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A Disturbance Observer-based Robust Tracking Controller for Uncertain Robot Manipulators

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Abstract: This paper considers the trajectory tracking problem for uncertain robot manipulators subject to external disturbance torques. The external disturbance torques are assumed to be unknown and time-varying. We present a disturbance observer-based controller which estimates the lumped disturbance (the external disturbance torque combined with the effect of plant uncertainties), and compensates it so that the overall closed-loop system behaves like the nominal closed-loop system that is composed of the nominal model of robot manipulator and the feedback linearization-based tracking controller. A simplified implementation of the proposed controller is also introduced. Simulation results on a robot manipulator are given to validate the performance of the proposed controller.

Keywords: Disturbance observer, robot manipulators, robust control, stability of nonlinear systems.

1. INTRODUCTION

Nowadays, robot manipulators are widely used in various fields such as automotive industry, semiconductor industry, etc., and their major tasks include moving objects, assembling parts, and painting. Usually, they are composed of several joints and links to have enough degrees of freedom required to conduct the desired tasks, which yields complex structure, and thus it is inevitable to have system uncertainties. In addition, they are subject to external disturbance torques due to interaction with objects or environment. To achieve the desired level of performance, it is therefore essential to develop robust control algorithms taking plant uncertainties and external disturbances into account and a number of results have been reported in the literature, e.g., adaptive control [1,2], sliding mode control [3], passivity-based control [4], H_∞ control [5], and disturbance observer-based control [6]; see also [7–9] and references therein.

The disturbance observer-based control approach is of particular interest in this paper. Among a number of disturbance observers introduced in the literature (see, e.g., [10] for details), the one introduced by Ohnishi [11] has been successfully applied to control problems for robot manipulators such as independent joint control [12], sensorless control [13], force control [14]. It these works, a disturbance observer is designed for each joint to estimate the disturbance torque which consists of not only the external torques applied on the joint but also the interactive torques between the joints arising from robot dynamics. Thus, even if the robot dynamics is fully known, the interactive torques are treated as disturbances, possibly resulting in excessive control effort.

Several results on the estimation of external disturbance torques considering the robot dynamics have been reported relatively recently. In [15], a nonlinear disturbance observer is constructed in a way that an estimate of disturbance torque is firstly obtained assuming that the angular acceleration is known, and then it is implemented without using the angular acceleration signal. Although the stability as well as convergence has been proved rigorously, the result can be applied to only 2-DOF manipulators and the disturbance should be constant. Time-varying disturbances with known disturbance model have been considered later in [16] where SISO nonlinear systems are considered. The result of [15] has been extended to n-DOF manipulators subject to constant disturbances in [17].

We note that the disturbance observers for robot manipulators developed so far mainly focus on the disturbance itself rather than plant uncertainties. Although plant uncertainties are considered in some works, e.g., in [17], their effect is lumped into the external disturbance and the lumped disturbance is assumed to be constant or slowly time-varying, which means that the effect of relatively fast motion cannot be covered.

In this paper, we present a disturbance observer-based

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robust tracking controller for uncertain robot manipulators, and the structure of disturbance observer is motivated by the one from [11] rather than [15, 16]. It should be noted that the idea of [11] has been generalized to SISO nonlinear systems [18] and MIMO nonlinear plants [19]. Although the work [19] considers the plant uncertainties as well as external disturbances, it is not applicable to our case because the nominal model should be linear. Assuming that angular displacements and angular velocities are available for feedback, we introduce a novel structure of disturbance observer-based controller which estimates the lumped disturbance torque (the external disturbance torque combined with the effect of plant uncertainties) and makes the real closed-loop system behave similar to the nominal closed-loop system which is composed of the disturbance free nominal robot manipulator and a feedback linearization-based tracking controller. The controller design is constructive in the sense that given a prescribed level of steady-state tracking error, a straightforward procedure is provided to tune the controller parameters. A simplified implementation which requires only n integrators for n-DOF manipulators is also presented.

