International Journal of Control, Automation and Systems 16(5) (2018) 2082-2092
http://dx.doi.org/10.1007/s12555-017-0662-5
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Asynchronous Switching Control for Continuous-time Switched Linear Systems with Output-feedback

Zhiyong Jiang and Peng Yan*

Abstract: This paper investigates the asynchronous switching control problem for continuous-time switched linear systems via dynamic output-feedback, where the dynamic output-feedback controller contains an impulsive reset law to reset the controller state. A time-varying multiple Lyapunov-like-function (MLF) approach is employed to analyze the stability and weighted L_2-gain of the closed-loop systems. The switching stability criteria for the closed-loop systems are established in terms of linear matrix inequalities (LMIs), which are dependent on the upper and lower bounds of the switching interval and the asynchronous delays. The switching logic is designed to guarantee the closed-loop systems achieving the weighted L_2-gain performance. Two numerical examples are provided to show the effectiveness of the proposed method.

Keywords: Asynchronous switching, average dwell time, dynamic output-feedback, switched linear system.

1. INTRODUCTION

The switched systems have been widely concerned by various scientific communities during the last decade due to its wide application in industry, such as biological systems [1], robotics [2], networked control [3], and so forth. At present, abundant results on the stability analysis of switched systems have been published with numerous practical analysis methods [4–6], such as common quadratic Lyapunov function (CLF) [7], multiple Lyapunov-like-function (MLF) [8], piecewise Lyapunov-Razumikhin functions [9], etc. Further, the control synthesis problem of switched systems has been extensively studied in recent years. The CLF approach was employed for switched linear systems with state-feedback control in [10], and a switching state-feedback robust control was designed in [11]. Moreover, the results on switched systems with output-feedback control have been published in recent literature. A hybrid control scheme was given in [12], and an extended linear-quadratic gaussian (LQG) design was proposed in [13].

However, the above mentioned results are based on the synchronous switching between the sub-controller and the subsystems, which ignores the time lag between the instant of the sub-controller activation and the subsystem switching instant. Therefore, the obtained results may be more conservative due to the delay of the sub-controller activation without consideration. Note that asynchronous switching control has received plenty of attention from researchers [14–18], since the concept of asynchronous switching was first proposed in [19]. The asynchronous switching state-feedback control problem for switched systems with a given maximal asynchronous delay was addressed in [20]. In addition, the dynamical output-feedback control problem for discrete-time switched linear systems was investigated under asynchronous switching in [21], where an approach was provided to address the bilinear problem. Note that the asynchronous dynamic output feedback control problem has been studied in the literature. However, the existing results, such as [22] and [23], are conservative and not very applicable for engineering applications, due to the non-convex form of the results and the conservative methods dealing with the asynchronous delays. In the present paper, we provided an alternative method with a time-varying MLF, which is capable of estimating the allowable upper and lower bounds of the asynchronous delays and decouples the bilinear matrix inequalities in asynchronous output feedback control design. The overall conservativeness is reduced theoretically, which is also verified by two numerical examples.

It is noticeable that the switching controllers with state reset exhibit more flexibility and potential advantages on controller solvability and performance improvement. Switching control with state reset has been extensively

Manuscript received October 26, 2017; revised January 28, 2018; accepted February 27, 2018. Recommended by Associate Editor Andrea Cristofaro under the direction of Editor Myo Taeg Lim. This work was supported in part by the National Natural Science Foundation of China under Grants 61327003 and 51775319.

Zhiyong Jiang is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, 100083, China (e-mail: jiangzy2013@163.com). Peng Yan is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing 100083, China and also with the Key Laboratory of High-efficiency and Clean Mechanical Manufacture, Ministry of Education, School of Mechanical Engineering, Shandong University, Jinan, Shandong 250061, China (e-mail: PengYan2007@gmail.com).
* Corresponding author.

studied in recent literatures such as [12,21,24], where such approach has been also reported with applications to F-16 aircraft [24]. Also note that the design of switching dynamic output feedback controller for the switched system *without state reset* may lead to non-convex control synthesis conditions, whereas the introduction of controller state reset for such problem can help to establish control synthesis conditions in terms of LMIs [12] more applicable for computations. With these motivations, we consider the design problem of a switching dynamic output feedback controller with state reset for switched linear systems. Moreover, a common assumption is that the upper bound of the asynchronous delays is given in advance in the current literature. How to determine the range of asynchronous delays is also a challenging problem.

In this paper, the asynchronous switching dynamic output-feedback control problem for continuous-time switched linear systems with average dwell time (ADT) is solved. A time-varying MLF is constructed to establish the control synthesis conditions that guarantee the closed-loop systems with the weighted L_2-gain performance. Note that the established conditions are dependent on the upper and lower bounds of the asynchronous delays and the switching interval, and the upper and lower bounds of the asynchronous delays can be estimated by solving the established control synthesis conditions.

The arrangement of this paper is as follows. The model of switched linear systems and the problem description are formulated in Section 2. The main theoretical results are presented in Section 3. Two numerical examples and conclusions are given in Section 4 and Section 5, respectively.

Notation: The following notations will be used in this paper. $\mathbb{R}_+$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the set of the nonnegative real numbers, the n-dimensional Euclidean space and the set of all $n\times m$ matrices, respectively. The Euclidean vector norm is represented by $\|\cdot\|$. The notation $\mathbb{N}$ represents the set of the nonnegative integers. For a real matrix M, M^{T} is said to be the transpose of M, and $He\{M\} = M + M^{\mathrm{T}}$. Further, if M is a real symmetric matrix, the symbol $M > (\geq, <, \leq) 0$ is expressed as a positive definite (positive semi-definite, negative definite, negative semi-definite) matrix. I_n, $I_{m\times n}$, 0_n and $0_{m\times n}$ are defined as the $n\times n$ identity matrix, $m\times n$ identity matrix, $n\times n$ zero matrix and $m\times n$ zero matrix, respectively. The notation $\mathbf{I}[N_1,N_2]$ is defined as the positive integers set $\{N_1, N_1+1, \cdots, N_2\}$ for two positive integers N_1, N_2 with $N_1 < N_2$. For two positive scalars δ_1, δ_2 with $\delta_1 \leq \delta_2$, let $\mathcal{S}(\delta_1,\delta_2)$ be the set of switching time sequences $\{t_k\}$ satisfying $\delta_1 \leq t_{k+1} - t_k \leq \delta_2$ for $\forall k \in \mathbb{N}$.

2. PROBLEM STATEMENT

Consider continuous-time switched linear systems of the following form:

$$\begin{cases} \dot{x}_p(t) = A_\sigma x_p(t) + B_{1,\sigma}u(t) + H_{1,\sigma}w(t), \\ z(t) = C_{1,\sigma}x_p(t) + B_{2,\sigma}u(t) + H_{2,\sigma}w(t), \\ y(t) = C_{2,\sigma}x_p(t) + H_{3,\sigma}w(t), \end{cases} \tag{1}$$

where $x_p(t) \in \mathbb{R}^n$ is the plant state, $u(t)$ is the control input, $z(t)$ is the controlled output, $y(t)$ is the measurement output, $w(t) \in L_2^p[0,\infty)$ is the disturbance input. The switching signal $\sigma(t)$, a piecewise constant function, takes positive integer value belonging to the finite set $\mathbf{I}[1,N_p]$ in the interval $[t_{k-1}, t_k), k \in \mathbb{N}$, where $N_p > 1$ is the number of subsystems. $\{t_k, k \in \mathbb{N}\}$ is a switching time sequence of $\sigma(t)$ satisfying $0 \leq t_0 < t_1 < \cdots < t_k < \cdots$, and $\lim_{k\to+\infty} t_k = +\infty$. In addition, $A_i \in \mathbb{R}^{n\times n}$, $B_{1,i} \in \mathbb{R}^{n\times n_u}$, $B_{2,i} \in \mathbb{R}^{p\times n_u}$, $C_{1,i} \in \mathbb{R}^{p\times n}$, $C_{2,i} \in \mathbb{R}^{q\times n}$, $H_{1,i} \in \mathbb{R}^{n\times n_w}$, $H_{2,i} \in \mathbb{R}^{p\times n_w}$, $H_{3,i} \in \mathbb{R}^{q\times n_w}, i \in \mathbf{I}[1,N_p]$ are known constant matrices. Similar to [12], we make the following assumption for each subsystem.

(A1) The triple $(A_i, B_{1,i}, C_{2,i})$ is stabilizable and detectable for $\forall i \in \mathbf{I}[1,N_p]$.

In this paper, our objective is to design a dynamical output-feedback switching controller with controller state reset and an admissible switching signal for the switched linear systems (1) such that the closed-loop systems is exponentially stable and has a weighted L_2-gain performance.

