

# Sampled-data Control of Fuzzy Systems Based on the Intelligent Digital Redesign Technique: An Input-delay Approach

Han Sol Kim, Jin Bae Park\*, and Young Hoon Joo\*

**Abstract:** In this paper, a novel intelligent digital redesign (IDR) method for a Takagi-Sugeno fuzzy system is proposed based on the guaranteed cost method. The objective of the IDR is to determine a sample-data data gain that achieves the same performance as a given continuous-time controller. Unlike previous works, we use the state-matching error cost function and develop an IDR technique without the use of any discretization methods. To this end, a sufficient condition guaranteeing both the asymptotic stabilization of the error dynamics model and the minimization of the upper bound of the error cost function is formulated in terms of linear matrix inequalities based on the input-delay approach. Finally, a simulation example validates the superiority of the proposed method.

**Keywords:** Guaranteed cost method, intelligent digital redesign (IDR), sampled-data control, Takagi-Sugeno (T-S) fuzzy system, time-dependent Lyapunov-Krasovskii functional (LKF).

## 1. INTRODUCTION

The Takagi-Sugeno (T-S) fuzzy model [1] has been intensively studied from the control community because it can express a given nonlinear dynamics model as a convex summation of sub-linear models and fuzzy weighting functions using the sector nonlinearity concept [2]. As a wide class of nonlinear systems can be modeled in the form of the T-S fuzzy model, it is possible to systematically design a controller for nonlinear dynamic systems based on the T-S fuzzy control approach. Accordingly, various studies have been performed regarding the control design of continuous-time T-S fuzzy models, such as observer-based control [3], robust control [4],  $\mathcal{H}_\infty$  control [5], and decentralized control [6].

At the same time, because of the advances of digital computing technologies, implementing a controller using a digital computer or microcontroller have also received significant attention. When the continuous-time system is controlled via a digital controller, such a control system is called a sampled-data (SD) control system. The most common approach, called the input-delay approach [7], designs an SD controller based on the Lyapunov-Krasovskii functional (LKF) after converting a given SD control system into an equivalent input-delay control system. While the input-delay approach has been successfully applied to the design of SD fuzzy controllers for

years [8–12], there exists a serious problem in that the approach relinquishes a large number of fruitful methods developed in the continuous-time domain.

As an alternative approach to the input-delay approach, an intelligent digital redesign (IDR) technique was proposed in [13]. In the IDR methods, an SD controller that has an equivalent performance with a pre-designed continuous-time controller is designed. Due to this characteristic, many IDR-based SD control approaches have been actively studied, example include [14, 15, 17]. In previous studies, the control systems were approximately discretized in advance, and the IDR problem was then addressed using the state-matching condition which guarantees the minimization of the norm distance between these closed-loop system matrices only at each sampling time. Thus, the discretization error together with the unaddressed inter-sampling time state-matching degrades the state-matching performance. Recently, a different type of IDR method was proposed based on the guaranteed cost method in [18]; however, this method could not obtain sufficient state-matching performance either because of the discretization error.

Motivated by the above considerations, this paper proposes an input-delay approach to the IDR of T-S fuzzy systems based on the guaranteed cost method. This is realized using the state-matching error cost function and a technique that minimizes the cost over the whole time

---

Manuscript received May 8, 2017; accepted June 6, 2017. Recommended by Associate Editor Ho Jae Lee under the direction of Editor Euntai Kim. This work was supported by Barun ICT Research Center at Yonsei University, and by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2016R1A6A1A03013567) and by the Ministry of Trade, Industry & Energy (MOTIE) of the Republic of Korea (No. 20174030201670).

Han Sol Kim and Jin Bae Park are with the School of Electrical and Electronic Engineering, Yonsei University, 50, Yonsei-ro, Seodaemun-gu, Seoul 03722, Korea (e-mails: {solsol, jbpark}@yonsei.ac.kr). Young Hoon Joo is with the Department of Control and Robotics Engineering, Kunsan National University, 558, Daehak-ro, Gunsan-si, Jeollabuk-do, 54150, Korea (e-mail: yhjoo@kunsan.ac.kr).

\* Corresponding authors.

domain. Linear matrix inequality (LMI) [19] conditions guaranteeing both the asymptotic stabilization of the error dynamics model and minimization of the upper bound of the error cost function is developed without the use of any discretization methods. By solving the performance degradation caused by the discretization error, an improved state-matching performance can be obtained. Finally, the superiority of the proposed method is validated using a simulation example.

## 2. PROBLEM FORMULATION

In this paper, we deal with the IDR problem of the T-S fuzzy model of the following form:

$$\dot{x}_a(t) = \sum_{i=1}^r w_i(x_a(t)) (A_i x_c(t) + B_i u_c(t)), \quad (1)$$

where  $i \in \mathcal{I}_r := \{1, 2, \dots, r\}$ ,  $x_a(t) \in \mathbb{R}^n$  and  $u_a(t) \in \mathbb{R}^m$  are the state and input vectors, respectively, in which the subscript ‘‘a’’ can be either both ‘‘c’’ or ‘‘d’’, the subscript ‘‘c’’ indicates the analog control while the subscript ‘‘d’’ denotes the SD control in the sequel,  $A_i$  and  $B_i$  are the system and input matrices, respectively, and  $w_i(x_a(t))$  is the scalar fuzzy weighting function satisfying the following properties:  $w_i(x_a(t)) \in [0, 1]$  and  $\sum_{i=1}^r w_i(x_a(t)) = 1$ .

We assume that the continuous-time fuzzy controller for stabilizing (1) was designed in advance and has the following form:

$$u_c(t) = \sum_{j=1}^r w_j(x_c(t)) K_j^c x_c(t), \quad (2)$$

where  $K_j^c$  for  $j \in \mathcal{I}_r$  is the predetermined continuous-time gain matrix.

Substituting (2) into (1) and for any matrix  $M_i$ s using the following notation:  $M(t) = \sum_{i=1}^r w_i(x_c(t)) M_i$ , we have the following continuous-time closed-loop fuzzy control system representation:

$$\dot{x}_c(t) = \left\{ A(t) + B(t)K^c(t) \right\} x_c(t). \quad (3)$$

Next, we employ an SD fuzzy controller of the following form:

$$\begin{aligned} u_d(t) = u_d(t_k) &= \sum_{j=1}^r w_j(x_d(t_k)) K_j^d x_d(t_k) \\ &= K^d(t_k) x_d(t_k), \end{aligned} \quad (4)$$

for  $t \in [t_k, t_{k+1})$ ,  $\forall k \in \mathbb{Z}_{\geq 0}$ , where  $t_k$  is the  $k$ th sampling time,  $h := t_{k+1} - t_k$  is a constant sampling period, and  $K_j^d$  for  $j \in \mathcal{I}_r$  is the SD gain matrix to be determined.

