International Journal of Control, Automation and Systems 16(4) (2018) 1483-1492
http://dx.doi.org/10.1007/s12555-017-0266-0
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

A Novel Leader-following Consensus of Multi-agent Systems with Smart Leader

Fu-Yong Wang, Zhong-Xin Liu*, and Zeng-Qiang Chen

Abstract: This article studies the leader-following consensus problem for mixed-order multi-agent systems with a leader. Different from the traditional leader which is independent of all the other agents, the leader, called smart leader, can obtain and utilize the feedback information from its neighbors at some disconnected time intervals. A new distributed consensus control protocol based on intermittent control is developed for leader-following consensus with a smart leader. Moreover, the smart leader can adjust the control protocol based on the feedback information from its neighbors. With the aid of Lyapunov function, some sufficient conditions are derived for leader-following consensus of multi-agent systems with mixed-order dynamics under fixed directed topology. In addition, the similar results are obtained under switching directed topology. Finally, simulation results are provided to verify the correctness and effectiveness of theoretical results.

Keywords: Distributed control, information feedback, leader-following consensus, mixed-order multi-agent systems, smart leader.

1. INTRODUCTION

With the rapid development of embedded systems and communication technology, multi-agent systems have drawn much attention of researchers from different scientific fields. A multi-agent system usually consists of multiple intelligent agents that can communication and cooperate with each other. A group of autonomous agents can cooperate with each other to finish some large or complex tasks that single agent cannot. More important, multi-agent systems have many advantages such as flexibility, robustness, reliability, scalability, reducing system complexity, saving system cost, and so on.

In recent years, distributed cooperative control of multi-agent systems has made profound and significant progress in broad areas, due to its potential in the study of animal group behaviors such as swarms [1], consensus [2], and flocking [3], and its broad applications in control scientific such as formation control of mobile robots, spacecraft formation flying, automatic highway systems, satellite cluster, and so on [4–13]. Motivated by the works [14, 15], the distributed cooperative control problem with random networks governed by a Markov chain can be addressed in multi-agent systems. Moreover, the robust and reliable H_∞ static output feedback control problem for the continuous-time nonlinear stochastic systems is discussed in [16], and the case of discrete-time systems is addressed in [17]. In [18], the adaptive reliable coordination control problem for a class of multi-agent systems with intermittent communications and actuator faults is studied under time-varying topology. The main focus of this article is on consensus which is one of the fundamental research issues in distributed cooperative control of multi-agent systems. A multi-agent system can achieve consensus means that all the agents asymptotically run in a certain common state that depends on the state of them, in which agents update their states based on local information exchange. For the consensus problem of multi-agent systems, a basic idea is how to design an appropriate distributed controller for consensus based on the relative local information of each agent. In the existing works, consensus problem in multi-agent systems has been extensively investigated from various perspectives. Consensus can be divided into leaderless consensus and leader-following consensus based on whether a leader exists in multi-agent systems. Many profound theoretical results have been obtained on leaderless consensus of first-order and second-order multi-agent systems [19–23]. Moreover, for the high-order multi-agent systems, the distributed reliable H_∞ consensus control problem with switching undirected topologies is investigated in [24]. However, in some practical applications, it is desirable that all the agents can accomplish the predefined tasks or track a given trajectory. Towards this end, a leader is designated for leader-

Manuscript received May 6, 2017; revised October 4, 2017; accepted December 28, 2017. Recommended by Associate Editor Hyo-Sung Ahn under the direction of Editor Yoshto Ohta. This work is supported by the National Natural Science Foundation of China (Nos. 61573200, 61573199).

Fu-Yong Wang, Zhong-Xin Liu, and Zeng-Qiang Chen are with the College of Computer and Control Engineering, Nankai University, Tianjin 300071, China (e-mails: wangfuyong0323@163.com, lzhx@nankai.edu.cn, chenzq@nankai.edu.cn).
* Corresponding author.

following consensus in multi-agent systems. In consideration of this practical situation, the consensus problem for multi-agent systems with an active leader and variable interconnection topology is addressed in [25], in which a distributed feedback along with a distributed state-estimation rule was proposed for leader-following consensus. Due to the possible slow process of interactions among the agents in communication networks, time delays are usually inevitable in multi-agent systems. In [26], the leader-following consensus problem with time-varying delays is considered, and some sufficient conditions are obtained for second-order multi-agent systems both with fixed and switching topologies. For the continuous-time single-integrator multi-agent systems under directed fixed topologies, a leader-following consensus control protocol with multiplicative measurement noises and time-delays is proposed in [27]. In consideration of the intermittent interaction of networked systems, a class of distributed leader-following consensus protocol only based on the relative local intermittent information is proposed for second-order multi-agent systems in [28], in which some sufficient conditions are derived for achieving leader-following consensus under a general fixed topology with completely and partly intermittent communication. For time-varying networks in multi-agent systems, a leader-following consensus protocol is derived via intermittent control in [29], and some sufficient conditions are presented based on the Lyapunov stability theory.

The above mentioned works focus on the leaderless consensus problem or traditional leader-following consensus problem. Compared with the traditional leader-follower configuration where the leader can affect their neighboring followers but there is no feedback from the followers to it, a novel leader-follower architecture refers to the leader can not only affect their neighboring followers but also intermittently obtain the information feedback from some of the neighboring followers. In the novel leader-follower architecture, the leader is possible to obtain the feedback information from some of the neighboring followers to optimize the performance of the team. For consensus problem with the novel leader-follower architecture in multi-agent systems, the tracking errors among the smart leader and its neighboring followers are reduced, which may contribute to improving the team cohesion. On the other hand, the energy consumption of the whole multi-agent systems can be reduced, as shown formally in Section 4.

In contrast to the traditional leader-following consensus problem where the leader is a special agent who is independent of all the other followers, a new kind of leader, called smart leader, can obtain and utilize the feedback information from its neighbors at some disconnected time intervals to improve the performance of multi-agent systems. In practice, whether the effective leadership and decision-making of leaders will be better implemented is depends on the feedback information from other members of the group. In the study of animal group behaviors, the researchers in [3] found that the leading pigeon can track the motions of the other pigeons behind it, which means that the leader not only decides the whole group's behaviors but also cares the behaviors of individuals behind it. Motivated by this fact and the modified leader-follower architectures which proposed in [30], in this article, a smart leader is introduced in multi-agent systems, which can adjust the control protocol based on the feedback information from its neighbors. Different from the work in [30], by considering the leader with second-order dynamics and intermittent communications but the followers with first-order dynamics and continuous communications, an observer-based leader-following consensus protocol is designed for the mixed-order multi-agent systems in this article.

Motivated by the above discussions, we consider the leader-following consensus problem for mixed-order multi-agent systems with a smart leader which can obtain and utilize the feedback information from its neighbors intermittently, including the two different cases of coupling topologies, fixed topology and switching topology. Different from the most previous works, we propose a new kind of leader-following consensus protocol for mixed-order multi-agent systems with a smart leader. On the other hand, to optimize the performance of the whole multi-agent system, the novel leader-follower architecture is used for leader-following consensus of multi-agent systems. Moreover, a new kind of distributed control strategy for the modified leader-following consensus is adopted by Lyapunov function and intermittent method, and some sufficient conditions are obtained such that each follower can track the smart leader. In addition, the smart leader is active, that is, its state variables keep changing, and all agents will track the smart leader with a different dynamics. The simulation results are provided to show the effectiveness of the obtained theory results.

