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Unscented Kalman Filtering for Nonlinear State Estimation with Correlated Noises and Missing Measurements

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Abstract: The unscented Kalman filtering problem is investigated for a class of nonlinear discrete stochastic systems subject to correlated noises and missing measurements. Here, a random variable obeying Bernoulli distribution with known conditional probability is introduced to depict the phenomenon of missing measurements occurring in a stochastic way. Due to taking the correlation of noises into account, a one-step predictor is designed by applying the innovative analysis and unscented transformation approach. And then, based on one-step predictor and the minimum mean square error principle, a new unscented Kalman filtering algorithm is proposed such that, for the correlated noises and missing measurements, the filtering error is minimized. By solving the recursive matrix equation, the filter gain matrices and the error covariance matrices can be obtained and the proposed results can be easily verified by using the standard numerical software. We finally provide a numerical example to show the performance of the proposed approach.

Keywords: Correlated noises, minimum mean square error, missing measurements, nonlinear discrete stochastic systems, unscented transformation.

1. INTRODUCTION

In the past few decades, the optimal estimation or filtering problems for stochastic systems were extensively investigated due to its wide application in a variety of directions such as tracking systems, communication and signal processing [1–6]. It is well known that Kalman first provided a new approach (Kalman filter) to investigate the estimate problem in [7] for the linear discrete stochastic systems by employing the principle of minimum mean square error and the projection theory. However, many practical systems are nonlinear and a large number of state estimation problems for the nonlinear systems can't be solved by applying traditional Kalman filter. Hence, unscented Kalman filter, as a new method for nonlinear filtering, was constructed for nonlinear systems in [8]. The unscented Kalman filtering is the combination of unscented transformation approach and standard Kalman filtering. The basic idea of unscented Kalman filtering algorithm is to make use of unscented transformation approach to approximate the mean and covariance in order to satisfy the minimum mean square error principle and the unscented Kalman filter can get higher accuracy than the extended Kalman filter. Thus, the unscented Kalman filter is considered to be an ideal alternative to the traditional extended Kalman filter method and the unscented Kalman filtering algorithm was also attracted wide attention.

Due to the certain factors of unreliable observations of the communication network, the phenomena of the correlation of noises and missing measurements are inevitable in many industrial process systems [9–26]. Generally speaking, the missing measurements are characterized by the Markov-chain or by the Bernoulli distributed random variable. Bernoulli distributed random variable is commonly utilized to characterize the phenomenon of missing measurements. For example, in [15], the estimation problem was considered for time-varying complex networks with missing measurements. The phenomenon of missing measurements in the measurement was certain probability of occurrence in the time-varying complex networks systems and the phenomenon of missing measurements was been described by the Bernoulli distributed random variable taking on values of 0 and 1. When Bernoulli distributed random variable is 1, it stands for that the sensor occurs the missing measurements; else, it represents that the sensor receives the data successfully. Therefore, the accuracy of system state estimation can be improved by reducing the impact of correlated noise and missing data. Over the past few years, a large number of efforts were made to solve the problems of the optimal filtering for the

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systems with correlated noises and missing measurements [21, 22]. More concretely, the problem of state estimation was solved in [23] for a class of linear discrete-time stochastic systems subject to missing data and correlated noises, where the estimators are unbiased and the estimation error covariances are minimized. Based on the state augmentation approach and the projection theory, the state estimation problem was investigated in [24] for linear uncertain systems with correlated noises and incomplete measurements. In [25], by using the measurement difference method, a recursive filtering algorithm was proposed for discrete-time linear systems with fading measurement and time-correlated channel noise. Based on the same approach as in [24], the optimal estimation problem was researched in [26] for a class of discrete-time stochastic systems subject to finite-step auto-correlated process noises, one-step randomly sensor delayed and data dropout. Compared with the existing results, the model in [26] can reflect the practical systems comprehensively and the system state was estimated more effective.

It is well known that the existence of the nonlinear phenomena would reduce the accuracy of filtering estimation [27–31]. Hence, it is necessary to handle the problem of the nonlinearity and a great number of results were presented for the nonlinear filtering problem of discrete stochastic systems. To mention just a few, in [32–34], by applying the unscented transformation approach, the unscented Kalman filter was designed in the sense of minimum mean square error for nonlinear discrete stochastic systems with random time delay observation. In [35], the problem of UKF-based nonlinear filtering was studied for general nonlinear systems over a wireless sensor network with fading channel and the stochastic stability analysis was investigated for unscented Kalman filtering algorithm. By using the unscented transformation approach, in [36], a new unscented Kalman filter was designed for unreliable communication networks with Markovian packet dropouts. The event-triggered nonlinear filtering problem was investigated in [37] for nonlinear dynamic systems over a wireless sensor network with packet dropout. In [41], the nonlinear filtering problem was addressed for a class of nonlinear discrete time stochastic systems with missing measurements by applying the extended and unscented Kalman filtering approach, respectively and it showed that the unscented Kalman filter is more effective than the extended Kalman filter. However, to the best of authors' knowledge, these researches do not pay much attention to the problem of the nonlinear filtering for nonlinear discrete stochastic systems subject to correlated noises and missing data in the measurement.

Based on the above discussion, in our paper, the purpose is to discuss the nonlinear filtering problem for discrete stochastic systems with correlated noises and missing measurements. The correlated noises and missing measurements exist in the system due to the certain unreliable factors. Based on the projection theory and the unscented transformation approach, the unscented Kalman filter is designed which can address the effects of the correlated noises and missing measurements in a unified framework. We can recursively compute the filter gain matrices and the error covariance matrices by using the new algorithm and Matlab software. Finally, we give a numerical example to verify the performance of the proposed filtering algorithm. The contribution of this paper: 1). The system model is extended from linear system to nonlinear system. 2). We make first attempt to propose the nonlinear filter for systems subject to correlated noises and missing measurements. 3). A new recursive algorithm is established to obtain the optimal nonlinear filter which is suitable for online applications.

Notation: The symbols used in the paper are standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. A^T represents the transpose of a matrix A. $\mathbb{E}\{x\}$ is the expectation of the random variable x. The identity matrix and the zero matrix are expressed by I and 0 with appropriate dimensions, respectively. δ_{k-j} is the Kronecker delta function. If $k=j$, then $\delta_{k-j}=1$; otherwise, $\delta_{k-j}=0$. If the dimensions of the matrices are not definitely stated, they are considered to be well-matched for algebraic operations.

