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Robust Finite-time Extended Dissipative Control for a Class of Uncertain Switched Delay Systems

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Abstract: This paper investigates the problem of finite-time extended dissipative analysis and control for a class of uncertain switched time delay systems, where the uncertainties satisfy the polytopic form. By using the average dwell-time and linear matrix inequality technique, some sufficient conditions are proposed to guarantee that the switched system is finite-time bounded and has finite-time extended dissipative performance, where the H_∞, L_2-L_∞, Passivity and (Q, S, R)-dissipativity performance can be solved simultaneously in a unified framework based on the concept of extended dissipative. Furthermore, a state feedback controller is presented to guarantee that the closed-loop system is finite-time bounded and satisfies the extended dissipative performance. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

Keywords: Average dwell-time, extended dissipative, finite-time, switch, time delay.

1. INTRODUCTION

In the last decades, switched systems have received considerable attention for their significant application in various fields [1–5]. Switched system is a special class of hybrid system, which consists of a finite number of subsystems and a logical rule that orchestrates switching between them. It is well known that time delay exists widely in many practical systems and may cause undesirable system performance or even instability [6]. Thus, great attention has been paid to switched delay systems in literatures [7–10]. For example, for several switched delay systems, the problem of stability and robust stabilization were considered in [8,9]. [7] focused on the study of H_∞ control. It should be noted that, all the above literature related to stability of switched systems focus on Lyapunov asymptotic stability, which is defined over an infinite time interval. However, in practice, the transient performance of a system is also of great significant. And numerous finite-time analysis and control for different switched systems were presented in recent literatures [11–14]. Specially, finite-time boundedness and L_2-gain analysis for switched delay systems was investigated in [11], robust finite-time H_∞ control was addressed in [12], etc.

On the other hand, an important concept named extended dissipative was firstly proposed by Zhang in [15]. The novel feature of the concept is that by adjusting weighting matrices, the extended dissipative covers some well-known performance indices such as H_∞ performance, L_2-L_∞ performance, Passivity performance and (Q, S, R)-dissipativity performance. Recently, the concept of extended dissipative concept has been effectively applied to several neural networks in reports [16–19]. To the best of our knowledge, the extended dissipative concept has not been applied to switched systems yet, and based on above discussion, finite-time analysis for switched systems is worth researthing. Can we design the controller for a class of switched time delay systems to make the closed loop systems be finite-time extended dissipative? This question is interesting and has not been investigated yet, which motivates our current study.

This paper is organized as follows: In Section 2, Preliminaries and Problem Statement are formulated. In Section 3, finite-time boundness and finite-time extended dissipative performance for switched delay systems are addressed, meanwhile, state feedback controllers are proposed in terms of a set of linear matrix inequalities. In Section 4, a numerical example is provided to show the effectiveness of the proposed approach. Finally, conclusions are given in Section 5. The most notable contributions of this paper can be summarized as follows: 1) The concept of extended dissipative is successfully applied to switched systems. 2) We addressed the problem of finite-time extended dissipative analysis and control for a class

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of delayed uncertain switched systems, where the uncertainties satisfy the polytopic form. 3) We create a novel designed controller method to deal with the polytopic uncertainties.

2. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following uncertain switched system with time delay:

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x(t)+B_{\sigma(t)}x(t-h(t))+D_{\sigma(t)}w(t)\\ &\quad +E_{\sigma(t)}u(t)+G_{\sigma(t)}\int_{t-r(t)}^{t}x(s)ds,\\ z(t) &= F_{\sigma(t)}x(t),\\ x(t_0+\theta) &= \varphi(\theta),\ \ \forall\theta\in[-\tau,0],\end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state vector, $u(t)$ the control input, $w(t)$ the exogenous disturbance which belongs to $L_2[0,\infty)$, $z(t)\in R^n$ the output. The switching signal $\sigma(t):[0,\infty)\mapsto M=\{1,2...l\}$ is a piecewise continuous function, where l is the number of subsystems, $\sigma(t)=i$ means that the ith subsystem is activated. For each $\sigma(t)=i$, the matrix parameters A_i,B_i,D_i,E_i,F_i,G_i belong to the following uncertainty polytype:

$$\begin{aligned}&[A_i,B_i,D_i,E_i,F_i,G_i]=\sum_{j=1}^{p}\theta_j[A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},G_{ij}],\\ &\sum_{j=1}^{p}\theta_j=1,\ \ \theta_j\geq 0,\end{aligned} \tag{2}$$

where $\theta_j(j\in\{1,2...p\})$ are time-invariant uncertainties, $\varphi(\theta)$ is the initial condition, $h(t)$, $r(t)$ denote the time-varying delay and satisfying

$$0\leq h(t)\leq h_m,\ \ \dot{h}(t)\leq\hat{h}<1,\ \ 0\leq r(t)\leq r_m. \tag{3}$$

Assumption 1 [11]**:** For a given time constant T_f, the external disturbance satisfies

$$\int_0^{T_f}w^T(t)w(t)dt\leq d,\ \ d\geq 0.$$

Assumption 2: For a given time constant T_f, the state vector $x(t)$ is time-varying and satisfies the constraint

$$\int_0^{T_f}x^T(t)x(t)dt\leq k,$$

where k is a fixed sufficient large constant number.

Assumption 3 [15]**:** Matrices $\psi_1,\psi_2,\psi_3,\psi_4$ satisfy the following conditions:

1) $\psi_1=\psi_1^T\leq 0,\psi_3=\psi_3^T>0,\psi_4=\psi_4^T\geq 0;$
2) $(\|\psi_1\|+\|\psi_2\|)\psi_4=0.$

Assumption 4: For $\forall\alpha\geq 0$, $\mu\geq 1$, $\forall t\in[0,T_f]$, we have

$$e^{\alpha t}\mu^{N_\sigma(0,t)}\leq b,$$

$N_\sigma(0,t)$ denote the switching number of $\sigma(t)$ over $(0,t)$, and b a positive number.

Definition 1: [11] Given three positive constants c_1,c_2,T_f with $c_1<c_2$, a positive definite matrix R and a switching signal $\sigma(t)$, assume that $\mu(t)\equiv 0,\forall t\in[0,T_f]$, switched system (1) is said to be finite-time bounded with respect to (c_1,c_2,R,T_f,σ), if $\forall t\in[0,T_f]$,

$$\begin{aligned}&\sup_{-\tau\leq\theta\leq 0}\{\,x^T(\theta)Rx(\theta),\dot{x}^T(\theta)R\dot{x}(\theta)\}\leq c_1\\ &\Rightarrow x^T(t)Rx(t)\leq c_2.\end{aligned}$$

If the condition above holds with $w(t)\equiv 0,\forall t\in[0,T_f]$, the system is said to be finite-time stable.

