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Variable Gains Sliding Mode Control

Sergio Alvarez-Rodríguez*, Gerardo Flores, and Noé Alcalá Ochoa

Abstract: In this paper a sliding mode control with variable gains is proposed. Such a controller has chattering-effect reduction without detriment to the robustness properties of the sliding modes. The key idea behind the control design is that the variable gains magnitude is proportional to the trajectory tracking error magnitude. Also, this design establishes a connection between both the first and the second order sliding modes control. It is demonstrated that the proposed controller does not overestimates disturbances, which significantly reduces the control energy used. Finally, a stability analysis in the sense of Lyapunov is developed to demonstrate finite time convergence to the origin; simulations experiments are carried out to show the effectiveness and robustness of the proposed controller.

Keywords: Chattering-effect, control variable gains, sliding Modes, twisting control.

1. INTRODUCTION

1.1. Background and motivation

In recent robust control literature, numerous approaches have been proposed in order to achieve satisfactory control performances for nonlinear systems. One of those approaches is the Sliding Modes (SM) technique, a particular class of variable structure control which was first introduced by Emelyanov [1]. As it has been set by Emelyanov, this control design framework has many attractive features including the ability to counteract the effect of uncertainties and disturbances, which are present in most practical systems. Nevertheless, it is well known that SM control (SMC) has a major drawback: the chattering-effect [2–4]. In practice, the chattering-effect is produced by the switching of the control signal [5, 6], causing high frequency mechanical vibrations, heat, mechanical wear, noise, among others. In order to overcome the chattering problem, the High Order Sliding Modes (HOSM) concept was introduced in the Ph.D. dissertation of A. Levant. Examples of HOSM are: Drift Algorithm, Twisting Algorithm (TA), Super-Twisting Algorithm (STA), and Integral Sliding Mode controllers [7, 8]. Nevertheless, in practice every SMC is susceptible to produce chattering, since it inherits such characteristic from the *variable structure systems*, a wider class of systems that includes SMC, which by nature present the chattering effect [9].

1.2. Literature review

The beginnings of adaptivity for SM control are established in [9], where it is treated the control gain proportional to the system state. This key idea of proposing the control gain as a function of system information is taken in several works. For instance, in [10], the control gain depends on the distance of the system state to a discontinuity surface. In the same way, [11] presents a Lyapunov-based variable gain for using STA approach applied to linear time invariant systems (LTIS); it provides an attenuation of the chattering-effect. A similar approach also applied to LTIS is presented in [12], where gain adaptation of STA is applied. This approach makes the control strategy global, but also provides attenuation of the chattering-effect. In the same sense, [13] presents an adaptive STA for the control of an electro-pneumatic actuator; this approach uses some dynamically adapted control gains. In the same way, in [14], an adaptive version of the TA is proposed, in which due to dynamic adaptation of the gains, the controller design does not require information about uncertainties and disturbances. Since chattering magnitude is proportional to the control discontinuity [15], one approach to reduce such chattering is by using adaptation methods. In this sense, some adaptive laws with SM approach have been investigated in several works such as [15–17]. In [15], the efforts were oriented to the application of adaptivity principles to reduce the consequences of chattering-effect. In [16], a systematic adaptive SM con-

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Sergio Alvarez-Rodríguez is with the Instituto Tecnológico José Mario Molina Pasquel y Henriquez, Lagos de Moreno, Jalisco, 47480 México (e-mail: ser@cio.mx). Gerardo Flores and Noé Alcalá Ochoa are with the Centro de Investigaciones en Óptica, León, Guanajuato, 37150 México (e-mails: {gflores, alon}@cio.mx).
* Corresponding author.

troller design is proposed for the robust control of nonlinear systems with uncertain parameters, where an adaptation tuning approach is developed to deal with unknown but bounded system uncertainties. Also, an adaptive law using integral sliding mode control concepts for disturbance rejection and considering system uncertainties is presented in [17].

In [18], the sliding-mode control problem is treated for T-S fuzzy-model-based nonlinear Markovian jump singular systems subject to matched/unmatched uncertainties. While in [19], dissipativity based fuzzy integral sliding mode control of continuous-time are addressed, taking Takagi-Sugeno fuzzy systems with matched/unmatched uncertainties and external disturbances. Despite these interesting approaches, the aim of the present work, is to increase conceptual basis of SM, taking generic control systems.

Another interesting idea to remove chattering is presented in [20], which uses an hyperbolic tangent function to reach the control law, however, it does not establishes a relationship between r-orders of SMC (see the last issue in next subsection: "Contribution" of the present work).

The problem studied herein can also be addressed by the ideal of other switching-based adaptive techniques as those proposed in [21–23], nevertheless, the present approach takes advantage of the robust properties of SM, whereas the last cited works use another control techniques.

1.3. Contribution

Conventional SM and TA are designed to operate with fixed parametric gains. Even when such gains are correctly tuned, it is possible that disturbances overestimation is presented, and thus generating additional chattering. A flexible controller with variable gains designed to avoid the chattering-effect, which increases system efficiency, is potentially attractive for the industrial sector. For that, in this work it is proposed a *variable-gain sliding mode control* (VGSM) based on the following goals:

- Design a variable gain structure for the sliding mode control.
- Decrease chattering-effect.
- Increase control efficiency.
- Establish a basic relationship between first and second order sliding mode control.

