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Integral Barrier Lyapunov Functions-based Neural Control for Strict-feedback Nonlinear Systems with Multi-constraint

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Abstract: A new robust tracking control approach is proposed for strict-feedback nonlinear systems with state and input constraints. The constraints are tackled by extending the control input as an extended state and introducing an integral barrier Lyapunov function (IBLF) to each step in a backstepping procedure. This extends current research on barrier Lyapunov functions(BLFs)-based control for nonlinear systems with state constraints to IBLF-based control for strict-feedback nonlinear systems with state and input constraints. Since the IBLF allows the original constraints to be mixed with the error terms, the use of IBLF decreases conservatism in barrier Lyapunov functions-based control. In the backstepping procedure, neural networks (NNs) with projection modifications are applied to estimate system uncertainties, due to their ability in guaranteeing estimators in a given bounded area. To facilitate the use of the once-differentiable NNs estimators in the backstepping procedure, the virtual controllers are passed through command filters. Finally, simulation results are presented to illustrate the feasibility and effectiveness of the proposed control.

Keywords: Barrier Lyapunov function, dynamic surface control, input saturation, neural networks, state constraints, strict-feedback nonlinear system.

1. INTRODUCTION

State constraints and control input constraints exist in many mechanical systems and process industries due to mechanical stoppages, safety specification, actuator saturation and so on. Ignoring state or input constraints may result in system performance degradation, closed-loop instability, and even disasters. Thus, more and more attentions have been payed to the control design for constrained systems [1–3].

The approaches used in control for constrained nonlinear systems mainly include anti-saturation control [4–8], model predictive control (MPC) [9–11], reference governor (RG) [12], and barrier Lyapunov functions (BLFs) [13, 14]. However, anti-saturation control can not tackle system state constraints. MPC and RG are well known as effective methods for control of constrained nonlinear systems. In MPC, system constraints are explicitly considered in receding horizon optimizations and control is obtained by solving the optimization online. In RG, system constraints are guaranteed by redesign of reference signal obtained by solving online optimization. However, the computational complexity of solving nonlinear optimizations brings difficulties for their applications in real-time control.

Recently, BLFs-based controllers have been developed for constrained nonlinear systems with time-invariant output constraints [13–15], time-varying output constraints [16], partial state constraints [17], and full state constraints [18–21]. Since the function values of BLFs will grow to infinity if the arguments approach the constraints boundary, the avoidance of constraints violation can be reached by bounding BLFs. To decrease conservatism of BLFs-based control due to imposing constraints on transformed errors not on original state constraints directly, integral BLFs-based control are designed for nonlinear systems with state constraints in [22,23]. However, input saturation is not considered in [13–23]. Therefore, integral BLFs-based control for nonlinear systems with state and input constraints deserves more research.

System uncertainties exist in most practical systems due to modeling errors and exogeneous disturbances. In [13–23], uncertainties are not considered or only uncertainties with linear parametric forms are considered except [14, 15, 19, 20, 23]. Adaptive RBF NNs [24–26] with σ modifications are used in [14, 15, 19, 20, 23] to estimate system uncertainties in constrained nonlinear systems. Though the use of σ modifications prevent NN

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weights from drifting to infinity, it can not guarantee the estimators confined in given bounded areas. In the BLFs-based backstepping control, the virtual controllers are required to be bounded by state constraint bound. Thus, compared with other NNs, NNs with projection modifications are applied in BLFs-based control to estimate uncertainties due to their ability in guaranteeing estimators in given bounded areas. But once-differentiability of the NNs with projection modifications prohibits their applications in high-order strict feedback uncertain nonlinear systems, since at least twice-differentiability of NN estimators are required in the backstepping procedure.

This paper proposes an integral BLFs-based adaptive NN control approach for a class of strict-feedback uncertain nonlinear systems with state and input constraints. The main contribution of this study include:

1) The system constraints are tackled by extending control input as an extended state and introducing an integral barrier Lyapunov function (IBLF) to each step in a backstepping procedure. This extends current research on IBLF-based control for nonlinear systems with state/output constraints to state constraints and symmetric control saturation.

2) Raised cosine RBF NNs with projection modifications are designed to estimate and compensate nonlinear system uncertainties. By properly choosing bounds of NN weights, the NN estimators can be confined in expected bounded areas and the virtual control can be easily bounded by the bounds of system states by properly choosing parameters.

3) To facilitate the use of the once-differentiable NN estimators in backstepping, dynamic surface control [27,28] is used by passing the virtual control through command filters, which also decrease the computation complexity in the backstepping procedure.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem statement

Consider the following nth order SISO nonlinear system:

$$\dot{x}_i = f_i(\bar{x}_i) + x_{i+1},\ i = 1,2,\cdots,n-1, \tag{1}$$

$$\dot{x}_n = f_n(\bar{x}_n) + u, \tag{2}$$

where $x_i \in R$ and $u \in R$ are the state variable and the system input, respectively, $f_i(\bar{x}_i)$ are unknown nonlinear functions with the state $\bar{x}_i = [x_1,\cdots,x_i]^T$ as the function variable. For system (1), the system constraints consist of state constraints and asymetric input constraint, which are described as follows:

$$|x_i| \le k_{ci}, |u| \le u_c,\ i = 1,2,..,n, \tag{3}$$

where k_{ci}, $-u_{c1}$ and u_{c2} are positive constants.

The objective of this paper is to design an IBLFs-based NN control $u(t)$ such that the system constraints (3) are satisfied, the signal of the closed-loop control system are bounded and the x_1 is driven to track the desired trajectory $y_d(t)$.

Assumption 1: The functions $f(x)$ is locally Lipschits continuous.

Assumption 2: The reference signal $y_d(t)$ and its jth-order time derivative $y_d^{(j)}(t)$, $j = 1,2,\cdots,n+1$ are known and satisfy $|y_d(t)| \le A_0 < k_{c1}$ and $|y_d^{(j)}| \le Y_j$, where $A_0, Y_1,\cdots,Y_{n+1}$ are positive constants.

