International Journal of Control, Automation and Systems 16(5) (2018) 2549-2558
http://dx.doi.org/10.1007/s12555-018-0118-6
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Adaptive Fuzzy Finite-time Control for Uncertain Nonlinear Systems with Dead-zone Input

Wenshun Lv, Fang Wang*, and Lili Zhang

Abstract: This paper presents a novel adaptive finite-time tracking control scheme for nonlinear systems. During the design process of control scheme, dead-zone input nonlinearity phenomena existing in the actuator is taken into account. Fuzzy logic systems are adopted to approximate the unknown nonlinear functions. This paper provides a new finite-time stability criterion, making the adaptive tracking control scheme more suitable in the practice than traditional methods. Under the presented controller, the desired system performance is realized in finite time. Finally, the validity and effectiveness of the proposed control method is validated by two examples.

Keywords: Adaptive tracking control, backstepping, dead-zone input, finite-time stability, fuzzy logic systems.

1. INTRODUCTION

In recent years, the adaptive control of nonlinear systems has achieved remarkable breakthroughs by combining with the backstepping technology [1–27]. Many of the technical limitations in traditional adaptive control, such as matching condition and relative-degree constraint, can be eliminated by adaptive backstepping control scheme. Fuzzy logic systems and neural networks(NNs) provide useful tools for designing control schemes of uncertain nonlinear systems, because of their capability of nonlinear approximation. The nonlinear control scheme using neural networks has been further improved by the introduction of adaptive algorithms for tuning the weighs of NNs [28]. By employing norms of unknown weight vectors as the estimated parameters, the huge computation problem of this method has also been resolved in [29] to a certain extent.It is worth mentioning that the control schemes proposed in the above literatures can only realize the desired system performance when the time tends to infinity. However, in practical engineering, the controlled systems are usually required to realize steady response from transient response quickly.

Finite-time control has received much attention because it can provide many benefits such as strong robustness and disturbance resistance capability [2, 3, 30, 31]. The Lyapunov theory of finite-time stability for nonlinear systems has been clearly established by several authors [32, 33]. It is necessary to point out that the nonlinear functions in these systems all meet the linear growth condition. In practice, the nonlinear functions are often completely unknown due to some constraints like the modeling method and unknown dynamic disturbances. In this case, the linear growth condition might not be satisfied. Consequently, these existed adaptive finite-time control methods are not suitable for the tracking control of uncertain nonlinear systems. To eliminate this limitation, a new finite-time stability criterion was proposed [34]. However, the effect of dead zone nonlinear factor on the system performance was not taken into account in [34], which would limit the application of the control method to some extent. In other words, there is still some room for improvement in making the finite-time control scheme implemented more efficiently.

In the field of practical application, the actuator of the system may encounter dead-zone input nonlinearity phenomena, which is marked by insensitivity for small control inputs. Dead zone in the actuator degrades the tracking performance of the system and it may even cause the instability of the system. Many results have been obtained for uncertain nonlinear systems with dead-zone input [35–40]. However, these control algorithms can only

Manuscript received February 27, 2018; revised April 28, 2018; accepted May 11, 2018. Recommended by Associate Editor Do Wan Kim under the direction of Editor Jessie (Ju H.) Park. This work was supported partially by the National Natural Science Foundation of China (Grant Nos. 61503223 and 61603098), in part by the Project of Shandong Province Higher Educational Science and Technology Program (J15LI09), and in part by China Postdoctoral Science Foundation-funded project 2016M592140, and partially by Shandong innovation postdoctoral program 201603066, and partially by the SDUST Research Fund (2014TDJH102).

Wenshun Lv is with the College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, Shandong, P. R. China, and also with the School of Automation, Nanjing University of Science and Technology, Nanjing, Jiangsu, P. R. China (e-mail: lvwenshun361@live.cn). Fang Wang is with the College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, Shandong, P. R. China (e-mails: sandywf75@126.com). Lili Zhang is with the School of Applied Mathematics, Guangdong University of Technology, Guangzhou, P. R. China (e-mail: 280305770@qq.com).
* Corresponding author.

drive the output tracking error to a small neighborhood of the origin when the time approaches to infinity. It is under current study to ensure that the performance of the system can be realized in finite time.

These facts motivate us to provide a new finite-time adaptive fuzzy control method for uncertain nonlinear systems with dead-zone input. In contrast with the existing literatures, this paper has the following contributions.

1) The traditional adaptive neural or fuzzy control strategies can only guarantee the system performance when time tends to infinity. These existing adaptive fuzzy control methods are not suitable for the finite-time tracking control of uncertain nonlinear system. Based on the Lyapunov theory of finite-time stability for nonlinear systems, this paper constructs an adaptive fuzzy controller which can ensure the tracking performance of the uncertain nonlinear system in finite time. Finite-time control scheme can also provide many benefits such as strong robustness and better disturbance resistance capability in presence of dead-zone input nonlinearity phenomena. Therefore, to a certain extent, the control strategy proposed in this paper is more meaningful than the control methods presented in [4, 6, 17, 41, 42] in the practical application fields.

2) In the available literature w.r.t. finite time stability and tracking control problems towards nonlinear systems, the nonlinear functions are assumed to satisfy the linear growth conditions and the bounding functions of those unknown system functions are known [30, 32, 33]. With the new adaptive control scheme proposed in this article, the nonlinear functions can be completely unknown and they are only required to be continuous. Consequently, in contrast with the finite-time control schemes in [30, 32–34], the control method in this note is more adaptable to the realistic systems. What is more, the stability analysis and control design depend on the finite-time stability condition presented in Lemma 5, i.e., $\dot{V} \leq -a_0V + b_0$ with $a_0, b_0 > 0$, rather than that in Lemma 3.6 proposed in [34], say $\dot{V} \leq -a_0V^{\wp} + d_0$ with $0 < \wp < 1$ and $d_0 > 0$. Accordingly, the difficulty of controller design is decreased without the introduce of the constant $0 < \wp < 1$ in control law and Lyapunov functions.

The paper is organized as follows. The control problem of the nonlinear system with dead-zone input is formulated in Section 2. In Section 3 and Section 4, finite-time adaptive fuzzy tracking control scheme is presented for a class of uncertain nonlinear systems with dead-zone input, and its stability is analyzed. Simulation results are presented in Section 5. The paper ends with the conclusion in Section 6.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. System description

The uncertain nonlinear systems with dead zone in this paper can be expressed as follows:

$$\begin{aligned} \dot{\chi}_\iota &= \chi_{\iota+1} + f_\iota(\tilde{\chi}_\iota) + p_\iota(t), \iota = 1,2,\ldots,n-1 \\ \dot{\chi}_n &= D(\nu) + f_n(\tilde{\chi}_n) + p_n(t) \\ y &= \chi_1, \end{aligned} \tag{1}$$

where $\tilde{\chi}_\iota = [\chi_1, \chi_2, ..., \chi_\iota]^T \in R^\iota$ and $\tilde{\chi}_n = [\chi_1, \chi_2, ..., \chi_n]^T \in R^n$ are the state vectors. $f_\iota(\tilde{\chi}_\iota)$, $\iota = 1,2,\ldots,n$ are unknown smooth functions, p_ι, $\iota = 1,2,\ldots,n$ denote bounded perturbations. ν is the control input signal to be designed. To facilitate the control system design, the dead-zone characteristic $D(\nu)$ is described as

$$D(\nu) = \begin{cases} \xi_1(\nu - \Im_1), & \nu \geq \Im_1, \\ 0, & \Im_1 < \nu < \Im_2, \\ \xi_2(\nu - \Im_2), & \nu \leq \Im_2. \end{cases} \tag{2}$$

The parameters ξ_1 ,ξ_2 ,$\Im_1$ and $\Im_2$ are unknown constants, and we suppose that there is the positive constant $\check{\xi}$ designed to meet

$$\begin{aligned} \check{\xi} &< \xi_1 < \hat{\xi}, \\ \check{\xi} &< \xi_2 < \hat{\xi}. \end{aligned}$$

