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Robust Control for a Class of Time-delay Nonlinear Systems via Output Feedback Strategy

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Abstract: This paper studies the robust control problem for time-delay systems with complicated inherent nonlinearities and unknown disturbances. Based on the modified homogeneous domination method and by constructing the proper Lyapunov-Krasovskii (L-K) functional, output feedback controllers are successfully constructed to guarantee the boundedness of all the states of the closed-loop system. The convergence of the states is also realizable when the L_2 norm of the disturbance exists. The presented method is also extended to solve the control problem of nonholonomic time-delay system. Simulation examples are given to show the effectiveness of the proposed theory.

Keywords: Disturbance, Lyapunov-Krasovskii functional, nonlinear time-delay system, output feedback.

1. INTRODUCTION

Nonlinear systems have received wide attentions these years, for instance, see [1–4]. One of the reasons for this fact is that nearly all the practical systems involve nonlinearities such as nonholonomic mechanical system, electric system, chemical reaction system and so on. Also, it should be mentioned that time delay often exists in such kind of systems [5], and plays a negative role in the control behavior. Consequently, the research of time-delay nonlinear system is becoming more and more important [6, 18, 19]. Actually, nonlinear systems sometimes may suffer from external disturbances [6–9] as well, which further affect the instability of the systems. It is important to study the control problems of this class of systems.

Over the past years, there have been reported many significant research. Specially, in [10], a reliable filter design method was proposed for semi-Markov jump delayed systems. Later in [11], the reliable mixed H_∞/passive control problem was investigated via semi-Markov jump model approach for Takagi-Sugeno (T-S) fuzzy delayed systems. Moreover, in [12], the authors presented a finite-time H_∞ fuzzy control strategy for a class of Markovian jump delayed systems. Later in [13], by employing the mismatched membership function approach and a new L-K functional, the sampled-data stabilization problem was discussed for T-S fuzzy systems with time-delay. For chaotic systems, to obtain a larger sampling period, a delay-dependent sampled-data control method was also proposed in [14]. For more delay-dependent stability criteria, see [15]. Besides, by introducing the sojourn probability approach, [16] further considered the reliable control design problem for a class of discrete-time switch time-delay system. Based on the adaptive control method, [17] considered the nonholonomic system with nonlinear drifts and unknown parameters. A homogeneous domination approach was also raised in [18] to solve the problem of a class of time-delay nonlinear systems, and later in [19] the authors extended the method to stochastic feedforward nonlinear systems with time-varying delay.

Recently, many excellent works focus on studying the stabilization problems by utilizing the output feedback control methods. For instance, [20] considered the MIMO nonlinear systems and proposed an adaptive output feedback controller such that all the states were bounded and the partial state tracking errors belonged to the prescribed bounds. [21] studied the output feedback control problem for a class of nonlinear systems with polynomial nonlinearity by using homogeneity and domination strategies. [22] constructed an output feedback controller for a class of stochastic nonholonomic systems. When the systems involves time-delay, there have been some important results [23–26]. Particularly, [23] studied the strict-feedback Markovian jump nonlinear systems with time-varying delay via the neural networks and backstepping design method. [24, 25] presented the output feedback control design approaches for feedforward time-delay nonlinear systems. [26] considered the nonholonomic time-delay system and raised an output feedback controller to guarantee the closed-loop systems to be

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global stable. However, there are few results for nonlinear systems with time-delay and disturbances [27]. When the systems contain complicated time-delay nonlinear terms and disturbances, the output control problem becomes nontrivial and remains open. Then, a very natural problem is

Is there an output feedback control strategy for time-delay system with complicated nonlinear terms and unknown disturbances? Can the control method be applied to nonholonomic time-delay system?

Practically speaking, many systems like those ones in [17, 20, 22, 25] may contain complicated nonlinear terms, time-delay and disturbances. Up to now, the existing studies have been focused on time-delay nonlinear systems with the linear growing conditions [8, 19] or some high-order nonlinear growing conditions [28, 29]. It remains difficult to design the output feedback controller when the nonlinear terms are polynomial. Due to the existence of time-delay and disturbances, it also needs to find an appropriate L-K functional to analyze the stability. Moreover, whether the control problem can be solved for time-delay nonholonomic system is still open. This further motivates us to proceed the study.

In this paper, we will focus on this problem and present a detailed design strategy. The contributions of this work lies in three aspects: (i) We consider the control design under more general conditions. As can be seen, in [19, 24, 28], the nonlinear terms needed to satisfy some linear growing conditions. In [17, 29], only some high-order nonlinear terms were considered. In [21], although the nonlinear terms were enough complicated, it did not consider time-delay and disturbances. In this paper, both polynomial time-delay terms and the disturbances are involved in the nonlinear systems. The control problem is hence more challenging. (ii) A new output feedback control method is proposed here. Compared with the existing studies [23-26], a new L-K functional is adopted to handle the disturbances here. Also, different from [19, 20, 23-26], we construct new virtual controllers to deal with the nonlinear bounds. (iii) Up to now, the existing researches for nonholonomic systems do not consider time-delay and disturbances, see [17, 22, 30]. In this paper, the presented method is applied to nonholonomic systems with time-delay and disturbances. Also, as a practical example, the mobile robot system is studied in the simulation.

2. OUTPUT FEEDBACK CONTROL DESIGN

In this paper, we study the following time-delay nonlinear system

$$\begin{aligned}
&\dot{x}_1(t) = x_2(t) + f_1(t, \bar{x}_1(t), \bar{x}_1(t-\tau_i)) + d_1(t),\\
&\dot{x}_2(t) = x_3(t) + f_2(t, \bar{x}_i(t), \bar{x}_i(t-\tau_i)) + d_2(t),\\
&\quad\vdots\\
&\dot{x}_n(t) = u(t) + f_n(t, \bar{x}_n(t), \bar{x}_n(t-\tau_i)) + d_n(t),\\
&y(t) = x_1(t),
\end{aligned} \tag{1}$$

where $\bar{x}_i(t-\tau_i) = (x_1(t-\tau_1), x_2(t-\tau_2), \dots, x_i(t-\tau_i))^T \in \mathcal{R}^i$, $\bar{x}_i = (x_1, x_2, \dots, x_i)^T \in \mathcal{R}^i$, $i = 1,,..,n$ are the system state vector, $u(t) \in \mathcal{R}$ and $y = x_1 \in \mathcal{R}$ are the control input and output, respectively. For $i = 1,,..,n$, $\tau_i \in [0, \tau_0]$ is the time-delay with τ_0 being a known constant, $f_i(\cdot)$ is uncertain nonlinear term, $d_i(t)$ is unknown disturbance, $x(\theta) = \xi_0(\theta)$, $\forall\, \theta \in [-\tau, 0]$ is system initial condition with $\xi_0(\cdot)$ being a specified continuous function.

Next, we will present an output feedback control scheme for system (1). The following assumptions are needed:

Assumption 1: For $i = 1, \dots, n$, there exist constants $C > 0, w_k \geq 0$ $(k = 1, \dots, m)$ such that

$$\begin{aligned}
|f_i(\cdot)| \leq & C \sum_{j=1}^{i} \Big(|x_j(t)| + |x_j(t-\tau_j)| \\
& + \sum_{k=1}^{m} (|x_j(t)|^{\frac{1+iw_k}{1+(j-1)w_k}} + |x_j(t-\tau_j)|^{\frac{1+iw_k}{1+(j-1)w_k}}) \Big).
\end{aligned}$$

Assumption 2: For $i = 1, \dots, n$, $d_i(t)$ is the uncertain external disturbance with $d_i(t) \in L_\infty$ and $\dot{d}_i(t) \in L_\infty$.

Remark 1: This conditions are much general than the existing ones. Assumption 1 indicates that the nonlinear terms of system (1) include linear term and high-order terms, and encompasses many assumptions in literature such as [8, 19, 21, 28]. Assumption 2 is widely used for nonlinear systems, see [8, 27]. Due to the inherent nonlinearity, time delay and multiple disturbances, the control problem is much more challenging.

Introduce the coordinate transformations

$$\eta_i(t) = \frac{x_i(t)}{r^{i-1}}, \;\; v = \frac{u}{r^n}, \;\; i = 1, 2, \dots, n, \tag{2}$$

where $r \geq 1$ is a constant. By using (2), the x-system can be transformed into

$$\begin{aligned}
&\dot{\eta}_i = r\eta_{i+1} + \bar{f}_i(t, \bar{\eta}_i, \bar{\eta}_i(t-\tau_i)) + \bar{d}_i, i = 1, \dots, n-1\\
&\dot{\eta}_n = rv + \bar{f}_n(t, \bar{\eta}_n, \bar{\eta}_n(t-\tau_n)) + \bar{d}_n,
\end{aligned} \tag{3}$$

where $\bar{f}_i = \frac{f_i(\cdot)}{r^{i-1}}$, $\bar{d}_i = \frac{d_i}{r^{i-1}}$, $\bar{\eta}_i = (\eta_1, \eta_2, \dots, \eta_i)^T \in \mathcal{R}^i$, $i = 1, \dots, n$.