As will be seen along this work, those goals are mutually complementary.

The proposed control law presents flexibility features, since it is possible that it behaves as a first or second order SM control. According to our knowledge, this is the first time that a SM controller with this property is proposed.

The remainder of this paper is organized as follows. Section 2, presents the system and the problem statement. In Section 3, the main result is presented including the control algorithm and the corresponding stability analysis of closed-loop control system. In Section 4, simulation results are obtained and a comparison between the conventional Twisting algorithm and the presented approach is investigated. Finally, Section 5 presents some concluding remarks and final comments.

2. PROBLEM SETTING

Let us consider the following nonlinear scalar system

$$\dot{x}_1 = x_2, \tag{1}$$
$$\dot{x}_2 = f(x_1, x_2, t) + bu, \tag{2}$$

where x_1, $x_2 \in \mathbb{R}$ are the system states; $u \in \mathbb{R}$ is the control input; $f(x_1,x_2,t) \in \mathbb{R}$ is an uncertain, measurable and bounded function which can represent non modeling terms and external disturbances, such a function satisfy $|f(x_1,x_2,t)| \leq C$ for $C \in \mathbb{R}^+$; and $b \in \mathbb{R}$ is a positive definite parameter. In this paper we consider the following problem setting:

Design a SM controller that behaves as first and second order, for solving the trajectory tracking problem for system (1,2), such that the chattering-effect be considerable reduced, and at the same time maintaining the robustness for the closed-loop system taking into account actuators bounds.

In this work the absolute value for the tracking error is used to dynamically adjust the variable gains magnitude. For this purpose, a sliding mode controller (SMC) is proposed, which is inspired in the conventional second order *twisting control* [7,14,24–26] and in the conventional *first order sliding mode control* [9]. The former has the form

$$u = -\mathrm{r}_1 \mathrm{sgn}(x_1) - \mathrm{r}_2 \mathrm{sgn}(x_2), \tag{3}$$

where $\mathrm{r}_1, r_2 \in \mathbb{R}^+$, and $\mathrm{r}_1 > \mathrm{r}_2$ holds; whereas the latter has the form

$$u = -\mathrm{r}\,\mathrm{sgn}(x_1), \tag{4}$$

where $\mathrm{r} \in \mathbb{R}^+$.

Remark 1: Note that (4) is included in (3) for the case of $\mathrm{r}_1 = \mathrm{r}$, $\mathrm{r}_2 = 0$. Also note that both the regulation problem and the trajectory tracking problem of system (1,2) can be solved either by the first or by the second order SMC.

The main disadvantage of controllers (3, 4) is that non-modeled dynamics and disturbances can be overestimated, producing unnecessary amounts of chattering, and thus, decreasing system efficiency. To overcome these drawbacks, it is proposed that r_1, r_2 and r be replaced by time-variable gains, specially designed to:

- Avoiding overestimation of internal and external disturbances.

- Avoiding overestimation of unmodeled dynamics.
- Reducing chattering-effect.
- Maintaining SMC robust properties.
- Reaching well performance on convergence rates.
- Obtaining adaptability and flexibility for controller operation.

3. MAIN RESULT

In this Section, the design of the variable-gain sliding mode control (VGSM) is addressed. Also, the stability analysis for the closed-loop system (1,2) with the VGSM controller is presented.

3.1. Controller design

The controller design is composed by the following three key ideas:

1) Consider the second order twisting control (3) with state-varying control gains, i.e.

$$u = -p_1(\cdot)\,\mathrm{sgn}(x_1) - p_2(\cdot)\,\mathrm{sgn}(x_2) \tag{5}$$

with $p_1(\cdot) : x_1 \to \mathbb{R}^+$ and $p_2(\cdot) : x_1 \to \mathbb{R}_0^+$, where $\mathbb{R}_0^+ = \{x|0 \le x < \infty\}$ and $p_1 > p_2 \ge 0$ holds. It is assumed that the first state x_1 represents the tracking error for the control system, i.e. $x_1 = X_1 - x_d$, where X_1 is the actual absolute value and x_d represents the desired value for the first state of the plant to be controlled. Thus, in order to solve the trajectory tracking problem, this proposal is focused on the positioning tracking error, avoiding the inclusion of x_2 (related to the velocity tracking) to design the variable gains.

Remark 2: The bounds for u, i.e.

$$\min(u) \le u \le \max(u)$$

can be obtained by taking two cases from (5):

a) when $\mathrm{sgn}(x_1) = \mathrm{sgn}(x_2) = +1$ giving rise to

$$\begin{aligned}\min(u) &= \min(-p_1(x_1) - p_2(x_1)) \\ &= -\max(p_1(x_1) + p_2(x_1)).\end{aligned}$$

b) When $\mathrm{sgn}(x_1) = \mathrm{sgn}(x_2) = -1$ giving place to

$$\max(u) = \max(p_1(x_1) + p_2(x_1)).$$

It is easy to see from (5), that for the cases $\mathrm{sgn}(x_1) \ne \mathrm{sgn}(x_2)$, and $\mathrm{sgn}(x_1) = \mathrm{sgn}(x_2) = 0$, $|u| \le |p_1(x_1(t)) + p_2(x_1(t))|, \forall t \ge 0$. Also note that for practical implementations, the expression $|u| \le U_0$ must be accomplished, where U_0 represents the operational limits of the plant.