2. PROBLEM FORMULATION

The above arguments are reflected in the following nonlinear discrete stochastic systems with correlated noises and missing measurements:

$$x_{k+1} = f_k(x_k) + \omega_k, \tag{1}$$
$$z_k = \lambda_k h_k(x_k) + \nu_k, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $z_k \in \mathbb{R}^m$ is the measured output, $f_k(\cdot): \mathbb{R}^n \to \mathbb{R}^n$ and $h_k(\cdot): \mathbb{R}^n \to \mathbb{R}^m$ are known nonlinear functions. $\omega_k \in \mathbb{R}^n$ and $\nu_k \in \mathbb{R}^m$ are correlated Gaussian white noises satisfying the following equations:

$$\begin{aligned}
&\mathbb{E}\{\omega_k\} = 0, \quad Cov(\omega_k, \omega_k) = Q_k = Q_k^T \geq 0,\\
&\mathbb{E}\{\nu_k\} = 0, \quad Cov(\nu_k, \nu_k) = R_k = R_k^T > 0,\\
&Cov(\omega_k, \nu_j) = S_k \delta_{k-j}.
\end{aligned}$$

The random variable λ_k, which describes the phenomenon of missing measurements, obeys the Bernoulli distribution and has the following statistical properties:

$$\begin{aligned}
&\text{Prob}\{\lambda_k = 1\} = \mathbb{E}\{\lambda_k\} = \alpha,\\
&\text{Prob}\{\lambda_k = 0\} = 1 - \mathbb{E}\{\lambda_k\} = 1 - \alpha,
\end{aligned}$$

where $\alpha \in [0,1]$ is a known scalar. In our paper, we assume that λ_k and other noise signals are mutually independent.

Remark 1: In the model (2), if $\lambda_k = 1$, $z_k = h_k(x_k) + \nu_k$, which represents that the sensor receives the data at time instant k successfully; if $\lambda_k = 0$, $z_k = v_k$, which stands for that the sensor receives the noises of the time instant k, i.e., the sensor occurs the phenomenon of missing data. The same description method was widely utilized in [9, 11, 13, 18].

The purpose of this paper is to design the unscented Kalman filter for nonlinear discrete stochastic systems (1)-(2) based on the minimum mean square error principle and the observation sequence $\{z_1, z_2, \cdots, z_k\}$.

3. MAIN RESULTS

In order to facilitate the subsequent developments, based on the observation sequence $\{z_1, z_2, \cdots, z_k\}$, we design the optimal one-step predictor for nonlinear discrete stochastic systems (1)-(2) by employing the unscented transformation approach and the method in the reference [7].

Lemma 1: The optimal one-step predictor for the systems (1)-(2) is given as follows:

$$\hat{x}_{k+1|k} = f_k\left(\hat{x}_{k|k-1}\right) + K_k^p \varepsilon_k, \tag{3}$$

$$\varepsilon_k = z_k - \alpha \hat{y}_{k|k-1}, \tag{4}$$

$$K_k^p = \left(\alpha P_{k|k-1}^{xy} + S_k\right)\left(\alpha P_{k|k-1}^{yy} + \alpha(1-\alpha)\hat{y}_{k|k-1}\left(\hat{y}_{k|k-1}\right)^T + R_k\right)^{-1}, \tag{5}$$

$$P_{k+1|k} = P_{k|k-1} - K_k^p\left(\alpha P_{k|k-1}^{xy} + S_k\right)^T + Q_k, \tag{6}$$

where

$$f_k\left(\hat{x}_{k|k-1}\right) = \mathbb{E}\left\{f_k(x_k)|Z_{k-1}\right\}, \tag{7}$$

$$\hat{y}_{k|k-1} = h_k\left(\hat{x}_{k|k-1}\right) = \mathbb{E}\left\{h_k(x_k)|Z_{k-1}\right\}, \tag{8}$$

$$P_{k|k-1}^{xy} = \mathbb{E}\left\{\left(f_k(x_k) - f_k\left(\hat{x}_{k|k-1}\right)\right) \times \left(y_k - \hat{y}_{k|k-1}\right)^T\right\}, \tag{9}$$

$$P_{k|k-1}^{yy} = \mathbb{E}\left\{\left(y_k - \hat{y}_{k|k-1}\right)\left(y_k - \hat{y}_{k|k-1}\right)^T\right\}, \tag{10}$$

$$P_{k|k-1} = \mathbb{E}\left\{\left(f_k(x_k) - f_k\left(\hat{x}_{k|k-1}\right)\right) \times \left(f_k(x_k) - f_k\left(\hat{x}_{k|k-1}\right)\right)^T\right\}, \tag{11}$$

K_k^p is the predictor gain matrix and ε_k is the innovation sequence. (7)-(11) is approximately computed by the unscented transformation approach.

Proof: Firstly, by applying the method in the reference [7], we have

$$\hat{z}_{k|k-1} = \mathbb{E}\left\{z_k|Z_{k-1}\right\} = \alpha\mathbb{E}\left\{h_k(x_k)|Z_{k-1}\right\}. \tag{12}$$

Letting $y_k = h_k(x_k)$, then

$$\hat{y}_{k|k-1} = h_k\left(\hat{x}_{k|k-1}\right) = \mathbb{E}\left\{h_k(x_k)|Z_{k-1}\right\}. \tag{13}$$

By the definition of the innovation sequence, equation (4) is satisfied as follows:

$$\begin{aligned}\varepsilon_k =& z_k - \hat{z}_{k|k-1} \\ =& z_k - \alpha h_k\left(\hat{x}_{k|k-1}\right) \\ =& \lambda_k h_k(x_k) - \alpha h_k\left(\hat{x}_{k|k-1}\right) + v_k \\ =& \lambda_k\left(h_k(x_k) - h_k\left(\hat{x}_{k|k-1}\right)\right) \\ &+ (\lambda_k - \alpha) h_k\left(\hat{x}_{k|k-1}\right) + v_k \\ =& \lambda_k\left(y_k - \hat{y}_{k|k-1}\right) + (\lambda_k - \alpha)\hat{y}_{k|k-1} + v_k.\end{aligned} \tag{14}$$