To construct the state observer, for $j = 2, 3, \dots, n, w = \max_{k\geq 1}\{w_k\}$, we define $\eta_j = (\rho_j + a_{j-1}\eta_{j-1})^{1+\frac{w}{1+(j-2)w}} + (\rho_j + a_{j-1}\eta_{j-1})$ with a_{j-1} being a constant to be designed later, then it can be seen that ρ_j exists. Similarly, define $\hat{\eta}_j = (\hat{\rho}_j + a_{j-1}\hat{\eta}_{j-1})^{1+\frac{w}{1+(j-2)w}} + (\hat{\rho}_j + a_{j-1}\hat{\eta}_{j-1})$. Then, similar to [21], we can construct the following reduced-order observer

$$\dot{\hat{\rho}}_j = -a_{j-1} r \hat{\eta}_j, \;\; j = 2, \dots, n. \tag{4}$$

For the control design later, we use the transformation as

$$\begin{cases}\xi_i(t)=\eta_i(t)-\lambda_{i-1}(t),\ \lambda_0=0,\\ \lambda_i(t)=-h_i\Big(\xi_i(t)+\xi_i^{\frac{1+iw}{1+(i-1)w}}(t)\Big),\\ \varepsilon_j=\rho_j-\hat{\rho}_j,\ i=1,\dots,n,\ j=2,\dots,n,\end{cases} \tag{5}$$

where λ_0, λ_i are the virtual controllers, $h_i>1$ is the constant to be specified. Then, one gets

$$\begin{aligned}&\dot{\xi}_i(t)=r(\xi_{i+1}+\lambda_i)-\sum_{l=1}^{i-1}\frac{\partial\lambda_{i-1}}{\partial\eta_l}r\eta_{l+1}+\tilde{f}_{1i}+\tilde{d}_{1i},\\ &\dot{\varepsilon}_j(t)=\tilde{f}_{2j}+\tilde{d}_{2j},\end{aligned} \tag{6}$$

where

$$\begin{aligned}\tilde{f}_{1i}&=\bar{f}_i-\sum_{l=1}^{i-1}\frac{\partial\lambda_{i-1}}{\partial\eta_l}\bar{f}_l,\\ \tilde{d}_{1i}&=\bar{d}_i-\sum_{l=1}^{i-1}\frac{\partial\lambda_{i-1}}{\partial\eta_l}\bar{d}_l,\ \ i=1,2,\dots,n,\\ \tilde{f}_{2j}&=\frac{r\eta_{j+1}+\bar{f}_j}{1+(1+\frac{w}{1+(j-2)w})(\rho_j+a_{j-1}\eta_j)^{\frac{w}{1+(j-2)w}}}\\ &\quad-a_{j-1}\bar{f}_{j-1}-a_{j-1}r(\eta_j-\hat{\eta}_j),\\ \tilde{d}_{2j}&=\frac{\bar{d}_j}{1+(1+\frac{w}{1+(j-2)w})(\rho_j+a_{j-1}\eta_j)^{\frac{w}{1+(j-2)w}}}\\ &\quad-a_{j-1}\bar{d}_{j-1},\ \ j=2,3,\dots,n.\end{aligned}$$

Now, define $g_{1i}=r(\xi_{i+1}+\lambda_i)-\sum_{l=1}^{i-1}\frac{\partial\lambda_{i-1}}{\partial\eta_l}\eta_{l+1}$ and

$$\begin{aligned}&Z=(\xi_1,\dots,\xi_n,\varepsilon_2,\dots,\varepsilon_n)^T,\\ &G(Z)=(g_{11},\dots,g_{1n},0,\dots,0)^T,\\ &F(Z,Z(t-\bar{\tau}))=(\tilde{f}_{11},\dots,\tilde{f}_{1n},\tilde{f}_{22},\dots,\tilde{f}_{2n})^T,\\ &D(t,Z)=(\tilde{d}_{11},\dots,\tilde{d}_{1n},\tilde{d}_{22},\dots,\tilde{d}_{2n})^T.\end{aligned} \tag{7}$$

Then, (6) can be rewritten to

$$\dot{Z}=G(Z)+F(Z,Z(t-\bar{\tau}))+D(t,Z), \tag{8}$$

where $Z(t-\bar{\tau}))=(Z_1(t-\tau_1)),\dots,Z_n(t-\tau_n)))$. Define the constant $\sigma\ge 1+nw$, and choose the Lyapunov function $U=V_n+T_n$ with

$$\begin{aligned}V_n&=\sum_{i=1}^{n}W_i(\eta_i),\\ W_i&=\frac{1}{2}\xi_i^2+\frac{1+(i-1)w}{2\sigma}\xi_i^{\frac{2\sigma}{1+(i-1)w}},\\ T_n&=\sum_{j=2}^{n}L_j(\varepsilon),\\ L_j&=\frac{\varepsilon_j^2}{2}+\int_{\rho_j+a_{j-1}\eta_{j-1}}^{\hat{\rho}_j+a_{j-1}\eta_{j-1}}\Big(s^{\frac{2\sigma-1-(j-2)w}{1+(j-2)w}}\\ &\quad-(\rho_j+a_{j-1}\eta_{j-1})^{\frac{2\sigma-1-(j-2)w}{1+(j-2)w}}\Big)\mathrm{d}s.\end{aligned} \tag{9}$$

The following lemmas are important for the subsequent control design and analysis. For details of the proof, please see the appendix.

Lemma 1: Under the output feedback control

$$v=-h_n\Big(\hat{\xi}_n+\hat{\xi}_n^{1+\frac{w}{1+(n-1)w}}\Big), \tag{10}$$

where $\hat{\xi}_1$, ..., $\hat{\xi}_n$ are defined as

$$\begin{aligned}&\hat{\xi}_1=\eta_1=x_1,\\ &\hat{\xi}_i=\hat{\eta}_i-\hat{\lambda}_{i-1}(\hat{\xi}_{i-1}),\ i=2,\dots,n-1,\\ &\hat{\lambda}_{i-1}=-h_{i-1}\Big(\hat{\xi}_{i-1}+\hat{\xi}_{i-1}^{1+\frac{w}{1+(i-2)w}}\Big),\\ &\hat{\xi}_n=\hat{\eta}_n-\hat{\lambda}_{n-1}(\hat{\xi}_{n-1}),\\ &\hat{\lambda}_{n-1}=-h_{n-1}\Big(\hat{\xi}_{n-1}+\hat{\xi}_{n-1}^{1+\frac{w}{1+(n-2)w}}\Big),\end{aligned}$$

and the derivative of $U(Z)$ along the solutions of the nominal system $\dot{Z}=G(Z)$ satisfies

$$\begin{aligned}\frac{\partial U(Z)}{\partial Z}G(Z)\le&-\frac{3}{4}r\sum_{i=1}^{n}\Big(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\Big)\\ &+r\delta_0\Big(\varepsilon_n^2+\varepsilon_n^{\frac{2\sigma+w}{1+(n-2)w}}\Big)\\ &+r\sum_{i=2}^{n-1}A_{n-1,i}\Big(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\Big).\end{aligned}$$

Lemma 2: The following inequality holds:

$$\begin{aligned}&\frac{\partial U(Z)}{\partial Z}\Big(F(Z,Z(t-\bar{\tau}))+D(t,Z)\Big)\\ &\le-rc\sum_{i=2}^{n-1}(a_{i-1}-B_{n-1,i})\Big(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\Big)\\ &\quad-cr(a_{n-1}-\rho)\Big(\varepsilon_n^2+\varepsilon_n^{\frac{2\sigma+w}{1+(i-2)w}}\Big)+r^{1-b}\delta_1\\ &\quad\times\sum_{i=1}^{n}\Big(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\Big)+r^{1-b}\delta_2\sum_{i=2}^{n}\Big(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\Big)\\ &\quad+r^{1-b}e^{-m\tau}\delta_3\sum_{i=1}^{n}\Big(\xi_i^2(t-\tau_i)+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}(t-\tau_i)\Big)\\ &\quad+r^{1-b}e^{-m\tau}\delta_4\sum_{i=1}^{n-1}\Big(\xi_i^2(t-\tau_{i+1})+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}(t-\tau_{i+1})\Big)\\ &\quad+\delta_5\sum_{i=1}^{n}\Big(d_i^2+d_i^{\frac{2\sigma+w}{1+iw}}\Big)+\frac{r}{4}\sum_{i=1}^{n}\big(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\big),\end{aligned}$$

where $c,m,\rho,\delta_k,B_{n-1,i}$ are some positive constants.