2) It is assumed that for the transient state, and also when a (sufficiently large) disturbance occurs at the steady state, $|x_1| \ne 0$ holds. Then the control u must be proportional to $|x_1|$. As such, let us propose that $p_1(\cdot) + p_2(\cdot) \propto |x_1|$. As a consequence we can define $p_1(\cdot), p_2(\cdot)$ as functions of x_1. Also, let k be a positive constant gain for the absolute value of the tracking error, and let S be a positive constant taken as an independent term to ensure that the sliding manifold occurs even at the steady state, such that

$$p_1(x_1) + p_2(x_1) = k\,|x_1| + S. \tag{6}$$

Remark 3: It is worth to mention that S should be selected in the interval $0 \le S \le U_0$ provided that the plant to control has actuators constraints (U_0), and must be noted that S with values near U_0 could produce saturation of $|u|$. Thus, important properties of S on control performance can be expressed as:

$$\lim_{x_1 \to 0} |u| = S, \quad \lim_{S \to \infty} |u| = U_0. \tag{7}$$

The main idea behind the correct selection of S, is to guarantee the minimum, necessary, but sufficient sliding mode, and must be tuned taking into account the needs of every specific implementation.

3) Let us define $w \in \mathbb{R}$ such that $0 \le w < 1$, to establish the following constant relationship between the variable gains,

$$p_2(x_1) = w p_1(x_1). \tag{8}$$

From the above, it follows that $p_1(x_1) > p_2(x_1) \ge 0$. Combining (6)-(8) the design for the variable gains is obtained as,

$$p_1(x_1) = \frac{k|x_1| + S}{1+w}, \tag{9}$$

$$p_2(x_1) = w\frac{k|x_1| + S}{1+w}. \tag{10}$$

Finally, substituting (9)-(10) in (5) the VGSM is given by

$$u = -\frac{k\,|x_1| + S}{1+w}\,\mathrm{sgn}(x_1) - w\,\frac{k\,|x_1| + S}{1+w}\,\mathrm{sgn}(x_2). \tag{11}$$

Remark 4: Controller gains in (11) includes two parts: a) the variable gains $(\frac{k|x_1|}{1+w}, \frac{wk|x_1|}{1+w})$ and b) the constant values $(\frac{S}{1+w}, \frac{wS}{1+w})$. Thus, $u = u_1 + u_2$, where $u_1 = -k|x_1|L$, $u_2 = -SL$, and $L = \dfrac{\mathrm{sgn}(x_1) + w\,\mathrm{sgn}(x_2)}{1+w}$. Then, parameter S affects the control performance only through u_2, and $S = -k|x_1| - \dfrac{u}{L}$.

Remark 5: It is important to mention that by choosing $w = 0$ in (11), the first order VGSM is obtained as

$$u = -(k\,|x_1| + S)\,\mathrm{sgn}(x_1). \tag{12}$$

While by selecting $k = 0$ the *variable gain sliding mode control* (VGSM) becomes

$$u = -\frac{S}{1+w}\,\mathrm{sgn}(x_1) - w\,\frac{S}{1+w}\,\mathrm{sgn}(x_2), \tag{13}$$

where $\frac{S}{1+w} > w\frac{S}{1+w} > 0, \quad \forall 0 < w < 1$, which is the conventional *twisting algorithm* (3), due to $\mathrm{r}_1 = \frac{S}{1+w}$, and $\mathrm{r}_2 = w\frac{S}{1+w}$. Also, by choosing $k = w = 0$ the first order VGSM becomes

$$u = -S\mathrm{sgn}(x_1), \tag{14}$$

which is the conventional first order sliding mode control (4) with $\mathrm{r} = S$. These facts show that *both the conventional SM and the TA are included in the VGSM*.

Next, we show the stability result of the closed-loop system (1)-(2)-(11).

Theorem 1: Consider the system (1,2) and the control given by

$$u = -p_1(x_1)\mathrm{sgn}(x_1) - p_2(x_1)\mathrm{sgn}(x_2),$$

where $p_1 : \mathbb{R} \to \mathbb{R}_+$, and $p_2 : \mathbb{R} \to \mathbb{R}_+$ are variable gains dependent of the state x_1, given as follows

$$p_1(x_1) = \frac{k|x_1| + S}{1+w}, \quad p_2(x_1) = w\frac{k|x_1| + S}{1+w},$$

then the origin of system (1,2) at closed-loop with (11) is globally stable with finite-time convergence

$$t_{reach} \leq \frac{2}{q}\sqrt{V(x_1(0), x_2(0), t_0)}.$$

Proof: The closed-loop system (1)-(2)-(11) is a Variable Structure System (VSS), and it is certainly not locally Lipschitz continuous, thus, existence and uniqueness of solutions must be understood in the sense of Filippov. In order to obtain a candidate Lyapunov function for the VGSM (a kind of VSS), a methodology based on [26, 27] is used in the proof. The method can be summarized in the next two steps:

- Propose the first time derivative $\dot{V}$ of the candidate Lyapunov function. Such that, $\dot{V}$ should be at least a negative semi-definite function.
- From the *Method of Characteristics* [28], it is obtained a partial differential equation generated by $\dot{V}$, then obtain the solution V which should be at least a positive semi-definite function.