Notation: For a vector $x \in \mathbb{R}^n$, $\|x\|$ denotes the Euclidean norm. For two vectors, $x \in \mathbb{R}^n, y \in \mathbb{R}^m$, we define $[x; y] = [x^\top \ y^\top]^\top$. For two matrices A and B having the same number of columns, $[A; B]$ is defined similarly. Given a matrix M whose eigenvalues are all real numbers, $\lambda_{\max}(M)$ (resp., $\lambda_{\min}(M)$) represents the maximum (resp., minimum) eigenvalue of M. The matrix I_n denotes the identity matrix in $\mathbb{R}^{n\times n}$ and n can be dropped if no confusion arises.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider an n-DOF robot manipulator whose dynamics is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_d, \tag{1}$$

where q, $\dot{q}$, τ, $\tau_d \in \mathbb{R}^n$ denote the angular displacement vector, angular velocity vector, the vector of control torques applied to the joints, and the vector of disturbance torques, respectively, and $M(q) \in \mathbb{R}^{n\times n}$, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$, and $G(q) \in \mathbb{R}^n$ are inertia matrix, Coriolis and centrifugal force matrix, and gravity vector, respectively. $M(q)$, $C(q,\dot{q})$, and $G(q)$ are assumed to be unknown while their nominal counterparts $\bar{M}(q)$, $\bar{C}(q,\dot{q})$, and $\bar{G}(q)$, respectively, are known. For simplicity, let $N(q,\dot{q}) = C(q,\dot{q})\dot{q} + G(q)$ and $\bar{N}(q,\dot{q})$ is defined similarly. It is assumed that the angular displacement vector $q(t)$ and the angular velocity vector $\dot{q}(t)$ are available for feedback.

The systems parameters such as mass, length, inertia, which determine $M(q)$, $C(q,\dot{q})$, and $G(q)$ are assumed to be unknown but belong to known compact sets. For example, the mass of ith link, denoted by m_i, satisfies $0 < m_i^- \le m_i \le m_i^+$ with m_i^- and m_i^+ being known. It is assumed that the nominal value for each parameter belongs to the the same set as the corresponding uncertain parameter.

Assumption 1: The uncertain inertia matrix $M(q)$ is symmetric positive definite and there exist positive constants m^- and m^+ such that $m^- I_n \le M(q) \le m^+ I_n, \forall q \in \mathbb{R}^n$. The nominal inertia matrix $\bar{M}(q)$ has the same properties as $M(q)$. ◇

In this paper, we aim to design a robust tracking controller such that the angular displacement vector $q(t)$ approximately converges to the reference trajectory $q_r(t)$, i.e., for any given $\varepsilon > 0$, the controller can be designed so that

$$\limsup_{t\to\infty} \|q_r(t) - q(t)\| \le \varepsilon. \tag{2}$$

Assumption 2: The reference trajectory $q_r(t)$ is a smooth function such that $\mathsf{q}_r(t) := [q_r(t); \dot{q}_r(t); \ddot{q}_r(t); q_r^{(3)}(t)]$ satisfies $\|\mathsf{q}_r(t)\| \le \mathsf{q}_r^+$, $\forall t \ge 0$, for some $\mathsf{q}_r^+ > 0$. The disturbance torque vector τ_d and its time derivative are uniformly bounded, i.e., there exists $\mathsf{t}_d^+ > 0$ such that the vector $\mathsf{t}_d(t) := [\tau_d(t); \dot{\tau}_d(t)]$ satisfies $\|\mathsf{t}_d(t)\| \le \mathsf{t}_d^+$, $\forall t \ge 0$. ◇