Obviously, $Z_k = (Z_{k-1}, z_k)$. By employing the method in the reference [7], the optimal one-step state prediction $\hat{x}_{k+1|k}$ at time instant $k+1$ is calculated as follows:

$$\begin{aligned}\hat{x}_{k+1|k} =& \mathbb{E}\left\{x_{k+1}|Z_k\right\} \\ =& \mathbb{E}\left\{x_{k+1}|Z_{k-1}\right\} + \mathbb{E}\left\{\tilde{x}_{k+1|k-1}|\tilde{z}_{k|k-1}\right\} \\ =& \mathbb{E}\left\{x_{k+1}|Z_{k-1}\right\} + \mathbb{E}\left\{\tilde{x}_{k+1|k-1}\tilde{z}_{k|k-1}^T\right\} \\ &\times\left(\mathbb{E}\left\{\tilde{z}_{k|k-1}\tilde{z}_{k|k-1}^T\right\}\right)^{-1}\tilde{z}_{k|k-1},\end{aligned} \tag{15}$$

where

$$\tilde{x}_{k+1|k-1} = x_{k+1} - \mathbb{E}\left\{x_{k+1}|Z_{k-1}\right\}, \tag{16}$$

$$\begin{aligned}\tilde{z}_{k|k-1} &= z_k - \mathbb{E}\left\{z_k|Z_{k-1}\right\} \\ &= z_k - \hat{z}_{k|k-1} = \varepsilon_k.\end{aligned} \tag{17}$$

By the same derivation of (12), we have

$$\begin{aligned}\mathbb{E}\left\{x_{k+1}|Z_{k-1}\right\} =& \mathbb{E}\left\{\left(f_k(x_k) + \omega_k\right)|Z_{k-1}\right\} \\ =& \mathbb{E}\left\{f_k(x_k)|Z_{k-1}\right\}.\end{aligned} \tag{18}$$

Then, let

$$f_k\left(\hat{x}_{k|k-1}\right) = \mathbb{E}\left\{f_k(x_k)|Z_{k-1}\right\}. \tag{19}$$

Substituting (18) and (19) into (16) leads to

$$\tilde{x}_{k+1|k-1} = f_k(x_k) - f_k\left(\hat{x}_{k|k-1}\right) + \omega_k. \tag{20}$$

By using (14), (17), (20) and $\mathbb{E}\{\lambda_k - \alpha\} = 0$, $\mathbb{E}\{\omega_k\} = \mathbb{E}\{v_k\} = 0$, it can be easily deduced that

$$\begin{aligned}&\mathbb{E}\left\{\tilde{x}_{k+1|k-1}\tilde{z}_{k|k-1}^T\right\} \\ &= \mathbb{E}\left\{\left(f_k(x_k) - f_k\left(\hat{x}_{k|k-1}\right) + \omega_k\right) \times \left(\lambda_k\left(y_k - \hat{y}_{k|k-1}\right) + (\lambda_k - \alpha)\hat{y}_{k|k-1} + v_k\right)^T\right\} \\ &= \alpha\mathbb{E}\left\{\left(f_k(x_k) - f_k\left(\hat{x}_{k|k-1}\right)\right)\left(y_k - \hat{y}_{k|k-1}\right)^T\right\}\end{aligned}$$

$$
\begin{aligned}
&+\mathbb{E}\left\{\omega_k v_k^T\right\}\\
&=\alpha P_{k|k-1}^{xy}+S_k, \qquad (21)\\
&\mathbb{E}\left\{\tilde{z}_{k|k-1}\tilde{z}_{k|k-1}^T\right\}=\mathbb{E}\left\{\varepsilon_k\varepsilon_k^T\right\}\\
&=\mathbb{E}\Big\{\left(\lambda_k\left(y_k-\hat{y}_{k|k-1}\right)+\left(\lambda_k-\alpha\right)\hat{y}_{k|k-1}+v_k\right)\\
&\quad\times\left(\lambda_k\left(y_k-\hat{y}_{k|k-1}\right)+\left(\lambda_k-\alpha\right)\hat{y}_{k|k-1}+v_k\right)^T\Big\}\\
&=\alpha\mathbb{E}\left\{\left(y_k-\hat{y}_{k|k-1}\right)\left(y_k-\hat{y}_{k|k-1}\right)^T\right\}\\
&\quad+\mathbb{E}\left\{\left(\lambda_k-\alpha\right)^2\right\}\hat{y}_{k|k-1}\left(\hat{y}_{k|k-1}\right)^T+\mathbb{E}\left\{v_k v_k^T\right\}\\
&=\alpha P_{k|k-1}^{yy}+\alpha(1-\alpha)\hat{y}_{k|k-1}\left(\hat{y}_{k|k-1}\right)^T+R_k, \qquad (22)
\end{aligned}
$$

where

$$
\begin{aligned}
P_{k|k-1}^{xy}&=\mathbb{E}\left\{\left(f_k\left(x_k\right)-f_k\left(\hat{x}_{k|k-1}\right)\right)\left(y_k-\hat{y}_{k|k-1}\right)^T\right\},\\
P_{k|k-1}^{yy}&=\mathbb{E}\left\{\left(y_k-\hat{y}_{k|k-1}\right)\left(y_k-\hat{y}_{k|k-1}\right)^T\right\}.
\end{aligned}
$$

Define the predictor gain matrix K_k^p as follows:

$$
K_k^p=\mathbb{E}\left\{\tilde{x}_{k+1|k-1}\tilde{z}_{k|k-1}^T\right\}\left(\mathbb{E}\left\{\tilde{z}_{k|k-1}\tilde{z}_{k|k-1}^T\right\}\right)^{-1}. \qquad (23)
$$

Obviously,

$$
\begin{aligned}
K_k^p=&\left(\alpha P_{k|k-1}^{xy}+S_k\right)\\
&\times\left(\alpha P_{k|k-1}^{yy}+\alpha(1-\alpha)\hat{y}_{k|k-1}\left(\hat{y}_{k|k-1}\right)^T+R_k\right)^{-1}.
\end{aligned} \qquad (24)
$$

Substituting (17), (18) and (24) into (15) yields

$$
\begin{aligned}
\hat{x}_{k+1|k}=&\mathbb{E}\left\{x_{k+1}|Z_{k-1}\right\}+K_k^p\tilde{z}_{k|k-1}\\
=&\mathbb{E}\left\{x_{k+1}|Z_{k-1}\right\}+K_k^p\varepsilon_k\\
=&f_k\left(\hat{x}_{k|k-1}\right)+K_k^p\varepsilon_k.
\end{aligned} \qquad (25)
$$