The rest of this article is organized as follows. In Section 2, some preliminaries and useful lemmas are introduced, and the novel leader-following consensus problem to be investigated is formulated. In Section 3, the main results for mixed-order multi-agent systems with a smart leader under the fixed and the switching topologies are obtained. Numerical simulation examples are given to validate the theoretical analysis in Section 4, and the concluding statements are drawn in Section 5.

Notation: The following notations will be used throughout this article. $\mathbb{N}$ and $\mathbb{R}$ denote the sets of integer and real number, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ represent the n dimensional real vector space and $n\times n$ real matrix space, respectively. 1_n is a vector with elements being all ones. I_n denotes the $n\times n$ identity matrix, and 0 represents the all-zero vector or a matrix with appropriate dimension. For a given vector or matrix θ, θ^{T} denotes its transpose.

$\lambda(\cdot)$ and $\lambda_{\max}(\cdot)$ ($\lambda_{\min}(\cdot)$) denotes the eigenvalue and the maximum (minimum) eigenvalue of the corresponding matrix, respectively.

2. PRELIMINARIES AND MODEL FORMULATION

In this section, some useful preliminaries and the model formulation for the novel leader-following consensus in multi-agent systems with mixed-order dynamics are introduced.

2.1. Preliminaries

Here, consider a multi-agent system with a leader and n follower agents, and let $\bar{G}$ be the interconnection topology graph for the multi-agent system. Graph $\bar{G}$ includes n followers related to graph G and a leader, and the directed graph $G=(W,E,A)$ will be utilized to model the interactions among n followers with the set of nodes $W=\{w_1,w_2,\cdots,w_n\}$, the set of directed edges $E=\{e_{ij}=(w_i,w_j)\}\subseteq W\times W$, and a weighted adjacency matrix $A=[a_{ij}]\in\mathbb{R}^{n\times n}$, where $a_{ij}>0$ if $e_{ij}\in E$, otherwise $a_{ij}=0$, and $a_{ii}=0$ for all $i=1,2,\cdots,n$. We assume that the node w_j is the neighbor of the node w_i if $(w_j,w_i)\in E$. The neighbor set of node w_i is denoted by $N_i=\{w_j\in W|e_{ji}\in E, j\neq i\}$. A directed path from node w_i to node w_j is denoted by a sequence of edges, $(w_i,w_{k1}),(w_{k1},w_{k2}),\cdots,(w_{kq},w_j)$, with distinct vertices w_{kp}, $p=1,2,\cdots,q$, Furthermore, the Laplacian matrix corresponding to the directed graph G is defined as $L=[l_{ij}]\in\mathbb{R}^{n\times n}$, where l_{ij} is defined as follows:

$$l_{ij}=\begin{cases}-a_{ij}, & i\neq j,\\ \sum\limits_{j\in N_i}a_{ij}, & i=j.\end{cases} \tag{1}$$

Moreover, when the communication topology of multi-agents systems is switching, we define a switching signal $\sigma(t):[0,\infty)\to\Gamma=\{1,2,\cdots,P\}$, $\sigma=\sigma(t)\in\Gamma$ which is a piecewise constant right continuous function, where P is the number of the switching topologies. In view of the interconnection topology is time-varying, the neighbour set of agent i, $N_i(t)$, the adjacent elements $a_{ij}(t)$, $i,j=1,2,\cdots,n$, Laplacian matrix $L_\sigma(\sigma\in\Gamma)$ and $B_\sigma(\sigma\in\Gamma)$ associated with the switching topology graph $G_\sigma(\sigma\in\Gamma)$ are time-varying.

Lemma 1 [31]**:** For any vectors x,y of appropriate dimensions and any symmetric positive definite matrix Z of appropriate dimension, the following inequality holds:

$$\pm2x^{\mathrm{T}}y\leq x^{\mathrm{T}}Zx+y^{\mathrm{T}}Z^{-1}y. \tag{2}$$

Lemma 2 (Schur complement) [32]**:** For a given symmetric matrix

$$S=\begin{bmatrix}M & C\\ C^{\mathrm{T}} & \Omega\end{bmatrix},$$

where Ω and M are square. $S>0$ is positive definite if and only if both M and $\Omega-C^{\mathrm{T}}M^{-1}C$ are positive definite.

2.2. Formulation of the model

Consider a multi-agent system consisting of n followers and one leader, the dynamics of each agent can be represented as follows

$$\begin{cases}\dot{x}_0(t)=v_0(t),\\ \dot{x}_i(t)=u_i(t), \quad i=1,2,\cdots,n,\end{cases} \tag{3}$$

where $x_i(t)\in\mathbb{R}^N$ and $u_i(t)\in\mathbb{R}^N$ are the position vector and the control input vector of the ith follower, respectively. $x_0(t)\in\mathbb{R}^N$ and $v_0(t)\in\mathbb{R}^N$ are respectively the position state and the velocity of the leader. For simplicity, we assume that $N=1$ in this article and the case of $N>1$ can be obtained with the Kronecker product.

In the traditional leader-follower configuration, the leader can affect their neighboring followers but there is no feedback from the followers to the leader. In practice, to optimize the performance of the team, the leader is possible to obtain the feedback from some of the neighboring followers to adjust its control protocol. This implies that the tracking errors among the smart leader and its neighboring followers are reduced, which may contribute to improving the team cohesion. On the other hand, the energy consumption of the whole multi-agent systems can be reduced, as shown formally in Section 4. Thus, novel leader-follower architecture is introduced in the present article, and a control algorithm with a kind of leader, called smart leader, is proposed for the novel leader-follower consensus of multi-agent systems.

The smart leader with intermittent local information feedback has the following control input

$$\begin{cases}\dot{v}_0(t)=a_0, & t\in T,\\ \dot{v}_0(t)=k_0\sum\limits_{j=1}^{n}b_j(x_j(t)-x_0(t))+a_0, & t\in\bar{T},\end{cases} \tag{4}$$

where $k_0>0$ is the control parameter, a_0 is the (acceleration) input of the smart leader, $b_j>0$ if and only if leader 0 can receive the feedback from agent j when $t\in\bar{T}$, otherwise, $b_j=0$, $j=1,2,\cdots,n$. T represents the union of time intervals over which the smart leader can only be affected by the external input, and $\bar{T}$ represents the union of time intervals over which the smart leader can not only be affected by the external input but also refer to the feedback information from its neighbors. Obviously, $T\cup\bar{T}=[0,+\infty)$.