Following the previous steps, let us propose $\dot{V}(x_1, x_2, t)$ as the first time derivative for the candidate Lyapunov function $V(x_1, x_2, t) : \mathbb{R}^2 \times \mathbb{R}^+ \to \mathbb{R}$, along the trajectories of system (1)-(2),

$$\dot{V} = \frac{\partial V}{\partial x_1}x_2 + \frac{\partial V}{\partial x_2}(f + bu) + \frac{\partial V}{\partial t}. \tag{15}$$

From the model presented in Section 2 where $|f(x_1, x_2, t)| \leq C$, it follows that

$$\dot{V} \leq \frac{\partial V}{\partial x_1}x_2 + \frac{\partial V}{\partial x_2}bu + C\left|\frac{\partial V}{\partial x_2}\right| + \frac{\partial V}{\partial t} \tag{16}$$

$$\leq \frac{\partial V}{\partial x_1}x_2 + \frac{\partial V}{\partial x_2}bu + C\frac{\partial V}{\partial x_2}\,\mathrm{sgn}\left(\frac{\partial V}{\partial x_2}\right) + \frac{\partial V}{\partial t}. \tag{17}$$

Taking q as a positive parameter, then, $-qV^{\frac{1}{2}} \leq 0$ is accomplished, thus, one needs to investigate a Lyapunov function $V(x_1, x_2, t)$ as an absolute continuous positive definite solution of the following PDE

$$-qV^{\frac{1}{2}} = \frac{\partial V}{\partial x_1}x_2 + \frac{\partial V}{\partial x_2}bu + C\frac{\partial V}{\partial x_2}\,\mathrm{sgn}\left(\frac{\partial V}{\partial x_2}\right) + \frac{\partial V}{\partial t}. \tag{18}$$

From the Method of Characteristics [28], the PDE (18) satisfies the following system

$$\frac{dx_1}{x_2} = \frac{dx_2}{Cv + bu} = \frac{dt}{1} = \frac{dV}{-qV^{\frac{1}{2}}}, \tag{19}$$

where $v = \mathrm{sgn}\left(\frac{\partial V}{\partial x_2}\right)$. Equation (19) holds for $|x_1|^2 + |x_2|^2 > 0$, then, $V(x_1, x_2, t)$ from (19) is a solution of (18) (to see the proof please refer to [28]). Note that both the first and the third members in (19) are redundant, since $x_2 = \frac{dx_1}{dt}$.

Introducing the control u (11) in (19), one obtains the following set of mutually coupled Ordinary Differential Equations (ODEs)

$$\frac{dx_1}{x_2} = \frac{(1+w)dx_2}{(1+w)Cv - b(k|x_1| + S)(\mathrm{sgn}(x_1) + w\mathrm{sgn}(x_2))}, \tag{20}$$

$$\frac{dx_1}{x_2} = dt = \frac{dV}{-qV^{\frac{1}{2}}}. \tag{21}$$

From (20)-(21) the following respective set of integrals is also obtained, giving rise to mutually dependent functions denoted by h_1, h_2 as follows

$$h_1 = h_a + h_b + h_c, \tag{22}$$

$$h_2 = \int dt + \frac{1}{q}\int V^{-\frac{1}{2}}dV, \tag{23}$$

where

$$h_a = bk\int |x_1|\mathrm{sgn}(x_1)dx_1 + bS\int \mathrm{sgn}(x_1)dx_1,$$

$$h_b = wbk\int |x_1|\mathrm{sgn}(x_2)dx_1 + wbS\int \mathrm{sgn}(x_2)dx_1,$$

$$h_c = (1+w)\left[\int x_2 dx_2 - Cv\int dx_1\right].$$

Solving the last integrals,

$$h_1 = h_d + h_e, \tag{24}$$

$$h_2 = t + \frac{2}{q}V^{\frac{1}{2}}, \tag{25}$$

where

$$h_d = b(1+wv)\left(\frac{1}{2}kx_1^2 + S|x_1|\right),$$
$$h_e = (1+w)\left(\frac{1}{2}x_2^2 - Cvx_1\right).$$

Now, selecting the linear dependency for h_1 and h_2 as $h_1 - Lh_2 = 0$, for some real L, it follows that

$$h_1 - Lt = \frac{2L}{q}V^{\frac{1}{2}}, \tag{26}$$

from which the Lyapunov function is obtained as

$$V(x_1, x_2, t) = \frac{q^2}{4L^2}\left[h_1(x_1, x_2) - Lt\right]^2. \tag{27}$$

Thus V is positive semi-definite and radially unbounded. Further, from (16)-(18) it follows that

$$\dot{V} \leq -qV^{1/2}. \tag{28}$$

Expression (28) can be rearranged as

$$\frac{dV}{-qV^{1/2}} \geq dt_{reach}. \tag{29}$$

Solving (29) for t_{reach} and taking the initial conditions $(x_1(0), x_2(0), t_0)$, the finite-time convergence is obtained and is given as follows

$$t_{reach} \leq \frac{2}{q}\sqrt{V(x_1(0), x_2(0), t_0)}. \tag{30}$$

Similar t_{reach} expressions (although for time-invariant systems) are presented in [26, 27]. □

4. SIMULATION RESULTS

Since (11) has a similar structure than (3), in this section, performance comparisons are made between VGSM and the conventional TA.
A linear piston in horizontal position was selected as the plant to control, with the following dynamics,

