was proposed for the nonlinear networked discrete-time system with missing measurements. With the help of dynamic output-feedback control technique, two-term approximation theory was investigated in [36] for the Markovian jump systems with time-varying delay and defective mode information. However, most of the above controllers are designed based on the static output feedback control. In addition, it should be mentioned that the performance of the T-S fuzzy systems have not been considered in the literatures. Very few results employed the fuzzy switched Lyapunov-Krasovskii functional method to estimate the attraction domain, and consider with the normalized membership functions in the nonlinear T-S fuzzy dynamic output-feedback control system. Especially the dynamic output feedback technique is more flexible and the control design conditions are relaxed. Therefore, the dynamic output-feedback control problem is investigated in this paper.

In this paper, the dynamic output-feedback control problem is considered for a class of nonlinear T-S fuzzy systems with multiple subsystems and normalized membership functions. Compared with the previous works, the developed controller in this paper is effective, which only uses the system output. And the control design conditions are relaxed because of the developed switching control law of the membership function with relaxation parameter. The contributions of this paper are summarized as follows:

1) The fuzzy switching control law is proposed based on the membership function and the minimal dwell time between two consecutive switching is guaranteed effectively. The relaxation parameter is introduced into this switching control law of the membership function and the control design conditions are relaxed in the control system design.

2) The dynamic output feedback controller is designed based on the switching control law and the relaxation parameter, such that the required conditions on the considered systems are less conservative. The estimate algorithm is introduced into the dynamic output feedback controller and the attraction domain can be estimated efficiently.

3) The switched Lyapunov-Krasovskii functional is employed and two stability criterions are obtained. By introducing the new Lyapunov-Krasovskii functional, it can be seen that the solutions of the closed-loop system converge to an adjustable bounded region. The results are further extended to the mobile robot case and the chemical process case.

This paper is organized as follows: The preliminary knowledge is presented in Section 2. Two stability criterions are presented for the closed-loop system in Section 3. The dynamic output feedback controller is designed in Section 4. Two simulation examples are performed in Section 5. Finally, Section 6 concludes with a summary of the obtained results.

2. PROBLEM FORMULATION

Consider a class of nonlinear T-S fuzzy system with multiple subsystems and normalized membership functions

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} f_i(t)\left(A_i x(t) + B_i u(t)\right), \\ y(t) = \sum_{i=1}^{r} f_i(t)\left(C_{1i} x(t)\right), \quad i \in [1,\ r], \end{cases} \tag{1}$$

where $x(t) \in R^{n_x}$, $u(t) \in R^{n_u}$ and $y(t) \in R^{n_y}$ are the state variable, control input and control output of the system (1), respectively. A_i, B_i and C_i are the gain matrices with appropriate dimensions for $i \in [1, r]$, where r is the numbers of IF-THEN rules (i.e., r is the numbers of subsystems). $f_i(t)$ are the normalized membership functions with premise variables in the fuzzy inference rules, such that the following conditions hold

$$\begin{cases} f_i(t) \geq 0, \\ \sum_{i=1}^{r} f_i(t) = 1. \end{cases} \tag{2}$$

For the system (1), the time derivatives of $f_i(t)$ satisfy

$$|\dot{f}_i(t)| \leq \varphi_i, \tag{3}$$

where φ_i are the upper bound parameters for the time derivatives of the membership functions $f_i(t)$.

With (1)-(3), the system (1) is rewritten [37]

$$\begin{aligned} \dot{x}(t) &= \bar{A}_i x(t) + \bar{B}_i u(t), \\ y(t) &= \bar{C}_{1i} x(t), \end{aligned} \tag{4}$$

where

$$\begin{aligned} \bar{A}_i &= \sum_{i=1}^{r} f_i(t)(A_i), \\ \bar{B}_i &= \sum_{i=1}^{r} f_i(t)(B_i), \\ \bar{C}_{1i} &= \sum_{i=1}^{r} f_i(t)(C_{1i}). \end{aligned} \tag{5}$$

For the T-S fuzzy system (4), the set of state variables is defined [38]

$$\amalg := \left\{ x(t) \in R^{n_x} \mid |x(t)| \leq \bar{x}_i, \quad i \in [1,\ r] \right\}, \tag{6}$$

where $\bar{x}_i$ is the upper bound value of $x(t)$.

With (1) and (3), the set of normalized membership functions is defined

$$\mathcal{R} := \left\{ f_i(t) \geq 0 \ \middle|\ \left|\dot{f}_i(t)\right| \leq \varphi_i,\ i \in [1,\ r] \right\}. \tag{7}$$

The objective of this paper is to design the dynamic output feedback controller, such that the solutions of the resultant closed-loop system converge to an adjustable bounded region. And the set of adjustable bounded region is defined

$$\mathcal{D} := \left\{ x(0) \in R^{n_x} \middle| \lim_{t \to +\infty} x(t) = 0 \right\}. \tag{8}$$

Assumption 1: For the system (1), if there exists a Lyapunov-Krasovskii functional $V(x(t)) : \mathcal{U} \to R$ for any $x(t) \in \mathcal{U} - \{0\}$ such that $\dot{V}(x(t)) \leq 0$, then the following condition holds

$$\mathcal{Z}(b) := \left\{ x(t) \in R^{n_x} \,|V(x(t)) \leq b \right\}, \tag{9}$$

where $\mathcal{Z}$ is an inner estimate of the attraction domain, and b is the attraction domain parameter.

Remark 1: Different from the previous works, the switching control law of the membership function, the relaxation parameter and the switched Lyapunov-Krasovskii functional are considered for the nonlinear T-S fuzzy systems with multiple subsystems and normalized membership functions in this paper. Although a membership-function-dependent switching method was proposed in [32], only the common Lyapunov function is considered. If the switching control law was given in the control system design, the switched Lyapunov-Krasovskii functional is more suitable [33]. With the above reasons, the LMIs sufficient conditions are derived by employing the switched Lyapunov-Krasovskii functional.

Remark 2: For the problem formulated, there are three challenging issues as follows. The first one is how to design the switching control law of the membership function based on the normalized membership functions for the nonlinear T-S fuzzy subsystems. The second one is how to introduce the relaxation parameter into the switching control law to guarantee the minimal dwell time between two consecutive switching. The third one is how to design the dynamic output feedback controller with the estimate algorithm to estimate the attraction domain. If the above three issues are solved, the controller will be designed with easy implementation in the practical control systems. Since the nonlinear membership functions and number of rules would make the controller design more difficult and lead to potentially conservative results. Thus the dynamic output feedback controller is constructed in this paper based on the switching control law and relaxation parameter for the nonlinear T-S fuzzy system via the estimate algorithm, such that the stability conditions in the form of LMIs are derived and the solutions of the resultant closed-loop system converge to an adjustable bounded region.

3. STABILITY ANALYSIS

In this section, the stability conditions are presented for the nonlinear T-S fuzzy system with multiple subsystems and normalized membership functions. In practice, the upper bound parameters φ_i for the time derivatives of the membership functions $f_i(t)$ may be available/unavailable. Thus, for the available and unavailable cases, two stability conditions are presented.

Remark 3: Compared with the previous works, the switched Lyapunov-Krasovskii functional $V_{\sigma(t)}(x(t))$ is introduced for the nonlinear T-S fuzzy systems, where $\sigma(t)$ is the switching control law. $\{t_0, t_1, ..., t_k, t_{k+1}\}$ are the switching sequence for $t_0 = 0$ is the initial time, and t_k is the kth switching instant. The using details of the parameters are shown in [17].

3.1. Stability conditions when φ_i are available

For the system (1), based on the normalized membership functions $f_i(t)$, the switching control law of the membership function is designed

$$\sigma(t) = \begin{cases} \arg\max\limits_{i\in[1,\,r]} f_i(0), & t = 0, \\ \sigma(t^-), & \lambda f_j(t) \geq \max\limits_{i\in[1,\,r]} f_i(t), \\ \arg\max\limits_{i\in[1,\,r]} f_i(t), & \lambda f_j(t) < \max\limits_{i\in[1,\,r]} f_i(t), \end{cases} \tag{10}$$

where t^- is the last time instant, λ is the relaxation parameter such that $\lambda \geq 1$.

By employing (10) for (1), the system (1) is written

$$\dot{x}(t) = \sum_{i=1}^{r} f_{\sigma(t)}(t)\left(A_i x(t) + B_i u(t)\right), \tag{11}$$

where $f_{\sigma(t)}(t)$ represents the membership function with switching control law.