Definition 2 [11]**:** For any $T_2>T_1\geq 0$, let $N_\sigma(T_1,T_2)$ denotes the switching number of $\sigma(t)$ over (T_1,T_2). If

$$N_\sigma(T_1,T_2)\leq N_0+\frac{T_2-T_1}{\tau_a}$$

holds for $\tau_a>0$ and an integer $N_0\geq 0$, then τ_a is called an average dwell-time. Without loss of generality, in this paper we choose $N_0=0$.

Definition 3 [15]**:** For given matrices ψ_1,ψ_2,ψ_3 and ψ_4 satisfying Assumption 3, system (1) is said to be extended dissipative if the following inequality holds for any $T_f\geq 0$ and all $\mu(t)\in L_2[0,\infty)$:

$$\int_0^{T_f}J(t)dt-\sup_{0\leq t\leq T_f}z^T(t)\psi_4 z(t)\geq 0, \tag{4}$$

where $J(t)=z^T(t)\psi_1 z(t)+2z^T(t)\psi_2 w(t)+w^T(t)\psi_3 w(t)$.

Lemma 1 [16]**:** For any real matrices a,b of appropriate dimensions, we have $2a^Tb\leq a^Ta+b^Tb$.

Lemma 2 [13]**:** For any positive definite symmetric matrix $N\in R^{n\times n}$, scalar $\tau>0$ and a vector function $x(\cdot):[-\tau,0]\to R^n$ the following integral inequality is satisfied

$$-\tau\int_{t-\tau}^{t}x^T(s)Nx(s)ds\leq-\int_{t-\tau}^{t}x^T(s)dsN\int_{t-\tau}^{t}x(s)ds.$$

3. MAIN RESULTS

3.1. Finite-time boundedness analysis

Consider the following unforced switched system with time delay

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x(t)+B_{\sigma(t)}x(t-h(t))+D_{\sigma(t)}w(t)\\ &\quad +G_{\sigma(t)}\int_{t-r(t)}^{t}x(s)ds,\\ z(t) &= F_{\sigma(t)}x(t),\\ x(t_0+\theta) &= \varphi(\theta),\forall\theta\in[-\tau,0].\end{aligned} \tag{5}$$

Theorem 1: Consider system (5). For given positive scalars $\alpha, \hat{h}, h_m, r_m$, if there exist positive definite symmetric matrices $\tilde{P}_i, \tilde{Q}_i, \tilde{M}_i$ and matrices N_i and Y_{jl} with $Y_{jj} = Y_{jj}^T, j = 1,2...,p, l = j,...,p$ with appropriate dimensions, such that

$$\Theta_i^{(jj)} < Y_{jj}, j = 1,2...p, \tag{6}$$

$$\Theta_i^{(jl)} + \Theta_i^{(lj)} < Y_{jl} + Y_{jl}^T, j < l, j,l = 1,2...p, \tag{7}$$

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} \\ * & Y_{22} & \cdots & Y_{2p} \\ * & * & \ddots & \vdots \\ * & * & * & Y_{pp} \end{bmatrix} \leq 0, \tag{8}$$

$$(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + r_m e^{\alpha r_m}\lambda_6)c_1 + \lambda_7 d < c_2\lambda_1 e^{-\alpha T_f}, \tag{9}$$

the average dwell-time satisfies

$$\tau_a > \tau_a^* = \frac{T_f ln\mu}{\begin{pmatrix} \ln(\lambda_1 c_2) \\ \quad -\ln[(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + r_m e^{\alpha r_m}\lambda_4)c_1 + \lambda_5 d] \\ -\alpha T_f \end{pmatrix}},$$

where $\mu > 1$ satisfies

$$\tilde{P}_i < \mu\tilde{P}_j, \tilde{Q}_i < \mu\tilde{Q}_j, \tilde{M}_i < \mu\tilde{M}_j, \forall i,j \in M. \tag{10}$$

And

$$\tilde{P}_i = R^{\frac{1}{2}}P_iR^{\frac{1}{2}}, \tilde{Q}_i = R^{\frac{1}{2}}Q_iR^{\frac{1}{2}}, \tilde{M}_i = R^{\frac{1}{2}}M_iR^{\frac{1}{2}},$$

where

$$[P_i, Q_i, M_i] = \sum_{j=1}^{p}\theta_j[P_{ij}, Q_{ij}, M_{ij}], \sum_{j=1}^{p}\theta_j = 1, \theta_j \geq 0,$$

and

$$\tilde{P}_{ij} = R^{\frac{1}{2}}P_{ij}R^{\frac{1}{2}}, \tilde{Q}_{ij} = R^{\frac{1}{2}}Q_{ij}R^{\frac{1}{2}}, \tilde{M}_{ij} = R^{\frac{1}{2}}M_{ij}R^{\frac{1}{2}}.$$

Furthermore,

$$\Theta_i^{(jj)} = \begin{bmatrix} \phi_{11}^{(jj)} & \tilde{P}_{ij}B_{ij} & \tilde{P}_{ij}D_{ij} & \tilde{P}_{ij}G_{ij} \\ * & -(1-\hat{h})\tilde{Q}_{ij} & 0 & 0 \\ * & * & -N_i & 0 \\ * & * & * & -\frac{\tilde{M}_{ij}}{r_m} \end{bmatrix},$$

$$\phi_{11}^{(jj)} = -\alpha\tilde{P}_{ij} + \tilde{P}_{ij}A_{ij} + A_{ij}^T\tilde{P}_{ij} + \tilde{Q}_{ij} + r_m\tilde{M}_{ij},$$

$$\Theta_i^{(jl)} = \begin{bmatrix} \phi_{11}^{(jl)} & \tilde{P}_{il}B_{ij} & \tilde{P}_{il}D_{ij} & \tilde{P}_{il}G_{ij} \\ * & -(1-\hat{h})\tilde{Q}_{il} & 0 & 0 \\ * & * & -N_i & 0 \\ * & * & * & -\frac{\tilde{M}_{il}}{r_m} \end{bmatrix},$$

$$\phi_{11}^{(jl)} = -\alpha\tilde{P}_{il} + \tilde{P}_{il}A_{ij} + A_{ij}^T\tilde{P}_{il} + \tilde{Q}_{il} + r_m\tilde{M}_{il},$$

$$\Theta_i^{(lj)} = \begin{bmatrix} \phi_{11}^{(lj)} & \tilde{P}_{ij}B_{il} & \tilde{P}_{ij}D_{il} & \tilde{P}_{ij}G_{il} \\ * & -(1-\hat{h})\tilde{Q}_{ij} & 0 & 0 \\ * & * & -N_i & 0 \\ * & * & * & -\frac{\tilde{M}_{ij}}{r_m} \end{bmatrix},$$

$$\phi_{11}^{(lj)} = -\alpha\tilde{P}_{ij} + \tilde{P}_{ij}A_{il} + A_{il}^T\tilde{P}_{ij} + \tilde{Q}_{ij} + r_m\tilde{M}_{ij},$$

and

$$\lambda_{min}(P_i) = \lambda_1,\ \lambda_{max}(P_i) = \lambda_2,\ \lambda_{max}(Q_i) = \lambda_3,$$
$$\lambda_{max}(M_i) = \lambda_4,\ \lambda_{max}(N_i) = \lambda_5. \tag{11}$$

Then switched system (5) is finite-time bounded with respect to $(c_1, c_2, d, R, T_f, \sigma)$.

