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Consensus Based Control Algorithm for Nonlinear Vehicle Platoons in the Presence of Time Delay

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Abstract: The platoon control problem for nonlinear vehicles in the presence of time delay is investigated in this paper, where both constant time delay and time-varying delay cases are considered. A linearized third-order vehicle dynamic model is firstly derived by deploying the exact feedback linearization technique and the vehicle platoon control problem is converted into a consensus-seeking problem. Then, a consensus based vehicle platoon control algorithm with time delay is proposed, which drives vehicles to form an equally spaced platoon with the same velocity. By deploying the Lyapunov-Razumikhin theorem, the upper bound of time delay for vehicle platoon with constant time delay is derived and the sufficient conditions that guarantee the stability of the vehicle platoon are obtained. Meanwhile, the sufficient conditions that ensure the stability of vehicle platoon with time-varying delay are acquired via the Lyapunov-Krasovskii theorem. Numerical demonstrations verify the feasibility and correctness of the theoretical results.

Keywords: Consensus, feedback linearization, nonlinear vehicle dynamics, time delay, vehicle platoon.

1. INTRODUCTION

Traffic congestion, road accident and air pollution have become a worldwide social problem with the ever increasing number of vehicles in large cities [1], which impel governments, automobile manufactures and academic researchers to make great efforts for the next generation of transportation systems [2]. Among all the feasible solutions, vehicle platoon [3], which requires vehicles to move in a string with predefined inter-vehicle space and the same velocity, has been identified as a promising alternative in future intelligent transportation systems (ITS) for its prominent advantages in enhancing traffic safety, improving highway capacity, increasing fuel economy and reducing carbon emissions [4–6].

Over the last decades, vehicle platoon has gained considerable interests in the academic community [3, 7, 8], various control schemes, such as sliding mode control [9–11], fuzzy logic control [12, 13], consensus based control [14, 15], model predictive control [16, 17] and so forth, have been developed. Among the existing methodologies, consensus based vehicle platoon control is more and more deployed as the control objective of vehicle platoon (i.e., all vehicles in the platoon move at the same velocity and maintain a desired inter-vehicle distance) can be easily formulated into a consensus-seeking problem [15]. For instance, a consensus based control scheme was proposed to evaluate the performance of vehicle platoon under different network topologies of initial states in [14]. Under a weighted and constrained consensus framework, the vehicle platoon control problem for enhancing highway safety and efficient utility was studied in [18]. Using a modified consensus-based control method, the vehicle platoon problem with absent velocity measurement and actuator saturation constrains was investigated in [19].

In practical applications, vehicles are required to get the state information of other vehicles via some wireless vehicle-to-vehicle (V2V) communication techniques like DSRC, VANET or 4G-LTE [4, 7, 20]. However, the information transmission between vehicles will inevitably induce the phenomenon of time delay due to the limited bandwidth or the congestion of communication channels [15, 21]. Time delay, which is known as a source of system instability, may degrade the performance of vehicle platoon and even cause the instability of the vehicle string [22, 23]. To this end, a consensus based control algorithm was developed for multi-platoon cooperative driving by considering the time delays in [15]. A distributed consensus strategy for vehicle platoon with time-varying heterogeneous communication delays was proposed in [24, 25].

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In addition, the stabilized parametric regions for vehicle strings with time delay were investigated via the cluster treatment of characteristic roots paradigm [22].

However, the above results primarily treat the vehicle dynamics as a second-order linear system, which may not well coincide with its real dynamic characteristics [15]. In fact, the vehicle dynamics is a typical high-order model with strong nonlinearity [26]. As far as we are concerned, literatures that specifically address the vehicle platoon control problem with nonlinearity, high order and time delays seem very few. Until very recently, a robust exponential H_∞ controller for vehicle platoon, where the vehicle nonlinearity, actuator saturation as well as time delay were considered, was proposed in [27].

Motivated by this fact, the platoon control of nonlinear vehicles with time delay is investigated in this paper. The main contributions of this paper are two-folds:

- The real nonlinear vehicle dynamic model is deployed and a feedback linearization method is adopted to convert the vehicle dynamics into a third-order linear system. Comparing with the existing results, like [15, 22, 24, 25], that simply take the vehicle dynamics as a second-order system, it is more akin to the real dynamics of a vehicle and of more practical significance in vehicle platoon control.
- Both constant time delay and time-varying delay in vehicle platoon control are investigated. Lyapunov-Razumikhin theorem and Lyapunov-Krasovskii functional are deployed to derive the upper bounds and the sufficient conditions for the stability of nonlinear vehicle platoon with constant time delay and time-varying delay, respectively.

The remainder of this paper is organized as follows: In section 2, some preliminary knowledge on graph theory, matrix theory and time-delay systems are introduced. In section 3, the linearized third-order vehicle dynamics is derived and a consensus based vehicle platoon control algorithm is designed. In section 4, the sufficient conditions for the stability of vehicle platoon with both constant time delay and time-varying delay are derived and the corresponding upper bounds of time delay are obtained. Numerical simulations are performed in section 5 to demonstrate the correctness of the theoretical results. Section 6 offers the concluding remarks and future work.

2. MATHEMATICAL PRELIMINARIES

Before addressing the vehicle platoon control problem with time delay, some mathematical preliminaries, including algebraic graph theory, matrix theory and time-delay systems, are firstly introduced.

2.1. Algebraic graph theory

In a vehicle platoon, if each vehicle is regarded as a node, the communication topology among vehicles can then be easily described by a neighboring graph $\mathcal{G}$. Here, graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is a directed graph consisting of a set of nodes (vehicles) $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$, a set of edges (communication links) $\mathcal{E} = \{(v_i, v_j) \in \mathcal{V} \times \mathcal{V} : v_i \sim v_j\}$ and an adjacency matrix $\mathcal{A} = [a_{ij}]$ with nonnegative adjacency elements a_{ij}. Here, we define $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise.

The set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$. Graph $\mathcal{G}$ is called an undirected graph if $(v_i, v_j) \in \mathcal{E} \Leftrightarrow (v_j, v_i) \in \mathcal{E}$. For an undirected graph, its adjacency matrix is symmetric (i.e., $\mathcal{A}^{\mathrm{T}} = \mathcal{A}$) and the corresponding Laplacian matrix $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \cdots, d_n\} \in \mathbb{R}^{n\times n}$ is the in-degree matrix of graph $\mathcal{G}$ with $d_i = \sum_{j=1}^{n} a_{ij}$ being the in-degree of node v_i. In addition, the Laplacian matrix $\mathcal{L}$ is symmetric and positive semi-definite with minimum eigenvalue zero and the corresponding eigenvector is $\mathbf{1} = [1, \cdots, 1]^{\mathrm{T}}$, i.e., $\mathcal{L}\mathbf{1} = 0$.

2.2. Matrix theory preliminaries

Lemma 1: For any vector x, y of appropriate dimensions and any symmetric positive definite matrix Z of appropriate dimension, the following inequality holds

$$\pm 2x^{\mathrm{T}}y \le x^{\mathrm{T}}Zx + y^{\mathrm{T}}Z^{-1}y. \tag{1}$$

Lemma 2 (Schur complement [28])**:** Given a symmetric matrix $F = F^{\mathrm{T}} \in \mathbb{R}^{(n+m)\times(n+m)}$ is partitioned as

$$F = \begin{bmatrix} A & B^{\mathrm{T}} \\ B & C \end{bmatrix}, \tag{2}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times n}$, $C \in \mathbb{R}^{m\times m}$. The following conditions are equivalent:

1) $F < 0$;
2) $C < 0$ and $A - B^{\mathrm{T}}C^{-1}B < 0$;
3) $A < 0$ and $C - BA^{-1}B^{\mathrm{T}} < 0$.