Next, compute the error covariance matrix $P_{k+1|k}$ of one-step prediction $\hat{x}_{k+1|k}$. By combining (20) and (25), we have the error $\tilde{x}_{k+1|k}$ of one-step prediction $\hat{x}_{k+1|k}$ as follows:

$$
\begin{aligned}
\tilde{x}_{k+1|k}=&x_{k+1}-\hat{x}_{k+1|k}\\
=&x_{k+1}-\mathbb{E}\left\{x_{k+1}|Z_{k-1}\right\}-K_k^p\tilde{z}_{k|k-1}\\
=&\tilde{x}_{k+1|k-1}-K_k^p\tilde{z}_{k|k-1}.
\end{aligned} \qquad (26)
$$

Accordingly,

$$
\begin{aligned}
&P_{k+1|k}\\
&=\mathbb{E}\left\{\tilde{x}_{k+1|k}\tilde{x}_{k+1|k}^T\right\}\\
&=\mathbb{E}\left\{\left(\tilde{x}_{k+1|k-1}-K_k^p\tilde{z}_{k|k-1}\right)\left(\tilde{x}_{k+1|k-1}-K_k^p\tilde{z}_{k|k-1}\right)^T\right\}\\
&=\mathbb{E}\left\{\tilde{x}_{k+1|k-1}\tilde{x}_{k+1|k-1}^T\right\}-\mathbb{E}\left\{\tilde{x}_{k+1|k-1}\tilde{z}_{k|k-1}^T\right\}\left(K_k^p\right)^T\\
&\quad-K_k^p\mathbb{E}\left\{\tilde{z}_{k|k-1}\tilde{x}_{k+1|k-1}^T\right\}\\
&\quad+K_k^p\mathbb{E}\left\{\tilde{z}_{k|k-1}\tilde{z}_{k|k-1}^T\right\}\left(K_k^p\right)^T\\
&=\mathbb{E}\Big\{\left(f_k\left(x_k\right)-f_k\left(\hat{x}_{k|k-1}\right)+\omega_k\right)\left(f_k\left(x_k\right)-f_k\left(\hat{x}_{k|k-1}\right)\right.\\
&\quad\left.+\omega_k\right)^T\Big\}-K_k^p\mathbb{E}\left\{\tilde{z}_{k|k-1}\tilde{x}_{k+1|k-1}^T\right\}\\
&=\mathbb{E}\left\{\left(f_k\left(x_k\right)-f_k\left(\hat{x}_{k|k-1}\right)\right)\left(f_k\left(x_k\right)-f_k\left(\hat{x}_{k|k-1}\right)\right)^T\right\}\\
&\quad+Q_k-K_k^p\left(\alpha P_{k|k-1}^{xy}+S_k\right)^T\\
&=P_{k|k-1}-K_k^p\left(\alpha P_{k|k-1}^{xy}+S_k\right)^T+Q_k, \qquad (27)
\end{aligned}
$$

where

$$
\begin{aligned}
&P_{k|k-1}\\
&=\mathbb{E}\left\{\left(f_k\left(x_k\right)-f_k\left(\hat{x}_{k|k-1}\right)\right)\left(f_k\left(x_k\right)-f_k\left(\hat{x}_{k|k-1}\right)\right)^T\right\}.
\end{aligned}
$$

The proof of this theorem is complete. □

Remark 2: It is worth noting that, in Theorem 1, the recursive optimal one-step predictor is designed for the addressed nonlinear discrete stochastic systems with correlated noises and missing measurements. Due to the factors of the sudden changes in the environment and unreliability of the communication network, the phenomena of correlated noises and missing measurements exist commonly. Therefore, in this paper, we consider the nonlinear discrete stochastic systems with correlated noises and missing measurements. In order to deal with the correlated noises and missing measurements, and then further improve the accuracy of filtering estimation for the systems, we take the innovative analysis approach to design optimal one-step predictor and the covariance matrix S_k of correlated noises is reflected by the predictor gain matrix K_k^p. Compared with the existing results without considering the correlation of noises, in this paper, the following nonlinear filter is designed based on one-step predictor.

Theorem 1: Based on the observation sequence $\{z_1,z_2,\cdots,z_k\}$ and the principle of minimum mean square error, the optimal filter for the nonlinear systems (1)-(2) is given as follows:

$$
\hat{x}_{k+1|k+1}=\hat{x}_{k+1|k}+K_{k+1}\varepsilon_{k+1}, \qquad (28)
$$

$$
\varepsilon_{k+1}=z_{k+1}-\alpha\hat{y}_{k+1|k}, \qquad (29)
$$

$$
\begin{aligned}
K_{k+1}=&\alpha P_{k+1|k}^{xy}\Big(\alpha P_{k+1|k}^{yy}\\
&+\alpha\left(1-\alpha\right)\hat{y}_{k+1|k}\left(\hat{y}_{k+1|k}\right)^T+R_{k+1}\Big)^{-1},
\end{aligned} \qquad (30)
$$

$$
P_{k+1|k+1}=P_{k+1|k}-\alpha K_{k+1}\left(P_{k+1|k}^{xy}\right)^T, \qquad (31)
$$

where K_{k+1} is the filter gain matrix.

Proof: Obviously, $Z_{k+1}=(Z_k,z_{k+1})$. By using the method in the reference [7], we can obtain the optimal estimation $\hat{x}_{k+1|k+1}$ at time instant $k+1$ as follows:

$$\begin{aligned}&\hat{x}_{k+1|k+1}\\&=\mathbb{E}\{x_{k+1}|Z_{k+1}\}\\&=\mathbb{E}\{x_{k+1}|Z_k\}+\mathbb{E}\{\tilde{x}_{k+1|k}|\tilde{z}_{k+1|k}\}\\&=\hat{x}_{k+1|k}+\mathbb{E}\left\{\tilde{x}_{k+1|k}\tilde{z}^T_{k+1|k}\right\}\\&\quad\times\left(\mathbb{E}\left\{\tilde{z}_{k+1|k}\tilde{z}^T_{k+1|k}\right\}\right)^{-1}\tilde{z}_{k+1|k},\end{aligned}\tag{32}$$

where

$$\tilde{x}_{k+1|k}=x_{k+1}-\hat{x}_{k+1|k},\tag{33}$$

$$\begin{aligned}\tilde{z}_{k+1|k}&=z_{k+1}-\hat{z}_{k+1|k}\\&=\varepsilon_{k+1}\\&=\lambda_{k+1}\left(y_{k+1}-\hat{y}_{k+1|k}\right)\\&\quad+(\lambda_{k+1}-\alpha)\hat{y}_{k+1|k}+\nu_{k+1}.\end{aligned}\tag{34}$$