The dynamic output-feedback switching controller with controller state reset is designed of the form:

$$\begin{cases} \dot{x}_c(t) = A_{c,\bar{\sigma}}x_c(t) + B_{c,\bar{\sigma}}y(t), \\ u(t) = C_{c,\bar{\sigma}}x_c(t) + D_{c,\bar{\sigma}}y(t), \\ x_c(t) = E_{ij}x_c(t^-), \quad t = t_k^c, \end{cases} \tag{2}$$

where $x_c(t) \in \mathbb{R}^n$ is the controller state. $E_{ij} \in \mathbb{R}^{n\times n}, i, j \in \mathbf{I}[1,N_p], i \neq j$ are defined as the controller reset matrices, which need to be designed. The subscript $\{ij\}$ is expressed as the switching from the sub-controller i to the sub-controller j. $\bar{\sigma}(t)$ is the switching signal of the controller (2). $\{t_k^c, k \in \mathbb{N}\}$ is the switching time sequence of $\bar{\sigma}(t)$. Moreover, assume that the solution of the controller (2) is right continuous, namely, $x_c(t_k) = x_c(t_k^+), k \in \mathbb{N}$. For the convenience of description, the notation $K_{c,\bar{\sigma}}$ is defines as

$$K_{c,\bar{\sigma}} = \begin{bmatrix} A_{c,\bar{\sigma}} & B_{c,\bar{\sigma}} \\ C_{c,\bar{\sigma}} & D_{c,\bar{\sigma}} \end{bmatrix}.$$

It is noted that the controller takes some time to identify and match the new subsystem when the subsystem switching occurs. Therefore, the switching time sequence of $\bar{\sigma}(t)$ can be expressed as $t_k^c = t_k + \triangle_k, k \in \mathbb{N}$, where $\triangle_k$ is asynchronous delay satisfying $\triangle_0 = 0$ and $0 < \triangle_k < t_{k+1} - t_k$ for $\forall k \in \mathbb{N}\backslash\{0\}$. Assume that there exist positive scalars $\delta_{c,l}, \delta_{p,l}$, $l = 1,2$ satisfying $0 < \delta_{c,1} \leq \delta_{c,2} < \max\{t_{k+1} - t_k, k \in \mathbb{N}\}$ and $0 < \delta_{p,1} \leq \delta_{p,2}$, such that $\delta_{c,1} \leq t_k^c - t_k \leq \delta_{c,2}$ and $\delta_{p,1} \leq t_{k+1} - t_k^c \leq \delta_{p,2}$ hold for $\forall k \in \mathbb{N}$. Then we have $\delta_1 \triangleq \delta_{c,1} + \delta_{p,1} \leq t_{k+1} - t_k \leq \delta_{p,2} + \delta_{c,2} \triangleq \delta_2$, $k \in \mathbb{N}$. For the system switching instant t_k, let $\sigma(t_k^-) = i$ and $\sigma(t_k) = j$. Thus, we have $\bar{\sigma}(t_k^{c-}) = i$ and $\bar{\sigma}(t_k^c) = j$ for the controller switching instant t_k^c.

Different from the synchronous switching control, the interval $[t_k, t_{k+1})$ is divided into the intervals $[t_k, t_k^c)$ and $[t_k^c, t_{k+1})$ in the asynchronous switching control for all $k \in \mathbb{N}$, where $[t_k, t_k^c)$ represents the unmatched interval of the sub-controller and subsystem, and the other is the matched interval of the sub-controller and subsystem. We do not require the Lyapunov function to decrease monotonically in the unmatched interval, and allow it to increase with a bounded increasing rate. In order to facilitate the presentation, let $T_\uparrow(t_s, t)$ be the length of the increasing interval of Lyapunov function within the interval $[t_s, t)$, for example, $T_\uparrow(t_k, t_{k+1}) = \triangle_k, \forall k \in \mathbb{N}$.

The closed-loop systems can be described in the following form by combining (1) and (2):

$$\begin{cases} \dot{x}(t) = A_{\sigma,\bar{\sigma}} x(t) + B_{\sigma,\bar{\sigma}} w(t), \\ z(t) = C_{\sigma,\bar{\sigma}} x(t) + D_{\sigma,\bar{\sigma}} w(t), \\ x(t) = \tilde{E}_{ij} x(t^-), \quad t = t_k^c, \end{cases} \tag{3}$$

where $x = \mathrm{col}\,(x_p, x_c)$. The matrices $A_{\sigma,\bar{\sigma}}, B_{\sigma,\bar{\sigma}}, C_{\sigma,\bar{\sigma}}, D_{\sigma,\bar{\sigma}}$ and $\tilde{E}_{ij}, i \neq j, i, j \in \mathbf{I}[1, N_p]$ are formulated as follows:

$$\begin{aligned} &A_{\sigma,\bar{\sigma}} = \tilde{A}_\sigma + \tilde{B}_{1,\sigma} K_{c,\bar{\sigma}} \tilde{C}_{2,\sigma}, \\ &B_{\sigma,\bar{\sigma}} = \tilde{H}_{1,\sigma} + \tilde{B}_{1,\sigma} K_{c,\bar{\sigma}} \tilde{H}_{3,\sigma}, \\ &C_{\sigma,\bar{\sigma}} = \tilde{C}_{1,\sigma} + \tilde{B}_{2,\sigma} K_{c,\bar{\sigma}} \tilde{C}_{2,\sigma}, \\ &D_{\sigma,\bar{\sigma}} = \tilde{H}_{2,\sigma} + \tilde{B}_{2,\sigma} K_{c,\bar{\sigma}} \tilde{H}_{3,\sigma}, \\ &\tilde{E}_{ij} = \mathrm{diag}\{I_n, E_{ij}\}, \quad i \neq j, \quad i, j \in \mathbf{I}[1, N_p], \end{aligned}$$

where

$$\begin{aligned} &\tilde{A}_\sigma = \begin{bmatrix} A_\sigma & 0_n \\ 0_n & 0_n \end{bmatrix}, \ \tilde{B}_{1,\sigma} = \begin{bmatrix} 0_n & B_{1,\sigma} \\ I_n & 0_{n\times n_u} \end{bmatrix}, \ \tilde{H}_{3,\sigma} = \begin{bmatrix} 0_{n\times n_w} \\ H_{3,\sigma} \end{bmatrix}, \\ &\tilde{C}_{2,\sigma} = \begin{bmatrix} 0_n & I_n \\ C_{2,\sigma} & 0_{q\times n} \end{bmatrix}, \ \tilde{H}_{1,\sigma} = \begin{bmatrix} H_{1,\sigma} \\ 0_{n\times n_w} \end{bmatrix}, \ \tilde{H}_{2,\sigma} = H_{2,\sigma}, \\ &\tilde{C}_{1,\sigma} = \begin{bmatrix} C_{1,\sigma} & 0_{p\times n} \end{bmatrix}, \quad \tilde{B}_{2,\sigma} = \begin{bmatrix} 0_{p\times n} & B_{2,\sigma} \end{bmatrix}. \end{aligned}$$

According to the properties of the switching signal, the closed-loop systems (3) can be rewritten as

$$\begin{cases} \begin{bmatrix} \dot{x}(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A_{i,i} & B_{i,i} \\ C_{i,i} & D_{i,i} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, \quad t \in [t_{k-1}^c, t_k), \\ \begin{bmatrix} \dot{x}(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A_{j,i} & B_{j,i} \\ C_{j,i} & D_{j,i} \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, \quad t \in [t_k, t_k^c), \\ x(t_k^c) = \tilde{E}_{ij} x(t_k^{c-}), \quad k \in \mathbb{N}. \end{cases} \tag{4}$$

The following definitions and lemmas will be used in this paper.

Definition 1 [19]**:** For a switching signal $\sigma(t)$ and two positive scalars s_1, s_2 with $t_0 \leq s_1 \leq s_2$, $N_{\sigma(t)}(s_1, s_2)$ is defined as the switching number of $\sigma(t)$ over the interval $[s_1, s_2)$. If there exist nonnegative integer N_0 and scalar $\tau_a > 0$ such that $N_{\sigma(t)}(s_1, s_2) \leq N_0 + (s_2 - s_1)/\tau_a$ holds, then τ_a and N_0 are called the ADT and the chatter bound, respectively.

Definition 2: Let $x(t, t_0, x(t_0))$ be the state trajectory of the closed-loop systems (4) with $w(t) \equiv 0$ through $(t_0, x(t_0))$. Given a class $\mathcal{S}(\delta_1, \delta_2)$ of switching time sequences, the closed-loop systems (4) with $w(t) \equiv 0$ is uniformly globally exponentially stable (UGES) over $\mathcal{S}(\delta_1, \delta_2)$, if there exist positive constants ε and $\mathcal{K}$ such that the following inequality holds:

$$\|x(t, t_0, x_0)\| \leq \mathcal{K} \|x(t_0)\| \mathrm{e}^{-\varepsilon(t - t_0)}, \quad t \geq t_0 \geq 0.$$

Definition 3: For given positive scalars α_0 and γ, the closed-loop systems (4) is said to be internally UGES and has a weighted L_2-gain less than γ, if the closed-loop systems (4) with $w(t) \equiv 0$ is UGES and under the zero initial condition, the controlled output $z(t)$ and the disturbance input $w(t)$ satisfy

$$\int_{t_0}^{\infty} \mathrm{e}^{-\alpha_0(s - t_0)} z^{\mathrm{T}}(s) z(s) ds \leq \gamma^2 \int_{t_0}^{\infty} w^{\mathrm{T}}(s) w(s) ds, \ t_0 \geq 0.$$

Lemma 1: Consider the closed-loop systems (4) satisfying (**A1**). For given scalars $\alpha > 0$, $\beta \geq 0$, $\gamma > 0$, $\gamma_l \geq 1$, $\mu_l \in (0, 1]$, $l = 1, 2$, nonnegative integer N_0, and a class $\mathcal{S}(\delta_1, \delta_2)$ of switching time sequences with the ADT τ_a, the closed-loop systems (4) is internally UGES and has a weighted L_2-gain less than γ over $\mathcal{S}(\delta_1, \delta_2)$, if there exist functions $V_i(t, x(t)) : \mathbb{R}_+ \times \mathbb{R}^{2n} \to \mathbb{R}_+$ with $V_{\sigma(t_0)}(t_0, x_0) \equiv 0$ and $V_{ij}(t, x(t)) : \mathbb{R}_+ \times \mathbb{R}^{2n} \to \mathbb{R}_+$, $i, j \in \mathbf{I}[1, N_p], i \neq j$ such that for $\forall i, j \in \mathbf{I}[1, N_p], i \neq j$, one has

$$\begin{cases} \dot{V}_i(t, x(t)) \leq -\alpha V_i(t, x(t)) - \Gamma(t), \ t \in [t_{k-1}^c, t_k), \\ \dot{V}_{ij}(t, x(t)) \leq \beta V_{ij}(t, x(t)) - \Gamma(t), \ t \in [t_k, t_k^c), \end{cases} \tag{5}$$

and

$$\begin{aligned} &V_{ij}(t_k, x(t_k)) \leq \gamma_1 V_i(t_k^-, x(t_k^-)), \\ &V_i(t_k^c, x(t_k^c)) \leq \gamma_2 V_{ij}(t_k^{c-}, x(t_k^{c-})), \quad k \in \mathbb{N}, \end{aligned} \tag{6}$$

where $\Gamma(t) = \varphi(t)(\frac{1}{\gamma} z^{\mathrm{T}}(t) z(t) - \theta \gamma w^{\mathrm{T}}(t) w(t))$, $\theta = \mu_0 \mathrm{e}^{-N_0(\ln(\gamma_1\gamma_2) + \delta_{c,2}(\alpha+\beta)) + \frac{\delta_{c,1} \ln \gamma_2}{\tau_a}}$, and

$$\begin{aligned} &\tau_a = \frac{\ln(\gamma_1\gamma_2) + (\alpha + \beta)\delta_{c,2}}{\alpha_0}, \\ &\varphi(t) = \begin{cases} \varphi_1(t), & t \in [t_{k-1}^c, t_k), \\ \varphi_2(t), & t \in [t_k, t_k^c), \end{cases} \end{aligned} \tag{7}$$

in which $\varphi_1(t)$ and $\varphi_2(t)$ are defined in (14), $\mu_0 = \min\{\mu_1, \mu_2\}$, and $\alpha_0 \in (0, \alpha)$.