Proof: Choose the Lyapunov-Krasovskii functional candidate as

$$V(t) = V_{\sigma(t)}(t) = V_i(t) = V_{1i}(t) + V_{2i}(t) + V_{3i}(t), \tag{12}$$

where

$$\begin{aligned} V_{1i}(t) &= x^T(t)\tilde{P}_ix(t), \\ V_{2i}(t) &= \int_{t-h(t)}^{t} e^{\alpha(t-s)}x^T(s)\tilde{Q}_ix(s)ds, \\ V_{3i}(t) &= \int_{-r_m}^{0}\int_{t+\varepsilon}^{t} e^{\alpha(t-s)}x^T(s)\tilde{M}_ix(s)dsd\varepsilon, \end{aligned} \tag{13}$$

in which α is a given scalar and $\tilde{P}_i, \tilde{Q}_i, \tilde{M}_i$ are positive definite matrices to be determined.

Taking the derivative of $V(t)$ with respect to t along the trajectory of system (5) yields

$$\begin{aligned} \dot{V}_{1i}(t) &= 2x^T(t)\tilde{P}_i\dot{x}(t), \\ \dot{V}_{2i}(t) &= \alpha V_{2i}(t) + x^T(t)\tilde{Q}_ix(t) \\ &\quad - e^{\alpha h(t)}(1-\dot{h}(t))x^T(t-h(t))\tilde{Q}_ix(t-h(t)) \\ &\leq \alpha V_{2i}(t) + x^T(t)\tilde{Q}_ix(t) \\ &\quad - (1-\hat{h})x^T(t-h(t))\tilde{Q}_ix(t-h(t)), \\ \dot{V}_{3i}(t) &= \alpha V_{3i}(t) + r_mx^T(t)\tilde{M}_ix(t) \\ &\quad - \int_{t-r(t)}^{t} e^{\alpha(t-s)}x^T(s)\tilde{M}_ix(s)ds \\ &\leq \alpha V_{3i}(t) + r_mx^T(t)\tilde{M}_ix(t) \\ &\quad - \int_{t-r(t)}^{t} x^T(s)\tilde{M}_ix(s)ds \end{aligned}$$

by Lemma 2, it is easy to obtain that

$$\begin{aligned} &-\int_{t-r(t)}^{t} x^T(s)\tilde{M}_ix(s)ds \\ &\leq -\frac{1}{r_m}\int_{t-r(t)}^{t} x^T(s)ds\tilde{M}_i\int_{t-r(t)}^{t} x(s)ds. \end{aligned}$$

Thus

$$\dot{V}(t)-\alpha V(t)-w^T(t)N_iw(t)\leq X^T(t)\Theta_iX(t),$$

where

$$X(t)=\left[\begin{array}{cccc} x^T(t) & x^T(t-h(t)) & w^T(t) & \int_{t-r(t)}^{t}x^T(s)ds \end{array}\right]^T,$$

meanwhile, we obtain from (6)-(8) that

$$\begin{aligned}\Theta_i&=\sum_{j=1}^{p}\theta_{ij}^2\Theta_i^{(jj)}+\sum_{j=1}^{p-1}\sum_{l=j+1}^{p}\theta_{(ij)}\theta_{(il)}(\Theta_i^{(jl)}+\Theta_i^{(lj)})\\ &<\sum_{j=1}^{p}\theta_{ij}^2Y_{jj}+\sum_{j=1}^{p-1}\sum_{l=j+1}^{p}\theta_{(ij)}\theta_{(il)}(Y_{jl}+Y_{lj}^T)\\ &=\Delta\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p}\\ * & Y_{22} & \cdots & Y_{2p}\\ * & * & \ddots & \vdots\\ * & * & * & Y_{pp}\end{bmatrix}\Delta^T\leq 0, \qquad (14)\end{aligned}$$

where

$$\Theta_i=\begin{bmatrix}\Theta_{11} & \tilde{P}_iB_i & \tilde{P}_iD_i & \tilde{P}_iG_i\\ * & -(1-\hat{h})\tilde{Q}_i & 0 & 0\\ * & * & -N_i & 0\\ * & * & * & -\frac{\tilde{M}_i}{r_m}\end{bmatrix},$$

$$\Theta_{11}=-\alpha\tilde{P}_i+\tilde{P}_iA_i+A_i^T\tilde{P}_i+\tilde{Q}_i+r_m\tilde{M}_i,$$

and

$$\Delta=[\theta_{i1}I,\theta_{i2}I,\cdots,\theta_{ip}I],$$

Therefore, from (12) to (14), we can obtain that

$$\dot{V}(t)-\alpha V(t)-w^T(t)N_iw(t)<0. \qquad (15)$$

Integrating (15), it can be obtained from (10) and (15) that, for $\forall t\in[t_k,t_{k+1})$,

$$\begin{aligned}V(t)<&e^{\alpha(t-t_k)}V(t_k)+\int_{t_k}^{t}e^{\alpha(t-s)}w^T(s)N_iw(s)ds\\ <&e^{\alpha(t-t_k)}\mu V(t_k^-)+\int_{t_k}^{t}e^{\alpha(t-s)}w^T(s)N_iw(s)ds\\ <&e^{\alpha(t-t_k)}\mu[e^{\alpha(t_k-t_{k-1})}V(t_{k-1})\\ &+\int_{t_{k-1}}^{t_k}e^{\alpha(t_k-s)}w^T(s)N_iw(s)ds]\\ &+\int_{t_k}^{t}e^{\alpha(t-s)}w^T(s)N_iw(s)ds\\ <&e^{\alpha(t-0)}\mu^{N_\sigma(0,t)}V(0)\\ &+\mu^{N_\sigma(0,t)}\int_0^{t_1}e^{\alpha(t-s)}w^T(s)N_iw(s)ds\\ &+\mu^{N_\sigma(t_1,t)}\int_{t_1}^{t_2}e^{\alpha(t-s)}w^T(s)N_iw(s)ds+\cdots\\ &+\mu\int_{t_{k-1}}^{t_k}e^{\alpha(t-s)}w^T(s)N_iw(s)ds\\ &+\int_{t_k}^{t}e^{\alpha(t-s)}w^T(s)N_iw(s)ds\\ =&e^{\alpha(t-0)}\mu^{N_\sigma(0,t)}V(0)\\ &+\int_0^{t}e^{\alpha(t-s)}\mu^{N_\sigma(s,t)}w^T(s)N_iw(s)ds\\ <&e^{\alpha t}\mu^{N_\sigma(0,t)}V(0)\\ &+\mu^{N_\sigma(0,t)}e^{\alpha t}\int_0^{t}w^T(s)N_iw(s)ds\\ <&e^{\alpha T_f}\mu^{N_\sigma(0,T_f)}[V(0)+\int_0^{T_f}w^T(s)N_iw(s)ds]\\ <&e^{\alpha T_f}\mu^{N_\sigma(0,T_f)}[V(0)+\lambda_{max}(N_i)d]. \qquad (16)\end{aligned}$$