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = I^{-1}(-fx_2 + f_{NOISE} + u), \tag{31}$$

where x_1, $x_2 \in \mathbb{R}$ are the variable system states for position and velocity, respectively, and $x_1 = x_1(t), x_2 = x_2(t)$; $f_{NOISE} \in \mathbb{R}$ represents a white noise function with an amplitude bounded by 2.1×10^{-4} Nm, and an averaging frequency of 650 Hz; $I = 0.42$ Kg·m, is the inertia value; $f = 0.14$ Kg·m/s; and u is the control signal. The task of the control is to solve the trajectory tracking problem, where the linear piston is intended to track a square wave form as the reference signal, which has an amplitude of 1 m (i.e., from +0.5 to -0.5 m) and frequency of 0.4 Hz (i.e., the period is 2.5 s). Initial conditions for the actuator are selected as $x_1(0) = 1.5$ rad, and $x_2(0) = 0$ rad/s.

4.1. Preliminary simulations

In order to select parameters for the TA, one first simulation is given to show the effect of increasing the sum $r_1 + r_2$, for the closed-loop control of plant (31). In Fig. 1, the graphic where $r_1 = 4, r_2 = 2$ is presented in thin line (in black), the performance for $r_1 = 8, r_2 = 4$ is shown in medium line (in red), and for parametric values $r_1 = 30, r_2 = 15$ the behavior of the TA is given in thick line (in bluish green). It can be seen that the reference signal (the square wave in dashed line), results to be a difficult signal to track by the closed-loop control of plant (31) with (3), e.g., for the first valley of the wave form, there is not convergence at all, while for the first peak of the wave form, the convergence is reached only by $r_1 = 30, r_2 = 15$, nearly at the end of square wave.

Observe that increasing the value for the sum $r_1 + r_2$, the control signal u is also increased, and as a consequence, the time convergence becomes reduced. In order to understand and to select the S value, Fig. 2 shows the closed-loop control of plant (31) with (11), taking $S = 0, 6, 18, 36$ with $w = 0.5$ and $k = 5000$. From this result, it is clear that low values for S, delay the time convergence, while high values for this parameter, make robust the control performance.

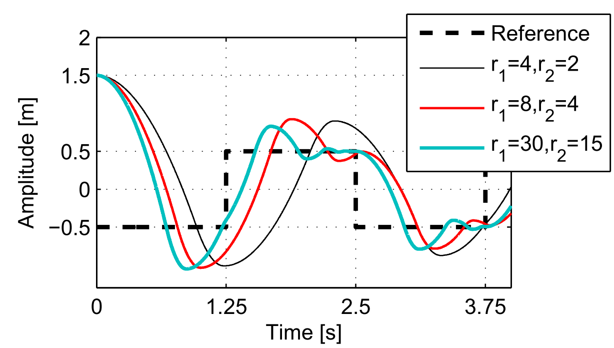

Fig. 1. Performance of conventional TA under three different values for $r_{1,2}$.

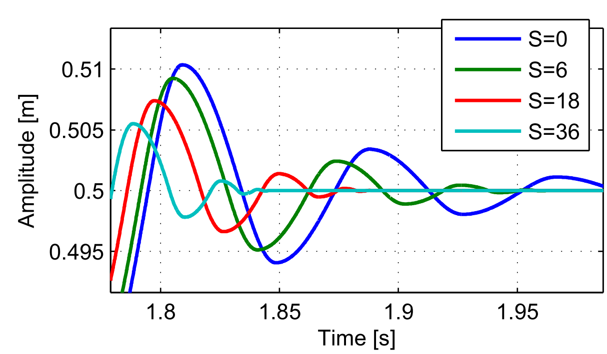

Fig. 2. Behavior of the VGSM under different values of S: 0, 6, 18, and 36 Nm.

4.2. Selection of parameters

As the VGSM controller has adaptivity properties, it is possible to select an only one set of parametric values to work under a wide range of operational conditions. Nevertheless, certain guidelines must be followed in order to tune it adequately:

- Verify the amount of noise that your application can absorb, without suffering damages.
- Verify the control robustness that your application requires.
- Take into account that when k increases, the VGSM reacts more powerfully to perturbations and noise.
- Take into account that increasing the value for S, both robustness and chattering also increase their magnitudes.
- Take into account that when w increases its value, the VGSM approaches to the conventional TA performance, while when w decreases, the VGSM approaches to conventional SMC.

Considering these guidelines for the plant (31), it was found that: in the range $1000 \leq k \leq 9000$, the controller reacts with enough power to reject disturbances; in the range $4 \leq S \leq 8$, the chattering-effect is reduced to negligible levels, but preserving the steady state convergence; and working with $0.3 \leq w \leq 0.7$, the VGSM takes advantage from both the SMC and the TA. As such, the averages for each of these ranges were selected to tune the controller: $k = 5000$ [N], $S = 6$ Nm, $w = 0.5$. Even more, actuator maximum operating limit is selected with $U_0 = 75$ Nm, which is an average value for commercial applications. These parametric values for the VGSM, are used in the reminder of this section.

Conventional TA is not endowed with flexibility and adaptability, thus, to obtain interesting comparisons with the VGSM, two cases are considered:

Case 1: Selecting the TA parameters to obtain low chattering, i.e., the same chattering level than that produced by the VGSM. This is in order to compare the convergence time, and disturbances rejection of both controllers. For this case $r_1 = 12$ and $r_2 = 6$ are proposed, which are equivalent to $\max|u| = 18$ Nm.