2.3. Theorems for time-delay systems

Consider the following system

$$\begin{cases} \dot{x}(t) = f(t, x_t), & t > 0, \\ x(\theta) = \varphi(\theta), & \theta \in [-\tau, 0], \end{cases} \tag{3}$$

where $x_t(\theta) = x(t+\theta), \forall \theta \in [-\tau, 0]$ and $f(t, 0) = 0$. Let $C([-\tau, 0], \mathbb{R}^n)$ be a Banach space of continuous functions defined on an interval $[-\tau, 0]$, taking values in $\mathbb{R}^n$ with the topology of uniform convergence, and with a norm $\|\varphi\|_c = \max_{\theta\in[-\tau,0]} \|\varphi(\theta)\|$. Then, the following results are for the stability of system (3):

Lemma 3 (Lyapunov-Razumikhin Theorem [29])**:** Let ϕ_1, ϕ_2 and ϕ_3 be continuous, nonnegative, nondecreasing functions with $\phi_1 > 0, \phi_2 > 0, \phi_3 > 0$ for $s > 0$ and $\phi_1(0) = \phi_2(0) = 0$. For system (3), suppose that the function $f : C([-\tau,0],\mathbb{R}^n) \to R$ takes bounded sets of $C([-\tau,0],\mathbb{R}^n)$ in bounded sets of $\mathbb{R}^n$. If there is a continuous function $V(t,x)$ such that

$$\phi_1(\|x\|) \le V(t,x) \le \phi_2(\|x\|),\ t \in R,\ x \in \mathbb{R}^n. \tag{4}$$

In addition, there exists a continuous nondecreasing function $\phi(s)$ with $\phi(s) > 0, s > 0$ such that

$$\begin{aligned} &\dot{V}(t,x) \le -\phi_3(\|x\|) \ \text{ if} \\ &V(t+\theta, x(t+\theta)) < \phi(V(t,x(t))),\ \theta \in [-\tau,0], \end{aligned} \tag{5}$$

then the solution $x = 0$ is uniformly asymptotically stable.

Usually, $V(t,x)$ is called Lyapunov-Razumikhin function if it satisfies (4) and (5) in Lemma 3.

Remark 1: It can be seen in Lemma 3 that one only needs to consider the initial data if a trajectory of (3) starting from these initial data is "diverging" rather than to require that $\dot{V}(t,x)$ be non-positive for all initial data in order to have the stability of system (3).

Consider the following differential equation with time delay

$$\dot{x}(t) = f(t,x_t),\ t \ge t_0, \tag{6}$$

where $x(t) \in \mathbb{R}^n$ is a state vector. In addition, $x_t(\theta)$ denotes a transfer operator of state trajectory for $[-\tau,0]$ and is defined as $x_t(\theta) = x(t+\theta)$ for $\forall\theta \in [-\tau,0]$. Functional $f(t,x_t)$ is continuous for x_t and satisfies $f(t,0) = 0$.

Lemma 4 (Lyapunov-Krasovskii Theorem) [29]**:** Supposing the mapping $f : \mathbb{R}\times\mathbb{C} \to \mathbb{R}^n$ is continuous and nondecreasing ($u,v,w : \bar{\mathbb{R}}_+ \to \bar{\mathbb{R}}_+$. For $s > 0$, $u(s) > 0$, $v(s) > 0$; for $s = 0$, $u(s) = v(s) = 0$), the stability of system (6) can be proved if it satisfies

1) $V : \mathbb{R}\times\mathbb{C} \to \mathbb{R}$ is continuous and differentiable;
2) $u\|\phi(0)\| \le V(t,\phi) \le v\|\phi\|_c$;
3) $\dot{V}(t,\phi) \le -\varpi(\|\phi(0)\|)$.

In addition, the solution $x = 0$ is uniformly asymptotically stable for $s > 0$, $\varpi(s) > 0$ and globally uniformly asymptotically stable for $\lim_{s\to\infty} u(s) = \infty$.

3. PROBLEM FORMULATION

3.1. Vehicle dynamics modeling

Consider a group of vehicles moving in a 1-D longitudinal path in a leader-follower fashion. Vehicles are assumed to be equipped with on-board sensors (e.g., inertial measurement unit, GPS and radar) to measure the position, velocity, acceleration of itself and its preceding neighbors [25]. In addition, each vehicle can get the state information of other vehicles via a V2V communication paradigm, the topological structure of the vehicle platoon is illustrated in Fig 1.

According to Newton's law, the dynamic equation for each vehicle can be formulated as [26]

$$\begin{cases} \dot{x}_i = v_i, \\ m_i\dot{v}_i = F_i + F_i^{\text{g}} + F_i^{\text{aero}} + F_i^{\text{drag}}, \end{cases} \tag{7}$$

where x_i, v_i and m_i are the position, velocity and mass of vehicle i, respectively.

The right hand side for the second equation of (7) represents the force acting on vehicle i. To be specific,

- $F_i = m_i\xi_i$ is the force produced by the engine, and $\dot{\xi}_i = -\frac{\xi_i}{\mu_i(v_i)} + \frac{\alpha_i}{m_i\mu_i(v_i)}$ models the engine dynamics, where $\mu_i(v_i)$ denotes the vehicle's engine time-constant with speed v_i and α_i represents the throttle input to vehicle's engine.
- $F_i^{\text{g}} = -m_i g\sin(\theta_i)$ represents the vehicle's weight parallel to the road surface, where g is the acceleration of gravity and θ_i denotes the angle between the road surface and the horizontal plane.
- $F_i^{\text{aero}} = -\frac{\rho A_i C_{di}}{2}(v_i + V_{\text{wind}})^2\text{sgn}(v_i + V_{\text{wind}})$ is the aerodynamic force, where ρ is the specific mass of air, A_i is the cross-sectional area of the vehicle, C_{di} is the drag coefficient and V_{wind} denotes the velocity of the wind gust.
- $F_i^{\text{drag}} = -d_{mi}$ is a constant, which represents the amplitude of the mechanical drag force.

Consequently, (7) can be rewritten as

$$\begin{cases} \dot{x}_i = v_i, \\ m_i\dot{v}_i = m_i\xi_i - m_i g\sin(\theta_i) - d_{mi} \\ \qquad\quad - \dfrac{\rho A_i C_{di}}{2}(v_i + V_{\text{wind}})^2\text{sgn}(v_i + V_{\text{wind}}), \\ \dot{\xi}_i = -\dfrac{\xi_i}{\mu_i(v_i)} + \dfrac{\alpha_i}{m_i\mu_i(v_i)}. \end{cases} \tag{8}$$

Assumption 1: The road surface is horizontal and there is no wind gust, then we have $\theta_i = 0$ and $V_{\text{wind}} = 0$.

Assumption 2: All vehicles are moving along the same direction, it can be obtained that $\text{sgn}(v_i) = 1$.

Then, the vehicle dynamics (8) can be simplified as

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = \xi_i - \dfrac{\rho A_i C_{di}}{2m_i}{v_i}^2 - \dfrac{d_{mi}}{m_i}, \\ \dot{\xi}_i = -\dfrac{\xi_i}{\mu_i(v_i)} + \dfrac{\alpha_i}{m_i\mu_i(v_i)}. \end{cases} \tag{9}$$

Remark 2: It is well known that the force of a vehicle is produced by its engine system [26]. In (9), the control input for vehicle i is the engine's throttle input α_i, which is more akin to the real dynamics of a vehicle than the previous mentioned literatures [15, 22, 24, 25] as they merely take the vehicle's acceleration (driving or braking force) as the control input.

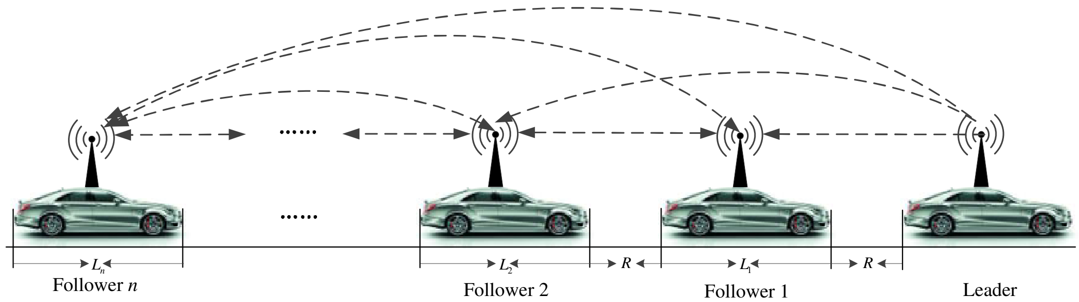

Fig. 1. The topology structure of vehicle platoon.