From Definition 3, we know

$$N_\sigma(0,T_f)<\frac{T_f}{\tau_a},$$

such that

$$V(t)<e^{(\alpha+\frac{ln\mu}{\tau_a})T_f}[V(0)+\lambda_5d]. \qquad (17)$$

On the other hand,

$$\begin{aligned}V(t)&>x^T(t)\tilde{P}_ix(t)=x^T(t)R^{\frac{1}{2}}P_iR^{\frac{1}{2}}x(t)\\ &\geq\lambda_{min}(P_i)x^T(t)Rx(t)=\lambda_1x^T(t)Rx(t), \qquad (18)\\ V(0)&\\ \leq&\lambda_{max}(P_i)x^T(0)Rx(0)\\ &+h_me^{\alpha h_m}\lambda_{max}(Q_i)\sup_{-\tau\leq\theta\leq0}\{x^T(\theta)Rx(\theta),\dot{x}^T(\theta)R\dot{x}(\theta)\}\\ &+r_me^{\alpha r_m}\lambda_{max}(M_i)\sup_{-\tau\leq\theta\leq0}\{x^T(\theta)Rx(\theta),\dot{x}^T(\theta)R\dot{x}(\theta)\}\\ \leq&[\lambda_{max}(P_i)+h_me^{\alpha h_m}\lambda_{max}(Q_i)+r_me^{\alpha r_m}\lambda_{max}(M_i)]c_1\\ \leq&[\lambda_2+h_me^{\alpha h_m}\lambda_3+r_me^{\alpha r_m}\lambda_4]c_1. \qquad (19)\end{aligned}$$

From (17)-(19), we have

$$\begin{aligned}&x^T(t)Rx(t)\\ &\leq\frac{V(t)}{\lambda_1} \qquad (20)\\ &<\frac{[\lambda_2+h_me^{\alpha h_m}\lambda_3+r_me^{\alpha r_m}\lambda_4]c_1+\lambda_5d}{\lambda_1}e^{\alpha T_f}\mu^{\frac{T_f}{\tau_a}}. \qquad (21)\end{aligned}$$

By virtue of (9), we have

$$\begin{aligned}&ln(\lambda_1c_2)-\ln[(\lambda_2+h_me^{\alpha h_m}\lambda_3+r_me^{\alpha r_m}\lambda_4)c_1+\lambda_5d]\\ &\quad-\alpha T_f>0.\end{aligned}$$

Considering (10), we have

$$\begin{aligned}&\frac{T_f}{\tau_a}<\frac{\ln(\lambda_1c_2)-\alpha T_f}{ln(\mu)}, \qquad (22)\\ &-\frac{\ln[(\lambda_2+h_me^{\alpha h_m}\lambda_3+r_me^{\alpha r_m}\lambda_4)c_1+\lambda_5d]}{\ln(\mu)}, \qquad (23)\end{aligned}$$

$$= \frac{\ln[\frac{\lambda_1 c_2 e^{-\alpha T_f}}{(\lambda_2+h_m e^{\alpha h_m}\lambda_3+r_m e^{\alpha r_m}\lambda_4)c_1+\lambda_5 d}]}{\ln(\mu)}. \quad (24)$$

Substituting (24) into (21) yields

$$\begin{aligned} & x^T(t)Rx(t) \\ & < [\frac{[\lambda_2+h_m e^{\alpha h_m}\lambda_3+r_m e^{\alpha r_m}\lambda_4]c_1+\lambda_5 d}{\lambda_1}]e^{\alpha T_f}, \\ & \left[\frac{\lambda_1 c_2 e^{-\alpha T_f}}{(\lambda_2+h_m e^{\alpha h_m}\lambda_3+r_m e^{\alpha r_m}\lambda_4)c_1+\lambda_5 d}\right] = c_2. \end{aligned}$$

Therefore, system (5) is finite-time bounded with respect to $(c_1, c_2, d, R, T_f, \sigma)$ from Definition 1. The proof is completed. □

3.2. Finite time extended dissipative analysis

Theorem 2: Consider system (5). For given positive scalars $\alpha, \hat{h}, h_m, r_m, b$, if there exist positive definite symmetric matrices $\tilde{P}_i, \tilde{Q}_i, \tilde{M}_i$ and matrices Y_{jl} with $Y_{jj} = Y_{jj}^T, j = 1,2...,p, l = j,...,p$ with appropriate dimensions, such that

$$\frac{1}{b}\tilde{P}_i - F_i^T\psi_4 F_i > 0, \quad (25)$$

$$\Phi_i^{(jj)} < Y_{jj}, \quad j = 1,2,...,p, \quad (26)$$

$$\Phi_i^{(jl)} + \Phi_i^{(lj)} < Y_{jl} + Y_{jl}^T, \quad j < l, \quad j,l = 1,2,...,p, \quad (27)$$

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} \\ * & Y_{22} & \cdots & Y_{2p} \\ * & * & \ddots & \vdots \\ * & * & * & Y_{pp} \end{bmatrix} \le 0, \quad (28)$$

the average dwell time satisfies

$$\tau_a > \tau_a^* = \frac{T_f ln\mu}{ln(\lambda_1 c_2) - ln[\lambda_6 k + (\lambda_7+\lambda_8)d] - \alpha T_f}, \quad (29)$$

where

$$\tilde{P}_i = R^{\frac{1}{2}}P_i R^{\frac{1}{2}}, \tilde{Q}_i = R^{\frac{1}{2}}Q_i R^{\frac{1}{2}}, \tilde{M}_i = R^{\frac{1}{2}}M_i R^{\frac{1}{2}},$$

we denote

$$[P_i, Q_i, M_i] = \sum_{j=1}^{p}\theta_j[P_{ij}, Q_{ij}, M_{ij}], \sum_{j=1}^{p}\theta_j = 1, \theta_j \ge 0,$$

and

$$\tilde{P}_{ij} = R^{\frac{1}{2}}P_{ij}R^{\frac{1}{2}}, \tilde{Q}_{ij} = R^{\frac{1}{2}}Q_{ij}R^{\frac{1}{2}}, \tilde{M}_{ij} = R^{\frac{1}{2}}M_{ij}R^{\frac{1}{2}}.$$