Since $v_0(t)$ cannot be measured even when the followers are connected to the leader, its value cannot be used in the control design. Instead, we have to estimate $v_0(t)$ during the evolution. Note that, each follower has to estimate $v_0(t)$ only by the information obtained from its neighbors in a decentralized way. The estimate of $v_0(t)$ by agent i is denoted by $\bar{v}_i(t)$, $i = 1,2,\cdots,n$. Therefore, for each follower, the local control algorithm is designed as follows:

$$u_i(t) = -k_1[\sum_{j\in N_i} a_{ij}(x_i(t) - x_j(t)) + b_i(x_i(t) - x_0(t))] + \bar{v}_i(t), \tag{5}$$

$$\dot{\bar{v}}_i(t) = -\alpha k_1[\sum_{j\in N_i} a_{ij}(x_i(t) - x_j(t)) + b_i(x_i(t) - x_0(t))] + a_0, \tag{6}$$

where $k_1 > 0$ and $0 < \alpha < 1$ denote the control parameters, $b_i > 0$ if and only if agent i is connected to leader 0 when $t \in [0,+\infty)$, otherwise, $b_i = 0$, $i = 1,2,\cdots,n$.

Remark 1: If $T = [0,+\infty)$ in system (4), that is, the leader's behavior has no relation with the feedback from its neighbors, then the consensus problem for systems (3) and (4) becomes the typical leader-following consensus problem for second-order multi-agent systems in previous works.

Remark 2: For the convenience of theoretical analysis, it is assumed that the smart leader in system (4) could sense the measurements of relative states between their own and the neighboring followers synchronously. According to [33], this assumption is crucial for possibly constructing a common Lyapunov function for the switching system (4).

In the following, we will derive sufficient conditions for leader-following consensus of the above multi-agent systems, which can guarantee that each follower's state will finally converge to the leader's as time going on, and the estimations of $v_0(t)$ by the followers will converge to the leader's velocity, i.e.,

$$\begin{cases} \lim_{t\to\infty} x_i(t) = x_0(t), \\ \lim_{t\to\infty} \bar{v}_i(t) = v_0(t),\ i = 1,2,\cdots,n. \end{cases} \tag{7}$$

3. MAIN RESULTS

In this section, the main results of the novel leader-following consensus problem with the smart leader under mixed-order dynamics are presented.

Denote $\tilde{x}_i(t) = x_i(t) - x_0(t)$ and $\tilde{v}_i(t) = \bar{v}_i(t) - v_0(t)$. We can obtain the error dynamics of (3)-(6) as follows

$$\dot{\tilde{x}}_i(t) = -k_1[\sum_{j\in N_i} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)) + b_i\tilde{x}_i(t)] + \tilde{v}_i(t),\ i = 1,2,\cdots,n, \tag{8}$$

$$\begin{cases} \dot{\tilde{v}}_i(t) = -\alpha k_1[\sum_{j\in N_i} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)) + b_i\tilde{x}_i(t)], & t \in T, \\ \dot{\tilde{v}}_i(t) = -\alpha k_1[\sum_{j\in N_i} a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)) + b_i\tilde{x}_i(t)] - k_0 \sum_{j=1}^{n} b_j\tilde{x}_j(t), & t \in \bar{T}. \end{cases} \tag{9}$$

Let $\tilde{x}(t) = [\tilde{x}_1^{\mathrm{T}}(t), \tilde{x}_2^{\mathrm{T}}(t), \cdots, \tilde{x}_n^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\tilde{v}(t) = [\tilde{v}_1^{\mathrm{T}}(t), \tilde{v}_2^{\mathrm{T}}(t), \cdots, \tilde{v}_n^{\mathrm{T}}(t)]^{\mathrm{T}}$, and $\tilde{y}(t) = [\tilde{x}^{\mathrm{T}}(t), \tilde{v}^{\mathrm{T}}(t)]^{\mathrm{T}}$. The dynamics of (8)-(9) can be rewritten in a matrix form

$$\begin{cases} \dot{\tilde{y}}(t) = H_1\tilde{y}(t),\ t \in T, \\ \dot{\tilde{y}}(t) = H_2\tilde{y}(t),\ t \in \bar{T}, \end{cases} \tag{10}$$

where

$$H_1 = \begin{bmatrix} -k_1\hat{L} & I_n \\ -\alpha k_1\hat{L} & 0_n \end{bmatrix},$$

$$H_2 = \begin{bmatrix} -k_1\hat{L} & I_n \\ -\alpha k_1\hat{L} - k_0 D & 0_n \end{bmatrix},$$

and $\hat{L} = L + B$, $D = 1_n[1_n^{\mathrm{T}}B]$, $B = diag\{b_1,\cdots,b_n\}$.

Definition 1 [34]**:** For an infinite time sequence of uniformly bounded, non-overlapping time intervals $[t_r,t_{r+1})$, $r \in \mathbb{N}$, let δ_r represents the Lebesgue measure of set $\{t|t \in [t_r,t_{r+1}) \cap T\}$, $\omega_r = t_{r+1} - t_r$ represents the total time over $[t_r,t_{r+1})$.

Theorem 1: Consider the multi-agent systems (3)-(4) with a smart leader, and suppose that there is a directed path from the smart leader to each follower. Then, the novel leader-following consensus is achieved if there exists an infinite time sequence of uniformly bounded, non-overlapping time intervals $[t_r,t_{r+1})$, $r \in \mathbb{N}$, with $t_1 = 0$, such that for each time interval $[t_r,t_{r+1})$, $r \in \mathbb{N}$, the following conditions hold

(a) $0 < \alpha < 1$,

(b) $k_1 > \dfrac{1}{2\alpha(1-\alpha^2)\lambda_\rho}$,

(c) $k_0 < \dfrac{\xi}{\min\{1,(\alpha\lambda_\theta + \lambda_\vartheta)\}}$,

where $\xi = \alpha + \frac{k_1(1-\alpha^2)}{2}\lambda_\rho - \frac{\sqrt{[2\alpha - k_1(1-\alpha^2)\lambda_\rho]^2+4}}{2}$, $\lambda_\rho = \lambda_{\min}(\hat{L} + \hat{L}^{\mathrm{T}})$, $\lambda_\theta = \lambda_{\max}(D + D^{\mathrm{T}})$, $\lambda_\vartheta = \lambda_{\max}(D^{\mathrm{T}}D)$.

Proof: Construct the following Lyapunov function candidate for the switching system (10)

$$V(t) = \tilde{y}^{\mathrm{T}}(t)Q\tilde{y}(t), \tag{11}$$

where $Q = \begin{bmatrix} I_n & -\alpha I_n \\ -\alpha I_n & I_n \end{bmatrix}$ is positive definite due to condition (a). As there is a directed path from the smart leader to each followers, $\hat{L}$ is a nonsingular M-matrix [35]. Thus, there exists a positive vector $1_n = [1,1,\cdots,1]^{\mathrm{T}}$, such that $\hat{L}1_n = 1_n$ and $\hat{L} + \hat{L}^{\mathrm{T}} > 0$ [35,36]. It is easy to verify that $V(t) \geq 0$ and $V(t) = 0$ if and only if $x_i(t) = x_0(t)$, $\bar{v}_i(t) = v_0(t)$ for all $i,j = 1,2,\cdots,n$. Thus, $V(t)$ is a valid Lyapunov function for system (10).