Case 2: Tuning the TA to reach the same convergence time and robustness than than obtained by the VGSM. This is to compare the levels of chattering-effect produced by both controllers. To not overload $U_0 = 75$ Nm, $r_1 = 50$ and $r_2 = 25$ are proposed.

4.3. Case 1

Fig. 3 exhibits performance of both the VGSM (thick line) and the conventional TA (thin line). From initial conditions, the piston must travel a distance of 2 m to reach the first valley of the wave form, thus, the inertial motion produces large overshoots for both controllers, however, the VGSM recovers faster than the conventional TA.

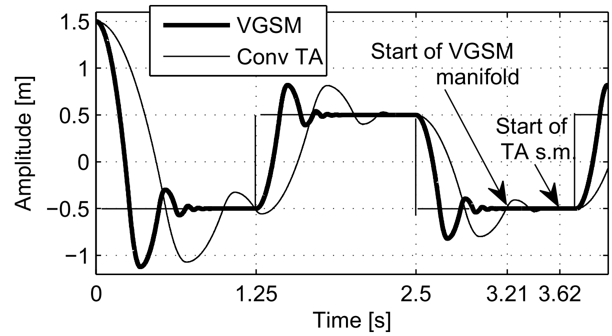

Fig. 3. Tracking a square wave form: thick line is the performance of VGSM, while thin line is the conventional TA with $r_1 = 12$ and $r_2 = 6$.

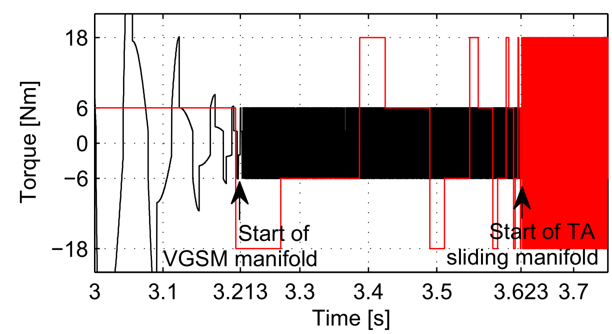

Fig. 4. Control signal produced by both VGSM and conventional TA, for the tracking process of Fig. 3.

Fig. 4 is an enlarged view of the control signals produced by both controllers. It can be seen that the proposed controller reaches the sliding manifold, along with the SM properties, faster than the TA. Control goals are achieved by the VGSM at 3.213 s, while the conventional TA at 3.623 s, thus, considering that the reference signal changes from $+0.5$ to -0.5 m at 2.5 s, the time difference (0.41 s) represents a reduction of $\approxeq 36.5\%$ for the time convergence. Also observe that from 3.213 to 3.623 s, the TA is out from control goals (e.g., convergence, robustness, sliding manifold).

Remember that the chattering-effect is a phenomena produced by the combination of both the dynamics of the plant to control and the output signal generated by the controller. The real amplitude, frequency and shape of the chattering, can be visualized by making a zoom view inside the simulation of Fig. 3. Once the conventional TA reaches the sliding manifold, it can be compared with the VGSM. Fig. 5 is an enlarged view of Fig. 3, to show the actual chattering waveform produced by the dynamics of the piston combined with the control signal presented by Fig. 4. Observe that for this operating conditions, the

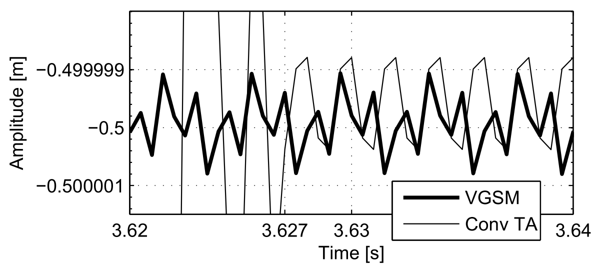

Fig. 5. Enlarged view of Fig. 3 from 3.62 to 3.64 s, to show the actual chattering produced by the control signals of VGSM and TA.

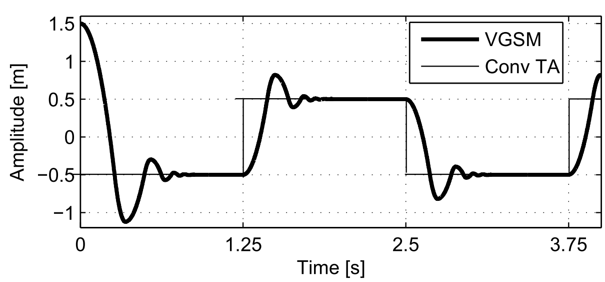

Fig. 6. Tracking a square wave form: thick line is the performance of VGSM, while thin line is the conventional TA with $r_1 = 50$ and $r_2 = 25$ (both lines are overlap with each other).

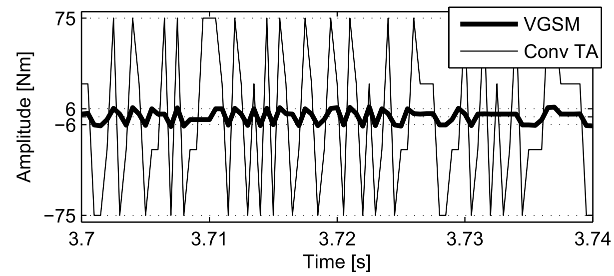

Fig. 7. Control signal produced by both VGSM and conventional TA for the tracking process given in Fig. 6.