For vehicle dynamics (9), an exogenous input u_i is introduced as the new control input. Then, by deploying the exact feedback linearization method [27], the linearized model of a single vehicle can be represented as

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = a_i, \\ \dot{a}_i = u_i, \end{cases} \tag{10}$$

where u_i is the newly defined control input, x_i, v_i and a_i denote the position, velocity and acceleration of vehicle i, respectively.

Remark 3: From (10) it can be seen that the linearized vehicle model is a third-order linear dynamic system, which is more simple than the nonlinear vehicle dynamic model (9). In the following, the vehicle platoon control problem with time delay will be investigated on the basis of (10).

3.2. Consensus based vehicle platoon algorithm

Given the third-order vehicle dynamic model (10), the vehicle platoon problem for maintaining a desired distance with successive vehicles and having the same velocity and acceleration with the leader vehicle can therefore be formulated as a consensus problem [24]. Then, the aim of vehicle platoon is to drive the positions, velocities and accelerations of all the vehicles towards the following equations

$$\begin{aligned} &\lim_{t\to\infty} \|x_i - x_j\| = R_{ij}, \\ &\lim_{t\to\infty} \|v_i - v_L\| = 0, \\ &\lim_{t\to\infty} \|a_i - a_L\| = 0, \end{aligned} \tag{11}$$

where $R_{ij} = |i-j| \cdot (R+L_i)$ represents the distance between vehicle i and j. Here, L_i is the body length of vehicle i and R is the safe distance between two consecutive vehicles. In addition, v_L and a_L are the velocity and acceleration of the leader vehicle, respectively.

If time delay is considered in the state information exchange between vehicles, the control objective (11) can be rewritten as

$$\begin{aligned} &\lim_{t\to\infty} \|x_i(t-\Psi(t)) - x_j(t-\Psi(t))\| = R_{ij}, \\ &\lim_{t\to\infty} \|v_i(t-\Psi(t)) - v_L(t-\Psi(t))\| = 0, \\ &\lim_{t\to\infty} \|a_i(t-\Psi(t)) - a_L(t-\Psi(t))\| = 0, \end{aligned} \tag{12}$$

where $\Psi(t)$ represents the time delay.

Here, by deploying the consensus strategy, the distributed controller for vehicle platoon with time delay is designed as

$$\begin{aligned} u_i = &-\sum_{j=1}^{n} k_{ij}\left[x_i(t-\Psi(t)) - x_j(t-\Psi(t)) - R_{ij}\right] \\ &-\left[x_i(t-\Psi(t)) - x_L(t-\Psi(t)) - R_{iL}\right] \\ &-\sum_{j=1}^{n} k_{ij}\left[v_i(t-\Psi(t)) - v_j(t-\Psi(t))\right] \\ &-\left[v_i(t-\Psi(t)) - v_L(t-\Psi(t))\right] \\ &-\beta\sum_{j=1}^{n} k_{ij}\left[a_i(t-\Psi(t)) - a_j(t-\Psi(t))\right] \\ &-\left[a_i(t-\Psi(t)) - a_L(t-\Psi(t))\right], \end{aligned} \tag{13}$$

where $R_{iL} = i \cdot (R+L_i)$ is the distance between vehicle i and the leader, $k_{ij} > 0$ is the weighted adjacency matrix, $\beta > 0$ denotes the acceleration damping gain.

Let $\tilde{x}_i = x_i(t-\Psi(t)) - x_L(t-\Psi(t)) - R_{iL}$, $\tilde{v}_i = v_i(t-\Psi(t)) - v_L(t-\Psi(t))$, $\tilde{a}_i = a_i(t-\Psi(t)) - a_L(t-\Psi(t))$ be the position error, velocity error and acceleration error with respect to the leader vehicle, respectively. In addition, define

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_n \end{bmatrix}, \quad \tilde{v} = \begin{bmatrix} \tilde{v}_1 \\ \tilde{v}_2 \\ \vdots \\ \tilde{v}_n \end{bmatrix}, \quad \tilde{a} = \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \vdots \\ \tilde{a}_n \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}.$$

Then, the control input (13) in terms of error expressions can be represented in the compact form

$$u = -\mathcal{L}\tilde{x}(t-\Psi(t)) - \mathcal{L}\tilde{v}(t-\Psi(t))$$

$$
\begin{aligned}
&-\beta\mathcal{L}\tilde{a}(t-\Psi(t)) - \tilde{x}(t-\Psi(t))\\
&-\tilde{v}(t-\Psi(t)) - \beta\tilde{a}(t-\Psi(t))\\
=&-(\mathcal{L}+I_n)\tilde{x}(t-\Psi(t)) - (\mathcal{L}+I_n)\tilde{v}(t-\Psi(t))\\
&-\beta(\mathcal{L}+I_n)\tilde{a}(t-\Psi(t)), \qquad (14)
\end{aligned}
$$

where $\mathcal{L}$ is the Laplacian matrix of the vehicle platoon, I_n is the adjacency matrix of the leader vehicle.

In the following, some sufficient conditions that guarantee the stability of the proposed vehicle platoon control algorithm (14) in the presence of time delay will be derived.

4. MAIN RESULTS

In this section, the sufficient conditions for the stability of vehicle platoon control algorithm with constant time delay are firstly addressed. After that, the case with time-varying delay is investigated.

4.1. Vehicle platoon with constant time delay

In the case of constant time delay, $\Psi(t) = \tau$ is assumed to be a constant value. Then, (14) can be rewritten as

$$
\begin{aligned}
u =&-(\mathcal{L}+I_n)\tilde{x}(t-\tau) - (\mathcal{L}+I_n)\tilde{v}(t-\tau)\\
&-\beta(\mathcal{L}+I_n)\tilde{a}(t-\tau). \qquad (15)
\end{aligned}
$$

Writing (15) in the state space form yields

$$
\begin{bmatrix}\dot{\tilde{x}}\\ \dot{\tilde{v}}\\ \dot{\tilde{a}}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & I_n & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & I_n\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{v}\\ \tilde{a}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -H & -H & -\beta H\end{bmatrix}\begin{bmatrix}\tilde{x}(t-\tau)\\ \tilde{v}(t-\tau)\\ \tilde{a}(t-\tau)\end{bmatrix}, \qquad (16)
$$

where $H = \mathcal{L}+I_n$.
Let $\varepsilon = [\tilde{x}^{\mathrm{T}} \quad \tilde{v}^{\mathrm{T}} \quad \tilde{a}^{\mathrm{T}}]^{\mathrm{T}}$, it can be obtained that

$$
\dot{\varepsilon}(t) = C_0\varepsilon(t) + C_1\varepsilon(t-\tau), \qquad (17)
$$

where

$$
C_0 = \begin{bmatrix}\mathbf{0} & I_n & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & I_n\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},\ C_1 = \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -H & -H & -\beta H\end{bmatrix}.
$$

Then, the following theorem for the stability of vehicle platoon with constant time delay is proposed.

Theorem 1: Under the control protocol (15), the vehicle platoon (10) with constant time delay will be asymptotically stable if the time delay τ satisfies

$$
\tau < \tau^* = \frac{\lambda_{\min}(F)}{\|Q_1\| + \|Q_2\| + 2r\|P\|}, \qquad (18)
$$

where $F = -[P(C_0+C_1) + (C_0+C_1)^{\mathrm{T}}P]$ is a positive definite symmetric matrix, Q_1 and Q_2 are defined as $Q_1 = PC_1C_0P^{-1}C_0^{\mathrm{T}}C_1^{\mathrm{T}}P$ and $Q_2 = PC_1^2P^{-1}(C_1^2)^{\mathrm{T}}P$, respectively.

