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Normalized Learning Rule for Iterative Learning Control

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Abstract: The iterative learning control (ILC) is attractive for its simple structure, easy implementation. So the ILC is applied to various fields. But the unexpected huge overshoot can be observed as iteration repeat when we use the ILC to the real world applications. Such bad transient becomes an obstacle for using the ILC in the real field. Designers use a projection method to avoid the bad transient usually. However, the projection method does not show a good error performance enough. Therefore we propose a new learning rule to reduce such a bad transient effectively. The simple normalized learning rules for P-type and PD-type are presented and we prove their convergence. Numerical examples are given to show the effectiveness of the proposed learning control algorithms.

Keywords: Huge overshoot, iterative learning, nomalized learning rule, P-type and PD-type control.

1. INTRODUCTION

Iterative learning control methods handle systems that perform the same tasks repeatedly over finite time intervals. The basic idea behind ILC is that the information obtained from a previous trial is used to improve the control input for the current trial. Because the ILC has very simple structure and is easy to implement, the ILC is attractive to a field engineer. Besides, the ILC can be applied to a nonlinear system effectively although we do not know plant information exactly. Therefore there are many examples that ILC is applied to a real field [1–4]. But, we can observe bad transient of ILC easily by repeating the iteration number [5–7]. This bad transient is caused by wrong design of a learning rule parameter and basically open-loop structure. Those phenomenon can be an obstacle for using the ILC to real examples. Therefore it is needed to analyze why this situation happens and propose a new learning rule to overcome bad transient.

Many researches have suggested a source of such bad transient. In summary, there are four kinds of view. First one is relation between lambda-norm and sup-norm [5], second one is about to time domain aspect [6], third one is about to frequency domain aspect [6], and last one is about to monotonic convergence [7]. In research of first view, they proposed a exponentially variable gain method to avoid the bad transient. Because the source of bad transient is too much increase of the learning rule in terminal time, the proposed method stop the learning in nearby terminal time. This proposed learning rule shows a good performance to reject the bad transient when we design the parameter of exponential term optimally.

However, we can not estimate how the output error increases, it is very hard to design the parameter adequately. Therefore we propose a normalized learning rule in this paper. This method normalize the output error in a certain range and it is possible to estimate the error value. This method includes the exponentially variable gain method and we can design the parameter of exponential term easily because of the normalized term. We proposed this normalized learning rule with PD-type and P-type. P-type learning rule is about to a parametric uncertainty problem.

This paper is organized as follows. Section 2 describes a source of the bad transient and the exponentially variable gain method. In Section 3, we propose a PD-type normalized learning rule and prove its convergence. In Section 4, we propose a P-type normalized learning rule and prove its convergence. Section 5 verifies the performance of the proposed learning rule by using the computer simulations. Finally, the conclusion is given is Section 6.

In this paper, we use the following notations. $\mathbb{R}^n$ is the n-dimensional Euclidean space with norm $||x|| = (x^T x)^{1/2}$ for $x \in \mathbb{R}^n$. $C \in \mathbb{R}^{p\times m}$ indicates C is an $(p \times m$-dimensional matrix with real elements and we use $||C|| = \sqrt{\lambda_{max}(C^T C)}$ as the norm for matrices. Let $\mathbb{N}$ be the set of positive integers $0, 1, 2, \ldots, n$. Finally, the λ-norm is defined for a function $z : [0,T] \to \mathbb{R}^r$ as

$$||z(\cdot)||_\lambda = \sup_{t\in[0,T]} e^{-\lambda t} ||z(t)||.$$

2. STUDY ON BAD TRANSIENT OF ILC

Example 1: Consider the following linear time-invariant system:

$$\dot{x}_k(t) = Ax_k(t) + Bu_k(t),$$

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$$y_k(t) = Cx_k(t), \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Let the desired output trajectory be given as

$$y_d(t) = 12t(1-t),\ 0 \le t \le 1. \tag{2}$$

Suppose that the PD-type learning rule is applied:

$$u_{k+1}(t) = u_k(t) + \Gamma[\dot{e}_k(t) - Re_k(t)], \tag{3}$$

where

$$e_k(t) = y_d(t) - y_k(t).$$

If

$$||I - \Gamma CB||_\infty < 1, \tag{4}$$

then it is already known that the update law makes the error between $y_d(t)$ and $y_k(t)$ approach zero as $k \to \infty$ [8].

Now let us assume that Γ and R are chosen as 1/1.3 and -55 respectively. Note that the large value of R does not affect the condition of convergence. Fig. 1 shows that there is a huge overshoot in the sense of the sup-norm, even though $||y_d(t) - y_k(t)||_\lambda$ monotonically decreases.

Example 2: Consider the following dynamics of a single-link robot manipulator

$$\ddot{\theta}(t) = \frac{1}{J}(0.5m_0 + M_0)\, gl \sin\theta(t) + \frac{1}{J}\tau(t) + d_1(t),$$
$$y(t) = \dot{\theta}(t) + d_2(t), \tag{5}$$

where $\theta(t)$ is the angular position of the manipulator, $\dot{\theta}(t)$ is the angular velocity of the manipulator, $\tau(t)$ is the joint torque, $d_1(t)$ is the state disturbance , $d_2(t)$ is the output disturbance and J is the moment of inertia of the joint, i.e., $J = M_0 l^2 + m_0 l^2/3$. The desired output trajectory is given as follows:

$$y_d(t) = \dot{\theta}_d(t) = 50\left(\frac{3}{8}t^2 - \frac{3}{8}t^3 + \frac{3}{32}t^4\right),$$
$$0 \le t \le 2. \tag{6}$$

Suppose that the PD-type learning rule is applied. If

$$||I - \Gamma g_x(x,t)B(x,t)||_\infty < 1, \tag{7}$$

then it is already known that the update law makes the error between $y_d(t)$ and $y_k(t)$ approach zero as $k \to \infty$. Fig. 2 shows that if we assume that Γ and R are chosen as $1/1.3$ and -55, respectively, then we can see a huge overshoot like as Example 1.

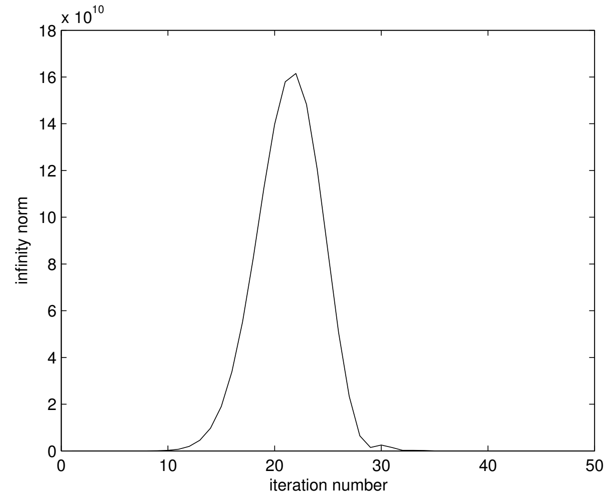

Fig. 1. Sup-norm when $\Gamma = 1/1.3$, $R = -55$ (linear system).