2.2. RBF neural networks

Define $D_i = \{[x_1,\cdots,x_i]^T \in R^i : |x_j| \le k_{cj}, j = 1,\cdots,i\}$. The locally Lipschits continuous functions $f_i(\bar{x}_i) : D_i \to R$ can be expressed by RBF NN as

$$f_i(\bar{x}_i) = \theta_i^T \Phi_i(\bar{x}_i) + \varepsilon_i(\bar{x}_i), \tag{4}$$

where W_i is the vector of optimal weight, $\varepsilon_i(\bar{x}_i)$ is the optimal estimation error and $\Phi_i(\bar{x}_i) = [\phi_{i1}(\bar{x}_i),\cdots,\phi_{i,N_i}(\bar{x}_i)]^T$ with $\phi_{is}(\bar{x}_i), s = 1,\cdots,N_i$ defined as

$$\phi_{is}(\bar{x}_i) = \prod_{j=1}^{i} \xi_{is,j}(x_j), \tag{5}$$

$$\xi_{is,j}(x_j) = \begin{cases} \frac{1}{2}\left(1+\cos\left(\frac{\pi(x_j - c_{is,j})}{\sigma_{is,j}}\right)\right), \\ \qquad\qquad \text{if } |x_j - c_{is,j}| \le \sigma_{is,j}, \\ 0, \qquad\quad \text{if } |x_j - c_{is,j}| > \sigma_{is,j}, \end{cases} \tag{6}$$

with $c_{is,j}$ and $\sigma_{is,j}$ being the center and radius, respectively.

The estimation of $f_i(\bar{x}_i)$ by the raised-cosine RBF can be expressed as

$$\hat{f}_i(\bar{x}_i) = \hat{\theta}_i^T \Phi_i(\bar{x}_i), \tag{7}$$

where $\hat{\theta}_i$ is the estimation of optimal weight.

Define the compact sets $\Omega_i := \{\hat{\theta}_i : ||\hat{\theta}_i|| \le c_{\theta i}\}$, $i = 1,\cdots,n$, where $c_{\theta i}$ are prespecified finite constants. Then

$$\theta_i = \underset{\hat{\theta}_j\in\Omega_i}{argmax}\left(\sup_{\bar{x}_i\in D_i} |f_i(\bar{x}_i) - \hat{f}(\bar{x}_i)|\right). \tag{8}$$

Define

$$c_i = \max_{\theta_i,\hat{\theta}_i\in\Omega_i} \tilde{\theta}_i^T \tilde{\theta}_i / p_i, \tag{9}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and p_i is a positive design parameter.

3. CONTROL DESIGN AND STABILITY ANALYSIS

Step 1: Define $z_1 = x_1 - y_d$ as the tracking error. Consider the following IBLF:

$$V_1 = \int_0^{z_1} \frac{\sigma k_{c1}^2}{k_{c1}^2 - (\sigma + y_d)^2} d\sigma + \frac{1}{2p_1} \tilde{\theta}_1^T \tilde{\theta}_1. \tag{10}$$

Taking time derivative of V_1, yields

$$\dot{V}_1 = \frac{k_{c1}^2 z_1}{k_{c1}^2 - x_1^2}(f_1(\bar{x}_1) + x_2 - \dot{y}_d) + \frac{\partial V_1}{\partial y_d}\dot{y}_d, \tag{11}$$

where $\partial V_1/\partial y_d$ is calculated as [22, 23]:

$$\frac{\partial V_1}{\partial y_d} = z_1\left(\frac{k_{c1}^2}{k_{c1}^2 - x_1^2} - \rho_1(z_1, y_d)\right), \tag{12}$$

and

$$\rho_1(z_1, y_d) = \int_0^1 \frac{k_{c1}^2}{k_{c1}^2 - (\beta z_1 + y_d)^2} d\beta \tag{13}$$

with

$$\rho_1(0, y_d) = \frac{k_{c1}^2}{k_{c1}^2 - y_d^2}, \tag{14}$$

which means that $\rho_1(z_1, y_d)$ is well defined and bounded around the neighborhood of $z_1 = 0$.

Define the virtual control α_1 as

$$\alpha_1 = -\hat{f}_1 - k_1 z_1 - \frac{1}{2}\frac{k_{c1}^2 z_1}{k_{c1}^2 - x_1^2} + \frac{k_{c1}^2 - x_1^2}{k_{c1}^2}\dot{y}_d\rho_1. \tag{15}$$

Passing α_1 through a command filter, yields

$$\tau_1\dot{\alpha}_{1d} = -\alpha_{1d} + \alpha_1. \tag{16}$$

Define $z_2 = x_2 - \alpha_{1d}$ and $\tilde{\alpha}_1 = \alpha_1 - \alpha_{1d}$. Then,

$$\begin{aligned}\dot{V}_1 = &-\frac{1}{2}\left(\frac{k_{c1}^2 z_1}{k_{c1}^2 - x_1^2}\right)^2 + \frac{k_{c1}^2 z_1}{k_{c1}^2 - x_1^2}(\varepsilon_1 + \tilde{\alpha}_1 + z_2)\\ &- \tilde{\theta}_1^T(\dot{\hat{\theta}}_1/p_1 - \Phi_1\frac{k_{c1}^2 z_1}{k_{c1}^2 - x_1^2}) - \frac{k_{c1}^2 k_1 z_1^2}{k_{c1}^2 - x_1^2}.\end{aligned} \tag{17}$$

Using Young's inequality,

$$\frac{k_{c1}^2 z_1}{k_{c1}^2 - x_1^2}(\varepsilon_1 + \tilde{\alpha}_1) \le (\varepsilon_{1d})^2 + (\tilde{\alpha}_1)^2 + \frac{1}{2}\left(\frac{k_{c1}^2 z_1}{k_{c1}^2 - x_1^2}\right)^2. \tag{18}$$