Closing (1) with (4), the closed-loop SD fuzzy control system becomes

$$\dot{x}_d(t) = A(t)x_d(t) + B(t)K^d(t_k)x_d(t_k)$$

$$\begin{aligned} &= \left\{ A(t) + B(t)K^d(t_k) \right\} x_d(t) \\ &\quad - (t - t_k)B(t)K^d(t_k)\bar{x}_d(t), \end{aligned} \quad (5)$$

where  $(t - t_k)\bar{x}_d(t) := x_d(t) - x_d(t_k)$ .

The proposed method is derived based on the following assumption:

**Assumption 1:** In this paper, we assume that the difference between the continuous-time and sampled-time fuzzy weighting functions is small enough to be ignored; in other words,  $w_i(x_c(t)) \simeq w_i(x_d(t))$  for  $i \in \mathcal{I}_r$  and  $\forall t$ . This is valid when the state-matching error between  $x_c(t)$  and  $x_d(t)$  is small enough, which is the objective of this paper.

The purpose of this paper is to minimize the state-matching error  $e(t) = x_c(t) - x_d(t)$  so that  $x_d(t)$  closely matches  $x_c(t)$  for all  $t$ . The error dynamics model can be configured as follows:

$$\begin{aligned} \dot{e}(t) &= \dot{x}_c(t) - \dot{x}_d(t) \\ &= \left\{ A(t) + B(t)K^c(t) \right\} e(t) + B(t)K^c(t)x_d(t) \\ &\quad - B(t)K^d(t_k)x_d(t) + (t - t_k)B(t)K^d(t_k)\bar{x}_d(t) \\ &= \left\{ A(t) + B(t)K^c(t) \right\} e(t) \\ &\quad + B(t) \left\{ K^c(t) - K^d(t_k) \right\} x_d(t) \\ &\quad + (t - t_k)B(t)K^d(t_k)\bar{x}_d(t). \end{aligned} \quad (6)$$

By defining the augmented state vector as  $\chi(t) = \text{col}\{x_d(t), e(t)\}$ , we have

$$\dot{\chi}(t) = \mathcal{A}(t, t_k)\chi(t) - (t - t_k)\mathcal{B}(t, t_k)\bar{\chi}(t), \quad (7)$$

where

$$\begin{aligned} \mathcal{A}(t, t_k) &= \begin{bmatrix} A(t) + B(t)K^d(t_k) & \mathbf{0} \\ B(t)\{K^c(t) - K^d(t_k)\} & A(t) + B(t)K^c(t) \end{bmatrix}, \\ \mathcal{B}(t, t_k) &= \begin{bmatrix} B(t)K^d(t_k) & \mathbf{0} \\ -B(t)K^d(t_k) & \mathbf{0} \end{bmatrix}, \text{ and} \\ \bar{\chi}(t) &:= \frac{1}{t - t_k} \left( \chi(t) - \chi(t_k) \right). \end{aligned} \quad (8)$$

Finally, the IDR problem considered in this paper is summarized in the following statements:

**Problem 1:** Assuming that there exists a well-constructed continuous-time gain matrix  $K^c(t)$ , find an SD gain matrix  $K^d(t_k)$  so that the following conditions can be satisfied simultaneously:

- 1) The upper bound of the error cost function  $J(\infty)$  is less than a predefined level  $\gamma \in \mathbb{R}_{>0}$ , where

$$J(t) = \int_{t_0}^t e^T(s) Q e(s) ds, \quad (9)$$

in which  $0 \prec Q = Q^T \in \mathbb{R}^{n \times n}$  is predefined positive definite matrix of an appropriate dimension.

- 2) The equilibrium of (5) is asymptotically stable.

### 3. THE INPUT-DELAY APPROACH TO THE IDR METHOD

In this section, we derive the proposed IDR method based on the input-delay approach. First, we briefly review lemmas required to derive the proposed method.

**Lemma 1 [22]:** Given any vector function  $v(t)$ , positive definite matrix  $M = M^T \succ 0$ , and positive scalars  $t_0$  and  $t_f$  with  $t_0 < t_f$ , the following inequality always holds:

$$\left\{ \int_{t_0}^{t_f} v(s) ds \right\}^T M \left\{ \int_{t_0}^{t_f} v(s) ds \right\} \leq (t_f - t_0) \int_{t_0}^{t_f} v^T(s) M v(s) ds. \quad (10)$$

**Lemma 2 [23]:** The following two statements are equivalent:

- (1) Find  $P = P^T \succ 0$  such that  $A^T P A - H \prec 0$ .
- (2) Find  $P = P^T \succ 0$  and  $G$  such that

$$\begin{bmatrix} -H & * \\ GA & -G - G^T + P \end{bmatrix} \prec 0.$$

The proposed IDR method is summarized in the following theorem:

**Theorem 1:** For a given sampling period  $h$  and a positive scalar  $\alpha > 0$ , state vectors  $x_c(t)$  of (3) and  $x_d(t)$  of (5) satisfy the conditions given in Problem 1 if there exist positive definite matrices  $\bar{P}$  and  $\bar{R}$ , full-rank matrices  $\bar{F}_1, \bar{F}_2, \bar{Z}_1, \bar{Z}_2, \bar{Y}_1, \bar{Y}_2, \bar{Y}_3$ , and matrix  $\bar{K}_i^d$  such that the following LMIs are satisfied:

$$\begin{aligned} \min_{\bar{X}} \gamma, \bar{X} \in \{\bar{P}, \bar{R}, \bar{F}_1, \bar{F}_2, \bar{Z}_1, \bar{Z}_2, \bar{Y}_1, \bar{Y}_2, \bar{Y}_3, \bar{K}_i^d\} \\ \text{subject to} \\ \mathcal{M}_{iiq}^l \prec 0 \text{ for } (i, q, l) \in \mathcal{I}_r \times \mathcal{I}_r \times \mathcal{I}_2, \quad (11) \\ \mathcal{M}_{ijq}^l + \mathcal{M}_{jiq}^l \prec 0 \text{ for } i < j \in \mathcal{I}_r, (q, l) \in \mathcal{I}_r \times \mathcal{I}_2, \quad (12) \end{aligned}$$

$$\begin{bmatrix} -\gamma & * \\ \chi(0) & -\bar{F} - \bar{F}^T + \bar{P} \end{bmatrix} \prec 0, \quad (13)$$

where the definition of  $\mathcal{M}_{ijq}^l$  is given in (15)-(17).