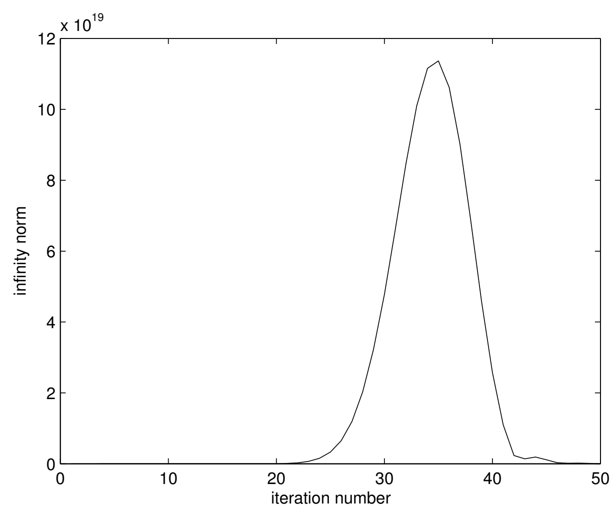

Fig. 2. Sup-norm when $\Gamma = 1/1.3$, $R = -55$ (nonlinear system).

One reason for this huge overshoot is that the amount of updated input is very large. Since the ILC is an open-loop-type controller, it is possible to have a large error at the terminal time. This large error becomes a large amount of the updated input with the help of Γ and R. And, at the next iteration, this large input makes the error larger and larger near the terminal time.

Note that, in calculating the λ-norm with a large value of λ, the errors near the terminal time are extremely less weighted than those near the start time. In general, for the proof of the convergence property of ILC, it is sufficient to prove the existence of a λ, but not much is known of the value of λ. So, if we apply an ILC that is proved to converge in the sense of the λ-norm, with a possibly large value of λ, we may have a huge tracking error, which is not allowable in practice or even for computer simulation.

Since the huge tracking error comes from the open-loop nature of ILC, one way of resolving this undesirable phenomenon is to use a projection method.

Example 3: simulation condition is same as Example 2, except to a learning rule. In general, projection method is follows:

$$u_{k+1}(t) = proj\left(u_k(t) + \Gamma[\dot{e}_k(t) - Re_k(t)]\right). \tag{8}$$

Fig. 3 shows that projection method prevent the output tracking error from grow rapidly, but the transient behavior and error performance is not good.

Therefore Lee and Bien [5] proposed a variable gain method that modifies the learning gain Γ. They replace Γ with a function of time $\Gamma(t)$, as long as $\Gamma(t)$ satisfies the convergence condition. One candidate for $\Gamma(t)$ has the form

$$\Gamma(t) = e^{-\gamma t}\Gamma, \tag{9}$$

where $\gamma > 0$. From above equation, one can easily find that the amount of updated input near at terminal time would be small. Therefore, even if a large error is produced near the terminal time, it does not become the large updated input. Fig. 4 shows that proposed variable gain method has a good performance for tracking error and transient behavior. However, it is hard to design gain γ because we do not know how much the error term increases and the proposed variable gain method is not appropriate in case of long interval system because the gain decreases exponentially.

3. PD-TYPE NORMALIZED LEARNING RULE FOR ILC

3.1. Problem formulation

Consider a class of discrete time-varying nonlinear systems that perform a given task repeatedly on a finite time interval $[0,T]$ (T is a positive integer). The systems can be described by the following different equations:

$$x_k(t+1) = f(t, x_k(t), u_k(t)) + \psi_k(t),$$
$$y_k(t) = g(t, x_k(t)) + \eta_k(t), \tag{10}$$

where k denotes the kth repetitive operation of the system and t is the discrete time index running from $t=0$ to $t=T$ to complete an operation. For all $t \in [0,T]$, $x_k(t) \in \mathbb{R}^n$, $u_k(t) \in \mathbb{R}^m$, $y_k(t) \in \mathbb{R}^r$, $\psi_k(t) \in \mathbb{R}^n$ and $\eta_k(t) \in \mathbb{R}^r$ are the states, inputs, outputs, state disturbances and output disturbances, respectively. Suppose that the initial state at first iteration is same with the desired initial state, i.e. $x_1(0) = x_d(0)$. The vector functions $f : \mathbb{R}^n \times \mathbb{R}^m \times N \times \to \mathbb{R}^n$, and $g : \mathbb{R}^n \times N \to \mathbb{R}^r$. Those functions are satisfy the properties and bounds stated as follow assumptions.

Assumption 1: The vector function $f(t,x,u)$ is globally uniformly Lipschitz in x, u on $[0,T]$ in the sense of

$$||f(t,x_1,u_1) - f(t,x_2,u_2)||$$
$$\le f_0||x_1 - x_2|| + f_0||u_1 - u_2||,$$
$$\forall x_1, x_2, u_1, u_2 \text{ for } t \in [0,T],$$

where f_0 is positive constants.

Assumption 2:

$$f(t,x,u) - f(t,x,u^*) = f_u(u-u^*),$$
$$g(t,x) - g(t,x^*) = g_x(x-x^*),$$

where

$$f_u = \frac{\partial f}{\partial u} = \begin{pmatrix} f_{1,u}(t,x,u+\xi_1(u^*-u)) \\ \vdots \\ f_{m,u}(t,x,u+\xi_m(u^*-u)) \end{pmatrix},$$

$$g_x = \frac{\partial g}{\partial x} = \begin{pmatrix} g_{1,x}(t,x+\zeta_1(x^*-x)) \\ \vdots \\ g_{n,x}(t,x+\zeta_n(x^*-x)) \end{pmatrix},$$

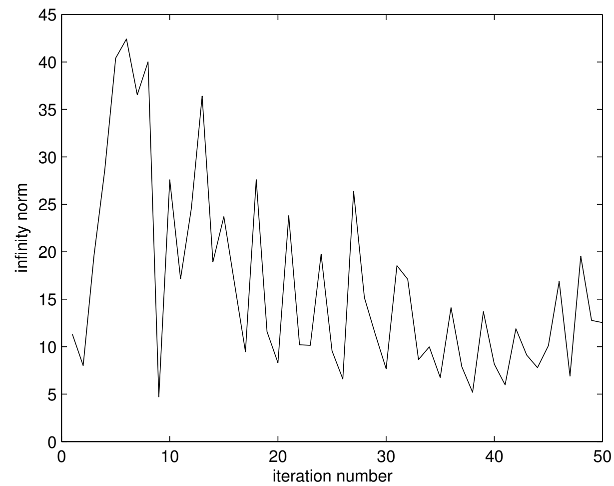

Fig. 3. Sup-norm when $\Gamma = 1/1.3$, $R = -55$ (nonlinear system with projection method).