Remark 4: Let $u(t) \equiv 0$, then the nonlinear T-S fuzzy system (11) can be rewritten

$$\dot{x}(t) = \sum_{i=1}^{r} f_{\sigma(t)}(t)\left(A_i x(t)\right). \tag{12}$$

With (4) and (5), the system (12) can be rewritten

$$\dot{x}(t) = \mathcal{A}_{\sigma(t)} x(t), \tag{13}$$

where $\mathcal{A}_{\sigma(t)} = \sum\limits_{i=1}^{r} f_{\sigma(t)}(t)(A_i)$. Then it can be seen that the system (12) is stable if there exists a common Lyapunov-Krasovskii functional. The common solutions can be obtained by employing the standard LMIs method in [39].

Assumption 2: For any switching signal $\sigma(t)$, if there exists a scalar $\mathcal{T}_d$ such that $\mathcal{T}_d > 0$, then the following inequality holds

$$\inf\{t_{k+1} - t_k\} \geq \mathcal{T}_d. \tag{14}$$

The minimal dwell time t_m is introduced

$$t_m = -\frac{(1-\lambda)}{r\varphi(1+\lambda)}, \tag{15}$$

where $\varphi = \min\{\varphi_i\}$.

With (3) and (10), one has

$$\begin{aligned} \lambda f_{\sigma(t_k)}(t) \geq& \lambda\left(f_{\sigma(t_k)}(t_k) - (t-t_k)\varphi\right) \\ &\geq \max f_i(t_k) + (\lambda-1) f_{\sigma(t_k)}(t_k) - \lambda(t-t_k)\varphi \\ &\geq \max f_i(t_k) + \varphi(1+\lambda)\left(\lambda - 1/r\varphi(\lambda+1)\right) \\ &\quad - \lambda(t-t_k)\varphi \\ &\geq \max f_i(t_k) + \varphi(1+\lambda)(t-t_k) - \lambda(t-t_k)\varphi \\ &\geq \max f_i(t_k) + (t-t_k)\varphi \\ &\geq \max f_i(t), \;\; t_m \geq t - t_k. \end{aligned} \tag{16}$$

With (4) and (16), one has

$$\inf\{t_{k+1} - t_k\} \geq \mathcal{T}_d \geq t_m. \tag{17}$$

Remark 5: From (10), it can be seen that if the relaxation parameter $\lambda = 1$, the switching control law (10) is reduced to the existing switching control law in [17,18,40]. Obviously, $\lambda > 1$ will ensure that $\mathcal{T}_d \geq t_m > 0$ which means that the switching control law (10) has a minimal dwell-time. As shown in [33], the switched control technique is a more effective way to deal with the dwell time switching. Therefore, the switching control law technique is adopted in this paper.

Theorem 1: For the given positive scalars $\lambda > 1$, $\mathcal{T}_d > 0$ and $K > 0$, if there exist the matrices $\mathcal{Q}_{jl,n}$, $\mathcal{S}_{ijl}$ and $\mathcal{N}_{il,n}$ such that $\mathcal{Q}_{jl,n} = \mathcal{Q}_{jl,n}^T > 0$, $\mathcal{S}_{ijl} = \mathcal{S}_{ijl}^T > 0$ and $\mathcal{N}_{il,n} = \mathcal{N}_{il,n}^T > 0$, then the following inequalities hold

$$\begin{cases} \mathcal{Q}_{jl,0} - \mathcal{Q}_{jq,K} < 0, & l \neq q, \\ \mathcal{Q}_{jl,n} - \mathcal{N}_{il,n} > 0, & \\ \mathcal{G}_{ijl,mm} + \mathcal{G}_{jil,mm} < 0, & i \leq j, \\ \mathcal{G}_{ijl,(m+1)m} + \mathcal{G}_{jil,(m+1)m} < 0, & i \leq j, \\ \mathcal{G}_{ijl,K} + \mathcal{G}_{jil,K} < 0, & i \leq j \end{cases} \tag{18}$$

with

$$\begin{cases} \mathcal{G}_{ijl,nm} = He(\mathcal{Q}_{jl,n}A_i) + \sum\limits_{q=1}^{r} \varphi_q(\mathcal{Q}_{ql,n} - \mathcal{N}_{il,n}) \\ \qquad + \left(K/\mathcal{T}_d\right)(\mathcal{Q}_{jl,m+1} - \mathcal{Q}_{jl,m}) \\ \qquad + a_{il}\lambda \sum\limits_{q=1}^{r} \mathcal{S}_{qjl} - \mathcal{S}_{ijl}, \\ \mathcal{G}_{ijl,K} = He(\mathcal{Q}_{jl,K}A_i) + \sum\limits_{q=1}^{r} \varphi_q(\mathcal{Q}_{ql,K} - \mathcal{N}_{il,K}) \\ \qquad + a_{il}\lambda \sum\limits_{q=1}^{r} \mathcal{S}_{qjl} - \mathcal{S}_{ijl}, \end{cases} \tag{19}$$

where $i,\, j,\, l,\, q \in [1,r]$, $m \in [1,K-1]$, $n \in [1,K]$ and

$$a_{ij} = \begin{cases} 0, & i \neq j, \\ 1, & i = l. \end{cases}$$

Thus, it can be seen that the solutions of the resultant closed-loop system converge to an adjustable bounded region.

Proof: Choose the switched Lyapunov-Krasovskii functional $V_{\sigma(t)}$

$$V_{\sigma(t)}(x(t)) = x^T(t)\mathcal{Q}_{f(t)\sigma(t)}(t)x(t). \tag{20}$$

With (17), one can obtain $t_{k+1} - t_k \geq \mathcal{T}_d$. Then the time interval $[t_k, t_k + \mathcal{T}_d)$ is divided into K segments, and described as $[t_k + \tau_m, t_k + \tau_{m+1})$ for $\tau_m = m\Delta T = m\mathcal{T}_D/K$. Note that

$$\mathcal{Q}_{f(t)l_k}(t_k + \tau_m) = \mathcal{Q}_{f(t)l_k,m}, \tag{21}$$

where l_k is a positive scalar representing the l_kth subsystem. It means that the l_k th subsystem is activated when $t \in [t_k, t_{k+1})$.

Based on the linear interpolation theories, one has

$$\begin{cases} \mathcal{Q}_{f(t)l_k}(t) = \mathcal{Q}_{f(t)l_k}\left(t_k + \tau_m + \alpha\left(\mathcal{T}_d/K\right)\right) \\ \qquad = (1-\alpha)\,\mathcal{Q}_{f(t)l_k,m} + \alpha\mathcal{Q}_{f(t)l_k,m+1}, \\ \qquad\qquad t_k \in [t_k + \tau_m,\ t_k + \tau_{m+1}), \\ \mathcal{Q}_{f(t)l_k}(t) = \mathcal{Q}_{f(t)l_k,K},\ \ t \in [t_k + \tau_k,\ t_{k+1}), \end{cases} \tag{22}$$

where $\alpha = \frac{(t-t_k-\tau_m)K}{\mathcal{T}_d}$.
With(21) and (22), equation (20) is rewritten

$$V_{\sigma(t)}(x(t)) = \begin{cases} x^T(t)\left((1-\alpha)\,\mathcal{Q}_{f(t)\sigma(t),m}\right. \\ \quad \left. +\alpha\mathcal{Q}_{f(t)\sigma(t),m+1}\right)x(t), \\ \qquad t \in [t_k + \tau_m,\ t_k + \tau_{m+1}), \\ x^T(t)\,\mathcal{Q}_{f(t)\sigma(t),K}x(t), \\ \qquad t \in [t_k + \tau_K,\ t_{k+1}). \end{cases} \tag{23}$$

The time derivative of (20) yields

$$\begin{aligned} \dot{V}_{\sigma(t)}(x(t)) =& 2\dot{x}^T(t)\,\mathcal{Q}_{f(t)\sigma(t)}(t)\,x(t) \\ & + x^T(t)\,\dot{\mathcal{Q}}_{f(t)\sigma(t)}(t)\,x(t) \\ =& x^T(t)\left(He\left(\mathcal{Q}_{f(t)\sigma(t)}(t)A_i\right)\right. \\ & \left. + \dot{\mathcal{Q}}_{f(t)\sigma(t)}(t)\right)x(t) \\ <& 0. \end{aligned} \tag{24}$$

Then, the proof of (24) is divided into three steps.