Remark 2: In the design procedure of the state observer, we introduce some parameters w, $\rho_j,$, $j=2,\dots,n$. Actually, for constants $w_a\le w_b$, $1\le k\le i$, it can be proved that $\frac{1+iw_a}{1+(k-1)w_a}\le\frac{1+iw_b}{1+(k-1)w_b}$. If we choose the constant w such that $w\ge\max_k\{w_k\}$, then by the definition of w, the above inequality and Lemma 4, Assumption 1 will be reduced to

$$|f_i(\cdot)|\le C\sum_{j=1}^{i}\Big(|x_j(t)|+|x_j(t-\tau_j)|$$

$$+\left(|x_j(t)|^{\frac{1+iw}{1+(j-1)w}}+|x_j(t-\tau_j)|^{\frac{1+iw}{1+(j-1)w}}\right)\Big).$$

This will simplify the control design. Besides, an extra condition $w\in R_{even}$ is required. In this case, for $j=2,\dots,n$, by using the definition $\eta_j=(\rho_j+a_{j-1}\eta_{j-1})^{1+\frac{w}{1+(j-2)w}}+(\rho_j+a_{j-1}\eta_{j-1})$ and the implicit function theorem, we can show that ρ_j exists. Since the estimation $\hat{\rho}_j$ is used for the construction of the control input u and ρ_j is not employed, it is not necessary to know ρ_j precisely.

3. MAIN RESULTS

Before giving the conclusion, we choose the following design parameters

$$\begin{aligned}&a_{n-1}=\rho+c^{-1}\delta_0+h,\\&a_{j-1}=A_{n-1,j}+B_{n-1,j}+h,\quad j=n-1,\dots,2,\end{aligned}\tag{11}$$

where $h>0$ is a constant. Now, the following theorem summarizes one of the main results.

Theorem 1: Under Assumptions 1-2, the perturbed time-delay nonlinear system (1) has an output feedback controller

$$u=-r^nh_n\left(\hat{\xi}_n+\hat{\xi}_n^{1+\frac{w}{1+(n-1)w}}\right),\tag{12}$$

which guarantees that all the states of the closed-loop system are bounded. Furthermore, if $d_i(t)\in L_2$, then $\lim_{t\to+\infty}x(t)=0$.

Proof: Firstly, we define the state $\Xi(t)=(\xi^T,\varepsilon^T)^T$ and choose the L-K functional $V=U+\hat{U}$ with $\hat{U}$ defined as

$$\begin{aligned}\hat{U}=&r^{1-b}\Bigg(\delta_3\sum_{i=1}^{n}\int_{t-\tau_i}^{t}e^{m(s-t)}\left(\xi_i^2(s)+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}(s)\right)\mathrm{d}s\\&+\delta_4\sum_{i=1}^{n-1}\int_{t-\tau_{i+1}}^{t}e^{m(s-t)}\left(\xi_i^2(s)+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}(s)\right)\mathrm{d}s\Bigg).\end{aligned}$$

By Lemmas 1 and 2, one gets

$$\begin{aligned}\dot{V}\le&-\frac{1}{2}r\sum_{i=1}^{n}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)+\delta_4\sum_{i=1}^{n}\left(d_i^2+d_i^{\frac{2\sigma+w}{1+iw}}\right)\\&-rc\sum_{i=2}^{n-1}(a_{i-1}-A_{n-1,i}-B_{n-1,i})\left(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\right)\\&-rc(a_{n-1}-\rho-c^{-1}\delta_0)\left(\varepsilon_n^2+\varepsilon_n^{\frac{2\sigma+w}{1+(i-2)w}}\right)\\&+r^{1-b}(\delta_1+\delta_3)\sum_{i=1}^{n}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)\\&+r^{1-b}\delta_2\sum_{i=2}^{n}\left(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\right)-m\hat{U}.\end{aligned}\tag{13}$$

Substituting (11) into (13), it yields that

$$\begin{aligned}\dot{V}\le&-\frac{r}{2}\left(1-\frac{\delta_1+\delta_3}{r^b}\right)\sum_{i=1}^{n}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)\\&-rc\sum_{i=2}^{n}\left(h-\frac{\delta_2}{r^b}\right)\left(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\right)\\&-m\hat{U}+\delta_4\sum_{i=1}^{n}\left(d_i^2+d_i^{\frac{2\sigma+w}{1+iw}}\right).\end{aligned}\tag{14}$$

Also, it can be deduced that

$$U\le\bar{c}\sum_{i=1}^{n}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)+\bar{c}\sum_{i=2}^{n}\left(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\right).\tag{15}$$

Choosing r such that $r^b>\{\delta_1+\delta_3,\frac{\delta_2}{h}\}$, then there exist positive constants β and c_v such that

$$\begin{aligned}\dot{V}\le&-r\beta\sum_{i=1}^{n}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)+\delta_4\sum_{i=1}^{n}\left(d_i^2+d_i^{\frac{2\sigma+w}{1+iw}}\right)\\&-r\beta\sum_{j=2}^{n}\left(\varepsilon_j^2+\varepsilon_j^{\frac{2\sigma+w}{1+(j-2)w}}\right)-m\hat{U}\\\le&-c_vV+\delta_4\sum_{i=1}^{n}\left(d_i^2+d_i^{\frac{2\sigma+w}{1+iw}}\right).\end{aligned}\tag{16}$$

By the definition of $V(\cdot)$, it follows that

$$\alpha_1\left(\|\Xi(t)\|\right)\le V\le\alpha_2\Big(\sup_{-\tau\le s\le0}\|\Xi(s+t)\|\Big),\tag{17}$$

where α_1, α_2 are class $\mathcal{K}_\infty$ functions. By Assumption 2, there exists a positive constant d such that $\delta_4\sum_{i=1}^n\left(d_i^2+d_i^{\frac{2\sigma+w}{1+iw}}\right)\le d$. Then, using (16), one obtains

$$V(t)\le e^{-c_vt}V(0)+\frac{d}{c_v}.\tag{18}$$

From (18), it follows that $\Xi(t)$ is bounded, which further shows that the states ξ and ε are bounded. By (5) and (6), it follows that $\dot{\xi}$ and $\dot{\varepsilon}$ are also bounded. If $d_i(t)\in L_2$, since $d_i(t)\in L_\infty$, one gets $\int_0^\infty\left(d_i^2+d_i^{\frac{2\sigma+w}{1+iw}}\right)<M$, where M is a constant. From (16), it follows that

$$\int_0^\infty\left(\xi_i^2(s)+\varepsilon_j^2(s)\right)ds\le V(0)+M,\tag{19}$$

which shows that $\xi\in L_2$ and $\varepsilon\in L_2$. Then, by utilizing Barbalat's lemma, it follows that $\lim_{t\to+\infty}\xi(t)=0$ and $\lim_{t\to+\infty}\varepsilon(t)=0$, which further implies $\lim_{t\to+\infty}x(t)=0$.

Remark 3: Consider the following time-delay system

$$\begin{aligned}&\dot{x}_1=x_2,\\&\dot{x}_2=u+f_1(t,x_1(t-\tau_1),x_2(t-\tau_2))+d_1(t),\end{aligned}\tag{20}$$

where x_1,x_2 are the states, u is the control input, τ_1,τ_2 are time delays. f_1,d_1 are the nonlinear term and disturbance, respectively. When $f_1=x_1(t-\tau_1)\sin t+x_2(t-\tau_2)$, $d_1=0$, the nonlinear term satisfies the linear growing condition in [8]. The control problem can be solved by using the method there. When $f_1=x_2^{7/5}(t-\tau_2)$, $d_1=0$, it can be verified that the nonlinear term satisfies the high-order growing condition in [28, 29], Hence, the control problem is also solvable with the methods there. However, if f_1 is more complicated and $d_1 \neq 0$, the output feedback control problem is challenging. For instance, $f_1 = x_1(t-\tau-1) + x_2^{7/5}(t-\tau_2)$, $d_1 = \sin t$, the existing methods cannot work anymore.

Remark 4: In this work, we will encounter many difficulties. As can be seen, compared with [17, 19, 24, 29], multiple time-delays and disturbances exist in the considered systems, we need to handle them effectively to construct the controller. Moreover, different from [25, 28, 29], the intricacy of polynomial nonlinear terms indeed make the stability analysis difficult. This is because the upper bounds for these terms should be found skillfully. Besides, what kind of the transformations should be introduced and how to construct the virtual controls are also not easy for the considered systems here.

4. EXTENSIONS

The method in the last section can be extended to a class of more general systems. Consider the following nonlinear time-delay systems with unmodeled dynamics

$$\begin{aligned}
&\dot{\zeta}(t) = f_0(t,\zeta,x),\\
&\dot{x}_1(t) = x_2(t) + f_1(t,\zeta(t),x(t),\zeta(t-\tau),x(t-\tau))\\
&\qquad + d_1(t),\\
&\dot{x}_2(t) = x_3(t) + f_2(t,\zeta(t),x(t),\zeta(t-\tau),x(t-\tau))\\
&\qquad + d_2(t),\\
&\vdots\\
&\dot{x}_n(t) = u(t) + f_n(t,\zeta(t),x(t),\zeta(t-\tau),x(t-\tau))\\
&\qquad + d_n(t),\\
&y(t) = x_1(t),
\end{aligned} \tag{21}$$

where $\zeta \in \mathcal{R}^m$, $x = (x_1, x_2, \ldots, x_n)^T \in \mathcal{R}^n$ and $x(t-\tau) = (x_1(t-\tau_1), x_2(t-\tau_2), \ldots, x_n(t-\tau_n))^T \in \mathcal{R}^n$ are the state vector, $u(t) = (u_0(t), u_1(t))^T \in \mathcal{R}^2$ and $y \in \mathcal{R}$ are the control input and the output, respectively, $\tau_1, \ldots, \tau_n$ are the time delay of the system state, $f_1(\cdot), \ldots, f_n(\cdot)$ are uncertain nonlinear terms, $X(\theta) = (\zeta^T(\theta), x^T(\theta))^T = \xi_0(\theta)$, $\forall \theta \in [-\tau, 0]$ is the system initial condition with $\xi_0(\cdot)$ being a specified continuous function.

