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Global Robust Synchronization of Fractional Order Complex Valued Neural Networks with Mixed Time Varying Delays and Impulses

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Abstract: In this article, we explore the theoretical issues on the drive-response synchronization of a class of fractional order uncertain complex valued neural networks (FOUCNNs) with mixed time varying delays and impulses. Based upon the contraction mapping principle, robust analysis techniques, as well as Riemann-Liouville (R-L) derivative, we derive a new set of novel sufficient conditions for the existence and uniqueness of equilibrium point of such neural network system, while by applying the Lyapunov functional approach, the global stability of the equilibrium solutions are obtained. Furthermore, the synchronization criterion of FOUCNNs is also attracted by means of the adaptive error feedback control strategy. Finally, two examples are provided along with the simulation results to demonstrate the effectiveness of our main proofs.

Keywords: Adaptive synchronization, asymptotic stability, complex valued neural networks, Riemann-Liouville derivative.

1. INTRODUCTION

The fresh concept of fractional order calculus and differential equations has three hundred years old of branch. For long period, the theory of fractional calculus is developed only on pure mathematics. Owing to lack of solution methods, the development of fractional order calculus has not much attracted more mathematicians in those periods. At present, fractional order dynamical system has attracted increasing interests of many researchers from various aspects such as porus media [2], viscoelasticity [19] and so on. As a extension of ordinary integer order calculus, fractional calculus has been acted more powerful tool because the results are more accurate than integer order in both theory part as well as application part. In continuous time integer order case, the common capacitance can be replaced by fractance, giving this issues is called the origin of non integer order neural network dynamical system [5]. Recently, fractional order calculus and their properties has been applied to neural networks, especially complex valued neural networks.

Complex valued neural network systems, the generalization of real valued recurrent neural network models is totally different properties and more complicated to real world neural network models because the connection weight parameters, activation functions and state variables are mainly chosen in complex values. Many author considered sigmoid activation functions in real world neural networks because these activation are continuously differentiable and bounded. Moreover, In complex valued neural networks, the continuously differentiable and bounded activation in complex domain is not convenient since they will reduce constants over entire $\mathbb{C}$ by means of Liouville's theorem [15]. In practice, detection of symmetry problem and XOR problem are not suitable for real world counterparts and it can be only solved by complex valued neural networks [32]. However, in practice, the stability may be affects the parameter uncertainty [4], discrete delay [7, 9], synchronization errors [13], distributed delay [33], diffusion effects [14], impulsive effects [22, 29], stochastic effects [34] and in electronic network implementations, these are widely exists to neural network models. For example, a lot of many excellent considerable results with fractional order and integer order complex valued neural networks have been reported in the existing literature, see [7, 8, 11, 12, 25].

Since the method of synchronization between two chaotic system with different initial conditions was governed in Pecora and Carroll [10]. In past few days, the study of fractional order complex valued synchronization

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has a burgeoning research topic in the area of neural networks and complex networks. Up to now, there are numerous types of synchronization in fractional order has been proposed projective synchronization [3], global Mittag Leffler synchronization [18], finite time synchronization [20], adaptive synchronization [23, 24], Finite time Mittag-Leffler synchronization [26], quasi uniform synchronization [27] and lag synchronization [28] by means of sliding mode control, state feedback control, adaptive controls and period intermittent control methods. There are few number of results are published in the synchronization analysis of complex valued neural networks with integer order and non integer order cases, see Ref [1, 31]. Through the review of literature, a few results have been available on robust stability and robust adaptive synchronization analysis of fractional order complex valued neural networks with mixed time delays and impulses. Motivated by the aforementioned arguments, the main objective of this article are listed as follows

- This is the first time to investigates the Riemann-Liouville sense for global robust synchronization of FOUCNNs with mixed time varying delay and impulses.
- By employing the contraction mapping principle, a suitable Lyapunov functional, Barbalat's Lemma and the properties of R-L derivative, some sufficient conditions which ensures the existence, uniqueness and stability of equilibrium point of the system are established.
- By means of adaptive feedback control, we have to show the occurrence of robust adaptive complex valued synchronization conditions between the drive-response systems and we have to introduces some special corollaries of obtained main results which is different from existing literatures.

Notations: Throughout this paper, $\mathbb{C}$ is the space of complex valued functions, $\mathbb{R}$ be the space of real valued functions, $\mathbb{C}^n$, $\mathbb{R}^n$ denotes n-dimensional unitary and n-dimensional Euclidean spaces. Let $x = p + iq$ be a complex number, where $i = \sqrt{-1}$ is the imaginary units, $p, q \in \mathbb{R}$. In this paper $\|.\|$ denotes $\|.\|_1$, $\mathbb{R}^{n\times n}$ and $\mathbb{C}^{n\times n}$ denotes the set of all $n \times n$ real and complex matrices. Let ${}^{RL}D^{\beta}$ is Riemann-Liouville operator, simply denoted by D^{β} and $C((-\infty, 0], \mathbb{R}^n)$ denotes the family of Banach space of all continuous functions mapping from $(-\infty, 0]$ to $\mathbb{R}^n$. Similarly, $C((-\infty, 0], \mathbb{C}^n)$ denotes the family of Banach space of all continuous functions mapping from $(-\infty, 0]$ to $\mathbb{C}^n$.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this work, we consider the impulsive fractional order uncertain complex valued neural networks (FOUCNN's) model as follows:

$$\begin{aligned}
D^{\beta}x_h(t) &= -r_h x_h(t) + \sum_{l=1}^{n}(u_{hl} + \Delta u_{hl})g_l(x_l(t)) \\
&\quad + \sum_{l=1}^{n}(v_{hl} + \Delta v_{hl})g_l(x_l(t - \tau_l(t))) \\
&\quad + \sum_{l=1}^{n}(w_{hl} + \Delta w_{hl})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s) \\
&\quad \times g_l(x_l(s))ds + I_h, \;\; t \neq t_k, \;\; t \geq 0, \\
\Delta x_h(t_k) &= x_h(t_k^+) - x_h(t_k^-) = \Phi_{hk}(x_h(t_k)), \qquad (1)
\end{aligned}$$

for $h = 1, 2, ..., n$, $k = 1, 2, ...,$ where $x_h(t) = (x_1(t), x_2(t), ..., x_n(t)) \in \mathbb{C}^n$ is the state vector of the of the h-th neuron at time t; D^{β} denotes the Riemann Liouville operator with fractional order $0 < \beta < 1$; $\tau_l(t) = (\tau_1(t), \tau_2(t), ..., \tau_n(t))$ denotes the corresponding discrete time varying delay of the l-th neuron to h-th neuron; $\mathbb{L}_{hl}(.)$ denotes the delay kernel of infinite distributed delay defined on $[0, +\infty)$ and non negative bounded function; $g_l : \mathbb{C}^n \to \mathbb{C}^n$ describes the nonlinear complex valued activation function; $R = diag\{r_h\} > 0$, $h = 1, 2, ..., n$ are constant matrices; $U = (u_{hl})_{n\times n} \in \mathbb{C}^{n\times n}$, $V = (v_{hl})_{n\times n} \in \mathbb{C}^{n\times n}$ and $W = (w_{hl})_{n\times n} \in \mathbb{C}^{n\times n}$ be the connection weight matrices; $I = (I_1, I_2, ..., I_n) \in \mathbb{C}^n$ denotes the external inputs; t_k denotes the impulsive perturbations and it satisfies $0 < t_1 < t_2 <, ..., \lim_{k\to\infty} t_k = +\infty$; Φ_{hk} denotes the impulsive jumps, $x_h(t_k^-) = \lim_{t\to t_k^-} x(t)$ and $x_h(t_k^+) = \lim_{t\to t_k^+} x(t)$ stands for the left and right limits on impulsive moments at time $t = t_k$. Without loss of generality, the solution of network system (1) is left continuous at time t_k. i.e., $x_i(t_k^-) = x_i(t_k)$; Δu_{hl}, Δv_{hl} and Δw_{hl} be the complex uncertain parameter. The initial value associated with the system (1) is

$$D^{-(1-\beta)}x_h(t) = \rho_h(t) \in C((-\infty, 0], \mathbb{C}^n).$$

There are two kinds of approaches to solved complex valued concepts in neural network systems. The first one is connection weight parameters, activation functions and state variables are all defined in complex domain and the results can be obtained straight forward [17, 25]. Another one is separation of complex valued neural networks into real and imaginary parts of neural networks, which is the twice that dimensional of real valued neural networks [1]. In this paper, we moves with second type of approaches. Let $x_h(t) = p_h(t) + iq_h(t)$, $g_h(x_h(t)) = g_h^R(p_h(t), q_h(t)) + ig_h^I(p_h(t), q_h(t))$, $I_h = I_h^R + I_h^I$, where $p_h(t), q_h(t) \in \mathbb{R}$, $g_h^R(p_h(t), q_h(t)), g_h^I(p_h(t), q_h(t)) : \mathbb{R}^2 \to \mathbb{R}$. Therefore, the equation (1) can be splitted into real and imaginary parts. We have that