Let $\mu_{i,j} > 0$, $i = 1,2,\ j = 1,2,\cdots,n$ denote the eigenvalues of Q which defined in (11). It follows that

$$\mu_{1,j} = 1 + \alpha,\ j = 1,2,\cdots,n, \tag{12}$$

$$\mu_{2,j} = 1 - \alpha,\ j = 1,2,\cdots,n. \tag{13}$$

Therefore,

$$\begin{aligned}V(t) \leq& \lambda_{max}(Q)\tilde{y}^{\mathrm{T}}(t)\tilde{y}(t)\\ \leq&(1+\alpha)\tilde{y}^{\mathrm{T}}(t)\tilde{y}(t)\\ =&\eta_1\tilde{y}^{\mathrm{T}}(t)\tilde{y}(t),\end{aligned}\tag{14}$$

where $\eta_1 = 1+\alpha > 0$.

For $t \in \{[t_r, t_{r+1}) \cap T\}$, for arbitrarily given $r \in \mathbb{N}$, taking the time derivative of $V(t)$ along the trajectories of (10) gives

$$\begin{aligned}\dot{V}(t) =& \tilde{y}^{\mathrm{T}}(t)[H_1^{\mathrm{T}}Q + QH_1]\tilde{y}(t)\\ =& -\tilde{y}^{\mathrm{T}}(t)\Xi\tilde{y}(t),\end{aligned}\tag{15}$$

where

$$\Xi = \begin{bmatrix} k_1(1-\alpha^2)(\hat{L}+\hat{L}^{\mathrm{T}}) & -I_n \\ -I_n & 2\alpha I_n \end{bmatrix}.$$

By Lemma 2, $\Xi > 0$ is equivalent to both $\alpha > 0$ and $k_1(1-\alpha^2)(\hat{L}+\hat{L}^{\mathrm{T}}) - \frac{1}{2\alpha}I_n > 0$, it follows that $\Xi > 0$ if $k_1 > \frac{1}{2\alpha(1-\alpha^2)\lambda_\rho}$, where $\lambda_\rho = \lambda_{\min}(\hat{L}+\hat{L}^{\mathrm{T}})$. Therefore, Ξ is positive definite due to condition (b).

Let $\nu_{i,j} > 0$, $i = 1,2$, $j = 1,2,\cdots,n$ denote the eigenvalues of Ξ which defined in (15). Since $\lambda_j(\hat{L}+\hat{L}^{\mathrm{T}}) > 0$, $j = 1,2,\cdots,n$, then

$$\nu_{1,j} = \alpha + \frac{k_1(1-\alpha^2)}{2}\lambda_j(\hat{L}+\hat{L}^{\mathrm{T}}) + \frac{\sqrt{\Delta}}{2},\tag{16}$$

$$\nu_{2,j} = \alpha + \frac{k_1(1-\alpha^2)}{2}\lambda_j(\hat{L}+\hat{L}^{\mathrm{T}}) - \frac{\sqrt{\Delta}}{2},\tag{17}$$

where $\Delta = [2\alpha - k_1(1-\alpha^2)\lambda_j(\hat{L}+\hat{L}^{\mathrm{T}})]^2 + 4$. Clearly, the minimum eigenvalue of Ξ will be found in $\nu_{2,j}$. Therefore, the minimum eigenvalue of Ξ will be no less than

$$\begin{aligned}\eta_2 =& \alpha + \frac{k_1(1-\alpha^2)}{2}\lambda_\rho\\ &-\frac{\sqrt{[2\alpha - k_1(1-\alpha^2)\lambda_\rho]^2+4}}{2}.\end{aligned}\tag{18}$$

Furthermore, $\eta_2 > 0$ is a valid eigenvalue for Ξ which is positive definite since $k_1 > \frac{1}{2\alpha(1-\alpha^2)\lambda_\rho}$. Therefore,

$$\begin{aligned}\dot{V}(t) \leq& -\lambda_{\min}(\Xi)\tilde{y}^{\mathrm{T}}(t)\tilde{y}(t)\\ =& -\eta_2\tilde{y}^{\mathrm{T}}(t)\tilde{y}(t).\end{aligned}\tag{19}$$

By (14) and (19), it follows that

$$\dot{V}(t) \leq -\gamma_1 V(t),\tag{20}$$

where $\gamma_1 = \eta_2/\eta_1 > 0$.

For $t \in \{[t_r, t_{r+1}) \cap \bar{T}\}$, for arbitrarily given $r \in \mathbb{N}$, taking the time derivative of $V(t)$ along the trajectories of (10) gives

$$\begin{aligned}\dot{V}(t) =& \tilde{y}^{\mathrm{T}}(t)[H_2^{\mathrm{T}}Q + QH_2]\tilde{y}(t)\\ =& \tilde{y}^{\mathrm{T}}(t)[-\Xi + \Psi]\tilde{y}(t),\end{aligned}\tag{21}$$

where $\Psi = \begin{bmatrix} \alpha k_0(D+D^{\mathrm{T}}) & -k_0D^{\mathrm{T}} \\ -k_0D & 0 \end{bmatrix}$.

By Lemma 1, it follows that

$$\begin{aligned}\tilde{y}^{\mathrm{T}}(t)\Psi\tilde{y}(t) =& \tilde{x}^{\mathrm{T}}(t)[\alpha k_0(D+D^{\mathrm{T}})]\tilde{x}(t)\\ &-2\tilde{x}^{\mathrm{T}}(t)[k_0D^{\mathrm{T}}]\tilde{v}(t)\\ \leq& \tilde{x}^{\mathrm{T}}(t)[\alpha k_0(D+D^{\mathrm{T}})]\tilde{x}(t)\\ &+k_0\tilde{x}^{\mathrm{T}}(t)[D^{\mathrm{T}}D]\tilde{x}(t) + k_0\tilde{v}^{\mathrm{T}}(t)\tilde{v}(t)\\ =& \tilde{y}^{\mathrm{T}}(t)\Phi\tilde{y}(t),\end{aligned}\tag{22}$$

where $\Phi = \begin{bmatrix} \alpha k_0(D+D^{\mathrm{T}}) + k_0(D^{\mathrm{T}}D) & 0_n \\ 0_n & k_0 I_n \end{bmatrix}$.

From the above derivation, then we get

$$\begin{aligned}\dot{V}(t) \leq& -\tilde{y}^{\mathrm{T}}(t)\Xi\tilde{y}(t) + \tilde{y}^{\mathrm{T}}(t)\Phi\tilde{y}(t)\\ \leq& -\lambda_{\min}(\Xi)\tilde{y}^{\mathrm{T}}(t)\tilde{y}(t) + \lambda_{\max}(\Phi)\tilde{y}^{\mathrm{T}}(t)\tilde{y}(t),\end{aligned}\tag{23}$$

where $\lambda_{\max}(\Phi) = k_0\max\{1, \alpha\lambda_{\max}(D+D^{\mathrm{T}}) + \lambda_{\max}(D^{\mathrm{T}}D)\}$.

Therefore,

$$\dot{V}(t) \leq -(\eta_2 - \eta_3)\tilde{y}^{\mathrm{T}}(t)\tilde{y}(t),\tag{24}$$

where $\eta_3 = \max\{k_0, (\alpha\lambda_\theta + \lambda_\vartheta)k_0\}$, $\lambda_\theta = \lambda_{\max}(D+D^{\mathrm{T}})$, $\lambda_\vartheta = \lambda_{\max}(D^{\mathrm{T}}D)$.