The following assumptions are needed:

Assumption 3: There exist class $\mathcal{K}_\infty$ functions $\alpha_1(\cdot)$, $\alpha_2(\cdot)$ and a Lyapunov function $V_0(\zeta) \in \mathcal{C}^1$ such that

$$\begin{aligned}
&\alpha_1(\|\zeta\|) \le V_0(\zeta) \le \alpha_2(\|\zeta\|),\\
&\frac{\partial V_0(\zeta)}{\partial \zeta} f_0(\cdot) \le -l_1(\|\zeta\|^2 + \|\zeta\|^{2a}) + l_2 x_1^2,
\end{aligned}$$

where $2a \ge 2\sigma + w$, $l_1 > 0$ and $l_2 > 0$ are constants.

Assumption 4: For $i = 1, \ldots, n$, there exist constants $C > 0, w_k \ge 0$ $(k = 1, \ldots, m)$ such that

$$\begin{aligned}
|f_i(\cdot)| \le\, & C\sum_{j=1}^{m}\left(\sum_{k=1}^{m}\left(|\zeta_j(t)|^{1+iw_k} + |\zeta_j(t-\tau_j)|^{1+iw_k}\right)\right.\\
&\left.+|\zeta_j(t)|\right) + C\sum_{j=1}^{i}\left(|x_j(t)| + \sum_{k=1}^{m}\left(|x_j(t)|^{\frac{1+iw_k}{1+(j-1)w_k}}\right.\right.\\
&\left.\left.+|x_j(t-\tau_j)|^{\frac{1+iw_k}{1+(j-1)w_k}}\right)\right).
\end{aligned}$$

The main result can be summarized as follows:

Theorem 2: Suppose that Assumptions 2-4 hold, then there exists an output feedback control for system (21) such that all the states of the closed-loop system are bounded. Moreover, if $d_i(t) \in L_2$, then $\lim_{t\to+\infty} x(t) = 0$.

Proof: For system (21), the unmodeled dynamics $\zeta \in \mathcal{R}^m$ will not be used for control design. Therefore, we need not to construct a state observer for these states. Considering this, we can use the steps (2)-(5) and get

$$\begin{aligned}
&\dot{\zeta}(t) = f_0(t,\zeta,x),\\
&\dot{\xi}_i(t) = r(\xi_{i+1} + \lambda_i) - \sum_{l=1}^{i-1}\frac{\partial \lambda_{i-1}}{\partial \eta_l} r\eta_{l+1} + \tilde{f}_{1i} + \tilde{d}_{1i},\\
&\dot{\varepsilon}_j(t) = \tilde{f}_{2j} + \tilde{d}_{2j},\;\; i = 1,2,\ldots,n,\;\; j = 2,3,\ldots,n.
\end{aligned}$$

Define

$$\begin{aligned}
&\bar{Z} = (\zeta, \xi_1, \ldots, \xi_n, \varepsilon_2, \ldots, \varepsilon_n)^T,\\
&\bar{G}(\bar{Z}) = (f_0, g_{11}, \ldots, g_{1n}, 0, \ldots, 0)^T,\\
&\bar{F}(\bar{Z}, \bar{Z}(t-\tau)) = (\tilde{f}_{11}, \ldots, \tilde{f}_{1n}, \tilde{f}_{22}, \ldots, \tilde{f}_{2n})^T,\\
&\bar{D}(t,\bar{Z}) = (\tilde{d}_{11}, \ldots, \tilde{d}_{1n}, \tilde{d}_{22}, \ldots, \tilde{d}_{2n})^T.
\end{aligned} \tag{22}$$

Then, it follows that

$$\dot{\bar{Z}} = \bar{G}(\bar{Z}) + \bar{F}(\bar{Z}, \bar{Z}(t-\bar{\tau})) + \bar{D}(t,\bar{Z}). \tag{23}$$

Now, choose $\bar{U} = \gamma V_0 + U$, $V = \bar{U} + \hat{U}$. The left proof is similar to that of Theorem 1, and hence it is omitted here.

5. APPLICATION TO NONHOLONOMIC SYSTEM

Consider the nonholonomic time-delay system

$$\begin{aligned}
&\dot{x}_0(t) = u_0(t) + d_0(t),\\
&\dot{x}_i(t) = x_{i+1}(t)u_0(t) + f_i(t, x_0(t), x(t), x(t-\tau))\\
&\qquad + s_i(x_0, d_i(t)), i = 1, \ldots, n-1,\\
&\dot{x}_n(t) = u_1(t) + f_n(t, x_0(t), x(t), x(t-\tau))\\
&\qquad + s_n(x_0, d_n(t)),
\end{aligned} \tag{24}$$

where $(x_0, x^T)^T = (x_0, x_1, \ldots, x_n)^T \in \mathcal{R}^{n+1}$, $x(t-\tau) = (x_1(t-\tau_1), x_2(t-\tau_2), \ldots, x_n(t-\tau_n))^T \in \mathcal{R}^n$ are the state vector and delayed state vector, $u(t) = (u_0(t), u_1(t))^T \in$

$\mathcal{R}^2$, and $y = (x_0, x_1)^T \in \mathcal{R}^2$ are the control input and output, respectively. For $i = 1, \ldots, n$, τ_i is the time-delay, $f_i(\cdot)$ is the uncertain delayed function, $s_i(x_0, d_i(t))$ is the disturbance term. $x(\theta) = \xi_0(\theta)$, $\forall \theta \in [-\tau_0, 0]$ is system initial condition with $\xi_0(\cdot)$ being a specified continuous function and $\tau_0 > 0$ being a constant. $0 < d_0(t) \leq \bar{d}_0$ is a bounded disturbance with $\bar{d}_0$ being a constant.

For system (24), the following assumption is imposed.

Assumption 5: For each $i = 1, \ldots, n$, there exists a nonnegative smooth function $C(x_0)$ and constants $w_k \geq 0$, $k = 1, \ldots, m$, such that

$$\begin{aligned}
&|s_i(\cdot)| \leq C(x_0) d_i(t), \\
&|f_i(\cdot)| \leq C(x_0) \sum_{j=1}^{i} \sum_{k=1}^{m} \Big(|x_j(t)|^{\frac{1+iw_k}{1+(j-1)w_k}} \\
&\qquad + |x_j(t-\tau_j)|^{\frac{1+iw_k}{1+(j-1)w_k}} \Big).
\end{aligned}$$

We have the same conclusion as follows:

Theorem 3: Under Assumptions 2 and 5, there exists an output feedback controller for system (24).

Proof: For the x_0-subsystem, we choose the Lyapunov candidate function $U_0(x_0) = \frac{1}{2} x_0^2$. Then, along the x_0-subsystem, the derivative of V_0 satisfies

$$\dot{U}_0 = x_0 u_0 + x_0 d_0(t). \tag{25}$$

Then, one can design the control u_0 as

$$u_0 = -\Big(c_0 + \frac{1}{2}\Big) x_0(t), \tag{26}$$

where c_0 is a positive constant. Then, by (25), it yields

$$\dot{V}_0 \leq -c_0 x_0^2 + \frac{\bar{d}_0^2}{2}. \tag{27}$$

From (27), we obtain that x_0 is global bounded. Similar to the proof of Theorem 1, it follows that $\lim_{t \to +\infty} x_0(t) = 0$. Furthermore, for all $t \in [t_0, +\infty)$, $x_0(t_0) \neq 0$, we have

$$x_0(t) = e^{-c_0(t-t_0)} x_0(t_0) + \int_0^t e^{-c_0(t-l)} d_0(l) dl \neq 0. \tag{28}$$