Step 1: (If the following inequality (27) holds, equation (24) will be holds) For (23) with $t \in [t_k + \tau_m, t_k + \tau_{m+1})$, the following equality holds

$$\begin{aligned} \dot{\mathcal{Q}}_{f(t)l_k}(t) =& \left(K/\mathcal{T}_d\right)\left(\mathcal{Q}_{f(t)l_k,m+1} - \mathcal{Q}_{f(t)l_k,m}\right) \\ & + \sum_{q=1}^{r} \dot{f}_q(t)\,\mathcal{Q}_{ql_k}(t), \\ & \qquad t_k \in [t_k + \tau_m,\ t_k + \tau_{m+1}), \end{aligned} \tag{25}$$

where

$$\mathcal{Q}_{ql_k}(t) = (1-\alpha)\mathcal{Q}_{ql_k,m} + \alpha\mathcal{Q}_{ql_k,m+1}. \tag{26}$$

With (25) and (26), one has

$$\begin{aligned} & \sum_{i=1}^{r}\sum_{j=1}^{r} f_i(t) f_j(t) \left(He\left(\mathcal{Q}_{jl_k}(t)A_i\right)\right. \\ & \quad \left. + \sum_{q=1}^{r} \dot{f}_q(t)\,\mathcal{Q}_{ql_k}(t) + \left(K/\mathcal{T}_d\right)\left(\mathcal{Q}_{jl_k,m+1} - \mathcal{Q}_{jl_k,m}\right)\right) \\ & < 0. \end{aligned} \tag{27}$$

Then, it can be seen that (24) holds.

Step 2: (If the following inequality (30) holds, the inequality (27) will hold) With (18), one has

$$\mathcal{Q}_{ql_k}(t) - \mathcal{N}_{il_k}(t) > 0. \tag{28}$$

With (2) and (28), one has

$$\begin{aligned} \sum_{q=1}^{r} \dot{f}_q(t)\,\mathcal{Q}_{ql_k}(t) =& \sum_{q=1}^{r} \dot{f}_q(t)\left(\mathcal{Q}_{ql_k}(t) + \mathcal{N}_{il_k}(t)\right) \\ \leq& \sum_{q=1}^{r} \varphi_q\left(\mathcal{Q}_{ql_k}(t) + \mathcal{N}_{il_k}(t)\right). \end{aligned} \tag{29}$$

For (27), if there exists the inequality

$$\begin{aligned} & \sum_{i=1}^{r}\sum_{j=1}^{r} f_i(t) f_j(t) \left(He\left(\mathcal{Q}_{jl_k}(t)A_i\right)\right. \\ & \quad + \sum_{q=1}^{r} \varphi_q\left(\mathcal{Q}_{ql_k}(t) - \mathcal{N}_{il_k}(t)\right) \\ & \quad \left. + \frac{K}{\mathcal{T}_d}\left(\mathcal{Q}_{jl_k,m+1} - \mathcal{Q}_{jl_k,m}\right) + a_{il_k}\lambda\sum_{q=1}^{r}\mathcal{S}_{qjl_k} - \mathcal{S}_{ijl_k}\right) \\ & = \sum_{i=1}^{r}\sum_{j=1}^{r} f_i(t) f_j(t)\left((1-\alpha)\mathcal{G}_{ijl_k,mm} + \alpha\mathcal{G}_{ijl_k,(m+1)m}\right) \\ & < 0. \end{aligned} \tag{30}$$

Then, it can be seen that inequality (27) holds.

Step 3: For (3) with $t \in [t_k + \tau_k, t_{k+1})$, and considering (22), one has

$$\dot{\mathcal{Q}}_{f(t)l_k}(t) = \sum_{q=1}^{r} \dot{f}_q(t)\,\mathcal{Q}_{ql_k,K},\ \ t \in [t_k + \tau_K,\ t_{k+1}). \tag{31}$$

Via the similar approach, (24) holds. Based on the condition $\mathcal{Q}_{jl,0} - \mathcal{Q}_{jq,K} < 0$ in (18), one has

$$V_{\sigma(t_{k+1})}(x(t_{k+1})) < V_{\sigma(t_k)}(x(t_{k+1}^-)), \tag{32}$$

where $V_{\sigma(t_{k+1})}$ is the value of the Lyapunov-Krasovskii functional at t_{k+1}. $V_{\sigma(t_k)}$ is the value of the Lyapunov-Krasovskii functional at t_{k+1}^-, and t_{k+1}^- is the last time instant before the $(k+1)$th switching. The proof of Theorem 1 is completed. □

Remark 6: The switching control law of the membership function is proposed based on the normalized membership functions $f_i(t)$ of the nonlinear T-S fuzzy system. Then the relaxation parameter λ in $a_{il}\lambda \sum_{q=1}^{r} \mathcal{S}_{qjl_k} - \mathcal{S}_{ijl_k}$ is introduced. Compared with the previous works, the developed Lyapunov-Krasovskii functional in this paper is switched, and the control design conditions are relaxed because of the developed relaxation parameter.

3.2. Stability conditions when φ_i are unavailable

From Theorem 1, it can be seen that the upper bound parameters φ_i for the time derivatives of the membership functions $f_i(t)$ should be available. However, for the practical systems, the upper bound parameters φ_i for the time derivatives of the membership functions $f_i(t)$ may be unavailable.

Theorem 2: For the given positive scalars $\lambda > 1$, $\mathcal{T}_d > 0$ and $K > 0$, if there exist the positive matrices $\mathcal{Q}_{l,n}$ and $\mathcal{S}_{il}$ such that $\mathcal{Q}_{l,n} = \mathcal{Q}_{l,n}^T > 0$ and $\mathcal{S}_{il} = \mathcal{S}_{il}^T > 0$, then the following inequalities hold

$$\begin{cases} \mathcal{Q}_{l,0} - \mathcal{Q}_{q,K} < 0, \quad l \neq q, \\ \mathcal{G}_{il,mm} < 0, \\ \mathcal{G}_{il,(m+1)m} < 0, \\ \mathcal{G}_{il,K} < 0 \end{cases} \tag{33}$$

with

$$\begin{cases} \mathcal{G}_{il,nm} = He\left(\mathcal{Q}_{l,n}A_i\right) + \left(K/\mathcal{T}_d\right)\left(\mathcal{Q}_{l,m+1} - \mathcal{Q}_{l,m}\right) \\ \qquad\qquad + a_{il}\lambda \sum_{q=1}^{r} \mathcal{S}_{ql} - \mathcal{S}_{il}, \\ \mathcal{G}_{il,K} = He\left(\mathcal{Q}_{l,K}A_i\right) + a_{il}\lambda \sum_{q=1}^{r} \mathcal{S}_{ql} - \mathcal{S}_{il}, \end{cases} \tag{34}$$

where i, l, $q \in [1,r]$, $m \in [1,K-1]$, $n \in [1,K]$, and $a_{ij} = \begin{cases} 0, & i \neq l, \\ 1, & i = l. \end{cases}$ Thus, it can be seen that the solutions of the closed-loop system converge to an adjustable bounded region.

Proof: Choose the switched Lyapunov-Krasovskii functional

$$V_{\sigma(t)}\left(x(t)\right) = x^T(t)\, Q_{f(t)\sigma(t)}(t)\, x(t). \tag{35}$$

Via the similar approach, one can obtain Theorem 2 based on the proof of Theorem 1. □

4. CONTROLLER DESIGN

In this section, based on the proposed switching control law $\sigma(t)$ and the relaxation parameter λ, the dynamic output feedback controller with the estimate algorithm is designed to estimate the attraction domain. The graphical abstract of the proposed methodology is shown in Fig. 1.

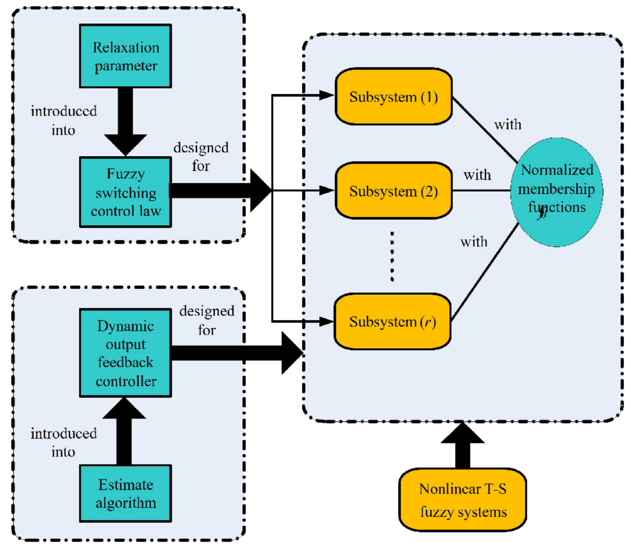

Fig. 1. The graphical abstract of proposed methodology.