By (14) and (24), it follows that

$$\dot{V}(t) \leq -\gamma_2 V(t),\tag{25}$$

where $\gamma_2 = (\eta_2 - \eta_3)/\eta_1 > 0$ due to condition (c). Moreover, it is easy to verify that $\gamma_2 < \gamma_1$.

For arbitrary $t > 0$, based on the above analysis, we have

$$\dot{V}(t) \leq \min\{-\gamma_1, -\gamma_2\}V(t).\tag{26}$$

Since $\gamma_2 < \gamma_1$, it follows that $\dot{V}(t) \leq -\gamma_1 V(t)$. Thus,

$$V(t) \leq e^{-\gamma_1 t}V(0),\tag{27}$$

which indicates that the states of agents of exponentially converge, thereby achieves consensus. Furthermore, the final consensus value of the position state $x_{con}(t) = x_0(t)$, and the final consensus value of velocity state $\bar{v}_{con}(t) = v_0(t)$. The proof of the Theorem 1 is thus completed. □

In practice, the communication topology of multi-agents systems may be variable. In the following part, we will discuss the leader-following consensus for multi-agent systems with switching directed topology.

Theorem 2: Consider the multi-agent systems (3)-(4) with switching topology $G_\sigma(\sigma \in \Gamma)$, and suppose that there exists an infinite time sequence of uniformly bounded, non-overlapping time intervals $[t_r, t_{r+1})$, $r \in \mathbb{N}$, with $t_1 = 0$. Assume that the directed topology is time-invariant over time intervals $[t_r, t_{r+1})$ and switches at time

t_r, $r = 1, 2, \cdots$, and there is a directed path from the smart leader to each follower for each time interval $[t_r, t_{r+1})$. Then, the novel leader-following consensus is achieved if for each time interval $[t_r, t_{r+1})$, the following conditions hold

(a') $0 < \alpha < 1$,

(b') $k_1 > \dfrac{1}{2\alpha(1-\alpha^2)\hat{\lambda}_\rho}$,

(c') $k_0 < \dfrac{\hat{\xi}}{\min\{1, (\alpha\hat{\lambda}_\theta + \hat{\lambda}_\vartheta)\}}$,

where $\hat{\xi} = \alpha + \frac{k_1(1-\alpha^2)}{2}\hat{\lambda}_\rho - \frac{\sqrt{[2\alpha - k_1(1-\alpha^2)\hat{\lambda}_\rho]^2 + 4}}{2}$, $\hat{\lambda}_\rho = \min\{\lambda_{\min}(\hat{L}_\sigma + \hat{L}_\sigma^{\mathrm{T}})\}$, $\hat{\lambda}_\theta = \max\{\lambda_{\max}(D_\sigma + D_\sigma^{\mathrm{T}})\}$, $\hat{\lambda}_\vartheta = \max\{\lambda_{\max}(D_\sigma^{\mathrm{T}} D_\sigma)\}$.

For the case of switching topology, the system (10) can be rewritten as

$$\begin{cases} \dot{\tilde{y}}(t) = H_{\sigma 1}\tilde{y}(t), & t \in T, \\ \dot{\tilde{y}}(t) = H_{\sigma 2}\tilde{y}(t), & t \in \bar{T}, \end{cases} \tag{28}$$

where

$$H_{\sigma 1} = \begin{bmatrix} -k_1\hat{L}_\sigma & I_n \\ -\alpha k_1 \hat{L}_\sigma & 0_n \end{bmatrix},$$

$$H_{\sigma 2} = \begin{bmatrix} -k_1\hat{L}_\sigma & I_n \\ -\alpha k_1 \hat{L}_\sigma - k_0 D_\sigma & 0_n \end{bmatrix},$$

and $\hat{L}_\sigma = L_\sigma + B_\sigma$, $D_\sigma = 1_n[1_n^{\mathrm{T}} B_\sigma]$, $B_\sigma = diag\{b_{\sigma 1}, b_{\sigma 2}, \cdots, b_{\sigma n}\}$.

The process of proof for Theorem 2 is similar to that of Theorem 1, which is thus omitted here.

Remark 3: From the condition (b') in Theorem 2, we can get the control parameter k_1 which is affected by the values of $\hat{\lambda}_\rho$, and we selected $\hat{\lambda}_\rho = \min\{\lambda_{\min}(\hat{L}_\sigma + \hat{L}_\sigma^{\mathrm{T}})\}$, $\sigma \in \Gamma$. The selection of control parameter k_0 is similar to that of k_1. Thus, the evolution of the system is not affected essentially by the varied communication relationships among agents due to the appropriate selection of control parameters. Therefore, the way of the proof of Theorem 1 can be extended to the proof of Theorem 2.

4. NUMERICAL RESULTS

In this section, some numerical examples will be provided to verify the effectiveness of the theoretical results in this article.

Example 1: Consider a multi-agent system consisting one smart leader and five followers with the fixed communication topology G as shown in Fig. 1, where the connection weights are all set to 1 in this article.

By the topology of graph G, we get

$$\hat{L} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix},$$

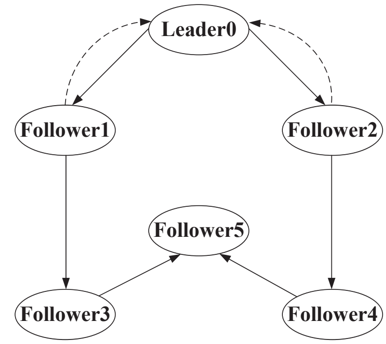


Fig. 1. Graph G of multi-agent systems with fixed topology.

$$D = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Some simple calculations give that $\lambda_\rho = 0.6571$, $\lambda_\theta = 5.1623$ and $\lambda_\vartheta = 10$. The control parameter α is taken as 0.5, which satisfies the condition (a) in Theorem 1. According to the condition (b) in Theorem 1, we get $k_1 > 2.0291$. Then, choose $k_1 = 2.5$, and we get $\xi = 0.1093$. According to condition (c) in Theorem 1, we get $k_0 < 0.1093$, and then choose $k_0 = 0.1$. Next, it is assumed that there exists an infinite time sequence of uniformly bounded and non-overlapping time intervals $[t_r, t_{r+1})$, with $t_{r+1} - t_r = \omega_r = 0.5$, $t_1 = 0$, for all $r \in \mathbb{N}$. Furthermore, δ_r, which is the communication duration over which the smart leader can only be affected by the external input, is randomly chosen within $\delta_r \in (0, 0.5)$, for all $r \in \mathbb{N}$. The acceleration of leaders a_0 equals 0.1, the initial positions of the leader and followers are selected as $x_0(0) = [5,3]^{\mathrm{T}}$, $x_1(0) = [1,2]^{\mathrm{T}}$, $x_2(0) = [2,1]^{\mathrm{T}}$, $x_3(0) = [3,4]^{\mathrm{T}}$, $x_4(0) = [4,5]^{\mathrm{T}}$, $x_5(0) = [4,1]^{\mathrm{T}}$. It is easy to show that the conditions of the Theorem 1 are satisfied, and the simulation results are presented in Figs. 2-4 which shows that the followers can track the smart leader and the leader-following consensus has been gained under the fixed topology.

