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Sliding Mode Control for A Class of Uncertain Discrete Switched Systems

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Abstract: This paper considers the discrete-time quasi-sliding mode control for a class of uncertain switched systems. Since the input matrices are different, a weighted sum approach is proposed such that a common sliding surface is designed. Moreover, by designing a sliding mode control law, the state trajectories are driven into a certain band of the sliding surface. Furthermore, to guarantee the exponential stability of the sliding mode dynamics, a sufficient condition based on the average dwell time technology is given. Finally, a simulation is given to demonstrate the efficiency of the proposed method.

Keywords: Average dwell time, discrete switched systems, quasi-sliding mode, sliding mode control.

1. INTRODUCTION

As a special class of hybrid dynamical systems, switched systems have received considerable attention during the last two decades. Many engineer systems such as power systems, aircraft control systems, chemical control processes and communication network systems [1], etc, can be transformed into certain types of switched systems. In addition, due to the existence of the switching signal, the dynamical performance of the switched systems is complex. Therefore, a lot of research have been focused on stability and stabilization of switched systems (see [2–5] and the reference therein).

On the other hand, owing to the widespread application of the digital controllers, many classic control strategies are implemented in discrete systems. Therefore, many results on stability and stabilization of the discrete switched systems [6–8] have been proposed. Among them, [6] investigated the qualitative properties of linear discrete switched systems based on the average dwell time technology. Later, Zhai [7] further considered quadratic stabilization via state and output feedback for discrete switched systems when all the subsystems are unstable. Recently, the results were further extended into [8], in which each subsystem is not required to be stable and the asynchronous switching may happen.

It is well known that sliding mode control (SMC) has many desirable properties, such as fast response, and strong robustness against uncertain parameters and external disturbances. Therefore, SMC strategy has been used widely in many complex systems, such as, stochastic systems [9, 10] and Markovian jumping systems [11, 12]. Recently, the problem of SMC on switched systems has received increasing attention [13–15]. Among them, Wu and Lam [13] considered SMC for a class of switched systems with state delay, whose method was further extended to stochastic switched systems in [14]. Besides, [15] discussed the robust H_∞ SMC for a class of uncertain switched systems. It is worth noting that, in the previous work [13–15], the input channel for each subsystem is required to be the same. To remove this constraint, [16] considered the switched systems with different input matrices and constructed a common sliding surface via the weighted sum approach. More recently, [17] further proposed an adaptive sliding mode controller for switched systems, by which the controller parameters can be updated automatically to compensate the actuator faults. However, as far as the author knows there are rare work on SMC of the discrete switched systems. This is because, in discrete sliding mode control, the sample time may results in the quasi-sliding mode. That is, the system state may not always remain onto the sliding surface but switchs around the sliding surface, which brings challenge in SMC of the controlled systems. Moreover, the existing works on sliding mode control of switched systems cannot be simply extended to discrete switched systems, due to the special characteristics of the switched systems.

Motivated by the above discussions, we consider the problem of SMC for a class of uncertain discrete switched systems. In the discrete switched systems under consideration, the input channel for each subsystem is not required to be the same, which is different from the existing work [13–15]. To design a common sliding surface, a weighted sum approach is presented. In addition, the average dwell time approach is employed to analyze the stability of the sliding mode dynamics. Furthermore, it is shown that the

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reachability of the quasi-sliding mode (QSM) can be guaranteed despite the presence of the sample time, parameter uncertainties and external disturbances.

The following notations are used throughout this paper: $\mathbb{R}^n$ denotes the real n-dimensional space; $\mathbb{R}^{m\times n}$ denotes the real $m\times n$ matrix space. For any vector $x\in\mathbb{R}^n$, $\|x\|$ denotes its Euclidean norm and x^T is its transpose. For a real symmetric matrix, $M>0$ means that M is positive definite; I is used to represent an identity matrix of appropriate dimensions. The vector $\mathbf{1_n}\in\mathbb{R}^{\mathbf{n}}$ is consisted of ones, and $e_i\in\mathbb{R}^n$ is the i-th standard base vector; $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denotes the maximum and minimum eigenvalue of a symmetric matrix, respectively, rank $(\cdot)$ represents the rank of a matrix, and the symmetry parts in a matrix are denoted by $*$. $\otimes$ stands for the Kronecker product. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. PROBLEM STATEMENT

Consider the following uncertain discrete switched systems

$$x(k+1)=(A_\sigma+\Delta A_\sigma)x(k)+B_\sigma(u(k)+f_\sigma(x(k))), \quad (1)$$

where $x(k)\in\mathbb{R}^n$ is the state, $u(k)\in\mathbb{R}^n$ is the control input of the system, A_σ and B_σ are known matrices and ΔA_i is the parameter uncertain, $f_\sigma(x(k))$ is the matched disturbance, $\{A_\sigma,B_\sigma:\sigma\in\Gamma\}$ is a family of matrices depending on an index set $\Gamma=\{1,2,\cdots,s\}$, and $\sigma(k):\mathbb{R}\to\Gamma$ is a piecewise constant function of time k called as switching signal.