Then, if time delay τ is smaller than τ^*, under the platoon control algorithm (15), the followers will track the trajectory of the leader with the desired distance, meanwhile keeping the same velocity and acceleration with it asymptotically.

Proof: Consider the following Lyapunov-Razumikhin function

$$
V_1(\varepsilon) = \varepsilon^{\mathrm{T}}P\varepsilon, \qquad (19)
$$

where P is a positive definite matrix.

Taking the time derivative of V_1, it obtains

$$
\dot{V}_1 = 2\varepsilon^{\mathrm{T}}P\dot{\varepsilon}. \qquad (20)
$$

According to the Leibniz-Newton formula and (17), it is known that

$$
\begin{aligned}
\varepsilon(t) - \varepsilon(t-\tau) &= \int_{t-\tau}^{t}\dot{\varepsilon}(s)ds\\
&= \int_{-\tau}^{0}[C_0\varepsilon(t+s) + C_1\varepsilon(t+s-\tau)]\,ds. \qquad (21)
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
\varepsilon(t-\tau) =&\varepsilon(t) - \int_{t-\tau}^{t}\dot{\varepsilon}(s)ds\\
=&\varepsilon(t) - C_0\int_{-\tau}^{0}\varepsilon(t+s)ds\\
&-C_1\int_{-2\tau}^{-\tau}\varepsilon(t+s)ds. \qquad (22)
\end{aligned}
$$

Then, equation (17) can be rewritten as

$$
\begin{aligned}
\dot{\varepsilon} =&C_0\varepsilon + C_1\left[\varepsilon - \int_{t-\tau}^{t}\dot{\varepsilon}(s)ds\right]\\
=&(C_0+C_1)\varepsilon - C_1C_0\int_{-\tau}^{0}\varepsilon(t+s)ds\\
&-C_1^2\int_{-2\tau}^{-\tau}\varepsilon(t+s)ds. \qquad (23)
\end{aligned}
$$

Invoking (23), the time derivative of V_1 is that

$$
\begin{aligned}
\dot{V}_1 =&2\varepsilon^{\mathrm{T}}P(C_0+C_1)\varepsilon\\
&-2\varepsilon^{\mathrm{T}}PC_1C_0\int_{-\tau}^{0}\varepsilon(t+s)ds\\
&-2\varepsilon^{\mathrm{T}}PC_1^2\int_{-2\tau}^{-\tau}\varepsilon(t+s)ds. \qquad (24)
\end{aligned}
$$

According to Lemma 1, the following inequalities hold

$$
\begin{aligned}
&-2\varepsilon^{\mathrm{T}}PC_1C_0\int_{-\tau}^{0}\varepsilon(t+s)ds\\
&\leq \tau\varepsilon^{\mathrm{T}}PC_1C_0P^{-1}C_0^{\mathrm{T}}C_1^{\mathrm{T}}P\varepsilon + \int_{-\tau}^{0}\varepsilon(t+s)^{\mathrm{T}}P\varepsilon(t+s)ds, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
&-2\varepsilon^{\mathrm{T}}PC_1^2\int_{-2\tau}^{-\tau}\varepsilon(t+s)ds \\
&\le \tau\varepsilon^{\mathrm{T}}PC_1^2P^{-1}(C_1^2)^{\mathrm{T}}P\varepsilon+\int_{-2\tau}^{-\tau}\varepsilon(t+s)^{\mathrm{T}}P\varepsilon(t+s)ds.
\end{aligned}
\tag{26}
$$

Therefore, equation (24) can be rewritten as

$$
\begin{aligned}
\dot{V}_1 =& -\varepsilon^{\mathrm{T}}F\varepsilon-2\varepsilon^{\mathrm{T}}PC_1C_0\int_{-\tau}^{0}\varepsilon(t+s)ds \\
& -2\varepsilon^{\mathrm{T}}PC_1^2\int_{-2\tau}^{-\tau}\varepsilon(t+s)ds \\
\le& -\varepsilon^{\mathrm{T}}F\varepsilon+\tau\varepsilon^{\mathrm{T}}PC_1C_0P^{-1}C_0^{\mathrm{T}}C_1^{\mathrm{T}}P\varepsilon \\
& +\int_{-\tau}^{0}\varepsilon(t+s)^{\mathrm{T}}P\varepsilon(t+s)ds \\
& +\tau\varepsilon^{\mathrm{T}}PC_1^2P^{-1}(C_1^2)^{\mathrm{T}}P\varepsilon \\
& +\int_{-2\tau}^{-\tau}\varepsilon(t+s)^{\mathrm{T}}P\varepsilon(t+s)ds \\
\le& -\varepsilon^{\mathrm{T}}F\varepsilon+\tau\varepsilon^{\mathrm{T}}PC_1C_0P^{-1}C_0^{\mathrm{T}}C_1^{\mathrm{T}}P\varepsilon \\
& +\tau\varepsilon^{\mathrm{T}}PC_1^2P^{-1}(C_1^2)^{\mathrm{T}}P\varepsilon \\
& +\int_{-2\tau}^{0}\varepsilon(t+s)^{\mathrm{T}}P\varepsilon(t+s)ds.
\end{aligned}
\tag{27}
$$

Let $\phi(s)=rs$ and $r>1$. According to Lemma 3, it can be obtained that

$$
\varepsilon(t+s)^{\mathrm{T}}P\varepsilon(t+s)\le r\varepsilon^{\mathrm{T}}P\varepsilon, \tag{28}
$$

when

$$
V(\varepsilon(t+\theta))<rV(\varepsilon(\theta)),\quad -\tau\le\theta\le 0. \tag{29}
$$

Note that (28) satisfies the following inequality according to the mean value theorems for definite integrals

$$
\int_{-2\tau}^{0}\varepsilon(t+s)^{\mathrm{T}}P\varepsilon(t+s)ds\le 2r\tau\varepsilon^{\mathrm{T}}P\varepsilon. \tag{30}
$$

Finally, invoking (27) and (30), $\dot{V}_1$ follows

$$
\begin{aligned}
\dot{V}_1 \le& -\varepsilon^{\mathrm{T}}[F-\tau(Q_1+Q_2+2rP)]\varepsilon \\
<& 0.
\end{aligned}
\tag{31}
$$

Thus, it is known that

$$
\lim_{x\to\infty}\varepsilon(t)=0, \tag{32}
$$

i.e., $\tilde{x}(t-\tau)=x_i(t-\tau)-x_L(t-\tau)-i\cdot R_{iL}=0$, $\tilde{v}(t-\tau)=v_i(t-\tau)-v_L(t-\tau)=0$, and $\tilde{a}(t-\tau)=a_i(t-\tau)-a_L(t-\tau)=0$, which means that vehicles in the platoon will keep the desired safe distance with other vehicles, and have the same velocity and acceleration with the leader vehicle in the presence of constant time delay. □

Remark 4: It is worth mentioning that Theorem 1 only applies for the case of constant time delay in vehicle platoon. However, the time delay in the information exchange between vehicles is associated with many factors like the deployed communication protocol, inter-vehicle distance, communication bandwidth and etc, which is uaually time-varying in the vehicle state transmission process [7]. In the following, a more general case for vehicle platoon with time-varying delay will be investigated.