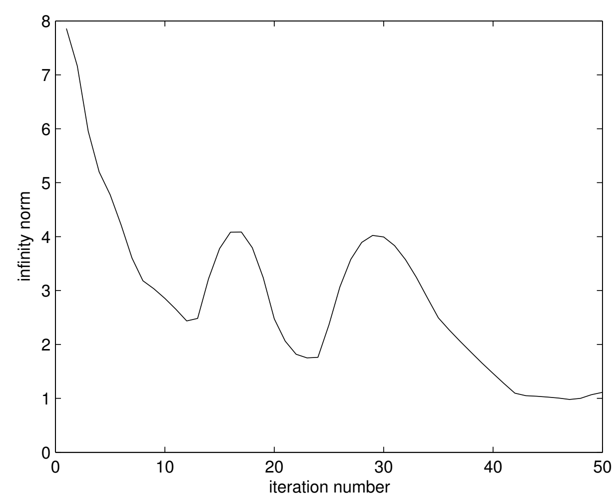

Fig. 4. Sup-norm when $\Gamma = 1/1.3$, $R = -55$ (nonlinear system with variable gain method).

$f_{m,u}$ is the partial derivative of the m^{th} element of f to u and $g_{n,x}$ is the partial derivative of the n^{th} element of g to x; $\xi_1,\ldots,\xi_m$ and $\zeta_1,\ldots,\zeta_n$ belong to (0,1). Since the mean value theorem will be used extensively here, the above notations are used to avoid the use of $\xi_1,\ldots,\xi_m$ and $\zeta_1,\ldots,\zeta_n$.

Assumption 3: The matrices f_u and g_x are defined with bounds as $||f_u|| \le b_f$, $||g_x|| \le b_g$, where b_f, b_g are positive constants. Furthermore, the product matrix $g_x f_u$ has full rank.

Assumption 4: The disturbances $\psi_k(t)$, $\eta_k(t)$ and initial state $x_k(0)$ satisfy:

$$||\psi_{k+1}(t)-\psi_k(t)|| \le d_\psi,$$
$$||\eta_{k+1}(t)-\eta_k(t)|| \le d_\eta,$$
$$||x_{k+1}(0)-x_k(0)|| \le d_0,$$

where d_ψ, d_η and d_0 are positive constants.

Those assumptions are natural in many repetitive dynamic systems such as the repeatability in robot specifications.

Here, we propose a new PD-type normalized learning rule. The previous example shows that the conventional PD-type learning rule can produce the unexpected huge overshoot when the design parameter increases. Therefore we propose a new learning rule to reduce those bad phenomenon. The proposed learning rule is follows:

$$u_k(t+1) = u_k(t) + \Gamma(t)\left(\frac{e_k(t)-e_k(t-m)}{m} - R\frac{e_k(t)}{1+e_k^2(t)}\right), \tag{11}$$

where $\Gamma(t)=\Gamma e^{-\gamma t}$, $\gamma>0$, $||\Gamma(t)|| \le b_\Gamma$ and $||\frac{1}{m}|| \le b_m$ for all $t \in [0,T]$. The normalized term of (11) prevent the output error from too much increasing. From the Fig. 5, we can see that the value of normalized term is bounded as $|0.5|$. The variable gain method that proposed by Lee and Bien [5], is hard to design the parameter γ because we don't know how the output error increase. But we can easily design the parameter γ in this normalized learning rule.

3.2. Main result

Theorem 1: Consider the discrete control system described by (10) and (11). Assume that the (A1)-(A4) are satisfied and the following inequality

$$\left|\left|I-\frac{g_x f_u \Gamma(t)}{m}\right|\right| \le \rho < 1 \tag{12}$$

holds for all x, u, and t, then the tracking error will be bounded when $k\to\infty$. If the initial state and all disturbances become repetitive gradually, i.e., $d_\psi = d_\eta = d_0 = 0$, then $y_k(t)$ will converge to $y_d(t)$ when $k\to\infty$.

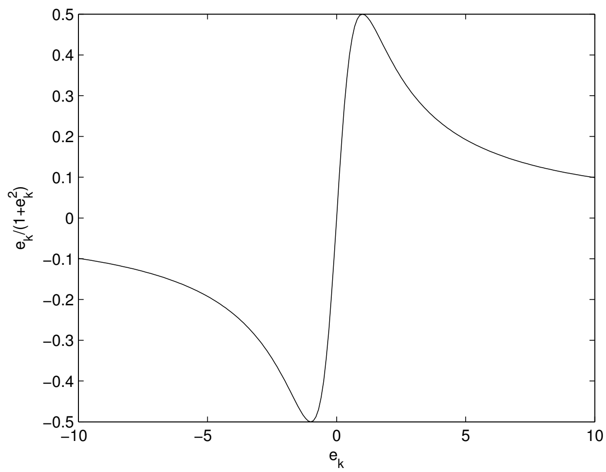

Fig. 5. Normalized term of the proposed learning rule.

Proof:

$$\begin{aligned}
&e_{k+1}(t)-e_k(t)\\
&= y_k(t)-y_{k+1}(t)\\
&= g(t,x_k(t))-g(t,x_{k+1}(t))+\eta_k(t)-\eta_{k+1}(t)\\
&= g_x(x_k(t)-x_{k+1}(t))+\eta_k(t)-\eta_{k+1}(t)\\
&= g_x(x_k(t)-x_{k+1}(t)+x_{k+1}(t+1)-x_{k+1}(t+1))\\
&\quad +\eta_k(t)-\eta_{k+1}(t)\\
&= -g_x f_u \Gamma(t)\left(\frac{e_k(t)-e_k(t-m)}{m}-R\frac{e_k(t)}{1+e_k^2(t)}\right)\\
&\quad - g_x(x_{k+1}(t)-x_k(t))\\
&\quad + g_x f_u(u_{k+1}(t)-u_k(t))+\eta_k(t)-\eta_{k+1}(t).
\end{aligned} \tag{13}$$

Equation (13) can be expressed as

$$\begin{aligned}
e_{k+1}(t) &= \left[I-\frac{g_x f_u \Gamma(t)}{m}\right]e_k(t)+\frac{g_x f_u \Gamma(t)}{m}e_k(t-m)\\
&\quad + g_x f_u(u_{k+1}(t)-u_k(t))\\
&\quad - g_x(x_{k+1}(t)-x_k(t))+g_x f_u\Gamma(t)R\frac{e_k(t)}{1+e_k^2(t)}\\
&\quad +\eta_k(t)-\eta_{k+1}.
\end{aligned} \tag{14}$$

Take norms on both sides of (14) and using the fact that $||\frac{e_k(t)}{1+e_k^2(t)}|| \le ||e_k(t)||$, we obtain

$$\begin{aligned}
||e_{k+1}(t)|| &\le \rho||e_k(t)||+b_0||e_k(t-m)||+b_1||e_k(t)||\\
&\quad + b_g b_f||u_{k+1}(t)-u_k(t)||\\
&\quad + b_g||x_{k+1}(t)-x_k(t)||+d_\eta,
\end{aligned} \tag{15}$$

where $b_0=\sup_{t\in[0,\,T]}\left|\left|\frac{g_x f_u\Gamma(t)}{m}\right|\right|$, $b_1=\sup_{t\in[0,\,T]}||g_x f_u \Gamma(t) R||$. From (11)

$$\begin{aligned}
&||u_{k+1}(t)-u_k(t)||\\
&= \left|\left|\Gamma(t)\left(\frac{e_k(t)-e_k(t-m)}{m}-R\frac{e_k(t)}{1+e_k^2(t)}\right)\right|\right|
\end{aligned}$$

$$\leq b_\Gamma(b_m+b_R)||e_k(t)||+b_\Gamma b_m||e_k(t-m)||. \tag{16}$$

From (10), we have

$$\begin{aligned}&x_{k+1}(t)-x_k(t)\\&=f(t-1,x_{k+1}(t-1),u_{k+1}(t-1))\\&\quad+\psi_{k+1}(t-1)-f(t-1,x_k(t-1),u_k(t-1))\\&-\psi_k(t-1).\end{aligned} \tag{17}$$