Let $e = [e_1; e_2]$ be the tracking error defined by $[e_1(t); e_2(t)] = [q_r(t) - q(t); \dot{q}_r(t) - \dot{q}(t)]$ and consider the disturbance-free nominal system given by

$$\bar{M}(\bar{q})\ddot{\bar{q}} + \bar{N}(\bar{q},\dot{\bar{q}}) = \tau_r, \quad [\bar{q}(0); \dot{\bar{q}}(0)] = [q(0); \dot{q}(0)]. \tag{3}$$

The signal τ_r is the control input designed for the nominal system, for example, the computed torque control [20] given by

$$\tau_r = \bar{M}(\bar{q})\left(\ddot{q}_r + K_p(q_r - \bar{q}) + K_d(\dot{q}_r - \dot{\bar{q}})\right) + \bar{N}(\bar{q},\dot{\bar{q}}) \tag{4}$$

where K_d and K_p are symmetric positive definite matrices. Under the control (4), the nominal closed-loop system (3)-(4) becomes

$$(\ddot{q}_r - \ddot{\bar{q}}) + K_d(\dot{q}_r - \dot{\bar{q}}) + K_p(q_r - \bar{q}) = 0,$$

which implies that the tracking error converges to zero exponentially.

In this paper, the goal (2) is achieved by designing a controller which makes the closed-loop system behave like the nominal closed-loop system (3)-(4). This can be done by estimating the lumped disturbance d given by

$$\mathsf{d} = \tau_d - (M(q) - \bar{M}(q))\ddot{q} - (N(q,\dot{q}) - \bar{N}(q,\dot{q})), \tag{5}$$

and applying the control $\tau = -\hat{\mathsf{d}} + \tau_r$, where $\hat{\mathsf{d}}$ is an estimate of d. One can easily show that this control input with $\hat{\mathsf{d}} = \mathsf{d}$ (d is known completely) and τ_r given by (4), makes

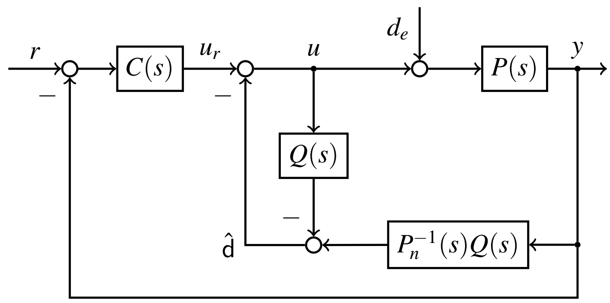

Fig. 1. Disturbance observer-based controller. $P(s)$: uncertain plant, $P_n(s)$: nominal plant, $Q(s)$: low pass filter, $C(s)$: outer-loop controller, r: reference input.

the system (1) become the nominal one (3) with $\bar{q} = q$, thus results in $\lim_{t\to\infty} \|q_r(t) - q(t)\| = 0$.

We close this section by introducing the disturbance observer by Ohnishi shown in Fig. 1. A rough description of the structure is given as follows. For an uncertain plant $P(s)$ with its nominal denoted by $P_n(s)$, the output y of $P(s)$ is fed to the filter $P_n^{-1}(s)Q(s)$ where $P_n^{-1}(s)$ is the inverse of $P_n(s)$ and $Q(s)$ is a low-pass filter with dc gain 1 such that $P_n^{-1}(s)Q(s)$ is implementable. With a properly chosen $Q(s)$, an estimate of the lumped disturbance, which is the external disturbance d_e combined with the effect of plant uncertainty, is given by $\hat{\mathsf{d}} = P_n^{-1}Q(s)y(s) - Q(s)u(s)$. Here the signal $P_n^{-1}(s)Q(s)y(s)$ is regarded as an estimate of $u + d_e$ (the actual input applied to $P(s)$), and $Q(s)u(s)$ an estimate of u.