The proof of Lemma 1 is presented in Appendix A.

Remark 1: The notations δ_1 and δ_2 in switching time sequence $\mathcal{S}(\delta_1, \delta_2)$ represent the minimum and maximum dwell time of the switching signal $\sigma(t)$, respectively. Then the ADT τ_a needs to satisfy $\delta_1 \le \tau_a \le \delta_2$. Obviously, τ_a also satisfies $\tau_a > \tau_a^* \triangleq \frac{\ln(\gamma_1\gamma_2)+(\alpha+\beta)\delta_{c,2}}{\alpha}$.

Lemma 2: For matrices $E=E^{\mathrm{T}}\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $K\in\mathbb{R}^{m\times p}$, $X\in\mathbb{R}^{p\times n}$, $p\le n$, if there exist $S\in\mathbb{R}^{p\times p}$ and positive scalar ε such that the following matrix inequality holds:

$$\begin{bmatrix} E+He\{BKS[I_p\ \tilde{I}]\} & X^{\mathrm{T}}-[I_p\ \tilde{I}]^{\mathrm{T}}S^{\mathrm{T}}+\varepsilon BKS \\ * & -\varepsilon He\{S\} \end{bmatrix} \le 0, \tag{8}$$

where $\tilde{I}=I_{p\times(n-p)}$. Then we have

$$E+BKX+(BKX)^{\mathrm{T}} \le 0. \tag{9}$$

The proof of Lemma 2 is given in Appendix A.

Lemma 3: ([25]) For any matrices $P>0$, U of appropriate dimensions, and positive scalar ν, the following matrix inequality holds:

$$UP^{-1}U^{\mathrm{T}} \ge \nu(U+U^{\mathrm{T}})-\nu^2P.$$

3. MAIN RESULTS

In this section, we employ different time-varying MLFs for the matched interval and the unmatched interval to analyze the stability and weighted L_2-gain of the closed-loop systems (4), which is less conservative than only constructing an identical MLFs. In the following, we introduce several piecewise auxiliary functions [25] in combination with switching time sequences $\{t_k, k\in\mathbb{N}\}$ to construct the time-varying MLFs.

For $t\in[t_{k-1}^c, t_k)$, we define $\bar{\rho}_1(t)=\frac{1}{t_k-t_{k-1}^c}$, and

$$\rho_1(t)=(t-t_{k-1}^c)\bar{\rho}_1(t),\ \ \rho_2(t)=(t_k-t)\bar{\rho}_1(t),\ \ k\in\mathbb{N}.$$

It follows that

$$\rho_1(t_k^-)=\rho_2(t_{k-1}^c)=1,\ \ \rho_1(t_{k-1}^c)=\rho_2(t_k^-)=0. \tag{10}$$

Let

$$\rho_{12}(t)=\begin{cases} \dfrac{1/\delta_{p,1}-\bar{\rho}_1(t)}{1/\delta_{p,1}-1/\delta_{p,2}}, & \delta_{p,2}>\delta_{p,1},\\ 0, & \delta_{p,2}=\delta_{p,1}, \end{cases}$$

and $\rho_{11}(t)=1-\rho_{12}(t)$. We have

$$\bar{\rho}_1(t)=\frac{1}{\delta_{p,1}}\rho_{11}(t)+\frac{1}{\delta_{p,2}}\rho_{12}(t). \tag{11}$$

Similarly, for $t\in[t_k, t_k^c)$, we define $\bar{\rho}_2(t)=\frac{1}{t_k^c-t_k}$, and

$$\tilde{\rho}_1(t)=(t-t_k)\bar{\rho}_2(t),\ \ \tilde{\rho}_2(t)=1-\tilde{\rho}_1(t),\ \ k\in\mathbb{N}.$$

Then, we obtain

$$\tilde{\rho}_1(t_k^{c-})=\tilde{\rho}_2(t_k)=1,\ \ \tilde{\rho}_1(t_k)=\tilde{\rho}_2(t_k^{c-})=0. \tag{12}$$

Let

$$\tilde{\rho}_{12}(t)=\begin{cases} \dfrac{1/\delta_{c,1}-\bar{\rho}_2(t)}{1/\delta_{c,1}-1/\delta_{c,2}}, & \delta_{c,2}>\delta_{c,1},\\ 0, & \delta_{c,2}=\delta_{c,1}, \end{cases}$$

and $\tilde{\rho}_{11}(t)=1-\tilde{\rho}_{12}(t)$. It implies that

$$\bar{\rho}_2(t)=\frac{1}{\delta_{c,1}}\tilde{\rho}_{11}(t)+\frac{1}{\delta_{c,2}}\tilde{\rho}_{12}(t). \tag{13}$$

Based on the introduction of auxiliary functions, we construct the following time-varying MLFs for $t\in[t_{k-1}^c, t_k^c)$, $k\in\mathbb{N}$,

$$\begin{cases} V_i(t,x(t))=\varphi_1(t)x^{\mathrm{T}}(t)P_i(t)x(t), & t\in[t_{k-1}^c, t_k),\\ V_{ij}(t,x(t))=\varphi_2(t)x^{\mathrm{T}}(t)P_{ij}(t)x(t), & t\in[t_k, t_k^c), \end{cases} \tag{14}$$

where $\varphi_1(t)=\mu_1^{\rho_1(t)}$, $P_i(t)=\rho_1(t)P_{i1}+\rho_2(t)P_{i2}$, $\varphi_2(t)=\mu_2^{\tilde{\rho}_1(t)}$, $P_{ij}(t)=\tilde{\rho}_1(t)P_{ij1}+\tilde{\rho}_2(t)P_{ij2}$, in which $0<\mu_l\le 1$, $P_{il}>0, P_{ijl}>0$, $l=1,2$, $i,j\in\mathbf{I}[1,N_p]$.

A sufficient condition is derived in the following theorem by the constructed time-varying MLFs (14) to guarantee the weighted L_2-gain performance for the closed-loop systems (4).

Theorem 1: Consider the closed-loop systems (4) satisfying (**A1**). Given scalars $\gamma_1\ge 1$, $\gamma_2\ge 1$, $\alpha>0$, $\beta\ge 0$, nonnegative integer N_0, and a class $\mathcal{S}(\delta_1,\delta_2)$ of switching time sequences with the ADT τ_a satisfying (7), the closed-loop systems (4) is internally UGES and has a weighted L_2-gain less than γ over $\mathcal{S}(\delta_1,\delta_2)$, if for prescribed positive scalars ε_{il}, ε_{ijl}, ν_i, ν_{ij}, η_{ij} and $\mu_l\in(0,1]$, there exist matrices $\bar{K}_i$, $\bar{E}_{ij}$, S_i, F_{ij}, $X_{il}>0$, $X_{ijl}>0$ of appropriate dimensions, and positive scalar γ, such that the following LMIs hold:

$$\Phi_{i1h}\le 0,\ \ \begin{bmatrix} \Phi_{i2h} & \mathbb{X}_{i2} \\ * & -\delta_{p,h}X_{i1} \end{bmatrix}\le 0, \tag{15}$$

$$\Phi_{ij1h}\le 0,\ \ \begin{bmatrix} \Phi_{ij2h} & \mathbb{X}_{ij2} \\ * & -\delta_{c,h}X_{ij1} \end{bmatrix}\le 0, \tag{16}$$

$$\left[\begin{array}{c|c|c} -\mu_2\gamma_2X_{ij1} & X_{ij1}\mathcal{I}_1+\mathcal{I}_3\bar{E}_{ij}^{\mathrm{T}}\mathcal{I}_2^{\mathrm{T}} & X_{ij1}\mathcal{I}_2-\mathcal{I}_3F_{ij}^{\mathrm{T}} \\ \hline * & -X_{j2} & \eta_{ij}\mathcal{I}_2\bar{E}_{ij} \\ * & * & -\eta_{ij}He\{F_{ij}\} \end{array}\right]\le 0, \tag{17}$$

$$X_{i1}\le\mu_1\gamma_1X_{ij2},\ \ i,j\in\mathbf{I}[1,N_p],\ \ i\ne j,\ \ l,h=1,2, \tag{18}$$

where

$$\Phi_{ilh}=\begin{bmatrix} \Sigma_{ilh}+He\{\mathcal{B}_i\bar{K}_i\mathcal{I}\} & \mathcal{X}_{il}\mathcal{H}_i^{\mathrm{T}}-\mathcal{I}^{\mathrm{T}}S_i^{\mathrm{T}}+\varepsilon_{il}\mathcal{B}_i\bar{K}_i \\ * & -\varepsilon_{il}He\{S_i\} \end{bmatrix},$$

$$\Phi_{ijlh}=\begin{bmatrix} \Sigma_{ijlh}+He\{\mathcal{B}_j\bar{K}_i\mathcal{I}\} & \mathcal{X}_{ijl}\mathcal{H}_j^{\mathrm{T}}-\mathcal{I}^{\mathrm{T}}S_i^{\mathrm{T}}+\varepsilon_{ijl}\mathcal{B}_j\bar{K}_i \\ * & -\varepsilon_{ijl}He\{S_i\} \end{bmatrix},$$

$$\Sigma_{ilh} = \begin{bmatrix} \Omega_{ilh} & \tilde{H}_{1,i} & X_{il}\tilde{C}_{1,i}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & \tilde{H}_{2,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix},$$