That is, $x_0(t) \neq 0$ for $t \in [t_0, +\infty)$, $x_0(t_0) \neq 0$. Therefore, for $i = 1, \ldots, n$, we can introduce the input-state scaling transformations:

$$\begin{aligned}
\zeta_i(t) &= \begin{cases} \dfrac{x_i(t)}{r^{i-1} u_0^{n-i}(t)}, & t \geq t_0, \\[2mm] \dfrac{x_i(t)}{r^{i-1}}, & t_0 - \tau_0 \leq t < t_0, \end{cases} \\
v_1(t) &= \frac{u_1(t)}{r^n},
\end{aligned} \tag{29}$$

where $r \geq 1$ is a positive constant. By using (29), the x-subsystem is transformed into

$$\begin{aligned}
\dot{\eta}_i(t) &= r\eta_{i+1}(t) + \hat{f}_i\big(t, x_0, \eta(t), \eta(t-\tau)\big) \\
&\quad + \hat{s}_i\big(x_0, d_i(t)\big), \\
\dot{\eta}_n(t) &= r v_1(t) + \hat{f}_n\big(t, x_0, \eta(t), \eta(t-\tau)\big) \\
&\quad + \hat{s}_n\big(x_0, d_n(t)\big),
\end{aligned} \tag{30}$$

where $\hat{f}_i = \frac{f_i(\cdot)}{r^{i-1} u_0^{n-i}(t)} - (n-i)\zeta_i(t) \frac{\dot{u}_0(t)}{u_0(t)}$, $\hat{s}_i(x_0, d_i(t)) = \frac{s_i(\cdot)}{r^{i-1} u_0^{n-i}(t)}$, $i = 1, \ldots, n-1$. It can be deduced that

$$\begin{aligned}
&|\hat{s}_i(\cdot)| \leq C|d_i(t)|, \\
&|\hat{f}_i(\cdot)| \leq C \sum_{j=1}^{i} \Bigg(|x_j(t)| + \sum_{k=1}^{m} \Big(|x_j(t)|^{\frac{1+iw_k}{1+(j-1)w_k}} \\
&\qquad + |x_j(t-\tau_j)|^{\frac{1+iw_k}{1+(j-1)w_k}} \Big) \Bigg).
\end{aligned} \tag{31}$$

As can be seen, system (30) has the similar nonlinear bounds as system (21). Then using the same design method in Section 3, we can construct the output feedback controller for it. The left proof is the same as that of Theorem 1. Hence, we omit it here for simplicity.

Remark 5: In this paper, we propose an output feedback control method for a class of time-delay nonlinear systems. It should be noted that the results here are somewhat conservative. For instance, under Assumption 2, the states of the systems are only guaranteed bounded. In Theorem 2, to ensure the convergent of the states, the disturbances $d_i(t), i = 1, 2, \ldots, n$ need to satisfy $d_i(t) \in L_2$. Also, nonlinear systems may contain unknown control coefficients, unknown parameters and random perturbations. To make the method much clear, we do not take into account these problems here. Future work will concentrate on studying the adaptive control problems for more general systems.

Remark 6: When the time delays are time-varying functions $\tau_i(t), i = 1, 2 \ldots, n$, we can design the controller similarly. Assume that $0 \leq \tau_i(t) \leq \mu_1$, $\dot{\tau}_i(t) \leq \mu_2 < 1$, then from (2) and (5), we can see that the time delays in terms of Lemma 2 will be time-varying functions. Choosing

$$\begin{aligned}
\hat{U} =& r^{1-b} \frac{1}{1-\mu_2} \Big(\sum_{i=1}^{n} \delta_3 \int_{t-\tau_i(t)}^{t} e^{m(s-t)} \big(\xi_i^2(s) \\
&+ \xi_i^{\frac{2\sigma+w}{1+(i-1)w}}(s) \big) \mathrm{d}s \\
&+ \sum_{i=1}^{n-1} \delta_4 \int_{t-\tau_{i+1}(t)}^{t} e^{m(s-t)} \big(\xi_i^2(s) + \xi_i^{\frac{2\sigma+w}{1+(i-1)w}}(s) \big) \mathrm{d}s \Big).
\end{aligned} \tag{32}$$

and using $V = U + \hat{U}$ to counteract the delay terms in procedure of stability analysis, we can obtain the same result as Theorem 1.

Remark 7: The novelty of this paper cover several aspects: (i) The homogeneous domination method is modified to deal with time-delay nonlinear systems with polynomial growing conditions. The existing work such as [17, 21, 29] are effective for systems with lower-order or high-order growing conditions. However, under Assumption 1, system (1) covers the two conditions and hence cannot be stabilized by the methods there. The strategy of this paper successfully constructs a robust output feedback controller. (ii) For systems with more complicated structure, we provide some extended results. Specifically, in Section 4, we show how to design robust control for nonlinear systems with unmodeled dynamics, which is more general than [8, 9]. In Section 5, we apply the presented method to nonholonomic systems. (iii) When the disturbances exist, the existing L-K functionals in many studies like [8, 23, 25] is not suitable for the systems here. We need to construct a new one to make its derivative satisfy (16).

6. SIMULATION EXAMPLES

Example 1: We consider the following system

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) + f_1(t,x,x(t-\tau)) + d_1(t),\\
\dot{x}_2(t) &= u(t) + f_2(t,x,x(t-\tau)) + d_2(t),\\
y(t) &= x_1(t),
\end{aligned} \tag{33}$$

where $x_i(t), i=1,2$ are system states, $u(t)$ is the control input, y is the system output. For $i=1,2$, f_i is the nonlinear term, $d_i(t)$ is the disturbance.

When $d_1 = d_2 = 0$, $f_1 = 0$, $f_2 = \frac{3x_1x_2^{\frac{4}{5}}(t-\tau)}{1+x_1^2}$, by the method in [31], we can design the controller

$$\begin{aligned}
u &= -r_1^2\Big(\beta_2(\hat{\rho}_1 + l_1x_1)^{\sigma} + \beta_2\beta_1x_1^{\sigma}\Big)^{\frac{1+2w}{\sigma}},\\
\dot{\hat{\rho}}_1 &= -l_1r(\hat{\rho}_1 + l_1x_1)^{1+w},
\end{aligned} \tag{34}$$

where r_1, l_1, β_1, β_2 are constants. By Theorem 1, one can construct the output feedback controller

$$\begin{aligned}
u &= -r^2h_2\Big(\hat{\xi}_2 + \hat{\xi}_2^{1+\frac{w}{1+(n-1)w}}\Big),\\
\dot{\hat{\rho}}_2 &= -a_1r_1\Big((\hat{\rho}_2 + a_1x_1)^{1+w} + (\hat{\rho}_2 + a_1x_1)\Big),
\end{aligned} \tag{35}$$

where $\hat{\xi}_2 = (\hat{\rho}_2 + m_1x_1)^{1+w} + (\hat{\rho}_2 + m_1x_1) + h_1(x_1 + x_1^{1+w})$.

For simplicity, the system parameters are chosen as $w = \frac{2}{3}$, $r_1 = 4$, $\sigma = 7/3$, $\beta_1 = 2$, $\beta_1 = 3$, $l_1 = 3$, $\tau = 0.5$. The initial conditions are $x_1(0) = 0.5$, $x_2(0) = -0.3$. Fig. 1 shows that the method here can achieve a faster convergent speed.

When $d_1(t) = d_2(t) = \frac{1}{1+t}$, $f_1 = x_1(t)\sin(x_2(t))$, $f_2 = \frac{3x_1x_2^{\frac{4}{5}}(t-\tau)}{1+x_1^2}$, choose $a_1 = 4$, $\sigma = 7/3$, $r = 1.5$, $h_1 = 4$, $h_2 = 4$, $n = 2$, $\hat{\rho}_2(0) = -0.6$. The left parameters are the same. As

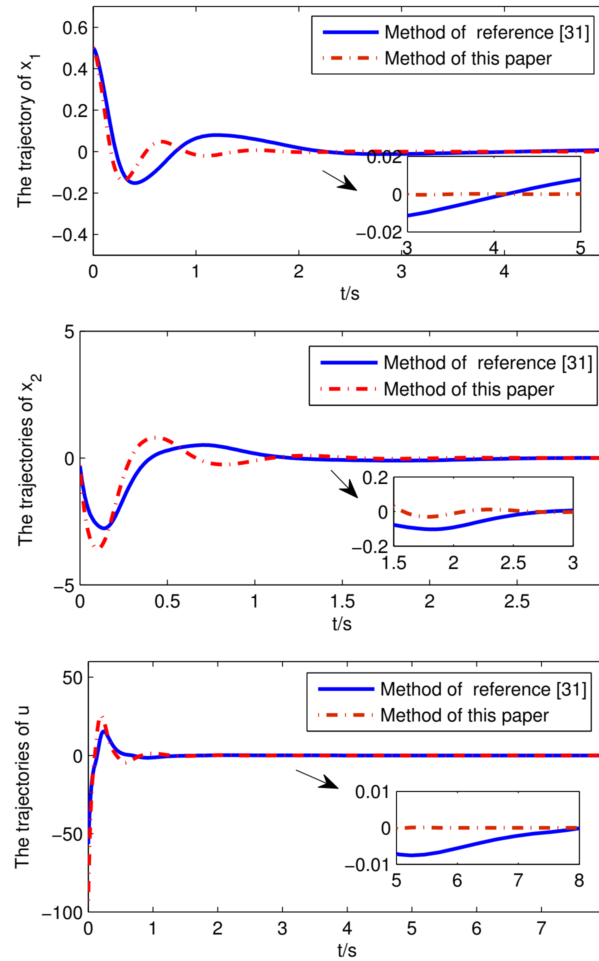

Fig. 1. The system Responses under control u_1 and u_2.

shown in Fig. 2, all the states are bounded and the system states x_k, $k=1,2$ converge to zero. Hence, the proposed control approach is effective.