$D(\nu)$ denotes the output of the dead zone, $\Im_1 > 0$ and $\Im_2 < 0$ stand for the breakpoints of the input nonlinearity. Then $D(\nu)$ can be reformulated as

$$D(\nu) = \xi(t)\nu + \varpi(t), \tag{3}$$

where

$$\xi(t) = \begin{cases} \xi_1, & \nu \geq \Im_1, \\ 0, & \Im_2 < \nu < \Im_1, \\ \xi_2, & \nu \leq \Im_2, \end{cases}$$

and

$$\varpi(t) = \begin{cases} \xi_1\Im_1, & \nu \geq \Im_1, \\ 0, & \Im_2 < \nu < \Im_1, \\ \xi_2\Im_2, & \nu \leq \Im_2. \end{cases}$$

Then, we have

$$\begin{aligned} &0 < \check{\xi} < \xi(t) < \hat{\xi} \\ &\text{and } |\varpi(t)| \leq \varpi_0 = max\{\hat{\xi}|\Im_1|, \hat{\xi}|\Im_2|\}. \end{aligned} \tag{4}$$

Define a vector function as $\bar{y}_{r_\iota} = [y_r, y_r^{(1)}, \ldots, y_r^{(\iota)}]^T \in \Omega_{r_\iota}$, where $y_r^{(\iota)}$ denotes the ιth time derivative of y_r and Ω_{r_ι} denotes a known compact set. The vectors $\bar{y}_{r_\iota}$, $\iota = 1,\ldots,n$ are continuous and available.

Assumption 1: There is an uncertain positive constant $\tilde{p}_\iota$ designed to meet

$$|p_\iota(t)| \leq \tilde{p}_\iota, \iota = 1,2,\ldots,n. \tag{5}$$

2.2. Fuzzy logic systems

The following fuzzy logic systems (FLSs) will be utilized to approximate the unknown function. Choose a collection of fuzzy rules as follows:

$$R^\iota: \text{ If } \chi_1 \text{ is } M_1^\iota \text{ and } \dots \text{ and } \chi_n \text{ is } M_n^\iota, \\ \text{then } y \text{ is } U^\iota,\ \iota = 1,2,\dots,N,$$

where $\tilde{\chi}_n = [\chi_1, \chi_2, ..., \chi_n]^T \in R^n$ and $y \in R$ are the input and the output of the fuzzy system, respectively. M_ℓ^ι and U_ι denote fuzzy sets composed of fuzzy membership functions $\mu_{M_\ell^\iota}(\chi_\ell)$ and $\mu_{U^\iota}(y)$. N is the number of the rules. Through the singleton fuzzifer, the product inference and the center-average defuzzifier, the fuzzy logic system is:

$$y(\chi) = \frac{\sum_{\iota=1}^N \varphi_\iota \prod_{\ell=1}^n \mu_{M_\ell^\iota}(\chi_\ell)}{\sum_{\iota=1}^N [\prod_{\ell=1}^n \mu_{M_\ell^\iota}(\chi_\ell)]},$$

where

$$\varphi_\iota = \max_{y\in R} \mu_{U^\iota}(y),\ \varphi = (\varphi_1, \varphi_2, \dots, \varphi_N)^T.$$

Let

$$\Re_\iota(\chi) = \frac{\prod_{\ell=1}^n \mu_{M_\ell^\iota}(\chi_\ell)}{\sum_{\iota=1}^N [\prod_{\ell=1}^n \mu_{M_\ell^\iota}(\chi_\ell)]},$$

where $\Re(\chi) = (\Re_1(\chi), \Re_2(\chi), \dots, \Re_N(\chi))^T$. We can override the fuzzy logic system as

$$y(\chi) = \varphi^T \Re(\chi). \tag{6}$$

Then, the definition of semi-globally uniformly finite-time bounded (SGUFB) and some related lemmas are introduced.

Definition 1: The solution $\{z(t), t \geq t_0\}$ of the nonlinear system $\dot{z} = f(z), f(0) = 0$ is SGUFB, if for any $z(t_0) = z_0 \in \Omega_0$ (some compact set containing the origin), there exists $\varepsilon > 0$ and a settling-time $T(\varepsilon, z_0) < \infty$, such that the following statements hold: $z \in \Omega_0 \cup \Omega$, for all $t \geq t_0$, where $\Omega = \{z | \|z(t)\| < \varepsilon\}$; $\|z(t)\| < \varepsilon$, for all $t \geq t_0 + T$.

Lemma 1 [43]**:** Let $f(\chi)$ be a continuous function defined on a compact set Ω. Then, for $\forall\varepsilon > 0$, there exists a FLS (6) such that

$$\sup_{\chi\in\Omega} |f(\chi) - \varphi^T \Re(\chi)| \leq \varepsilon. \tag{7}$$

Lemma 2 [44]**:** For $a_\ell \in R$, $q \in (0, 1]$, $\ell = 1, 2, \dots, d$, the following inequality is satisfied:

$$(\sum_{\ell=1}^d |a_\ell|)^q \leq \sum_{\ell=1}^d |a_\ell|^q \leq d^{1-q} (\sum_{\ell=1}^d |a_\ell|)^q. \tag{8}$$

Lemma 3 [45]**:** For any constants $\bar{s}$, $\bar{t}$, $\lambda > 0$, ρ, $\sigma \in R$ satisfy the following inequality:

$$|\rho|^{\bar{s}} |\sigma|^{\bar{t}} \leq \frac{\bar{s}}{\bar{s}+\bar{t}} \lambda |\rho|^{\bar{s}+\bar{t}} + \frac{\bar{t}}{\bar{s}+\bar{t}} \lambda^{-\frac{\bar{s}}{\bar{t}}} |\sigma|^{\bar{s}+\bar{t}}. \tag{9}$$

Lemma 4 [34]**:** Consider the system $\dot{z} = f(z, \nu)$, for smooth positive-definite function $V(z) \in C^1$, if there exist constants $a_0 > 0, 0 < \wp < 1$ and $d_0 > 0$ satisfying that

$$\dot{V}(z) \leq -a_0 V^\wp(z) + d_0, t \geq 0, \tag{10}$$

then the solution of the system $\dot{z} = f(z, \nu)$ is SGUFB.

Lemma 5: Consider the system $\dot{z} = f(z, \nu)$, for smooth positive-definite function $V(z) \in C^1$, if there exist constants $a_0 > 0$ and $b_0 > 0$ satisfying that

$$\dot{V}(z) \leq -a_0 V(z) + b_0,\ t \geq 0, \tag{11}$$

then we have

$$\dot{V}(z) \leq -a_0 V^\wp(z) + d_0,\ t \geq 0, \tag{12}$$

where $0 < \wp < 1$ and $d_0 = a_0(1-\wp)\wp^{\frac{\wp}{1-\wp}} + b_0$.

Proof: According to Lemma 3, for $\forall 0 < \wp < 1$, the following inequality is satisfied:

$$V^\wp(z) = V^\wp(z) 1^{1-\wp} \leq \wp\lambda V(z) + (1-\wp)\lambda^{\frac{-\wp}{1-\wp}}. \tag{13}$$

Let $\lambda = \wp^{-1}$, then one has

$$V^\wp(z) \leq V(z) + (1-\wp)\wp^{\frac{\wp}{1-\wp}}. \tag{14}$$

It follows that

$$-V(z) \leq -V^\wp(z) + (1-\wp)\wp^{\frac{\wp}{1-\wp}}. \tag{15}$$

From (11) and (15), we have

$$\dot{V}(z) \leq -a_0 V^\wp(z) + d_0,\ \ t \geq 0, \tag{16}$$

where $0 < \wp < 1$ and $d_0 = a_0(1-\wp)\wp^{\frac{\wp}{1-\wp}} + b_0$. □

3. ADAPTIVE TRACKING CONTROLLER DESIGN

The control goal of this manuscript is to establish an adaptive fuzzy controller $\nu(t)$, such that the desired control performance can be guaranteed in finite time.