Remark 7: In practice, the upper bound parameters φ_i for the time derivatives of the membership functions $f_i(t)$ are not easy to obtain. In addition, in order to obtain the inner estimate of the attraction domain $\mathcal{Z}(b^*)$ with $b^* = \max\{b \in R \mid \mathcal{Z}(b) \subseteq \mathcal{U}\}$, the values of φ_i are computed in the prescribed region $\mathcal{L}(\theta)$ [40].

$$\mathcal{L}(\theta) = \{x(t) \in R^{n_x} \mid |x_i(t)| \leq \theta, \quad i \in [1, r]\} \subseteq \mathcal{C}, \tag{36}$$

where $\theta = [\theta_1, \theta_2, ..., \theta_j]^T$.

For the nonlinear T-S fuzzy system (1), the dynamic output feedback controller is designed.

Step 1: For the system (1), there exist the scalars k, γ_j, λ, ε and vector $\theta^k = [\theta_1^k, \theta_2^k, ..., \theta_j^k]^T$ such that $k \geq 1$, $\gamma_j > 0$, $\lambda > 1$, $\varepsilon > 0$ and $\theta_j^k > 0$. With Theorems 1, 2 and (36), the upper bound parameters φ_i^k for the time derivatives of the membership functions $f_i(t)$ are described

$$\varphi_i^k = \left\{x(t) \in \mathcal{L}(\theta) \mid \max\left|\dot{f}_i(t)\right|, \quad i \in [1, r]\right\}. \tag{37}$$

Then, φ_i^k can be computed by the LMIs toolbox.

Step 2: With (10), (13) and (37), the minimal dwell time is designed

$$t_m^k = -\frac{1-\lambda}{r\varphi^k(1+\lambda)} \leq \mathcal{T}_d^k. \tag{38}$$

Step 3: With (37) and (38), the following conditions hold.

If $k = 1$, set $\theta_j^k = \theta_{j-1}^k + \gamma_j$, then one has

$$\begin{cases} \theta_j^k > \bar{x}_i, \text{ go to Step 4}, \\ \theta_j^k \leq \bar{x}_i, \text{ return to Step 1}. \end{cases}$$

If $k \neq 1$, set $\theta_j^k = \theta_{j-1}^k - \frac{\gamma_j}{2}$, then one has

$$\begin{cases} \theta_j^k > \varepsilon, \text{ return to Step 1,} \\ \theta_j^k \leq \varepsilon, \text{ go to Step 4.} \end{cases}$$

Step 4: Defining $\varphi_i^* = \varphi_i^{k-1}$, $t_m^* = t_m^{k-1} \leq \mathcal{T}_d^*$ and $\theta_j^* = \theta_j^{k-1}$, then the attraction domain parameter b^* can be described

$$b^* = \max\{b \in R \mid Z(b) \subseteq \mathcal{L}(\theta^*)\}. \tag{39}$$

Step 5: For the system (1), the dynamic output feedback controller is designed

$$\begin{cases} \dot{x}_d(t) = \mathcal{A}_{f(t)} x_d(t) + \mathcal{B}_{f(t)} y(t), \\ u(t) = \mathcal{C}_{f(t)} x_d(t) + \mathcal{D}_{f(t)} y(t), \end{cases} \tag{40}$$

where $x_d(t)$ is the state vector of the dynamic output-feedback controller. $\mathcal{A}_{f(t)}$, $\mathcal{B}_{f(t)}$, $\mathcal{C}_{f(t)}$ and $\mathcal{D}_{f(t)}$ are the gain matrices with appropriate dimensions. With (1) and (40), the nonlinear closed-loop system is obtained

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_d(t) \end{bmatrix} = \begin{bmatrix} \bar{A}_i + \bar{B}_i \mathcal{D}_{f(t)} \bar{C}_{1i} & \bar{B}_i \mathcal{C}_{f(t)} \\ \mathcal{B}_{f(t)} \bar{C}_{1i} & \mathcal{A}_{f(t)} \end{bmatrix} \begin{bmatrix} x(t) \\ x_d(t) \end{bmatrix}. \tag{41}$$

Theorem 3: For the given positive scalars $\lambda > 1$, $\mathcal{T}_d > 0$ and $K > 0$, if there exist the positive matrices $\mathcal{W}_{jl,n}$, $\bar{\mathcal{S}}_{ijl}$ and $\bar{\mathcal{N}}_{il,n}$ such that $\mathcal{W}_{jl,n} = \mathcal{W}_{jl,n}^T > 0$, $\bar{\mathcal{S}}_{ijl} = \bar{\mathcal{S}}_{ijl}^T$, and $\bar{\mathcal{N}}_{il,n} = \bar{\mathcal{N}}_{il,n}^T > 0$, then the following inequalities hold

$$\begin{cases} \mathcal{W}_{jl,K} - \mathcal{W}_{jq,0} < 0, \ \ l \neq q, \\ \mathcal{W}_{jl,n} - \bar{\mathcal{N}}_{il,n} > 0, \\ \bar{\mathcal{G}}_{ijl,mm} + \bar{\mathcal{G}}_{jil,mm} < 0, \\ \bar{\mathcal{G}}_{ijl,(m+1)m} + \bar{\mathcal{G}}_{jil,(m+1)m} < 0, \\ \bar{\mathcal{G}}_{ijl,K} + \bar{\mathcal{G}}_{jil,K} < 0 \end{cases} \tag{42}$$

with

$$\begin{cases} \bar{\mathcal{G}}_{ijl,mm} = He\big(\mathcal{A}_i \mathcal{W}_{jl,n} \\ \qquad + \mathcal{B}_i \mathcal{S}_{jl,n}\big) + \sum_{q=1}^{r} \varphi_q\left(\mathcal{W}_{ql,n} - \bar{\mathcal{N}}_{il,n}\right) \\ \qquad - K/\mathcal{T}_{\mathcal{D}}\left(\mathcal{W}_{jl,m+1} - \mathcal{W}_{jl,m}\right) \\ \qquad + a_{il}\lambda \sum_{q=1}^{r} \bar{\mathcal{S}}_{qjl} - \bar{\mathcal{S}}_{ijl}, \\ \bar{\mathcal{G}}_{ijl,K} = He\left(\mathcal{A}_i \mathcal{W}_{jl,K} + \mathcal{B}_i \mathcal{S}_{jl,K}\right) \\ \qquad + \sum_{q=1}^{r} \varphi_q\left(\mathcal{W}_{ql,K} - \bar{\mathcal{N}}_{il,K}\right) \\ \qquad + a_{il}\lambda \sum_{q=1}^{r} \bar{\mathcal{S}}_{qjl} - \bar{\mathcal{S}}_{ijl}, \end{cases} \tag{43}$$

where $i, j, l, q \in [1,r]$, $n \in [1,K]$, $m \in [1, K-1]$, $a_{ij} = \begin{cases} 0, & i \neq l, \\ 1, & i = l, \end{cases}$ $\mathcal{A}_i = \begin{bmatrix} \bar{A}_i & \bar{B}_i \mathcal{C}_{f(t)} \\ \mathcal{B}_{f(t)} \bar{C}_{1i} & \mathcal{A}_{f(t)} \end{bmatrix}$ and $\mathcal{B}_i = \begin{bmatrix} (\mathcal{B}_{f(t)} \bar{C}_{1i})^T & 0 \end{bmatrix}^T$.

Thus, it can be seen that the solutions of the closed-loop system converge to an adjustable bounded region.

Proof: The proof of Theorem 3 is divided into two steps.