Example 2: We consider the bilinear model of a mobile robot [17] described as

$$\begin{aligned}
\dot{x}_l(t) &= \Big(1 - \frac{\xi^2}{2}\Big)v(t) + d_0(t),\\
\dot{y}_l(t) &= \big(\theta_l(t) + \xi\big)v(t) + f_1\big(X(t), X(t-\tau)\big)\\
&\quad + s_1\big(x_l, d_1(t)\big),\\
\dot{\theta}_l(t) &= w(t) + f_2\big(X(t), X(t-\tau)\big) + s_2\big(x_l, d_2(t)\big),
\end{aligned} \tag{36}$$

where $x_l(t)$ and $y_l(t)$ denote the center position of mass of the robot; $\theta_l(t)$ represents heading angle of the robot; $X(t) = [x_l(t), y_l(t), \theta_l(t)]^T$; ξ is a positive constant, $v(t)$ is the forward velocity and $w(t)$ is the angular velocity of the robot, $f_1(\cdot), f_2(\cdot)$ and $s_1(\cdot), s_2(\cdot)$ are nonlinear terms, $d_0(\cdot)$, $d_1(\cdot)$ and $d_2(\cdot)$ are the disturbances.

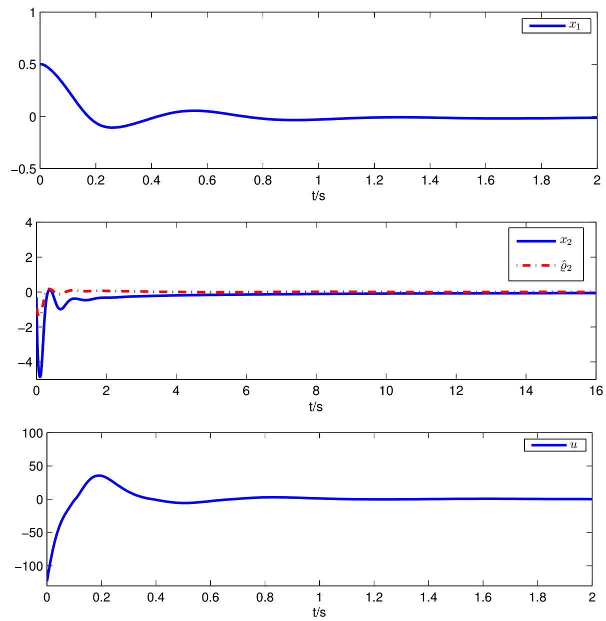

Fig. 2. Responses of the closed-loop system (33) and (35).

By introducing the transformations

$$x_0(t) = x_l(t),\ \ x_1(t) = y_l(t),\ \ x_2(t) = \frac{\theta_l(t)+\xi}{1-\frac{\xi^2}{2}},$$
$$u_0(t) = \big(1-\frac{\xi^2}{2}\big)v(t),\ \ u_1(t) = w(t),$$

and defining $\bar{X}(t) = [x_0(t), x_1(t), x_2(t)]^T$, the system can be transformed into

$$\begin{aligned}
\dot{x}_0(t) &= u_0(t) + d_0(t),\\
\dot{x}_1(t) &= x_2(t)u_0(t) + \bar{f}_1(\bar{X}(t), \bar{X}(t-\tau)) + \bar{s}_1(x_0, d_1(t)),\\
\dot{x}_2(t) &= u_1(t) + \bar{f}_1(\bar{X}(t), \bar{X}(t-\tau)) + \bar{s}_2(x_0, d_2(t)),
\end{aligned} \tag{37}$$

where $\bar{f}_1(\cdot) = f_1(\cdot)$, $\bar{s}_1(\cdot) = s_1(\cdot)$, $\bar{f}_2(\cdot) = \frac{2f_2(\cdot)}{2-\xi^2}$, $\bar{s}_2(\cdot) = \frac{2s_2(\cdot)}{2-\xi^2}$. When $\bar{f}_i(\cdot)$ and $\bar{s}_i(\cdot), i = 1,2$ satisfy Assumptions 2 and 5, the control problem of this system can be solved. Next, to verify the method of this paper, we assume $f_1 = x_0x_1(t-\tau)\sin x_2$, $d_1 = \frac{1}{1+t}x_0$, $f_2 = x_2(t-\tau)$ and $d_2 = \frac{\sin t}{1+2t}$.

By using Theorem 3, we can construct the output feedback control

$$\begin{aligned}
u_0 &= -2x_0(t),\\
u_1 &= -2r^2 g_2\Big(\hat{\rho}_2 + a_1(x_1/u_0) + g_1(x_1/u_0)\Big),\\
\dot{\hat{\rho}}_2 &= -2a_1 r(\rho_2 + a_1 x_1).
\end{aligned} \tag{38}$$

In this simulation, the parameters are selected as $r = 2$, $g_1 = 2$, $g_2 = 4$, $a_1 = 3$, $\tau = 0.6$. The initial conditions are $x_0(0) = 0.7$, $x_1(0) = 0.2$, $x_2(0) = -2$, $\hat{\rho}_2(0) = -1.5$. Fig. 3 shows that all the states are bounded and x_k, $k = 0,1,2$ converge to zero. Therefore, the control strategy here is valid.

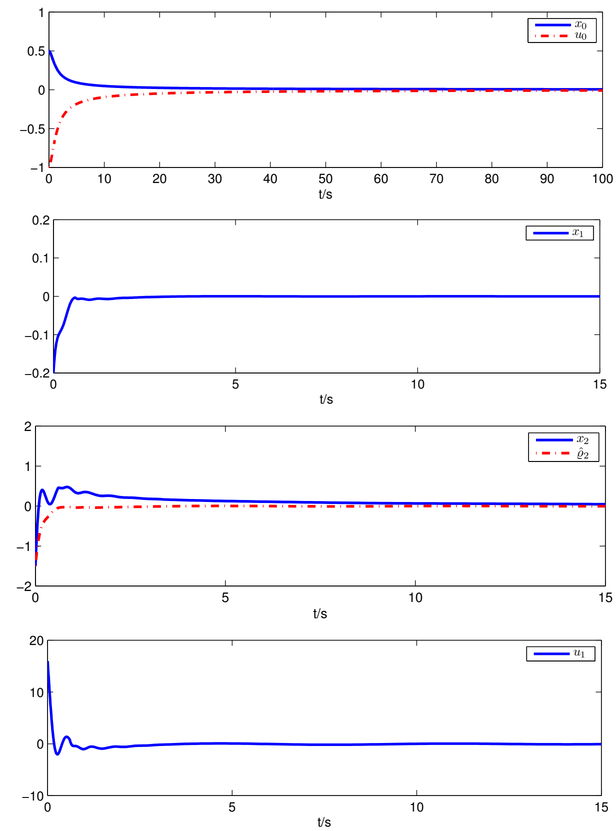

Fig. 3. Responses of the closed-loop system (37) and (38).

7. CONCLUSIONS

In this paper, by using the output control strategy, we discuss the robust control problem for a class of nonlinear time-delay system with unknown disturbances. Firstly, by using some transformations, the system is transformed into a new system. Then, a reduced-order observer is introduced. By utilizing the modified adding a power integrator method, we successfully construct an output feedback controller. The proposed method is then promoted to time-delay system with uncertain dynamics and nonholonomic time-delay system. An interesting problem is how to extend the method to nonlinear time-delay system with random disturbances.

APPENDIX A

Useful lemmas: The following lemmas are to be used throughout the paper.

Lemma 3 [21]**:** If $p \in \mathcal{R}^{\geq 1}_{\text{odd}}$, then the following inequalities hold for $\forall x \in \mathcal{R}$, $\forall y \in \mathcal{R}$:

$$(|x|+|y|)^{\frac{1}{p}} \leq |x|^{\frac{1}{p}} + |y|^{\frac{1}{p}} \leq 2^{\frac{p-1}{p}}(|x|+|y|)^{\frac{1}{p}},$$
$$|x-y|^p \leq 2^{p-1}|x^p - y^p|.$$

Lemma 4 [21]**:** For given real numbers δ_i, $i=1,\ldots,n$ satisfying $0\le\delta_1\le\cdots\le\delta_n$ and given nonnegative functions $c_i(x,y)$, $i=1,\ldots,n$, there hold

$$\begin{aligned}&c_1(x,y)|x|^{\delta_1}+c_n(x,y)|x|^{\delta_n}\\&\le\sum_{j=1}^{n}c_j(x,y)|x|^{\delta_j}\\&\le\left(|x|^{\delta_1}+|x|^{\delta_n}\right)\sum_{j=1}^{n}c_j(x,y),\ \forall x\in\mathcal{R}.\end{aligned}$$

Lemma 5 [25]**:** For given positive real numbers m,n and a given positive function $a(x,y)$, there exists a positive function $c(x,y)$ such that

$$\begin{aligned}&|a(x,y)x^m y^n|\\&\le c(x,y)|x|^{m+n}+\frac{n}{m+n}\left(\frac{m}{(m+n)c(x,y)}\right)^{\frac{m}{n}}\\&\quad\times|a(x,y)|^{\frac{m+n}{n}}|y|^{m+n},\ \forall x\in\mathcal{R},\ \forall y\in\mathcal{R}.\end{aligned}$$