From Fig. 2, we can see the initial positions of the followers and the smart leader are marked by arrows, respectively. Fig. 2 shows that the followers asymptotically track the smart leader under the fixed topology. Fig. 3 shows that the position errors $\tilde{x}_i(t) = x_i(t) - x_0(t)$, $i = 1, 2, 3, 4, 5$ are asymptotically convergent to zero under the fixed topology. Fig. 4 shows that the velocity errors $\tilde{v}_i(t) = \bar{v}_i(t) - v_0(t)$, $i = 1, 2, 3, 4, 5$ between the follower i and the smart leader are asymptotically convergent to zero under the fixed topology.

Example 2: Consider a multi-agent system consisting one smart leader and five followers. The network starts with topology G_1 and switches to another topology randomly chosen within $\{G_1, G_2, G_3, G_4\}$ at time kT, $k =$

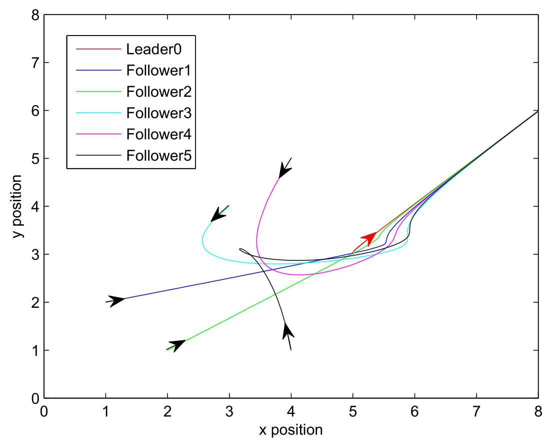


Fig. 2. The trajectories of agents in Example 1.

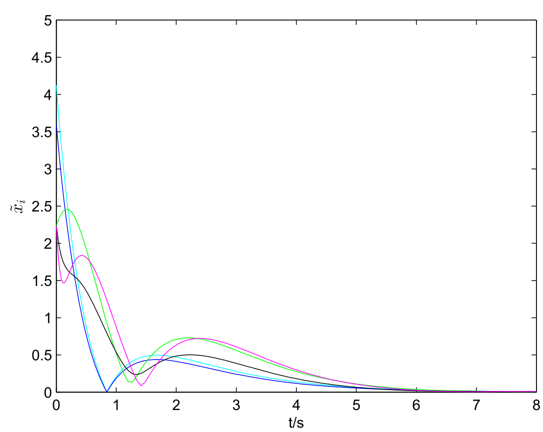


Fig. 3. The trajectories of position errors $\tilde{x}_i$, $i = 1, 2, 3, 4, 5$ in Example 1.

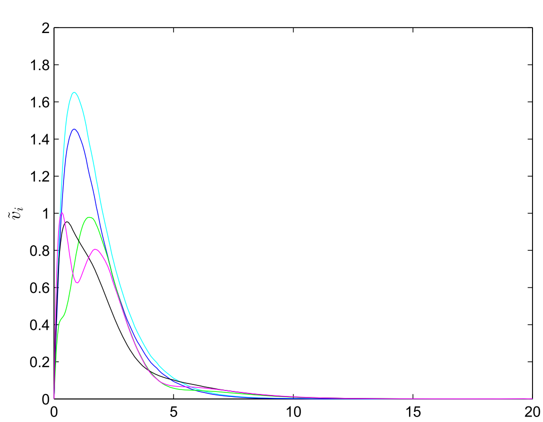


Fig. 4. The trajectories of velocity errors $\tilde{v}_i$, $i = 1, 2, 3, 4, 5$ in Example 1.

$1, 2, \cdots$, $T = 0.5s$, where the connection weights are all set to 1 in Fig. 5.

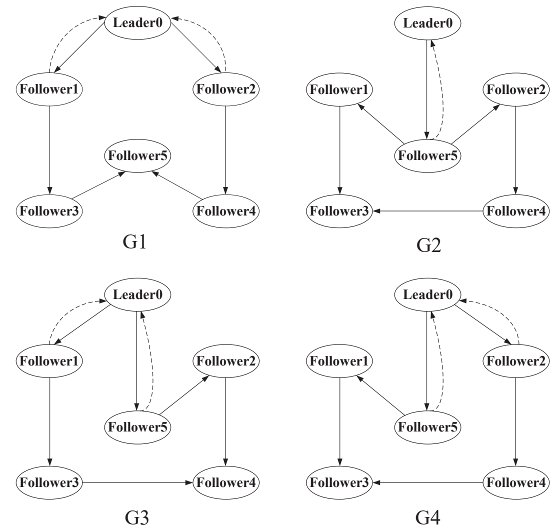


Fig. 5. Four possible graphs of multi-agent systems with switching topology.

By the topologies of $G_1 - G_4$, we get

$$\hat{L}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix}, \; D_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\hat{L}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; D_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\hat{L}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; D_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\hat{L}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; D_4 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

The control parameter α is selected the same as that in Example 1. Some simple calculations give that $\hat{\lambda}_\rho = 0.2087$, $\hat{\lambda}_\theta = 5.1623$, $\hat{\lambda}_\vartheta = 10$. By the condition (b') in Theorem 2, we get $k_1 > 6.3888$. Then, choose $k_1 = 7$, and we get $\hat{\xi} = 0.0467$. According to condition (c') in Theorem 2, we get $k_0 < 0.0467$, and then choose $k_0 = 0.04$. Next, it is assumed that there exists an infinite time sequence of uniformly bounded and non-overlapping time intervals $[t_r, t_{r+1})$, with $t_{r+1} - t_r = \omega_r = 0.5$, $t_1 = 0$, for all $r \in \mathbb{N}$. Furthermore, δ_r, which is the communication duration over which the smart leader can only be affected by

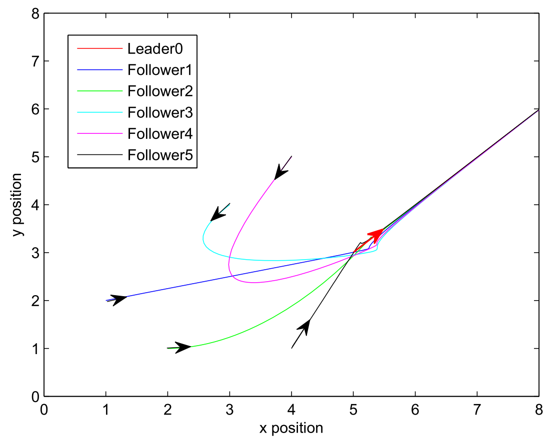


Fig. 6. The trajectories of agents in Example 2.