$$\Sigma_{ijlh} = \begin{bmatrix} \Omega_{ijlh} & \tilde{H}_{1,j} & X_{ijl}\tilde{C}_{1,j}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & \tilde{H}_{2,j}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix},$$

and $\Omega_{i1h} = (\frac{\ln\mu_1+1-2\nu_i}{\delta_{p,h}} + \alpha)X_{i1} + \frac{\nu_i^2}{\delta_{p,h}}X_{i2} + He\{\tilde{A}_iX_{i1}\}$, $\Omega_{i2h} = (\frac{\ln\mu_1-1}{\delta_{p,h}} + \alpha)X_{i2} + He\{\tilde{A}_iX_{i2}\}$, $\Omega_{ij1h} = (\frac{\ln\mu_2+1-2\nu_{ij}}{\delta_{c,h}} - \beta)X_{ij1} + \frac{\nu_{ij}^2}{\delta_{c,h}}X_{ij2} + He\{\tilde{A}_jX_{ij1}\}$, $\Omega_{ij2h} = (\frac{\ln\mu_2-1}{\delta_{c,h}} - \beta)X_{ij2} + He\{\tilde{A}_jX_{ij2}\}$, $\mathcal{B}_i = [\tilde{B}_{1,i}^{\mathrm{T}}\ 0\ \tilde{B}_{2,i}^{\mathrm{T}}]^{\mathrm{T}}$, $\mathcal{I} = [I_{n+q}\ \tilde{I}]$, $\mathcal{I}_1 = \mathrm{diag}\{I_n\ 0_n\}$, $\mathcal{I}_2 = [0_n\ I_n]^{\mathrm{T}}$, $\mathcal{I}_3 = [I_n\ I_n]^{\mathrm{T}}$, $\mathcal{X}_{il} = \mathrm{diag}\{X_{il}, I_{n_w}, I_p\}$, $\mathcal{X}_{ijl} = \mathrm{diag}\{X_{ijl}, I_{n_w}, I_p\}$, $\mathbb{X}_{i2} = [X_{i2}, 0]^{\mathrm{T}}$, $\mathbb{X}_{ij2} = [X_{ij2}, 0]^{\mathrm{T}}$, $\mathcal{H}_i = [\tilde{C}_{2,i}\ \tilde{H}_{3,i}\ 0]$, $\theta = \mu_0 e^{-N_0(\ln(\gamma_1\gamma_2)+\delta_{c,2}(\alpha+\beta))+\frac{\delta_{c,1}\ln\gamma_2}{\tau_a}}$, $\mu_0 = \min\{\mu_1,\mu_2\}$, $\tilde{I} = I_{(n+q)\times(n+n_w+p-q)}$. Furthermore, the gain matrices and reset matrices are presented as follows:

$$K_{c,i} = \bar{K}_iS_i^{-1},\ E_{ij} = \bar{E}_{ij}F_{ij}^{-1},\ i,j \in \mathbf{I}[1,N_p],\ i \neq j.$$

Proof: Before the next processing, set $P_{il} = X_{il}^{-1}$, $P_{ijl} = X_{ijl}^{-1}$, $i,j \in \mathbf{I}[1,N_p], i \neq j$, $l = 1,2$. If the left inequality of (15) holds, it follows by Lemma 2 that

$$\Sigma_{i1h} + He\{\mathcal{B}_iK_{c,i}\mathcal{H}_i\mathcal{X}_{i1}\} \le 0,$$

which is equivalent to

$$\begin{bmatrix} \tilde{\Omega}_{i1h} & B_{i,i} & X_{i1}C_{i,i}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & D_{i,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix} \le 0, \tag{19}$$

where $\tilde{\Omega}_{i1h} = \Omega_{i1h} + He\{A_{i,i}X_{i1}\} - He\{\tilde{A}_iX_{i1}\}$. By Lemma 3, it follows from (19) that

$$\begin{bmatrix} \bar{\Omega}_{i1h} & B_{i,i} & X_{i1}C_{i,i}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & D_{i,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix} \le 0, \tag{20}$$

where $\bar{\Omega}_{i1h} = (\frac{\ln\mu_1+1}{\delta_{p,h}} + \alpha)X_{i1} - \frac{1}{\delta_{p,h}}X_{i1}P_{i2}X_{i1} + He\{A_{i,i}X_{i1}\}$.

Pre-multiplying and Post-multiplying matrix $\mathrm{diag}\{P_{i1}, I_{n_w}, I_p\}$ on both sides of inequality (20), we have

$$\Xi_{i1h} = \begin{bmatrix} \hat{\Omega}_{i1h} & P_{i1}B_{i,i} & C_{i,i}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & D_{i,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix} \le 0, \tag{21}$$

where $\hat{\Omega}_{i1h} = (\frac{\ln\mu_1}{\delta_{p,h}} + \alpha)P_{i1} + \frac{1}{\delta_{p,h}}(P_{i1} - P_{i2}) + He\{P_{i1}A_{i,i}\}$.

Let $\mathfrak{I} = [I_{2n}\ 0_{2n\times n_w}\ 0_{2n\times p}]$. Applying Schur complement and Lemma 2 for the right inequality of (15), we have

$$\Sigma_{i2h} + \frac{1}{\delta_{p,h}}\mathfrak{I}^{\mathrm{T}}X_{i2}P_{i1}X_{i2}\mathfrak{I} + He\{\mathcal{B}_iK_{c,i}\mathcal{H}_i\mathcal{X}_{i2}\} \le 0,$$

which is equivalent to

$$\begin{bmatrix} \tilde{\Omega}_{i2h} & B_{i,i} & X_{i2}C_{i,i}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & D_{i,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix} \le 0, \tag{22}$$

where $\tilde{\Omega}_{i2h} = (\frac{\ln\mu_1-1}{\delta_{p,h}} + \alpha)X_{i2} + \frac{1}{\delta_{p,h}}X_{i2}P_{i1}X_{i2} + He\{A_{i,i}X_{i2}\}$. Multiplying matrix $\mathrm{diag}\{P_{i2}, I_{n_w}, I_p\}$ to the right and left of the inequality (22), we obtain

$$\Xi_{i2h} = \begin{bmatrix} \hat{\Omega}_{i2h} & P_{i2}B_{i,i} & C_{i,i}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & D_{i,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix} \le 0, \tag{23}$$

where $\hat{\Omega}_{i2h} = (\frac{\ln\mu_1}{\delta_{p,h}} + \alpha)P_{i2} + \frac{1}{\delta_{p,h}}(P_{i1} - P_{i2}) + He\{P_{i2}A_{i,i}\}$.

Analogous to the inequalities (21) and (23) derived from (15), the following inequality can be derived from the inequalities (16),

$$\Xi_{ijlh} = \begin{bmatrix} \hat{\Omega}_{ijlh} & P_{ijl}B_{j,i} & C_{j,i}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & D_{j,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix} \le 0, \tag{24}$$

where $\hat{\Omega}_{ijlh} = (\frac{\ln\mu_2}{\delta_{c,h}} - \beta)P_{ijl} + \frac{1}{\delta_{c,h}}(P_{ij1} - P_{ij2}) + He\{P_{ijl}A_{j,i}\}$.

According to the convex combination technique, the following inequalities are easy to be obtained from (21) and (23),

$$\Xi_i(t) = \sum_{l,h=1}^{2} \rho_l(t)\rho_{1h}(t)\Xi_{ilh} \le 0, \quad i \in \mathbf{I}[1,N_p].$$

Then, we have

$$\Xi_i(t) = \begin{bmatrix} \Psi_i(t) & P_i(t)B_{i,i} & C_{i,i}^{\mathrm{T}} \\ * & -\theta\gamma I_{n_w} & D_{i,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix} \le 0, \tag{25}$$

where $\Psi_i(t) = (\bar{\rho}_1(t)\ln\mu_1 + \alpha)P_i(t) + \bar{\rho}_1(t)(P_{i1} - P_{i2}) + He\{P_i(t)A_{i,i}\}$. Using Schur complement, we can conclude that (25) is equivalent to

$$\tilde{\Xi}_i(t) = \begin{bmatrix} \Psi_i(t) & P_i(t)B_{i,i} \\ * & -\theta\gamma I_{n_w} \end{bmatrix} + \frac{1}{\gamma}\begin{bmatrix} C_{i,i}^{\mathrm{T}} \\ D_{i,i}^{\mathrm{T}} \end{bmatrix}[C_{i,i}\ D_{i,i}] \le 0. \tag{26}$$

Moreover, it is easy to obtain from inequality (24) that

$$\begin{bmatrix} \Psi_{ij}(t) & P_{ij}(t)B_{j,i} \\ * & -\theta\gamma I_{n_w} \end{bmatrix} + \frac{1}{\gamma}\begin{bmatrix} C_{j,i}^{\mathrm{T}} \\ D_{j,i}^{\mathrm{T}} \end{bmatrix}[C_{j,i}\ D_{j,i}] \le 0, \tag{27}$$

where $\Psi_{ij}(t) = (\bar{\rho}_2(t)\ln\mu_2 - \beta)P_{ij}(t) + \bar{\rho}_2(t)(P_{ij1} - P_{ij2}) + He\{P_{ij}(t)A_{j,i}\}$.