4.2. Vehicle platoon with time-varying delay

In this subsection, we further consider the problem of vehicle platoon with time-varying delay, i.e., $\Psi(t)=\tau(t)$. Here, $\tau(t)$ is a continuously differentiable function satisfying $0\le\tau(t)\le\tau^*$, $\dot{\tau}(t)\le h<1$, for all $t\ge 0$.
Then, equation (14) is given as

$$
\begin{aligned}
u=&-(\mathcal{L}+I_n)\tilde{x}(t-\tau(t))-(\mathcal{L}+I_n)\tilde{v}(t-\tau(t)) \\
&-\beta(\mathcal{L}+I_n)\tilde{a}(t-\tau(t)).
\end{aligned}
\tag{33}
$$

Similarly, equation (33) can be written in the state space form

$$
\begin{aligned}
\begin{bmatrix}\dot{\tilde{x}}\\ \dot{\tilde{v}}\\ \dot{\tilde{a}}\end{bmatrix}
=& \begin{bmatrix}\mathbf{0} & I_n & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & I_n\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}
\begin{bmatrix}\tilde{x}\\ \tilde{v}\\ \tilde{a}\end{bmatrix} \\
&+\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -H & -H & -\beta H\end{bmatrix}
\begin{bmatrix}\tilde{x}(t-\tau(t))\\ \tilde{v}(t-\tau(t))\\ \tilde{a}(t-\tau(t))\end{bmatrix},
\end{aligned}
\tag{34}
$$

where $H=\mathcal{L}+I_n$.
Let $\varepsilon=[\tilde{x}^{\mathrm{T}}\quad \tilde{v}^{\mathrm{T}}\quad \tilde{a}^{\mathrm{T}}]^{\mathrm{T}}$, we have

$$
\dot{\varepsilon}(t)=C_0\varepsilon(t)+C_1\varepsilon(t-\tau(t)), \tag{35}
$$

where

$$
C_0=\begin{bmatrix}\mathbf{0} & I_n & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & I_n\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},\quad
C_1=\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -H & -H & -\beta H\end{bmatrix}.
$$

Then, the following theorem for the stability of vehicle platoon with time-varying delay is introduced.

Theorem 2: Under the control protocol (33), the vehicle platoon (10) with time-varying delay can be asymptotically stable if there exist symmetric positive definite matrices G, R, E of appropriate dimensions satisfying the following inequality

$$
\Lambda=\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & G & \tau^*GC_1\\ \Lambda_{21} & \Lambda_{22} & 0 & 0\\ G & 0 & -R & 0\\ \tau^*GC_1 & 0 & 0 & -\tau^*R\end{bmatrix}<0, \tag{36}
$$

where $\Lambda_{11}=(C_0+C_1)^{\mathrm{T}}G+G(C_0+C_1)+E+\tau^*(C_0+C_1)^{\mathrm{T}}R(C_0+C_1)-(1-h)E$, $\Lambda_{12}=(1-h)E-2\tau^*(C_0+C_1)^{\mathrm{T}}RC_1$, $\Lambda_{21}=(1-h)E$, $\Lambda_{22}=\tau^*{C_1}^{\mathrm{T}}RC_1-(1-h)E$.
Then, all the followers will track the leader vehicle with the same velocity and acceleration, meanwhile keeping the desired distance with its consecutive vehicles in the presence of time-varying delay.

Proof: Define the following Lyapunov-Krasovskii functional

$$V_2(t) = \varepsilon^{\mathrm{T}}(t)G\varepsilon(t) + \int_{t-\tau(t)}^{t} \varepsilon^{\mathrm{T}}(s)E\varepsilon(s)ds + \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} \varepsilon^{\mathrm{T}}(s)R\varepsilon(s)dsd\theta. \quad (37)$$

Taking the time derivative of $V_2(t)$ yields

$$\begin{aligned}
\dot{V}_2(t) =& \dot{\varepsilon}^{\mathrm{T}}(t)G\varepsilon(t) + \varepsilon^{\mathrm{T}}(t)G\dot{\varepsilon}(t) + \varepsilon^{\mathrm{T}}(t)E\varepsilon(t) \\
&- (1-\dot{\tau}(t))\varepsilon^{\mathrm{T}}(t-\tau(t))E\varepsilon(t-\tau(t)) \\
&+ \tau(t)\dot{\varepsilon}^{\mathrm{T}}(t)R\dot{\varepsilon}(t) - \int_{t-\tau(t)}^{t}\dot{\varepsilon}^{\mathrm{T}}(s)R\dot{\varepsilon}(s)ds \\
=& \left[(C_0+C_1)\varepsilon(t) - C_1\int_{t-\tau(t)}^{t}\dot{\varepsilon}(s)ds\right]^{\mathrm{T}} G\varepsilon(t) \\
&+ \varepsilon^{\mathrm{T}}(t)G\left[(C_0+C_1)\varepsilon(t) - C_1\int_{t-\tau(t)}^{t}\dot{\varepsilon}(s)ds\right] \\
&+ \varepsilon^{\mathrm{T}}(t)E\varepsilon(t) + \tau(t)\dot{\varepsilon}^{\mathrm{T}}(t)R\dot{\varepsilon}(t) \\
&- \int_{t-\tau(t)}^{t}\dot{\varepsilon}^{\mathrm{T}}(s)R\dot{\varepsilon}(s)ds \\
&- (1-\dot{\tau}(t))\varepsilon^{\mathrm{T}}(t-\tau(t))E\varepsilon(t-\tau(t)) \\
=& \varepsilon^{\mathrm{T}}(t)[(C_0+C_1)^{\mathrm{T}}G + G(C_0+C_1) + E]\varepsilon(t) \\
&- \int_{t-\tau(t)}^{t}\dot{\varepsilon}^{\mathrm{T}}(s)C_1^{\mathrm{T}}G\varepsilon(t)ds \\
&- \int_{t-\tau(t)}^{t}\varepsilon^{\mathrm{T}}(t)GC_1\dot{\varepsilon}(s)ds \\
&- (1-\dot{\tau}(t))\varepsilon^{\mathrm{T}}(t-\tau(t))E\varepsilon(t-\tau(t)) \\
&- \int_{t-\tau(t)}^{t}\dot{\varepsilon}^{\mathrm{T}}(s)R\dot{\varepsilon}(s)ds + \tau(t)\dot{\varepsilon}^{\mathrm{T}}(t)R\dot{\varepsilon}(t) \\
\leq& \varepsilon^{\mathrm{T}}(t)[(C_0+C_1)^{\mathrm{T}}G + G(C_0+C_1) + E]\varepsilon(t) \\
&+ \frac{1}{2}\int_{t-\tau(t)}^{t}\dot{\varepsilon}^{\mathrm{T}}(s)R\dot{\varepsilon}(s)ds \\
&+ \frac{1}{2}\int_{t-\tau(t)}^{t}\varepsilon^{\mathrm{T}}(t)GC_1R^{-1}C_1^{\mathrm{T}}G\varepsilon(t)ds \\
&+ \frac{1}{2}\int_{t-\tau(t)}^{t}\dot{\varepsilon}^{\mathrm{T}}(s)R\dot{\varepsilon}(s)ds \\
&+ \frac{1}{2}\int_{t-\tau(t)}^{t}\varepsilon^{\mathrm{T}}(t)GC_1R^{-1}C_1^{\mathrm{T}}G\varepsilon(t)ds \\
&- \int_{t-\tau(t)}^{t}\dot{\varepsilon}^{\mathrm{T}}(s)R\dot{\varepsilon}(s)ds \\
&- (1-h)\varepsilon^{\mathrm{T}}(t-\tau(t))E\varepsilon(t-\tau(t)) \\
&+ \tau(t)\dot{\varepsilon}^{\mathrm{T}}(t)R\dot{\varepsilon}(t) \\
\leq& \varepsilon^{\mathrm{T}}(t)[(C_0+C_1)^{\mathrm{T}}G + G(C_0+C_1) + E \\
&+ \tau(t)GC_1R^{-1}C_1^{\mathrm{T}}G + GR^{-1}G]\varepsilon(t) \\
&- (1-h)\varepsilon^{\mathrm{T}}(t-\tau(t))E\varepsilon(t-\tau(t)) \\
&+ \tau(t)\dot{\varepsilon}^{\mathrm{T}}(t)R\dot{\varepsilon}(t) \\
=& \varepsilon^{\mathrm{T}}(t)[(C_0+C_1)^{\mathrm{T}}G + G(C_0+C_1) + E \\
&+ \tau(t)GC_1R^{-1}C_1^{\mathrm{T}}G + GR^{-1}G]\varepsilon(t) \\
&- (1-h)\varepsilon^{\mathrm{T}}(t-\tau(t))E\varepsilon(t-\tau(t)) \\
&+ \tau(t)\left[(C_0+C_1)\varepsilon(t) - C_1\int_{t-\tau(t)}^{t}\dot{\varepsilon}(s)ds\right]^{\mathrm{T}} \\
&\times R\left[(C_0+C_1)\varepsilon(t) - C_1\int_{t-\tau(t)}^{t}\dot{\varepsilon}(s)ds\right] \\
=& \varepsilon^{\mathrm{T}}(t)[(C_0+C_1)^{\mathrm{T}}G + G(C_0+C_1) + E \\
&+ \tau(t)GC_1R^{-1}C_1^{\mathrm{T}}G + GR^{-1}G \\
&+ \tau(t)(C_0+C_1)^{\mathrm{T}}R(C_0+C_1) \\
&- 2\tau(t)(C_0+C_1)^{\mathrm{T}}RC_1 + \tau(t)C_1{}^{\mathrm{T}}RC_1]\varepsilon(t) \\
&- (1-h)\varepsilon^{\mathrm{T}}(t-\tau(t))E\varepsilon(t-\tau(t)) \\
&+ \tau(t)\varepsilon^{\mathrm{T}}(t-\tau(t))C_1{}^{\mathrm{T}}RC_1\varepsilon(t-\tau(t)) \\
&+ 2\tau(t)\varepsilon^{\mathrm{T}}(t)(C_0+C_1)^{\mathrm{T}}RC_1\varepsilon(t-\tau(t)) \\
&- \tau(t)\varepsilon^{\mathrm{T}}(t)C_1^{\mathrm{T}}RC_1\varepsilon(t-\tau(t)) \\
&- \tau(t)\varepsilon^{\mathrm{T}}(t-\tau(t))C_1^{\mathrm{T}}RC_1\varepsilon(t).
\end{aligned} \quad (38)$$