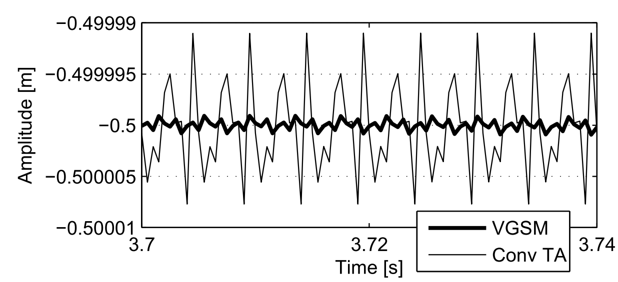

Fig. 8. Actual chattering produced by control signals presented in Fig. 7.

chattering produced by VGSM (thick line) is smaller than that produced by the conventional TA (thin line) before 3.627 s, and after this time both real vibrations are similar (which corresponds to the approach of this first case study).

This first case shows that the time convergence is much more faster for the VGSM than that of conventional TA, when both controllers have similar chattering in the steady state. Also shows that the proposed control reject disturbances faster.

4.4. Case 2

In order to increase robustness for the conventional TA to the same level of the one obtained by the VGSM, the Twisting parameters must be modified to the maximum allowed by the operational limits of the plant 75 Nm, e.g., $r_1 = 50$ and $r_2 = 25$. Under these new parameters, Fig. 6 shows that the thick and the thin lines are practically the same, which means that the tracking performance of both controllers is similar. Nevertheless, when robustness for the conventional TA is increased, also the chattering-effect is increased, because the control signals must be more powerful. Fig. 7 shows a zoom view of the control signal produced by VGSM (thick line) and conventional TA (thin line), for the steady state where both controllers have reached the sliding manifold. Fig. 8 is an enlarged view inside Fig. 6, to show the actual chattering produced by the dynamics of the plant combined with control signals of Fig. 7. The bounds of the vibrations produced by conventional TA are eight times larger than those produced by VGSM under the specified working conditions. At this point, it is important to note that the overshoots appearing at Figs. 3 and 6 can be reduced or eliminated by modifying the constraint of the operational limits of the actuator. Second case shows an outstanding reduction of the chattering-effect when the VGSM technique is used. Time convergence and disturbances rejection are similar for both controllers, nevertheless, the proposed controller is able to not-overestimate noise and perturbations, which makes the control work much more efficient. In Fig. 9 the behavior of the variable gains p_1 (thin line) and p_2 (thick line) is presented. This graphic was obtained for the tracking process of Fig. 6. After 0.9s, is clearly visible the effect produced by f_{NOISE} (when the noise function is suppressed, both p_1 and p_2 become plain). This Figures shows that the VGSM is sensitive to disturbances and noise.

4.5. Phase portraits for VGSM and conventional TA

Let us consider the following two closed-loop systems,

$$\dot{x}_1 = x_2, \tag{32}$$

$$\dot{x}_2 = -12\text{sgn}(x_1) - 6\text{sgn}(x_2), \tag{33}$$

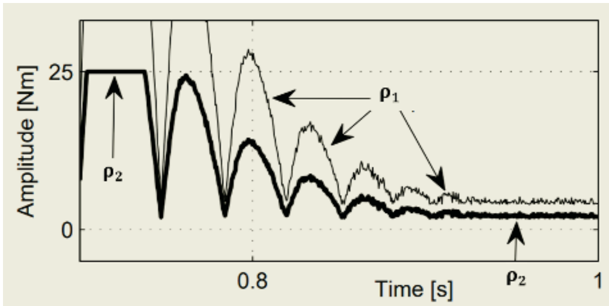

Fig. 9. Enlarged view to show the behavior of variable gains p_1 and p_2.

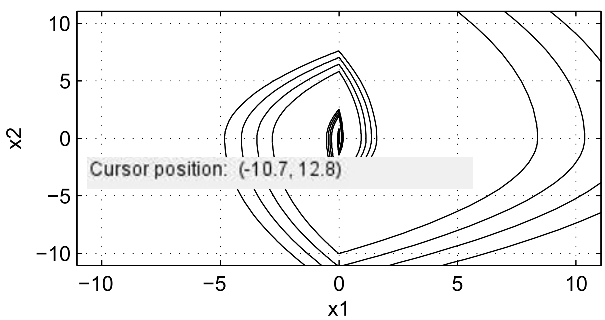

Fig. 10. Phase portrait for Twisting Algorithm for system (32).

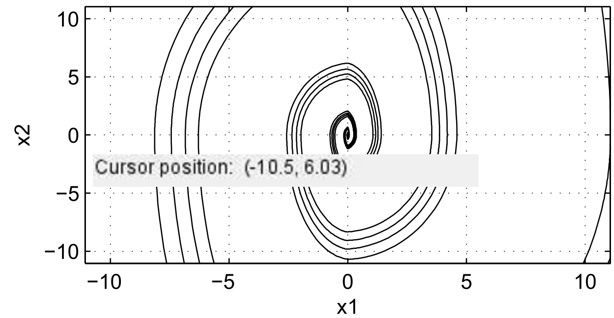

Fig. 11. Phase portrait for the VGSM algorithm corresponding to the closed-loop system (34).