Define $\hat{\Theta}_\iota$ as the estimation of Θ_ι, and $\check{\Theta}_\iota = \Theta_\iota - \hat{\Theta}_\iota$ with $\Theta_\iota = \|\varphi_\iota\|^2$.

The design procedure is based on the coordinate transformation as follows:

$$z_1 = \chi_1 - y_r, z_\iota = \chi_\iota - \alpha_{\iota-1},\ \ \iota = 2, \dots, n,$$

where $\alpha_{\iota-1}$ is an virtual controller.

Step 1: Consider the following Lyapunov function candidate:

$$V_1 = \frac{z_1^2}{2} + \frac{\check{\Theta}_1^2}{2\nu_1}.$$

Differentiating V_1 with respect to time t yields

$$\dot{V}_1 = z_1(\chi_2 + f_1(\chi_1) + p_1(t) - \dot{y}_r) - \frac{1}{v_1}\check{\Theta}_1\dot{\hat{\Theta}}_1. \tag{17}$$

Based on Lemma 3 , one has

$$z_1 z_2 \leq \frac{z_1^2}{2} + \frac{z_2^2}{2}, \tag{18}$$

and

$$z_1 p_1 \leq \frac{z_1^2}{2l_1} + \frac{l_1\tilde{p}_1^2}{2},$$

where $l_1 > 0$. Introduce one new function $\overline{f}_1 = f_1 + \frac{z_1}{2l_1} + z_1 - \dot{y}_r$, then it yields

$$\dot{V}_1 \leq z_1(\overline{f}_1 + \alpha_1) + \frac{l_1\tilde{p}_1^2}{2} - \frac{z_1^2}{2} + \frac{z_2^2}{2} - \frac{1}{v_1}\check{\Theta}_1\dot{\hat{\Theta}}_1. \tag{19}$$

According to Lemma 1, for $\forall\varepsilon_1 > 0$, there exists a FLS $\varphi_1^T\Re_1(X_1)$ satisfying

$$\overline{f}_1 = \varphi_1^T\Re_1(X_1) + \Delta_1(X_1), |\Delta_1| \leq \varepsilon_1, \tag{20}$$

where $X_1 = [\chi_1, y_r, \dot{y}_r]^T$. By using Lemma 3, we have

$$\begin{aligned} z_1\overline{f}_1 &= z_1\varphi_1^T\Re_1(X_1) + z_1\Delta_1(X_1) \\ &\leq \frac{1}{2m_1^2}z_1^2\Theta_1\Re_1^T\Re_1 + \frac{1}{2}m_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2. \end{aligned} \tag{21}$$

Choose a virtual control signal as

$$\alpha_1 = -k_1 z_1 - \frac{z_1}{2m_1^2}\hat{\Theta}_1\Re_1^T\Re_1. \tag{22}$$

Establish the adaptation law as

$$\dot{\hat{\Theta}}_1 = \frac{v_1}{2m_1^2}z_1^2\Re_1^T\Re_1 - \mu_1\hat{\Theta}_1, \tag{23}$$

where $\mu_1 > 0$. Then we have

$$\dot{V}_1 \leq -k_1 z_1^2 + \frac{1}{2}m_1^2 + \frac{1}{2}\varepsilon_1^2 + \frac{l_1\tilde{p}_1^2}{2} + \frac{z_2^2}{2} + \frac{\mu_1}{v_1}\check{\Theta}_1\dot{\hat{\Theta}}_1. \tag{24}$$

$\frac{z_2^2}{2}$ will be handled later on.

Step 2 : Construct the Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2v_2}\check{\Theta}_2^2. \tag{25}$$

Applying the same approach as step 1, we have

$$\dot{V}_2 = \dot{V}_1 + z_2(f_2 + z_3 + \alpha_2 + p_2 - \dot{\alpha}_1) - \frac{1}{v_2}\check{\Theta}_2\dot{\hat{\Theta}}_2. \tag{26}$$

It is noticed that

$$z_2 z_3 \leq \frac{z_2^2}{2} + \frac{z_3^2}{2}, \tag{27}$$

and

$$z_2 p_2 \leq \frac{z_2^2}{2l_2} + \frac{l_2\tilde{p}_2^2}{2}, \tag{28}$$

where $l_2 > 0$. Introduce one new function $\overline{f}_2 = f_2 + \frac{z_2}{2l_2} + \frac{3}{2}z_2 - \dot{\alpha}_1$, then it yields

$$\begin{aligned} \dot{V}_2 \leq& -k_1 z_1^2 + \frac{1}{2}m_1^2 + \frac{1}{2}\varepsilon_1^2 + \frac{l_1\tilde{p}_1^2}{2} + \frac{\mu_1}{v_1}\check{\Theta}_1\hat{\Theta}_1 + z_2\bar{f}_2 \\ &- \frac{1}{2}z_2^2 + z_2\alpha_2 + \frac{1}{2}z_3^2 + \frac{l_2}{2}\tilde{p}_2^2 - \frac{1}{v_2}\check{\Theta}_2\dot{\hat{\Theta}}_2. \end{aligned} \tag{29}$$

On the base of Lemma 1, for $\forall\varepsilon_2 > 0$, we can find a FLS $\varphi_2^T\Re_2(X_2)$ satisfying

$$\overline{f_2} = \varphi_2^T\Re_2(X_2) + \Delta_2(X_2), |\Delta_2| \leq \varepsilon_2, \tag{30}$$

where $X_2 = [\tilde{\chi}_2^T, \hat{\Theta}_1, \bar{y}_{r_2}^T]^T$. One has

$$\begin{aligned} z_2\overline{f}_2 &= z_2\varphi_2^T\Re_2(X_2) + z_2\Delta_2(X_2) \\ &\leq \frac{1}{2m_2^2}z_2^2\Theta_2\Re_2^T\Re_2 + \frac{1}{2}m_2^2 + \frac{1}{2}\varepsilon_2^2 + \frac{1}{2}z_2^2. \end{aligned} \tag{31}$$

Choose a virtual control signal is as

$$\alpha_2 = -k_2 z_2 - \frac{z_2}{2m_2^2}\hat{\Theta}_2\Re_2^T\Re_2. \tag{32}$$

Take the adaptation law as

$$\dot{\hat{\Theta}}_2 = \frac{v_2}{2m_2^2}z_2^2\Re_2^T\Re_2 - \mu_2\hat{\Theta}_2, \tag{33}$$

where $\mu_2 > 0$. Then we have

$$\begin{aligned} \dot{V}_2 \leq& \sum_{\ell=1}^{2}(-k_\ell z_\ell^2 + \frac{1}{2}m_\ell^2 + \frac{1}{2}\varepsilon_\ell^2 + \frac{l_\ell\tilde{p}_\ell^2}{2} \\ &+ \frac{\mu_\ell}{v_\ell}\check{\Theta}_\ell\hat{\Theta}_\ell) + \frac{1}{2}z_3^2. \end{aligned} \tag{34}$$

$\frac{z_3^2}{2}$ will be handled later on.