In addition, it means from inequality (17) combined with Lemma 2 that

$$\begin{bmatrix} -\mu_2\gamma_2X_{ij1} & X_{ij1}\tilde{E}_{ij}^{\mathrm{T}} \\ * & -X_{j2} \end{bmatrix} \le 0, \quad i,j \in \mathbf{I}[1,N_p],\ i \neq j. \tag{28}$$

Pre-multiplying and Post-multiplying $\text{diag}\{P_{ij1}, P_{j2}\}$ on both sides of inequality (28), and using Schur complement, we can derive the following matrix inequalities,

$$\tilde{E}_{ij}^{\mathrm{T}} P_{j2} \tilde{E}_{ij} \leq \mu_2 \gamma_2 P_{ij1}, \quad i,j \in \mathbf{I}[1,N_p],\ i \neq j. \tag{29}$$

Furthermore, the following inequalities can be derived from inequality (18),

$$P_{ij2} \leq \mu_1 \gamma_1 P_{i1}, \quad i,j \in \mathbf{I}[1,N_p],\ i \neq j. \tag{30}$$

Let $V_i(t) = V_i(t,x(t))$, $V_{ij}(t) = V_{ij}(t,x(t))$ and $\xi(t) = [x^{\mathrm{T}}(t)\ w^{\mathrm{T}}(t)]^{\mathrm{T}}$, where $V_i(t,x(t))$ and $V_{ij}(t,x(t))$ are defined in (14). For $t \in [t_{k-1}^c, t_k)$, taking the derivative of $V_i(t)$ along the trajectory of the closed-loop systems (4), we have

$$\begin{aligned}&\dot{V}_i(t) + \alpha V_i(t) + \varphi_1(t)(\frac{1}{\gamma} z^{\mathrm{T}}(t) z(t) - \theta \gamma w^{\mathrm{T}}(t) w(t))\\ &= \varphi_1(t) \xi^{\mathrm{T}}(t) \tilde{\Xi}_i(t) \xi(t).\end{aligned}$$

It is easy to derive from (26) that

$$\dot{V}_i(t) \leq -\alpha V_i(t) - \varphi_1(t)(\frac{1}{\gamma} z^{\mathrm{T}}(t) z(t) - \theta \gamma w^{\mathrm{T}}(t) w(t)),$$

which satisfies the condition (5) of Lemma 1.

In the following, we take the derivative of $V_{ij}(t)$ along the trajectory of the closed-loop systems (4) for $t \in [t_k, t_k^c)$. It implies from inequality (27) that

$$\dot{V}_{ij}(t) \leq \beta V_{ij}(t) - \varphi_2(t)(\frac{1}{\gamma} z^{\mathrm{T}}(t) z(t) - \theta \gamma w^{\mathrm{T}}(t) w(t)),$$

which, obviously, satisfies the condition (5) of Lemma 1.

On the other hand, by the properties (10) and (12) of the auxiliary functions, the following inequalities are derived from inequalities (29) and (30),

$$V_{ij}(t_k) \leq \gamma_1 V_i(t_k^-), \quad V_i(t_k^c) \leq \gamma_2 V_{ij}(t_k^{c-}),$$

which satisfies the condition (6) of Lemma 1.

Hence, all the conditions of Lemma 1 are satisfied. Namely, the closed-loop systems (4) is internally UGES and has a weighted L_2-gain less than γ over $\mathcal{S}(\delta_1, \delta_2)$. □

Note that the control synthesis conditions presented in Theorem 1 are with a form of LMIs, as opposite to the form of bilinear matrix inequalities in the existing results such as [22, 23]. Meanwhile, a time-varying multiple Lyapunov-like-function approach is employed in the present work such that the control synthesis conditions are established dependent on the upper and lower bounds of the switching interval. Therefore, more precise control synthesis conditions are obtained for different switching signals, which helps to reduce the conservativeness of the results.

Remark 2: Note that the above result is based on mode-independent ADT, where all subsystems have a uniform maximum, minimum and average dwell time. It is also possible to investigate the switching signal $\sigma(t)$ by following the mode-dependent ADT approach such as [6], where each subsystem has its own allowable maximum, minimum and average dwell time, expressed as τ_{i2}, τ_{i1} and τ_{ia}, $i \in \mathbf{I}[1,N_p]$, respectively. Extension of the present work to model-dependent ADT deserves separate studies in the future.

Remark 3: The LMIs of (15), (16) and (17)-(18) are the control synthesis conditions when the controller matches the subsystem, when the controller does not match the subsystem, and at the switching instant, respectively. Note that we assume that $(A_i, B_{1,i}, C_{2,i})$ is stabilizable and detectable. Thus, there exists a set of parameters $(\bar{K}_i, S_i, X_{il})$ such that the conditions (15) are solvable. Moreover, we can ensure that the conditions (16) and (17)-(18) hold by choosing β, γ_1 and γ_2 sufficiently large.

It is interesting to consider a special case of Theorem 1, where the time-varying MLFs (14) is reconstructed as a time-invariant MLFs as follows:

$$\begin{cases} V_i(x(t)) = x^{\mathrm{T}}(t) P_i x(t), & t \in [t_{k-1}^c, t_k), \\ V_{ij}(x(t)) = x^{\mathrm{T}}(t) P_{ij} x(t), & t \in [t_k, t_k^c), \end{cases}$$

where $P_i > 0$, $P_{ij} > 0$, $i,\ j \in [1,N_p]$, $i \neq j$. Let $X_i = P_i^{-1}$, $X_{ij} = P_{ij}^{-1}$, $i,\ j \in [1,N_p]$, $i \neq j$. Then, we can obtain the following corollary for the closed-loop systems (4).

Corollary 1: Consider the closed-loop systems (4) satisfying (**A1**). Given scalars $\gamma_1 \geq 1$, $\gamma_2 \geq 1$, $\alpha > 0$, $\beta \geq 0$, nonnegative integer N_0, and a class of switching signal $\sigma(t)$ with the ADT τ_a satisfying (7), the closed-loop systems (4) is internally UGES and has a weighted L_2-gain less than γ, if for prescribed positive scalars ε_i, ε_{ij}, and η_{ij}, there exist matrices $\bar{K}_i$, $\bar{E}_{ij}$, S_i, F_{ij}, $X_i > 0$, $X_{ij} > 0$ of appropriate dimensions, and positive scalar γ, such that the following LMIs hold:

$$\begin{bmatrix} \Sigma_i + He\{\mathcal{B}_i \bar{K}_i \mathcal{I}\} & \mathcal{X}_i \mathcal{H}_i^{\mathrm{T}} - \mathcal{I}^{\mathrm{T}} S_i^{\mathrm{T}} + \varepsilon_i \mathcal{B}_i \bar{K}_i \\ * & -\varepsilon_i He\{S_i\} \end{bmatrix} \leq 0,$$

$$\begin{bmatrix} \Sigma_{ij} + He\{\mathcal{B}_j \bar{K}_i \mathcal{I}\} & \mathcal{X}_{ij} \mathcal{H}_j^{\mathrm{T}} - \mathcal{I}^{\mathrm{T}} S_i^{\mathrm{T}} + \varepsilon_{ij} \mathcal{B}_j \bar{K}_i \\ * & -\varepsilon_{ij} He\{S_i\} \end{bmatrix} \leq 0,$$

$$\begin{bmatrix} -\gamma_2 X_{ij} & X_{ij}\mathcal{I}_1 + \mathcal{I}_3 \bar{E}_{ij}^{\mathrm{T}} \mathcal{I}_2^{\mathrm{T}} & X_{ij}\mathcal{I}_2 - \mathcal{I}_3 F_{ij}^{\mathrm{T}} \\ * & -X_j & \eta_{ij} \mathcal{I}_2 \bar{E}_{ij} \\ * & * & -\eta_{ij} He\{F_{ij}\} \end{bmatrix} \leq 0,$$

$$X_i \leq \gamma_1 X_{ij}, \qquad i,j \in \mathbf{I}[1,N_p],\ i \neq j,$$

where

$$\Sigma_i = \begin{bmatrix} \Omega_i & \tilde{H}_{1,i} & X_i \tilde{C}_{1,i}^{\mathrm{T}} \\ * & -\bar{\theta}\gamma I_{n_w} & \tilde{H}_{2,i}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix},$$

$$\Sigma_{ij} = \begin{bmatrix} \Omega_{ij} & \tilde{H}_{1,j} & X_{ij} \tilde{C}_{1,j}^{\mathrm{T}} \\ * & -\bar{\theta}\gamma I_{n_w} & \tilde{H}_{2,j}^{\mathrm{T}} \\ * & * & -\gamma I_p \end{bmatrix},$$

and $\Omega_i = \alpha X_i + He\{\tilde{A}_i X_i\}$, $\Omega_{ij} = -\beta X_{ij} + He\{\tilde{A}_j X_{ij}\}$, $\mathcal{X}_i = \mathrm{diag}\{X_i, I_{n_w}, I_p\}$, $\mathcal{X}_{ij} = \mathrm{diag}\{X_{ij}, I_{n_w}, I_p\}$, $\bar{\theta} = \frac{\theta}{\mu_0}$. $\mathcal{B}_i$, $\mathcal{I}$, $\mathcal{I}_1$, $\mathcal{I}_2$, $\mathcal{I}_3$, and $\mathcal{H}_i$ are defined as Theorem 1. Furthermore, the gain matrices and reset matrices are presented as follows:

$$K_{c,i} = \bar{K}_i S_i^{-1}, \quad E_{ij} = \bar{E}_{ij} F_{ij}^{-1}, \quad i,j \in \mathbf{I}[1,N_p],\ i \neq j.$$

The proof of Corollary 1 is similar to the proof of Theorem 1, and thus omitted here.

Based on the above analysis, we establish the control synthesis conditions in Theorem 1 and Corollary 1, which guarantee the switched linear systems (1) to achieve a weighted L_2-gain less than γ under the asynchronous switching controller (2). The control synthesis problem is reduced to an LMIs optimization problem to obtain the minimum γ. For given scalars α, β, γ_1 and γ_2, the LMIs optimization problem is expressed as follows:

$$\begin{aligned} &\text{Minimize} \quad \gamma, \\ &\text{subject to} \quad (15)-(18). \end{aligned} \tag{31}$$

4. NUMERICAL EXAMPLES

We consider the following two examples to demonstrate the effectiveness of our theoretical results.