3. ROBUST TRACKING CONTROLLER

In this section, we propose a robust tracking controller that adopts the idea of conventional disturbance observer shown in Fig. 1. We start by rewriting the actual system (1) using the lumped disturbance d defined in (5), namely,

$$\bar{M}(q)\ddot{q} + \bar{N}(q,\dot{q}) = \tau + \mathsf{d}. \tag{6}$$

If the signal $\ddot{q}$ were known, then $\tau + \mathsf{d}$ is completely known from (6) since q as well as $\dot{q}$ is measurable. Since the signal $\ddot{q}$ is not available in our case, it is replaced by $\dot{\zeta}$, which is generated from

$$\dot{\zeta} = -\frac{1}{\mu}\Gamma_\zeta(\zeta - \dot{q}), \quad \zeta(0) = \dot{q}(0), \tag{7}$$

where Γ_ζ is a symmetric positive definite matrix and $\mu > 0$ is a design parameter to be chosen later. We note that the transfer function from $\dot{q}$ to ζ is $Q_\zeta(s) = (\mu s I_n + \Gamma_\zeta)^{-1}\Gamma_\zeta$ which corresponds to $Q(s)$ (of $P_n^{-1}(s)Q(s)$) in Fig. 1. With $\dot{\zeta}$, an estimate of $\tau + \mathsf{d}$, which is denoted by $\hat{\tau}_p$ and corresponds to the signal $P_n^{-1}(s)Q(s)y(s)$ in Fig. 1, is constructed as follows.

$$\hat{\tau}_p = \bar{M}(q)\dot{\zeta} + \bar{N}(q,\dot{q}). \tag{8}$$

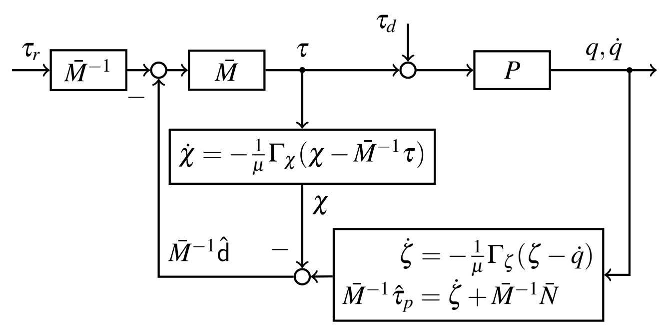

Fig. 2. Structure of proposed controller.

We now estimate the lumped disturbance as follows. If we follow the idea of disturbance observer, then $\hat{\mathsf{d}}$ would be chosen as $\hat{\mathsf{d}} = \hat{\tau}_p - Q(s)\tau$, where $Q(s)\tau$ instead of τ is used. (In fact, if we take $\hat{\mathsf{d}} = \hat{\tau}_p - \tau$ and compute the the control input τ as $\tau = -\hat{\mathsf{d}} + \tau_r$, then one has $0 \cdot \tau = -\hat{\tau}_p + \tau_r$, from which one cannot compute τ. This can be avoided by using $Q(s)\tau$.) The proposed controller, however, computes $\hat{\mathsf{d}}$ as

$$\hat{\mathsf{d}} = \hat{\tau}_p - \bar{M}(q)Q_\chi(s)(\bar{M}^{-1}(q)\tau), \tag{9}$$

where $Q_\chi(s) = (\mu s I_n + \Gamma_\chi)^{-1}\Gamma_\chi$ with Γ_χ being a symmetric positive definite matrix. We realize the filter $Q_\chi(s)$ as

$$\dot{\chi} = -\frac{1}{\mu}\Gamma_\chi\left(\chi - \bar{M}^{-1}(q)\tau\right), \quad \chi \in \mathbb{R}^n. \tag{10}$$

With χ, the signal $\hat{\mathsf{d}}$ is implemented as

$$\begin{aligned}\hat{\mathsf{d}} &= \bar{M}(q)\dot{\zeta} + \bar{N}(q,\dot{q}) - \bar{M}(q)\chi \\ &= \bar{M}(q)\left(-\frac{1}{\mu}\Gamma_\zeta(\zeta - \dot{q}) - \chi\right) + \bar{N}(q,\dot{q}).\end{aligned} \tag{11}$$