Take norms on both sides of (17), we have

$$\begin{aligned}&||x_{k+1}(t)-x_k(t)||\\&\leq f_0||x_{k+1}(t-1)-x_k(t-1)||\\&\quad+f_0||u_{k+1}(t-1)-u_k(t-1)||+d_\psi\\&\leq f_0||x_{k+1}(t-1)-x_k(t-1)||\\&\quad+f_0b_\Gamma\left((b_m+b_R)||e_k(t)||+b_m||e_k(t-m)||\right)+d_\psi\\&\leq f_0^2||x_{k+1}(t-2)-x_k(t-2)||\\&\quad+\left(f_0^2+f_0\right)b_\Gamma\left((b_m+b_R)||e_k(t)||+b_m||e_k(t-m)||\right)\\&\quad+(f_0+1)d_\psi\\&\leq\ \cdots\\&\leq f_0^t||x_{k+1}(0)-x_k(0)||\\&\quad+b_\Gamma(f_0^t+f_0^{t-1}+\cdots+f_0^2+f_0)((b_m+b_R)||e_k(t)||\\&\quad+b_m||e_k(t-m)||)\\&\quad+\left(f_0^{t-1}+f_0^{t-2}+\cdots+f_0^2+f_0+1\right)d_\psi\\&\leq m_1+m_2||e_k(t)||+m_3||e_k(t-m)||,\end{aligned} \tag{18}$$

where $m_1=f_0^td_0+\left(f_0^{t-1}+f_0^{t-2}+\cdots+f_0^2+f_0+1\right)d_\psi$, $m_2=b_\Gamma(b_m+b_R)\left(f_0^t+f_0^{t-1}+\cdots+f_0^2+f_0\right)$, and $m_3=b_\Gamma b_R\left(f_0^t+f_0^{t-1}+\cdots+f_0^2+f_0\right)$.

By (15), (16) and (18), we have

$$\begin{aligned}&||e_{k+1}(t)||\\&\leq(\rho+b_1+b_gb_fb_\Gamma(b_m+b_R)+b_gm_2)\,||e_k(t)||\\&\quad+(b_0+b_gb_fb_\Gamma b_m+b_gm_3)\,||e_k(t-m)||\\&\quad+b_gm_1+d_\eta.\end{aligned} \tag{19}$$

Multiplying $e^{-\lambda t}$ on both sides of (19), we have

$$\begin{aligned}&||e_{k+1}||_\lambda\\&\leq[\rho+b_1+b_gb_fb_\Gamma(b_m+b_R)+b_gm_2\\&\quad+e^{-\lambda m}(b_0+b_gb_fb_\Gamma b_m+b_gm_3)]||e_k||_\lambda\\&\quad+b_gm_1+d_\eta.\end{aligned} \tag{20}$$

Choose λ large enough so that

$$\begin{aligned}\hat{\rho}=&[\rho+b_1+b_gb_fb_\Gamma(b_m+b_R)+b_gm_2\\&+e^{-\lambda m}(b_0+b_gb_fb_\Gamma b_m+b_gm_3)]<1.\end{aligned} \tag{21}$$

Thus, we have

$$||e_{k+1}||_\lambda\leq\hat{\rho}||e_k||_\lambda+M, \tag{22}$$

where $M=b_gm_1+d_\eta$ and $0<\hat{\rho}<1$, $M>0$.
From inequality (22), we have

$$\begin{aligned}||e_{k+1}||_\lambda&\leq\hat{\rho}^k||e_1||_\lambda+M\sum_{j=0}^{k-1}\hat{\rho}^j\\&=\hat{\rho}^k||e_1||_\lambda+\frac{M\left(1-\hat{\rho}^k\right)}{1-\hat{\rho}}.\end{aligned} \tag{23}$$

Thus

$$\lim_{k\to\infty}||e_k||_\lambda\leq\frac{M}{1-\hat{\rho}}. \tag{24}$$

If the initial state and all disturbances become repetitive gradually, i.e., d_ψ, d_η, $d_0\to0$ as $k\to\infty$, then $M\to0$, $||e_k||_\lambda\to0$.

By the definition of $||\cdot||_\lambda$, we know that

$$\sup_{t\in[0,T]}||e_k||\leq e^{\lambda T}||e_k||_\lambda. \tag{25}$$

Therefore,

$$\sup_{t\in[0,T]}||e_k||\to0\text{ as }k\to\infty, \tag{26}$$

which means that $y_k(t)\to y_d(t)$ when $k\to\infty$ on $t\in[0,T]$. □

4. P-TYPE NORMALIZED LEARNING RULE WITH PARAMETRIC UNCERTAINTY

4.1. Problem formulation

In a nonlinear system, many problems can be represented by parametric uncertainty form. Usually in a general nonlinear problem, the design parameter of P-type learning rule is included in the stability condition. Therefore the parameter can not too much increases. However, because the design parameter of P-type learning rule with parametric uncertainty problem is not included in the stability condition, the parameter can increases by the designer, i.e., a huge overshoot can be existed. Therefore we propose a P-type normalized learning rule with parametric uncertainty problem and prove its convergence. In previous sections, the convergence is proved by the time-weighted norm (λ-norm), but in this problem, we prove the convergence with Lyapunov approach.

Consider simple first order nonlinear dynamic system in the i-th iteration

$$\dot{x}_i(t)=\theta(t)\xi(x_i(t),t)+u_i(t),\ \ x(0)=x_0, \tag{27}$$

where $\xi(x_i(t),t)$ is a known nonlinear function which can be local Lipschitzian and $\theta(t)\leq\theta_M\in[0,\ T]$ is the unknown time-varying parameter.

The reference trajectory is generated by a dynamics

$$\dot{x}_r(t)=f(x_r,r,t), \tag{28}$$

where $f_r = f(x_r, r, t)$ is a known smooth function, r is a reference input which yields a bounded state $x_r(t)$ over the interval $[0,\ T]$. The tracking error is defined as $e_i(t) = x_r(t) - x_i(t)$.

The objective of ILC is to find a sequence of appropriate control input $u_i(t)$ for $t \in [0,\ T]$ such that the system state $x_i(t)$ tracks the reference trajectory x_r as $i \to \infty$.