And

$$\Phi_i^{(jj)} = \begin{bmatrix} \Xi_{11}^{(jj)} & \tilde{P}_{ij}B_{ij} & \tilde{P}_{ij}D_{ij} - F_{ij}^T\psi_2 & \tilde{P}_{ij}G_{ij} \\ * & -(1-\hat{h})\tilde{Q}_{ij} & 0 & 0 \\ * & * & -\psi_3 & 0 \\ * & * & * & -\frac{\tilde{M}_{ij}}{r_m} \end{bmatrix},$$

$$\Xi_{11}^{(jj)} = -\alpha\tilde{P}_{ij} + \tilde{P}_{ij}A_{ij} + A_{ij}^T\tilde{P}_{ij} + \tilde{Q}_{ij} - F_{ij}^T\psi_1 F_{ij} + r_m\tilde{M}_{ij},$$

$$\Phi_i^{(jl)} = \begin{bmatrix} \Xi_{11}^{(jl)} & \tilde{P}_{il}B_{ij} & \tilde{P}_{il}D_{ij} - F_{ij}^T\psi_2 & \tilde{P}_{il}G_{ij} \\ * & -(1-\hat{h})\tilde{Q}_{il} & 0 & 0 \\ * & * & -\psi_3 & 0 \\ * & * & * & -\frac{\tilde{M}_{il}}{r_m} \end{bmatrix},$$

$$\Xi_{11}^{(jl)} = -\alpha\tilde{P}_{il} + \tilde{P}_{il}A_{ij} + A_{ij}^T\tilde{P}_{il} + \tilde{Q}_{il} - F_{ij}^T\psi_1 F_{ij} + r_m\tilde{M}_{il},$$

$$\Phi_i^{(lj)} = \begin{bmatrix} \Xi_{11}^{(lj)} & \tilde{P}_{ij}B_{il} & \tilde{P}_{ij}D_{il} - F_{il}^T\psi_2 & \tilde{P}_{ij}G_{il} \\ * & -(1-\hat{h})\tilde{Q}_{ij} & 0 & 0 \\ * & * & -\psi_3 & 0 \\ * & * & * & -\frac{\tilde{M}_{ij}}{r_m} \end{bmatrix},$$

$$\Xi_{11}^{(lj)} = -\alpha\tilde{P}_{ij} + \tilde{P}_{ij}A_{il} + A_{il}^T\tilde{P}_{ij} + \tilde{Q}_{ij} - F_{il}^T\psi_1 F_{il} + r_m\tilde{M}_{ij},$$

and

$$\lambda_{min}(P_i) = \lambda_1, \lambda_{max}(F_i^T F_i) = \lambda_6, \lambda_{max}(\psi_2^T\psi_2) = \lambda_7,$$
$$\lambda_{max}(\psi_3) = \lambda_8.$$

Then the system is finite-time bounded and satisfies the extended dissipative performance.

Proof: Choose the same Lyapunov-Krasovskii function as in (12)-(13), similar to the proof of Theorem 1, we have

$$\dot{V}(t) - \alpha V(t) - J(t) \le X^T(t)\Phi_i X(t)$$

where

$$X(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & w^T(t) & \int_{t-r(t)}^{t} x^T(s)ds \end{bmatrix}^T,$$

by (26)-(28), we can obtain

$$\begin{aligned} \Phi_i &= \sum_{j=1}^{p}\theta_{ij}^2\Phi_i^{(jj)} + \sum_{j=1}^{p-1}\sum_{l=j+1}^{p}\theta_{(ij)}\theta_{(il)}(\Phi_i^{(jl)} + \Phi_i^{(lj)}) \\ &< \sum_{j=1}^{p}\theta_{ij}^2 Y_{jj} + \sum_{j=1}^{p-1}\sum_{l=j+1}^{p}\theta_{(ij)}\theta_{(il)}(Y_{jl} + Y_{lj}^T) \\ &= \Delta \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} \\ * & Y_{22} & \cdots & Y_{2p} \\ * & * & \ddots & \vdots \\ * & * & * & Y_{pp} \end{bmatrix} \Delta^T \le 0, \end{aligned} \quad (30)$$

where

$$\Phi_i = \begin{bmatrix} \Phi_{11} & \tilde{P}_i B_i & \tilde{P}_i D_i - F_i^T\psi_2 & \tilde{P}_i G_i \\ * & -(1-\hat{h})\tilde{Q}_i & 0 & 0 \\ * & * & -\psi_3 & 0 \\ * & * & * & -\frac{\tilde{M}_i}{r_m} \end{bmatrix},$$

$$\Phi_{11} = -\alpha\tilde{P}_i + \tilde{P}_i A_i + A_i^T \tilde{P}_i + \tilde{Q}_i - F_i^T \psi_1 F_i + r_m \tilde{M}_i,$$

and

$$\Delta = [\theta_{i1} I, \theta_{i2} I, \cdots, \theta_{ip} I].$$

Therefore, from (30), we have that

$$\dot{V}(t) - \alpha V(t) - J(t) < 0,$$

following the proof line of (16), it is easy to obtain the following inequality

$$V(t) < e^{\alpha t}\mu^{N_\sigma(0,t)} V(0) + \int_0^t e^{\alpha(t-s)}\mu^{N_\sigma(s,t)} J(s)ds,$$

under zero initial condition V(0)=0, it can be calculated that

$$V(t) < e^{\alpha t}\mu^{N_\sigma(0,t)} \int_0^t J(s)ds,$$

by Assumption 4 we have

$$\int_0^t J(s)ds > \frac{V(t)}{b} > \frac{1}{b} x^T(t)\tilde{P}_i x(t) > 0,$$

considering inequality

$$\int_0^{T_f} J(t)dt - \sup_{0\le t\le T_f} z^T(t)\psi_4 z(t) \ge 0,$$

when $\psi_4 = 0$, it is apparent that $\int_0^{T_f} J(t)dt \ge 0$, when $\psi_4 > 0$, it can be obtained by Assumption 3 that $\psi_1 = 0, \psi_2 = 0, \psi_3 > 0$, so

$$\int_0^t J(s)ds = \int_0^t w^T(s)\psi_3 w(s)ds,$$

thus for $\forall t \in [0, T_f]$,

$$\int_0^{T_f} J(s)ds > \int_0^t J(s)ds \ge \frac{1}{b} x^T(t)\tilde{P}_i x(t) > 0,$$