Finally, an SD fuzzy gain matrix  $K_q^d$  for  $q \in \mathcal{I}_r$  can be obtained by

$$K_q^d = \bar{K}_q^d \bar{F}_1^{-1}. \quad (14)$$

**Proof:** Consider the following LKF [7]:

$$\begin{aligned} V(t) = & \chi^T(t) P \chi(t) + (t_{k+1} - t) \int_{t_k}^t \dot{\chi}^T(s) R \dot{\chi}(s) ds \\ & + (t_{k+1} - t) \begin{bmatrix} \chi(t) \\ \chi(t_k) \end{bmatrix}^T \begin{bmatrix} \mathcal{H}_{11} & * \\ \mathcal{H}_{21} & \mathcal{H}_{22} \end{bmatrix} \begin{bmatrix} \chi(t) \\ \chi(t_k) \end{bmatrix} \end{aligned} \quad (18)$$

for  $t \in [t_k, t_{k+1})$ , where  $0 \prec P = P^T \in \mathbb{R}^{2n \times 2n}$  and  $0 \prec R = R^T \in \mathbb{R}^{2n \times 2n}$  are positive definite matrices to be determined,  $\mathcal{H}_{11} = 0.5(Z_1 + Z_1^T)$ ,  $\mathcal{H}_{21} = -Z_1^T + Z_2^T$ , and  $\mathcal{H}_{22} = -Z_2 - Z_2^T + 0.5(Z_1 + Z_1^T)$ , in which  $Z_1 \in \mathbb{R}^{2n \times 2n}$  and  $Z_2 \in \mathbb{R}^{2n \times 2n}$  are full-rank matrices to be determined.

The time derivative of (18) along the trajectories of (7) for  $t \in (t_k, t_{k+1})$  is as follows:

$$\begin{aligned} \dot{V}(t) = & 2\chi^T(t) P \dot{\chi}(t) \\ & + (t_{k+1} - t) \dot{\chi}^T(t) R \dot{\chi}(t) - \int_{t_k}^t \dot{\chi}^T(s) R \dot{\chi}(s) ds \\ & + 2(t_{k+1} - t) \left\{ \chi^T(t) \mathcal{H}_{11} \dot{\chi}(t) + \chi^T(t_k) \mathcal{H}_{21} \dot{\chi}(t) \right\} \\ & - \begin{bmatrix} \chi(t) \\ \chi(t_k) \end{bmatrix}^T \begin{bmatrix} \mathcal{H}_{11} & * \\ \mathcal{H}_{21} & \mathcal{H}_{22} \end{bmatrix} \begin{bmatrix} \chi(t) \\ \chi(t_k) \end{bmatrix}. \end{aligned} \quad (19)$$

From Lemma 1, we know that

$$- \int_{t_k}^t \dot{\chi}^T(s) R \dot{\chi}(s) ds \leq -(t - t_k) \bar{\chi}^T(t) R \bar{\chi}(t) \quad (20)$$

holds; thus, from (19) and (20), we have

$$\begin{aligned} \dot{V}(t) \leq & 2\chi^T(t) P \dot{\chi}(t) \\ & + (t_{k+1} - t) \dot{\chi}^T(t) R \dot{\chi}(t) - (t - t_k) \bar{\chi}^T(t) R \bar{\chi}(t) \\ & + 2(t_{k+1} - t) \left\{ \chi^T(t) \mathcal{H}_{11} \dot{\chi}(t) + \chi^T(t_k) \mathcal{H}_{21} \dot{\chi}(t) \right\} \\ & - \begin{bmatrix} \chi(t) \\ \chi(t_k) \end{bmatrix}^T \begin{bmatrix} \mathcal{H}_{11} & * \\ \mathcal{H}_{21} & \mathcal{H}_{22} \end{bmatrix} \begin{bmatrix} \chi(t) \\ \chi(t_k) \end{bmatrix}. \end{aligned} \quad (21)$$

On the other hand, the following null expressions are obvious from (7) and (8):

$$\begin{aligned} 0 = & 2 \left\{ \chi^T(t) Y_1^T + \dot{\chi}^T(t) Y_2^T + \chi^T(t_k) Y_3^T \right\} \\ & \times \left\{ -\chi(t) + \chi(t_k) + (t - t_k) \bar{\chi}(t) \right\}, \end{aligned} \quad (22)$$

$$\begin{aligned} 0 = & 2 \left\{ \chi^T(t) F^T + \alpha \dot{\chi}^T(t) F^T \right\} \\ & \times \left\{ -\dot{\chi}(t) + \mathcal{A}(t, t_k) \chi(t) - (t - t_k) \mathcal{B}(t, t_k) \bar{\chi}(t) \right\}, \end{aligned} \quad (23)$$

where  $Y_1, Y_2$ , and  $Y_3$  are  $2n \times 2n$  full-rank matrices to be determined,  $F = \text{diag}\{F_1, F_2\}$ , in which  $F_1$  and  $F_2$  are  $n \times n$  matrices to be determined, and  $\alpha \in \mathbb{R}_{>0}$  is a predefined scalar.

Combining (21)-(23) yields

$$\begin{aligned} \dot{V}(t) \leq & \eta^T(t) \left\{ \Theta^1(t, t_k) + (t_{k+1} - t) \Theta^2 \right. \\ & \left. + (t - t_k) \Theta^3(t, t_k) \right\} \eta(t), \end{aligned} \quad (24)$$

where  $\eta(t) = \text{col}\{\chi(t), \dot{\chi}(t), \chi(t_k), \bar{\chi}(t)\}$ ,

$$\Theta^1(t, t_k) = \begin{bmatrix} He(F^T \mathcal{A}(t, t_k) - Y_1) - \mathcal{H}_{11} \\ P - Y_2^T - F + \alpha F^T \mathcal{A}(t, t_k) \\ -Y_3^T - \mathcal{H}_{21} + Y_1 \\ \mathbf{0} \end{bmatrix}$$