$$\begin{aligned}
&D^{\beta}p_h(t) \\
&= -r_h p_h(t) + \sum_{l=1}^{n}(u_{hl}^R + \Delta u_{hl}^R)g_l^R(p_l(t), q_l(t))
\end{aligned}$$

$$
\begin{aligned}
& -\sum_{l=1}^{n}(u_{hl}^{I}+\Delta u_{hl}^{I})g_{l}^{I}(p_{l}(t),q_{l}(t)) \\
& +\sum_{l=1}^{n}(v_{hl}^{R}+\Delta v_{hl}^{R})g_{l}^{R}(p_{l}(t-\tau_{l}(t))),q_{l}(t-\tau_{l}(t)))) \\
& -\sum_{l=1}^{n}(v_{hl}^{I}+\Delta v_{hl}^{I})g_{l}^{I}(p_{l}(t-\tau_{l}(t))),q_{l}(t-\tau_{l}(t))) \\
& +\sum_{l=1}^{n}(w_{hl}^{R}+\Delta w_{hl}^{R})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)g_{l}^{R}(p_{l}(s),q_{l}(s))ds \\
& -\sum_{l=1}^{n}(w_{hl}^{I}+\Delta w_{hl}^{I})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)g_{l}^{I}(p_{l}(s),q_{l}(s))ds \\
& +I_{h}^{R},\ \ t\neq t_{k},\ \ t\geq 0, \\
\Delta p_{h}(t_{k}) & = p_{h}(t_{k}^{+})-p_{h}(t_{k}^{-}) \\
& = \Phi_{hk}^{R}(p_{h}(t_{k})),\ \ k=1,2,..., \qquad (2)
\end{aligned}
$$

and

$$
\begin{aligned}
& D^{\beta}q_{h}(t) \\
& =-r_{h}q_{h}(t)+\sum_{l=1}^{n}(u_{hl}^{I}+\Delta u_{hl}^{I})g_{l}^{R}(p_{l}(t),q_{l}(t)) \\
& +\sum_{l=1}^{n}(u_{hl}^{R}+\Delta u_{hl}^{R})g_{l}^{I}(p_{l}(t),q_{l}(t)) \\
& +\sum_{l=1}^{n}(v_{hl}^{I}+\Delta v_{hl}^{I})g_{l}^{R}(p_{l}(t-\tau_{l}(t)),q_{l}(t-\tau_{l}(t))) \\
& +\sum_{l=1}^{n}(v_{hl}^{R}+\Delta v_{hl}^{R})g_{l}^{I}(p_{l}(t-\tau_{l}(t))),q_{l}(t-\tau_{l}(t)))) \\
& +\sum_{l=1}^{n}(w_{hl}^{I}+\Delta w_{hl}^{I})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s) \\
& \times g_{l}^{R}(p_{l}(s),q_{l}(s))ds+\sum_{l=1}^{n}(w_{hl}^{R}+\Delta w_{hl}^{R}) \\
& \times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)g_{l}^{I}(p_{l}(s),q_{l}(s))ds \\
& +I_{h}^{I},\ \ t\neq t_{k},\ \ t\geq 0, \\
\Delta q_{h}(t_{k}) & = q_{h}(t_{k}^{+})-q_{h}(t_{k}^{-}) \\
& = \Phi_{hk}^{I}(q_{h}(t_{k})),\ \ k=1,2,.... \qquad (3)
\end{aligned}
$$

The initial conditions of separating neural drive system (2)-(3) are defined as

$$
\begin{aligned}
& D^{-(1-\beta)}p_{h}(t)=\varphi_{h}(t)\in C((-\infty,0],\mathbb{R}^{n}), \\
& D^{-(1-\beta)}q_{h}(t)=\chi_{h}(t)\in C((-\infty,0],\mathbb{R}^{n}).
\end{aligned}
$$

Definition 1 [16]**:** The fractional order integral of order β for an integral function $y:[t_0,t)\longrightarrow\mathbb{R}$ is defined as

$$
D^{-\beta}y(t)=\frac{1}{\Gamma(\beta)}\int_{t_0}^{t}(t-m)^{\beta-1}y(m)dm,
$$

where $\beta>0$ and $\Gamma(.)$ is the Gamma function.

Definition 2 [16]**:** The Riemann-Lioville (R-L) derivative of order β for a function $y(t)$ is defined as

$$
D^{\beta}y(t)=\frac{1}{\Gamma(n-\beta)}\frac{d^{n}}{dt^{n}}\int_{t_0}^{t}(t-m)^{n-\beta-1}y(m)dm,
$$

where $t\geq t_0$ and n is the positive integer such that $n-1<\beta<n$.
Particularly, when $0<\beta<1$,

$$
D^{\beta}y(t)=\frac{1}{\Gamma(1-\beta)}\frac{d}{dt}\int_{t_0}^{t}(t-m)^{-\beta}y(m)dm.
$$

Definition 3: Let $x^{*}=(x_1^{*},x_2^{*},...,x_n^{*})$ is an equilibrium point of the system (1), if and only if

$$
\begin{aligned}
0= & -r_{h}x_{h}^{*}+\sum_{l=1}^{n}(u_{hl}+\Delta u_{hl}+v_{hl}+\Delta v_{hl})g_{l}(x_{l}^{*}) \\
& +\sum_{l=1}^{n}(w_{hl}+\Delta w_{hl})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)g_{l}(x_{l}^{*})ds+I_{h}, \\
0= & \Phi_{hk}(x_{h}^{*}),\ \ h=1,2,...,n,\ \ k=1,2,.... \qquad (4)
\end{aligned}
$$

Lemma 1 [6]**:** If the time dependent differential function $\sigma(t)$ has finite limit as $t\to+\infty$, and if $\frac{d\sigma}{dt}$ is uniformly continuous, then $\frac{d\sigma}{dt}\to 0$ as $t\to+\infty$.

Assumption 1: Let $x_h(t)=p_h(t)+iq_h(t)$ and $\tilde{x}_h(t)=\tilde{p}_h(t)+i\tilde{q}_h(t)$. The nonlinear Lipschitz continuous activation function are given as follows: $g_h(x_h(t))=g_h^R(p_h(t),q_h(t))+ig_h^I(p_h(t),q_h(t))$, where $g_h^R(.,.):\mathbb{R}^2\to\mathbb{R}$, $g_h^I(.,.):\mathbb{R}^2\to\mathbb{R}$ with $p_h(t),\ q_h(t),\ \tilde{p}_h(t),\ \tilde{q}_h(t)\in\mathbb{R}$, there exist some positive constants κ^{RR}, κ^{RI}, κ^{IR}, κ^{II} such that

$$
\begin{aligned}
& |g_h^R(\tilde{p}_h(t),\tilde{q}_h(t))-g_h^R(p_h(t),q_h(t))| \\
& \leq\kappa_h^{RR}|\tilde{p}_h(t)-p_h(t)|+\kappa_h^{RI}|\tilde{q}_h(t)-q_h(t)|, \\
& |g_h^I(\tilde{p}_h(t),\tilde{q}_h(t))-g_h^I(p_h(t),q_h(t))| \\
& \leq\kappa_h^{IR}|\tilde{p}_h(t)-p_h(t)|+\kappa_h^{II}|\tilde{q}_h(t)-q_h(t)|.
\end{aligned}
$$

Assumption 2: There exist some positive scalars $\check{\tau}$ and $\hat{\tau}$ such that $0<\tau_l(t)<\check{\tau}$, $\dot{\tau}_l(t)\leq\hat{\tau}<1$, $l=1,2,...,n$.

Assumption 3: For any h, l = 1, 2, ..., n, there exists a positive constant $\boldsymbol{\zeta_{hl}}$ such that

$$
\int_{0}^{+\infty}\mathbb{L}_{hl}(m)dm=\zeta_{hl}.
$$

Assumption 4: $\forall h$, k = 1, 2, ..., n, there exist real scalars μ_{hl}^{α}, ϑ_{hl}^{α}, $\omega_{hl}^{\alpha}(\alpha=R,I)$ such that

$$
\Delta u_{hl}^{\alpha}=\mu_{hl}^{\alpha}\eta_{hl}^{\alpha}(t),\ \ \Delta v_{hl}^{\alpha}=\vartheta_{hl}^{\alpha}\hat{\eta}_{hl}^{\alpha}(t),\ \ \Delta w_{hl}^{\alpha}=\omega_{hl}^{\alpha}\check{\eta}_{hl}^{\alpha}(t).
$$

where the time-varying uncertain real function η_{hl}^{α}, $\hat{\eta}_{hl}^{\alpha}$ and $\check{\eta}_{hl}^{\alpha}(\alpha=R,I)$ satisfies the conditions $[\eta_{hl}^{\alpha}(t)]^2\leq 1$, $[\hat{\eta}_{hl}^{\alpha}(t)]^2\leq 1$, $[\check{\eta}_{hl}^{\alpha}(t)]^2\leq 1$, h,l = 1, 2, ..., n.