In this work, it is assumed that the admissible uncertainty ΔA_σ, $\sigma\in\Gamma$, satisfies

$$\Delta A_\sigma=E_\sigma G_\sigma(k)N_\sigma,$$

where E_σ and N_σ are constant matrices, and $G_\sigma(k)$ is an unknown matrix function satisfying $G_\sigma^T(k)G_\sigma(k)\le I$. Besides, the disturbance $f_\sigma(x(k))$ is assumed to be norm bounded, that is,

$$\|f_\sigma(x(k))\|<d_\sigma\|x(k)\|$$

with a known scalar $d_\sigma>0$.

For convenience, we will denote the system associated with the i-th subsystem by

$$A_\sigma\triangleq A_i,\ \Delta A_\sigma=\Delta A_i,\ B_\sigma\triangleq B_i,\ f_\sigma(x(k))\triangleq f_i(k).$$

Thus, for $\sigma(k)=i$, system (1) can be expressed as

$$x(k+1)=(A_i+\Delta A_i)x(k)+B_i(u(k)+f_i(k)). \quad (2)$$

It is noted that the input matrix $B_i\in\mathbb{R}^{n\times m}$ for each subsystem is not required to be the same, which brings challenge in designing a common sliding surface. To overcome the difficulty, the following weighted sum approach of the input matrices, as in [16], is introduced

$$\begin{aligned}
&B\triangleq\sum_{i=1}^{s}\alpha_iB_i\in\mathbb{R}^{n\times m},\\
&M\triangleq\frac{1}{2}\left[(B-s\alpha_1B_1)\ (B-s\alpha_2B_2)\ \cdots\ (B-s\alpha_nB_s)\right],\\
&L(i)\triangleq(I_s-2e_ie_i^T)\otimes I_s\in\mathbb{R}^{ms\times ms},\\
&N\triangleq\mathbf{1_s}\otimes I_m\in\mathbb{R}^{ms\times m},\\
&\mathcal{M}\triangleq\begin{bmatrix}M & \alpha I_n\end{bmatrix},\ \mathcal{L}(i)\triangleq\begin{bmatrix}L(i) & 0\\ 0 & \frac{1}{\alpha}(1-s\alpha_i)B_i\end{bmatrix},\\
&\mathcal{N}\triangleq\begin{bmatrix}N\\ I_m\end{bmatrix},\\
&\alpha\triangleq\max\{|s\underline{\alpha}-1|,|s\overline{\alpha}-1|\}\cdot\max_{1\le i\le s}\{\|B_i\|\},
\end{aligned}$$

where $\alpha_i\in\mathbb{R}$ is a bounded scalar satisfying

$$\underline{\alpha}\le\alpha_i\le\overline{\alpha},\ i=1,2,\ldots,s, \quad (3)$$

with $\underline{\alpha}$ and $\overline{\alpha}$ being known scalars.

Remark 1: It is shown that the input matrix B_i for each subsystem is not required to be the same. Therefore, the proposed method is more applicable than the existing ones [13–15].

Remark 2: It is worth noting that for

$$B=\alpha_1B_1+\alpha_2B_2+\cdots+\alpha_sB_s, \quad (4)$$

with B_i full column rank, B is generally full column rank. That is, for general choice of scalars α_i, $i=1,\ldots,s$, it can be obtained that B is full column rank.

By employing the weighted sum approach, it is can be shown that $B_i=B+\mathcal{M}\mathcal{L}(i)\mathcal{N}$, with $\|\mathcal{L}(i)\|\le1$. In the following parts, it can be shown that the weighted sum approach plays an important role in designing a common sliding surface.

The control objective of this work is to design a SMC law such that the switched system (1) is exponentially stable despite the presence of the sample time, parameter uncertainties and external disturbances. To this end, some necessary assumption and lemma are introduced as follows.

Assumption 1: The matrix B_i is full column rank, that is, $\mathrm{rank}(B_i)=m$.