Step 1: Based on the conditions $\bar{\mathcal{G}}_{ijl,mm} + \bar{\mathcal{G}}_{jil,mm} < 0$ and $\bar{\mathcal{G}}_{ijl,(m+1)m} + \bar{\mathcal{G}}_{jil,(m+1)m} < 0$ in (42), one has

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r} f_i(t) f_j(t) \big(He\left(A_i \mathcal{W}_{jl_k} + B_i \mathcal{W}_{jl_k}\right) \\ &\quad + \sum_{q=1}^{r} \varphi_q\left(\mathcal{W}_{ql_k} - \bar{\mathcal{N}}_{il_k}\right) - \frac{K}{\mathcal{T}_d}\left(\mathcal{W}_{jl_k,m+1} - \mathcal{W}_{jl_k,m}\right)\big) \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} f_i(t) f_j(t) \left(a_{il_k}\lambda \sum_{q=1}^{r} \bar{\mathcal{S}}_{qjl_k} - \bar{\mathcal{S}}_{ijl_k}\right) < 0, \end{aligned} \tag{44}$$

where

$$\sum_{i=1}^{r}\sum_{j=1}^{r} f_i(t) f_j(t) \left(a_{il_k}\lambda \sum_{q=1}^{r} \bar{\mathcal{S}}_{qjl_k} - \bar{\mathcal{S}}_{ijl_k}\right) > 0, \tag{45}$$

$$\mathcal{W}_{ql_k} = (1-\alpha)\mathcal{W}_{ql_k,m} + \alpha \mathcal{W}_{ql_k,m+1}. \tag{46}$$

Based on the conditions (2) and (3), with (40), (45) and (46), inequality (44) can be rewritten

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r} f_i(t) f_j(t) \big(He\left(A_i \mathcal{W}_{jl_k} + B_i \mathcal{W}_{jl_k}\right) \\ &\quad - \sum_{q=1}^{r} \dot{f}_q(t) \mathcal{W}_{ql_k} - \frac{K}{\mathcal{T}_d}\left(\mathcal{W}_{jl_k,m+1} - \mathcal{W}_{jl_k,m}\right)\big) \\ &= He\left(\mathcal{A}_{f(t)} \mathcal{X}_{f(t)l_k}(t) + \mathcal{B}_{f(t)} \mathcal{X}_{f(t)l_k}(t)\right) - \dot{\mathcal{X}}_{f(t)l_k}(t) \\ &< 0, \end{aligned} \tag{47}$$

where $\mathcal{X}_{f(t)l_k}(t)$ is a function matrix and defined

$$\mathcal{X}_{f(t)l_k}(t) = \frac{K}{\mathcal{T}_d}\left(\mathcal{X}_{f(t)l_k,m+1} - \mathcal{X}_{f(t)l_k,m}\right) + \sum_{i=1}^{r} \dot{f}_i(t) \mathcal{W}_{il_k}. \tag{48}$$

Let

$$\mathcal{H}_{l_k}(t) = \mathcal{X}_{f(t)l_k}^{-1}(t). \tag{49}$$

Then, the time derivative of (49) yields

$$\dot{\mathcal{H}}_{l_k}(t) = \dot{\mathcal{X}}_{f(t)l_k}^{-1}(t) = -\mathcal{X}_{f(t)l_k}^{-1}(t)\dot{\mathcal{X}}_{f(t)l_k}(t)\mathcal{X}_{f(t)l_k}^{-1}(t). \tag{50}$$

Substituting (46) into (47), with (49) and (50), one has

$$\begin{aligned} &He\left(\mathcal{H}_{l_k}(t)\left(\mathcal{A}_{f(t)} + \mathcal{B}_{f(t)}\left(\mathcal{C}_{f(t)} x_d(t) + \mathcal{D}_{f(t)} y(t)\right)\right)\right) \\ &\quad + \dot{\mathcal{H}}_{l_k}(t) < 0, \ \ t \in [t_k + \tau_m, t_k + \tau_{m+1}). \end{aligned} \tag{51}$$

Step 2: Via the similar way as shown in the first step, based on the condition $\bar{\mathcal{G}}_{ijk,K} + \bar{\mathcal{G}}_{jil,K} < 0$ in (42), the following inequality holds

$$He\left(\mathcal{H}_{l_k}(t)\left(\mathcal{A}_{f(t)} + \mathcal{B}_{f(t)}\left(\mathcal{C}_{f(t)} x_d(t) + \mathcal{D}_{f(t)} y(t)\right)\right)\right)$$

$$+\dot{\mathcal{H}}_{l_k}(t) < 0,\ t \in [t_k + \tau_k,\ t_{k+1}). \tag{52}$$

For the system (41), choose the switched Lyapunov-Krasovskii functional

$$V_{\sigma(t)}(x(t)) = x^T(t)\mathcal{H}_{f(t)\sigma(t)}(t)x(t). \tag{53}$$

Then, the time derivative of (53) yields

$$\begin{aligned}\dot{V}_{\sigma(t)}(x(t)) =& \dot{x}^T(t)\mathcal{H}_{f(t)\sigma(t)}(t)x(t)\\ &+x^T(t)\dot{\mathcal{H}}_{f(t)\sigma(t)}(t)\\ &+x^T(t)\mathcal{H}_{f(t)\sigma(t)}(t)\dot{x}(t).\end{aligned} \tag{54}$$

Based on the condition $\mathcal{W}_{jl,K} - \mathcal{W}_{jq,0} < 0$ in (42), with (51) and (54), one has

$$\begin{cases}\dot{V}_{\sigma(t)}(x(t)) < 0,\\ V_{\sigma(t_{k+1})}(x(t_{k+1})) - V_{\sigma(t_k)}\left(x\left(t_{k+1}^-\right)\right) < 0.\end{cases} \tag{55}$$

Then it can be seen that the solutions of the closed-loop system (41) converge to an adjustable bounded region. The proof of the Theorem 3 is completed. □

5. SIMULATIONS

In this section, two simulation examples are performed to verify the feasibility and effectiveness of the proposed method.

5.1. Example 1

Consider a class of nonlinear T-S fuzzy mobile robot system

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix} = \sum_{i=1}^{r} f_i(t)\left(A_i\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix} + B_i u\right), \tag{56}$$

where x_1 is the direction angle of the mobile robot, x_2 and x_3 are the mobile robot position coordinates. u is the control input of the system. For the simulation, the gain matrices are defined $A_1 = \begin{bmatrix}1 & 0 & 0.5\\ 0 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$, $A_2 = \begin{bmatrix}0.5 & 1 & 05\\ 1 & 0.5 & 1\\ 1 & 2 & 0.5\end{bmatrix}$ and $B_1 = B_2 = \begin{bmatrix}0.5\\ 0.8\\ 1\end{bmatrix}$.

For the system (56), the dynamic output feedback controller with the estimate algorithm is designed.

Step 1: The parameters k, γ_1, γ_2, γ_3,λ, ε and θ_1^1 are given as $k = 1$, $\gamma_1 = \gamma_2 = \gamma_3 = 0.3$, $\lambda = 1.5$, $\varepsilon = 0.003$ and $\theta_1^1 = 1$. With Theorems 1, 2 and (36), the upper bound parameters φ_i^k for the time derivatives of the membership functions $f_i(t)$ are described as follows:

$$\varphi_i^k = \left\{x(t) \in L(\theta) \mid \max\left|\dot{f}_i(t)\right|,\ i = 1,2,3\right\}. \tag{57}$$

In this case, the upper bound parameters φ_1^1, φ_2^1 and φ_3^1 are given

$$\begin{cases}\varphi_1^1 = \max\left|\dot{f}_1(t)\right| = 0.125,\\ \varphi_2^1 = \max\left|\dot{f}_2(t)\right| = 0.148,\\ \varphi_3^1 = \max\left|\dot{f}_3(t)\right| = 0.130.\end{cases}$$

Step 2: With (10) and (57), the minimal dwell time is designed

$$t_m^k = -\frac{1-\lambda}{r\phi^k(1+\lambda)} \le \mathcal{T}_d^k. \tag{58}$$

Step 3: With (57) and (58), the following conditions hold.