Then,

$$\begin{aligned}\dot{V}_1 \le &-\frac{k_{c1}^2 k_1 z_1^2}{k_{c1}^2 - x_1^2} + \frac{k_{c1}^2 z_1 z_2}{k_{c1}^2 - x_1^2} - \tilde{\theta}_1^T(\dot{\hat{\theta}}_1/p_1 - \Phi_1\frac{k_{c1}^2 z_1}{k_{c1}^2 - x_1^2})\\ &+ (\tilde{\alpha}_1)^2 + (\varepsilon_{1d})^2.\end{aligned} \tag{19}$$

Step i ($i = 2, \cdots, n$)**:** Define $z_i = x_i - \alpha_{i-1,d}$. Consider the candidate Lyapunov function

$$V_i = V_{i-1} + \Lambda_i, \tag{20}$$

where

$$\Lambda_i = \int_0^{z_i} \frac{\sigma k_{ci}^2}{k_{ci}^2 - (\sigma + \alpha_{i-1,d})^2} d\sigma + \frac{1}{2p_i}\tilde{\theta}_i^T\tilde{\theta}_i. \tag{21}$$

Taking time derivative of Λ_i, yields

$$\begin{aligned}\dot{\Lambda}_i = &\frac{k_{ci}^2 z_i}{k_{ci}^2 - x_i^2}(f_i + x_{i+1} - \dot{\alpha}_{i-1,d})\\ &+ \frac{\partial\Lambda_i}{\partial\alpha_{i-1,d}}\dot{\alpha}_{i-1,d} - \frac{1}{2p_i}\tilde{\theta}_i^T\dot{\hat{\theta}}_i,\end{aligned} \tag{22}$$

where $\frac{\partial\Lambda_i}{\partial\alpha_{i-1,d}}$ is calculated as

$$\frac{\partial\Lambda_i}{\partial\alpha_{i-1,d}} = z_i\left(\frac{k_{ci}^2}{k_{ci}^2 - x_i^2} - \rho_i(z_i, \alpha_{i-1,d})\right), \tag{23}$$

and

$$\rho_i(z_i, \alpha_{i-1,d}) = \int_0^1 \frac{k_{ci}^2}{k_{ci}^2 - (\beta z_i + \alpha_{i-1,d})^2} d\beta \tag{24}$$

with

$$\rho_i(0, \alpha_{i-1,d}) = \frac{k_{ci}^2}{k_{ci}^2 - \alpha_{i-1,d}{}^2}, \tag{25}$$

which means that $\rho_i(0, \alpha_{i-1,d})$ is well defined and bounded around the neighborhood of $z_i = 0$.

Lemma 1 [22]**:** The function $\rho_i(z_i, \alpha_{i-1,d})$ is C^1 in the set $\Psi_i = \{z_i \in R, \alpha_{i-1,d} \in R : |\alpha_{i-1,d}| \le k_{ci}, |z_i + \alpha_{i-1}| \le k_{ci}\}$.

Define the virtual control α_i as

$$\begin{aligned}\alpha_i = &-\hat{f}_i - k_i z_i - \frac{1}{2}\frac{k_{ci}^2 z_i}{k_{ci}^2 - x_i^2} + \frac{k_{ci}^2 - x_i^2}{k_{ci}^2}\dot{\alpha}_{i-1,d}\rho_i\\ &- \frac{k_{c,i-1}^2(k_{ci}^2 - x_i^2)z_{i-1}}{k_{ci}^2(k_{c,i-1}^2 - x_{i-1}^2)}.\end{aligned} \tag{26}$$

Passing α_i through a command filter, yields

$$\tau_i\dot{\alpha}_{id} = -\alpha_{id} + \alpha_i. \tag{27}$$

Define $\tilde{\alpha}_i = \alpha_i - \alpha_{id}$ and $z_{n+1} = u - \alpha_{nd}$. Then,

$$\begin{aligned}\dot{\Lambda}_i = &-\frac{k_{ci}^2 k_i z_i^2}{k_{ci}^2 - x_i^2} - \frac{1}{2}\left(\frac{k_{ci}^2 z_i}{k_{ci}^2 - x_i^2}\right)^2 + \frac{k_{ci}^2 z_i}{k_{ci}^2 - x_i^2}(\varepsilon_i + \tilde{\alpha}_i + z_{i+1})\\ &- \tilde{\theta}_i^T(\dot{\hat{\theta}}_i/p_i - \Phi_i\frac{k_{ci}^2 z_i}{k_{ci}^2 - x_i^2}) - \frac{k_{c,i-1}^2 z_{i-1} z_i}{k_{c,i-1}^2 - x_{i-1}^2}.\end{aligned} \tag{28}$$

Then, we have

$$\begin{aligned}\dot{\Lambda}_i \le &-\frac{k_{ci}^2 k_i z_i^2}{k_{ci}^2 - x_i^2} + \frac{k_{ci}^2 z_i z_{i+1}}{k_{ci}^2 - x_i^2} - \tilde{\theta}_i^T(\dot{\hat{\theta}}_i/p_i - \Phi_i\frac{k_{ci}^2 z_i}{k_{ci}^2 - x_i^2})\\ &+ (\tilde{\alpha}_i)^2 + (\varepsilon_{id})^2 - \frac{k_{c,i-1}^2 z_{i-1} z_i}{k_{c,i-1}^2 - x_{i-1}^2}.\end{aligned} \tag{29}$$