by (32) we have

$$\begin{aligned}\int_0^{T_f} J(s)ds \ge \frac{1}{b} x^T(t)\tilde{P}_i x(t) &\ge x^T(t) F_i^T \psi_4 F_i x(t) \\ &= z^T(t)\psi_4 z(t),\end{aligned}$$

so we get

$$\int_0^{T_f} J(t)dt - \sup_{0\le t\le T_f} z^T(t)\psi_4 z(t) \ge 0.$$

Thus the proof of extended dissipative is completed. □

Next, we proof finite-time boundedness. Following the above proof, we have

$$V(t) < e^{\alpha t}\mu^{N_\sigma(0,t)} \int_0^t J(s)ds,$$

and

$$V(t) < e^{(\alpha + \frac{ln\mu}{\tau_a})T_f} \int_0^{T_f} J(s)ds,$$

for $\psi_1 \le 0$, we can obtain

$$\begin{aligned}\int_0^{T_f} J(s)ds \le \int_0^{T_f} [&2z^T(s)\psi_2 w(s) \\ &+ w^T(s)\psi_3 w(s)]ds,\end{aligned}$$

and

$$\begin{aligned}V(t) < e^{(\alpha + \frac{ln\mu}{\tau_a})T_f} [\int_0^{T_f} &[2z^T(s)\psi_2 w(s) \\ &+ w^T(s)\psi_3 w(s)]ds],\end{aligned}$$

so we get

$$\begin{aligned}x^T(t)Rx(t) \le &\frac{V(t)}{\lambda_1} \\ &< \frac{e^{(\alpha + \frac{ln\mu}{\tau_a})T_f}}{\lambda_1} [\int_0^{T_f} [2x^T(s)F_i^T \psi_2 w(s) \\ &\quad + w^T(s)\psi_3 w(s)]ds],\end{aligned}$$

by Lemma 1 we have

$$2x^T(s)F_i^T \psi_2 w(s) \le x^T(s)F_i^T F_i x(s) + w^T(s)\psi_2^T \psi_2 w(s).$$

By (37) we can obtain that

$$\begin{aligned}x^T(t)Rx(t) \le &\frac{V(t)}{\lambda_1} \\ &< \frac{e^{(\alpha + \frac{ln\mu}{\tau_a})T_f}}{\lambda_1} [\int_0^{T_f} [x^T(s)F_i^T F_i x(s) \\ &\quad + w^T(s)\psi_2^T \psi_2 w(s) + w^T(s)\psi_3 w(s)]ds] \\ &< \frac{e^{(\alpha + \frac{ln\mu}{\tau_a})T_f}}{\lambda_1} [\lambda_6 k + (\lambda_7 + \lambda_8) d].\end{aligned}$$

Considering (36), we have

$$x^T(t)Rx(t) \le c_2.$$

Thus the proof is completed.

3.3. Robust finite-time extended dissipative control

Consider system (1), under the controller $\mu(t) = K_{\sigma(t)} x(t)$, the corresponding closed-loop system is given by

$$\begin{aligned}\dot{x}(t) &= (A_{\sigma(t)} + E_{\sigma(t)} K_{\sigma(t)}) x(t) + B_{\sigma(t)} x(t - h(t)) \\ &\quad + D_{\sigma(t)} w(t) + G_{\sigma(t)} \int_{t-r(t)}^t x(s)ds, \\ z(t) &= F_{\sigma(t)} x(t), \\ x(t_0 + \theta) &= \varphi(\theta), \forall\theta \in [-\tau, 0].\end{aligned} \tag{31}$$

Theorem 3: Consider system (31). For given positive scalars $\alpha, \hat{h}, h_m, r_m, b$, if there exist positive definite symmetric matrices $\tilde{P}_i, \tilde{Q}_i, \tilde{M}_i$ and matrices Y_{jl} with $Y_{jj} = Y_{jj}^T, j = 1,2...,p, l = j,...,p$ with appropriate dimensions, such that

$$\frac{1}{b}\tilde{P}_i - F_i^T\psi_4 F_i > 0, \tag{32}$$

$$\Pi_i^{(jj)} < Y_{jj}, \quad j = 1,2,...,p, \tag{33}$$

$$\Pi_i^{(jl)} + \Pi_i^{(lj)} < Y_{jl} + Y_{jl}^T, \quad j < l, \quad j,l = 1,2,...,p, \tag{34}$$

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} \\ * & Y_{22} & \cdots & Y_{2p} \\ * & * & \ddots & \vdots \\ * & * & * & Y_{pp} \end{bmatrix} \leq 0, \tag{35}$$

the average dwell time satisfies

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_1 c_2) - \ln[\lambda_6 k + (\lambda_7 + \lambda_8)d] - \alpha T_f}, \tag{36}$$

we denote

$$[P_i, Q_i, M_i] = \sum_{j=1}^{p} \theta_j [P_{ij}, Q_{ij}, M_{ij}], \sum_{j=1}^{p} \theta_j = 1, \theta_j \geq 0,$$

and

$$[R_i, \hat{Q}_i, \hat{M}_i] = \sum_{j=1}^{p} \theta_j [R_{ij}, \hat{Q}_{ij}, \hat{M}_{ij}], \sum_{j=1}^{p} \theta_j = 1, \theta_j \geq 0,$$

where

$$\Pi_i^{(jj)} = \begin{bmatrix} \phi_{11}^{(jj)} & B_{ij}R_{ij} & \phi_{13}^{(jj)} & G_{ij} & R_{ij}F_{ij}^T & \phi_{16}^{(jj)} \\ * & \phi_{22}^{(jj)} & 0 & 0 & 0 & 0 \\ * & * & -\psi_3 & 0 & 0 & 0 \\ * & * & * & -\frac{\tilde{M}_{ij}}{r_m} & 0 & 0 \\ * & * & * & * & \psi_1^{-1} & 0 \\ * & * & * & * & * & S_i + S_i^T \end{bmatrix},$$

$$\phi_{11}^{(jj)} = -\alpha R_{ij} + A_{ij}R_{ij} + R_{ij}A_{ij}^T + E_{ij}Y_i + Y_i^T E_{ij}^T + \hat{Q}_{ij} + r_m \hat{M}_{ij},$$