$$\mathcal{M}_{ijq}^1 = \begin{bmatrix} He(\bar{\mathcal{A}}_{ijq} - \bar{Y}_1) - \bar{\mathcal{H}}_{11} & * & * & * \\ \bar{P} - \bar{Y}_2^T - \bar{F}^T + \alpha \bar{\mathcal{A}}_{ijq} + h \bar{\mathcal{H}}_{11}^T & -\alpha(\bar{F} + \bar{F}^T) + h \bar{R} & * & * \\ -\bar{Y}_3^T - \bar{\mathcal{H}}_{21} + \bar{Y}_1 & \bar{Y}_2 + h \bar{\mathcal{H}}_{21} & -\bar{\mathcal{H}}_{22} + \bar{Y}_3 + \bar{Y}_3^T & * \\ \begin{bmatrix} \mathbf{0} & \bar{F}_2 \end{bmatrix} & \mathbf{0} & \mathbf{0} & -Q^{-1} \end{bmatrix}, \quad (15)$$

$$\mathcal{M}_{ijq}^2 = \begin{bmatrix} He(\bar{\mathcal{A}}_{ijq} - \bar{Y}_1) - \bar{\mathcal{H}}_{11} & * & * & * & * \\ \bar{P} - \bar{Y}_2^T - \bar{F}^T + \alpha \bar{\mathcal{A}}_{ijq} & -\alpha(\bar{F} + \bar{F}^T) & * & * & * \\ -\bar{Y}_3 - \bar{\mathcal{H}}_{21} + \bar{Y}_1 & \bar{Y}_2 & -\bar{\mathcal{H}}_{22} + \bar{Y}_3 + \bar{Y}_3^T & * & * \\ h(\bar{Y}_1 - \bar{\mathcal{B}}_{iq}^T) & h(\bar{Y}_2 - \alpha \bar{\mathcal{B}}_{iq}^T) & h \bar{Y}_3 & -h \bar{R} & * \\ \begin{bmatrix} \mathbf{0} & \bar{F}_2 \end{bmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -Q^{-1} \end{bmatrix}, \quad (16)$$

$$\bar{\mathcal{A}}_{ijq} = \begin{bmatrix} A_i \bar{F}_1 + B_i \bar{K}_q^d & \mathbf{0} \\ B_i K_j^c \bar{F}_1 - B_i \bar{K}_q^d & (A_i + B_i K_j^c) \bar{F}_2 \end{bmatrix}, \quad \bar{\mathcal{B}}_{iq} = \begin{bmatrix} B_i \bar{K}_q^d & \mathbf{0} \\ -B_i \bar{K}_q^d & \mathbf{0} \end{bmatrix}, \quad \text{and } \bar{F} = \begin{bmatrix} \bar{F}_1 & \mathbf{0} \\ \mathbf{0} & \bar{F}_2 \end{bmatrix}$$

for  $(i, j, q, l) \in \mathcal{I}_r \times \mathcal{I}_r \times \mathcal{I}_r \times \mathcal{I}_2$ ,  $\bar{\mathcal{H}}_{11} = 0.5(\bar{Z}_1 + \bar{Z}_1^T)$ ,  $\bar{\mathcal{H}}_{21} = -\bar{Z}_1^T + \bar{Z}_2^T$ , and  $\bar{\mathcal{H}}_{22} = \text{He}(-\bar{Z}_2 + 0.5\bar{Z})$ . (17)

$$\Theta^2 = \begin{bmatrix} * & * & * \\ -\alpha(\bar{F} + \bar{F}^T) & * & * \\ Y_2 & -\bar{\mathcal{H}}_{22} + Y_3 + Y_3^T & * \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix}, \quad \text{and}$$

$$\Theta^3(t, t_k) = \begin{bmatrix} \mathbf{0} & * & * & * \\ \mathcal{H}_{11}^T & R & * & * \\ \mathbf{0} & \mathcal{H}_{21} & \mathbf{0} & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \text{and}$$

$$\Theta^3(t, t_k) = \begin{bmatrix} \mathbf{0} & * \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ Y_1 - \mathcal{B}^T(t, t_k)F & Y_2 - \alpha \mathcal{B}^T(t, t_k)F \\ * & * \\ * & * \\ \mathbf{0} & * \\ Y_3 & -R \end{bmatrix}.$$

Adding  $e^T(t)Qe(t) = \eta^T(t)\hat{I}Q\hat{I}^T\eta(t) \geq 0$  to (24) on both left- and right-hand sides, we have

$$\begin{aligned} \hat{V}(t) + e^T(t)Qe(t) &= \hat{V}(t) + \eta^T(t)\hat{I}Q\hat{I}^T\eta(t) \\ &\leq \eta^T(t) \left[ \Theta^1(t, t_k) + \hat{I}Q\hat{I}^T \right. \\ &\quad \left. + (t_{k+1} - t)\Theta^2 \right. \\ &\quad \left. + (t - t_k)\Theta^3(t, t_k) \right] \eta(t), \quad (25) \end{aligned}$$

where  $\hat{I} = \text{col}\{\begin{bmatrix} \mathbf{0}_n & I \end{bmatrix}^T, \mathbf{0}_{2n}, \mathbf{0}_{2n}, \mathbf{0}_{2n}\}$ , in which  $I$  is an  $n \times n$  identity matrix, and  $\mathbf{0}_n$  and  $\mathbf{0}_{2n}$  stand for  $n \times n$  and  $2n \times n$  zero matrices, respectively.

Thus,  $\hat{V}(t) + e^T(t)Qe(t) \leq 0$  for  $t \in (t_k, t_{k+1})$  if and only if the following matrix inequality holds:

$$\hat{V}(t) := \Theta^1(t, t_k) + (t_{k+1} - t)\Theta^2 + (t - t_k)\Theta^3(t, t_k) + \hat{I}Q\hat{I}^T \prec 0. \quad (26)$$

Moreover, we can reformulate  $\hat{V}(t)$  as the following

convex sum representation:

$$\begin{aligned} \hat{V}(t) &= \frac{t_{k+1} - t}{h} \left[ \Theta^1(t, t_k) + h\Theta^2 + \hat{I}Q\hat{I}^T \right] \\ &\quad + \frac{t - t_k}{h} \left[ \Theta^1(t, t_k) + h\Theta^3(t, t_k) + \hat{I}Q\hat{I}^T \right] \prec 0. \quad (27) \end{aligned}$$

From the above, we can know that (27) holds if the following matrix inequalities are satisfied simultaneously:

$$\Theta^1(t, t_k) + h\Theta^2 + \hat{I}Q\hat{I}^T \prec 0, \quad (28)$$

$$\Theta^1(t, t_k) + h\Theta^3(t, t_k) + \hat{I}Q\hat{I}^T \prec 0, \quad (29)$$

because  $t \in (t_k, t_{k+1})$  and  $h = t_{k+1} - t_k, \forall k \in \mathbb{Z}_{\geq 0}$ .