Based on (19) and (29),

$$\dot{V}_i \le \frac{k_{ci}^2 z_i z_{i+1}}{k_{ci}^2 - x_i^2} - \sum_{j=1}^{i}\frac{k_{ci}^2 k_i z_i^2}{k_{ci}^2 - x_i^2} - \sum_{j=1}^{i}((\tilde{\alpha}_i)^2 + (\varepsilon_{id})^2)$$

$$+\sum_{j=1}^{i}\tilde{\theta}_i^T(\dot{\hat{\theta}}_i/p_i-\Phi_i\frac{k_{ci}^2z_i}{k_{ci}^2-x_i^2}). \quad (30)$$

Step $n+1$: Consider the candidate Lyapunov function:

$$V_{n+1}=V_n+\Lambda_{n+1}, \quad (31)$$

where

$$\Lambda_{n+1}=\int_0^{z_{n+1}}\frac{u_c^2}{u_c^2-(\sigma+\alpha_{n,d})^2}d\sigma. \quad (32)$$

Taking time derivative of Λ_{n+1}, yields

$$\dot{\Lambda}_{n+1}=\frac{u_c^2z_{n+1}}{u_c^2-u^2}(\dot{u}-\dot{\alpha}_{n,d})+\frac{\partial\Lambda_{n+1}}{\partial\alpha_{n,d}}\dot{\alpha}_{n,d}, \quad (33)$$

where $\partial\Lambda_{n+1}/\partial\alpha_{n,d}$ is calculated as:

$$\frac{\partial\Lambda_{n+1}}{\partial\alpha_{n,d}}=z_{n+1}\left(\frac{u_c^2}{u_c^2-u^2}-\rho_{n+1}(z_{n+1},\alpha_{n,d})\right). \quad (34)$$

Define system control u such that

$$\dot{u}=-k_{n+1}z_{n+1}+\frac{u_c^2-u^2}{u_c^2}\dot{\alpha}_{n,d}\rho_{n+1} -\frac{k_{c,n}^2(u_c^2-u^2)z_n}{u_c^2(k_{c,n}^2-x_n^2)}. \quad (35)$$

Then,

$$\dot{\Lambda}_{n+1}=-\frac{u_c^2k_{n+1}z_{n+1}^2}{u_c^2-u^2}-\frac{k_{cn}^2z_nz_{n+1}}{k_{cn}^2-x_n^2}. \quad (36)$$

Based on (30) and (36), one can obtain

$$\dot{V}_{n+1}\le-\sum_{j=1}^{n}\frac{k_{ci}^2k_iz_i^2}{k_{ci}^2-x_i^2}-\frac{u_c^2k_{n+1}z_{n+1}^2}{u_c^2-u^2}-\sum_{j=1}^{n}(\tilde{\alpha}_i)^2 -\sum_{j=1}^{n}(\varepsilon_{id})^2+\sum_{j=1}^{n}\tilde{\theta}_i^T(\dot{\hat{\theta}}_i/p_i-\Phi_i\frac{k_{ci}^2z_i}{k_{ci}^2-x_i^2}), \quad (37)$$

where $k_{c,n+1}=u_c$.

Design the adaptive law of $\hat{\theta}_i$ as

$$\dot{\hat{\theta}}_i=\mathrm{Proj}\left(p_i\Phi_i\frac{k_{ci}^2z_i}{k_{ci}^2-x_i^2}\right), \quad (38)$$

where $Proj(\bullet)$ is a projection operator given by [26]

$$Proj(\bullet)=\begin{cases}\bullet, & \text{if } ||\hat{\theta}_i||<c_{\theta i} \\ & \text{or } ||\hat{\theta}_i||=c_{\theta i}\ \&\ \hat{\theta}_i^T\bullet\le 0, \\ \bullet-\hat{\theta}_i\hat{\theta}_i^T\bullet/||\hat{\theta}_i|^2|, & \\ & \text{if } ||\hat{\theta}_i||=c_{\theta i}\ \&\ \hat{\theta}_i^T\bullet>0.\end{cases} \quad (39)$$

Lemma 2 [22]**:** The first part of (21) satisfies

$$\int_0^{z_i}\frac{\sigma k_{ci}^2}{k_{ci}^2-(\sigma+\alpha_{i-1,d})^2}d\sigma\le\frac{k_{ci}^2z_i^2}{k_{ci}^2-x_i^2} \quad (40)$$

for $|x_i|<k_{ci}$, and

$$\int_0^{z_{n+1}}\frac{u_c^2}{u_c^2-(\sigma+\alpha_{n,d})^2}d\sigma\le\frac{u_c^2z_{n+1}^2}{u_c^2-u^2}. \quad (41)$$

Lemma 3 [28]**:** For the filters (27), if $\tilde{\alpha}_i(0)=0$ and $x(t)\in D_n,\forall t\in[0,T_f]$, then given $\mu\in R^+$, there exist $\tau_i>0$ such that $|\tilde{\alpha}_i(t)|\le\mu,\forall t\in[0,T_f],i=1,2,\cdots,n$.

Theorem 1: Consider the system (1-2) under Assumption 1-2, control law described in (13) and initial condition $x(0)\in D_n$, $|u(0)|\le u_c$. Let

$$A_i=\max_{(\bar{z}_i,\bar{y}_{di})\in\Gamma_i}|\alpha_i(\bar{z}_i,\bar{y}_{di})|,\ A_i^{'}=\max_{(\bar{z}_i,\bar{y}_{di})\in\Gamma_i}|\alpha_{di}|, \quad i=1,\cdots,n,$$

where $\bar{z}_i=[z_1,\cdots,z_i]^T,\bar{y}_{di}=[y_d,y_d^{(1)},\cdots,y_d^{(i-1)}]^T$, and

$$\Gamma_i=\{\bar{z}_i\in R^i,\bar{y}_{di}\in R^i:||z_j|\le\sqrt{2V|_{t=0}}, j=1,\cdots,i,|y_d|\le A_0,\cdots,|y_d^{(i-1)}|\le Y_{i-1}\}. \quad (42)$$

If there exist $k_1,\cdots,k_{n+1}$ and $\tau_1,\cdots,\tau_n>0$ such that

$$k_{ci}>A_{i-1},k_{ci}>A_{i-1}^{'},i=1,2,\cdots,n,u_c>A_n, \quad (43)$$

then the system constraints (3) are satisfied, $z_i,\tilde{\theta}_i,\hat{\theta}_i\in L_\infty,i=1,\cdots,n$, and the tracking error z_1 converge to a bounded compact set.