If $k = 1$, set $\theta_j^1 = \theta_{j-1}^1 + \gamma_j = 1.3$, then one has

$$\begin{cases}\theta_j^k > \bar{x}_i, \text{ go to Step 4},\\ \theta_j^k \le \bar{x}_i, \text{ return to Step 1}.\end{cases}$$

If $k = 9 \neq 1$, set $\theta_j^9 = \theta_{j-1}^9 - \frac{\gamma_j}{2}$, then one has

$$\begin{cases}\theta_j^k > \varepsilon, \text{ return to Step 1},\\ \theta_j^k \le \varepsilon, \text{ go to Step 4}.\end{cases}$$

Step 4: For $k = 9$, one can obtain $\varphi_i^* = \varphi_i^{k-1} = \varphi_i^8 = 0.8329$, $t_m^* = t_m^{k-1} = t_m^8 = 0.0286 \le \mathcal{T}_d^*$ and $\theta_j^* = \theta_j^{k-1} = \theta_j^8 = 1.2875$, then the attraction domain parameter b^* can be designed

$$b^* = \max\{b \in R \mid \mathcal{Z}(b) \subseteq \mathcal{L}(\theta^*)\} = 0.4180. \tag{59}$$

Step 5: For the system (56), the dynamic output feedback controller is designed

$$\begin{cases}\dot{x}_d(t) = \mathcal{A}_{f(t)}x_d(t) + \mathcal{B}_{f(t)}y(t),\\ u(t) = \mathcal{C}_{f(t)}x_d(t) + \mathcal{D}_{f(t)}y(t),\end{cases} \tag{60}$$

where $\mathcal{A}_{f(t)} = \begin{bmatrix}-0.3200 & 0.3171 & 0.1573\\ -0.1047 & 0.1957 & 0.0265\\ -0.1732 & 0.5010 & 0.6142\end{bmatrix}$, $\mathcal{B}_{f(t)} = \begin{bmatrix}-0.1915\\ -1.1800\\ -0.0025\end{bmatrix}$, $\mathcal{C}_{f(t)} = \begin{bmatrix}3.5400 & -0.8120 & -0.7058\end{bmatrix}$ and $\mathcal{D}_{f(t)} = -0.1672$.

For the simulation, the initial state are chosen as $x(0) = [0.5\ 0\ 0]^T$. The responses of the system state variables $x_1(t)$, $x_2(t)$ and $x_3(t)$ for system (56) are shown in Figs. 2 and 3. The control inputs are shown in Fig. 4. From three figures, it can be seen that the proposed method is effective and can stabilize the mobile robot system quickly.

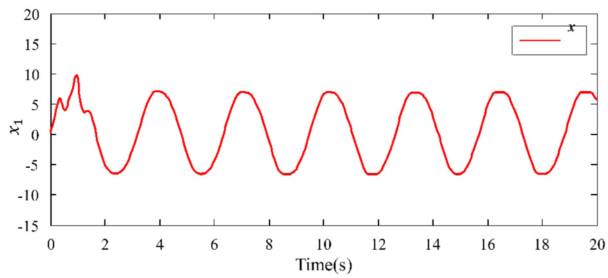

Fig. 2. The response of system state variables x_1.

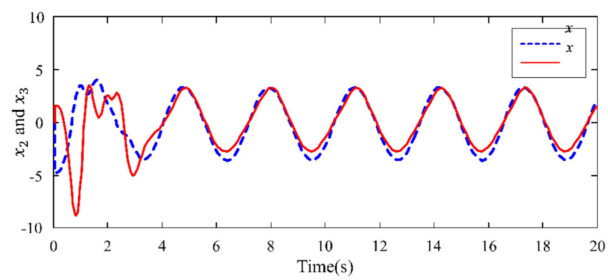

Fig. 3. The responses of system state variables x_2 and x_3.

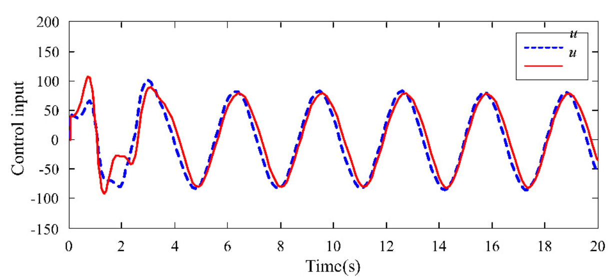

Fig. 4. The responses of system control inputs.

5.2. Example 2

Consider a class of nonlinear T-S fuzzy chemical stirred tank reactor system [41]

$$\begin{cases} \dot{C}_A = qV^{-1}\left(C_{Af_{si}k} - C_A\right) - a_0 \exp\left(-\dfrac{E}{RT}\right) C_A, \\ \dot{T} = qV^{-1}\left(T_{f_{si}k} - T\right) - a_1 \exp\left(-\dfrac{E}{RT}\right) C_A \\ \qquad + a_2\left(T_C - T\right), \end{cases} \tag{61}$$

where $k = 1$, 2 is the mode index, f_{si} is the feed stream index, and the use details of parameters k and f_{si} were shown in [41]. C_A, T and T_C are the reactant concentration, reactor temperature and coolant temperature, respectively. In this paper, the mode index k is chosen as $k = 1$ [41], then the system (61) is rewritten

$$\begin{cases} \dot{C}_A = qV^{-1}\left(C_{Af_{si}1} - C_A\right) - a_0 \exp\left(-\dfrac{E}{RT}\right) C_A, \\ \dot{T} = qV^{-1}\left(T_{f_{si}1} - T\right) - a_1 \exp\left(-\dfrac{E}{RT}\right) C_A \\ \qquad + a_2\left(T_C - T\right). \end{cases} \tag{62}$$

In addition, the using details of parameters in (62) were shown in [41].

For the chemical stirred tank reactor system (62), the desired nominal operating values are $C_A^* = 0.5$ mod/L, $T^* = 350$ K and $T_C^* = 350$ K. In this case, $x_1 = C_A - C_A^*$ and $x_2 = T - T^*$ are the state variables. With $k = 1$ and the Table 1 as shown in [41], the system (62) is rewritten [41]

$$(\kappa = 1):\ \begin{cases} \dot{x}_1(t) = x_2(t) + 0.5\left(1.5 - x_1(t)\right) \\ \qquad - a_0 x_1(t) e^{-8750/(x_2(t)+350)} \\ \qquad - x_2(t), \\ \dot{x}_2(t) = a_2 u(t) - 2.592 x_2(t) \\ \qquad - a_1 x_1(t) e^{-8750/(x_2(t)+350)} \\ \qquad - 104.6. \end{cases} \tag{63}$$

Remark 8: The schematic diagram of the chemical stirred tank reactor is shown in Fig. 5. It consists of a constant volume CSTR fed by a single inlet stream through a selector valve. Suppose that the position of the selector valve at each time is determined by a supervisory mechanism based on an objective. In other words, at each time the reactor is fed by one of the source streams according to the decision made by the supervisor. Since the source streams have different parameters, the parameters of the feed of the reactor can change instantaneously. The reactor is cooled by a coolant stream with a constant flow rate and a variable temperature T_C [41].

For the simulation, the gain matrices are defined

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and $B_2 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$.

For the system (63), the dynamic output feedback controller with the estimate algorithm is designed.

Step 1: The parameters k, γ_1, γ_2, λ, ε and θ_1^1 are given as $k = 1$, $\gamma_1 = \gamma_2 = 0.5$, $\lambda = 2$, $\varepsilon = 0.003$ and $\theta_1^1 = 1$. With Theorems 1, 2 and (36), the upper bound parameters φ_1^k for the time derivatives of the membership functions $f_i(t)$ are described

$$\varphi_i^k = \{x(t) \in \mathcal{L}(\theta) \mid \max|\dot{f}_i(t)|,\ \ i = 1,2\}. \tag{64}$$

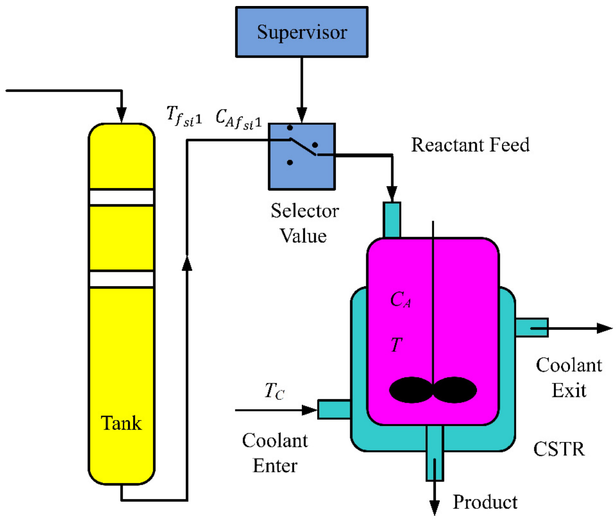

Fig. 5. The schematic diagram of the chemical stirred tank reactor.