Thus, applying the Schur complement to (28) and (29), we have

$$(28) \Leftrightarrow \begin{bmatrix} \Theta^1(t, t_k) + h\Theta^2 & * \\ \hat{I}^T & -Q^{-1} \end{bmatrix} \prec 0, \quad \text{and} \quad (30)$$

$$(29) \Leftrightarrow \begin{bmatrix} \Theta^1(t, t_k) + h\Theta^3(t, t_k) & * \\ \hat{I}^T & -Q^{-1} \end{bmatrix} \prec 0, \quad (31)$$

respectively.

Before proceeding next, we define

$$\begin{aligned} \bar{F}_1 &= F_1^{-1}, \quad \bar{F}_2 = F_2^{-1}, \quad \bar{F} = F^{-1} = \text{diag}\{\bar{F}_1, \bar{F}_2\}, \\ \bar{Y}_1 &= \bar{F}^T Y_1 \bar{F}, \quad \bar{Y}_2 = \bar{F}^T Y_2 \bar{F}, \quad \bar{Y}_3 = \bar{F}^T Y_3 \bar{F}, \\ \bar{Z}_1 &= \bar{F}^T Z_1 \bar{F}, \quad \bar{Z}_2 = \bar{F}^T Z_2 \bar{F}, \quad \bar{P} = \bar{F}^T P \bar{F}, \quad \bar{R} = \bar{F}^T R \bar{F}, \\ \bar{\mathcal{H}}_{11} &= 0.5(\bar{Z}_1 + \bar{Z}_1^T), \quad \bar{\mathcal{H}}_{21} = -\bar{Z}_1^T + \bar{Z}_2^T, \\ \bar{\mathcal{H}}_{22} &= -\bar{Z}_2 - \bar{Z}_2^T + 0.5(\bar{Z}_1 + \bar{Z}_1^T), \quad \bar{K}^d(t_k) = K^d(t_k) \bar{F}_1, \\ \bar{\mathcal{A}}(t, t_k) &= \mathcal{A}(t, t_k) \bar{F} = \begin{bmatrix} A(t) \bar{F}_1 + B(t) \bar{K}^d(t_k) \\ B(t) K^c(t) \bar{F}_1 - B(t) \bar{K}^d(t_k) \end{bmatrix}, \quad \text{and} \\ &\quad (A(t) + B(t) K^c(t)) \bar{F}_2, \\ \bar{\mathcal{B}}(t, t_k) &= \mathcal{B}(t, t_k) \bar{F} = \begin{bmatrix} B(t) \bar{K}^d(t_k) & \mathbf{0} \\ -B(t) \bar{K}^d(t_k) & \mathbf{0} \end{bmatrix}. \end{aligned}$$

Finally, using the above relationships and applying the congruence transformation to (30) and (31) with  $\text{diag}\{\bar{F}, \bar{F}, \bar{F}, I\}$  and  $\text{diag}\{\bar{F}, \bar{F}, \bar{F}, I\}$ , we conclude that  $\dot{V}(t) + e^T(t)Qe(t) \leq 0$  for  $t \in (t_k, t_{k+1})$  if

$$\mathcal{M}^l(t, t_k) \prec 0, \text{ for } l \in \mathcal{I}_2, \quad (32)$$

which is equivalent to

$$\begin{aligned} & \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t)w_q(t_k)\mathcal{M}_{ijq}^l \\ &= \sum_{i=1}^r \sum_{q=1}^r w_i^2(t)w_q(t_k)\mathcal{M}_{iiq}^l \\ &+ \sum_{i < j}^r \sum_{q=1}^r w_i(t)w_j(t)w_q(t_k)[\mathcal{M}_{ijq}^l + \mathcal{M}_{jiq}^l] \prec 0, \quad (33) \end{aligned}$$

for  $l \in \mathcal{I}_2$ , where the definitions of  $\mathcal{M}_{ijq}^l$  are defined in (15) and (16).

Therefore, if the LMIs of (11) and (12) are satisfied, then the equilibrium of (5) is asymptotically stable, which means that condition 2 of Problem 1 is achieved.

Moreover, integrating  $\dot{V}(t) + e^T(t)Qe(t)$  from  $t_k^+$  to  $t_{k+1}^-$  with respect to time  $t$  yields

$$\int_{t_k^+}^{t_{k+1}^-} [\dot{V}(t) + e^T(t)Qe(t)] dt \leq 0. \quad (34)$$

Because  $V(t_k^-) = V(t_k^+) = V(t_k)$  and  $e(t_k^-) = e(t_k^+) = e(t_k)$ , summing (34) from  $k = 0$  to  $\infty$ , we have

$$\begin{aligned} & \sum_{k=0}^{\infty} \left[ \int_{t_k^+}^{t_{k+1}^-} \left\{ \dot{V}(t) + e^T(t)Qe(t) \right\} dt \right] \\ &= \sum_{k=0}^{\infty} \left[ V(t_{k+1}^-) - V(t_k^+) + \int_{t_k^+}^{t_{k+1}^-} e^T(t)Qe(t) dt \right] \\ &= V(\infty) - \dots - V(t_0^+) + V(t_0^-) - V(t_{-1}^+) + \dots - V(t_0^+) \\ &+ \int_{t_0}^{\infty} e^T(t)Qe(t) dt \\ &= V(\infty) - V(t_0) + \int_{t_0}^{\infty} e^T(t)Qe(t) dt \leq 0, \quad (35) \end{aligned}$$

which yields

$$\int_{t_0}^{\infty} e^T(t)Qe(t) dt \leq V(t_0) - V(\infty) \leq V(t_0). \quad (36)$$

Thus, the upper bound of the state-matching error cost function is  $V(t_0)$ . Now, assume that the following holds:

$$V(t_0) = \mathcal{X}^T(0)P\mathcal{X}(0) \leq \gamma, \quad (37)$$

where  $\gamma$  is a positive scalar to be determined, from which we have

$$\mathcal{X}^T(0)P\mathcal{X}(0) - \gamma \leq 0. \quad (38)$$

Applying Lemma 2 to (38) yields

$$\begin{bmatrix} -\gamma & * \\ F\mathcal{X}(0) & -F - F^T + P \end{bmatrix} \prec 0. \quad (39)$$

Finally, applying the congruence transformation with  $\text{col}\{1, \bar{F}\}$ , we have the LMI condition (13). Therefore, if there exists a numerical solution to the optimization problem with LMIs (11)-(13), conditions of Problem 1 are achieved, and the upper bound of the state-matching error cost function is minimized below  $\gamma$ . This concludes the proof.  $\square$

**Remark 1:** Using Theorem 1, a sufficient condition guaranteeing the asymptotic stabilization of (7) and the minimization of the upper bound of the error cost function (9) is proposed in terms of LMIs. However, because of the mismatched information between  $w_i(t)$  (or  $w_j(t)$ ) and  $w_q(t_k)$ , the LMIs in Theorem 1 are somewhat conservative. Thus, in the following, this paper derives a relaxation method which is an extended version of the conventional one.