Lemma 1: Let D, H, and $G(t)$ be real matrices of appropriate dimensions with $G(t)$ satisfying $G(t)^TG(t)\le I$. Then, for any $\varepsilon>0$, we have

$$DG(t)H+H^TG^T(t)D^T\le\varepsilon^{-1}DD^T+\varepsilon H^TH.$$

For convenience of our later development, the following concepts are proposed.

Definition 1: [18] The sliding motion in the δ vicinity of the sliding surface $S(k)=0$ satisfying

$$\|S(k)\| \le \delta$$

is called a quasi-sliding mode. In addition, the parameter $\delta>0$ is called the quasi-sliding mode band width.

Definition 2: [19] For any $k_v>k_s>k_0$, let $N_{\sigma(k)}(k_s,k_v)$ denote the number of switchings of σ over $[k_s,k_v]$. If

$$N_{\sigma(k)}(k_s,k_v) \le N_0 + \frac{(k_s-k_v)}{T_\sigma}$$

holds for $T_\sigma>0$ and $N_0\ge 0$, then T_σ is called the average dwell time.

In this work, let $N_0=0$, as is usually used in the previous works.

Definition 3: The equilibrium $x^*=0$ of system (1) is said to be exponentially stable under switching signal $\sigma(k)$ if there exist scalars $\rho>0$, $0<\beta<1$ such that the solution $x(k)$ satisfies

$$\|x(k)\| \le \rho\beta^{(k-k_0)}\|x(k_0)\|, \quad \forall\, k\ge k_0.$$

3. SLIDING SURFACE AND QUASI-SLIDING MODE DYNAMICS

In this section, a common sliding surface will be designed and the stability of the quasi-sliding mode dynamics will be analyzed.

The common sliding surface is designed as

$$S(k)=Dx(k), \tag{5}$$

where $D=(B^TB)^{-1}B^T$. It can be seen from Assumption 1 that B_i is full column rank, which implies that B is generally full column rank. Therefore, the non-singularity of B^TB can be guaranteed.

Remark 3: It is worth noting that the sliding surface (5) is different from the previous work [20], in which the sliding surface is mode-depended. If a mode-depended sliding surface is designed, it will be difficult to analyze the stability of the state trajectory as it jumps from one sliding surface to another. Therefore, a common sliding surface is designed.

It is noted that the ideal sliding mode satisfies

$$S(k+1)=S(k)=0, \tag{6}$$

In view of (5) and (6), one has

$$D(A_i+\Delta A_i)x(k)+DB_i(u_i(k)+f_i(k))=0. \tag{7}$$

Thus, the following equivalent controller $u_{eq}(k)$ is obtained

$$u_{eq}(k)=-(DB_i)^{-1}D(A_i+\Delta A_i)x(k)-f_i(k). \tag{8}$$

Substituting (8) into system (2), the ideal sliding mode is obtained as follows

$$x(k+1)=(I-B_i(DB_i)^{-1}D)(A_i+\Delta A_i)x(k). \tag{9}$$

Remark 4: The matrix $DB_i=I+D\mathcal{M}\mathcal{L}(i)\mathcal{N}$ is required to be non-singular. Hence, the parameters $\alpha_i, i=1,2,\cdots,s$, should be designed such that the non-singularity condition is satisfied.

In the above discussion, a common sliding surface has been designed and the corresponding idea sliding mode (9) is obtained. In the sequel, by employing the average dwell time method, we will discuss the exponential stability of the sliding mode dynamics (9).

Theorem 1: Consider the switched systems in (1) satisfying the Assumption 1. For the given scalar $0<\gamma<1$, if there exist matrice $P_i>0$, and parameters $\varepsilon_{i1}>0$, $\varepsilon_{i2}>0$, $i\in\Gamma$, satisfying the following linear matrix inequalities (LMIs)

$$\begin{bmatrix}\Theta_{i1} & \Theta_{i2}\\ * & -\varepsilon_{i1}I\end{bmatrix}<0, \tag{10}$$

$$\begin{bmatrix}-P_i & \Theta_{i3}\\ * & -\varepsilon_{i2}I\end{bmatrix}<0, \quad i\in\Gamma, \tag{11}$$

where

$$\begin{aligned}\Theta_{i1} &= ((I-B_i(DB_i)^{-1}D)A_i)^TP_i((I-B_i(DB_i)^{-1}D)A_i)\\ &\quad -\gamma P_i+\varepsilon_{i1}N_i^TN_i+\varepsilon_{i2}N_i^TN_i,\\ \Theta_{i2} &= ((I-B_i(DB_i)^{-1}D)A_i)^TP_i(I-B_i(DB_i)^{-1}D)E_i,\\ \Theta_{i3} &= P_i(I-B_i(DB_i)^{-1}D)E_i,\end{aligned}$$