Let $\Gamma(t) = \varepsilon(t) - \varepsilon(t-\tau)$, we have

$$\dot{V}_2(t) \leq \begin{bmatrix}\varepsilon^{\mathrm{T}}(t) & \Gamma^{\mathrm{T}}(t)\end{bmatrix} \times \begin{bmatrix}\Lambda_{11} + \tau^* GC_1R^{-1}C_1^{\mathrm{T}}G + GR^{-1}G & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22}\end{bmatrix} \times \begin{bmatrix}\varepsilon(t) \\ \Gamma(t)\end{bmatrix}. \quad (39)$$

According to inequality (36) and Lemma 2, it follows that

$$\begin{bmatrix}\Lambda_{11} + \tau^* GC_1R^{-1}C_1^{\mathrm{T}}G + GR^{-1}G & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22}\end{bmatrix} < 0. \quad (40)$$

Hence, $\dot{V}_2(t) < 0$. According to Lyapunov theory, it can be concluded that the vehicle platoon is asymptotically stable and $\lim_{x\to\infty}\varepsilon(t) = 0$, i.e., $\tilde{x}(t-\tau(t)) = x_i(t-\tau(t)) - x_L(t-\tau(t)) - i\cdot R_{iL} = 0$, $\tilde{v}(t-\tau(t)) = v_i(t-\tau(t)) - v_L(t-\tau(t)) = 0$, $\tilde{a}(t-\tau(t)) = a_i(t-\tau(t)) - a_L(t-\tau(t)) = 0$.

Therefore, vehicles in the platoon can keep the desired safe distance with the consecutive vehicles and have the same velocity and acceleration with the leader in the presence of time-varying delay. □

5. SIMULATION STUDIES

Numerical simulations are performed to verify the effectiveness of the proposed vehicle platoon algorithm in the presence of time delay. Here, one leader and 4 followers are chosen to form the platoon, the reference velocity of the leader is 20m/s.

The parameters of vehicles are listed as follows, where $\rho = 1.293$ m/s^3, $A = 2.5$ m^2, $C_{\mathrm{d}} = 0.45$, $d_{\mathrm{m}} = 5N$, $m = 1775$ kg, $\mu(v) = 0.1$. Substituting these parameters into

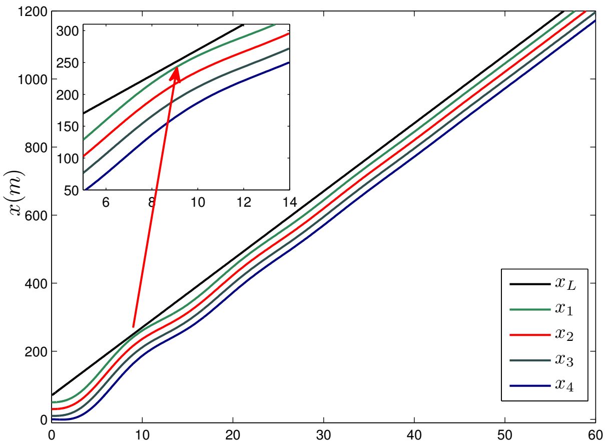

Fig. 2. Positions for four follower vehicles and the leader with constant time delay.

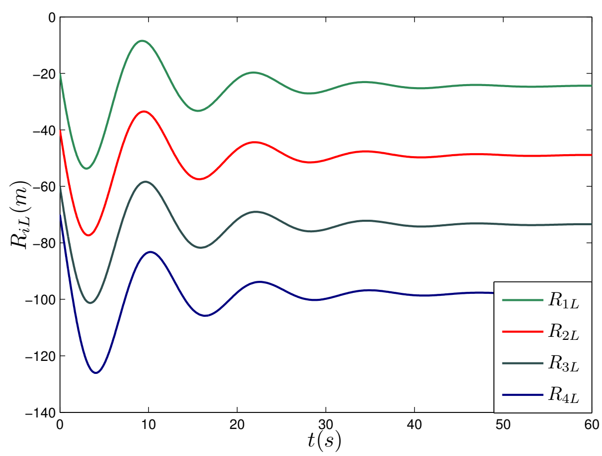

Fig. 3. Inter-vehicle spaces between four follower vehicles and the leader with constant time delay.

the simplified nonlinear vehicle dynamics (9), it obtains

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = \xi_i - \dfrac{1.293 \times 2.5 \times 1.45}{2 \times 1775} {v_i}^2 - \dfrac{5}{1775}, \\ \dot{\xi}_i = -\dfrac{\xi_i}{0.1} + \dfrac{\alpha_i}{1775 \times 0.1}. \end{cases} \tag{41}$$

In addition, the initial conditions for the leader and follower vehicles are $x_L(0) = 70$ m, $x_i(0) = [0; 10; 30; 50]$ m, $v_L(0) = 20$ m/s, $v_i(0) = 0$ m/s, $a_L(0) = a_i(0) = 0$ m/s^2, the body length of each vehicle is $L_i = 4.5$ m and the desired distance between consecutive vehicles is $R = 20$ m.

5.1. Constant time delay case

Numerical simulations are conducted to illustrate the feasibility of the platoon control algorithm in (15) and the effectiveness of Theorem 1. Here, we let $\beta = 4$, $\mathcal{L}$ and I_n

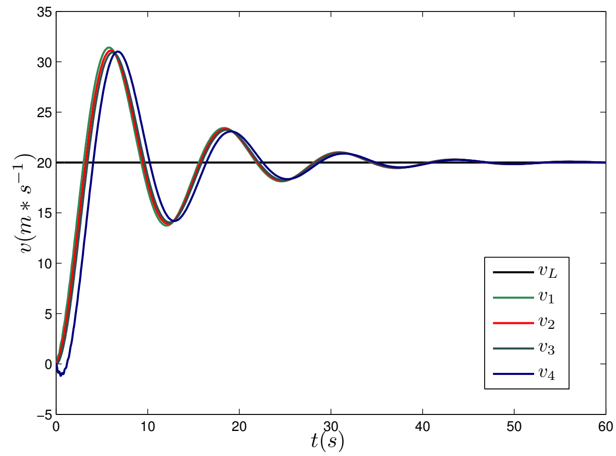

Fig. 4. Velocities for four follower vehicles and the leader with constant time delay.