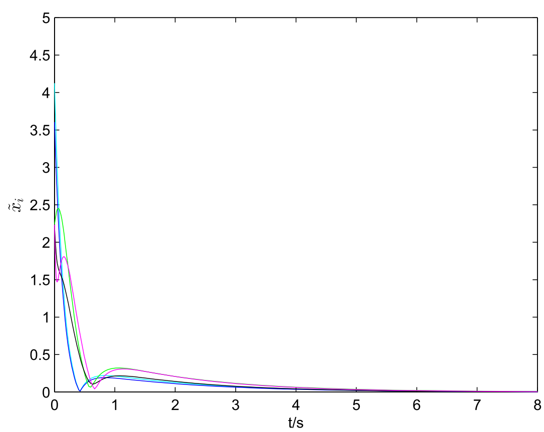


Fig. 7. The trajectories of position errors $\tilde{x}_i$, $i = 1, 2, 3, 4, 5$ in Example 2.

the external input, is randomly chosen within $\delta_r \in (0, 0.5)$, for all $r \in \mathbb{N}$. The initial states of leaders and followers are selected the same as those in Example 1. It is easy to show that the conditions of the Theorem 2 are satisfied, and the simulation results are presented in Figs. 6-8 which shows that the followers can track the smart leader and the leader-following consensus has been gained under the switching topology.

Considering an infinite time sequence of uniformly bounded and non-overlapping time intervals $[t_s, t_{s+1})$, with $t_1 = 0$, for all $s \in \mathbb{N}$. Let $E_i(t_s) = \int_{t_s}^{t_s+\Delta t} u_i^2(t)dt$ represents the energy consumption of the agent i on time t_s, $i = 0, 1, 2, 3, 4, 5$. It is easy to verify that $E(t) = \sum_{i=0}^{i=5} [E_i(t_s) + E_i(t_s + \Delta t) + \cdots + E_i(t_{s+1})]$ is the energy consumption of the multi-agent systems during $[t_s, t_{s+1})$. Then, $E = \sum_{i=0}^{i=5} \int_0^\infty u_i^2(t)dt$ for $t \in (0, +\infty)$. If the control parameters and the initial conditions are selected the same as those in Example 1, then Fig. 9(a) shows

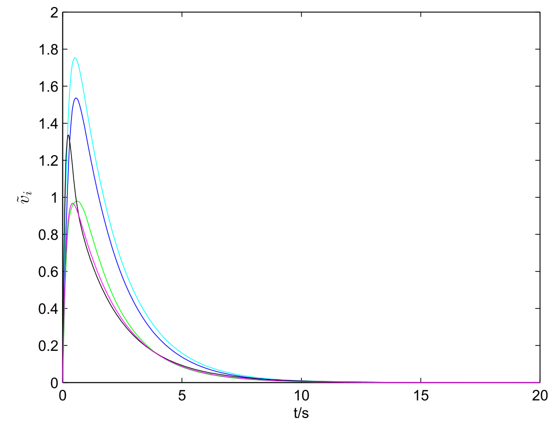
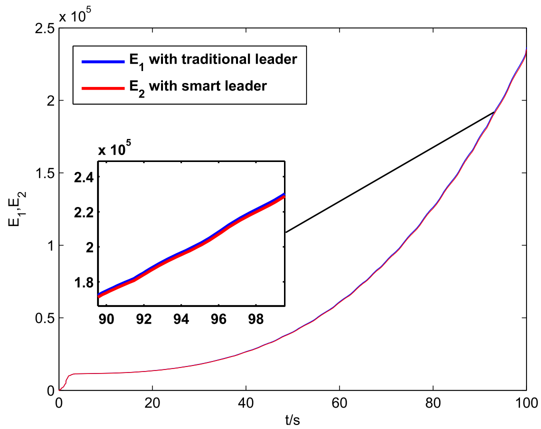


Fig. 8. The trajectories of velocity errors $\tilde{v}_i$, $i = 1, 2, 3, 4, 5$ in Example 2.

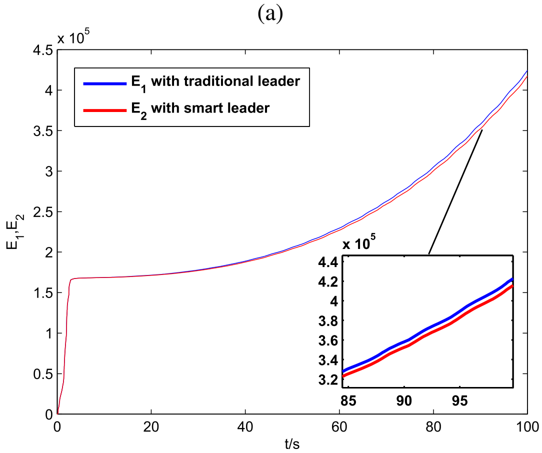
the curves of energy consumption of multi-agent systems with smart leader and traditional leader. If the initial positions of the leader and followers are selected as $x_0(0) = [50, 25]^T$, $x_1(0) = [1, 2]^T$, $x_2(0) = [2, 3]^T$, $x_3(0) = [3, 1]^T$, $x_4(0) = [3, 5]^T$, $x_5(0) = [5, 2]^T$, then Fig. 9(b) shows the curves of energy consumption of two kinds multi-agent systems. From the Fig. 9, we can see the multi-agent system with a smart leader has a lower consumption than the multi-agent system with a traditional leader. Furthermore, under the assumption that the communication topology of multi-agent systems is strongly connected, the greater the space distances between the leader and the followers, the more obvious our advantage.

5. CONCLUSION

In this article, considering a kind of leader, called smart leader, which can obtain and utilize the feedback information from its neighbors only during a sequence of disconnected time intervals, we aim to update the tradition model of the leader for the leader-following consensus problem of multi-agent systems. Based on intermittent control, a new kind of consensus control protocol for leader-following consensus is proposed to guarantee that each follower can track the active leader, and the control protocol can be adjust by using the feedback information. With the help of Lyapunov function technology, some sufficient conditions are obtained for consensus of mixed-order multi-agent systems with a smart leader under fixed directed topology. Similarly, by constructing Lyapunov function, some sufficient conditions are given under switching directed topology. Finally, some numerical examples are presented for illustration. It is the defect of this article that the point of switch time and the feedback duration for the smart leader are not given. In the future, we will optimize consensus control protocol for the multi-agent systems with a smart leader, and solve the de-



(a)



(b)

Fig. 9. The curves of energy consumption for two kinds of multi-agent systems.

fect in the current work. Moreover, consider the fact that the system states are generally partially measurable, we will, in the future, discuss the leader-following consensus problem for the multi-agent systems with a smart leader via output feedback control.

REFERENCES

[1] C. M. Topaz and A. L. Bertozzi, "Swarming patterns in a two-dimensional kinematic model for biological groups," *SIAM Journal on Applied Mathematics*, vol. 65, no. 1, pp. 152-174, September 2004.

[2] I. D. Couzin, J. Krause, N. R. Franks, and S. A. Levin, "Effective leadership and decision-making in animal groups on the move," *Nature*, vol. 433, no. 7025, pp. 513-516, February 2005.

[3] M. Nagy, Z. Akos, D. Biro, and T. Vicsek, "Hierarchical group dynamics in pigeon flocks," *Nature*, vol. 464, no. 7290, pp. 890-893, April 2010.

[4] F. Giulietti, L. Pollini, and M. Innocenti, "Autonomous formation flight," *IEEE Control Systems*, vol. 20, no. 6, pp. 34-44, December 2000.

[5] R. W. Beard, J. Lawton, and F. Y. Hadaegh, "A coordination architecture for spacecraft formation control," *IEEE Transactions on Control Systems Technology*, vol. 9, no. 6, pp. 777-790, November 2001.