Proof of Lemma 1: By the definition of W_i, one can deduce that

$$\begin{aligned}&\left|\frac{\partial W_i}{\partial\eta_i}\right|\le|\xi_i|+|\xi_i|^{\frac{2\sigma-1-(i-1)w}{1+(i-1)w}},\\&\left|\frac{\partial W_i}{\partial\eta_j}\right|\le\left(|\xi_i|+|\xi_i|^{\frac{2\sigma-1-(i-1)w}{1+(i-1)w}}\right)\left|\frac{\partial\lambda_{i-1}(\bar\eta_{i-1})}{\partial\eta_j}\right|,\\&j=1,\ldots,i-1.\end{aligned}\tag{A.1}$$

Step 1: Define the function $U_1(\xi_1)=W_1$. Then, along the system $\dot{\mathcal{Z}}=\mathcal{G}(\mathcal{Z})$, the derivative of U_1 satisfies

$$\dot U_1=r(\xi_1+\xi_1^{2\sigma-1})(\eta_2-\lambda_1)+r(\xi_1+\xi_1^{2\sigma-1})\lambda_1.\tag{A.2}$$

The virtual controller λ_1 in this step can be selected as

$$\lambda_1(\zeta_1)=-n(\xi_1+\xi_1^{1+w})=:-h_1(\xi_1+\xi_1^{1+w}),\tag{A.3}$$

where $h_1=n$ is a constant. Using $\xi_1^{2\sigma}\ge0$ and $\xi_1^{2+w}\ge0$, one can deduce that

$$\dot U_1\le-nr(\xi_1^2+\xi_1^{2\sigma+w})+r(\xi_1+\xi_1^{2\sigma-1})(\eta_2-\lambda_1).\tag{A.4}$$

Step k $(k=2,3,\ldots,n)$**:** Suppose that for step $k-1$, along the solutions of system $\dot{\mathcal{Z}}=\mathcal{G}(\mathcal{Z})$, there holds

$$\begin{aligned}\dot U_{k-1}\le&-(n-k+2)r\sum_{i=1}^{k-1}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)\\&+r\left(\xi_{k-1}+\xi_{k-1}^{\frac{2\sigma-1-(k-2)w}{1+(k-2)w}}\right)(\eta_k-\lambda_{k-1}).\end{aligned}\tag{A.5}$$

Then we choose the function $U_k=U_{k-1}+W_k$. Using (A.1), Lemmas 3-4, and following similar design method in [21], it yields that $U_k(\bar\eta_k)$ satisfies (A.6) at step k.

Hence, when $k=n$, if we choose the Lyapunov function $U_n(\bar\eta)$ and use $\lambda_n:\mathcal{R}^n\to\mathcal{R}$, we obtain

$$\begin{aligned}\dot U_n\le&r\left(\xi_n+\xi_n^{\frac{2\sigma-1-(n-1)w}{1+(n-1)w}}\right)(v-\lambda_n)\\&-r\sum_{i=1}^{n}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right).\end{aligned}\tag{A.6}$$

By Lemma 3, it follows that

$$|v-\lambda_n|\le s_0|\xi_n-\hat\xi_n|\left(1+|\xi_n|^{\frac{w}{1+(n-1)w}}+|\hat\xi_n|^{\frac{w}{1+(n-1)w}}\right).\tag{A.7}$$

By Lemma 3, it follows that

$$|v-\lambda_n|\le s_0|\xi_n-\hat\xi_n|\left(1+|\xi_n|^{\frac{w}{1+(n-1)w}}+|\hat\xi_n|^{\frac{w}{1+(n-1)w}}\right).\tag{A.8}$$

From the definition of $\hat\xi_n$ and Lemmas 3-4, one can deduce that

$$\begin{aligned}&r\left(\xi_n+\xi_n^{\frac{2\sigma-1-(n-1)w}{1+(n-1)w}}\right)(v-\lambda_n)\\&\le s_0r\left(|\xi_n-\hat\xi_n|^2+|\xi_n-\hat\xi_n|^{\frac{2\sigma+w}{1+(n-1)w}}\right)\\&\quad+\frac{r}{8}\sum_{j=1}^{n}\left(\xi_j^2+\xi_j^{\frac{2\sigma+w}{1+(j-1)w}}\right)+rs_1\left(\varepsilon_n^{\frac{2\sigma+w}{1+(n-2)w}}+\varepsilon_n^2\right)\\&\quad+r\sum_{j=2}^{n-1}\bar A_{n-1,j}\left(\varepsilon_j^2+\varepsilon_j^{\frac{2\sigma+w}{1+(j-2)w}}\right).\end{aligned}\tag{A.9}$$

From Lemmas 3-5, one gets

$$\begin{aligned}&s_0r\left(|\xi_n-\hat\xi_n|^2+|\xi_n-\hat\xi_n|^{\frac{2\sigma+w}{1+(n-1)w}}\right)\\&\le\frac{r}{8}\sum_{j=1}^{n}\left(\xi_j^2+\xi_j^{\frac{2\sigma+w}{1+(j-1)w}}\right)+rs_2\left(\varepsilon_n^2+\varepsilon_n^{\frac{2\sigma+w}{1+(n-2)w}}\right)\\&\quad+r\sum_{j=2}^{n-1}\tilde A_{n-1,j}\left(\varepsilon_j^2+\varepsilon_j^{\frac{2\sigma+w}{1+(j-2)w}}\right).\end{aligned}\tag{A.10}$$

Defining $\delta_0=s_1+s_2$, $A_{n-1,j}=\bar A_{n-1,j}+\tilde A_{n-1,j}$, and combing (A.9) and (A.10), it follows that

$$\begin{aligned}&r\left(\xi_n+\xi_n^{\frac{2\sigma-1-(n-1)w}{1+(n-1)w}}\right)(v-\lambda_n)\\&\le r\delta_0\left(\varepsilon_n^2+\varepsilon_n^{\frac{2\sigma+w}{1+(n-2)w}}\right)+\frac{r}{4}\sum_{i=1}^{n}\left(z_i^2+z_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)\\&\quad+r\sum_{j=2}^{n-1}A_{n-1,j}\left(\varepsilon_j^2+\varepsilon_j^{\frac{2\sigma+w}{1+(j-2)w}}\right).\end{aligned}\tag{A.11}$$

Then, we have

$$\begin{aligned}\dot U_n\le&-\frac{3}{4}r\sum_{i=1}^{n}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)+r\delta_0\left(\varepsilon_n^2+\varepsilon_n^{\frac{2\sigma+w}{1+(n-2)w}}\right)\\&+r\sum_{j=2}^{n-1}A_{n-1,j}\left(\varepsilon_j^2+\varepsilon_j^{\frac{2\sigma+w}{1+(j-2)w}}\right).\end{aligned}\tag{A.12}$$

Proof of Lemma 2: By the definition of U, it follows that

$$\begin{aligned}
&\frac{\partial U(\mathcal{Z})}{\partial \mathcal{Z}}\Big(\mathcal{F}(\mathcal{Z},\mathcal{Z}(t-\bar{\tau}))+\mathcal{D}(t,\mathcal{Z})\Big)\\
&=-r\sum_{i=2}^{n}a_{i-1}\Big(\varepsilon_i+\big(\rho_i+a_{i-1}\eta_{i-1}\big)^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\\
&\quad-\big(\hat{\rho}_i+a_{i-1}\eta_{i-1}\big)^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\Big)(\eta_i-\hat{\eta}_i)\\
&\quad+\sum_{i=1}^{n}\frac{\partial W_i}{\partial \xi_i}\big(\tilde{f}_{1i}+\tilde{d}_{1i}\big)+\sum_{i=2}^{n}\big(r\eta_{i+1}+\bar{f}_i+\bar{d}_i\big)\varepsilon_i\\
&\quad\times\frac{1+\alpha_{i1}(\rho_i+m_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}-1}}{1+\alpha_{i2}(\rho_i+m_{i-1}\eta_{i-1})^{\frac{w}{1+(i-2)w}}}\\
&\quad-\sum_{i=2}^{n}a_{i-1}\big(\bar{f}_{i-1}+\bar{d}_{i-1}\big)\Big(\big(\rho_i+a_{i-1}\eta_{i-1}\big)^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\\
&\quad+\varepsilon_i-\big(\hat{\rho}_i+a_{i-1}\eta_{i-1}\big)^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\Big).
\end{aligned}\tag{B.1}$$

It can be deduced that

$$\frac{\partial W_i}{\partial \xi_i}=\xi_i+\xi_i^{\frac{2\sigma-1-(i-1)w}{1+(i-1)w}},\ \tilde{f}_{1i}=\bar{f}_i-\sum_{l=1}^{i-1}\frac{\partial \lambda_{i-1}}{\partial \eta_l}\bar{f}_l.\tag{B.2}$$