then with the parameter

$$\mu=\max_{i,j\in\Gamma,i\ne j}\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}, \tag{12}$$

and the average dwell time T_σ satisfying

$$T_\sigma \ge T_{\sigma^*} > -\frac{\ln\mu}{\ln\gamma}, \tag{13}$$

the switched system (9) is exponentially stable. Furthermore, the state is estimated by

$$\|x(k)\| \le \rho\beta^{k-k_0}\|x(k_0)\|, \tag{14}$$

with the parameters

$$\begin{aligned}&\beta=\sqrt{\gamma\mu^{1/T_\sigma}}, \quad \rho=\sqrt{\frac{b}{a}}\ge 1,\\ &a=\min_{i\in\Gamma}\lambda_{\min}(P_i), \quad b=\max_{i\in\Gamma}\lambda_{\max}(P_i).\end{aligned} \tag{15}$$

Proof: For the switched systems (9), consider the Lyapunov function of the i-th subsystem as

$$V_i(k)=x^T(k)P_ix(k). \tag{16}$$

Thus, it follows (2) and Lemma 1 that

$$
\begin{aligned}
&V_i(k+1)-\gamma V_i(k)\\
&=x^T(k)((I-B_i(DB_i)^{-1}D)(A_i+\Delta A_i))^T P_i\\
&\quad\times(I-B_i(DB_i)^{-1}D)(A_i+\Delta A_i)x(k)-\gamma x^T(k)P_i x(k)\\
&\le x^T(k)((I-B_i(DB_i)^{-1}D)A_i)^T P_i\\
&\quad\times(I-B_i(DB_i)^{-1}D)A_i)x(k)+\varepsilon_{i1}x^T(k)N_1^T N_1 x(k)\\
&\quad+\varepsilon_{i1}^{-1}x^T(k)((I-B_i(DB_i)^{-1}D)A)^T P_i\\
&\quad\times(I-B_i(DB_i)^{-1}D)E_i(P_i(I-B_i(DB_i)^{-1}D)E_i)^T\\
&\quad\times(I-B_i(DB_i)^{-1}D)Ax(k)+x^T(k)\\
&\quad\times((I-B_i(DB_i)^{-1}D)\Delta A_i)^T P_i\\
&\quad\times(I-B_i(DB_i)^{-1}D)\Delta A_i x(k)-\gamma x^T(k)P_i x(k). \qquad (17)
\end{aligned}
$$

It can be shown from (17) and Schur's complement that

$$V_i(k+1)-\gamma V_i(k)<0, \qquad (18)$$

can be implied by

$$\begin{bmatrix}\Theta_{i4} & \Theta_{i2} & \Theta_{i5}\\ * & -\varepsilon_{i1}I & 0\\ * & * & -P_i\end{bmatrix}<0, \qquad (19)$$

where

$$
\begin{aligned}
\Theta_{i4}=&((I-B_i(DB_i)^{-1}D)A_i)^T P_i((I-B_i(DB_i)^{-1}D)A_i)\\
&-\gamma P_i+\varepsilon_{i1}N_i^T N_i,\\
\Theta_{i5}=&((I-B_i(DB_i)^{-1}D)\Delta A_i)^T P_i.
\end{aligned}
$$

The expression (19) can be rewritten as follows

$$\begin{bmatrix}\Theta_{i4} & \Theta_{i2} & 0\\ * & -\varepsilon_{i1}I & 0\\ * & * & -P_i\end{bmatrix}+\Xi\mathbb{G}\Pi+\Pi^T\mathbb{G}^T\Xi^T<0, \qquad (20)$$

where $\Xi=\begin{bmatrix}N_i & 0 & 0\end{bmatrix}^T$, $\mathbb{G}=G^T(k)$ and $\Pi=[0\ 0\ \ ((I-B_i(DB_i)^{-1}D)E_i)^T P_i]$.
In view of Schur's complement, it can be seen that (20) is implied by (10) and (11).