Assumption 5: For any h = 1, 2, ..., n and k = 1, 2, ..., there exist positive constants Υ_{hk}^R and Υ_{hk}^I such that functions Φ_{hk}^R and Φ_{hk}^I satisfying

$$
\begin{aligned}
& \Phi_{hk}^R=-\Upsilon_{hk}^R(y_h(t)),\ \ \Phi_{hk}^I=-\Upsilon_{hk}^I(z_h(t)), \\
& 0<\Upsilon_{hk}^R<2,\ \ 0<\Upsilon_{hk}^I<2,
\end{aligned}
$$

where $y_h(t)$ and $z_h(t)$ are defined in later.

3. MINE RESULTS

3.1. Existence and uniqueness of the equilibrium point

Theorem 1: Suppose Assumptions 1, 2, 3, 4, and 5 hold, then there exists a unique equilibrium point of the system (1), $x^* = (p^*, q^*)$ is globally robust stable if $\Lambda_1 > 0$, $\Lambda_2 > 0$ and

$$\hat{\Upsilon} = \max_{1\le h\le n}\Big\{\sum_{l=1}^{n}\Big([\hat{u}_{lh}^R+\hat{u}_{lh}^I]\kappa_h^R+[\hat{v}_{lh}^R+\hat{v}_{lh}^I]\kappa_h^R + \zeta_{lh}[\hat{w}_{lh}^R+\hat{w}_{lh}^I]\kappa_h^R\Big), \sum_{l=1}^{n}\Big([\hat{u}_{lh}^R+\hat{u}_{lh}^I]\kappa_h^I+[\hat{v}_{lh}^R + \hat{v}_{lh}^I]\kappa_h^I+\zeta_{lh}[\hat{w}_{lh}^R+\hat{w}_{lh}^I]\kappa_h^I\Big)\Big\} < \min_{1\le h\le n}\{r_h\}, \quad (5)$$

where

$$\Lambda_1 = \min_{1\le h\le n}\Big\{r_h - \sum_{l=1}^{n}\{\hat{u}_{lh}^R\kappa_h^R+\hat{u}_{lh}^I\kappa_h^R\} - \sum_{l=1}^{n}\frac{\{\hat{v}_{lh}^R\kappa_h^R+\hat{v}_{lh}^I\kappa_h^R\}}{1-\hat{\tau}} - \sum_{l=1}^{n}\{\hat{w}_{lh}^R\kappa_h^R+\hat{w}_{lh}^I\kappa_h^R\}\zeta_{lh}\Big\},$$

$$\Lambda_2 = \min_{1\le h\le n}\Big\{r_h - \sum_{l=1}^{n}\{\hat{u}_{lh}^R\kappa_h^I+\hat{u}_{lh}^I\kappa_h^I\} - \sum_{l=1}^{n}\frac{\{\hat{v}_{lh}^R\kappa_h^I+\hat{v}_{lh}^I\kappa_h^I\}}{1-\hat{\tau}} - \sum_{l=1}^{n}\{\hat{w}_{lh}^R\kappa_h^I+\hat{w}_{lh}^I\kappa_h^I\}\zeta_{lh}\Big\},$$

where $\hat{u}_{lh}^\alpha = |u_{lh}^\alpha|+|\mu_{lh}^\alpha|$, $\hat{v}_{lh}^\alpha = |v_{lh}^\alpha|+|\vartheta_{lh}^\alpha|$, $\hat{w}_{lh}^\alpha = |w_{lh}^\alpha|+|\omega_{lh}^\alpha|$ $(\alpha = R, I)$, $\kappa_h^R = \kappa_h^{RR}+\kappa_h^{IR}$, $\kappa_h^I = \kappa_h^{IR}+\kappa_h^{II}$.

Proof: First, we can prove the existence and uniqueness of the equilibrium point.

Let $r_h p_h = \psi_h$ and $r_h q_h = \varepsilon_h$. Consider a mapping $\Pi : \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ is $\Pi(\psi,\varepsilon) = \Big(\Pi_1(\psi,\varepsilon),\ ...,\ \Pi_n(\psi,\varepsilon), \Pi_{n+1}(\psi,\varepsilon), ..., \Pi_{2n}(\psi,\varepsilon)\Big)^T$ defined by

$$\begin{aligned}\Pi_h(\psi_h,\varepsilon_h) = &\sum_{l=1}^{n}(u_{hl}^R+\Delta u_{hl}^R)g_l^R\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big) - \sum_{l=1}^{n}(u_{hl}^I+\Delta u_{hl}^I)g_l^I\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\ &+\sum_{l=1}^{n}(v_{hl}^R+\Delta v_{hl}^R)g_l^R\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big) - \sum_{l=1}^{n}(v_{hl}^I+\Delta v_{hl}^I)g_l^I\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\ &+\sum_{l=1}^{n}(w_{hl}^R+\Delta w_{hl}^R)\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s) \times g_l^R\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)ds - \sum_{l=1}^{n}(w_{hl}^I+\Delta w_{hl}^I)\\ &\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)g_l^I\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)ds + I_h^R,\end{aligned}$$

$$\begin{aligned}\Pi_{n+h}(\psi_h,\varepsilon_h) = &\sum_{l=1}^{n}(u_{hl}^I+\Delta u_{hl}^I)g_l^R\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big) + \sum_{l=1}^{n}(u_{hl}^R+\Delta u_{hl}^R)g_l^I\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\ &+\sum_{l=1}^{n}(v_{hl}^I+\Delta v_{hl}^I)g_l^R\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big) + \sum_{l=1}^{n}(v_{hl}^R+\Delta v_{hl}^R)g_l^I\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\ &+\sum_{l=1}^{n}(w_{hl}^I+\Delta w_{hl}^I)\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s) \times g_l^R\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)ds + \sum_{l=1}^{n}(w_{hl}^R+\Delta w_{hl}^R)\\ &\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)g_l^I\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)ds + I_h^I.\end{aligned}$$

Let us take any two complex vectors (ψ,ε) and $(\tilde{\psi},\tilde{\varepsilon})$. Based Assumptions 1, 3, and 4, we gain

$$\begin{aligned}&\Big\|\Pi(\tilde{\psi},\tilde{\varepsilon})-\Pi(\psi,\varepsilon)\Big\|\\ &\le \sum_{h=1}^{n}\Big[\sum_{l=1}^{n}\frac{1}{r_l}\Big((|u_{hl}^R|+|\mu_{hl}^R|)\kappa_l^R+(|v_{hl}^R|+|\vartheta_{hl}^R|)\kappa_l^R + \zeta_{hl}(|w_{hl}^R|+|\omega_{hl}^R|)\kappa_l^R+(|u_{hl}^I|+|\mu_{hl}^I|)\kappa_l^R+(|v_{hl}^I| + |\vartheta_{hl}^I|)\kappa_l^R+\zeta_{hl}(|w_{hl}^I|+|\omega_{hl}^I|)\kappa_l^R\Big)|\tilde{\psi}_l-\psi_l|\Big]\\ &+\sum_{h=1}^{n}\Big[\sum_{l=1}^{n}\frac{1}{r_l}\Big((|u_{hl}^I|+|\mu_{hl}^I|)\kappa_l^I+(|v_{hl}^I|+|\vartheta_{hl}^I|)\kappa_l^I + \zeta_{hl}(|w_{hl}^I|+|\omega_{hl}^I|)\kappa_l^I+(|u_{hl}^R|+|\mu_{hl}^R|)\kappa_l^I+(|v_{hl}^R| + |\vartheta_{hl}^R|)\kappa_l^I+\zeta_{hl}(|w_{hl}^R|+|\omega_{hl}^R|)\kappa_l^I\Big)|\tilde{\varepsilon}_l-\varepsilon_l|\Big]\\ &= \sum_{h=1}^{n}\frac{\sum_{l=1}^{n}\Lambda_{1l}}{r_h}|\tilde{\psi}_h-\psi_h| + \sum_{h=1}^{n}\frac{\sum_{l=1}^{n}\Lambda_{2l}}{r_h}|\tilde{\varepsilon}_h-\varepsilon_h|,\end{aligned}$$

where $\Lambda_{1l} = \Big([\hat{u}_{lh}^R+\hat{u}_{lh}^I]\kappa_h^R+[\hat{v}_{lh}^R+\hat{v}_{lh}^I]\kappa_h^R+\zeta_{lh}[\hat{w}_{lh}^R+\hat{w}_{lh}^I]\kappa_h^R\Big)$, $\Lambda_{2l} = \Big([\hat{u}_{lh}^R+\hat{u}_{lh}^I]\kappa_h^I+[\hat{v}_{lh}^R+\hat{v}_{lh}^I]\kappa_h^I+\zeta_{lh}[\hat{w}_{lh}^R+\hat{w}_{lh}^I]\kappa_h^I\Big)$.