The error dynamics at the i-th iteration can be expressed as

$$\dot{e}_i(t) = f_r - \theta(t)\xi_i - u_i. \tag{29}$$

The learning control mechanism consists of the control law

$$u_i = ke_i + f_r - \hat{\theta}_i(t)\xi_i, \tag{30}$$

and the parametric learning law

$$\hat{\theta}_i(t) = \hat{\theta}_{i-1}(t) - \gamma_1 \frac{\xi_i}{1+\xi_i^2}\frac{e_i(t)}{1+e_i(t)^2} - \gamma_2\xi_i e_i(t),$$
$$\hat{\theta}_{-1}(t) = 0. \tag{31}$$

In a previous research, Xu, Yan, and Chen [9] proposed the parametric learning law

$$\hat{\theta}_i(t) = proj(\hat{\theta}_{i-1}(t)) - \gamma\xi_i e_i(t). \tag{32}$$

On the other hand, we proposed a normalized parametric learning law (31). The designer can increase γ_1 arbitrarily, but γ_2 should be maintain a normal value. The normalized learing rule is can be used not only for general ILC problems, but also for parametric uncertainties. The proposed learing rule is based on conventional several parametric uncertainties laws, but the normalized term is added to the problem solutions, resulting maintains a robust output in an abnormal learning gain.

Substituting the learning control law (30) into the error dynamics (29) yields the closed-loop error dynamics

$$\dot{e}_i(t) = -ke_i - \phi_i(t)\xi_i, \tag{33}$$

where $\phi_i(t) \triangleq \theta(t) - \hat{\theta}_i(t)$.

4.2. Main result

First derive the boundedness of tracking error e_i and parameter estimate $\hat{\theta}_i$ under learning control law (30) and (31). Note that at the initial iteration $i = 0$, there is no parametric learning as $\hat{\theta}_{-1} = 0$. Hence we have to derive the boundedness of $(e_0,\ \hat{\theta}_0)$ in a way different from that for $(e_i,\ \hat{\theta}_i)$ with $i \geq 1$.

Proposition 1: $(e_0,\ \hat{\theta}_0)$ is bounded for $t \in [0,\ T]$.

Proof: Choose Lyapunov functional

$$V_0(t) = \frac{1}{2}e_0^2(t) + \frac{1}{2}\int_0^t \phi_0^2(\tau)d\tau. \tag{34}$$

The upper right hand derivative of V_0 is

$$\dot{V}_0 = e_0\dot{e}_0 + \frac{1}{2}\phi_0^2 = -ke_0^2 - \phi_0\xi_0 e_0 + \frac{1}{2}\phi_0^2. \tag{35}$$

Noticing that $\hat{\theta}_0 = -\gamma_1 \frac{\xi_0}{1+\xi_0^2}\frac{e_0}{1+e_0^2} - \gamma_2\xi_0 e_0$, V_0 becomes

$$\begin{aligned}\dot{V}_0 &= -ke_0^2 + \bar{r}\phi_0\hat{\theta}_0 + \frac{1}{2}\phi_0^2 \\ &= -ke_0^2 - \frac{1}{2}\phi_0^2 + \phi_0\bar{\theta},\end{aligned} \tag{36}$$

where

$$\bar{r} = \frac{1}{\frac{\gamma_1}{(1+\xi_0^2)(1+e_0^2)} + \gamma_2} > 0,$$
$$\bar{\theta} = \theta + (\bar{r} - 1)\hat{\theta}_0 < \bar{\theta}_m.$$

Using Young's inequality, for any $c > 0$ we have $\phi_0\bar{\theta} \leq c\phi_0^2 + \frac{1}{4c}\bar{\theta}^2$. Let $0 < c < \frac{1}{2}$, $\dot{V}_0 \leq -ke_0^2 - (\frac{1}{2} - c)\phi_0^2 + \frac{1}{4c}\bar{\theta}^2$. Thus $\dot{V}_0$ is negative definite outside the region $\{(e_0,\ \phi_0) \in \mathcal{D} | ke_0^2 + (\frac{1}{2} - c)\phi_0^2 \leq \frac{1}{4c}\bar{\theta}^2\}$ which specifies the bound of $V_0(t)$ in the finite interval $[0,\ T]$. The boundedness of $V_0(t)$ implies the boundedness of e_0 in the sequel the boundedness of x_0, ξ_0, and $\hat{\theta}_0$. □

Now we can prove the boundedness of $(e_i,\ \hat{\theta}_i)$, which is summarized in the following theorem.

Theorem 2: Let the ILC scheme be composed of the control law (30),parametric learning law (31). If the following inequality holds:

$$|\phi_i| \leq \frac{4k\gamma_2 + 2(\gamma_1/4 + \gamma_2)^2}{\gamma_1},$$

then the error dynamic system (33) is bounded and converges as follows:

(i) $(e_i,\ \hat{\theta}_i)$ is bounded for all $i \geq 1$.
(ii) $\lim_{i\to\infty} e_i(t) = 0$ for all $t \in [0,\ T]$.

Proof: Define the following Lyapunov functional

$$\begin{aligned}V(e_i, \phi_i, \phi_{i-1}, t) =& \frac{1}{2}e_i^2 + \frac{1}{2\gamma_2}\int_0^t \phi_i^2(\tau)d\tau \\ &+ \frac{1}{2\gamma_2}\int_t^T \phi_{i-1}^2(\tau)d\tau.\end{aligned} \tag{37}$$

The upper right hand derivative of $V(e_i, \phi_i, \phi_{i-1}, t)$ is

$$\dot{V}(e_i, \phi_i, \phi_{i-1}, t) = e_i\dot{e}_i + \frac{1}{2\gamma_2}\left(\phi_i^2 - \phi_{i-1}^2\right). \tag{38}$$

Substituting the closed-loop error dynamics (33), the first term on the right hand side of (38) is

$$e_i\dot{e}_i = -ke_i^2 - \phi_i\xi_i e_i. \tag{39}$$

Next substituting the parametric learning law (31) into the second term on the right hand side of (38), using the relations $(a-b)^2-(a-c)^2=-2(a-b)(b-c)-(b-c)^2$, we have

$$\begin{aligned}
&\frac{1}{2\gamma_2}\left(\phi_i^2-\phi_{i-1}^2\right)\\
&=\frac{1}{2\gamma_2}\left[(\theta-\hat{\theta}_i)^2-(\theta-\hat{\theta}_{i-1})^2\right]\\
&=-\frac{1}{\gamma_2}(\theta-\hat{\theta}_i)(\hat{\theta}_i-\hat{\theta}_{i-1})-\frac{1}{2\gamma_2}(\hat{\theta}_i-\hat{\theta}_{i-1})^2\\
&=\frac{\gamma_1}{\gamma_2}\frac{\phi_i\xi_ie_i}{(1+\xi_i^2)(1+e_i^2)}+\phi_i\xi_ie_i\\
&\quad-\frac{\xi_i^2e_i^2}{2\gamma_2}\left[\frac{\gamma_1}{(1+\xi_i^2)(1+e_i^2)}+\gamma_2\right]^2. \qquad (40)
\end{aligned}$$