It can be shown from (18) that

$$V_i(k+1)\le\gamma V_i(k). \qquad (21)$$

Thus, for any $k\in[k_l,k_{l+1})$, it can be derived from (21) that

$$V_{\sigma(k)}(k)\le\gamma^{(k-k_l)}V_{\sigma(k_l)}(k_l). \qquad (22)$$

According to (12) and (22), there holds

$$
\begin{aligned}
V_{\sigma(k)}(k)\le&\gamma^{(k-k_l)}\mu V_{\sigma(k_l-1)}(k_l)\\
&\vdots\\
\le&\gamma^{(k-k_0)}\mu^{(k-k_0)/T_\sigma}V_{\sigma(k_0)}(k_0)\\
\le&(\gamma\mu^{1/T_\sigma})^{(k-k_0)}V_{\sigma(k_0)}(k_0). \qquad (23)
\end{aligned}
$$

Considering (15), one has

$$a\|x(k)\|^2\le V_{\sigma(k)}(k), \qquad (24)$$

and

$$V_{\sigma(k_0)}(k_0)\le b\|x(k_0)\|^2. \qquad (25)$$

It can be shown from (13) that

$$\alpha\mu^{1/T_\sigma}\le\gamma\mu^{-\ln\gamma/\ln\mu}\le 1. \qquad (26)$$

Combining (23)-(26), yields

$$
\begin{aligned}
\|x(k)\|^2\le&\frac{1}{a}V_{\sigma(k)}(k)\\
\le&\frac{b}{a}\beta^{2(k-k_0)}\|x(k_0)\|^2. \qquad (27)
\end{aligned}
$$

Therefore, the ideal sliding mode (9) is exponentially stable, which completes the proof. □

4. SLIDING MODE CONTROLLER DESIGN

In the following part, a SMC law will be designed to ensure the reachability of the quasi-sliding mode. To realize the control purpose, the SMC law is designed as follows:

$$S(k+1)-S(k)=-\varpi T\operatorname{sgn}(S(k))-qTS(k), \qquad (28)$$

where T is the sample time, ϖ and q are scalars that satisfying $0<\varpi<1$ and $1-qT>0$.

In view of (7) and (28), the sliding mode controller is obtained as follows:

$$
\begin{aligned}
u(k)=&-(DB_i)^{-1}(DA_i-(1-qT)D)x(k)\\
&-(DB_i)^{-1}\varpi T\operatorname{sgn}(S(k))-F_i(k), \qquad (29)
\end{aligned}
$$

where $F_i(k)=(DB_i)^{-1}D\Delta A_i x(k)+f_i(k)$.

Since the controller (29) contains the uncertain $F_i(k)$, it is not applicable in practice. Thus, the following SMC law is proposed

$$
\begin{aligned}
u(k)=&-(DB_i)^{-1}(DA_i-(1-qT)D)x(k)\\
&-(DB_i)^{-1}\varpi T\operatorname{sgn}(S(k))-\tilde{u}_i(k), \qquad (30)
\end{aligned}
$$

where

$$
\tilde{u}_i(k)=\begin{cases}(\alpha_i(k)+(DB_i)^{-1}\|DB_i\|\eta_i)\dfrac{S(k)}{\|S(k)\|}, & \\ & \text{if}\,\|S(k)\|\neq 0,\\ 0, & \text{if}\,\|S(x(k))=0\|,\end{cases}
$$

$$
\begin{aligned}
\alpha_i(k)=&(DB_i)^{-1}\|DE_i\|\|H_i x(k)\\
&+(DB_i)^{-1}\|DB_i\|d_i\|x(k)\|, \qquad (31)
\end{aligned}
$$

and η is a positive scalar.

In the following part, the result on reachability of the QSM will be analyzed.

Theorem 2: Consider the switched system (1) satisfying Assumption 1. If the SMC law is designed as (30), the QSM domain

$$\Omega=\Big\{\|S(k)\|<\zeta\mid$$
$$\zeta=\min_{i\in\Gamma}\Big\{\frac{(\varpi T)^2+\eta_i\|DB_i\|(2\varpi T+\eta_i\|DB_i\|)}{2\varpi T(1-qT)+2\eta_i(1-qT)\|DB_i\|}\Big\}\Big\} \tag{32}$$

can be reached in finite time. Moreover, the state trajectory will not escape from the domain once it enters there.

Proof: Choose the following Lyapunov function

$$V(k)=S^T(k)S(k). \tag{33}$$

According to (5) and the SMC law (30), one has

$$\begin{aligned}\Delta V(k)=&V(k+1)-V(k)\\ =&\big[(1-qT)S(k)-\varpi T\,\mathrm{sgn}(S(k))\big]^T\\ &\times\big[(1-qT)S(k)-\varpi T\,\mathrm{sgn}(S(k))\big]\\ &-2\varpi T\,\mathrm{sgn}(S(k))DB_i(F_i(k)-\tilde{u}_i(k))\\ &+2(1-qT)\big[DB_i(F_i(k)-\tilde{u}_i(k))\big]^TS(k)\\ &+\big[DB_i(F_i(k)-\tilde{u}_i(k))\big]^T\big[DB_i(F_i(k)-\tilde{u}_i(k))\big]\\ &-S^T(k)S(k).\end{aligned} \tag{34}$$