$$\phi_{13}^{(jj)} = D_{ij} - R_{ij}F_{ij}^T\psi_2,$$

$$\phi_{16}^{(jj)} = E_{ij}Y_i + S_i^T - R_{ij},$$

$$\phi_{22}^{(jj)} = -(1-\hat{h})\hat{Q}_{ij},$$

$$\Pi_i^{(jl)} = \begin{bmatrix} \phi_{11}^{(jl)} & B_{ij}R_{il} & \phi_{13}^{(jl)} & G_{ij} & R_{il}F_{ij}^T & \phi_{16}^{(jl)} \\ * & \phi_{22}^{(jl)} & 0 & 0 & 0 & 0 \\ * & * & -\psi_3 & 0 & 0 & 0 \\ * & * & * & -\frac{\tilde{M}_{il}}{r_m} & 0 & 0 \\ * & * & * & * & \psi_1^{-1} & 0 \\ * & * & * & * & * & S_i + S_i^T \end{bmatrix},$$

$$\phi_{11}^{(jl)} = -\alpha R_{il} + A_{ij}R_{il} + R_{il}A_{ij}^T + E_{ij}Y_i + Y_i^T E_{ij}^T + \hat{Q}_{il} + r_m \hat{M}_{il},$$

$$\phi_{13}^{(jl)} = D_{ij} - R_{il}F_{ij}^T\psi_2,$$

$$\phi_{16}^{(jl)} = E_{ij}Y_i + S_i^T - R_{il},$$

$$\phi_{22}^{(jl)} = -(1-\hat{h})\hat{Q}_{il},$$

$$\Pi_i^{(lj)} = \begin{bmatrix} \phi_{11}^{(lj)} & B_{il}R_{ij} & \phi_{13}^{(lj)} & G_{il} & R_{ij}F_{il}^T & \phi_{16}^{(lj)} \\ * & \phi_{22}^{(lj)} & 0 & 0 & 0 & 0 \\ * & * & -\psi_3 & 0 & 0 & 0 \\ * & * & * & -\frac{\tilde{M}_{ij}}{r_m} & 0 & 0 \\ * & * & * & * & \psi_1^{-1} & 0 \\ * & * & * & * & * & S_i + S_i^T \end{bmatrix},$$

$$\phi_{11}^{(lj)} = -\alpha R_{ij} + A_{il}R_{ij} + R_{ij}A_{il}^T + E_{il}Y_i + Y_i^T E_{il}^T + \hat{Q}_{ij} + r_m \hat{M}_{ij},$$

$$\phi_{13}^{(lj)} = D_{il} - R_{ij}F_{il}^T\psi_2,$$

$$\phi_{16}^{(lj)} = E_{il}Y_i + S_i^T - R_{ij},$$

$$\phi_{22}^{(lj)} = -(1-\hat{h})\hat{Q}_{ij},$$

and the matrices and parameters are defined as follows:

$$\tilde{P}_i = R^{\frac{1}{2}}P_iR^{\frac{1}{2}}, \ \tilde{Q}_i = R^{\frac{1}{2}}Q_iR^{\frac{1}{2}}, \ \tilde{M}_i = R^{\frac{1}{2}}M_iR^{\frac{1}{2}},$$
$$\tilde{P}_{ij} = R^{\frac{1}{2}}P_{ij}R^{\frac{1}{2}}, \ \tilde{Q}_{ij} = R^{\frac{1}{2}}Q_{ij}R^{\frac{1}{2}}, \ \tilde{M}_{ij} = R^{\frac{1}{2}}M_{ij}R^{\frac{1}{2}},$$
$$\tilde{P}_i^{-1} = R_i, \ \tilde{P}_i^{-1}\tilde{Q}_i\tilde{P}_i^{-1} = \hat{Q}_i, \ \tilde{P}_i^{-1}\tilde{M}_i\tilde{P}_i^{-1} = \hat{M}_i,$$
$$\lambda_{min}(P_i) = \lambda_1, \ \lambda_{max}(F_i^TF_i) = \lambda_6, \ \lambda_{max}(\psi_2^T\psi_2) = \lambda_7,$$
$$\lambda_{max}(\psi_3) = \lambda_8.$$

Then the switched linear system is finite-time bounded and satisfies the extended dissipative performance under the controller $\mu(t) = K_{\sigma(t)}x(t)$. The controller gains can be given by $K_i = Y_iS_i^{-1}$.

Proof: By (33)-(35), we get the following inequality

$$\begin{aligned} \Pi_i &= \sum_{j=1}^{p} \theta_{ij}^2 \Pi_i^{(jj)} + \sum_{j=1}^{p-1}\sum_{l=j+1}^{p} \theta_{(ij)}\theta_{(il)}(\Pi_i^{(jl)} + \Pi_i^{(lj)}) \\ &< \sum_{j=1}^{p} \theta_{ij}^2 Y_{jj} + \sum_{j=1}^{p-1}\sum_{l=j+1}^{p} \theta_{(ij)}\theta_{(il)}(Y_{jl} + Y_{lj}^T) \\ &= \Delta \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1p} \\ * & Y_{22} & \cdots & Y_{2p} \\ * & * & \ddots & \vdots \\ * & * & * & Y_{pp} \end{bmatrix} \Delta^T \leq 0, \end{aligned} \tag{37}$$

where

$$\Pi_i$$

$$= \begin{bmatrix} \phi_{11} & B_iR_i & \phi_{13} & G_i & R_iF_i^T & \phi_{16} \\ * & -(1-\hat{h})\hat{Q}_i & 0 & 0 & 0 & 0 \\ * & * & -\psi_3 & 0 & 0 & 0 \\ * & * & * & -\frac{\tilde{M}_i}{r_m} & 0 & 0 \\ * & * & * & * & \psi_1^{-1} & 0 \\ * & * & * & * & * & S_i+S_i^T \end{bmatrix},$$

$$\phi_{11} = -\alpha R_i + A_iR_i + R_iA_i^T + E_iY_i + Y_i^TE_i^T + \hat{Q}_i + r_m\hat{M}_i,$$
$$\phi_{13} = D_i - R_iF_i^T\psi_2,$$
$$\phi_{16} = E_iY_i + S_i^T - R_i,$$

and

$$\Delta = [\theta_{i1}I, \theta_{i2}I, \cdots, \theta_{ip}I],$$

pre- and post-multiply (37) by

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 & -E_iK_i \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}$$

and its transpose, respectively, we have

$$\begin{bmatrix} \phi_{11} & B_iR_i & D_i - R_iF_i^T\psi_2 & G_i & R_iF_i^T \\ * & -(1-\hat{h})\hat{Q}_i & 0 & 0 & 0 \\ * & * & -\psi_3 & 0 & 0 \\ * & * & * & -\frac{\tilde{M}_i}{r_m} & 0 \\ * & * & * & * & \psi_1^{-1} \end{bmatrix} < 0, \tag{38}$$