Along the same method of the derivation of (21) and (22), we obtain

$$\begin{aligned}&\mathbb{E}\left\{\tilde{x}_{k+1|k}\tilde{z}^T_{k+1|k}\right\}\\&=\mathbb{E}\left\{\tilde{x}_{k+1|k}\Big(\lambda_{k+1}\left(y_{k+1}-\hat{y}_{k+1|k}\right)\right.\\&\qquad\left.+(\lambda_{k+1}-\alpha)\hat{y}_{k+1|k}+\nu_{k+1}\Big)^T\right\}\\&=\alpha\mathbb{E}\left\{\left(x_{k+1}-\hat{x}_{k+1|k}\right)\left(y_{k+1}-\hat{y}_{k+1|k}\right)^T\right\}\\&=\alpha P^{xy}_{k+1|k},\end{aligned}\tag{35}$$

$$\begin{aligned}&\mathbb{E}\left\{\tilde{z}_{k+1|k}\tilde{z}^T_{k+1|k}\right\}\\&=\alpha P^{yy}_{k+1|k}\\&\quad+\alpha(1-\alpha)\hat{y}_{k+1|k}\left(\hat{y}_{k+1|k}\right)^T+R_{k+1}.\end{aligned}\tag{36}$$

Define the filter gain matrix K_{k+1} as follows:

$$K_{k+1}=\mathbb{E}\left\{\tilde{x}_{k+1|k}\tilde{z}^T_{k+1|k}\right\}\left(\mathbb{E}\left\{\tilde{z}_{k+1|k}\tilde{z}^T_{k+1|k}\right\}\right)^{-1}.\tag{37}$$

Obviously,

$$\begin{aligned}K_{k+1}=&\alpha P^{xy}_{k+1|k}\Big(\alpha P^{yy}_{k+1|k}\\&+\alpha(1-\alpha)\hat{y}_{k+1|k}\left(\hat{y}_{k+1|k}\right)^T+R_{k+1}\Big)^{-1}.\end{aligned}\tag{38}$$

Hence, (32) is equivalent to the following equation:

$$\begin{aligned}\hat{x}_{k+1|k+1}&=\hat{x}_{k+1|k}+K_{k+1}\tilde{z}_{k+1|k}\\&=\hat{x}_{k+1|k}+K_{k+1}\varepsilon_{k+1}.\end{aligned}\tag{39}$$

Subsequently, the following derivations are given to obtain the error covariance matrix $P_{k+1|k+1}$ of optimal estimation $\hat{x}_{k+1|k+1}$. By using the equations (33) and (39), one has

$$\begin{aligned}\tilde{x}_{k+1|k+1}&=x_{k+1}-\hat{x}_{k+1|k+1}\\&=x_{k+1}-\hat{x}_{k+1|k}-K_{k+1}\tilde{z}_{k+1|k}\\&=\tilde{x}_{k+1|k}-K_{k+1}\tilde{z}_{k+1|k}.\end{aligned}\tag{40}$$

Accordingly,

$$\begin{aligned}&P_{k+1|k+1}\\&=\mathbb{E}\left\{\tilde{x}_{k+1|k+1}\tilde{x}^T_{k+1|k+1}\right\}\\&=\mathbb{E}\left\{\left(\tilde{x}_{k+1|k}-K_{k+1}\tilde{z}_{k+1|k}\right)\left(\tilde{x}_{k+1|k}-K_{k+1}\tilde{z}_{k+1|k}\right)^T\right\}\\&=\mathbb{E}\left\{\tilde{x}_{k+1|k}\tilde{x}^T_{k+1|k}\right\}-\mathbb{E}\left\{\tilde{x}_{k+1|k}\tilde{z}^T_{k+1|k}\right\}K^T_{k+1}\\&\quad-K_{k+1}\mathbb{E}\left\{\tilde{z}_{k+1|}\tilde{x}^T_{k+1|k}\right\}+K_{k+1}\mathbb{E}\left\{\tilde{z}_{k+1|}\tilde{z}^T_{k+1|k}\right\}K^T_{k+1}\\&=P_{k+1|k}-K_{k+1}\mathbb{E}\left\{\tilde{z}_{k+1|}\tilde{x}^T_{k+1|k}\right\}\\&=P_{k+1|k}-\alpha K_{k+1|k}\left(P^{xy}_{k+1|k}\right)^T.\end{aligned}\tag{41}$$

Then, the proof of this theorem is complete. □

According to Theorem 1, a new recursive algorithm can be established to obtain the optimal nonlinear filter for the addressed discrete stochastic nonlinear systems with correlate noises and missing measurements. The following algorithm shows how to design the unscented Kalman filter in Theorem 1.

Algorithm: The steps of the design of the unscented Kalman filter are shown as follows:

Step 1: Choose the sigma points.

We choose $2n+1$ points as a sigma points set, i.e.

$$\begin{aligned}&\chi^0_{k-1|k-1}=\hat{x}_{k-1|k-1},\\&\chi^s_{k-1|k-1}=\hat{x}_{k-1|k-1}+\left(\sqrt{(n+\kappa)P_{k-1|k-1}}\right)_s,\\&\qquad\qquad s=1,\cdots,n,\\&\chi^s_{k-1|k-1}=\hat{x}_{k-1|k-1}-\left(\sqrt{(n+\kappa)P_{k-1|k-1}}\right)_{s-n},\\&\qquad\qquad s=n+1,\cdots,2n,\end{aligned}$$

where κ is the scaling factor and $\left(\sqrt{(n+\kappa)P_{k-1|k-1}}\right)_s$ is either the s-th row or the s-th column of the matrix square root of $(n+\kappa)P_{k-1|k-1}$.