As can be seen from (11), the proposed estimate $\hat{\mathsf{d}}$ given in (9) contains the term $\frac{1}{\mu}\Gamma_\zeta\zeta + \chi$ which facilitates the stability analysis and enables us to simplify the controller structure. Finally, the control input is given by

$$\begin{aligned}\tau &= -\hat{\mathsf{d}} + \tau_r \\ &= -\bar{M}(q)\left(-\frac{1}{\mu}\Gamma_\zeta(\zeta - \dot{q}) - \chi\right) - \bar{N}(q,\dot{q}) + \tau_r.\end{aligned} \tag{12}$$

Note that τ_r is the control input designed for the nominal system and in this paper, the control (4), with $(\bar{q}, \dot{\bar{q}})$ being replaced by $(q, \dot{q})$, is employed. Fig. 2 describes the structure of proposed controller.

Now we are ready to analyze the stability of the closed-loop system under the controller (7), (10), and (12). Firstly, the closed-loop system is rewritten in the coordinates $(e, \xi, \chi) := (e, \frac{1}{\mu}(\zeta - \dot{q}), \chi)$ as follows:

$$\dot{e}_1 = e_2,$$

$$\dot{e}_2 = \ddot{q}_r - M^{-1}\left(\bar{M}(\Gamma_\zeta \xi + \chi) - N - \bar{N} + \tau_r + \tau_d\right),$$
$$\mu\dot{\xi} = -(I + M^{-1}\bar{M})\Gamma_\zeta \xi - M^{-1}(\bar{M}\chi - N - \bar{N} + \tau_r + \tau_d), \quad \xi(0) = 0,$$
$$\mu\dot{\chi} = \Gamma_\chi \Gamma_\zeta \xi + \Gamma_\chi \bar{M}^{-1}(\tau_r - \bar{N}). \tag{13}$$

To derive (13), we first note that using (12) the signal $\ddot{q}$ is written as

$$\ddot{q} = M^{-1}(-N + \tau + \tau_d) = M^{-1}(\bar{M}(\Gamma_\zeta \xi + \chi) - N - \bar{N} + \tau_r + \tau_d). \tag{14}$$

The dynamics of e_1 is trivially obtained and one can derive the dynamics of e_2, ξ, and χ using the relations (12) and (14).

Note that the dynamics (13) is in the standard singular perturbation form with μ the time separation parameter [21]. Thus, it is expected that for sufficiently small μ, the variables ξ and χ approach their quasi-steady-states ξ^* and χ^*, respectively, and these vectors depend on 'slow variables' such as e, q_r, $\dot{q}_r$, τ_r, and τ_d. We find the vector $[\xi^*;\chi^*]$ by computing the equilibrium point of the (ξ,χ)-dynamics with $\mu = 0$, and it turns out that

$$\xi^* = \Gamma_\zeta^{-1}\bar{M}^{-1}(\bar{N} - \tau_r),$$
$$\chi^* = \bar{M}^{-1}M\left(\bar{M}^{-1}(\tau_r - \bar{N}) + M^{-1}(N - \tau_d)\right). \tag{15}$$

Define $\tilde{\xi} = \xi - \xi^*$ and $\tilde{\chi} = \chi - \chi^*$. In $(e,\tilde{\xi},\tilde{\chi})$-coordinates, the dynamics (13) becomes

$$\dot{e} = A_e e + B_e[\tilde{\xi};\tilde{\chi}],$$
$$\mu\dot{\tilde{\xi}} = -(I + M^{-1}\bar{M})\Gamma_\zeta \tilde{\xi} - M^{-1}\bar{M}\tilde{\chi} - \mu\dot{\xi}^*,$$
$$\mu\dot{\tilde{\chi}} = \Gamma_\chi \Gamma_\zeta \tilde{\xi} - \mu\dot{\chi}^*, \tag{16}$$

where $\tilde{\xi}(0) = -\xi^*(0)$, and

$$A_e = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix}, \quad B_e = \begin{bmatrix} 0 & 0 \\ -M^{-1}\bar{M}\Gamma_\zeta & -M^{-1}\bar{M} \end{bmatrix}.$$