Example 1: Consider the continuous-time switched linear systems (1) with $N_p = 2$, where the system matrices are given as follows

$$A_1 = \begin{bmatrix} -0.2274 & -0.0594 & 0.53 \\ -0.2667 & -0.883 & 0.081 \\ 0.133 & -0.74 & 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.1135 & 0.3756 & -0.21 \\ 0.6404 & -0.2835 & 0.33 \\ 0.1 & -0.344 & -0.1 \end{bmatrix},$$

$$\begin{aligned} &H_{1,1} = -[0.4396\ 0.6347\ 0.1]^{\mathrm{T}}, \quad H_{2,1} = H_{2,2} = 0, \\ &H_{1,2} = [0.0813\ 0.6236\ 0.1]^{\mathrm{T}}, \quad H_{3,1} = H_{3,2} = 0.01, \\ &C_{1,1} = [0.01\ 0.06\ 0.02],\ B_{1,2} = -[0.886\ 1.725\ 2.5]^{\mathrm{T}}, \\ &C_{2,1} = [-0.05\ 0.02\ 0.05], \quad C_{1,2} = [0.01\ 0.02\ 0.05], \\ &B_{1,1} = -[1.8555\ 1.204\ 2.2]^{\mathrm{T}}, \quad B_{2,1} = B_{2,2} = 0.1, \\ &C_{2,2} = [-0.08\ 0.15\ -0.01]. \end{aligned}$$

The disturbance input $w(t)$ is considered as follows:

$$w(t) = \begin{cases} 0.5, & t \in [2.9, 6]\mathrm{s}, \\ -2, & t \in [10, 10.5]\mathrm{s}, \\ 0, & \text{otherwise}. \end{cases}$$

It is easy to verify from Fig. 1 that the two subsystems of the open-loop systems (1) are unstable. In the following, we design a dynamic output-feedback switching controller with controller state reset to achieve the weighted L_2-gain performance of the closed-loop systems.

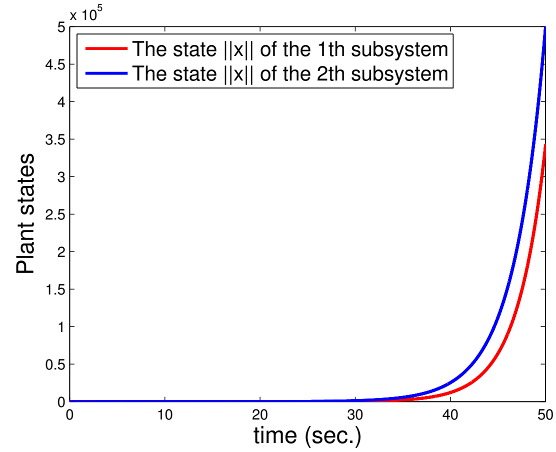


Fig. 1. The state trajectories of open-loop system.

Table 1. The weighted L_2-gain γ for different choices of α, β, μ_1, $\delta_{p,1}$ and $\delta_{p,2}$.

α	β	μ_1	$\delta_{p,1}$	$\delta_{p,2}$	τ_a^*	γ
1.0	0.1	0.737	0.2	0.2630	0.2828	5.1990
1.0	0.2	0.735	0.2	0.2650	0.2848	9.4938
1.0	0.3	0.733	0.2	0.2669	0.2868	44.0420
1.5	0.1	0.761	0.1	0.1754	0.1952	0.7137
1.5	0.2	0.760	0.1	0.1767	0.1965	0.7473
1.5	0.3	0.758	0.1	0.1781	0.1979	0.7946
2.0	0.1	0.769	0.1	0.1316	0.1514	0.5593
2.0	0.2	0.767	0.1	0.1326	0.1524	0.5838
2.0	0.3	0.766	0.1	0.1336	0.1534	0.6104

We fix parameters $\gamma_1 = 1.18$, $\gamma_2 = 1.1$, $\mu_2 = 1$ and $\varepsilon_{il} = 1$, $\varepsilon_{ijl} = 1$, $\nu_{ij} = 1$, $\nu_i = 1$, $\eta_{ij} = 1$, $i,j \in \mathbf{I}[1,N_p], i \neq j$, $l = 1,2$. Moreover, we select the maximum and minimum values of asynchronous delays as $\delta_{c,2} = 0.02$ and $\delta_{c,1} = 0.01$, respectively. By solving the optimization problem (31), we obtain the minimum value of γ for different choices of $(\alpha, \beta, \mu_1, \delta_{p,1}, \delta_{p,2})$ as listed in Table 1. We can see that the value of γ decreases as the value of α increases, and increases with the increase of the value of β.

For simulation studies, we choose the parameters $\alpha = 2$, $\beta = 0.1$, $\mu_1 = 0.764$, $\delta_{p,1} = 0.1$ and $\delta_{p,2} = 0.135$ to verify the validity of the theoretical results. Solving the optimization problem (31) with the above given parameters, we obtain that the minimum value of γ is 1.1417.

We design the switching time signals $\sigma(t)$ and $\bar{\sigma}(t)$ to guarantee the ADT τ_a satisfying $\tau_a^* = 0.1514 \leq \tau_a \leq 0.1550 = \delta_{c,2} + \delta_{p,2}$. The designed switching time signals $\sigma(t)$ and $\bar{\sigma}(t)$ are displayed in Fig. 2. The time evolution curves of the control input $u(t)$ and the controlled output $z(t)$ are shown in Fig. 3. It illustrates from Fig. 3 that the designed switching controller (2) guarantee the closed-loop systems (4) achieving the weighted L_2-gain less than

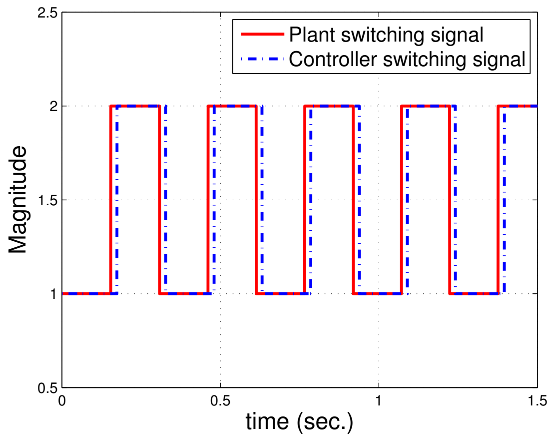


Fig. 2. The switching signal $\sigma(t)$ and $\bar{\sigma}(t)$.

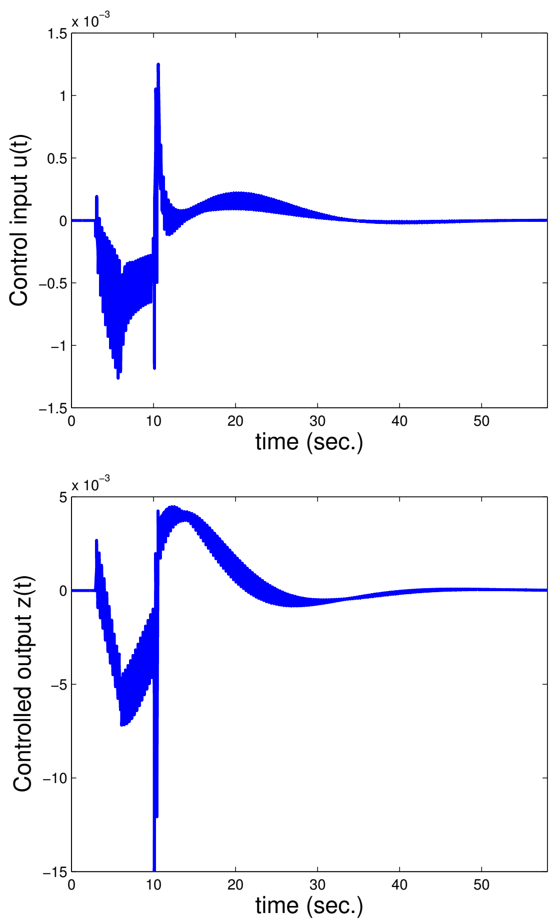


Fig. 3. The time response curves of the control input $u(t)$ and the controlled output $z(t)$ under the designed switching signal.

1.1417 under asynchronous switching with maximum delay 0.02s.

For the purpose of comparison with [22, 23], we consider the following numerical example to demonstrate the effectiveness of the main results in this paper.

Table 2. The weighted L_2-gain γ and the minimum ADT τ_a^* for different choices of $(\alpha, \mu_1, \delta_{p,2})$.

α	μ_1	$\delta_{p,2}$	γ	τ_a^*
0.10	0.34	3.0	6.6271	2.6482
0.25	0.31	1.5	1.4675	1.0713
0.50	0.35	0.6	0.1825	0.5456
0.75	0.38	0.5	0.1427	0.3704
1.00	0.44	0.4	0.1020	0.2828
1.50	0.55	0.3	0.0782	0.1952
2.00	0.76	0.2	0.0550	0.1514

Example 2: Consider the continuous-time switched linear systems (1) with the following system matrices

$$A_1 = \begin{bmatrix} -9 & 0.2 \\ 0.3 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -5 & 0 \\ 0 & -2 \end{bmatrix},$$
$$H_{1,1} = [-0.4 \ -0.1]^{\mathrm{T}}, \ C_{1,1} = [0\ 1], \ C_{1,2} = [0.5\ 0.1],$$
$$H_{1,2} = [-0.1\ 0.1]^{\mathrm{T}}, \ C_{2,1} = [1\ 0], \ C_{2,2} = [0.5\ 0.1],$$
$$B_{1,1} = [-0.5\ 0.1]^{\mathrm{T}}, \ B_{1,2} = [-0.1\ 0.2]^{\mathrm{T}},$$
$$H_{2,1} = H_{2,2} = 0, \ B_{2,1} = B_{2,2} = 0.1,$$
$$H_{3,1} = H_{3,2} = 0.01.$$

The disturbance input $w(t)$ is considered as follows:

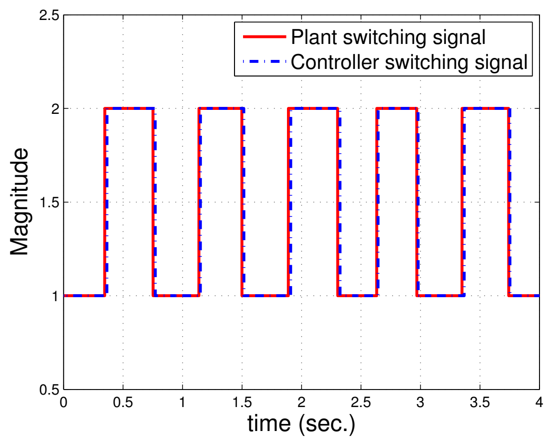
$$w(t) = \begin{cases} 0.5, & t \in [2.9, 6]\mathrm{s}, \\ -1, & t \in [10, 10.5]\mathrm{s}, \\ 0, & \text{otherwise}. \end{cases}$$

In the following, a dynamic output-feedback switching controller with controller state reset is designed to achieve the weighted L_2-gain performance of the closed-loop systems.