$\phi_{11} = -\alpha R_i + A_iR_i + R_iA_i^T + E_iK_iR_i + R_iK_i^TE_i^T + \hat{Q}_i + r_m\hat{M}_i$ pre- and post-multiply (38) by $diag\{\tilde{P}_i, \tilde{P}_i, I, I, I\}$ and its transpose, respectively, by Schur complement, we have

$$\begin{bmatrix} \phi_{11} & \tilde{P}_iB_i & \tilde{P}_iD_i - F_i^T\psi_2 & \tilde{P}_iG_i \\ * & -(1-\hat{h})\tilde{Q}_i & 0 & 0 \\ * & * & -\psi_3 & 0 \\ * & * & * & -\frac{\tilde{M}_i}{r_m} \end{bmatrix} < 0, \tag{39}$$

where $\phi_{11} = -\alpha\tilde{P}_i + \tilde{P}_i(A_i+E_iK_i) + (A_i+E_iK_i)^T\tilde{P}_i + \tilde{Q}_i + r_m\tilde{M}_i - F_i^T\psi_1F_i$. Similar to the proof of Theorem 2, we can obtain that

$$\dot{V}(t) - \alpha V(t) - J(t) \le X^T(t)\Phi_iX(t),$$

where

$$X(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & w^T(t) & \int_{t-r(t)}^t x^T(s)ds \end{bmatrix}^T,$$

Table 1. Matrices for each case.

Analysis	Ψ_1	Ψ_2	Ψ_3	Ψ_4
$L_2 - L_\infty$ performance	0	0	$\gamma^2 I$	I
H_∞ performance	$-I$	0	$\gamma^2 I$	0
Passivity	0	I	γ	0
Dissipativity	$-I$	I	$I-\beta * I$	0

where (39) is equivalent to Φ_i. The following proof is similar to that of Theorem 2, it is omitted here.

Remark 1: The above results could be easily extended to the other systems and models, for example the neural networks [22, 23], Markovian jump delayed systems, sampled-data systems [20], Lur'e systems [21], and so on, which deserve further study.

4. NUMERICAL EXAMPLE

In this section, we present an example to illustrate the effectiveness of the controller design method.

Consider system (31) with two subsystems, each subsystem has two vertices to represent polytopic uncertain system:

Subsystem 1:

$$A_{11} = \begin{bmatrix} 2 & 0 \\ 3 & 3 \end{bmatrix}, \; B_{11} = \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix},$$
$$C_{11} = \begin{bmatrix} 0.3 & 0 \\ 0 & -0.2 \end{bmatrix}, \; D_{11} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix},$$
$$E_{11} = \begin{bmatrix} 3 & -3 \\ 0 & 4 \end{bmatrix}, \; G_{11} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.6 \end{bmatrix}, \; F_{11} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}, \; B_{12} = \begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix},$$
$$C_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.3 \end{bmatrix}, \; D_{12} = \begin{bmatrix} -1 & 0 \\ 2 & 0.8 \end{bmatrix},$$
$$E_{12} = \begin{bmatrix} 4 & -1 \\ 1 & 6 \end{bmatrix}, \; G_{12} = \begin{bmatrix} 0.7 & 0 \\ 1 & 0.5 \end{bmatrix}, \; F_{12} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}.$$

Subsystem 2:

$$A_{21} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \; B_{21} = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix}, \; C_{21} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$
$$D_{21} = \begin{bmatrix} -1.5 & -1 \\ 0 & -0.9 \end{bmatrix}, \; E_{21} = \begin{bmatrix} -0.7 & 0 \\ 0.2 & 1.4 \end{bmatrix},$$
$$G_{21} = \begin{bmatrix} 0.3 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \; F_{21} = \begin{bmatrix} 0.1 & 0 \\ 0.4 & 1 \end{bmatrix},$$
$$A_{22} = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}, \; B_{22} = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}, \; C_{22} = \begin{bmatrix} -3 & 2 \\ 0 & 3 \end{bmatrix},$$
$$D_{22} = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.7 \end{bmatrix}, \; E_{22} = \begin{bmatrix} 0.5 & -0.4 \\ 0 & 2 \end{bmatrix},$$
$$G_{22} = \begin{bmatrix} 0.7 & 0 \\ 0.4 & 0.3 \end{bmatrix}, \; F_{22} = \begin{bmatrix} 0.3 & 0 \\ 0.2 & 3 \end{bmatrix}.$$

Furthermore, we assume the uncertain parameters to be $\theta_{ij} = 0.5$, $i = 1, 2$, $j = 1, 2$. And $\hat{h} = 0.01$, $h_m = 0.01$, $r_m = 0.01$, $\alpha = 0.01$. For extended dissipative control, choose matrices for each case in Table 1:

By solving the LMIs from (33)-(35) presented in Theorem 3, we can obtain the optimized variables of four performances in Table 2, and controller gain for each case in Table 3, respectively.

Table 2. Optimized variable for each case.

$L_2 - L_\infty$ performance	H_∞ performance
$\gamma^2_{min} = 1 * 10^{-7}$	$\gamma^2_{min} = 1 * 10^{-7}$
Passivity	Dissipativity
$\gamma_{min} = 1 * 10^{-7}$	$\beta_{max} = 0.9999999$

Table 3. Controller gain for each case.

Subsystem	1
$L_2 - L_\infty$	$K_1 = \begin{bmatrix} 0.2985 & 0.0520 \\ 0.0445 & 0.2233 \end{bmatrix}$
H_∞ performance	$K_1 = \begin{bmatrix} 0.3146 & 0.0503 \\ 0.0458 & 0.2340 \end{bmatrix}$
Passivity	$K_1 = \begin{bmatrix} 0.4951 & 0.0457 \\ 0.0356 & 0.3400 \end{bmatrix}$
Dissipativity	$K_1 = \begin{bmatrix} 0.4956 & 0.0449 \\ 0.0353 & 0.3419 \end{bmatrix}$
Subsystem	2
$L_2 - L_\infty$	$K_2 = \begin{bmatrix} -0.6627 & -0.0232 \\ -0.0256 & 0.6866 \end{bmatrix}$
H_∞ performance	$K_2 = \begin{bmatrix} -0.6869 & -0.0207 \\ -0.0440 & 0.7234 \end{bmatrix}$
Passivity	$K_2 = \begin{bmatrix} -0.5140 & -0.0486 \\ -0.0711 & 0.8044 \end{bmatrix}$
Dissipativity	$K_2 = \begin{bmatrix} -0.5388 & -0.0392 \\ -0.0669 & 0.8064 \end{bmatrix}$

5. CONCLUSION

In this paper, we have investigated the problem of finite time extended dissipative analysis and control of switched systems with time delay. Based on extended dissipative performance, we can solve the H_∞, $L_2 - L_\infty$, Passivity and (Q, S, R)-dissipativity performance in a unified framework. All the results are given in terms of linear matrix inequalities (LMIs), numerical examples are provided to show the effectiveness of the proposed method.

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