Step $\imath$ ($3 \leq \imath \leq n-1$)**:** Establish the Lyapunov function as

$$V_\imath = V_{\imath-1} + \frac{1}{2}z_\imath^2 + \frac{1}{2v_\imath}\check{\Theta}_\imath^2, \tag{35}$$

where $v_\imath > 0$ and $\check{\Theta}_\imath = \Theta_\imath - \hat{\Theta}_\imath$ is the parameter error. Similar to the approach in step 1, we have

$$\begin{aligned} \dot{V}_\imath =& \dot{V}_{\imath-1} + z_\imath(f_\imath + z_{\imath+1} + \alpha_\imath + p_\imath - \dot{\alpha}_{\imath-1}) \\ &- \frac{1}{v_\imath}\check{\Theta}_\imath\dot{\hat{\Theta}}_\imath. \end{aligned} \tag{36}$$

It is noticed that

$$z_\imath z_{\imath+1} \leq \frac{z_\imath^2}{2} + \frac{z_{\imath+1}^2}{2} \tag{37}$$

and

$$z_\iota p_\iota \leq \frac{z_\iota^2}{2l_\iota} + \frac{l_\iota \tilde{p}_\iota^2}{2}, \tag{38}$$

where $l_\iota > 0$. Define a new function $\overline{f}_\iota = f_\iota + \frac{z_\iota}{2l_\iota} + \frac{3}{2}z_\iota - \dot{\alpha}_{\iota-1}$, then plugging (37), (38) into (36) yields

$$\begin{aligned}\dot{V}_\iota \leq &\sum_{\ell=1}^{\iota-1}(-k_\ell z_\ell^2 + \frac{1}{2m_\ell^2} + \frac{1}{2\varepsilon_\ell^2} + \frac{l_\ell \tilde{p}_\ell^2}{2} + \frac{\mu_\ell}{v_\ell}\check{\Theta}_\ell\hat{\Theta}_\ell) \\ &+ z_\iota \bar{f}_\iota - \frac{1}{2}z_\iota^2 + z_\iota\alpha_\iota + \frac{1}{2}z_{\iota+1}^2 + \frac{l_\iota}{2}\tilde{p}_\iota^2 - \frac{1}{v_\iota}\check{\Theta}_\iota\hat{\Theta}_\iota.\end{aligned} \tag{39}$$

On the basis of Lemma 1, for $\forall\varepsilon_\iota > 0$, we can find a FLS $\varphi_\iota^T\Re_\iota(X_\iota)$ satisfying

$$\overline{f_\iota} = \varphi_\iota^T\Re_\iota(X_\iota) + \Delta_\iota(X_\iota), |\Delta_\iota| \leq \varepsilon_\iota \tag{40}$$

where $X_\iota = [\tilde{\chi}_\iota^T, \hat{\Theta}_{\iota-1}, \bar{y}_{r_\iota}^T]^T$. One has

$$\begin{aligned}z_\iota\overline{f}_\iota &= z_\iota\varphi_\iota^T\Re_\iota(X_\iota) + z_\iota\Delta_\iota(X_\iota) \\ &\leq \frac{1}{2m_\iota^2}z_\iota^2\Theta_\iota\Re_\iota^T\Re_\iota + \frac{1}{2}m_\iota^2 + \frac{1}{2}\varepsilon_\iota^2 + \frac{1}{2}z_\iota^2.\end{aligned} \tag{41}$$

An virtual control signal is taken as

$$\alpha_\iota = -k_\iota z_\iota - \frac{z_\iota}{2m_\iota^2}\hat{\Theta}_\iota\Re_\iota^T\Re_\iota. \tag{42}$$

Take the adaptation law as

$$\dot{\hat{\Theta}}_\iota = \frac{v_\iota}{2m_\iota^2}z_\iota^2\Re_\iota^T\Re_\iota - \mu_\iota\hat{\Theta}_\iota, \tag{43}$$

where $\mu_\iota > 0$. Plugging (41)-(43) into (39) , we have

$$\begin{aligned}\dot{V}_\iota \leq &\sum_{\ell=1}^{\iota}(-k_\ell z_\ell^2 + \frac{1}{2}m_\ell^2 + \frac{1}{2}\varepsilon_\ell^2 + \frac{l_\ell\tilde{p}_\ell^2}{2} \\ &+ \frac{\mu_\ell}{v_\ell}\check{\Theta}_\ell\hat{\Theta}_\ell) + \frac{1}{2}z_{\iota+1}^2.\end{aligned} \tag{44}$$

Step *n*: Establish the Lyapunov function candidate as follows:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2v_n}\check{\Theta}_n^2. \tag{45}$$

Differentiating V_n with respect to time t yields

$$\begin{aligned}\dot{V}_n =&\dot{V}_{n-1} + z_n(f_n + D(v) + p_n - \dot{\alpha}_{n-1}) \\ &- \frac{1}{v_n}\check{\Theta}_n\dot{\hat{\Theta}}_n.\end{aligned} \tag{46}$$

It is noticed that

$$z_n p_n \leq \frac{z_n^2}{2l_n} + \frac{l_n\tilde{p}_n^2}{2}, \tag{47}$$

where $l_n > 0$. Define a new function $\overline{f}_n = f_n + \frac{z_n}{2l_\iota} + z_n - \dot{\alpha}_{n-1}$, then plugging (47) into (46) yields

$$\begin{aligned}\dot{V}_n \leq &\sum_{\ell=1}^{n-1}(-k_\ell z_\ell^2 + \frac{1}{2m_\ell^2} + \frac{1}{2\varepsilon_\ell^2} + \frac{l_\ell\tilde{p}_\ell^2}{2} + \frac{\mu_\ell}{v_\ell}\check{\Theta}_\ell\hat{\Theta}_\ell) \\ &+ z_n\bar{f}_n - \frac{1}{2}z_n^2 + z_nD(v) + \frac{l_n}{2}\tilde{p}_n^2 - \frac{1}{v_n}\check{\Theta}_n\hat{\Theta}_n.\end{aligned} \tag{48}$$

On the basis of Lemma 1, for $\forall\varepsilon_n > 0$, we can find a FLS $\varphi_n^T\Re_n(X_n)$ satisfying

$$\overline{f_n} = \varphi_n^T\Re_n(X_n) + \Delta_n(X_n), |\Delta_n| \leq \varepsilon_n \tag{49}$$

where $X_n = [\bar{\chi}_n^T, \hat{\Theta}_n, \bar{y}_{r_n}^T]^T$.

Based on Lemma 3 , one has

$$\begin{aligned}z_n\overline{f}_n &= z_n\varphi_n^T\Re_n(X_n) + z_n\Delta_n(X_n) \\ &\leq \frac{1}{2m_n^2}z_n^2\Theta_n\Re_n^T\Re_n + \frac{1}{2}m_n^2 + \frac{1}{2}\varepsilon_n^2 + \frac{1}{2}z_n^2.\end{aligned} \tag{50}$$

The controller law is chosen as

$$v = -\frac{k_n}{\check{\xi}}z_n - \frac{1}{2m_n^2\check{\xi}}z_n\hat{\Theta}_n\Re_n^T\Re_n. \tag{51}$$

Take the adaptation law as

$$\dot{\hat{\Theta}}_n = \frac{v_n}{2m_n^2}z_n^2\Re_n^T\Re_n - \mu_n\hat{\Theta}_n. \tag{52}$$

According to [46], for any given bounded initial condition $\hat{\Theta}_n(t_0) \geq 0$, one has $\hat{\Theta}_n(t) \geq 0$ for $\forall t \geq 0$.

Plugging (50)-(52) into (48), we have

$$\begin{aligned}\dot{V}_n \leq &\sum_{\ell=1}^{n} -k_\ell z_\ell^2 + \sum_{\ell=1}^{n}\frac{1}{2}m_\ell^2 + \sum_{\ell=1}^{n}\frac{1}{2}\varepsilon_\ell^2 \\ &+ \sum_{\ell=1}^{n}\frac{l_\ell\tilde{p}_\ell^2}{2} + \sum_{\ell=1}^{n}\frac{\mu_\ell}{v_\ell}\check{\Theta}_\ell\hat{\Theta}_\ell + \frac{1}{2}\varpi_0^2.\end{aligned} \tag{53}$$

4. STABILITY ANALYSIS

Theorem 1: Consider the uncertain nonlinear system (1) with Assumption 1. Under control law (51) and adaptive laws (23), (33), (43), (52), all the signals in the system are semi-globally uniformly finite-time bounded (SGUFB) for any bounded initial conditions.

Proof: Choose a Lyapunov function candidate $V = V_n$ for system (1). From (53), we have

$$\begin{aligned}\dot{V} \leq &-\check{k}\sum_{\ell=1}^{n}z_\ell^2 + \sum_{\ell=1}^{n}(\frac{1}{2}m_\ell^2 + \frac{1}{2}\varepsilon_\ell^2 + \frac{l_\ell\tilde{p}_\ell^2}{2} \\ &+ \frac{\mu_\ell}{v_\ell}\check{\Theta}_\ell\hat{\Theta}_\ell) + \frac{1}{2}\varpi_0^2,\end{aligned} \tag{54}$$

where $\check{k} = min_{\ell=1,\dots,n}k_\ell$.