In view of (31), we get

$$\begin{aligned}&2(1-qT)\big[DB_i(F_i(k)-\tilde{u}_i(k))\big]^TS(k)\\ &\le-2\eta_i(1-qT)\|DB_i\|\|S(k)\|,\end{aligned} \tag{35}$$

$$\begin{aligned}&-2\big[DB_i(F_i(k)-\tilde{u}_i(k))\big]^T\varepsilon T\,\mathrm{sgn}(S(k))\\ &+\big[DB_i(F_i(k)-\tilde{u}_i(k))\big]^T\big[DB_i(F_i(k)-\tilde{u}_i(k))\big]\\ &\le\eta_i\|DB_i\|(2\varpi T+\eta_i\|DB_i\|).\end{aligned} \tag{36}$$

Combining (34)-(36), we have

$$\begin{aligned}\Delta V(k)=&(1-qT)^2S^T(k)S(k)\\ &-2\varpi T(1-qT)\|(S(k)\|\\ &+(\varpi T)^2-S^T(k)S(k)\\ &-2\eta_i(1-qT)\|DB_i\|\|S(k)\|\\ &+\eta_i\|DB_i\|(2\varpi T+\eta_i\|DB_i\|)\\ =&((1-qT)^2-1)S^T(k)S(k)-\big[2\varpi T(1-qT)\\ &+2\eta_i(1-qT)\|DB_i\|\big]\|S(k)\|\\ &+(\varpi T)^2+\eta_i\|DB_i\|(2\varpi T+\eta_i\|DB_i\|).\end{aligned} \tag{37}$$

It can be shown from (32) and (37) that, when $\|S(k)\|>\zeta$, there holds

$$\Delta V(k)\le 0.$$

This means that the QSM domain can be reached in finite time and the state trajectories of system (9) will remain in the QSM domain all the time, which completes the proof. □

Remark 5: It is shown that outside the domain of QSM there holds $\|S(k+1)\|<\|S(k)\|$, which means that $\|S(k)\|$ will finally enter into the QSM domain in finite time. Moreover, the quasi-sliding mode band width satisfies $\zeta=\min_{i\in\Gamma}\Big\{\frac{(\varpi T)^2+\eta_i\|DB_i\|(2\varpi T+\eta_i\|DB_i\|)}{2\varpi T(1-qT)+2\eta_i(1-qT)\|DB_i\|}\Big\}$.

5. SIMULATION

Consider a switched system as in (1) with two modes and parameters as follows:

Subsystem 1:

$$A_1=\begin{bmatrix}0.3&-0.3&0.4\\-0.1&-0.1&-0.2\\-0.4&0.2&-0.4\end{bmatrix},\ B_1=\begin{bmatrix}1&0.5\\1&-1\\0&1.5\end{bmatrix},$$
$$E_1=\begin{bmatrix}-0.5\\-0.5\\0.5\end{bmatrix},$$
$$G_1(k)=0.5\sin(t),\ N_1=\begin{bmatrix}-0.2&-0.2&0.2\end{bmatrix},$$
$$f_1(x)=\begin{bmatrix}\frac{1}{1+t^2}\\0\end{bmatrix}.$$

Subsystem 2:

$$A_2=\begin{bmatrix}0.1&-0.5&0.4\\0.5&-0.2&0.2\\0.4&0.5&0.2\end{bmatrix},\ B_2=\begin{bmatrix}-0.5&1.5\\1&-2\\1&0.5\end{bmatrix},$$
$$E_2=\begin{bmatrix}0.5\\-0.5\\0.5\end{bmatrix},$$
$$G_2(k)=0.5\cos(t),\ N_2=\begin{bmatrix}0.2&-0.2&-0.2\end{bmatrix},$$
$$f_2(x)=\begin{bmatrix}0\\\frac{1}{1+t^2}\end{bmatrix}.$$

By choosing $\alpha_1=\alpha_2=\frac{1}{2}$, it can be shown that $B_i=ML(i)N$. Moreover, the non-singular of the matrix DB_i can be guaranteed. For scalar $\gamma=0.9$, solving LMIs (10) and (11) yields

$$\varepsilon_{11}=32.8497,\ \varepsilon_{12}=32.2462,$$
$$\varepsilon_{21}=37.7794,\ \varepsilon_{22}=37.2609,$$
$$P_1=\begin{bmatrix}24.5019&-0.7601&7.7072\\-0.7601&35.1248&0.4201\\7.7072&0.4201&28.6393\end{bmatrix},$$
$$P_2=\begin{bmatrix}30.4217&-2.3981&6.9465\\-2.3981&40.2809&2.0903\\6.9465&2.0903&34.2725\end{bmatrix}.$$