By using (5), we know that

$$\frac{\sum_{l=1}^{n}\Lambda_{1l}}{r_h} < 1, \quad (6)$$

$$\frac{\sum_{l=1}^{n}\Lambda_{2l}}{r_h} < 1. \quad (7)$$

By using the inequality (6) and (7), we get

$$\begin{aligned}\|\Pi(\tilde{\psi},\tilde{\varepsilon})-\Pi(\psi,\varepsilon)\| &< \sum_{h=1}^{n}|\tilde{\psi}_h-\psi_h| + \sum_{h=1}^{n}|\tilde{\varepsilon}_h-\varepsilon_h|\\ &= \|(\tilde{\psi},\psi)-(\tilde{\varepsilon},\varepsilon)\|. \quad (8)\end{aligned}$$

It follows that $\Pi : \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ is a contraction mapping on $\mathbb{R}^{2n}$. Thus there exists a unique fixed point such that (ψ^*,ε^*) such that $\Pi(\psi^*,\varepsilon^*) = (\psi^*,\varepsilon^*)$, that is

$$\psi_h^* = \sum_{l=1}^{n}(u_{hl}^R+\Delta u_{hl}^R)g_l^R\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)$$

$$
\begin{aligned}
&-\sum_{l=1}^{n}(u_{hl}^{I}+\Delta u_{hl}^{I})g_l^{I}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\
&+\sum_{l=1}^{n}(v_{hl}^{R}+\Delta v_{hl}^{R})g_l^{R}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\
&-\sum_{l=1}^{n}(v_{hl}^{I}+\Delta v_{hl}^{I})g_l^{I}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\
&+\sum_{l=1}^{n}(w_{hl}^{R}+\Delta w_{hl}^{R})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\\
&\times g_l^{R}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)ds-\sum_{l=1}^{n}(w_{hl}^{I}+\Delta w_{hl}^{I})\\
&\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)g_l^{I}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)ds+I_h^{R},\\
\varepsilon_h^{*}=&\sum_{l=1}^{n}(u_{hl}^{I}+\Delta u_{hl}^{I})g_l^{R}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\
&+\sum_{l=1}^{n}(u_{hl}^{R}+\Delta u_{hl}^{R})g_l^{I}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\
&+\sum_{l=1}^{n}(v_{hl}^{I}+\Delta v_{hl}^{I})g_l^{R}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\
&+\sum_{l=1}^{n}(v_{hl}^{R}+\Delta v_{hl}^{R})g_l^{I}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)\\
&+\sum_{l=1}^{n}(w_{hl}^{I}+\Delta w_{hl}^{I})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\\
&\times g_l^{R}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)ds+\sum_{l=1}^{n}(w_{hl}^{R}+\Delta w_{hl}^{R})\\
&\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)g_l^{I}\Big(\frac{\psi_l}{r_l},\frac{\varepsilon_l}{r_l}\Big)ds+I_h^{I}.
\end{aligned}
$$

Substituting $\psi_h^*=r_hp_h^*$, $\varepsilon_h^*=r_hq_h^*$ into above equalities and by virtue of Definition 4, we conclude that, there exists a unique equilibrium point $x^*=(p^*,q^*)$ of a system (1).

Next, we shall prove to the unique equilibrium point of the system (1) is globally stable. Let $\hat{x}_h(t)=x_h(t)-x_h^*=y_h(t)+iz_h(t)$, where $y_h(t)=p_h(t)-p^*$, $z_h(t)=q_h(t)-q^*$. From (2)-(3), the error system can be obtained by

$$
\begin{aligned}
D^{\beta}y_h(t)=&-r_hy_h(t)+\sum_{l=1}^{n}(u_{hl}^{R}+\Delta u_{hl}^{R})\Big[g_l^{R}(y_l(t)\\
&+p_l^*,z_l(t)+q_l^*)-g_l^{R}(p_l^*,q_l^*)\Big]\\
&-\sum_{l=1}^{n}(u_{hl}^{I}+\Delta u_{hl}^{I})\Big[g_l^{I}(y_l(t)+p_l^*,z_l(t)+q_l^*)\\
&-g_l^{I}(p_l^*,q_l^*)\Big]+\sum_{l=1}^{n}(v_{hl}^{R}+\Delta v_{hl}^{R})\\
&\times\Big[g_l^{R}(y_l(t-\tau_l(t)))+p_l^*,z_l(t-\tau_l(t)))\\
&+q_l^*)-g_l^{R}(p_l^*,q_l^*)\Big]-\sum_{l=1}^{n}(v_{hl}^{I}+\Delta v_{hl}^{I})\\
&\times\Big[g_l^{I}(y_l(t-\tau_l(t)))+p_l^*,z_l(t-\tau_l(t)))\\
&+q_l^*)-g_l^{I}(p_l^*,q_l^*)\Big]+\sum_{l=1}^{n}(w_{hl}^{R}+\Delta w_{hl}^{R})\\
&\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\Big[g_l^{R}(y_l(s)+p_l^*,z_l(s)+q_l^*)\\
&-g_l^{R}(p_l^*,q_l^*)\Big]ds-\sum_{l=1}^{n}(w_{hl}^{I}+\Delta w_{hl}^{I})\\
&\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\Big[g_l^{I}(y_l(s)+p_l^*,z_l(s)+q_l^*)\\
&-g_l^{I}(p_l^*,q_l^*)\Big]ds,\ t\neq t_k,\ \ t\geq 0,\\
\Delta y_h(t_k)=&\ \Phi_{hk}^{R}(y_h(t_k)),\ \ k=1,2,..., \qquad (9)\\
D^{\beta}z_h(t)=&-r_hz_h(t)+\sum_{l=1}^{n}(u_{hl}^{I}+\Delta u_{hl}^{I})\Big[g_l^{R}(y_l(t)\\
&+p_l^*,z_l(t)+q_l^*)-g_l^{R}(p_l^*,q_l^*)]+\sum_{l=1}^{n}(u_{hl}^{R}\\
&+\Delta u_{hl}^{R})[g_l^{I}(y_l(t)+p_l^*,z_l(t)+q_l^*)\\
&-g_l^{I}(p_l^*,q_l^*)]+\sum_{l=1}^{n}(v_{hl}^{I}+\Delta v_{hl}^{I})\\
&\times[g_l^{R}(y_l(t-\tau_l(t)))+p_l^*,z_l(t-\tau_l(t)))\\
&+q_l^*)-g_l^{R}(p_l^*,q_l^*)]+\sum_{l=1}^{n}(v_{hl}^{R}+\Delta v_{hl}^{R})\\
&\times[g_l^{I}(y_l(t-\tau_l(t)))+p_l^*,z_l(t-\tau_l(t)))\\
&+q_l^*)-g_l^{I}(p_l^*,q_l^*)]+\sum_{l=1}^{n}(w_{hl}^{I}+\Delta w_{hl}^{I})\\
&\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)[g_l^{R}(y_l(s)+p_l^*,z_l(s)+q_l^*)\\
&-g_l^{R}(p_l^*,q_l^*)]ds+\sum_{l=1}^{n}(w_{hl}^{R}+\Delta w_{hl}^{R})\\
&\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)[g_l^{I}(y_l(s)+p_l^*,z_l(s)+q_l^*)\\
&-g_l^{I}(p_l^*,q_l^*)]ds,\ \ t\neq t_k,\ \ t\geq 0,\\
\Delta z_h(t_k)=&\ \Phi_{hk}^{I}(z_h(t_k)),\ \ k=1,2,.... \qquad (10)
\end{aligned}
$$

Construct the following Lyapunov function:

$$
\begin{aligned}
V(t)=&D^{-(1-\beta)}\Big[\sum_{h=1}^{n}|y_h(t)|+\sum_{h=1}^{n}|z_h(t)|\Big]\\
&+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_1\int_{t-\tau_h(t)}^{t}|y_h(m)|dm\\
&+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_2\int_{t-\tau_h(t)}^{t}|z_h(m)|dm\\
&+\sum_{h=1}^{n}\Theta_3\int_{-\infty}^{0}\int_{t+s}^{t}\mathbb{L}_{lh}(-s)|y_h(m)|dmds\\
&+\sum_{h=1}^{n}\Theta_4\int_{-\infty}^{0}\int_{t+s}^{t}\mathbb{L}_{lh}(-s)|z_h(m)|dmds, \qquad (11)
\end{aligned}
$$

where $\Theta_1=\sum_{l=1}^{n}\{\hat{v}_{lh}^{R}\kappa_h^{R}+\hat{v}_{lh}^{I}\kappa_h^{R}\}$, $\Theta_2=\sum_{l=1}^{n}\{\hat{v}_{lh}^{R}\kappa_h^{I}+\hat{v}_{lh}^{I}\kappa_h^{I}\}$, $\Theta_3=\sum_{l=1}^{n}\{\hat{w}_{lh}^{R}\kappa_h^{R}+\hat{w}_{lh}^{I}\kappa_h^{R}\}$, $\Theta_4=\sum_{l=1}^{n}\{\hat{w}_{lh}^{R}\kappa_h^{I}+$