Nothing that $\check{\Theta}_\ell = \Theta_\ell - \hat{\Theta}_\ell$, one has

$$g_\ell\check{\Theta}_\ell\hat{\Theta}_\ell = g_\ell\check{\Theta}_\ell(-\check{\Theta}_\ell + \Theta_\ell) = g_\ell(-\check{\Theta}_\ell^2 + \check{\Theta}_\ell\Theta_\ell)$$

$$\le g_\ell(-\check{\Theta}_\ell^2+\frac{1}{2s}\check{\Theta}_\ell^2+\frac{s}{2}\Theta_\ell^2)$$
$$=\frac{-g_\ell(2s-1)}{2s}\check{\Theta}_\ell^2+\frac{sg_\ell}{2}\Theta_\ell^2, \quad (55)$$

where s is a positive constant satisfying $s\ge\frac{1}{2}$, and $g_\ell=\frac{\mu_\ell}{v_\ell}$. Substituting (55) into (54) yields

$$\dot{V}\le -a_0V+b_0, \quad (56)$$

where $a_0=min\{2\check{k},\frac{\mu_1(2s-1)}{s},\dots,\frac{\mu_n(2s-1)}{s}\}$ with $s>\frac{1}{2}$ and

$$b_0=\sum_{\ell=1}^{n}(\frac{s\mu_\ell}{2v_\ell}\Theta_\ell^2+\frac{1}{2}m_\ell^2+\frac{1}{2}\varepsilon_\ell^2+\frac{l_\ell\tilde{p}_\ell^2}{2})+\frac{1}{2}\varpi_0^2. \quad (57)$$

According to Lemma 5 and (56), one has

$$\dot{V}\le -a_0V^{\wp}+d_0, \quad (58)$$

where

$$d_0=a_0(1-\wp)\wp^{\frac{\wp}{1-\wp}}+b_0. \quad (59)$$

Define a positive constant $\varsigma_0=\frac{d_0}{(1-\zeta_0)a_0}$, where ζ_0 is a constant which satisfies $0<\zeta_0<1$.

Let

$$T_r=\frac{1}{(1-\wp)\zeta_0a_0}[V^{1-\wp}(X(0))-\varsigma_0^{\frac{1-\wp}{\wp}}], \quad (60)$$

where $V(X(0))$ represents the initial of $V(X)$ with $X=[\bar{\chi}_n^T,\hat{\Theta}_n,\bar{y}_{r_n}^T]^T$.

Then according to Lemma 4, If $z_t\in\tilde{\Omega}_X=\{X|V^{\wp}(X)\ge\frac{d_0}{(1-\zeta_0)a_0}\}$, we have

$$\dot{V}(X)\le -\zeta_0a_0V^{\wp}(X)\le 0,\ \ t\in[t_0,T_r). \quad (61)$$

The time to reach the set $X(t)\in\Omega_X$, is bounded as T_r where $\Omega_X=\{X|V^{\wp}(X)\le\frac{d_0}{(1-\zeta_0)a_0}\}$. It follows that $z\in\Omega_I\cup\Omega_X$, for all $t\ge t_0$, where $\Omega_I=\{X|V^{\wp}(X)\le V_n(X(0))\}$.

If $X(0)\in\Omega_X$, $X(t)(t\ge t_0)$ does not exceed the set Ω_X. Consequently, all signals in the resulting system are SGUFB.

It can be seen from (57) and (59) that d_0 depends upon the parameters $s,\mu_\ell,v_\ell,m_\ell,\varepsilon_\ell,l_\ell,\varpi_0,\ell=1,2,\dots,n$. By appropriately choosing these parameters, for example, increasing v_ℓ, decreasing μ_ℓ and m_ℓ, d_0 will be reduced. On the other hand, a smaller μ_ℓ will lead to a smaller a_0, then by (60), the convergent rate of the state will become smaller. Accordingly, there is a tradeoff between the bound of the neighborhood and the convergent rate. □

5. SIMULATION EXAMPLE

In this section, to testify the results obtained, two simulation examples are performed.

5.1. Example 1

The nonlinear system with dead-zone input is described as follows:

$$\dot{\chi}_1=\chi_2+(1+\sin^2(\chi_1))\chi_2+\frac{\cos(t)}{10},$$
$$\dot{\chi}_2=D(v)-\frac{5\chi_2}{2}+\chi_1\chi_2^2+\frac{\sin(2t)}{10},$$
$$y=\chi_1, \quad (62)$$

where $D(v)$ is defined as (2).

Choose the dead-zone parameters as $\Im_1=1$, $\Im_2=-1$, $\xi_1=\xi_2=1.5$. The reference signal is chosen as $y_r=\sin(\frac{t}{2})+\frac{t}{2}\sin(t)$.

Based on Theorem 1, the adaptive laws (23,52), the intermediate control function (22) and the control law (51) are chosen.

The related simulation parameters are selected as $m_1=0.8$, $m_2=1$, $\mu_1=1$, $\mu_2=2$, $v_1=20$, $v_2=25$. Choose the initial conditions as $\chi_1(0)=1$, $\chi_2(0)=-0.5$, $\hat{\Theta}_1(0)=0.1$ and $\hat{\Theta}_2(0)=0.2$.

The fuzzy membership functions are chosen as

$$\mu_{M_n^1}(\chi)=(1+\exp(5\chi+10))^{-1},$$
$$\mu_{M_n^k}(\chi)=\exp(-(\chi+3.5-k)^2),\ \ k=2,3,4,5,$$
$$\mu_{M_n^6}(\chi)=(1+\exp(-5\chi+10))^{-1}. \quad (63)$$

The results of simulation are shown in Figs. 1-4.

To give some suggestions in choosing the design parameters, we choose two groups of variables as $v_1=20$, $v_2=25$, $m_1=0.8$, $m_2=1$ and $v_1=5$, $v_2=5$, $m_1=10$, $m_2=10$, respectively, while the rest of parameters remain the same. Compared with the existing control strategies, a previous adaptive control scheme proposed in [31] is also utilized to control this system with the same controller parameters and $\wp=0.93$. The simulation results are shown in Figs. 5-7. It can be seen from Figs. 5-6 that the convergence region of the tracking error in Fig. 5 is smaller than

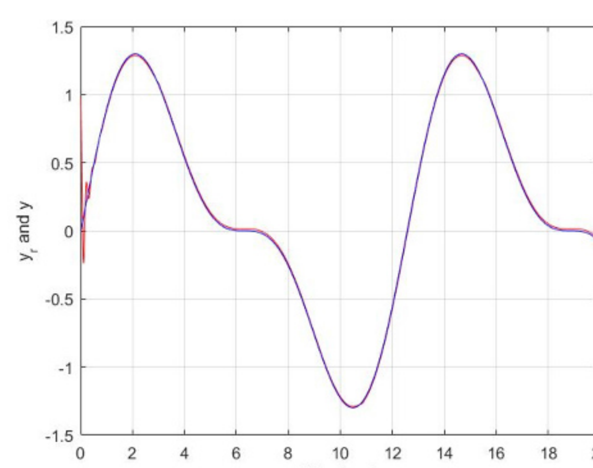


Fig. 1. y and y_r.

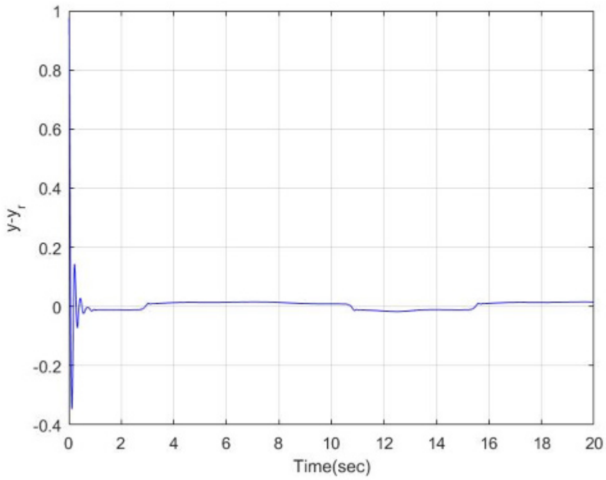


Fig. 2. $y - y_r$.