Now, compute the transformed values of the sigma points by using the nonlinear function $f_{k-1}(x_{k-1})$.

$$\chi^s_{k|k-1}=f_{k-1}\left(\chi^s_{k-1|k-1}\right),\quad s=0,1,\cdots,2n,$$

Then, the one-step prediction $\hat{x}_{k|k-1}$ and error covariance matrix $P_{k|k-1}$ can be calculated by recombining the weighted sigma points as follows:

$$\begin{aligned}&\hat{x}_{k|k-1}=\sum_{s=0}^{2n}W^s\chi^s_{k|k-1},\\&P_{k|k-1}=\sum_{s=0}^{2n}W^s\left(\chi^s_{k|k-1}-\hat{x}_{k|k-1}\right)\left(\chi^s_{k|k-1}-\hat{x}_{k|k-1}\right)^T\end{aligned}$$

$$+Q_{k-1},$$

according to weights

$$W^s = \begin{cases} \frac{\kappa}{n+\kappa}, & s=0 \\ \frac{1}{2(n+\kappa)}, & s=1,2,\cdots,2n \end{cases}. \tag{42}$$

On the other hand, we choose another $2n+1$ points as another sigma points set. Then,

$$\begin{aligned} \delta^0_{k|k-1} &= \hat{x}_{k|k-1}, \\ \delta^s_{k|k-1} &= \hat{x}_{k|k-1} + \left(\sqrt{(n+\kappa)P_{k|k-1}}\right)_s, \\ &\qquad s=1,\cdots,n, \\ \delta^s_{k|k-1} &= \hat{x}_{k|k-1} - \left(\sqrt{(n+\kappa)P_{k|k-1}}\right)_{s-n}, \\ &\qquad s=n+1,\cdots,2n. \end{aligned} \tag{43}$$

Step 2: Calculate the one-step predictor $\hat{x}_{k+1|k}$ at time instant $k+1$.

From Theorem 1, we can obtain one-step prediction $\hat{x}_{k+1|k}$ at time instant $k+1$ by applying the unscented transformation approach to compute the mean and covariance of one-step prediction $\hat{x}_{k|k-1}$ at time instant k.

By (43), the sigma points $\delta^s_{k|k-1}$, which are computed by $\hat{x}_{k|k-1}$ and $P_{k|k-1}$, are transmitted to the sigma points $\xi^s_{k|k-1}$ and $\gamma^s_{k|k-1}$ by the state function $f_k(\cdot)$ and observation function $h_k(\cdot)$, respectively. We apply the $\xi^s_{k|k-1}$ and $\gamma^s_{k|k-1}$ to calculate the following mean $f_k\left(\hat{x}_{k|k-1}\right)$, $h_k\left(\hat{x}_{k|k-1}\right)$ and covariances $P_{k|k-1}$, $P^{yy}_{k|k-1}$ and cross covariance $P^{xy}_{k|k-1}$

$$\xi^s_{k|k-1} = f_k\left(\delta^s_{k|k-1}\right), \quad s=0,1,\cdots,2n, \tag{44}$$

$$\gamma^s_{k|k-1} = h_k\left(\delta^s_{k|k-1}\right), \quad s=0,1,\cdots,2n, \tag{45}$$

$$f_k\left(\hat{x}_{k|k-1}\right) = \sum_{s=0}^{2n} W^s \xi^s_{k|k-1} = \sum_{s=0}^{2n} W^s f_k\left(\delta^s_{k|k-1}\right), \tag{46}$$

$$\begin{aligned} \hat{y}_{k|k-1} = h_k\left(\hat{x}_{k|k-1}\right) &= \sum_{s=0}^{2n} W^s \gamma^s_{k|k-1} \\ &= \sum_{s=0}^{2n} W^s h_k\left(\delta^s_{k|k-1}\right), \end{aligned} \tag{47}$$

$$\begin{aligned} P_{k|k-1} = \sum_{s=0}^{2n} W^s &\left(\xi^s_{k|k-1} - f_k\left(\hat{x}_{k|k-1}\right)\right) \\ &\times \left(\xi^s_{k|k-1} - f_k\left(\hat{x}_{k|k-1}\right)\right)^T, \end{aligned} \tag{48}$$

$$\begin{aligned} P^{yy}_{k|k-1} = \sum_{s=0}^{2n} W^s &\left(\gamma^s_{k|k-1} - \hat{y}_{k|k-1}\right) \\ &\times \left(\gamma^s_{k|k-1} - \hat{y}_{k|k-1}\right)^T, \end{aligned} \tag{49}$$

$$\begin{aligned} P^{xy}_{k|k-1} = \sum_{s=0}^{2n} W^s &\left(\xi^s_{k|k-1} - f_k\left(\hat{x}_{k|k-1}\right)\right) \\ &\times \left(\gamma^s_{k|k-1} - \hat{y}_{k|k-1}\right)^T, \end{aligned} \tag{50}$$

where W^s satisfies the equation (42). Substituting (44)-(50) into (3)-(6), we can compute the one-step prediction $\hat{x}_{k+1|k}$ and error covariance matrix $P_{k+1|k}$.

Step 3: Calculate the estimation $\hat{x}_{k+1|k+1}$ at time instant $k+1$.

By Theorem 1, the estimation $\hat{x}_{k+1|k+1}$ at time instant $k+1$ can be computed by applying the unscented transformation approach to calculate the mean and covariance of one-step prediction $\hat{x}_{k+1|k}$ at time instant $k+1$.

Hence, we choose the sigma points, which are computed by $\hat{x}_{k+1|k}$ and $P_{k+1|k}$, as follows:

$$\begin{aligned} \eta^0_{k+1|k} &= \hat{x}_{k+1|k}, \\ \eta^s_{k+1|k} &= \hat{x}_{k+1|k} + \left(\sqrt{(n+\kappa)P_{k+1|k}}\right)_s, \\ &\qquad s=1,\cdots,n, \\ \eta^s_{k+1|k} &= \hat{x}_{k+1|k} - \left(\sqrt{(n+\kappa)P_{k+1|k}}\right)_{s-n}, \\ &\qquad s=n+1,\cdots,2n, \end{aligned}$$

Then, the sigma points $\eta^s_{k+1|k}$ are transmitted to the sigma points $\sigma^s_{k+1|k}$ by the observation function $h_{k+1}(\cdot)$. The mean $\hat{y}_{k+1|k}$, covariance $P^{yy}_{k+1|k}$ and cross covariance $P^{xy}_{k+1|k}$ can be obtained by the sigma points $\sigma^s_{k+1|k}$ as follows:

$$\sigma^s_{k+1|k} = h_{k+1}\left(\eta^s_{k+1|k}\right), \quad s=0,1,\cdots,2n, \tag{51}$$

$$\begin{aligned} \hat{y}_{k+1|k} &= h_{k+1}\left(\hat{x}_{k+1|k}\right) \\ &= \sum_{s=0}^{2n} W^s \sigma^s_{k+1|k} \\ &= \sum_{s=0}^{2n} W^s h_{k+1}\left(\sigma^s_{k+1|k}\right), \end{aligned} \tag{52}$$

$$P^{yy}_{k+1|k} = \sum_{s=0}^{2n} W^s \left(\sigma^s_{k+1|k} - \hat{y}_{k+1|k}\right)\left(\sigma^s_{k+1|k} - \hat{y}_{k+1|k}\right)^T, \tag{53}$$

$$P^{xy}_{k+1|k} = \sum_{s=0}^{2n} W^s \left(\eta^s_{k+1|k} - \hat{x}_{k+1|k}\right)\left(\sigma^s_{k+1|k} - \hat{y}_{k+1|k}\right)^T, \tag{54}$$

where W^s satisfies (42). The estimation $\hat{x}_{k+1|k+1}$ and error covariance matrix $P_{k+1|k+1}$ are computed by substituting (51)-(54) into (28)-(31).