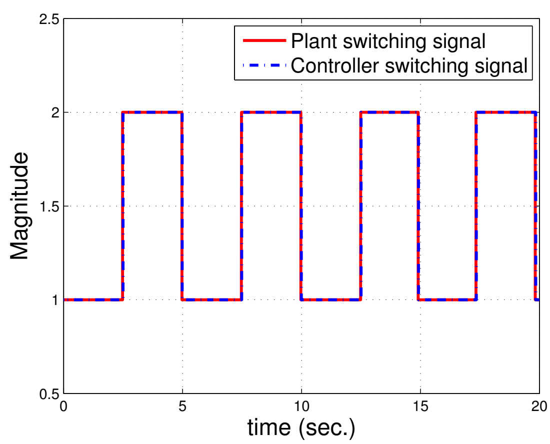
The parameters γ_l, μ_2, $\delta_{c,1}$, $\delta_{c,2}$ and ε_{il}, ε_{ijl}, ν_{ij}, ν_i, η_{ij}, $i, j \in \mathbf{I}[1, N_p]$, $i \neq j$, $l = 1, 2$ are given as Example 1. Without loss of generality, we choose $\beta = 0.1$ and $\delta_{p,1} = 0.1$. By solving the optimization problem (31), we obtain the minimum value of γ and the minimum ADT τ_a^* for different choices of $(\alpha, \mu_1, \delta_{p,2})$ as presented in Table 2.

For comparison purposes, we establish the control synthesis conditions of the closed-loop systems by the method in [22, 23]. Based on the algorithm provided in [22, 23], the minimum ADT $\bar{\tau}_a^*$ for different choices of (α, γ) can be obtained by solving the established control synthesis conditions, as shown in Table 3. It is easy to see from Table 2 and 3 that a smaller minimum ADT can be obtained by the proposed method in the present paper when the same weighted L_2-gain performance is achieved. Therefore, the proposed method is less conservative than the results proposed in [22, 23].

For simulation studies, we choose the parameters $\alpha = 1$, $\mu_1 = 0.44$, and $\delta_{p,2} = 0.4$ to illustrate the effectiveness



(a) The switching signals of the plant and the controller with the method proposed in this paper.



(b) The switching signals of the plant and the controller with the method in [22,23].

Fig. 4. The switching signals of the plant and the controller.

Table 3. The minimum ADT $\bar{\tau}_a^*$ for different choices of (α,γ).

α	γ	β	γ_1	γ_2	$\bar{\tau}_a^*$
0.10	6.6271	4.5200	1.5207	1.0054	5.1694
0.25	1.4675	13.7961	1.6693	1.0046	3.1914
0.50	0.1825	7.0088	2.5013	1.0190	2.1716
0.75	0.1427	9.8454	3.6924	1.0104	2.0380
1.00	0.1020	73.1115	2.4587	1.0141	2.3959
1.50	0.0782	87.4341	55.9393	1.1408	3.9564
2.00	0.0550	237.2338	31.6316	1.0576	4.1476

of the theoretical results. We obtain from Table 2 that the minimum value of γ and the minimum ADT τ_a^* are $\gamma = 0.1020$ and $\tau_a^* = 0.2828$, respectively.

Here, we design the switching signal $\sigma(t)$ of the plant to guarantee the ADT τ_a satisfying $\tau_a^* = 0.2828 \leq \tau_a \leq$

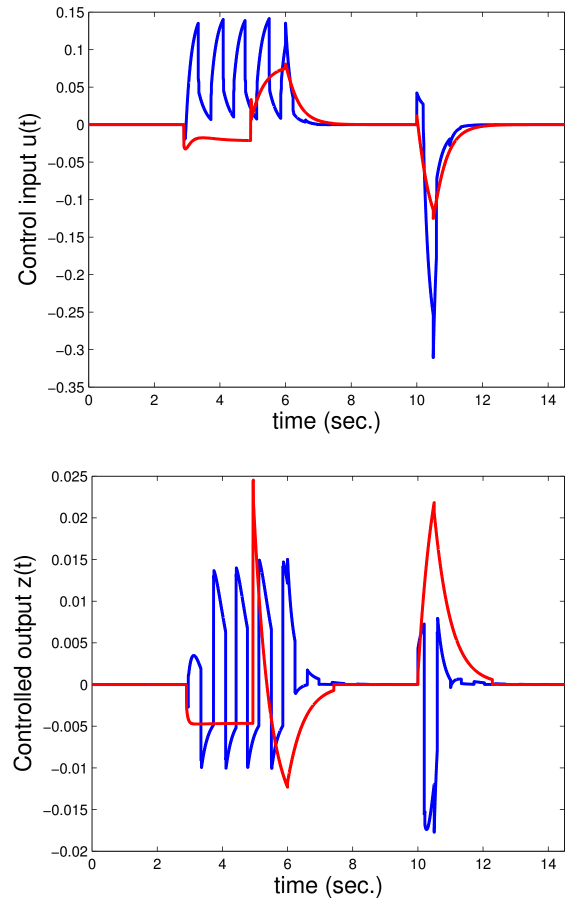


Fig. 5. The time response curves of the control input $u(t)$ and the controlled output $z(t)$.

$0.42 = \delta_{c,2} + \delta_{p,2}$, and design the switching signal $\bar{\sigma}(t)$ of the controller to satisfy $0.01 \leq \Delta_k \leq 0.02$, $k \in \mathbb{N}$. In addition, the switching signals $\sigma(t)$ and $\bar{\sigma}(t)$ for the method in [22,23] are also designed to satisfy $\tau_a^* = 2.3959 \leq \tau_a$ and $0.01 \leq \Delta_k \leq 0.02$, $k \in \mathbb{N}$. The designed switching signals are shown in Fig. 4. The time evolution curves of the control input $u(t)$ and the controlled output $z(t)$ are shown in Fig. 5. In Fig. 5, it should be noted that the blue lines represent the time evolution curves obtained by the method proposed in this paper, and the red lines indicate the time evolution curves obtained by the method in [22,23]. It can be seen from Fig. 5 that the controller designed with all the methods can achieve the weighted L_2-gain less than 0.1020 under asynchronous switching with maximum delay $0.02s$, where the method proposed in this paper has a smaller ADT τ_a.

5. CONCLUSIONS

In this paper, the asynchronous switching problem for continuous-time switched linear systems via dynamic output feedback control has been solved. Based on the convex combination technique, a time-varying MLFs was constructed to establish the control synthesis conditions for the closed-loop systems. It is worth noting that the derived conditions are related to the upper and lower bounds of asynchronous delays. Moreover, the upper and lower bounds of asynchronous delays can be estimated by solving the feasibility of LMIs (15)-(18). Finally, two numerical examples are provided to demonstrate the feasibility of our proposed method.

APPENDIX A

A.1. The proof of Lemma 1

First, let $V_i(t) = V_i(t,x(t))$ and $V_{ij}(t) = V_{ij}(t,x(t))$. Note that $\mu_0 < \varphi(t) \le 1$, $\forall t \in [t_0,\infty)$. For $t \in [t^c_{k-1},t_k)$, we can obtain from inequalities (5) and (6) that

$$\begin{aligned} V_i(t) \le& V_i(t^c_{k-1})\mathrm{e}^{-\alpha(t-t_{k-1})+\alpha\triangle_{k-1}} - \int_{t^c_{k-1}}^{t} \mathrm{e}^{-\alpha(t-s)}\Gamma(s)ds \\ \le& \gamma_2 V_{ij}(t^{c-}_{k-1})\mathrm{e}^{-\alpha(t-t_{k-1})+\alpha\triangle_{k-1}} - \int_{t^c_{k-1}}^{t} \mathrm{e}^{-\alpha(t-s)}\Gamma(s)ds \\ \le& V_\sigma(t_0)\mathrm{e}^{-\alpha(t-t_0)+\Pi(t_0,t)} - \int_{t_0}^{t}\mathrm{e}^{-\alpha(t-s)+\Pi(s,t)}\Gamma'(s)ds, \end{aligned} \tag{A.1}$$

where $\Pi(s,t) = (\alpha+\beta)T_\uparrow(s,t) + N_\sigma(s,t)\ln\gamma_1 + N_{\bar\sigma}(s,t)\ln\gamma_2$, $\Gamma'(t) = \frac{\mu_0}{\gamma}z^{\mathrm{T}}(t)z(t) - \theta\gamma w^{\mathrm{T}}(t)w(t)$.

Next, we multiply on both sides of (A.1) by $\mathrm{e}^{-\Pi(t_0,t)}$ to obtain

$$\mathrm{e}^{-\Pi(t_0,t)}V_i(t) \le V_\sigma(t_0)\mathrm{e}^{\alpha(t_0-t)} - \int_{t_0}^{t}\mathrm{e}^{-\alpha(t-s)-\Pi(t_0,s)}\Gamma'(s)ds.$$

Similarly, for $t \in [t_k,t^c_k)$, we have

$$\mathrm{e}^{-\Pi(t_0,t)}V_{ij}(t) \le V_\sigma(t_0)\mathrm{e}^{\alpha(t_0-t)} - \int_{t_0}^{t}\mathrm{e}^{-\alpha(t-s)-\Pi(t_0,s)}\Gamma'(s)ds.$$

It is noted that $V_i(t) > 0$, $V_{ij}(t) > 0$ and $V_\sigma(t_0) = 0$. Then, for $\forall t > t_0$, we have

$$\int_{t_0}^{t}\mathrm{e}^{-\alpha(t-s)}[\mathrm{e}^{-\Pi(t_0,s)}z^{\mathrm{T}}(s)z(s) - \frac{\theta\gamma^2}{\mu_0}w^{\mathrm{T}}(s)w(s)]ds \le 0.$$

In addition, there is a fact that $T_\uparrow(t_0,s) \le N_\sigma(t_0,s)\delta_{c,2}$ and $N_{\bar\sigma}(t_0,s) \le N_\sigma(t_0,s-\delta_{c,1})$. We choose a positive constant α_0 satisfying $\alpha_0 < \alpha$ such that the following inequality holds:

$$\Pi(t_0,s) \le \ln\mu_0 - \ln\theta + \alpha_0(s-t_0).$$

Then we can consider the ADT τ_a taking value as (7). Thus, we have

$$\int_{t_0}^{t}\mathrm{e}^{\alpha(s-t)-\alpha_0(s-t_0)}z^{\mathrm{T}}(s)z(s)ds \le \int_{t_0}^{t}\gamma^2\mathrm{e}^{\alpha(s-t)}w^{\mathrm{T}}(s)w(s)ds.$$

We integrate on both sides of the above inequality from $t = t_0$ to ∞ to get

$$\int_{t_0}^{\infty}\mathrm{e}^{-\alpha_0(s-t_0)}z^{\mathrm{T}}(s)z(s)ds \le \gamma^2\int_{t_0}^{\infty}w^{\mathrm{T}}(s)w(s)ds.$$

Thus, the closed-loop systems (4) can achieve a weighted L_2-gain less than γ over $\mathcal{S}(\delta_1,\delta_2)$ under the zero initial condition.