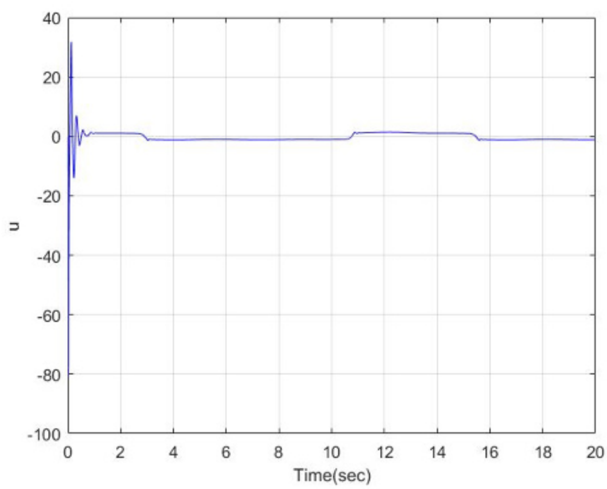


Fig. 3. u.

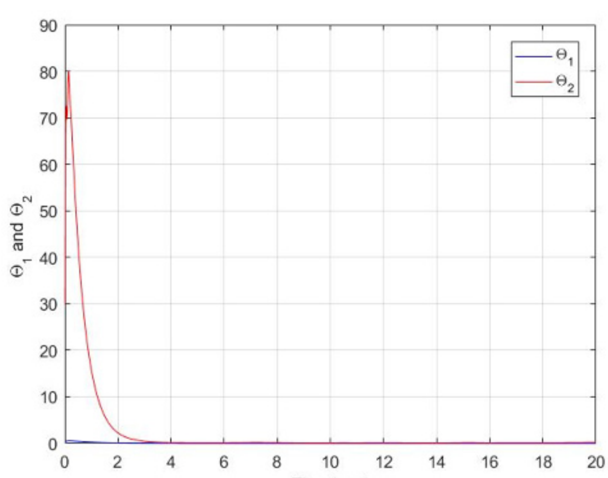


Fig. 4. Θ_1 and Θ_2.

that in Fig. 6. From Figs. 5 and 7, we see that the tracking errors converge to a small neighborhood of the origin in finite time $T_r \approx 0.65s$ and $T_r \approx 4.60s$, respectively.

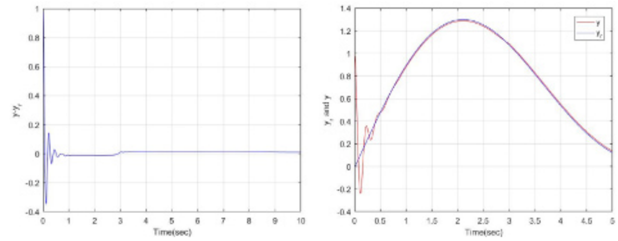


Fig. 5. $y - y_r$, y and y_r with $v_1 = 20, v_2 = 25, m_1 = 0.8, m_2 = 1$.

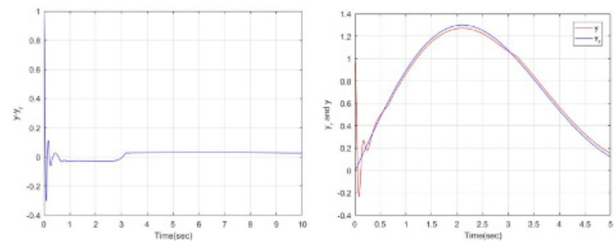


Fig. 6. $y - y_r$, y and y_r with $v_1 = 5, v_2 = 5, m_1 = 10, m_2 = 10$.

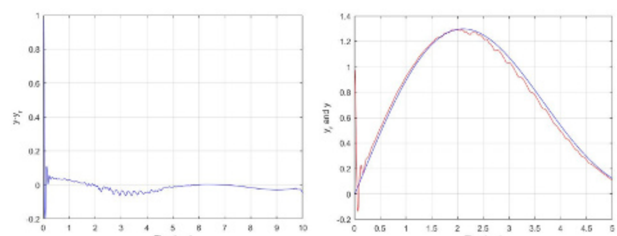


Fig. 7. $y - y_r$, y and y_r with the controller in [31].

5.2. Example 2

Consider the tracking control of a one-link manipulator actuated by a brush dc motor [41]. The nonlinear system is described as:

$$\begin{aligned} &R\ddot{q} + S\dot{q} + W sin(q) = p + 4sin(t), \\ &C\dot{p} = Ap + L_m\dot{q} + I, \end{aligned} \tag{64}$$

where q, $\dot{q}$, $\ddot{q}$ denote the link angular position, velocity and acceleration, respectively. p is the motor current. The disturbance is chosen as $4sin(t)$. I is the input voltage. Choose the appropriate parameters as $R = 1$, $S = 1$, $C = 1$, $A = -0.5$, $W = 2.2$ and $L_m = -5$.

Let $\chi_1 = q$, $\chi_2 = \dot{q}$, $\chi_3 = p$ and $I = D(v)$. (64) can be rewritten as

$$\begin{aligned} &\dot{\chi}_1 = \chi_2, \\ &\dot{\chi}_2 = -W\sin(\chi_1) - \chi_2 + \chi_3 + 4\sin(t), \\ &\dot{\chi}_3 = L_m\chi_2 + A\chi_3 + D(v), \\ &y = \chi_1. \end{aligned} \tag{65}$$

$D(v)$ is defined as (2). Choose the dead-zone parameters as $\Im_1 = 0.7$, $\Im_2 = 0.4$, $\xi_1 = \xi_2 = 1.5$. The reference

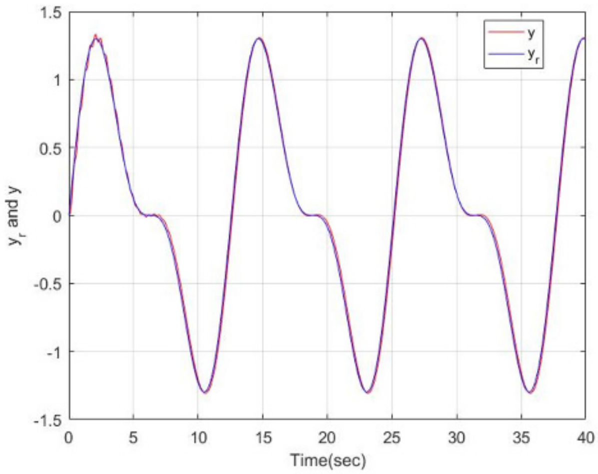


Fig. 8. y and y_r.

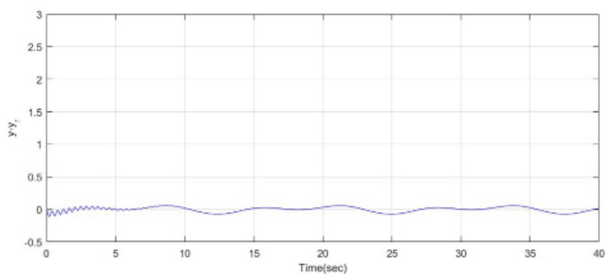


Fig. 9. $y - y_r$.

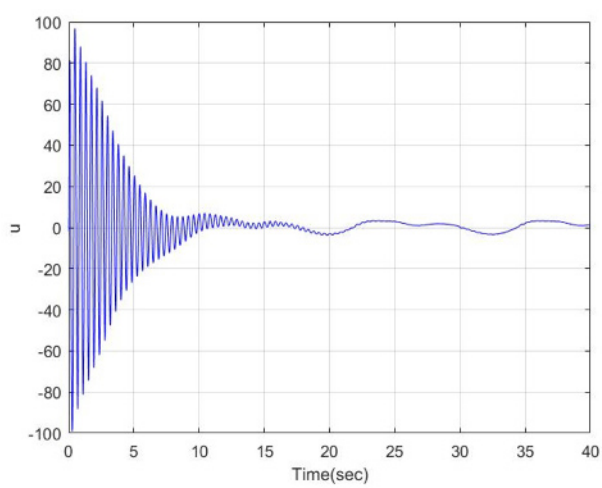


Fig. 10. u.

signal is chosen as $y_r = \sin(\frac{t}{2}) + \frac{1}{2}\sin(t)$. The fuzzy membership functions are chosen as (63).