$\hat{w}^I_{lh}\kappa^I_h\}$. On the other hand, from using Assumption 5 and we consider the case $t = t_k$, $k = 1, 2, 3, ...$ and $t > 0$, one has

$$\begin{aligned}
V(t_k^+) = {}& D^{-(1-\beta)}\Big[\sum_{h=1}^{n}|1-\Upsilon^R_{hk}||y_h(t_k^-)| \\
&+\sum_{h=1}^{n}|1-\Upsilon^I_{hk}||z_h(t_k^-)|\Big] \\
&+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_1\int_{t_k^+-\tau_h(t_k^+)}^{t_k^+}|y_h(m)|dm \\
&+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_2\int_{t_k^+-\tau_h(t_k^+)}^{t_k^+}|z_h(m)|dm \\
&+\sum_{h=1}^{n}\Theta_3\int_{-\infty}^{0}\int_{t_k^++s}^{t_k^+}\mathbb{L}_{lh}(-s)|y_h(m)|dm\,ds \\
&+\sum_{h=1}^{n}\Theta_4\int_{-\infty}^{0}\int_{t_k^++s}^{t_k^+}\mathbb{L}_{lh}(-s)|z_h(m)|dm\,ds \\
<{}& V(t_k^-), \\
\dot{V}(t) \leq {}& \sum_{h=1}^{n}sgn(y_h(t))D^\beta y_h(t)+\sum_{h=1}^{n}sgn(z_h(t)D^\beta z_h(t) \\
&+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_1|y_h(t)|+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_2|z_h(t)| \\
&-\sum_{h=1}^{n}\Theta_1|y_h(t-\tau_h(t))|-\sum_{h=1}^{n}\Theta_2|z_h(t-\tau_h(t))| \\
&+\sum_{h=1}^{n}\Theta_3\zeta_{lh}|y_h(t)|+\sum_{h=1}^{n}\Theta_4\zeta_{lh}|z_h(t)| \\
&-\sum_{h=1}^{n}\Theta_3\int_{-\infty}^{t}\mathbb{L}_{lh}(t-s)|y_h(s)|ds \\
&-\sum_{h=1}^{n}\Theta_4\int_{-\infty}^{t}\mathbb{L}_{lh}(t-s)|z_h(s)|ds. \qquad (12)
\end{aligned}$$

Noticing the error dynamical system (9)-(10), Assumptions 1 and 4, we can finally obtain the following inequality:

$$\begin{aligned}
&\sum_{h=1}^{n}sgn(y_h(t))D^\beta y_h(t)+\sum_{h=1}^{n}sgn(z_h(t))D^\beta z_h(t) \\
&\leq\sum_{h=1}^{n}\Big\{-r_h+\sum_{l=1}^{n}\{\hat{u}^R_{lh}\kappa^R_h+\hat{u}^I_{lh}\kappa^R_h\}|y_h(t)| \\
&\quad+\sum_{l=1}^{n}\{\hat{v}^R_{lh}\kappa^R_h+\hat{v}^I_{lh}\kappa^R_h\}|y_h(t-\tau_h(t))| \\
&\quad+\sum_{l=1}^{n}\{\hat{w}^R_{lh}\kappa^R_h+\hat{w}^I_{lh}\kappa^R_h\}\int_{-\infty}^{t}\mathbb{L}_{lh}(t-s)|y_h(s)|ds\Big\} \\
&\quad+\sum_{h=1}^{n}\Big\{-r_h+\sum_{l=1}^{n}\{\hat{u}^R_{lh}\kappa^I_h+\hat{u}^I_{lh}\kappa^I_h\}|z_h(t)| \\
&\quad+\sum_{l=1}^{n}\{\hat{v}^R_{lh}\kappa^I_h+\hat{v}^I_{lh}\kappa^I_h\}|z_h(t-\tau_h(t))| \\
&\quad+\sum_{l=1}^{n}\{\hat{w}^R_{lh}\kappa^I_h+\hat{w}^I_{lh}\kappa^I_h\}\int_{-\infty}^{t}\mathbb{L}_{lh}(t-s)|z_h(s)|ds\Big\}. \qquad (13)
\end{aligned}$$

From (13) substitute in (12), one has

$$\dot{V}(t)\leq-\Lambda\Big[\sum_{l=1}^{n}|y_h(t)|+\sum_{l=1}^{n}|z_h(t)|\Big]\ \forall\, t\in[t_{k-1},t_k), \qquad (14)$$

where $\Lambda=\min\{\Lambda_1,\Lambda_2\}$. Taking integral on both sides of (14) from t to t_{k-1}, we get

$$\begin{aligned}
V(t)\leq{}&V(t_{k-1})-\int_{t_{k-1}}^{t}\Big[\sum_{l=1}^{n}|y_h(s)|+\sum_{l=1}^{n}|z_h(s)|\Big]ds, \\
V(t)\leq{}&V(t_{k-1}^+)-\int_{t_{k-1}}^{t}\Big[\sum_{l=1}^{n}|y_h(s)|+\sum_{l=1}^{n}|z_h(s)|\Big]ds \\
\leq{}&V(t_{k-1}^-)-\int_{t_{k-1}}^{t}\Big[\sum_{l=1}^{n}|y_h(s)|+\sum_{l=1}^{n}|z_h(s)|\Big]ds \\
\leq{}&...\leq V(t_0)-\int_{t_0}^{t}\Big[\sum_{l=1}^{n}|y_h(s)|+\sum_{l=1}^{n}|z_h(s)|\Big]ds.
\end{aligned}$$

Thus, we have

$$V(t)+\int_{t_0}^{t}\Big[\sum_{l=1}^{n}|y_h(s)|+\sum_{l=1}^{n}|z_h(s)|\Big]ds\leq V(t_0). \qquad (15)$$

Let $H(t)=\Big[\sum_{l=1}^{n}|y_h(s)|+\sum_{l=1}^{n}|z_h(s)|\Big]$. It easy to we can obtain, $\int_{t_0}^{t}H(s)ds$ has finite limit and $H(t)$ is bounded, it follows that $y_h(t)$ and $z_h(t)$ is also bounded. According to Eq. (9)-(10) and based on the previous description, there exists a scalars ς such that $|D^\beta H(t)|\leq\varsigma$. Next we will prove to $H(t)$ is uniformly continuous on the basis of Barbalat's lemma (2.7). For $t_0\leq t_1\leq t_2$, pointed has

$$|H(t_1)-H(t_2)|\leq2\varsigma\frac{(t_2-t_1)^\beta}{\Gamma(\beta+1)}, \qquad (16)$$

where $|t_2-t_1|<\theta(\varepsilon)=[\varepsilon\frac{\Gamma(\beta+1)}{2\varsigma}]^{\frac{1}{\beta}}$. By virtue the definition of uniformly continuous, $H(t)$ is uniformly continuous.

According to Barbalat's lemma (2.7), we can get

$$\lim_{t\to\infty}\Big[\|y(t)\|+\|z(t)\|\Big]=0.$$

Therefore, the equilibrium point $x^*=(p^*,q^*)$ of the system (1) is globally stable.

Remark 1: In [21], author addressed the asymptotic stability of delayed fractional-order neural networks with impulsive effects. In [22], asymptotic stability of delayed fractional-order BAM neural networks with impulsive effects were studied. In that two results, by applying Riemann-Liouville definitions and suitable Lyapunov approach, the equilibrium point of global asymptotic stability conditions was inspected. So, the main point in this work is to apply complex valued properties, parameter uncertainty and infinite time distributed delays.

3.2. Synchronization condition under adaptive feedback control

In this section, a novel sufficient conditions are established to ensure the global synchronization of fractional order UCNNs based on the adaptive feedback control. Next we shall consider the complex valued response system as follows:

$$\begin{aligned}D^{\beta}\tilde{x}_h(t) &= -r_h\tilde{x}_h(t) + \sum_{l=1}^{n}(u_{hl}+\Delta u_{hl})g_l(\tilde{x}_l(t))\\ &\quad + \sum_{l=1}^{n}(v_{hl}+\Delta v_{hl})g_l(\tilde{x}_l(t-\tau_l(t)))\\ &\quad + \sum_{l=1}^{n}(w_{hl}+\Delta w_{hl})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\\ &\quad \times g_l(\tilde{x}_l(s))ds + I_h + m_h(t),\ \ t\neq t_k,\ \ t\geq 0,\\ \Delta\tilde{x}_l(t_k) &= \tilde{x}_l(t_k^{+}) - \tilde{x}_l(t_k^{-}) = \Phi_{hk}(\tilde{x}_l(t_k)),\ k=1,...,\end{aligned} \tag{17}$$

where $\tilde{x}_h(t) = (\tilde{x}_1(t),...,\tilde{x}_n(t)) \in \mathbb{C}^n$ denotes the state variable of the drive response system and $m_h(t) = (m_1(t),...,m_n(t))$ denotes new designed controllers. All others are similar to defined in (1). The initial values associated with the system (17) is $D^{-(1-\beta)}\tilde{x}_h(t) = \tilde{\rho}_h(t) = (\tilde{\rho}_1(t),....,\tilde{\rho}_n(t))^T \in C((-\infty,0],\mathbb{C}^n)$.
Denote $\tilde{x}_h(t) = \tilde{p}_h(t) + i\tilde{q}_h(t)$, $g_h(\tilde{x}_h(t)) = g_h^R(\tilde{p}_h(t), \tilde{q}_h(t)) + ig_h^I(\tilde{p}_h(t),\tilde{q}_h(t))$, where $\tilde{p}_h(t)$, $\tilde{q}_h(t) \in \mathbb{R}$, $g_h^R(\tilde{p}_h(t), \tilde{q}_h(t))$, $g_h^I(\tilde{p}_h(t), \tilde{q}_h(t)) : \mathbb{R}^2 \to \mathbb{R}$. The adaptive linear feedback controller is defined as