Proof: From the expression of $A_i^{'},i=1,\cdots,n$ and their boundedness, the errors $\tilde{\alpha}_i$ are bounded and there exist α_D such that $|\tilde{\alpha}_i|\le\alpha_D$. Substituting (38) into (37), yields

$$\begin{aligned}\dot{V}_{n+1}\le&-\sum_{j=1}^{n}\frac{k_{ci}^2k_iz_i^2}{k_{ci}^2-x_i^2}-\frac{u_c^2k_{n+1}z_{n+1}^2}{u_c^2-u^2}\\&+\sum_{j=1}^{n}((\tilde{\alpha}_i)^2+(\varepsilon_{id})^2)\\\le&\sum_{i=1}^{n}\left(\alpha_D^2+\varepsilon_D^2+\frac{1}{2}kc_i\right)-k\sum_{i=1}^{n}\frac{1}{2p_i}\tilde{\theta}_i^T\tilde{\theta}_i\\&-k\left(\sum_{j=1}^{n}\frac{k_{ci}^2z_i^2}{k_{ci}^2-x_i^2}+\frac{u_c^2z_{n+1}^2}{u_c^2-u^2}\right)\\\le&-\frac{1}{2}kV_{n+1}\\&-\frac{1}{2}\left(kV_{n+1}-2\sum_{i=1}^{n}\left(\alpha_D^2+\varepsilon_D^2+\frac{1}{2}kc_i\right)\right),\end{aligned} \quad (44)$$

where $k=\min_i\{k_i\}$. Then,

$$\dot{V}_{n+1}\le-\frac{1}{2}\min_i\{k_i\}V_{n+1},\text{if } V_{n+1}(t)\ge\zeta, \quad (45)$$

where $\zeta=2\sum_{i=1}^n(\alpha_D^2+\varepsilon_D^2+\frac{1}{2}kc_i)/k$. Therefore, $V_{n+1}(t)$ is bounded for all $t\in R^+$, from which $x(t)\in D_n,\forall t\in R^+$ and $|u(t)|\le u_c$ can be concluded.

It is derived from Lemma 3 that given $\tilde{\alpha}_i(0) = 0$ and $\mu > 0$, there exist τ_i such that $|\tilde{\alpha}_i(t)| \leq \mu, \forall t \in [0, t_f]$. Therefore,

$$\dot{V}_{n+1} \leq -\frac{1}{2}\min_i\{k_i\}V_{n+1}, \text{if } V_{n+1}(t) \leq \zeta_1, \forall t \in [0, t_f], \tag{46}$$

where $\zeta_1 = 2\sum_{i=1}^{n}(\mu^2 + \varepsilon_D^2 + \frac{1}{2}kc_i/p_i)/k$. Therefore, $\tilde{x}$ and $\tilde{\theta}_i$ are bounded over any finite time interval, by [29, Theorem 3.3], the solution exists for $t \in [0, \infty)(i.e. T_f = \infty)$.

Solving the inequality (46), yields

$$||\bar{z}_n(t)|| \leq \sqrt{2V_{n+1}(0)}\exp(-kt/2) + \sqrt{2\zeta_1} \tag{47}$$

from which one can conclude that the tracking error z_1 converges to a small neighborhood of zero by properly choosing k_i, τ_i, p_i, $i = 1, \cdots, n$, and k_{n+1}.

4. SIMULATION RESULTS

To illustrate the effectiveness of the proposed IBLFs-based adaptive NN control, simulations are carried out for a constrained nonlinear system and a single-link robot manipulator.

4.1. Case 1

The dynamics of the considered nonlinear system is given by

$$\dot{x}_1 = x_1^2 + x_2, \tag{48}$$

$$\dot{x}_2 = x_1x_2 + 2x_1 + u(t), \tag{49}$$

$$|x_1| < 0.8, |x_2| < 2.5, |u| < 10. \tag{50}$$

Denote $x_3 = u$ and choose the initial system states as $x_1(0) = 0.3, x_2(0) = 0.2, x_3(0) = 0$ and the reference trajectory as $y_d = 0.5\sin(0.2t)$.

In the simulation, the control gains are chosen according to dynamic frequency responses of the control system. The dynamic frequency response of x_2 should be much faster than responses of x_1. Thus, we choose $k_2 = 2.5$, which is 5 times of $k_1 = 0.5$, and choose $1/\tau_2 = 30$, which is two times of $1/\tau_1 = 15$. To avoid too fast variation of the control law $u(t)$, choose $k_3 = k_2 = 2.5$.