Clearly $\phi_i\xi_ie_i$ appears in (39) and (40) with opposite signs. Therefore, the upper right hand derivative of $V(e_i,\phi_i,\phi_{i-1},t)$ is

$$\begin{aligned}
\dot{V}(e_i,\phi_i,\phi_{i-1},t)=&-ke_i^2+\frac{\gamma_1}{\gamma_2}\frac{\phi_i\xi_ie_i}{(1+\xi_i^2)(1+e_i^2)}\\
&-\frac{\xi_i^2e_i^2}{2\gamma_2}\left[\frac{\gamma_1}{(1+\xi_i^2)(1+e_i^2)}+\gamma_2\right]^2. \qquad (41)
\end{aligned}$$

If we design k, γ_1, γ_2 sufficiently large such that

$$\begin{aligned}
&\max_{\xi_i,\,e_i}\left|\frac{\gamma_1}{\gamma_2}\frac{\phi_i\xi_ie_i}{(1+\xi_i^2)(1+e_i^2)}\right|\\
&\le ke_i^2+\frac{\xi_i^2e_i^2}{2\gamma_2}\left[\frac{\gamma_1}{(1+\xi_i^2)(1+e_i^2)}+\gamma_2\right]^2, \qquad (42)
\end{aligned}$$

then

$$\dot{V}(e_i,\phi_i,\phi_{i-1},t)\le 0. \qquad (43)$$

Note that from (42), $\max_{\xi_i,\,e_i}\left|\frac{\gamma_1}{\gamma_2}\frac{\phi_i\xi_ie_i}{(1+\xi_i^2)(1+e_i^2)}\right|=\frac{1}{4}$, when $\xi_i=e_i=\pm1$.

Therefore (42) is satisfied when

$$|\phi_i|\le\frac{4k\gamma_2+2(\gamma_1/4+\gamma_2)^2}{\gamma_1}. \qquad (44)$$

Integrating the derivative of V, using the negativeness of $\dot{V}$, the boundedness of e_i and $\hat{\theta}_i$ can be derived if $V(e_i(0),\phi_i(0),\phi_{i-1}(0))$ is bounded, i.e.,

$$\begin{aligned}
&V(e_i(t),\phi_i(t),\phi_{i-1}(t),t)\\
&=V(e_i(0),\phi_i(0),\phi_{i-1}(0),0)+\int_0^t\dot{V}dt\\
&\le V(e_i(0),\phi_i(0),\phi_{i-1}(0),0). \qquad (45)
\end{aligned}$$

Note that

$$\begin{aligned}
&V(e_i(0),\phi_i(0),\phi_{i-1}(0),0)\\
&=\frac{1}{2}e_i^2(0)+\frac{1}{2}\int_0^T\phi_{i-1}^2(\tau)d\tau, \qquad (46)
\end{aligned}$$

and $e_i(0)$ is bounded. Let us look at the first iteration $i=1$.

$$V(e_1(0),\phi_1(0),\phi_0(0),0)=\frac{1}{2}e_1^2(0)+\frac{1}{2}\int_0^T\phi_0^2(\tau)d\tau \qquad (47)$$

is bounded because $\phi_0(t)$ is bounded according to Proposition 1. In the sequel $V(e_1(t),\phi_1(t),\phi_0(t),t)\le V(e_1(0),\phi_1(0),\phi_0(0),0)$ is bounded. From the parametric learning law (31), the boundedness of e_1 warrants the boundedness of $\hat{\theta}_1$.

Now assume that $(e_{i-1},\ \hat{\theta}_{i-1})$ are bounded for all $t\in[0,\ T]$, so is $V(e_i(0),\phi_i(0),\phi_{i-1}(0),0)$. From (45), $V(e_i(t),\phi_i(t),\phi_{i-1}(t),t)$ is bounded. Similarly, from the boundedness of e_i and the parametric learning law (31) we can derive the boundedness of $\hat{\theta}_i$. By the Mathematical Induction, the quantities $(e_i,\ \hat{\theta}_i)$ are bounded for any $i\ge 0$, which is first part of Theorem 2.

In order to prove (ii), let's define a Lyapunov function as

$$V_i(t)=\frac{1}{2}e_i^2+\frac{1}{2\gamma_2}\int_0^t\phi_i^2(\tau)d\tau. \qquad (48)$$

The difference between V_i and V_{i-1} is

$$\begin{aligned}
\Delta V_i=&V_i-V_{i-1}\\
=&\frac{1}{2}e_i^2+\frac{1}{2\gamma_2}\int_0^t\left(\phi_i^2(\tau)-\phi_{i-1}^2(\tau)\right)d\tau\\
&-\frac{1}{2}e_{i-1}^2. \qquad (49)
\end{aligned}$$

The first term on the right side of (49) is

$$\begin{aligned}
\frac{1}{2}e_i^2=&\int_0^t e_i\dot{e}_id\tau+\frac{1}{2}e_i^2(0)\\
=&\int_0^t(-ke_i^2-\phi_i\xi_ie_i)d\tau+\frac{1}{2}e_i^2(0).
\end{aligned}$$

Similarly as (40), the second term on the right side of (49) can be expressed as

$$\begin{aligned}
&\frac{1}{2\gamma_2}\int_0^t\left(\phi_i^2-\phi_{i-1}^2\right)d\tau\\
&=\int_0^t\left[\frac{\gamma_1}{\gamma_2}\frac{\phi_i\xi_ie_i}{(1+\xi_i^2)(1+e_i^2)}+\phi_i\xi_ie_i\right]d\tau\\
&\quad-\int_0^t\frac{\xi_i^2e_i^2}{2\gamma_2}\left[\frac{\gamma_1}{(1+\xi_i^2)(1+e_i^2)}+\gamma_2\right]^2d\tau.
\end{aligned}$$

Therefore, the difference become

$$\begin{aligned}
\Delta V_i=&-\int_0^t ke_i^2d\tau+\int_0^t\left[\frac{\gamma_1}{\gamma_2}\frac{\phi_i\xi_ie_i}{(1+\xi_i^2)(1+e_i^2)}\right]d\tau\\
&-\int_0^t\frac{\xi_i^2e_i^2}{2\gamma_2}\left[\frac{\gamma_1}{(1+\xi_i^2)(1+e_i^2)}+\gamma_2\right]^2d\tau-\frac{1}{2}e_{i-1}^2
\end{aligned}$$

$$+\frac{1}{2}e_i^2(0). \tag{50}$$

If we design d, γ_1, and γ_2 sufficiently large such that (44) is satisfied,

$$\Delta V_i \leq -\frac{1}{2}e_{i-1}^2 + \frac{1}{2}e_i^2(0). \tag{51}$$

Applying (51) repeatedly we have

$$\begin{aligned} V_i(t) &= V_0(t) + \sum_{j=1}^{i} \Delta V_j \\ &\leq V_0(t) + \frac{1}{2}\sum_{j=1}^{i} e_j^2(0) - \frac{1}{2}\sum_{j=1}^{i-1} e_j^2(t), \\ \lim_{i\to\infty} V_i(t) &\leq V_0(t) + \lim_{i\to\infty}\frac{1}{2}\sum_{j=1}^{i} e_j^2(0) - \lim_{i\to\infty}\frac{1}{2}\sum_{j=1}^{i-1} e_j^2(t). \end{aligned} \tag{52}$$