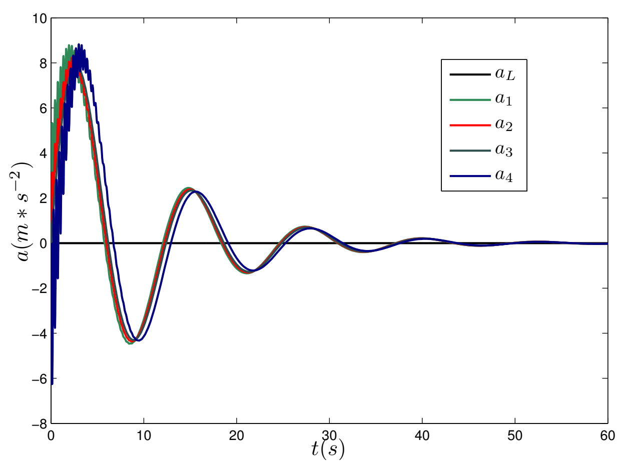

Fig. 5. Accelerations for four follower vehicles and the leader with constant time delay.

can be written in the following form

$$\mathcal{L} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}, \quad I_n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{42}$$

According to Theorem 1, the maximum upper bound for constant time delay can be solved by resorting to the LMI Toolbox in MATLAB, which follows $r = 2$ and $\tau^* = 3.345 \times 10^{-7}$ s.

Let $\tau = 3 \times 10^{-7}\mathrm{s} < \tau^*$ be the constant time delay, under the vehicle platoon control algorithm (15), the position, inter-vehicle space, velocity and acceleration curves of vehicles are shown in Figs. 2-5.

From Fig. 2 and Fig. 3 it can be seen that the followers will track the leader vehicle and the distance between consecutive vehicles will converge to the desired value.

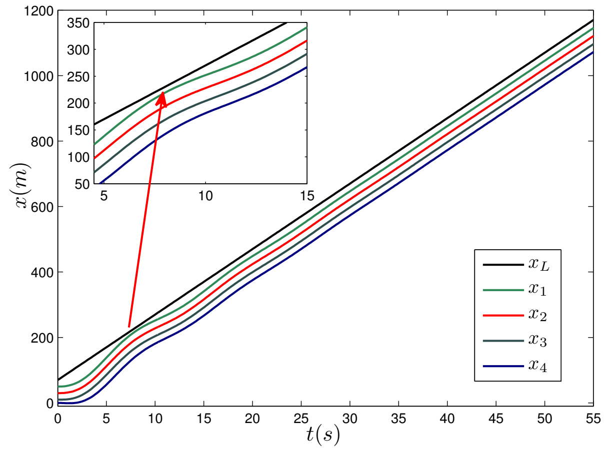

Fig. 6. Positions for four follower vehicles and the leader with time-varying delay.

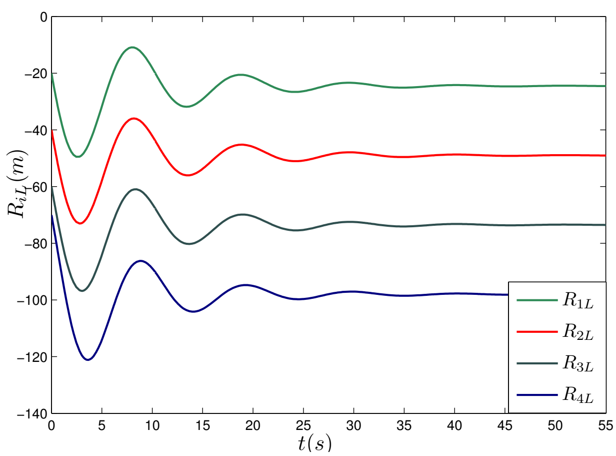

Fig. 7. Inter-vehicle spaces between four follower vehicles and the leader with time-varying delay.

Meanwhile, there is no collision between any two consecutive vehicles during this process. Fig. 4 shows that the velocity of followers will converge to that of the leader vehicle (20m/s). In addition, the acceleration curves of vehicles in Fig. 5 demonstrate that the acceleration of all the vehicles will asymptotically converge to zero, suggesting that the whole platoon moves in a uniform motion with the desired velocity in the steady state.

Therefore, the proposed control algorithm (15) is able to achieve the stable vehicle platoon in the presence of constant time delay, as long as the time delay τ satisfies (18).

Remark 5: According to the above simulation results, it is known that the vehicle platoon (10) with control protocol (15) can achieve the platoon formation with the desired inter-vehicle distance and velocity in the presence of constant time delay. Particularly, the longitudinal distance between consecutive vehicles can maintain the predefined space R_{ij} and the velocity of each vehicle can track the

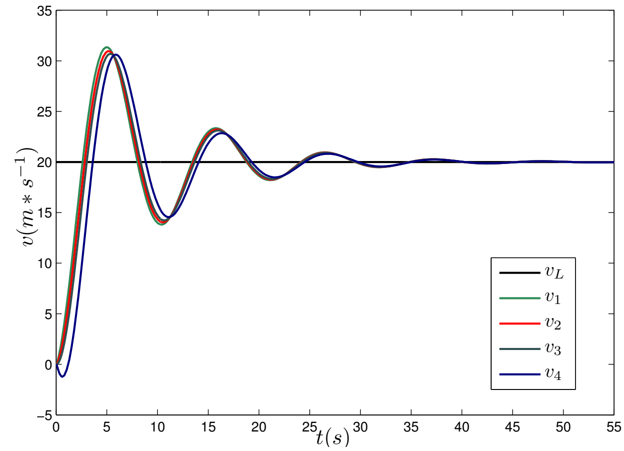

Fig. 8. Velocities for four follower vehicles and the leader with time-varying delay.

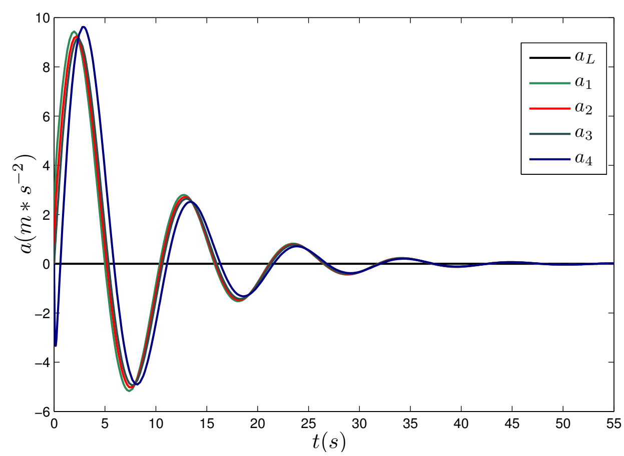

Fig. 9. Accelerations for four follower vehicles and the leader with time-varying delay.

desired velocity v_L.

Remark 6: In Theorem 1, the feasible solutions for the positive definite matrixes P and F are solved by the resorting to the LMI tool box in MATLAB. Then, by substituting the feasible solutions of P and F into (18), the upper bond of constant time delay τ^* can be obtained.

5.2. Time-varying delay case

For the case of vehicle platoon with time-varying delay, let $\beta = 1.5$ and $\tau(t) = 0.085|\sin(t)|$, it can be derived that $\tau^* = 0.085$ s together with $h = 0.085$. According to Theorem 2, the feasible solutions of G, E, R can also be given by solving the LMIs.