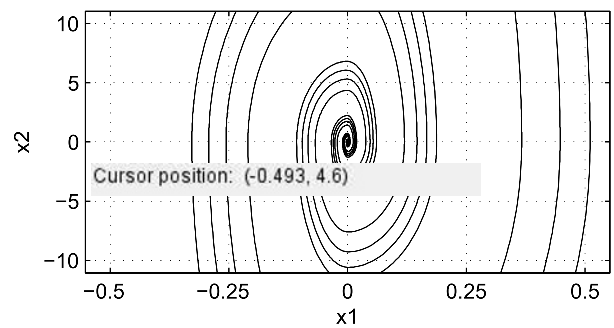

Fig. 12. Phase portrait for the VGSM controller at closed-loop with system (31).

and

$$\dot{x_1} = x_2, \tag{34}$$

$$\dot{x_2} = -(10|x_1|+2)\text{sgn}(x_1) - (5|x_1|+1)\text{sgn}(x_2). \tag{35}$$

System (32) corresponds to TA of (3) with $r_1 = 12$ and $r_2 = 6$. System (34) corresponds to the structure of equation (11), with $k = 15$, $S = 3$ and $w = 0.5$. The phase portraits of both systems are shown in Figs. 10-11 respectively, for comparison purposes. Fig. 10, exhibits that for $(0,x_2)$ the trajectories of TA present discontinuities. In Fig. 11 it can be observed that the trajectories for VGSM are soft functions for all (x_1,x_2).

Finally, Fig. 12, shows the behaviour of the closed-loop control system (31), where it is possible to observe a similar performance than in Fig. 11, nevertheless, the time convergence is faster in Fig. 12 (observe that the scale for x_1 in Fig. 12, is from -0.5 to 0.5, while in Fig. 11, is from -10 to 10).

5. CONCLUSIONS

The VGSM controller can be tuned manually to increase its robustness via increasing the value of S. Also, it automatically increases its robustness when a disturbance occurs, because the amplitude of the sliding manifold is proportional to the absolute value of the state ($|x_1|$). The conventional SM is included in VGSM when $k = 0$, since under this case the variable terms in (11) are switched off. Results exhibited by Figs. 3, 4 and 5, show evidence that VGSM has a better performance than TA in tracking the reference signal, in the case when TA is tuned to produce similar levels of chattering than VGSM. Results given by Figs. 6, 7 and 8, show evidence that VGSM produces only a small fraction (around $1/8$) of the chattering produced by the conventional TA, when TA is tuned to be as robust as VGSM in tracking the reference signal. In order to match the tracking performance of the VGSM, TA inevitably must increase the chattering-effect. Fig. 9 shows actual behaviour of the variable gains, which does not overestimate disturbances.

According to the Stability analysis, the control system at closed-loop presents finite time convergence under the action of the VGSM controller. This claim is also shown by results exhibited in Figs. 3, 6, 11, and 12. Since VGSM is designed for operational flexibility and adaptability, it has the possibility to be tuned in order to improve the performances for both the tracking processes and the permitted levels of chattering, according to specific needs of the plant to be controlled or to the specific application where it is going to be implemented.

As such, the novel VGSM controller has the potential to be successfully used all over the industrial processes and practical applications where a robust, adaptable, and flexible control law is required. Even more, this controller is environmentally friendly, since it does not overestimates perturbations and noise.

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Sergio Alvarez-Rodríguez received the post-graduated degree in mechatronics engineering from Centro Nacional de Investigación y Desarrollo Tecnológico, México, in 2007 and the Ph.D. degree in Science and Technology from Universidad de Guadalajara, México, in 2014, from February 2016 to July 2017 he made a post-doctoral stay at Optical Research Center, AC. He is currenlty full time researcher/professor at TecMM, Lagos de Moreno, Jalisco, México. His areas of interest are on robotics, control theory, and sensors for instrumentation.

Gerardo Flores received the B.S degree in Electronic Engineering with honors from the Instituto Tecnológico de Saltillo, México in 2000; the M.S. degree in Automatic Control from CINVESTAV-IPN, Mexico City, in 2010; and the Ph.D. degree in Systems and Information Technology from the Heudiasyc Laboratory of the Université de Technologie de Compiègne - Sorbonne Universités, France in October 2014. From November 2014 to July 2016, he was a post-doctoral researcher with the Centro de Investigacién y de Estudios Avanzados del Instituto Politécnico Nacional, Mexico City. Since August 2016, he has been full time researcher and Head of the Perception and Robotics LAB with the Center for Research in Optics, León Guanajuato, Mexico. His research interests are focused on the theoretical and practical problems arising from the development of autonomous robotic systems and vision systems. He is especially interested in: artificial vision, theory and applications of intelligent systems, nonlinear control, design, conception and control of UAVs. Dr. Flores has published more than 30 papers in the areas of control systems, computer vision and robotics. He has been member of the *Sistema Nacional de Investigadores* since 2014.

Noé Alcalá Ochoa received his BSc degree in physics with a major in mathematics from the Universidad Michoacana México, in 1986, and his MSc and PhD degrees from Optical Research Center (CIO), México, in 1990 and 1997, respectively. Since then, he has been a researcher at CIO. His main research interests include nondestructive optical testing (ESPI, moiré, interferometry, profilometry), optical superresolution, and microscopy.