Remark 3: In general, if the square root A of the matrix P is the form of $P = A^T A$, the sigma points are formed from the row of A. Otherwise, the columns of A are employed if $P = AA^T$ [8]. Therefore, we utilize the s-th column of the matrix square root of $(n+\kappa)P_{k-1|k-1}$ in our algorithm. In addition, it can be seen clearly from the algorithm that unscented Kalman filter with correlated noises and missing measurements could be implemented by two blocks: one is state prediction and the other is state estimation. The state prediction can be calculated independently. However, the state estimation can be derived by employing the state prediction.

Remark 4: From the above algorithm, the estimation $\hat{x}_{k+1|k+1}$ can be computed by using the one-step prediction $\hat{x}_{k+1|k}$. Hence, in **Step 1**, we choose the sigma points $\chi^s_{k-1|k-1}$ by using the value of estimation $\hat{x}_{k-1|k-1}$. Based on the nonlinear state function $f_{k-1}(\cdot)$ and $\chi^s_{k-1|k-1}$, compute the value of $\hat{x}_{k|k-1}$ and $P_{k|k-1}$. Then, the sigma points $\delta^s_{k|k-1}$ are chosen by using the value of $\hat{x}_{k|k-1}$ and $P_{k|k-1}$; In **Step 2**, by the state function $f_k(\cdot)$ and observation function $h_k(\cdot)$, the sigma points $\delta^s_{k|k-1}$ are transmitted to the sigma points $\xi^s_{k|k-1}$ and $\gamma^s_{k|k-1}$, respectively. Therefore, the one-step predictor $\hat{x}_{k+1|k}$ can be calculated based on the mean $f_k\left(\hat{x}_{k|k-1}\right)$, $h_k\left(\hat{x}_{k|k-1}\right)$ and covariances $P_{k|k-1}$, $P^{yy}_{k|k-1}$ and cross covariance $P^{xy}_{k|k-1}$; In **Step 3**, we compute the estimation $\hat{x}_{k+1|k+1}$ by applying the unscented transformation approach and the one-step prediction $\hat{x}_{k+1|k}$.

Remark 5: Note that, compared with linear system with correlated noises and missing measurements, we design the unscented Kalman filter to solve the state estimation problem for nonlinear system based on the unscented transformation approach. The technical difficulties are to deal with the correlated noises and missing measurements by using the unscented transformation method. In order to deal with the correlated noises and missing measurements, and then further improve the accuracy of filtering estimation for the systems, we compute the parameters $f_k\left(\hat{x}_{k|k-1}\right)$, $\hat{y}_{k|k-1}$, $P^{xy}_{k|k-1}$, $P^{yy}_{k|k-1}$, and $P_{k|k-1}$ to obtain one-step predictor $\hat{x}_{k+1|k}$. Then, by using the one-step predictor $\hat{x}_{k+1|k}$ and calculating the parameter $P^{xy}_{k+1|k}$, the new recursive unscented Kalman filtering algorithm has been employed to estimate the system state. On the other hand, in the former results, these researches do not pay much attention to the problem of the nonlinear filtering for nonlinear discrete stochastic systems with correlated noises and missing data in the measurement. Hence, we make the first attempt to propose the nonlinear filter for systems subject to correlated noises and missing measurements and proposed filtering algorithm accuracy can be improved because we have made great efforts to compensate the effects from correlated noises and missing measurements. In the following part, we make the comparison with the unscented Kalman filter and extended Kalman filter in [41] for nonlinear systems with missing measurements.

4. AN ILLUSTRATIVE EXAMPLE

We give a simulation example to illustrate the performance of the developed filtering algorithm in this section.

Consider the following nonlinear discrete stochastic systems:

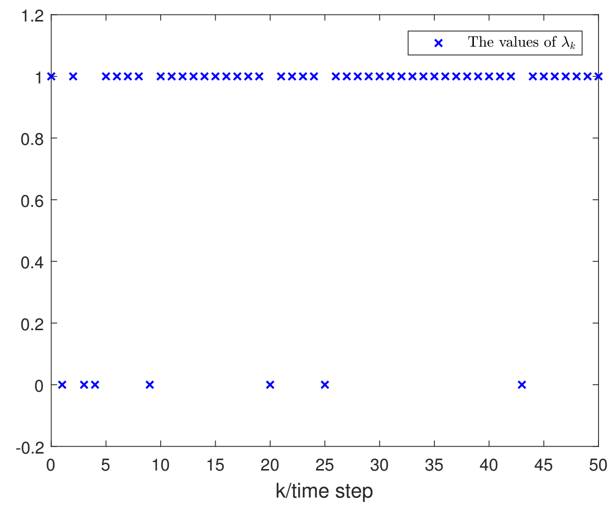

Fig. 1. The trajectories of λ_k.

$$x_{k+1} = \begin{bmatrix} \sin(x_k^1)x_k^1 + \cos(2k)x_k^2 \\ \cos(x_k^2)x_k^2 + 0.75x_k^2 \end{bmatrix} + \omega_k$$

$$z_k = \lambda_k \begin{bmatrix} \sin(2k)x_k^1 + x_k^1 x_k^2 \\ \cos(2x_k^2)x_k^1 + \sin(k)x_k^2 \end{bmatrix} + \nu_k$$

where $x_k = \begin{bmatrix} x_k^1 & x_k^2 \end{bmatrix}^T$, ω_k and ν_k are correlated Gaussian white noises with covariances $Q_k = 0.2I_2$, $R_k = 0.1I_2$ and cross covariance $S_k = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.05 \end{bmatrix}$.

Let

$$x_0 = \begin{bmatrix} 2 & 2 \end{bmatrix}^T; \ \hat{x}_{0|0} = \begin{bmatrix} 2 & 2 \end{bmatrix}^T;$$
$$P_{0|0} = I_2; \ \alpha = 0.85$$

and e_k^i denote the error for the estimation of x_k^i, i.e., $e_k^i = x_k^i - \hat{x}^i_{k|k}$, where $i = 1, 2$.