In this case, the upper bound parameters φ_1^1 and φ_2^1 are given

$$\begin{cases} \varphi_1^1 = \max|\dot{f}_1(t)| = 0.187, \\ \varphi_2^1 = \max|\dot{f}_2(t)| = 0.209. \end{cases}$$

Step 2: With (10) and (64), the minimal dwell time is designed

$$t_m^k = -\frac{1-\lambda}{r\phi^k(1+\lambda)} \leq \mathcal{T}_d^k. \tag{65}$$

Step 3: With (64) and (65), the following conditions hold.

If $k=1$, set $\theta_j^1 = \theta_{j-1}^1 + \gamma_j = 1.5$, then one has

$$\begin{cases} \theta_j^k > \bar{x}_i, \text{ go to Step 4,} \\ \theta_j^k \leq \bar{x}_i, \text{ return to Step 1.} \end{cases}$$

If $k = 9 \neq 1$, set $\theta_j^9 = \theta_{j-1}^9 - \frac{\gamma_j}{2} = 1.2096$, then one has

$$\begin{cases} \theta + J^k > \varepsilon, \text{ return to Step 1,} \\ \theta_j^k \leq \varepsilon, \text{ go to Step 4.} \end{cases}$$

Step 4: For $k=9$, one can obtain $\varphi_i^* = \varphi_i^8 = 0.5111$, $t_m^* = t_m^8 - 0.1913 \leq \mathcal{T}_d^*$ and $\theta_j^* = \theta_j^8 = 1.2875$, then the attraction domain parameter b^* can be designed

$$b^* = \max\{b \in R \mid \mathcal{Z}(b) \subseteq \mathcal{L}(\theta^*)\} = 0.3947. \tag{66}$$

Step 5: For the system (63), the dynamic output feedback controller is designed

$$\begin{cases} \dot{x}_d(t) = \mathcal{A}_{f(t)} x_d(t) + \mathcal{B}_{f(t)} y(t), \\ u(t) = \mathcal{C}_{f(t)} x_d(t) + \mathcal{D}_{f(t)} y(t), \end{cases} \tag{67}$$

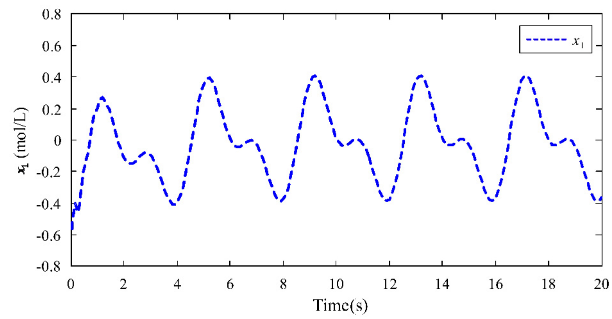

Fig. 6. The response of system state variable x_1.

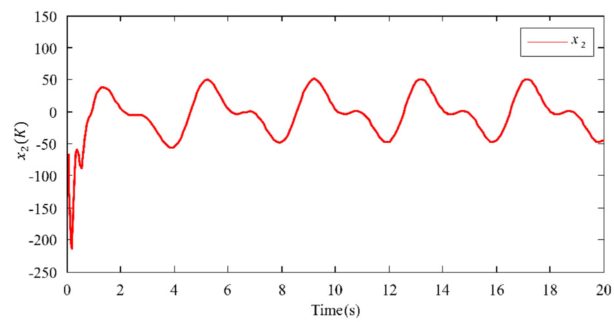

Fig. 7. The response of system state variable x_2.

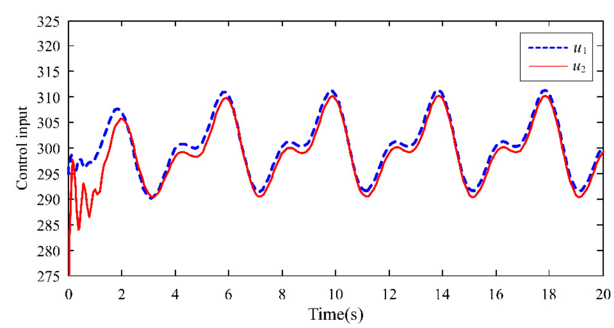

Fig. 8. The responses of system control inputs.

where $\mathcal{A}_{f(t)} = \begin{bmatrix} -0.1506 & 0.2295 \\ -0.5009 & 0.1981 \end{bmatrix}$, $\mathcal{B}_{f(t)} = \begin{bmatrix} 0.1050 \\ -0.1873 \end{bmatrix}$, $\mathcal{C}_{f(t)} = \begin{bmatrix} 2.4300 & -0.7910 \end{bmatrix}$ and $\mathcal{D}_{f(t)} = -0.1517$.

For the simulation, the initial state are chosen as $x(0) = [-0.57\ -69]^T$. The responses of the system state variables $x_1(t)$ and $x_2(t)$ for system (63) are shown in Figs. 6 and 7. The control inputs are shown in Fig. 8. From the three figures, it can be seen that the proposed method is effective and can stabilize the chemical stirred tank reactor system quickly.

6. CONCLUSION

This paper addresses the stability analysis and dynamic output-feedback control problems for a class of nonlinear T-S fuzzy system with multiple subsystems and normalized membership functions. The upper bound parameters for the time derivatives of the membership functions are available/unavailable and bounded by the linear matrix inequalities technique. The switching control law of the membership function with the relaxation parameter is designed, and the minimal dwell time between the two consecutive switching is guaranteed. The dynamic output feedback controller with the estimate algorithm is designed, and the attraction domain can be estimate effectively. With the help of the proposed switched Lyapunov-Krasovskii functional, it can be seen that the designed controller renders that the closed-loop system has better transient state performance and better steady state performance. Finally, two simulation examples are performed to show the effectiveness of the proposed method. With the help of the minimal dwell time, the switched Lyapunov-Krasovskii functional technique can be adopted to obtain the less conservativeness conditions. Moreover, a more effective algorithm is obtained based on the new stability conditions to estimate the attraction domain for the T-S fuzzy systems. In the future work, the proposed method will be applied to the finite/fixed-time stabilization of the T-S fuzzy system with multiple time-varying delays and the obtained results will further extended to the actual application of the mobile robot system.

REFERENCES

[1] S. Park, H. Lee, S. Han, and J. Lee, "Adaptive fuzzy super-twisting backstepping control design for MIMO nonlinear strict feedback systems," *International Journal of Control Automation & Systems*, vol. 16, no. 3, pp. 1165-1178, April 2018.

[2] J. B. Qiu, S. X. Ding, L. L. Li, and S. N. Yin, "Reliable fuzzy output feedback control of nonlinear parabolic distributed parameter systems with sensor faults1," *Journal of Intelligent & Fuzzy Systems*, vol. 29, no. 3, pp. 1197-1208, October 2015.

[3] C. C. Hua, Q. G. Wang, and X. P. Guan, "Adaptive fuzzy output-feedback controller design for nonlinear time-delay systems with unknown control direction," *IEEE Transactions on Systems Man & Cybernetics Part B*, vol. 39, no.2, pp. 363-374, April 2009.

[4] C. C. Hua and X. P. Guan, "Smooth dynamic output feedback control for multiple time-delay systems with nonlinear uncertainties," *Automatica*, vol. 68, pp. 1-8, June 2016.

[5] Y. L. Wei, J. B. Qiu, P. Shi, and M. Chadli, "Fixed-order piecewise-affine output feedback controller for fuzzy-affine-model-based nonlinear systems with time-varying delay," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 64, no.4, pp. 945-958, April 2017.

[6] Y. L. Wei, J. B. Qiu, and H. R. Karimi, "Reliable output feedback control of discrete-time fuzzy affine systems with actuator faults," *IEEE Transactions on Circuits & Systems I Regular Papers*, vol. 64, no. 1 pp. 170-181, July 2017.

[7] Y. L. Wei, J. B. Qiu, P. Shi, and H. K. Lam, "A new design of H-infinity piecewise filtering for discrete-time nonlinear time-varying delay systems via T-S fuzzy affine models," *IEEE Transactions on Systems Man & Cybernetics Systems*, vol. 47 no. 8, pp. 2034-2047, August 2017.

[8] C. Z. Zhang, J. F. Hu, J. B. Qiu, and Q. J. Chen, "Event-triggered nonsynchronized H- filtering for discrete-time T-S fuzzy systems based on piecewise Lyapunov functions," *IEEE Transactions on Systems Man & Cybernetics Systems*, vol. 47, no. 8, pp. 2330-2341, August 2017.

[9] W. Yu, F. O. Rodríguez, and M. Armendariz, "Hierarchical fuzzy CMAC for nonlinear systems modeling," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 5, pp. 1302-1314, June 2008.