In what follows, we prove that the closed-loop systems (4) with $w(t) = 0$ is UGES over $\mathcal{S}(\delta_1,\delta_2)$. First, let $\sigma(t_0) = i_0$, $i_0 \in \mathbf{I}[1,N_p]$, $\lambda_0 = \max\{\lambda_{\max}(P_{i_0l}), l = 1,2\}$, $\lambda_1 = \min\{\lambda_{\min}(P_{il}), \lambda_{\min}(P_{ijl}), i,j \in \mathbf{I}[1,N_p],\ l = 1,2\}$. Let $x(t) \triangleq x(t,t_0,x(t_0))$ be the solution of the closed-loop systems (4) with $w(t) = 0$ through $(t_0,x(t_0))$. For $t \in [t^c_{k-1},t_k), k \in \mathbb{N}$, by virtue of (A.1), we have

$$V_i(t) \le V_{i_0}(t_0)\mathrm{e}^{-\alpha(t-t_0)+\Pi(t_0,t)}, \quad i \in \mathbf{I}[1,N_p].$$

In the same way, for $t \in [t_k,t^c_k), k \in \mathbb{N}$, we obtain

$$V_{ij}(t) \le V_{i_0}(t_0)\mathrm{e}^{-\alpha(t-t_0)+\Pi(t_0,t)}, \quad i,j \in \mathbf{I}[1,N_p],\ i \ne j.$$

Hence, for $t \in [t_k,t_{k+1})$, $k \in \mathbb{N}$, one has

$$\|x(t)\|^2 \le \mathcal{K}^2\|x(t_0)\|^2\mathrm{e}^{-(\alpha-\alpha_0)(t-t_0)},$$

where $\mathcal{K} = \sqrt{\frac{\lambda_0}{\lambda_1\theta}}$. It indicates that

$$\|x(t)\| \le \mathcal{K}\|x(t_0)\|\mathrm{e}^{-\frac{(\alpha-\alpha_0)}{2}(t-t_0)},\ \ t \ge t_0.$$

Thus, the closed-loop systems (4) with $w(t) = 0$ is UGES over $\mathcal{S}(\delta_1,\delta_2)$. □

A.2. The proof of Lemma 2

Multiplying the matrix $[I_n\ BK]$ and its transpose to the left and right sides of inequality (8), respectively, we have $E + BKX + (BKX)^{\mathrm{T}} \le 0$. Thus, inequality (9) can be derived from inequality (8). □

REFERENCES

[1] N. H. El-Farra, A. Gani, and P. D. Christofides, "A switched systems approach for the analysis and control of mode transitions in biological networks," *Proc. of American Control Conference*, pp. 3247-3252, 2005.

[2] N. R. Gans and S. A. Hutchinson, "Stable visual servoing through hybrid switched-system control," *IEEE Trans. on Robotics*, vol. 23, no. 3, pp. 530-540, June 2007.

[3] G. S. Deaecto, M. Souza, and J. C. Geromel, "Discrete-time switched linear systems state feedback design with application to networked control," *IEEE Trans. on Automatic Control*, vol. 60, no. 3, pp. 877-881, March 2015.

[4] L. Vu and K. A. Morgansen, "Stability of time-delay feedback switched linear systems," *IEEE Trans. on Automatic Control*, vol. 55, no. 10, pp. 2385-2390, October 2010.

[5] M. Wakaiki and Y. Yamamoto, "Stability analysis of sampled-data switched systems with quantization," *Automatica*, vol. 69, pp. 157-168, July 2016.

[6] X. Zhao, L. Zhang, and P. Shi , "Stability and stabilization of switched linear systems with mode-dependent average dwell time," *IEEE Trans. on Automatic Control*, vol. 57, no. 7, pp. 1809-1815, July 2012.

[7] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Systems Magazine*, vol. 19, no. 5, pp. 59-70, October 1999.

[8] X. Ding and X. Liu, "Stability analysis for switched positive linear systems under state-dependent switching," *International Journal of Control, Automation and Systems*, vol. 15, no. 2, pp. 481-488, April 2017.

[9] P. Yan and H. Özbay, "Stability analysis of switched time delay systems," *SIAM Journal on Control and Optimization*, vol. 47, no. 2, pp. 936-949, February 2008.

[10] V. F. Montagner, V. J. S. Leite, and R. Oliveira, "State feedback control of switched linear systems: An LMI approach," *Journal of Computational and Applied Mathematics*, vol. 194, no. 2, pp. 192-206, October 2006.

[11] J. C. Geromel and G. S. Deaecto, "Switched state feedback control for continuous-time uncertain systems," *Automatica*, vol. 45, no. 2, pp. 593-597, February 2009.

[12] C. Z. Yuan and F. Wu, "Hybrid control for switched linear systems with average dwell time," *IEEE Trans. on Automatic Control*, vol. 60, no. 1, pp. 240-245, January 2015.

[13] P. Riedinger and J. C. Vivalda, "Dynamic output feedback for switched linear systems based on a LQG design," *Automatica*, vol. 54, pp. 235-245, April 2015.

[14] Q. Zheng, H. Zhang, and D. Zheng, "Stability and asynchronous stabilization for a class of discrete-time switched nonlinear systems with stable and unstable subsystems," *International Journal of Control, Automation and Systems*, vol. 15, no. 3, pp. 986-994, June 2017.

[15] Y. E. Wang, J. Zhao, and B. Jiang, "Stabilization of a class of switched linear neutral systems under asynchronous switching," *IEEE Trans. on Automatic Control*, vol. 58, no. 8, pp. 2114-2119, August 2013.

[16] D. Ma and J. Zhao, "Stabilization of networked switched linear systems: An asynchronous switching delay system approach," *Systems & Control Letters*, vol. 77, pp. 46-54, March 2015.

[17] J. Cheng, J. H. Park, and H. R. Karimi, "Static output feedback control of nonhomogeneous Markovian jump systems with asynchronous time delays," *Information Sciences*, vol. 399, pp. 219-238, August 2017.

[18] J. Cheng, J. H. Park, and L. Zhang, "An asynchronous operation approach to event-triggered control for fuzzy Markovian jump systems with general switching policies," *IEEE Trans. on Fuzzy Systems*, vol. 26, no. 1, pp. 6-18, February 2018.

[19] L. X. Zhang and P. Shi, "Stability, l_2-gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time," *IEEE Trans. on Automatic Control*, vol. 54, no. 9, pp. 2192-2199, September 2009.

[20] L. X. Zhang and H. Gao, "Asynchronously switched control of switched linear systems with average dwell time," *Automatica*, vol. 46, no. 5, pp. 953-958, May 2010.

[21] C. Z. Yuan and F. Wu, "Asynchronous switching output feedback control of discrete-time switched linear systems," *International Journal of Control*, vol. 88, no. 9, pp. 1766-1774, March 2015.

[22] R. Wang, Z. G. Wu, and P. Shi, "Dynamic output feedback control for a class of switched delay systems under asynchronous switching," *Information Sciences*, vol. 225, pp. 72-80, March 2013.

[23] X. H. Wang, G. D. Zong, and H. B. Sun, "Asynchronous finite-time dynamic output feedback control for switched time-delay systems with non-linear disturbances," *IET Control Theory & Applications*, vol. 10, no. 10, pp. 1142-1150, June 2016.

[24] B. Lu, F. Wu, and S. W. Kim, "Switching LPV control of an F-16 aircraft via controller state reset," *IEEE trans. on control systems technology*, vol. 14, no. 2, pp. 267-277, March 2006.

[25] W-H. Chen, Z. Y. Jiang, and J. C. Zhong, "On designing decentralized impulsive controllers for synchronization of complex dynamical networks with nonidentical nodes and coupling delays," *Journal of the franklin institute*, vol. 351, no. 8, pp. 4084-4110, August 2014.

Zhiyong Jiang received the B.S. degree in information and computing science from Hunan university of Arts and Science, Changde, China, in 2012, and the M.S. degree in operational research and cybernetics from Guangxi University, Nanning, China, in 2015. He is currently pursuing the Ph.D. degree at the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China. His research interests include complex dynamical networks, hybrid systems, impulsive control, and anti-windup control.

Peng Yan received the B.S. and M.S. degrees in electrical engineering from Southeast University, Nanjing, China, in 1997 and 1999, respectively, and the Ph.D. degree in electrical engineering from The Ohio State University, Columbus, OH, in 2003. He is currently a full Professor with the School of Mechanical Engineering, Shandong University, Jinan, China, and also with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China. He has worked with various industry positions before joining Shandong University, including a Staff Scientist at United Technologies Research Center at East Hartford, CT, from 2010 to 2011, and a Senior Staff Engineer at Seagate Technology at Twin Cities, MN from 2005 to 2010. His current research interests include robust control, hybrid systems, and control of high precision mechatronics. He has authored more than 70 scientific papers, and more than 30 granted/pending patents.