[6] A. Pant, P. Seiler, and K. Hedrick, "Mesh stability of look-ahead interconnected systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 2, pp. 403-407, February 2002.

[7] R. W. Beard, T. W. McLain, M. A. Goodrich, and E. P. Anderson, "Coordinated target assignment and intercept for unmanned air vehicles," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 6, pp. 911-922, December 2002.

[8] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1465-1476, September 2004.

[9] J. Cortes and F. Bullo, "Coordination and geometric optimization via distributed dynamical systems," *SIAM Journal on Control and Optimization*, vol. 44, no. 5, pp. 1543-1574, November 2005.

[10] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401-420, March 2006.

[11] N. E. Leonard, D. A. Paley, F. Lekien, and R. E. Davis, "Collective motion, sensor networks, and ocean sampling," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 48-74, January 2007.

[12] H. T. Zhang, C. Zhai, and Z. Chen, "A general alignment repulsion algorithm for flocking of multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 2, pp. 430-435, February 2011.

[13] T. F. Liu and Z. P. Jiang, "Distributed formation control of nonholonomic mobile robots without global position measurements," *Automatica*, vol. 49, no. 2, pp. 592-600, 2013.

[14] Y. L. Wei, J. B. Qiu, H. R. Karimi, and M. Wang, "H_∞ model reduction for continuous-time Markovian jump systems with incomplete statistics of mode information," *International Journal of Systems Science*, vol. 45, no. 7, pp. 1496-1507, September 2014.

[15] Y. L. Wei, J. B. Qiu, and H. R. Karimi, "Quantized H_∞ filtering for continuous-time Markovian jump systems with deficient mode information," *Asian Journal of Control*, vol. 17, no. 5, pp. 1914-1923, 2015.

[16] Y. L. Wei, J. B. Qiu, H. K. Lam, and L. G. Wu, "Approaches to T-S fuzzy-affine-model-based reliable output feedback control for nonlinear *Itô* stochastic systems," *IEEE Transactions on Circuits and Systems*, vol. 25, no. 3, pp. 569-583, June 2017.

[17] Y. L. Wei, J. B. Qiu, and H. R. Karimi, "Reliable output feedback control of discrete-time fuzzy affine systems with actuator faults," *IEEE Transactions on Fuzzy Systems*, vol. 64, no. 1, pp. 170-181, January 2017.

[18] X. Wang and G. H. Yang, "Adaptive reliable coordination control for linear agent networks with intermittent communication constraints," *IEEE Transactions on Control of Network Systems*, DOI:10.1109/TCNS.2017.2687818.

[19] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655-661, May 2005.

[20] P. Lin and Y. M. Jia, "Consensus of a class of second-order multi-agent systems with time-delay and jointly-connected topologies," *IEEE Transactions on Automatic Control*, vol. 55, no. 3, pp. 778-784, March 2010.

[21] Y. Z. Song, "Consensus of agents with mixed linear discrete dynamics," *International Journal of Control, Automation and Systems*, vol. 14, no. 4, pp. 1139-1143, August 2016.

[22] J. Liu, P. Ming, and S. Li, "Consensus gain conditions of stochastic multi-agent system with communication noise," *International Journal of Control, Automation and Systems*, vol. 14, no. 5, pp. 1223-1230, October 2016.

[23] J. H. Qin, H. J. Gao, T. Hayat, and F. E. Alsaadi, "Synchronising second-order multi-agent systems under dynamic topology via reference model-based algorithm," *Journal of Control and Decision*, vol. 1, no. 3, pp. 214-225, 2014.

[24] X. Wang and G. H. Yang, "Distributed reliable consensus control for a class of multi-agent systems under switching networks: A topology-based average dwell time approach," *International Journal of Robust and Nonlinear Control*, vol. 26, no. 13, pp. 2767-2787, 2016.

[25] Y. G. Hong, J. P. Hu, and L. X. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177-1182, 2006.

[26] W. Zhu and D. Cheng, "Leader-following consensus of second-order agents with multiple time-varying delays," *Automatica*, vol. 46, no. 12, pp. 1994-1999, 2010.

[27] S. Djaidja and Q. H. Wu, "Leader-following consensus of single-integrator multi-agent systems under noisy and delayed communication," *International Journal of Control, Automation, and Systems*, vol. 14, no. 2, pp. 357-366, April 2016.

[28] N. Huang, Z. S. Duan, and Y. Zhao, "Leader-following consensus of second-order non-linear multi-agent systems with directed intermittent communication," *IET Control Theory & Applications*, vol. 8, no. 10, pp. 782-795, July 2014.

[29] A. H. Hu, J. D. Cao, and M. Hu, "Consensus of leader-following multi-agent systems in time-varying networks via intermittent control," *International Journal of Control, Automation and Systems*, vol. 12, no. 5, pp. 969-976, October 2014.

[30] E. Semsar-Kazerooni and K. Khorasani, "Optimal consensus algorithms for cooperative team of agents subject to partial information," *Automatica*, vol. 44, no. 11, pp. 2766-2777, 2008.

[31] M. Wu, Y. He, and J. H. She, *Stability Analysis and Robust Control of Time-delay Systems*, Springer, Berlin, 2010.

[32] B. Boyd, L. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

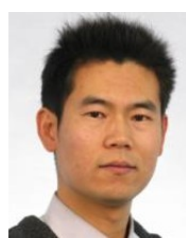
[33] A. Olshevsky and J. N. Tsitsiklis, "On the nonexistence of quadratic Lyapunov functions for consensus algorithms," *IEEE Transactions on Automatic Control*, vol. 53, no. 11, pp. 2642-2645, December 2008.

[34] G. H. Wen, Z. S. Duan, W. W Yu, and G. R. Chen, "Consensus in multi-agent systems with communication constraints," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 2, pp. 170-182, 2012.

[35] Z. H. Qu, *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*, Springer Science & Business Media, 2009.

[36] H. W. Zhang, F. L. Lewis, and Z. H. Qu, "Lyapunov, adaptive, and optimal design techniques for cooperative systems on directed communication graphs," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 7, pp. 3026-3041, July 2012.

Fu-Yong Wang received his B.S. degree in electrical engineering and automation and his M.S. degree in computer application technology from the Ludong University, Yantai, China, in 2013 and 2016, respectively. He is now pursuing the Ph.D. degree in the College of Computer and Control Engineering, Nankai University, Tianjin, China. His research interest covers coordination of multi-agent systems.



Zhong-Xin Liu received his B.S. degree in automation and his Ph.D. degree in control theory and control engineering from the Nankai University, Tianjin, China, in 1997 and 2002, respectively. He has been at Nankai University, where he is currently a Professor in the Department of Automation. His main areas of research are in predictive control, complex networks and multi-agents system.

Zeng-Qiang Chen received his B.S. degree in mathematics, his M.S. and Ph.D. degrees in control theory and control engineering from the Nankai University, Tianjin, China, in 1987, 1990 and 1997, respectively. He has been at Nankai University, where he is currently a Professor in the Department of Automation. His main areas of research are in neural network control, complex networks and multi-agents system.