$$\begin{cases} m_h^R(t) = -\lambda_h(t)[y_h(t)],\ \ m_h^I(t) = -\upsilon_h(t)[z_h(t)],\\ \dot{\lambda}_h(t) = \dfrac{-\xi_h\varepsilon_h|y_h(t)| - \xi_h\varepsilon_h|y_h(t-\tau_h(t))|}{\lambda_h(t)}\\ \qquad + \xi_h|y_h(t)|,\\ \dot{\upsilon}_h(t) = \dfrac{-\delta_h\phi_h|z_h(t)| - \delta_h\psi_h|z_h(t-\tau_h(t))|}{\upsilon_h(t)}\\ \qquad + \delta_h|z_h(t)|,\end{cases} \tag{18}$$

where $\lambda_h(t)$, $\upsilon_h(t)$ denotes the adaptive coupling strengths, ε_h, ε_h, ϕ_h, ψ_h, ξ_h and δ_h are arbitrary positive scalars.
Let $\hat{x}_h(t) = \tilde{x}_h(t) - x_h(t) = y_h(t) + iz_h(t)$, where $y_h(t) = \tilde{p}_h(t) - p_h(t)$, $z_h(t) = \tilde{q}_h(t) - q_h(t)$. Based on drive-response systems (1) and (17), the error dynamical system can be expressed by the following form:

$$\begin{aligned}D^{\beta}y_h(t) &= -(r_h+\lambda_h(t))y_h(t) + \sum_{l=1}^{n}(u_{hl}^R+\Delta u_{hl}^R)\\ &\quad\times\Big[g_l^R(y_l(t)+p_l(t),z_l(t)+q_l(t))\\ &\quad - g_l^R(p_l(t),q_l(t))\Big] - \sum_{l=1}^{n}(u_{hl}^I+\Delta u_{hl}^I)\\ &\quad\times\Big[g_l^I(y_l(t)+p_l(t),z_l(t)+q_l(t))\\ &\quad - g_l^I(p_l(t),q_l(t))\Big] + \sum_{l=1}^{n}(v_{hl}^R+\Delta v_{hl}^R)\\ &\quad\times\Big[g_l^R(y_l(t-\tau_l(t))+p_l(t-\tau_l(t)),\\ &\quad z_l(t-\tau_l(t))+q_l(t-\tau_l(t)))\\ &\quad - g_l^R(p_l(t-\tau_l(t)),q_l(t-\tau_l(t)))\Big]\\ &\quad - \sum_{l=1}^{n}(v_{hl}^I+\Delta v_{hl}^I)\Big[g_l^I(y_l(t-\tau_l(t))\\ &\quad + p_l(t-\tau_l(t)),z_l(t-\tau_l(t))\\ &\quad + q_l(t-\tau_l(t))) - g_l^I(p_l(t-\tau_l(t)),\\ &\quad q_l(t-\tau_l(t)))\Big] + \sum_{l=1}^{n}(w_{hl}^R+\Delta w_{hl}^R)\\ &\quad\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\Big[g_l^R(y_l(s)+p_l(s),z_l(s)\\ &\quad + q_l(s)) - g_l^R(p_l(s),q_l(s))\Big]ds\\ &\quad - \sum_{l=1}^{n}(w_{hl}^I+\Delta w_{hl}^I)\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\\ &\quad\times\Big[g_l^I(y_l(s)+p_l(s),z_l(s)+q_l(s))\\ &\quad - g_l^I(p_l(s),q_l(s))\Big]ds,\ t\neq t_k,\ t\geq 0,\\ \Delta y_h(t_k) &= \Phi_{hk}^R(y_h(t_k)),\ \ k=1,2,...,\end{aligned} \tag{19}$$

and

$$\begin{aligned}D^{\beta}z_h(t) &= -(r_h+\upsilon_h(t))z_h(t) + \sum_{l=1}^{n}(u_{hl}^I+\Delta u_{hl}^I)\\ &\quad\times\Big[g_l^R(y_l(t)+p_l(t),z_l(t)+q_l(t))\\ &\quad - g_l^R(p_l(t),q_l(t))\Big] + \sum_{l=1}^{n}(u_{hl}^R+\Delta u_{hl}^R)\\ &\quad\times\Big[g_l^I(y_l(t)+p_l(t),z_l(t)+q_l(t))\\ &\quad - g_l^I(p_l(t),q_l(t))\Big] + \sum_{l=1}^{n}(v_{hl}^I+\Delta v_{hl}^I)\\ &\quad\times\Big[g_l^R(y_l(t-\tau_l(t))+p_l(t-\tau_l(t)),\\ &\quad z_l(t-\tau_l(t))+q_l(t-\tau_l(t)))\\ &\quad - g_l^R(p_l(t-\tau_l(t)),q_l(t-\tau_l(t)))\Big]\\ &\quad + \sum_{l=1}^{n}(v_{hl}^R+\Delta v_{hl}^R)\Big[g_l^I(y_l(t-\tau_l(t))\\ &\quad + p_l(t-\tau_l(t)),z_l(t-\tau_l(t))\\ &\quad + q_l(t-\tau_l(t))) - g_l^I(p_l(t-\tau_l(t)),\\ &\quad q_l(t-\tau_l(t)))\Big] + \sum_{l=1}^{n}(w_{hl}^I+\Delta w_{hl}^I)\\ &\quad\times\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\Big[g_l^R(y_l(s)+p_l(s),z_l(s)\\ &\quad + q_l(s)) - g_l^R(p_l(s),q_l(s))\Big]ds\end{aligned}$$

$$+\sum_{l=1}^{n}(w_{hl}^{R}+\Delta w_{hl}^{R})\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\times\Big[g_{l}^{I}(y_{l}(s)+p_{l}(s),z_{l}(s)+q_{l}(s))-g_{l}^{I}(p_{l}(s),q_{l}(s))\Big]ds,\ t\neq t_k,\ t\geq 0,$$

$$\Delta z_h(t_k)=\Phi_{hk}^{I}(z_h(t_k)),\quad k=1,2,\ldots. \tag{20}$$

□

Theorem 2: Suppose Assumption 1-5 hold. Then the drive-response system (1) and (17) can be globally robust synchronized under the adaptive controller (18) if $\Lambda_1>0$ and $\Lambda_2>0$, where

$$\Lambda_1=\min_{1\leq h\leq n}\Big\{r_h+\varepsilon_h+\varepsilon_h-\sum_{l=1}^{n}\{\hat{u}_{lh}^{R}+\hat{u}_{lh}^{I}\}\kappa_h^{R}-\sum_{l=1}^{n}\frac{\{\hat{v}_{lh}^{R}+\hat{v}_{lh}^{I}\}\kappa_h^{R}}{1-\hat{\tau}}-\sum_{l=1}^{n}\{\hat{w}_{lh}^{R}+\hat{w}_{lh}^{I}\}\kappa_h^{R}\zeta_{lh}\Big\},$$

$$\Lambda_2=\min_{1\leq h\leq n}\Big\{r_h+\phi_h+\psi_h-\sum_{l=1}^{n}\{\hat{u}_{lh}^{R}\kappa_h^{I}+\hat{u}_{lh}^{I}\kappa_h^{I}\}-\sum_{l=1}^{n}\frac{\{\hat{v}_{lh}^{R}+\hat{v}_{lh}^{I}\}\kappa_h^{I}}{1-\hat{\tau}}-\sum_{l=1}^{n}\{\hat{w}_{lh}^{R}+\hat{w}_{lh}^{I}\}\kappa_h^{I}\zeta_{lh}\Big\},$$

$$\hat{u}_{lh}^{\alpha}=|u_{lh}^{\alpha}|+|\mu_{lh}^{\alpha}|,\ \hat{v}_{lh}^{\alpha}=|v_{lh}^{\alpha}|+|\vartheta_{lh}^{\alpha}|,\ \hat{w}_{lh}^{\alpha}=|w_{lh}^{\alpha}|+|\omega_{lh}^{\alpha}|,\ \kappa_h^{R}=\kappa_h^{RR}+\kappa_h^{IR},\ \kappa_h^{I}=\kappa_h^{IR}+\kappa_h^{II},\ (\alpha=R,I),$$