The control is designed to satisfy

$$\dot{u} = -2z_3 - \frac{2.5^2(10^2 - x_3^2)z_2}{10^2(2.5^2 - x_2^2)} + \frac{10^2 - x_3^2}{10^2}\dot{\alpha}_{2d}\rho_3, \tag{51}$$

where $z_2 = x_2 - \alpha_{1d}$ and $z_3 = x_3 - \alpha_{2d}$ with α_{1d} and α_{2d} obtained by passing virtual control α_1, α_2 through the following command filters

$$\dot{\alpha}_{1d} = -15(\alpha_{1d} - \alpha_1),\ \dot{\alpha}_{2d} = -30(\alpha_{2d} - \alpha_2). \tag{52}$$

The virtual control α_1, α_2 are chosen as

$$\alpha_1 = -\hat{f}_1 - 0.4z_1 + \frac{0.8^2 - x_1^2}{0.64}\dot{y}_d\rho_1 - 0.5\frac{0.8^2 z_1}{0.8^2 - x_1^2}, \tag{53}$$

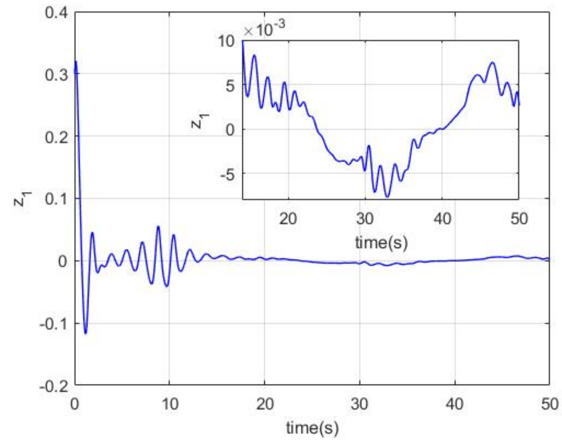

Fig. 1. Trajectory tracking errors $z_1 = x_1 - y_d$.

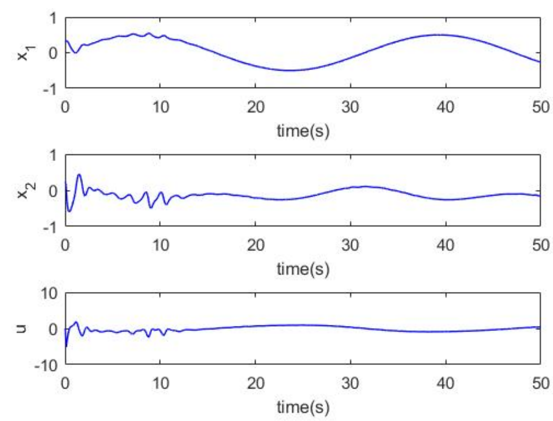

Fig. 2. The state x_1, x_2 and control input u.

$$\alpha_2 = -\hat{f}_2 - 2z_2 - 0.5\frac{2.5^2 z_2}{2.5^2 - x_2^2} + \frac{2.5^2 - x_2^2}{2.5^2}\dot{\alpha}_{1d}\rho_2 - \frac{0.8^2(2.5^2 - x_2^2)}{2.5^2(0.8^2 - x_1^2)}z_1. \tag{54}$$

In NN weights update law (38), $p_1 = 10$ and $p_2 = 10$. In the NN approximation, the $c_{\theta 1}$ and $c_{\theta 2}$ in (39) are chosen as 0.3 and 3, respectively. The $c_{1s,1}$, $\sigma_{1s,1}$ in (6) are chosen as $c_{1s,1} = -0.8 + 2(s-1)$, $\sigma_{1s,1} = 0.4$, $s = 1, 2, \cdots, 9$, and $\sigma_{2s,1} = 0.6$, $c_{2s,1} = -0.8, -0.4, 0, 0.4, 0.8$ for $s \bmod 5 = 1$, 2, 3, 4, 5, respectively, and $\sigma_{2s,2} = 0.7$, $c_{2s,2} = -2.5 + 0.5[s/6]$, $s = 1, 2, \cdots, 55$.

The simulation results are presented in Figs. 1-5, where Fig. 1 presents the trajectory tracking error z_1 in case 1, Fig. 2 depicts the performance of system states and input in case 1, Fig. 3 describes the NNs approximation performance in Case 1, and Figs. 4-5 present f_1, $\hat{f}_1$ and f_2, $\hat{f}_2$ in Case 1, respectively.

From Fig. 1, the tracking error will be less than 0.015 after 15 seconds and the constraints satisfaction $|x_1| \leq 0.8$,

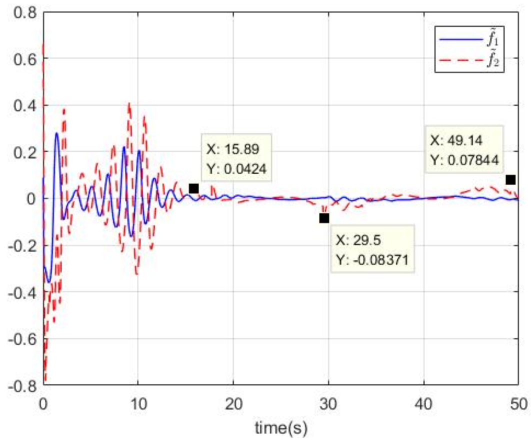

Fig. 3. The NN estimation errors $\tilde{f}_1$ and $\tilde{f}_2$.

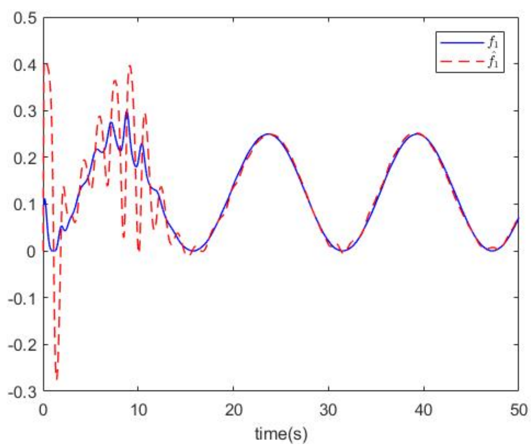

Fig. 4. The uncertainty f_1 and its NN estimator $\hat{f}_1$.