Thus, the desired sliding surface (5) is designed as

$$S(t) = \begin{bmatrix} 0.3614 & 0.6231 & 0.5732 \\ 0.2991 & -0.2430 & 0.3364 \end{bmatrix} x(t).$$

According to Theorem 1, the parameters μ and T_σ are designed, respectively, as follows:

$$\mu = \max_{i,j\in\Gamma} \frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}, \quad \text{and} \quad {}_{\sigma} > \frac{\ln\mu}{\ln\lambda}.$$

Therefore, it can be obtained that the parameters

$$\mu = 1.1635,$$

and the average dwell time can be designed as

$$T_\sigma = 1.5.$$

Assume the sampling time $T = 0.1$ s, choose the parameter $\varpi = 0.5$ and $q = 2$. In view of (30)-(31), the SMC law is designed as

$$u(k) = \begin{cases} \begin{bmatrix} 0.3203 & 0.5203 & 0.4957 \\ 0.2029 & -0.1971 & 0.1565 \end{bmatrix} x(k) \\ \quad + \begin{bmatrix} -0.1043 & 0.0486 \\ 0.0065 & -0.1145 \end{bmatrix} \operatorname{sgn}(S(k)), \\ \quad \text{if} \quad i = 1 \text{ and } \|S(k)\| = 0, \\ \begin{bmatrix} -0.3102 & 0.3136 & -0.0159 \\ 0.1080 & -0.1773 & 0.0568 \end{bmatrix} x(k) \\ \quad + \begin{bmatrix} -0.1006 & -0.0381 \\ -0.0051 & -0.0926 \end{bmatrix} \operatorname{sgn}(S(k)), \\ \quad \text{if} \quad i = 2 \text{ and } \|S(k)\| = 0, \end{cases}$$

$$u(k) = \begin{cases} \begin{bmatrix} 0.3203 & 0.5203 & 0.4957 \\ 0.2029 & -0.1971 & 0.1565 \end{bmatrix} x(k) \\ \quad + \begin{bmatrix} -0.1043 & 0.0486 \\ 0.0065 & -0.1145 \end{bmatrix} \operatorname{sgn}(S(k)) \\ \quad - \Bigg(\begin{bmatrix} 0.2597 & -0.1208 \\ -0.0162 & 0.2849 \end{bmatrix} \\ \quad \times \left\| \begin{bmatrix} -0.2 & -0.2 & 0.2 \end{bmatrix} x(k) \right\| \\ \quad + \begin{bmatrix} 1.2509 & -0.5820 \\ -0.078 & 1.3725 \end{bmatrix} (d_1 \|x(k)\| \\ \quad + \eta_1) \Bigg) \frac{S(k)}{\|S(k)\|}, \\ \quad \text{if} \quad i = 1 \text{ and } \|S(k)\| \neq 0, \\ \begin{bmatrix} -0.3102 & 0.3136 & -0.0159 \\ 0.1080 & -0.1773 & 0.0568 \end{bmatrix} x(k) \\ \quad + \begin{bmatrix} -0.1006 & -0.0381 \\ -0.0051 & -0.0926 \end{bmatrix} \operatorname{sgn}(S(k)) \\ \quad - \Bigg(\begin{bmatrix} 0.4687 & 0.1774 \\ 0.0238 & 0.4316 \end{bmatrix} \\ \quad \left\| \begin{bmatrix} -0.2 & -0.2 & 0.2 \end{bmatrix} x(k) \right\| \\ \quad + \begin{bmatrix} 1.3227 & 0.5007 \\ 0.0673 & 1.2181 \end{bmatrix} (d_2 \|x(k)\| \\ \quad + \eta_2) \Bigg) \frac{S(k)}{\|S(k)\|}, \\ \quad \text{if} \quad i = 2 \text{ and } \|S(k)\| \neq 0. \end{cases}$$

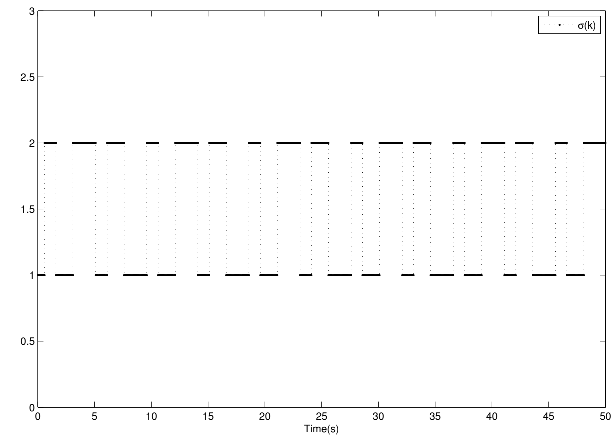

Fig. 1. Switching signal $\sigma(k)$.