Simulation results are depicted in Figs. 6-9, from which it can be seen that the vehicle platoon will be in the stable state after 40 s, i.e., the inter-vehicle space between any two consecutive vehicles will maintain the desired distance, the velocity of followers will converge to that of the leader vehicle and the acceleration of all vehicles

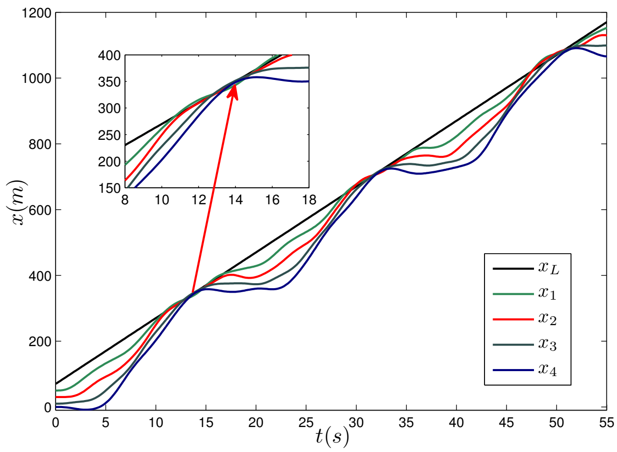

Fig. 10. Positions for four follower vehicles and the leader in [27].

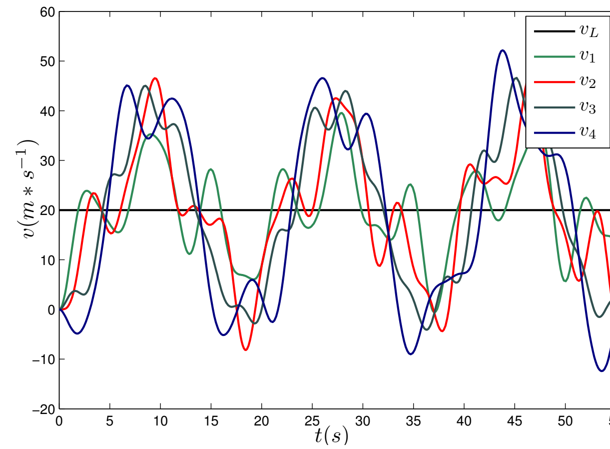

Fig. 12. Velocities for four follower vehicles and the leader in [27].

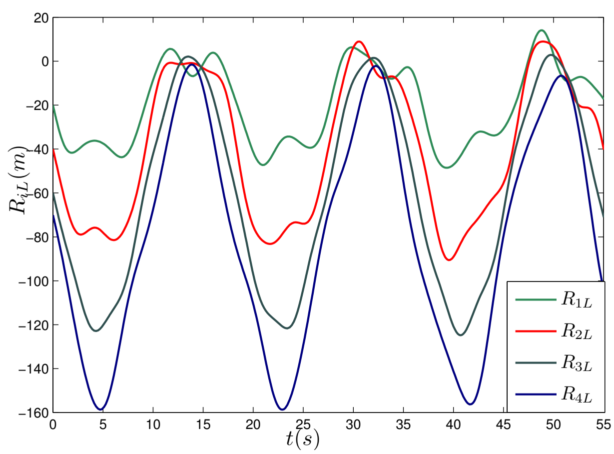

Fig. 11. Inter-vehicle spaces between four follower vehicles and the leader in [27].

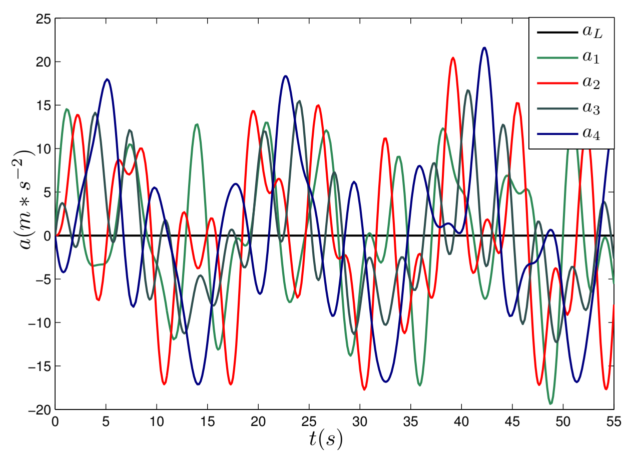

Fig. 13. Accelerations for four follower vehicles and the leader in [27].

will converge to zero eventually, which verify the feasibility and correctness of the proposed control algorithm (33) and sufficient conditions (36) for vehicle platoon in the presence of time-varying delay.

Remark 7: According to the above simulation results, it is known that the vehicle platoon (10) with control protocol (33) can achieve the platoon formation with the desired inter-vehicle distance and velocity in the presence of time-varying delay. Particularly, the longitudinal distance between consecutive vehicles can maintain the predefined space R_{ij} and the velocity of each vehicle can track the desired velocity v_L.

Remark 8: It is worth noting that the upper bound for time delay in Theorem 2 is much larger than that of Theorem 1, which suggests that the sufficient conditions, derived by Lyapunov-Razumikhin Theorem, for the stability of vehicle platoon with constant time delay, are more conservative than that are given by Lyapunov-Krasovskii Theorem. Therefore, Theorem 2 is more robust against time delay than Theorem 1 and it is more suitable for the vehicle platoon control with time delay.

5.3. Comparative analysis

In order to illustrate the advantage of the proposed algorithm, a comparative study is carried out between Theorem 2 and [27], where a robust exponential H_∞ controller for vehicle platoon with time-varying delay is proposed in [27].

In particular, the time delay for [27] is chosen as $\tau(t) = 0.085|\sin(t)|$, other simulation parameters can be referred to the above simulation cases. The simulation results of [27] are shown in Figs. 10-13.

From Figs. 10-13, it can be clearly seen that the vehicle platoon control algorithm in [27] is unable to realize the stable vehicle platoon when the time delay follows $\tau(t) = 0.085|\sin(t)|$. To be specific, the position curves of vehicle in Fig. 10 demonstrate that vehicles can not form the platoon formation. Some overlapped positions also indicate that collision may occur in the platoon, which is not allowed for vehicle platoon from the safety perspective. From Fig. 11, it can be seen that the inter-vehicle spaces fluctuate drastically and can not maintain the desired distance. The velocity and acceleration curves in Fig. 12 and Fig. 13 suggest that the velocity of followers can not converge to that of the leader. Meanwhile, vehicles are frequently accelerating or decelerating, which may cause discomfort for passengers and may also consume more fuel that uniform motion.

Remark 9: Form the above comparative analysis, it is known that Theorem 2 performs better than that of [27] under the same time-varying delay. Moreover, the upper bound of time delay is $\tau^* = 20$ ms in the simulation of [27], which is much smaller than that of Theorem 2. Therefore, the results of [27] are more conservative than our algorithm and our results are more robust against time delay.

6. CONCLUSIONS AND FUTURE WORK

The platoon control problem for nonlinear vehicles with time delay is studied in this paper. A feedback linearization approach is firstly deployed to convert the nonlinear vehicle dynamics into a third-order linear dynamic system and the vehicle platoon control algorithm is designed based on the consensus theory. Then, two time delay cases with, respectively, constant time delay and time-varying delay are investigated under the proposed control algorithm. The sufficient conditions for the stability of vehicle platoon with constant time delay and time-varying delay are derived by deploying the Lyapunov-Razumikhin theorem and the Lyapunov-Krasovskii theorem, respectively. Various simulation cases demonstrate the feasibility and effectiveness of the theoretical results.

In the future, the following issues will be further investigated:

- The linearized vehicle dynamic model (10) adopted in this paper neglects some intrinsic properties of vehicles such as the actuator (gas pedal or break system) saturation and time lag, which may also degrade the performance of vehicle platoon in practical applications. Therefore, we will improve our algorithm and investigate the actuator saturation and time lag problems of vehicle platoon.
- This paper only considers the time delay issue of communication networks in vehicle platoon control. However, the packet loss problem is another critical issue in networked control systems [30] that may also degrade the performance of vehicle platoon. In the future, the vehicle platoon control with packet loss will be further investigated.
- The topological structure of vehicle platoon in this paper is assumed to be fixed, which is not well coincide with the actual characteristic of vehicle platoon. As the topology of vehicle platoon is time-varying with the cruising of vehicles [31], the vehicle platoon control under switching topology is our new concern in the future.

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