According to Theorem 1, the nonlinear recursive filter can be constructed by applying the unscented transformation approach and MMSE estimation principle. The filter gain matrices K_{k+1} and the error covariance matrices $P_{k+1|k+1}$ at every time step can be recursively computed by utilizing the given algorithm and Matlab software. The results are shown in Figs. 1-5. The trajectories of random variable λ_k are plotted in Fig. 1. We can easily know that λ_k is 1 or 0 at every time step. Fig. 2 and Fig. 3 plot the filtering errors e_k^i $(i = 1, 2)$. From the simulations, we can see that the range of error fluctuation in our paper is relatively small compared with the error of UKF and EKF in the references [41]. The actual system states x_k^i and their estimates $\hat{x}^i_{k|k}$ $(i = 1, 2)$ are plotted in Fig. 4 and Fig. 5. It is easily seen that, due to making a lot of efforts to reduce the effects from correlate noises and missing measurements, the proposed filter can estimate the system state effectively and the recursive algorithm is feasible.

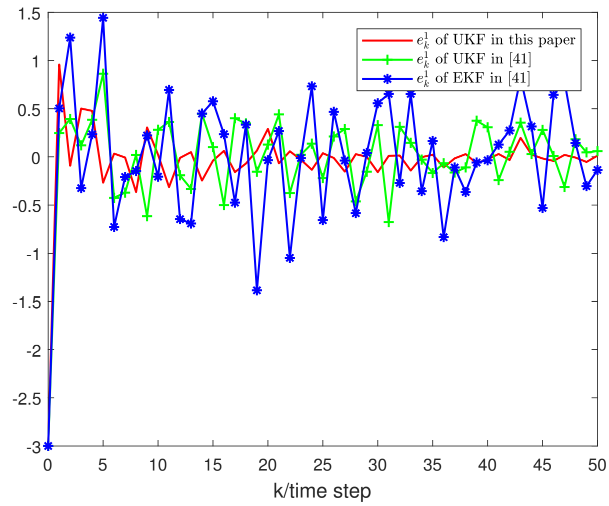

Fig. 2. The error e_k^1.

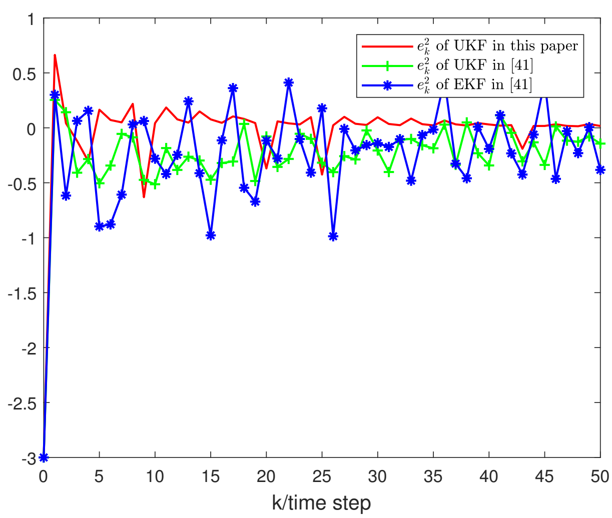

Fig. 3. The error e_k^2.

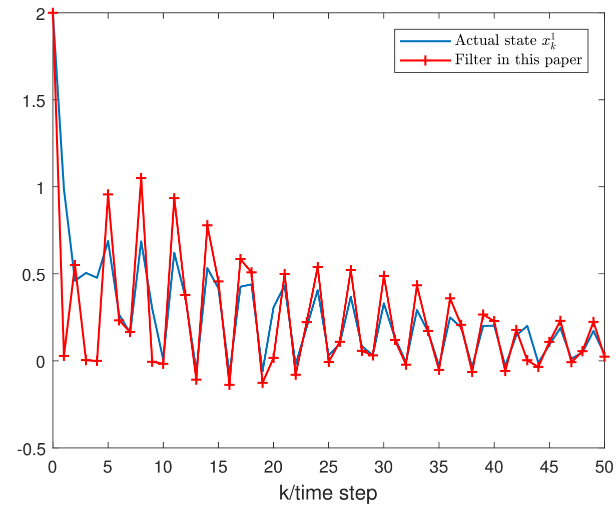

Fig. 4. The trajectories of x_k^1 and $\hat{x}^1_{k|k}$.

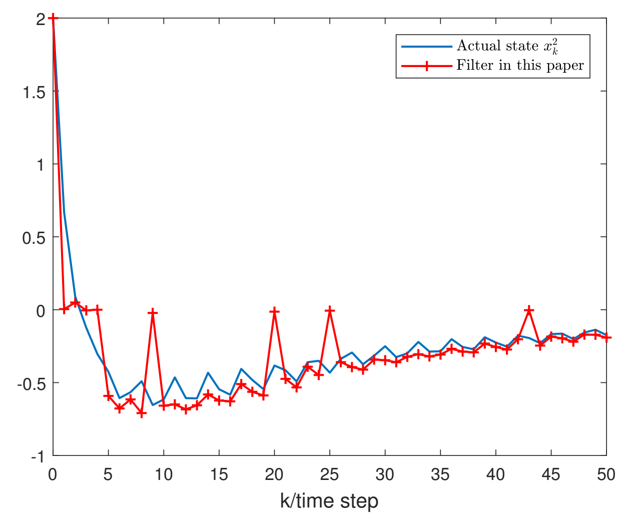

Fig. 5. The trajectories of x_k^2 and $\hat{x}^2_{k|k}$.

5. CONCLUSION

The problem of nonlinear estimation has been investigated for a class of nonlinear discrete stochastic systems subject to correlated noises and missing measurements. Firstly, by applying the innovative analysis and unscented transformation approach, an optimal one-step predictor has been designed to address the effects of the correlated noises and missing measurements. Then, based on the one-step predictor, the projection theory and the minimum mean square error (MMSE) principle, the nonlinear unscented Kalman filter has been constructed which can estimate the system state effectively. Finally, a recursive algorithm has been given to design the nonlinear filter and a simulation example has been given to show the feasibility and usefulness of the proposed approach. Further research topics include the stability analysis of the proposed algorithm and dealing with the UKF with missing measurements described by Markov-chain. The corresponding results will appear in the near future.

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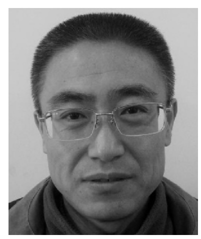

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