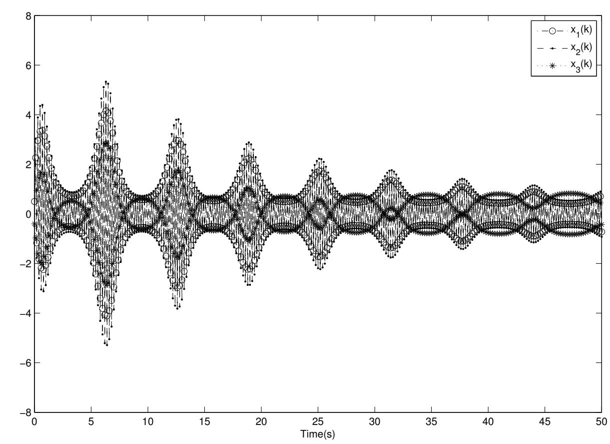

Fig. 2. State trajectories $x(k)$.

To eliminate the chattering phenomenon, the sign function $\operatorname{sgn}(S(k))$ is replaced by $\frac{S(k)}{\|S(k)\|+0.1}$. Then, For the initial states $x(0) = \begin{bmatrix} 0.5 & -0.5 & -0.4 \end{bmatrix}^T$, choose the parameter $d_1 = d_2 = 2$ and $\eta_1 = \eta_2 = 0.8$, the simulation results with the proposed sliding mode controller can be seen in Figs. 1-4. The switching signal is given in Fig. 1. The control signal is depicted in Fig. 4. It can be seen from Figs. 2 and 3 that the states $x_1(k)$, $x_2(k)$ and $x_3(k)$ will be driven onto the domain of the sliding mode and exponentially tend to zero.

6. CONCLUSION

In this paper, we have discussed SMC for a class of uncertain discrete switched systems. By employing the weighted sum approach, a common sliding surface is designed. Moreover, the exponential stability of the sliding mode dynamics is analyzed by adopting the average dwell time strategy. Besides, it is shown that the state trajectories can be driven onto the domain of the quasi-sliding

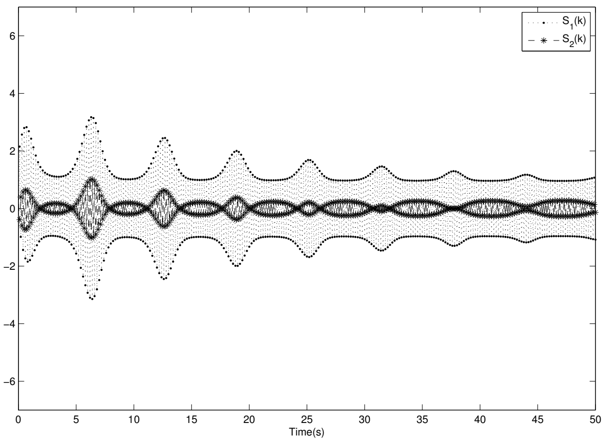

Fig. 3. Sliding surface $S(k)$.

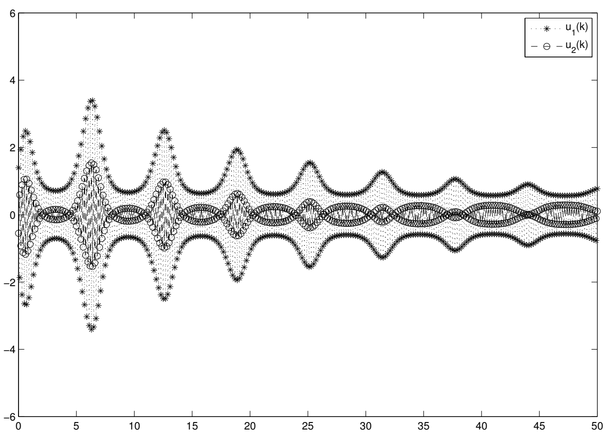

Fig. 4. Control input $u(k)$.

mode despite the presence of the sample time, parameter uncertainties and external disturbances. However, for physical models, there are many more complex phenomena such as stochastic disturbance effect [21]. These may be further considered in the future research.

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