Proof: Construct the following Lyapunov functions:

$$\begin{aligned}V(t)=&D^{-(1-\beta)}\Big[\sum_{h=1}^{n}|y_h(t)|+\sum_{h=1}^{n}|z_h(t)|\Big]+\sum_{h=1}^{n}\frac{1}{2\xi_h}[\lambda_h(t)]^2+\sum_{h=1}^{n}\frac{1}{2\delta_h}[\upsilon_h(t)]^2\\&+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_1\int_{t}^{t-\tau_h(t)}|y_h(m)|dm+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_2\int_{t}^{t-\tau_h(t)}|z_h(m)|dm\\&+\sum_{h=1}^{n}\Theta_3\int_{-\infty}^{0}\int_{t+s}^{t}\mathbb{L}_{lh}(-s)|y_h(m)|dmds+\sum_{h=1}^{n}\Theta_4\int_{-\infty}^{0}\int_{t+s}^{t}\mathbb{L}_{lh}(-s)|z_h(m)|dmds,\end{aligned}\tag{21}$$

where

$$\Theta_1=\varepsilon_h-\sum_{l=1}^{n}\{\hat{v}_{lh}^{R}+\hat{v}_{lh}^{I}\}\kappa_h^{R},\quad \Theta_3=\sum_{l=1}^{n}\{\hat{w}_{lh}^{R}+\hat{w}_{lh}^{I}\}\kappa_h^{R},$$

$$\Theta_2=\psi_h-\sum_{l=1}^{n}\{\hat{v}_{lh}^{R}+\hat{v}_{lh}^{I}\}\kappa_h^{I},\quad \Theta_4=\sum_{l=1}^{n}\{\hat{w}_{lh}^{R}+\hat{w}_{lh}^{I}\}\kappa_h^{I}.$$

On the other hand, according to Assumption 5 and we consider the case $t=t_k,\ k=1,2,3,\ldots$ and $t>0$, one has

$$\begin{aligned}V(t_k^{+})<&D^{-(1-\beta)}\Big[\sum_{h=1}^{n}|y_h(t_k^{-})|+\sum_{h=1}^{n}|z_h(t_k^{-})|\Big]+\sum_{h=1}^{n}\frac{1}{2\xi_h}[\lambda_h(t_k^{+})]^2+\sum_{h=1}^{n}\frac{1}{2\delta_h}[\upsilon_h(t_k^{+})]^2\\&+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_1\int_{t_k^{+}}^{t_k^{+}-\tau_h(t_k^{+})}|y_h(m)|dm+\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_2\int_{t_k^{+}}^{t_k^{+}-\tau_h(t_k^{+})}|z_h(m)|dm\\&+\sum_{h=1}^{n}\Theta_3\int_{-\infty}^{0}\int_{t_k^{+}+s}^{t_k^{+}}\mathbb{L}_{lh}(-s)|y_h(m)|dmds+\sum_{h=1}^{n}\Theta_4\int_{-\infty}^{0}\int_{t_k^{+}+s}^{t_k^{+}}\mathbb{L}_{lh}(-s)|z_h(m)|dmds\\=&V(t_k^{-}),\end{aligned}$$

$$\begin{aligned}\dot{V}(t)\leq&\sum_{h=1}^{n}sgn(y_h(t))D^{\beta}y_h(t)+\sum_{h=1}^{n}sgn(z_h(t))D^{\beta}z_h(t)\\&-\sum_{h=1}^{n}[\varepsilon_h|y_h(t)|+\varepsilon_h|y_h(t-\tau_h(t))|+\lambda_h(t)\times|y_h(t)|\Big]-\sum_{h=1}^{n}\Big[\phi_h|z_h(t)|+\psi_h|z_h(t-\tau_h(t))|)\\&-\upsilon_h(t)|z_h(t)|\Big]+\sum_{h=1}^{n}\Theta_1|y_h(t-\tau_h(t))|-\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_1|y_h(t)|+\sum_{h=1}^{n}\Theta_2|z_h(t-\tau_h(t))|\\&-\frac{1}{(1-\hat{\tau})}\sum_{h=1}^{n}\Theta_2|z_h(t)|+\sum_{h=1}^{n}\Theta_3\zeta_{lh}|y_h(t)|+\sum_{h=1}^{n}\Theta_4\zeta_{lh}|z_h(t)|-\sum_{h=1}^{n}\Theta_3\int_{-\infty}^{t}\mathbb{L}_{lh}(t-s)\\&\times|y_h(s)|ds-\sum_{h=1}^{n}\Theta_4\int_{-\infty}^{t}\mathbb{L}_{lh}(t-s)|z_h(s)|ds.\end{aligned}\tag{22}$$

According to Assumptions 1, 4 and by proceeding the similar way of Theorem 1, we can get

$$\begin{aligned}&\sum_{h=1}^{n}sgn(y_h(t))D^{\beta}y_h(t)\\&\leq\sum_{h=1}^{n}\Big\{-[r_h+\lambda_h(t)]|y_h(t)|+\sum_{l=1}^{n}\hat{u}_{hl}^{R}\Big[\kappa_l^{RR}|y_l(t)|+\kappa_l^{RI}|z_l(t)|\Big]+\sum_{l=1}^{n}\hat{u}_{hl}^{I}\times\Big[\kappa_l^{IR}|y_l(t)|+\kappa_l^{II}|z_l(t)|\Big]\\&+\sum_{l=1}^{n}\hat{v}_{hl}^{R}\Big[\kappa_l^{RR}|y_l(t-\tau_l(t))|+\kappa_l^{RI}|z_l(t-\tau_l(t))|\Big]+\sum_{l=1}^{n}\hat{v}_{hl}^{I}|\Big[\kappa_l^{IR}|y_l(t-\tau_l(t))|+\kappa_l^{II}|z_l(t-\tau_l(t))|\Big]\\&+\sum_{l=1}^{n}\hat{w}_{hl}^{R}\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\Big[\kappa_l^{RR}|y_l(s)|+\kappa_l^{RI}|z_l(s)|\Big]ds\end{aligned}$$

$$+\sum_{l=1}^{n}\hat{w}_{hl}^{I}\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\Big[\kappa_l^{IR}|y_l(s)|+\kappa_l^{II}|z_l(s)|\Big]ds\Big\}, \quad (23)$$

$$\begin{aligned}&\sum_{h=1}^{n}sgn(z_h(t))D^{\beta}z_h(t)\\ &\leq\sum_{h=1}^{n}\Big\{-[r_h+\upsilon_h(t)]|z_h(t)|+\sum_{l=1}^{n}\hat{u}_{hl}^{I}\Big[\kappa_l^{RR}|y_l(t)|\\ &\quad+\kappa_l^{RI}|z_l(t)|\Big]+\sum_{l=1}^{n}\hat{u}_{hl}^{R}\Big[\kappa_l^{IR}|y_l(t)|+\kappa_l^{II}|z_l(t)|\Big]\\ &\quad+\sum_{l=1}^{n}\hat{v}_{hl}^{I}\Big[\kappa_l^{RR}|y_l(t-\tau_l(t))|+\kappa_l^{RI}|z_l(t-\tau_l(t))|\Big]\\ &\quad+\sum_{l=1}^{n}\hat{v}_{hl}^{R}\Big[\kappa_l^{IR}|y_l(t-\tau_l(t))|+\kappa_l^{II}|z_l(t-\tau_l(t))|\Big]\\ &\quad+\sum_{l=1}^{n}\hat{w}_{hl}^{I}\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\Big[\kappa_l^{RR}|y_l(s)|+\kappa_l^{RI}|z_l(s)|\Big]ds\\ &\quad+\sum_{l=1}^{n}\hat{w}_{hl}^{R}\int_{-\infty}^{t}\mathbb{L}_{hl}(t-s)\Big[\kappa_l^{IR}|y_l(s)|+\kappa_l^{II}|z_l(s)|\Big]ds\Big\}. \end{aligned} \quad (24)$$

From (23) and (24) to (22), it yields that

$$\dot{V}(t)\leq-\Lambda\Big[\sum_{l=1}^{n}|y_h(t)|+\sum_{l=1}^{n}|z_h(t)|\Big]\ \ \forall t\in[t_{k-1},t_k).$$

Where $\Lambda=\min\{\Lambda_1,\Lambda_2\}$. By proceeding the similar way to the proof of Theorem 1 and we conclude that $\lim_{t\to\infty}\Big[\|y(t)\|+\|z(t)\|\Big]=0$. Therefore, the drive-response systems are globally synchronized based on the controller (18). □

Remark 2: In [1], authors proposed Caputo sense of synchronization conditions for fractional order delayed complex valued neural networks and derived some synchronization criteria. In [30], authors studied the complex projective synchronization of integer order complex-valued neural network with structure identification. In above mentioned papers, impulses, infinite time distributed delay, parameter uncertainty was ignored, while these parameters are considered in in this paper by employing the adaptive feedback control and R-L properties. When the state variable, connection weights, activations are assumed to real values, the problem turn to robust adaptive synchronization of fractional order real valued neural networks with mixed time varying delays and impulses.

3.3. Numerical examples

In this section, two numerical simulations are presented to prove the effectiveness of the proposed main results.