The following theorem gives the method for handling the mismatched fuzzy weighting functions:

**Theorem 2:** The LMIs of (11) and (12) hold if the following LMIs are satisfied:

$$\mathcal{M}_{ijq}^l + \mathcal{M}_{jiq}^l + \Omega_{ij}^l + \Omega_{ji}^l \prec 0, \quad (40)$$

$$\Xi_{ijq}^l + \Xi_{iqj}^l + \Xi_{jiq}^l + \Xi_{jq i}^l + \Xi_{qij}^l + \Xi_{qji}^l \prec 0, \quad (41)$$

for  $(i, j, q, l) \in \mathcal{I}_r \times \mathcal{I}_r \times \mathcal{I}_r \times \mathcal{I}_2$ , where  $\Omega_{ij}^l$  is a symmetric matrix of an appropriate dimension to be determined,

$$\Xi_{ijq}^l := \mathcal{M}_{ijq}^l - \sum_{v=1}^r \sigma_v \left( \mathcal{M}_{ijv}^l + \Omega_{ij}^l \right),$$

and  $\sigma_v > 0$  is a predefined scalar satisfying  $w_v(t_k) - w_v(t) + \sigma_v \geq 0$ .

**Proof:** From (33), we have

$$\begin{aligned} & \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t)w_q(t_k)\mathcal{M}_{ijq}^l \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t)(w_q(t_k) + w_q(t) - w_q(t) \\ &+ \sigma_q - \sigma_q)\mathcal{M}_{ijq}^l \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t)w_q(t) \left( \mathcal{M}_{ijq}^l - \sum_{v=1}^r \sigma_v \mathcal{M}_{ijv}^l \right) \\ &+ \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t) \\ &\times (w_q(t_k) - w_q(t) + \sigma_q)\mathcal{M}_{ijq}^l \prec 0. \quad (42) \end{aligned}$$

For an arbitrary symmetric matrix  $\Omega_{ij}^l$  of an appropriate dimension, the following is obvious:

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t) \times (w_q(t_k) - w_q(t) + \sigma_q - \sigma_q)\Omega_{ij}^l = \mathbf{0}. \quad (43)$$

Combining (42) and (43) yields

$$\begin{aligned} &= \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t)w_q(t) \left\{ \mathcal{M}_{ijq}^l \right. \\ &\quad \left. - \sum_{v=1}^r \sigma_v \left( \mathcal{M}_{ijv}^l + \Omega_{ij}^l \right) \right\} \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t)(w_q(t_k) - w_q(t) + \sigma_q) \\ &\quad \times \left( \mathcal{M}_{ijq}^l + \Omega_{ij}^l \right) \prec 0. \end{aligned} \quad (44)$$

We can say that the above matrix inequality is governed by the following matrix inequalities:

$$\begin{aligned} &\frac{1}{6} \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t)w_q(t) \left( \Xi_{ijq}^l + \Xi_{iqj}^l \right. \\ &\quad \left. + \Xi_{jiq}^l + \Xi_{jq i}^l + \Xi_{qij}^l + \Xi_{qji}^l \right) \prec 0 \text{ and} \end{aligned} \quad (45)$$

$$\begin{aligned} &\frac{1}{2} \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r w_i(t)w_j(t)(w_q(t_k) - w_q(t) + \sigma_q) \\ &\quad \times \left( \mathcal{M}_{ijq}^l + \mathcal{M}_{jiq}^l + \Omega_{ij}^l + \Omega_{ji}^l \right) \prec 0. \end{aligned} \quad (46)$$

Because we have chosen  $w_q(t)$  with  $q \in \mathcal{I}_r$  to satisfy  $w_q(t_k) - w_q(t) + \sigma_q \geq 0$ , the matrix inequalities of (45) and (46) hold if and only if the LMIs of (40) and (41) are satisfied. This concludes the proof.  $\square$

**Remark 2:** The main contributions of this paper are summarized as follows:

- 1) Unlike previous works, the IDR problem is addressed without the use of any discretization methods.
- 2) By minimizing the upper bound of the error cost function, the state-matching performance is considered over the whole time interval.
- 3) The relaxed stabilization condition was developed based on the proposed extended relaxation technique, which handles the mismatched information of the continuous-time and sampled-time fuzzy weighting functions.

#### 4. NUMERICAL EXAMPLES

In this section, we conduct a numerical simulation using YALMIP [20] running on MATLAB 2016a in order to demonstrate the effectiveness of the proposed IDR

method. In this example, an inverted pendulum on a cart [21] is chosen that can be represented by the T-S fuzzy system (1) with

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3-aml} & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3-aml\beta^2)} & 0 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0 \\ -\frac{a}{4l/3-aml} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3-aml\beta^2} \end{bmatrix}, \\ w_1(t) &= \begin{cases} 1 - \frac{2}{\pi}x_1, & 0 \leq x_1 \leq \frac{\pi}{2} \\ 1 + \frac{2}{\pi}x_1, & -\frac{\pi}{2} \leq x_1 \leq 0 \end{cases}, \end{aligned}$$

$w_2(t) = 1 - w_1(t)$ ,  $x_1$  is the angle of the pendulum from the vertical rad,  $x_2$  is the angular velocity rad/s,  $g = 9.8 \text{ m/s}^2$ ,  $a = 1/(m + M)$ ,  $M = 8.0 \text{ kg}$ ,  $m = 2.0 \text{ kg}$ ,  $2l = 1.0 \text{ m}$ , and  $\beta = \cos(88^\circ)$ .