Using (5) and Assumption 1, one gets

$$\begin{aligned}
&\big(\xi_i+\xi_i^{\frac{2\sigma-1-(i-1)w}{1+(i-1)w}}\big)\bar{f}_i\\
&\le r^{1-b}e^{-m\tau_0}\gamma_{i1}\sum_{l=1}^{i}\Big(\xi_l^2(t-\tau_l)+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}(t-\tau_l)\Big)\\
&\quad+r^{1-b}e^{-m\tau_0}\bar{\gamma}_{i1}\sum_{l=1}^{i-1}\Big(\xi_l^2(t-\tau_{l+1})+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}(t-\tau_{l+1})\Big)\\
&\quad+r^{1-b}m_{i1}\sum_{l=1}^{i}\Big(\xi_l^2+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}\Big),
\end{aligned}\tag{B.3}$$

and

$$\begin{aligned}
&-\Big(z_i+z_i^{\frac{2\sigma-1-(i-1)w}{1+(i-1)w}}\Big)\sum_{l=1}^{i-1}\frac{\partial \lambda_{i-1}}{\partial \eta_l}\bar{f}_l\\
&\le r^{1-b}e^{-m\tau_0}\gamma_{i2}\sum_{l=1}^{i-1}\Big(\xi_l^2(t-\tau)+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}(t-\tau)\Big)\\
&\quad+r^{1-b}e^{-m\tau_0}\bar{\gamma}_{i2}\sum_{l=1}^{i-2}\Big(\xi_l^2(t-\tau)+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}(t-\tau)\Big)\\
&\quad+r^{1-b}m_{i2}\sum_{l=1}^{i}\Big(\xi_l^2+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}\Big),
\end{aligned}\tag{B.4}$$

where m_{il}, γ_{il}, and $\bar{\gamma}_{il}$, $l=1,2$ are positive constants. Define $m_0=\sum_{l=1}^{i}(m_{i1}+m_{i2})$, $\gamma_0=\sum_{l=1}^{i}(\gamma_{i1}+\gamma_{i2})$, $\bar{\gamma}_0=\sum_{l=1}^{i}(\bar{\gamma}_{i1}+\bar{\gamma}_{i2})$, we have

$$\begin{aligned}
&\sum_{i=1}^{n}\frac{\partial W_i}{\partial \xi_i}\tilde{f}_{1i}\\
&\le r^{1-b}m_0\sum_{l=1}^{n}\Big(\xi_l^2(t)+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}(t)\Big)\\
&\quad+r^{1-b}e^{-m\tau_0}\bar{\gamma}_0\sum_{l=1}^{n-1}\Big(\xi_l^2(t-\tau_{l+1})+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}(t-\tau_{l+1})\Big)\\
&\quad+r^{1-b}e^{-m\tau_0}\gamma_0\sum_{l=1}^{n}\Big(\xi_l^2(t-\tau_l)+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}(t-\tau_l)\Big).
\end{aligned}\tag{B.5}$$

Similarly, one gets

$$\begin{aligned}
\sum_{i=1}^{n}\frac{\partial W_i}{\partial \xi_i}\tilde{d}_{1i}&\le r^{1-b}\bar{m}_0\sum_{l=1}^{n}\Big(\xi_l^2(t)+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}(t)\Big)\\
&\quad+\bar{\gamma}_0\sum_{l=1}^{n}\Big(d_l^2(t)+d_l^{\frac{2\sigma+w}{1+(l-1)w}}(t)\Big).
\end{aligned}\tag{B.6}$$

Let $\bar{\eta}_i=\hat{\rho}_j+m_{j-1}\eta_{j-1}+(\hat{\rho}_j+m_{j-1}\eta_{j-1})^{1+\frac{w}{1+(i-2)w}}$, it follows that

$$\begin{aligned}
&-a_{i-1}r\Big(\varepsilon_i+(\rho_i+a_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\\
&\quad-(\hat{\rho}_i+a_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\Big)(\eta_i-\bar{\eta}_i)\\
&\le -ca_{i-1}r\Big(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\Big).
\end{aligned}\tag{B.7}$$

Using Lemmas 4-5, one obtains

$$\begin{aligned}
&-a_{i-1}r\Big(\varepsilon_i+(\rho_i+a_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\\
&\quad-(\hat{\rho}_i+m_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\Big)(\bar{\eta}_i-\hat{\eta}_i)\\
&\le r\rho_i\Big(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\Big)+r\sum_{l=2}^{i-1}\bar{B}_{j-1,l}\Big(\varepsilon_l^2+\varepsilon_l^{\frac{2\sigma+w}{1+(i-2)w}}\Big)\\
&\quad+\frac{r}{4n}\sum_{l=1}^{i}\Big(\xi_l^2+\xi_l^{\frac{2\sigma+w}{1+(l-1)w}}\Big).
\end{aligned}\tag{B.8}$$

Combining (B.7) and (B.8), we have

$$\begin{aligned}
&-r\sum_{i=2}^{n}a_{i-1}\Big(\varepsilon_i+(\chi_i+a_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\\
&\quad-(\hat{\rho}_i+a_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\Big)(\eta_i-\hat{\eta}_i)\\
&\le -rc\sum_{i=2}^{n-1}\big(a_{i-1}-\bar{B}_{n-1,i}-\rho_i\big)\Big(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\Big)\\
&\quad-cr(a_{n-1}-\rho_n)\Big(\varepsilon_n^2+\varepsilon_n^{\frac{2\sigma+w}{1+(i-2)w}}\Big)\\
&\quad+\frac{r}{4}\sum_{i=1}^{n}\Big(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\Big).
\end{aligned}\tag{B.9}$$

By utilizing Lemmas 3, 4, and 5, one has

$$\begin{aligned}
&-a_{i-1}\big(\bar{f}_{i-1}+\bar{d}_{i-1}\big)\Big(\varepsilon_i+(\rho_i+a_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\\
&\quad-(\hat{\rho}_i+a_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}}\Big)\\
&\le\frac{r^{1-b}}{n}e^{-m\tau_0}\sum_{j=1}^{i-1}\big(\xi_j^2(t-\tau_j)+\xi_j^{\frac{2\sigma+w}{1+(j-1)w}}(t-\tau_j)\big)
\end{aligned}$$

$$+\frac{r^{1-b}}{n}e^{-m\tau_0}\sum_{j=1}^{i-2}\left(\xi_j^2(t-\tau_{j+1})+\xi_j^{\frac{2\sigma+w}{1+(j-1)w}}(t-\tau_{j+1})\right)$$
$$+\pi_0 r^{1-b}a_{i-1}\left(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}+\sum_{j=1}^{i}\left(\xi_i^2+\xi_i^{\frac{2\sigma+w}{1+(i-1)w}}\right)\right)$$
$$+\pi_{i1}\left(d_{i-1}^2+d_{i-1}^{\frac{2\sigma+w}{1+(i-1)w}}\right), \tag{B.10}$$

where π_0 and π_1 are positive constants. By Lemmas 4 and 5, it yields that

$$\frac{1+\alpha_{i1}(\rho_i+a_{i-1}\eta_{i-1})^{\frac{2\sigma-1-(i-1)w}{1+(i-2)w}-1}}{1+\alpha_{i2}(\rho_i+a_{i-1}\eta_{i-1})^{\frac{w}{1+(i-2)w}}}(r\eta_{i+1}+\bar{f}_i+\bar{d}_i)\varepsilon_i$$
$$\leq\frac{r^{1-b}}{n-1}e^{-m\tau_0}\sum_{j=1}^{i}\left(\xi_j^2(t-\tau_j)+\xi_j^{\frac{2\sigma+w}{1+(j-1)w}}(t-\tau_j)\right)$$
$$+\frac{r^{1-b}}{n-1}e^{-m\tau_0}\sum_{j=1}^{i}\left(\xi_j^2(t-\tau_{j+1})+\xi_j^{\frac{2\sigma+w}{1+(j-1)w}}(t-\tau_{j+1})\right)$$
$$+\pi_{i2}\left(d_i^2+d_i^{\frac{2\sigma+w}{1+iw}}\right)+\frac{r}{6(n-1)}\sum_{j=1}^{i+1}\left(\xi_j^2+\xi_j^{\frac{2\sigma+w}{1+(j-1)w}}\right)$$
$$+r\tilde{\rho}_i\left(\varepsilon_i^2+\varepsilon_i^{\frac{2\sigma+w}{1+(i-2)w}}\right), \tag{B.11}$$

where π_{i2}, $\tilde{\rho}_i$ are constants. Define $B_{n-1,i}=\bar{B}_{n-1,i}+\rho_i+\tilde{\rho}_i$, $\rho=\rho_n$, $\delta_1=m_1+\bar{m}_0+\sum_{i=2}^{n}a_{i-1}\pi_0$, $\delta_2=\sum_{i=2}^{n}a_{i-1}\pi_0$, $\delta_3=\gamma_0+2$ and $\delta_4=\bar{\gamma}_0+2$,$\delta_5=\bar{\gamma}_0+\sum_{i=1}^{n}(\pi_{i1}+\pi_{i2})$. The above deduction indicates that Lemma 2 holds.

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