Based on Theorem 1, adaptive laws (23,52) and the intermediate control function (22) and the control law (51) are chosen. The related simulation parameters are selected as $m_1 = \frac{4}{5}$, $m_2 = 1$, $m_3 = \frac{4}{5}$, $\mu_1 = 1$, $\mu_2 = 2$, $\mu_3 = 2$, $v_1 = 20$, $v_2 = 25$, $v_3 = 25$. Choose the initial conditions as $\chi_1(0) = \frac{1}{100}$, $\chi_2(0) = \frac{1}{100}$, $\chi_3(0) = \frac{1}{100}$, $\hat{\Theta}_1(0) = \frac{1}{10}$, $\hat{\Theta}_2(0) = \frac{1}{5}$ and $\hat{\Theta}_3(0) = \frac{1}{10}$. Simulation results are shown in Figs. 8-11.

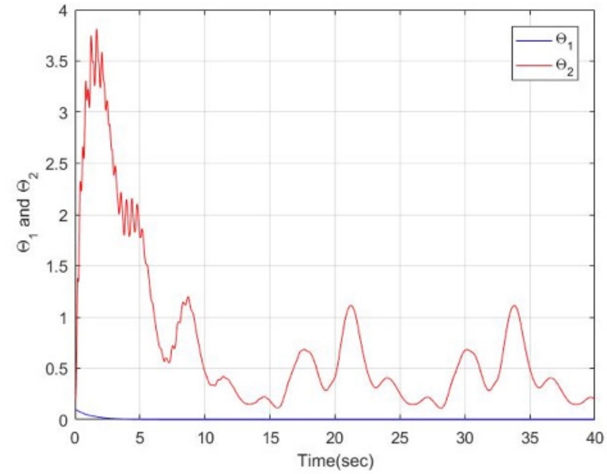


Fig. 11. Θ_1 and Θ_2.

6. CONCLUSION

Combining backstepping technique and FlSs, an adaptive controller of uncertain nonlinear system with deadzone input has been proposed. Finite time stability of the nonlinear system was ensured by using a novel Lyapunov theory. In the simulation, the proposed control scheme was proved to be able to achieve the desired tracking performance of the system in finite time.

It is known that there are many uncertainties in practical nonlinear systems, such as time delays uncertainties, unknown dynamics disturbances, unmodeled dynamics and so on [47]. Therefore, it is an interesting issue to apply the control scheme proposed in this paper for systems with these certainties in the future. As one of the most famous nonlinear control strategies, SMC technique has the advantages of quick response and strong robustness [48–52]. So We would like to combine the SMC technique with the proposed finite time adaptive control method in the future.

REFERENCES

[1] Y. M. Li, S. C. Tong, L. Liu, and G. Feng, "Adaptive output-feedback control design with prescribed performance for switched nonlinear systems," *Automatica*, vol. 80, pp. 225-231, 2017.

[2] F. Wang, X. Y. Zhang, B. Chen, C. Lin, X. H. Li, and J. Zhang, "Adaptive finite-time tracking control of switched nonlinear systems," *Inf. Sci.*, vol. 421, pp. 126-135, 2017.

[3] F. Wang, B. Chen, C. Lin, J. Zhang, and X. Meng, "Adaptive neural network finite-time output feedback control of quantized nonlinear systems," *IEEE Trans. Cybern.*, vol. 48, no. 6, pp. 1839-1848, 2017.

[4] Y. Yu and Y. S. Zhong, "Semiglobal robust backstepping output tracking for strict-feedback form systems with nonlinear uncertainty," *Int. J. Control Autom. Syst.*, vol. 9, no. 2, pp. 366-375, 2011.

[5] Y. M. Li, L. Liu, and G. Feng, "A robust adaptive output feedback control approach to a class of non-triangular stochastic nonlinear systems," *Automatica*, vol. 89, pp. 325-332, 2018.

[6] X. Zhang, X. Liu, and Y. Li, "Adaptive fuzzy tracking control for nonlinear strict-feedback systems with unmodeled dynamics via backstepping technique," *Neurocomputing*, vol. 235, pp. 182-191, 2017.

[7] S. C. Tong, Y. M. Li, and S. Sui, "Adaptive fuzzy tracking control design for SISO uncertain non-strict feedback nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 6, pp.1441-1454, 2016.

[8] Y. J. Liu, M. Z. Gong, S. C. Tong, C. L. P. Chen, and D. J. Li, "Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints," *IEEE Trans. Fuzzy Syst.*, 2018.

[9] Q. Zhou, H. Li, L. Wang, and R. Lu, "Prescribed performance observer-based adaptive fuzzy control for nonstrict-feedback stochastic nonlinear systems," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. PP, no. 99, pp. 1-12, 2017.

[10] L. Wang, M. Basin, H. Li, and R. Lu, "Observer-based composite adaptive fuzzy control for nonstrict-feedback systems with actuator failures," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 4, pp. 2338-2347, 2017.

[11] Z. Tang, J. H. Park, and T. H. lee, " Dynamic output-feedback-based H∞ design for networked control systems with multipath packet dropouts," *Appl. Math Comput.*, vol. 275, pp. 121-133, 2016.

[12] C. D. Li, J. L. Gao, J. Q. Yi, and G. Q. Zhang, "Analysis and design of functionally weighted single-Input-Rule-Modules connected fuzzy inference systems," *IEEE Trans. Fuzzy Syst.*, vol. 26 no. 1, pp. 56-71, 2018.

[13] C. D. Li, Z. X. Ding, D. B. Zhao, J. Q. Yi, and G. Q. Zhang, "Building energy consumption prediction: an extreme deep learning approach," *Energies*, vol. 10, no. 10, pp. 1-20, 2017.

[14] Y. M. Li and S. C. Tong, "Fuzzy adaptive control design strategy of nonlinear switched large-scale systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. PP, no. 99, pp. 1-11, 2017.

[15] Y. M. Li and S. C. Tong, "Adaptive neural networks prescribed performance control design for switched interconnected uncertain nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 7, pp. 3059-3068, 2017.

[16] Y. M. Li, Z. Y. Ma, and S. C. Tong, "Adaptive fuzzy fault-tolerant control of non-triangular structure nonlinear systems with error-constraint," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 4, pp. 2062-2074, 2017.

[17] R. R. Liu, Y. Li, and X. K. Liu, "Linear-quadratic optimal control for unknown mean-field stochastic discrete-time system via adaptive dynamic programming approach," *Neurocomputing*, vol. 282, pp.16-24, 2018.

[18] S. C. Tong, K. K. Sun, and S. Sui, "Observer-based adaptive fuzzy decentralized optimal control design for strict feedback nonlinear large-scale systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 569-584, 2018.

[19] J. M. Wang, H. D. Cheng, Y. Li, and X. N. Zhang, "The geometrical analysis of a predator-prey model with multi-state dependent impulses," *J. Appl. Anal. Comput.*, vol. 8, no. 2, pp. 427-442, 2018.

[20] Y. J. Liu, S.M. Lu, S. C. Tong, X. K. Chen, C. L. P. Chen, and D. J. Li. "Adaptive control-based Barrier Lyapunov functions for a class of stochastic nonlinear systems with full state constraints," *Automatica*, vol. 87, pp. 83-93, 2018.

[21] F. Liu, "Continuity and approximate differentiability of multisublinear fractional maximal functions," *Math. Inequal. Appl.*, vol. 21, no. 1, pp. 25-40, 2018.

[22] Y. Li, H. Cheng, J. Wang, and Y. Wang, "Dynamic analysis of unilateral diffusion Gompertz model with impulsive control strategy," *Adv. Differ. Equ.*, vol. 2018, no. 32, 2018.