From [2], the continuous-time fuzzy controller for this system can be obtained as follows:

$$K_1^c = [120.6667 \quad 22.6667], \quad K_2^c = [2551.6 \quad 764.0].$$

In this example, we assume the operating regions of the state variables as

$$\begin{aligned} x_1(t) &\in \{-\Delta_{x1}, \Delta_{x1}\} := \left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}, \text{ and} \\ x_2(t) &\in \{-\Delta_{x2}, \Delta_{x2}\} := \left\{ -\frac{1000}{180}\pi, \frac{1000}{180}\pi \right\}. \end{aligned} \quad (47)$$

Under this assumption, we compute minimum required value for  $\sigma_i$  with  $i \in \mathcal{I}_r$  using the following process: First, because of the system state vector, it is obvious that  $\dot{x}_{a1}(t) = x_{a2}(t)$ ; thus, we have

$$\begin{aligned} x_{a1}(t) &= x_{a1}(t_k) + \int_{t_k}^t \dot{x}_{a1}(t)dt = x_{a1}(t_k) + \int_{t_k}^t x_{a2}(t)dt \\ &\leq x_{a1}(t_k) + \int_{t_k}^t \Delta_{x2}dt = x_{a1}(t_k) + (t - t_k)\Delta_{x2} \\ &\leq x_{a1}(t_k) + h\Delta_{x2}, \end{aligned} \quad (48)$$

where subscript "a" can be either "c" or "d".

From the above, the relationship between  $x_{a1}(t)$  and  $x_{a1}(t_k)$  becomes

$$x_{a1}(t) \in [x_{a1}(t_k) - h\Delta_{x2}, x_{a1}(t_k) + h\Delta_{x2}]. \quad (49)$$

Assuming the maximum allowable sampling period  $h$  is 0.015 s and based on the relation (49), we numerically found the minimum required value for  $\sigma_i$  for  $i \in \mathcal{I}_r$  via a gridding procedure on the operating regions (47). The result is  $\sigma_i \geq 0.1667$ .

We set the initial conditions for  $x_d(t)$  and  $x_c(t)$  as  $x_d(0) = x_c(0) = \text{col}\{1.2, 1\}$  and the termination time  $t_f = 5[s]$ . To quantitatively show the state-matching performance, we employ the following performance index:

$$\mathcal{P} = \sum_{i=1}^2 \left\{ \int_0^5 |x_{ci}(t) - x_{di}(t)| dt \right\}. \quad (50)$$

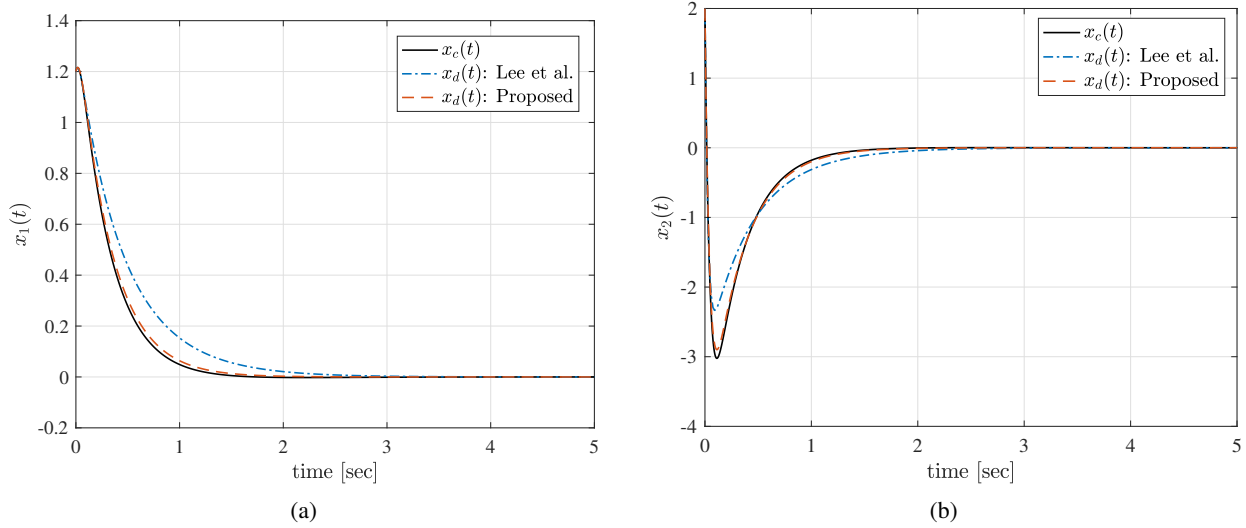


Fig. 1. Time responses of  $x_c(t)$  and  $x_d(t)$  for various methods at  $T = 0.015[s]$ .

For 0.015 s, we used the previous method given in [13] as well as the proposed Theorem 2 to compute the SD fuzzy gain matrices. For the proposed method, the parameters were chosen as follows:  $\sigma_i = 0.2$  with  $i \in \mathcal{I}_r$ ,  $Q = 10^{-1}I$ , and  $\alpha = 10^{-2}$ . Other parameters for the previous method were chosen appropriately to be able to sufficiently show its effectiveness. The control gains are obtained as follows from Theorem 2

$$K_1^d = [473.78 \quad 142.84], \quad K_2^d = [2290.06 \quad 743.06],$$

and from [13]

$$K_1^d = [611.34 \quad 275.95], \quad K_2^d = [1991.88 \quad 808.43].$$

Using the above gains, we measured  $\mathcal{P}$ , and the results are as follows:  $\mathcal{P} = 0.1279$  for Theorem 2 and  $\mathcal{P} = 0.5108$  for [13], in which the lower means the better performance. Moreover, we depicted the time responses of  $x_c(t)$  and  $x_d(t)$  in Fig. 1. The figure clearly shows that the state-matching error performance of the proposed method is better than the other method. From the results, we can see that the proposed method provides better performance than the existing method. This is mainly because the proposed method directly minimizes the error cost function  $J(t)$  without the use of any discretization methods, while the previous approach approximately discretizes a given T-S fuzzy model prior to apply their methods.

## 5. CONCLUSIONS

In this paper, we proposed an input-delay approach to the IDR method for T-S fuzzy systems. Previous IDR methods only minimized the state-matching error at each sampling time, and were derived based on the discretized model. However, in this paper, the main contributions are

that the performance is considered over the whole continuous time interval and no discretization procedures were used. Moreover, a sufficient condition guaranteeing both the asymptotic stabilization and minimization of the cost function was derived in terms of the LMIs. Finally, we conducted a simulation, from which we conclude that the proposed method provides better state-matching performance than conventional ones.