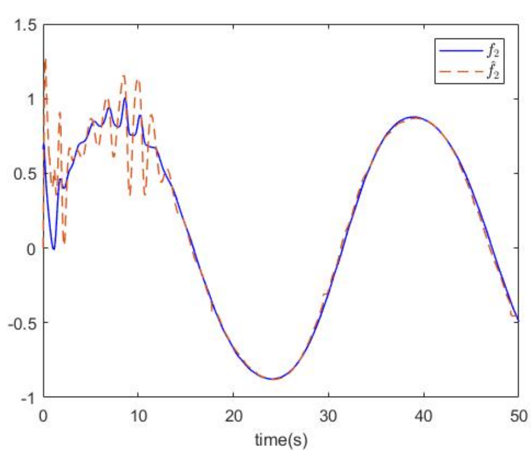

Fig. 5. The uncertainty f_2 and its NN estimator $\hat{f}_2$.

$|x_2| \leq 2.5$, $|u| \leq 10$ can be easily seen from Fig. 2. From Figures 3-5, one can see that the absolute values of the approximation errors $\tilde{f}_1$ and $\tilde{f}_2$ will be no more than 0.01

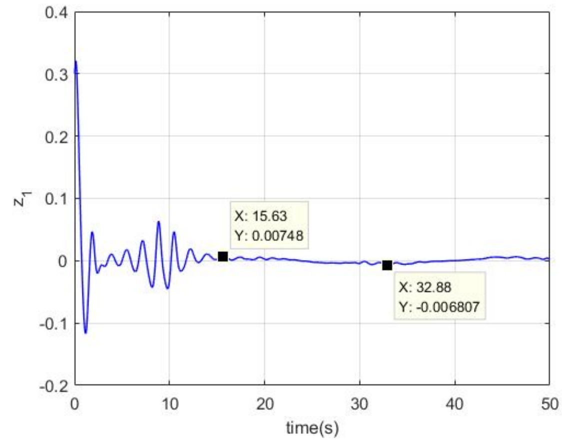

Fig. 6. The tracking error performance z_1 for the system disturbed by $d(t) = 0.2\sin(0.2t)$.

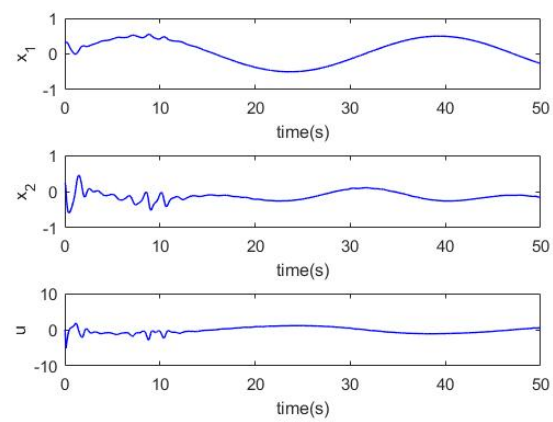

Fig. 7. The state and the control of the system disturbed by $d(t) = 0.2\sin(0.2t)$.

after 15 seconds and the designed NNs can well approximate the uncertainties f_1 and f_2. Therefore, the designed IBLFs-based adaptive NN control makes the system state and control input constraints are satisfied and the trajectory tracking error converge to a small neighborhood of zero. Figs. 6-7 illustrate the tracking error performance and the performance of states and control of the system (48) and (49) disturbed by $d(t) = 0.2\sin(0.2t)$. From the two figures, we can see that the system tracks the desired trajectory with good performance without violation of constraints, which illustrate control robustness to the disturbances.

4.2. Case 2

Consider a single-link robot manipulator with the following dynamics

$$M\ddot{q} + \frac{1}{2}mgl\sin q = u, \tag{55}$$

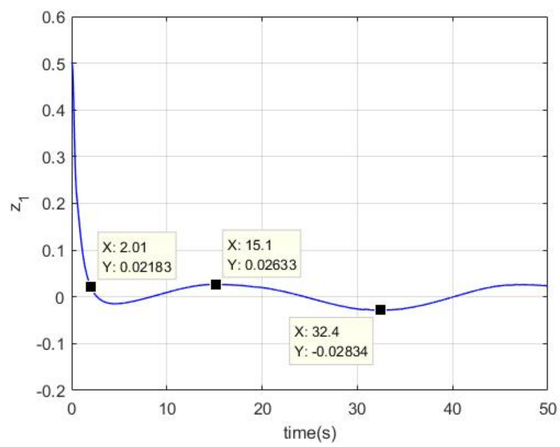

Fig. 8. Trajectory tracking errors $z_1 = x_1 - y_d$.

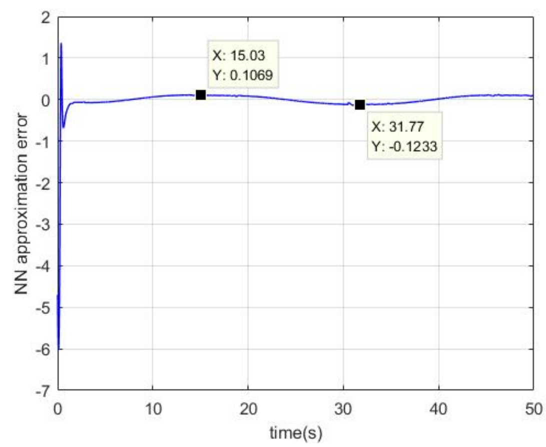

Fig. 10. The NN estimation errors $\tilde{f}$.

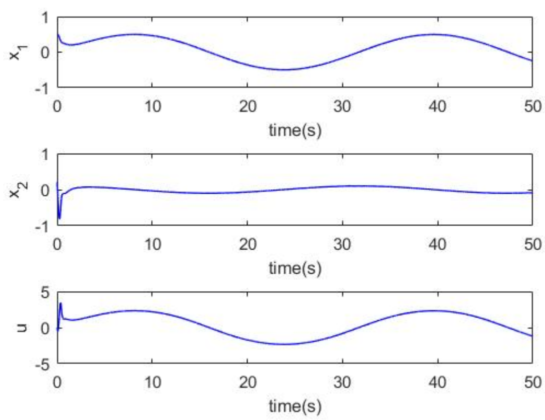

Fig. 9. The state x_1, x_2 and control input u.