[23] X. P. Li, X. Y. Lin, and Y. Q. Lin, "Lyapunov-type conditions and stochastic differential equations driven by G-brownian motion," *J. Math. Anal. Appl.*, vol. 439, no. 1, pp. 235-255, 2016.

[24] F. F. Bian, W. C. Zhao, Y. Song, and R. Yue, "Dynamical analysis of a class of prey-predator model with Beddington-DeAngelis functional response, stochastic perturbation, and impulsive toxicant Input," *Complexity*, vol. 2017, Article ID 3742197, 2017.

[25] B. Niu, H. Li, T. Qin, and H. R. Karimi, "Adaptive NN dynamic surface controller design for nonlinear pure-feedback switched systems with time-delays and quantized input," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. PP, no. 99, pp. 1-13, 2017.

[26] F. Liu and H. X. Wu, "Regularity of discrete multisublinear fractional maximal functions," *Sci. China Math.*, vol. 60, no. 8, pp. 1461-1476, 2017.

[27] B. Niu and L. Li, "Adaptive backstepping-based neural tracking control for MIMO nonlinear switched systems subject to input delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 6, pp. 2638-2644, 2017.

[28] T. Zhang, S. S. Ge, and C. C. Hang, "Adaptive neural network control for strict-feedback nonlinear systems using backstepping design," *Automatica*, vol. 36, no. 12, pp. 1835-1846, 2000.

[29] M. Chen, S. S. Ge, and B. B. Ren, "Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints," *Automatica*, vol. 47, no. 3, pp. 452-465, 2011.

[30] Y. Yang, C. Hua, and X.Guan, "Adaptive fuzzy finite-time coordination control for networked nonlinear bilateral teleoperation system," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 3, pp. 631-641, 2014.

[31] Y. M. Sun, B. Chen, C. Lin, and H. H. Wang, "Finite-time adaptive control for a class of nonlinear systems with nonstric," *IEEE Trans. Cybern.*, vol. PP, no. 99, pp. 1-9, 2017.

[32] S. P. Bhat and D. S. Bernstein, "Continuous finite-time stabilization of the translational and rotational double integrators," *IEEE Trans. Autom. Control*, vol. 43, no. 5, pp. 678-682, 1998.

[33] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751-766, 2000.

[34] Z. Zhu, Y. Q. Xia, and M. Y. Fu, "Attitude stabilization of rigid spacecraft with finite-time convergence," *Int. J. Robust Nonlinear Control*, vol. 21, no. 6, pp. 686-702, 2011.

[35] S. C. Tong and Y. M. Li, "Adaptive fuzzy output feedback tracking backstepping control of strict-feedback nonlinear systems with unknown dead zones," *IEEE Trans. Fuzzy Syst.* vol. 20, no. 1, pp. 168-179, 2012.

[36] S. C. Tong and Y. M. Li, "Adaptive fuzzy output feedback control of MIMO nonlinear systems with unknown dead-zone input," *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 1, pp. 134-146, 2013.

[37] J. Zhou, C. Wen, and Y. Zhang, "Adaptive output control of nonlinear systems with uncertain dead-zone nonlinearity," *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 504-511, 2006.

[38] W. J. Si, X. D. Dong, and F. F. Yang, "Adaptive neural dynamic surface control for a general class of stochastic nonlinear systems with time delays and input dead-zone," *Int. J. Control Autom. Syst.*, vol. 15, no. 5, pp. 2416-2424, 2017.

[39] H. Su and W. H. Zhang, "A combined backstepping and dynamic surface control to adaptive fuzzy state-feedback control," *Int. J. Control Autom. Syst.*, vol. 31, no. 11, pp. 1666-1685, 2017.

[40] Z. Q. Zhang, J. H. Park, K. M. Zhang, and J. W. Lu, "Adaptive control for a class of nonlinear time-delay systems with dead-zone input," *J. Franklin Inst.*, vol. 353, no. 17, pp. 4400-4421, 2016.

[41] Y. Cui, H. Zhang, Y. Wang, and W. Gao, "Adaptive control for a class of uncertain strict-feedback nonlinear systems based on a generalized fuzzy hyperbolic model," *Fuzzy Sets Syst.*, vol. 302, pp. 52-64, 2016.

[42] M. Z. Hou and G. R. Duan, "Robust adaptive dynamic surface control of uncertain nonlinear systems," *Int. J. Control Autom. Syst.*, vol. 9, no. 1, pp. 161-168, 2011.

[43] L. X. Wang and J. M. Mendel, "Fuzzy basis functions, universal approximation, and orthogonal least-squares learning," *IEEE Trans. Neural Netw.*, vol. 3, no. 5, pp. 807-814, 1992.

[44] G. H. Hardy, J. E. Littlewood, and G. Polya, *Inequalities*, Cambridge University Press, Cambridge, 1952.

[45] C. Qian and W. Lin, "Non-Lipshitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization," *Syst. Control Lett.*, vol. 42, no. 3, pp. 185-200, 2001.

[46] X. C. Shi, S. Y. Xu, Y. M. Li, W. M. Chen, and Y.M. Chu, "Robust adaptive control of strict-feedback nonlinear systems with unmodelled dynamics and time-varying delays," *Int. J. Control*, vol. 90, no. 2, pp. 334-347, 2016.

[47] J. H. Park and P. G. Park, "Special issue on time-delay systems and their applications" *Int. J. Control Autom. Syst.*, vol. 14, no. 1, pp. 1-2, 2016.

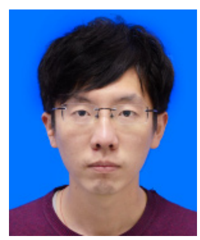
[48] Y. Y. Wang, H. Shen, H. R. Karimi, and D. P. Duan,"Dissipativity-based Fuzzy Integral Sliding Mode Control of Continuous-Time T-S Fuzzy Systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1164-1176, 2017.

[49] Y. Y. Wang, Y. B. Gao, H. R. Karimi, H. Shen, Z. J. Fang, and Senior, "Sliding mode control of fuzzy singularly perturbed systems with application to electric circuit," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. PP, no. 99, pp. 1-9, 2017.

[50] Y. Y. Wang, Y. Q. Qing, H. Shen, and P. F. Zhou, "SMC design for robust stabilization of nonlinear Markovian jump singular systems," *IEEE Trans. Autom. Control*, vol. 63, no. 1, pp. 219-224, 2018.

[51] X. Y. Chen, J. H. Park, J. D. Cao, and J. L. Qiu, "Adaptive synchronization of multiple uncertain coupled chaotic systems via sliding mode control," *Neurocomputing*, vol. 273, pp. 9-21, 2018.

[52] T. H. Lee and J. H. Park, "Improved stability conditions of time-varying delay systems based on new Lyapunov functionals," *J. Franklin Inst.*, vol. 335, no. 3, pp. 1176-1191, 2018.



Wenshun Lv received his B.S. degree from Shandong University of Science and Technology, China, in 2015. He is currently pursuing an M.S. degree with the College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, China. His research interests include fuzzy control, neural network control, backstepping control, and adaptive control.

Fang Wang received his B.S. degree from the Qufu Normal University, Qufu, China, an M.S. degree from Shandong Normal University, Jinan, China, and a Ph.D. degree from Guangdong University of Technology, Guangzhou, China, in 1997, 2004, and 2015, respectively. Since 2005, she has been at the Shandong University of Science and Technology, Qingdao, China. Her current research interests include stochastic nonlinear control systems, multi-agent systems, quantized control, and adaptive fuzzy control.

Lili Zhang received her M.S. degree in applied mathematics from University of Science and Technology Beijing, Beijing, P. R. China, in 2004, and her Ph.D degree in school of automation, Guangdong University of Technology, Guangzhou, China, in 2014. She is currently an associate professor in School of Applied Mathematics, Guangdong University of Technology, Guangzhou, China. Her research interests include control and synchronization for complex dynamical networks and chaotic systems.