Consider the positiveness of V_i and boundedness of V_0, the sequence $e_i(t)$ converges to zero pointwisely as $i \to \infty$ and (ii) of Theorem 2 follows. □

5. SIMULATION RESULTS

5.1. Simulation for PD-type normalized learning rule

Consider the following dynamics of a single-link robot manipulator

$$\ddot{\theta}(t) = \frac{1}{J}(0.5m_0 + M_0)\,gl\sin\theta(t) + \frac{1}{J}\tau(t) + d_1(t),$$
$$y(t) = \dot{\theta}(t) + d_2(t),$$

where $\theta(t)$ is the angular position of the manipulator, $\dot{\theta}(t)$ is the angular velocity of the manipulator, $\tau(t)$ is the joint torque, $d_1(t)$ is the state disturbance , $d_2(t)$ is the output disturbance and J is the moment of inertia of the joint, i.e., $J = M_0 l^2 + m_0 l^2/3$. The parameters are given in Table 1. The desired output trajectory is given as follows:

$$y_d(t) = \dot{\theta}_d(t) = 50\left(\frac{3}{8}t^2 - \frac{3}{8}t^3 + \frac{3}{32}t^4\right),\quad 0 \leq t \leq 2.$$

The disturbances are given as

$$d_1(t) = 5\sin(40\pi t),\ d_2(t) = 0.05\sin(100\pi t).$$

To measure the performance, we calculate the RMS error. The error tolerance $\varepsilon = 0.005$ and the iteration limit is 200. In order to compare the conventional method and the proposed one, five PD-type methods are used. The five PD-type methods are conventional ILC, projection based ILC, variable gain based ILC, normalized based ILC with fixed gain (Normalized), and normalized based ILC with variable gain (Proposed).

We performed these simulations with three cases. The first case is about to a normal gain with $\Gamma = 0.77$, $R = -5$.

Table 1. Robot manipulator parameters.

m_0	the mass of the link	2 kg
l	the length of the link	0.5 m
M_0	the tip load	4 kg
g	the gravitational acceleration	9.8 ms^{-2}

Table 2. Performance comparison with various PD-type learning rules.

Method	Case 1	Case 2	Case 3
Conventional	9	x (inf/0.60)	x (inf/661.95)
Projection	9	x (17.20/0.09)	x (63.32/46.35)
Variable	9	53(10.93/0.0)	x (inf/580.51)
Normalized	9	x (7.18/0.14)	x (33.82/0.89)
Proposed	9	52(2.91/0.0)	x (36.52/0.59)

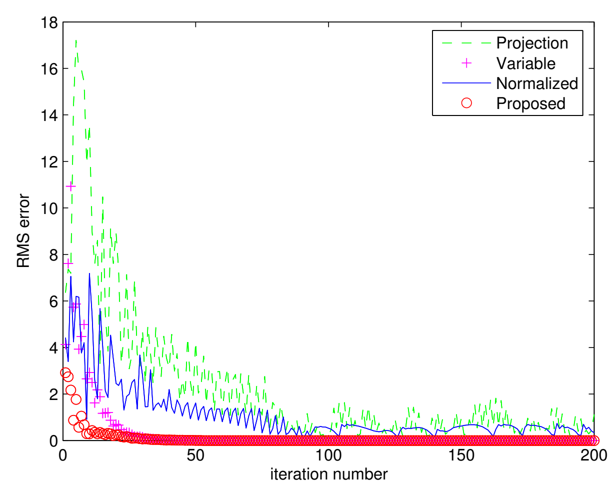

Fig. 6. RMS error in Case 2.

The second case is about to a large gain with $\Gamma = 0.77$, $R = -50$. This large gain can cause a huge overshoot. The third case is same as case 2, but the tracking interval is long ($0 \leq t \leq 4$).

In Table 2, first column means the method of learning rule. The numbers in table means the iteration number to achieve RMS error within error tolerance and 'x' indicate the RMS error can not reduced by error tolerance within iteration limit. Also, the numbers within a parenthesis mean the maximum/minimum RMS error within iteration limit. For example, Case 2, Variable mathod is reached to a RMS error within error tolerance in 53 iteration and maximum RMS error is 10.93, minimum RMS error is 0,0. We used $\gamma = 0.0001$ for case 1 and $\gamma = 0.008$ for case 2,3. In Table 2 and Fig. 6, we see that the proposed method shows a better performance than other methods for the large gain.

5.2. Simulation for P-type normalized learning rule with parametric uncertainty

Consider the system

$$\dot{x} = (1+\sin \pi t)x^2 + u, \;\; x(0) = 1.$$

The reference model is

$$\dot{x}_r = -x_r + \sin^2 \pi t + 2, \;\; x_r(0) = 1.$$

The tracking interval is $[0,\ 2]$. Throughout the simulation, choose the feedback gain $k = 1$. To measure the performance, we calculate the RMS error and the error tolerance $\varepsilon = 0.001$. The iteration limit is 200.

In order to compare the conventional method and the proposed one, four P-type methods are used. The four P-type methods are conventional ILC, projection based ILC, variable gain based ILC, normalized based ILC (proposed).

We performed these simulations with seven cases. The Case 1: $(\gamma_1 = 10,\ \gamma_2 = 10)$ and Case 2: $(\gamma_1 = 8,\ \gamma_2 = 8)$ are about to normal gain. The Case 3: $(\gamma_1 = 21,\ \gamma_2 = 10)$ and Case 4: $(\gamma_1 = 30,\ \gamma_2 = 10)$ are about to large gain. The Case 5: $(\gamma_1 = 21,\ \gamma_2 = 1)$ considers a long period $([0,\ 100])$ with a large gain. Finally, Case 6 and 7 consider a fixed $(e_i(0) = 0.2)$ and random $(e_i(0) = [0,\ 0.2])$ initial error with the iteration limit 100, respectively. In Table 3, first column means the method of learning rule. The numbers in table means the iteration number and 'x' indicate the RMS error can not reduced by error tolerance within iteration limit.

In Table 3, we see that the proposed method shows a better performance than other methods for all cases. Fig. 7 shows the RMS error in Case 4 and Fig. 8 shows that the values of θ and $\hat{\theta}$ in Case 4. Fig. 9, 10 show the output tracking performance of projection method and proposed one in Case 5, respectively. Although the period of the task becomes long, the proposed method has a small error than other methods. Figs. 11 and 12 show that the values of theta with projection method and proposed one in Case 5, respectively.

In Table 4, the performance comparison between the projection method and the proposed one with the fixed and random initial error is presented. The numbers in second and third column is minimum RMS error in iteration limit. The fourth and fifth columns means that gain condition for Case 6 and 7, respectively. We can see that the proposed method has better robust performance than the projection method for non-zero initial error.

6. CONCLUSION

In this paper, the source of bad transients is analyzed from several examples and P-type and PD-type normalized learning rules are proposed to avoid an unexpected

Table 3. Performance comparison with various P-type learning rules.