[10] J. L. Chang and T. C. Wu, "Disturbance observer based output feedback controller design for systems with mismatched disturbance," *International Journal of Control Automation & Systems*, vol. 16, no. 4, pp. 1775-1782, July 2018.

[11] C. Z. Zhang, J. F. Hu, J. B. Qiu, and Q. J. Chen, "Reliable output feedback control for T-S fuzzy systems with decentralized event triggering communication and actuator failures," *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2592-2602, February 2017.

[12] G. B. Koo, B. P. Jin, and Y. H. Joo, "Intelligent digital redesign for sampled-data fuzzy control systems based on state-matching error cost function approach," *International Journal of Control Automation & Systems*, vol. 16, no. 1, pp. 350-359, March 2018.

[13] C. J. Lee and M. T. Lim, "Fuzzy FIR filtering for TS fuzzy systems with quantization and packet dropout," *International Journal of Control Automation & Systems*, vol. 13, no. 3, pp. 961-971, May 2018.

[14] S. C. Tong, Y. M. Li, G. Feng, and T. S. Li, "Observer-based adaptive fuzzy back-stepping dynamic surface control for a class of MIMO nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B*, vol. 41, no. 4, pp. 1124-1135, August 2011.

[15] Y. Li, Y. M. Li, and S. C. Tong, "Adaptive fuzzy decentralized output feedback control for stochastic nonlinear large-scale systems," *Neurocomputing*, vol. 23, no. 4, pp. 381-399, April 2013.

[16] Y. L. Wei, J. B. Qiu, H. K. Lam, and L. G. Wu, "Approaches to T-S fuzzy-affine-model-based reliable output feedback control for nonlinear Ito stochastic systems," *IEEE Transactions on fuzzy systems*, vol. 25, no. 3, pp. 569-583, May 2017.

[17] T. C. Wang and S. C. Tong, "Observer-based output-feedback asynchronous control for switched fuzzy systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2579-2591, May 2017.

[18] C. J. Lee and M. T. Lim, "Synergetic adaptive fuzzy control for a class of nonlinear discrete-time systems," *International Journal of Control Automation & Systems*, vol. 16, no. 4, pp. 1981-1988, July 2018.

[19] Y. M. Li, S. C. Tong, L. Liu, and G. Feng, "Adaptive output-feedback control design with prescribed performance for switched nonlinear systems," *Automatica*, vol. 80, pp. 225-231, June 2017.

[20] D. Zhai, A. Y. Lu, J. H. Li, and Q. L. Zhang, "Simultaneous fault detection and control for switched linear systems with mode-dependent average dwell-time," *Applied Mathematics & Computation*, vol. 273, pp. 767-792, January 2016.

[21] S. C. Tong, L. L. Zhang, and Y. M. Li, "Observed-based adaptive fuzzy decentralized tracking control for switched uncertain nonlinear large-scale systems with dead zones," *IEEE Transactions on Systems Man & Cybernetics Systems*, vol. 46, no. 1, pp. 37-47, January 2016.

[22] T. Wang, Y. F. Zhang, J. B. Qiu, and H. J. Gao, "Adaptive fuzzy backstepping control for a class of nonlinear systems with sampled and delayed measurements," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 2, pp. 302-312, April 2015.

[23] Y. N. Sun, G. B. Koo, B. P. Jin, and Y. H. Joo, "I-∞ adaptive fuzzy filter design for nonlinear systems with missing measurements: fuzzy basis-dependent Lyapunov function approach," *International Journal of Control Automation & Systems*, vol. 14, no. 2, pp. 425-434, Febrary 2017.

[24] C. W. Wu, J. X. Liu, Y. Y. Xiong, and L. G. Wu, "Observer-based adaptive fault-tolerant tracking control of nonlinear nonstrict-feedback systems," *IEEE Transactions on Neural Networks & Learning Systems*, vol. 29, no. 7, pp. 3022-3033, July 2018.

[25] C. W. Wu, J. X. Liu, X. J. Jing, H. Y. Li, and L. G. Wu, "Adaptive fuzzy control for nonlinear networked control systems," *IEEE Transactions on Systems Man & Cybernetics Systems*, vol. 47, no. 8, pp. 2420-2430, August 2017.

[26] Q. Zhou, H. Y. Li, L. J. Wang, and R. Q. Lu, "Prescribed performance observer-based adaptive fuzzy control for nonstrict-feedback stochastic nonlinear systems," *IEEE Transactions on Systems Man & Cybernetics Systems*, vol. 48, no. 10, pp. 1747-1758, October 2018.

[27] J. B. Qiu, S. X. Ding, H. J. Gao, and S. Yin, "Fuzzy-model-based reliable static output feedback H-∞ control of nonlinear hyperbolic PDE systems," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 2, pp. 388-400, April 2016.

[28] Y. M. Li, S. C. Tong, Y. J. Liu, and T. Li, "Adaptive fuzzy robust output feedback control of nonlinear systems with unknown dead zones based on a small-gain approach," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 1, pp. 164-176, February 2014.

[29] S. C. Tong, S. Sui, and Y. Li, "Fuzzy adaptive output feedback control of MIMO nonlinear systems with partial tracking errors constrained," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 4, pp. 729-742, August 2015.

[30] Y. M. Li and S. C. Tong, "Adaptive fuzzy output-feedback stabilization control for a class of switched nonstrict-feedback nonlinear systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 4, pp. 1007-1016, April 2017.

[31] T. Wang, J. B. Qiu, H. J. Gao, and C. H. Wang, "Network-based fuzzy control for nonlinear industrial processes with predictive compensation strategy," *IEEE Transactions on Systems Man & Cybernetics Systems*, vol. 47, no. 8, pp. 2137-2147, August 2017.

[32] J. B. Qiu, G. Feng, and H. Gao, "Static-output-feedback control of continuous-time T-S fuzzy affine systems via piecewise Lyapunov functions," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 2, pp. 245-261, April 2013.

[33] T. Wang, J. B. Qiu, S. S. Fu, and W. Q. Ji, "Distributed fuzzy H-∞ filtering for nonlinear multirate networked double-layer industrial processes," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 5203-5211, June 2017.

[34] T. Wang, H. J. Gao, and J. B. Qiu, "A combined fault-tolerant and predictive control for network-based industrial processes," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 4, pp. 2529-2536, April 2016.

[35] H. Y. Li, C. Wu, and Z. Feng, "Fuzzy dynamic output-feedback control of non-linear networked discrete-time system with missing measurements," *IET Control Theory & Applications*, vol. 9, no. 3, pp. 327-335, February 2015.

[36] Y. L. Wei, J. B. Qiu, H. R. Karimi, and M. Wang, "New results on H-∞ dynamic output feedback control for Markovian jump systems with time-varying delay and defective mode information," *Optimal Control Applications & Methods*, vol. 35, no. 6, pp. 656-675, November 2015.

[37] X. H. Chang, J. Xiong, and J. H. Park, "Fuzzy robust dynamic output feedback control of nonlinear systems with linear fractional parametric uncertainties," *Applied Mathematics & Computation*, vol. 291, pp. 213-225, December 2016.

[38] D. H. Lee, J. B. Park, and Y. H. Joo, "A fuzzy Lyapunov function approach to estimating the domain of attraction for continuous-time Takagi-Sugeno fuzzy systems," *Information Sciences*, vol. 185, no. 1, pp. 230-248, February 2012.

[39] A. Zemouche and M. Boutayeb, "On LMI conditions to design observers for Lipschitz nonlinear systems," *Automatica*, vol. 49, no. 2, pp. 585-591, February 2013.

[40] D. Zhai, A. Y. Lu, J. Dong, and Q. L. Zhang, "Stability analysis and state feedback control of continuous-time T-S fuzzy systems via anew switched fuzzy Lyapunov function approach," *Applied Mathematics & Computation*, vol. 293, no. C, pp. 586-599, January 2017.

[41] M. B. Yazdi and M. R. Jahed-Motlagh, "Stabilization of a CSTR with two arbitrarily switching modes using modal state feedback linearization," *Chemical Engineering Journal*, vol. 155, no. 3, pp. 838-843, December 2016.

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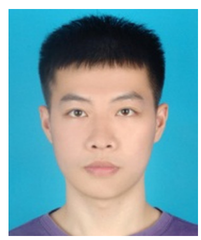

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