Example 1: In system (1), choose $\beta=0.97$, $x(t)=(x_1(t),\ x_2(t))^T$, $x_l(t)=p_l(t)+iq_l(t)$, $\tau(t)=\frac{\exp(t)}{1+\exp(t)}$, $\Upsilon_{1k}^R=\Upsilon_{2k}^R=1.4$, $\Upsilon_{1k}^I=\Upsilon_{2k}^I=0.9$, $\dot{\tau}_j(t)\leq\frac{1}{5}<\hat{\tau}=$

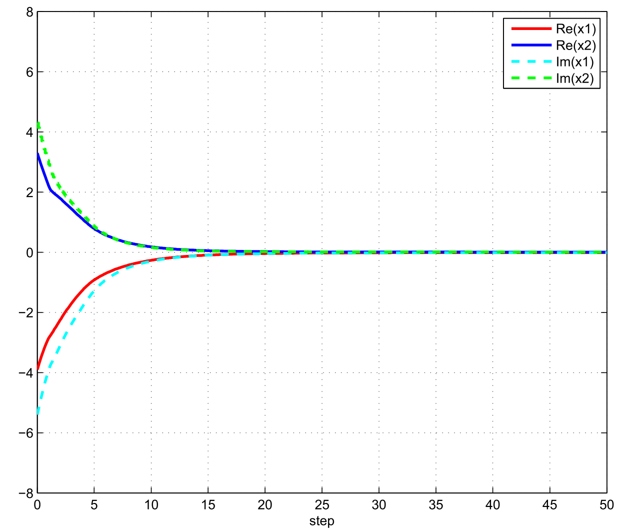

Fig. 1. Transient states of equilibrium point of the system in Example 1.

0.5, $I_h=\frac{1}{7}(0.7+1.2i,0.6i)^T$, $f_l(x_l(t))=\frac{1-\exp(-p_l(t))}{1+\exp(-p_l(t))}+i\frac{1}{1+\exp(-q_l(t))}$, $(l=1,2)$, and

$$R=\begin{bmatrix}7&0\\0&9\end{bmatrix},\ U=\frac{1}{7}\begin{bmatrix}1-2i&-0.5+3i\\-2+7i&0.7+5i\end{bmatrix},$$
$$V=\frac{1}{7}\begin{bmatrix}1+3i&-0.2+5i\\0.4+2i&-0.9-1.2i\end{bmatrix},$$
$$W=\frac{1}{9}\begin{bmatrix}-1.8+2i&0.8+1.4i\\0.3+0.9i&-1-0.8i\end{bmatrix},$$
$$\Delta U=(0.1\sin(t)-0.3i\cos(t))\begin{bmatrix}1&1\\1&1\end{bmatrix},$$
$$\Delta V=(0.4\sin(t)-0.2i\cos(t))\begin{bmatrix}1&1\\1&1\end{bmatrix},$$
$$\Delta W=(0.3\sin(t)-0.6i\cos(t))\begin{bmatrix}1&1\\1&1\end{bmatrix}.$$

From Assumption 4, we can choose the uncertain parameter values given by $\mu_{hl}^R=0.1$, $\mu_{hl}^I=-0.3$, $\vartheta_{hl}^R=0.4$, $\vartheta_{hl}^I=-0.2$, $\omega_{hl}^R=0.3$ and $\omega_{hl}^I=-0.6$, while we select $\kappa_h^{RR}=0.75$, $\kappa_h^{II}=1$, $\kappa_h^{RI}=\kappa_h^{IR}=0$ and $\zeta_{lh}=1$ for h, $l=1,2$, then Assumptions 1-5 are satisfied. After the simple manipulation, it is easy to derive that the conditions of Theorem 1 are $5.238<7$, $\Lambda_1=0.6765>0$ and $\Lambda_2=0.721>0$. Thus, the equilibrium point of the FOUCNNs system (1) is globally robust stable, which is depicted in Fig. 1.

Example 2: In drive system (1), choose $\beta=0.98$, $x(t)=(x_1(t),x_2(t))^T$, $x_l(t)=p_l(t)+q_l(t)$, $\tau(t)=\frac{\exp(t)}{1+\exp(t)}$, $\Upsilon_{1k}^R=0.8$, $\Upsilon_{2k}^R=1.6$, $\Upsilon_{1k}^I=1.3$, $\Upsilon_{2k}^I=0.9$, $\dot{\tau}_j(t)\leq\frac{1}{4}<\hat{\tau}=0.4$, $I_h=(0.8,1.7+0.6i)^T$, $f_l(x_l(t))=\frac{1-\exp(-p_l(t))}{1+\exp(-p_l(t))}+$

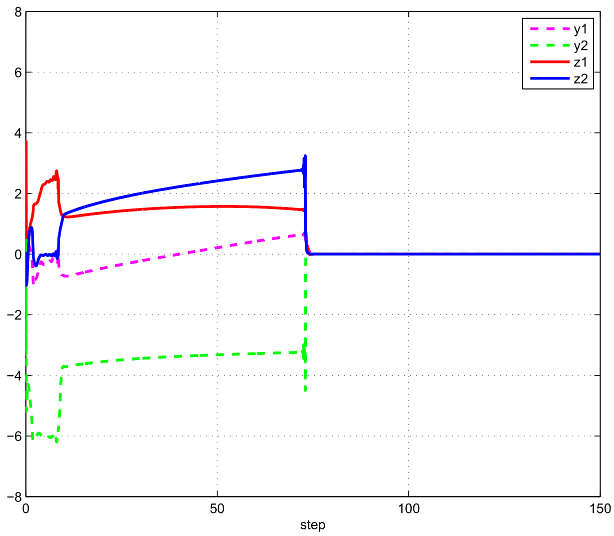

Fig. 2. Transient states of drive-response synchronization errors in Example 2.

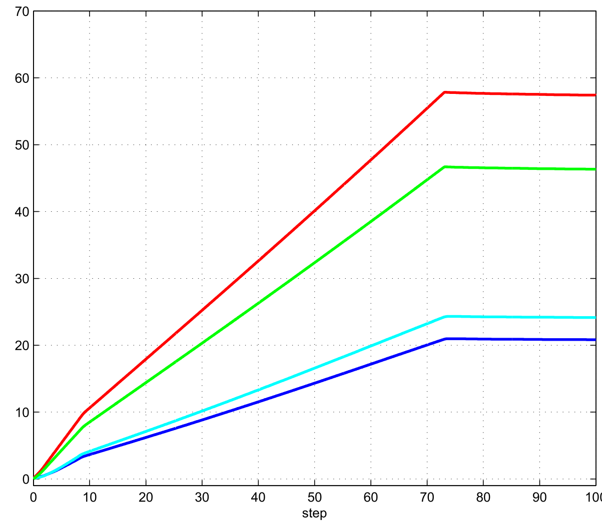

Fig. 3. Time response of $\lambda_h(t)$ and $\upsilon_h(t)$ in Example 2.

$i\frac{1}{1+\exp(-q_l(t))}$, $(l=1,2)$,

$$R=\begin{bmatrix}3.5 & 0\\ 0 & 4\end{bmatrix},\ U=\begin{bmatrix}1+2i & -1.5+2i\\ 0.5+i & 1.2+0.8i\end{bmatrix},$$
$$V=\begin{bmatrix}-1.3+i & 2.2+1.3i\\ 0.8+2i & 0.5+1.2i\end{bmatrix},$$
$$W=\begin{bmatrix}3+1.3i & -0.9-2i\\ 0.7+1.5i & 1.7+2.1i\end{bmatrix},$$
$$\Delta U=(0.4\sin(t)-0.2i\cos(t))\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},$$
$$\Delta V=(0.9\sin(t)-2.1i\cos(t))\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},$$
$$\Delta W=(1.4\sin(t)-1.9i\cos(t))\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}.$$

From Assumption 4, we can choose the uncertain parameter values are $\mu_{hl}^R=0.4$, $\mu_{hl}^I=-1.2$, $\vartheta_{hl}^R=0.9$, $\vartheta_{hl}^I=-2.1$, $\omega_{hl}^R=1.4$ and $\omega_{hl}^I=-1.9$, while we select $\kappa_h^{RR}=0.25$, $\kappa_h^{II}=0.5$, $\kappa_h^{RI}=\kappa_h^{IR}=0$ and $\zeta_{lh}=1.5$ for $h,\ l=1,\ 2$. Assumptions 1-5 hold. In controller (18), we choose the control gains are $\lambda_h(0)=\upsilon_h(0)=0.1$ $(h=1,2)$, $\varepsilon_1=4.25$, $\varepsilon_2=3.5$, $\varepsilon_1=4.7$, $\varepsilon_2=3.8$, $\phi_1=9.1$, $\phi_2=7.6$, $\psi_1=8.2$, $\psi_2=8.8$. Thus the conditions of Theorem 2, it is easily to verified that $\Lambda_1=1.8>0$ and $\Lambda_2=1.375>0$. The simulations of the synchronization errors are showed in Fig. 3. Therefore the drive-response FOUCNNs systems are achieved to globally robust adaptive synchronization and the adaptive coupling strengths converges to some positive scalars, which confirm the effectiveness of Theorem 2.

4. CONCLUSIONS

In this article, Robust adaptive synchronization of fractional order complex valued neural networks with mixed varying delays and impulses is investigated. By employing the Banach contraction mapping principle, the existence, uniqueness and global stability of equilibrium point for such UCNNs are established. A new sufficient criteria ensuring the robust synchronization of UCNNs have been investigated based on the valid adaptive control, Barbalat's lemma and the application of robust stability principle. At the end, we have presented two numerical simulations to provide the obtained theoretical main results. Furthermore, the adaptive state feedback control approach proposed in this paper can be applied for solving global projective synchronization of complex-valued competitive-type neural networks, and we will consider the interesting issue for future work.

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