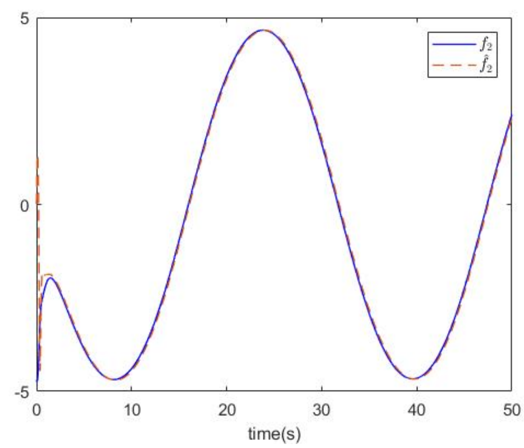

Fig. 11. The uncertainty f and its NN estimator $\hat{f}$.

where $M = 0.5$ is the inertial moment, q is the angle, $m = 1$ Kg is the mass of the link, $l = 1$ m is the length of the link, and $g = 9.8$ m/s^2 is the gravity acceleration. Denote $x_1 = q$, $x_2 = \dot{q}$, and $x_3 = u$. Then, the dynamics (55) can be represented as

$$\dot{x}_1 = x_2, \tag{56}$$

$$\dot{x}_2 = f(x_1, x_2) + 2x_3, \tag{57}$$

$$\dot{x}_3 = \dot{u}, \tag{58}$$

where $f(x_1, x_2) = -9.8\sin(x_1)$ is the uncertainty term, and the system constraints are as follows: $|x_1| < 1$, $|x_2| < 1$, $|u| < 5$.

In the simulation, the initial system states as $x_1(0) = 0.5$, $x_2(0) = 0.2$, $x_3(0) = 0$ and the reference trajectory as $y_d = 0.5\sin(0.2t)$. The control gains are chosen according to dynamic frequency responses of the control system. The dynamic frequency response of x_2 should be much faster than responses of x_1. Thus, we choose $k_2 = 2.5$, which is 5 times of $k_1 = 0.5$, and choose $1/\tau_2 = 45$, which is two times of $1/\tau_1 = 15$. To avoid too fast variation of the control law $u(t)$, choose $k_3 = k_2 = 2.5$.

The simulation results in Case 2 are presented in Figs. 8-11, where Fig. 8 presents the trajectory tracking error z_1 in case 2, Fig. 9 depicts the performance of system states and input in Case 2, Fig. 10 describes the NN approximation performance in Case 2, and Fig. 11 presents f and $\hat{f}$ in Case 2.

From Fig. 8, the tracking error will be less than 0.03 after 2 seconds and satisfaction of the constraints $|x_1| \le 1$, $|x_2| \le 1$, $|u| \le 5$ can be easily seen from Fig. 9. From Fig. 10-11, one can see that the absolute value of the approximation error $\tilde{f}$ will be no more than 0.12 after 2 seconds and the designed NN can well approximate the uncertainties f. Therefore, the designed IBLFs-based adaptive NN control makes the system state and control input constraints be satisfied and the trajectory tracking error converge to a small neighborhood of zero. Figs. 12-13 illustrate the tracking error performance and the performance of states and control of the system (56-58) dis-

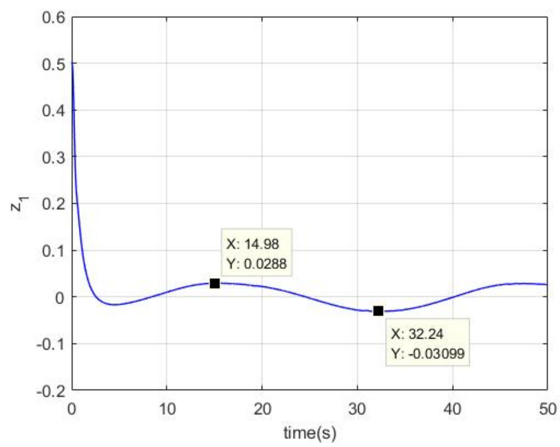

Fig. 12. The tracking error performance z_1 for the system disturbed by $d(t) = -0.5\sin(0.2t)$.

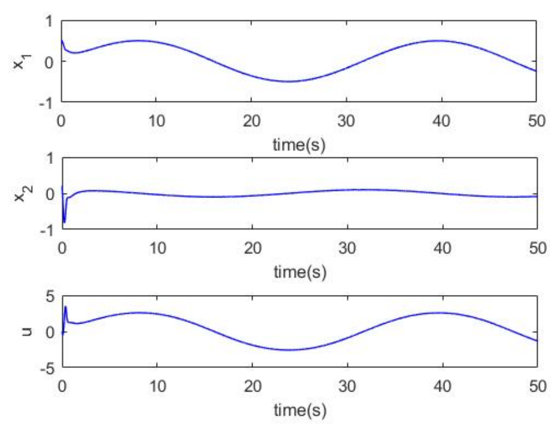

Fig. 13. The state and the control of the system disturbed by $d(t) = -0.5\sin(0.2t)$.

turbed by $d(t) = -0.5\sin(0.2t)$. From the two figures, we can see that the system tracks the desired trajectory with good performance without violation of constraints, which illustrate control robustness to the disturbances.

5. CONCLUSIONS

In this paper, a IBLFs-based adaptive NN control was designed for a class of strict-feedback nonlinear systems with state and input constraints. The paper extended the current results on BLFs-based control for systems with output and state constraints to systems with state and input constraints, by considering the control input as an extended state. In the control, the system uncertainties were estimated and compensated by NNs with projection modifications. From simulation results, one can see that the tracking error and NN approximation error is accurate for the constrained system. Therefore, based on theoretical analysis and simulation results, the effectiveness of the proposed control scheme can be concluded.

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