Method	Case 1	Case 2	Case 3	Case 4	Case 5
Conventional	13	19	x	x	x
Projection	13	19	15	x	x
Variable	13	19	4	3	x
Proposed	7	10	4	3	2

Table 4. Performance comparison with fixed, random initial error.

Method	Case 6	Case 7	gain (Case 6)	gain (Case 7)
Projection	0.0312	0.1137	$k = 1$, $\gamma = 10$	$k = 20$, $\gamma = 10$
Proposed	0.0161	0.0971	$k = 1$, $\gamma_1 = 21$, $\gamma_2 = 1$	$k = 20$, $\gamma_1 = 21$, $\gamma_2 = 1$

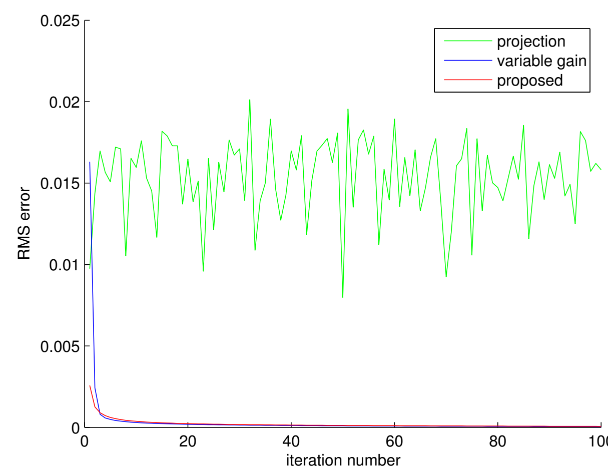

Fig. 7. RMS error in Case 4.

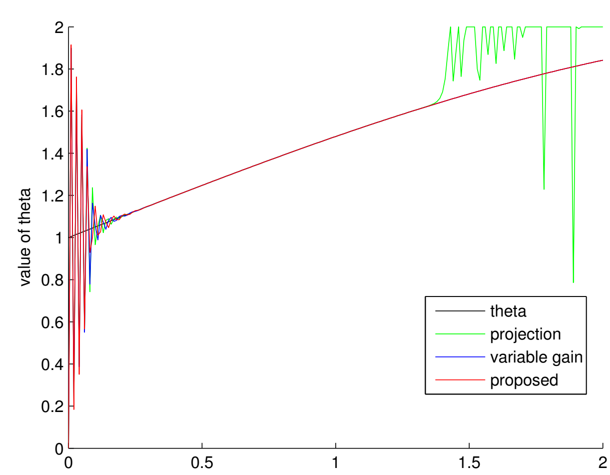

Fig. 8. Values of θ and $\hat{\theta}$ in Case 4.

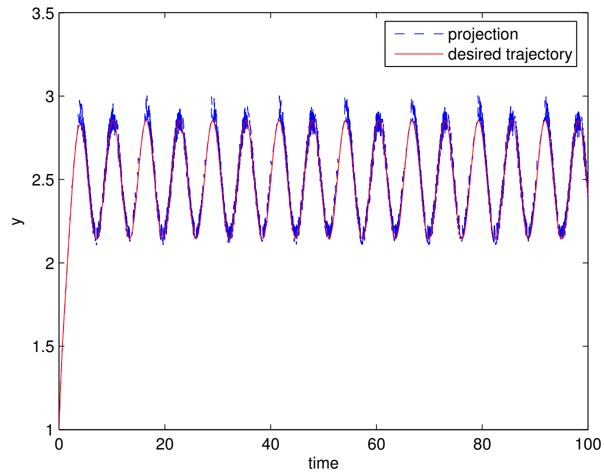

Fig. 9. Output tracking of the projection method in Case 5.

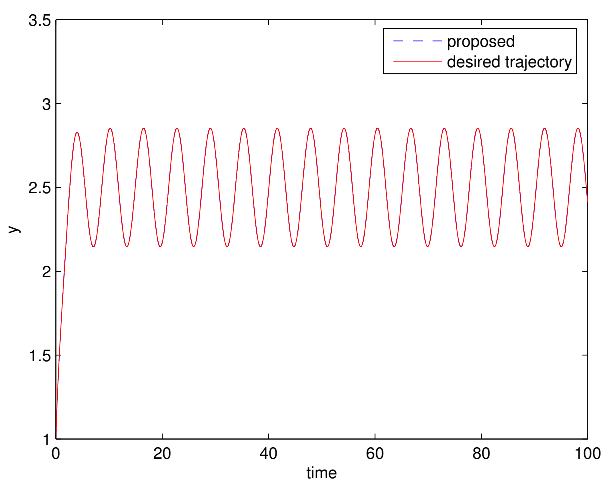

Fig. 10. Output tracking of the proposed method in Case 5.

huge overshoot or even divergence. In the PD-type normalized learning rule, the variable gain can be designed easily by using the normalized error term. In a parametric uncertainty problem, instead of using the lambda norm, we proved the boundedness and convergence of the closed loop system based on Lyapunov theory. For two proposed learning rules, it is shown that the tracking error converges to zero along the iteration axis. The performance of the proposed method is illustrated by a computer simulations.

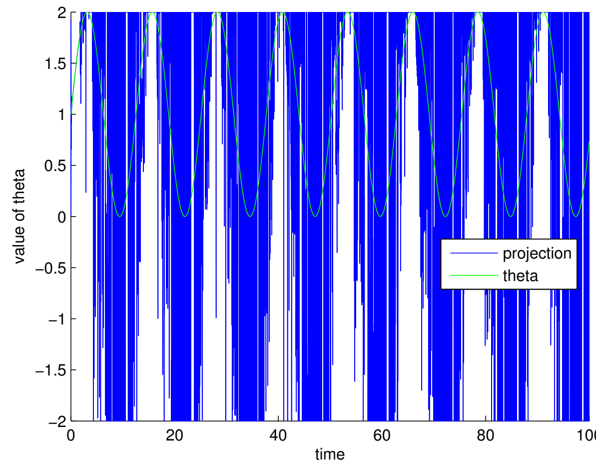

Fig. 11. Values of theta with projection method in Case 5.

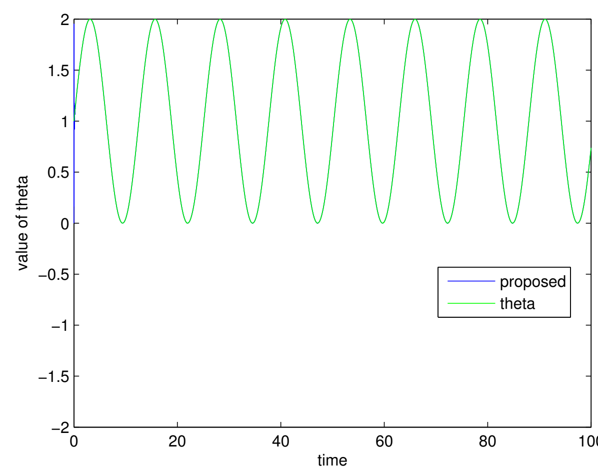

Fig. 12. Values of theta with proposed method in Case 5.

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