## REFERENCES

- [1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116-132, 1985. [click]
- [2] K. Tanaka and H. O. Wang, "Fuzzy control systems design and analysis: a linear matrix inequality approach," *Wiley*, 2001.
- [3] K. Tanaka, T. Ikeda, and H. O. Wang, "Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs," *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 2, pp. 250-265, 1998. [click]
- [4] H. J. Lee, J. B. Park, and Y. H. Joo, "Robust fuzzy control of nonlinear systems with parametric uncertainties," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 2, pp. 369-379, 2001. [click]
- [5] H. S. Kim, J. B. Park, and Y. H. Joo, "A systematic approach to fuzzy-model-based robust  $\mathcal{H}_\infty$  control design for a quadrotor UAV under imperfect premise matching," *to appear in Int. J. Fuzzy Syst.*, doi:10.1007/s40815-016-0233-6, 2016.
- [6] G. B. Koo, J. B. Park, and Y. H. Joo, "Robust fuzzy controller for large-scale nonlinear systems using decentralized static output-feedback," *Int. J. Control, Autom, Syst.*, vol. 9, no. 4, pp. 649-658, 2011. [click]



- [7] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, pp.421-427, 2010. [click]
- [8] X. L. Zhu, B. Chen, D. Yue, and Y. Wang, "An improved input delay approach to stabilization of fuzzy systems under variable sampling," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 330-341, 2012. [click]
- [9] J. Yoneyama, "Robust sampled-data stabilization of uncertain fuzzy systems via input delay approach," *Inform. Sci.*, vol. 198, pp. 169-176, 2012.
- [10] Z. G. Wu, P. Shi, H. Su, and J. Chu, "Sampled-data fuzzy control of chaotic systems based on a T-S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 153-163, 2014. [click]
- [11] F. Yang, H. Zhang, and Y. Wang, "An enhanced input-delay approach to sampled-data stabilization of T-S fuzzy systems via mixed convex combination," *Nonlinear Dyn.*, vol. 75, pp. 501-512, 2014. [click]
- [12] H. K. Lam, "Stabilization of nonlinear systems using sampled-data output-feedback fuzzy controller based on polynomial-fuzzy-model-based control approach," *IEEE Trans. Syst., Man, Cybern.*, vol. 42, no. 1, pp. 258-267, 2012. [click]
- [13] H. J. Lee, H. Kim, Y. H. Joo, W. Chang, and J. B. Park, "A new intelligent digital redesign for TS fuzzy systems: global approach," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 2, pp. 274-284, 2004. [click]
- [14] H. J. Lee, J. B. Park, and Y. H. Joo, "Digitalizing a fuzzy observer-based output-feedback control: intelligent digital redesign approach," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 5, pp. 701-716, 2005. [click]
- [15] H. C. Sung, J. B. Park, and Y. H. Joo, "Robust digital control of fuzzy systems with parametric uncertainties: LMI-based digital redesign approach," *Fuzzy Sets Syst.*, vol. 161, no. 6, pp. 919-933, 2010. [click]
- [16] H. J. Kim, J. B. Park, and Y. H. Joo, "Fuzzy filter for nonlinear sampled-data Systems: Intelligent digital redesign approach," *Int. J. Cont., Autom., Syst.*, vol. 15, no. 2, pp. 603-610, 2017. [click]
- [17] G. B. Koo, J. B. Park, and Y. H. Joo, "Intelligent digital redesign for nonlinear interconnected systems using decentralized fuzzy control," *J. Elect. Eng. Tech.*, vol. 7, no. 3, pp. 420-428, 2012.
- [18] G. B. Koo, J. B. Park, and Y. H. Joo, "Intelligent digital redesign for nonlinear systems using a guaranteed cost control method," *Int. J. Cont., Autom., Syst.*, vol. 11, no. 6, pp. 1075-1083, 2013. [click]
- [19] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, "Linear matrix inequalities in system and control theory," *SIAM Studies in Applied Mathematics*, SIAM, Philadelphia, Pennsylvania, 1994.
- [20] J. Löfberg, "YALMIP: a toolbox for modeling and optimization in MATLAB," *Proc. CACSD Conf.*, pp. 284-289, 2004.
- [21] S. H. Zak, "Stabilizing fuzzy system models using linear controller," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 2, pp. 236-240, 1999. [click]
- [22] K. Gu, "An integral inequality in the stability problem of time-delay systems," *Proc. 39th IEEE Conf. Dec. Cont.*, pp. 2805-2810, 2000.
- [23] J. V. Oliveira, J. Bernussou, and J. C. Geromel, "A new discrete-time robust stability condition," *Syst. Cont. Lett.*, vol. 37, no. 4, pp. 261-265, 1999. [click]



**Han Sol Kim** received his B.S. degree in Electronic and Computer Engineering from Hanyang University, Korea, in 2011 and his M.S. degree in Electrical and Electronic Engineering, Yonsei University, Korea, in 2012. From 2012, he is working toward a Ph.D. degree in Electrical and Electronic Engineering, Yonsei University, Korea. His current research interests include sampled-data control of fuzzy systems, fuzzy-model-based control, and interconnected fuzzy systems.



**Jin Bae Park** received his B.S. degree in Electrical Engineering from Yonsei University, Korea, and his M.S. and Ph.D. degrees in Electrical Engineering from Kansas State University, Manhattan, KS, USA, in 1977, 1985, and 1990, respectively. Since 1992, he has been with the School of Electrical and Electronic Engineering, Yonsei University, where he is

currently a Professor. His major research interests include robust control and filtering, nonlinear control, intelligent mobile robot, drone, fuzzy control, neural networks, adaptive dynamic programming, chaos theory, and genetic algorithms. Dr. Park served as the Editor-in-Chief for the International Journal of Control, Automation, and Systems (2006-2010) and the President for the Institute of Control, Robot, and Systems Engineers (2013). He served as the Senior Vice-President for Yonsei University (2014-2015).



**Young Hoon Joo** received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Yonsei University, Korea, in 1982, 1984, and 1995, respectively. He worked with Samsung Electronics Company, Korea, from 1986 to 1995, as a Project Manager. He was with the University of Houston, Houston, TX, USA, from 1998 to 1999, as a Visiting Professor

with the Department of Electrical and Computer Engineering. He is currently a Professor with the Department of Control and Robotics Engineering, Kunsan National University, Korea. His major research interests include the field of intelligent robot, robot vision, intelligent control, humanrobot interaction, wind-farm control, and intelligent surveillance systems. Dr. Joo served as the President for the Korea Institute of Intelligent Systems (2008-2009), the Vice-President for the Korean Institute of Electrical Engineers (2013-2014), and the Editor-in-Chief for the International Journal of Control, Automation, and Systems (2014-2017).