To proceed, we define sets $S_e(\delta_e)$, $S_\chi(\delta_\chi)$, and Ω_l, where the sets $S_e(\delta_e)$ and $S_\chi(\delta_\chi)$ stand for the sets of the initial tracking errors and initial values of χ, respectively, and Ω_l represents the set where the trajectory of the closed-loop system belongs to. For given constants $\delta_e > 0$ and $\delta_\chi > 0$, define $S_e(\delta_e) = \left\{e \in \mathbb{R}^{2n} | \|e\| \le \delta_e\right\}$ and $S_\chi(\delta_\chi) = \left\{\chi \in \mathbb{R}^n | \|\chi\| \le \delta_\chi\right\}$. Since $\bar{M}$, M, and N are continuous with respect to $(q,\dot{q})$ and the signals q_r, $\dot{q}_r$, and τ_d are uniformly bounded, it follows that there exists a constant $\delta_{\tilde{\chi}} > 0$ such that $\tilde{\chi}(0) \in S_{\tilde{\chi}}(\delta_{\tilde{\chi}}) := \{\tilde{\chi} \in \mathbb{R}^n | \|\tilde{\chi}\| \le \delta_{\tilde{\chi}}\}$ for any $e(0) \in S_e(\delta_e)$ and $\chi(0) \in S_\chi(\delta_\chi)$. Similarly, there exists $\delta_{\tilde{\xi}} > 0$ such that $\tilde{\xi}(0) = -\xi^*(0) \in S_{\tilde{\xi}}(\delta_{\tilde{\xi}}) := \{\tilde{\xi} \in \mathbb{R}^n | \|\tilde{\xi}\| \le \delta_{\tilde{\xi}}\}$.

Let P_e be the solution of $P_e A_e + A_e^\top P_e = -I$ and define a Lyapunov function candidate

$$V(e,\tilde{\xi},\tilde{\chi}) = e^\top P_e e + \frac{1}{2}\tilde{\xi}^\top \tilde{\xi} + \frac{1}{2}\alpha(\tilde{\chi} + \Gamma_\zeta \tilde{\xi})^\top (\tilde{\chi} + \Gamma_\zeta \tilde{\xi}). \tag{17}$$

The constant α in V is given by

$$\alpha > \frac{(\alpha_1 + \alpha_0\|\Gamma_\zeta\|)^2}{4\lambda_{\min}(\Gamma_\zeta)\alpha_2}, \tag{18}$$

where $\alpha_0 > 0$, $\alpha_1 \ge \max_{q\in\mathbb{R}^n} \lambda_{\max}(M^{-1}(q)\bar{M}(q))$, and $0 < \alpha_2 \le \min_{q\in\mathbb{R}^n} \lambda_{\min}(\Gamma_\zeta M^{-1}(q)\bar{M}(q))$. Given $l > 0$, we define a level set for V by

$$\Omega_l = \left\{(e,\tilde{\xi},\tilde{\chi}) | V(e,\tilde{\xi},\tilde{\chi}) \le l\right\}. \tag{19}$$

Theorem 1: Let $\delta_e > 0$ and $\delta_\chi > 0$ be given and suppose $e(0) \in S_e(\delta_e)$ and $\chi(0) \in S_\chi(\delta_\chi)$. Consider the Lyapunov function V given by (17), which is defined in terms of Γ_ζ and α, and let $l > 0$ be such that $S_e(\delta_e) \times S_{\tilde{\xi}}(\delta_{\tilde{\xi}}) \times S_{\tilde{\chi}}(\delta_{\tilde{\chi}}) \subset \Omega_l$. Then, for any given $\varepsilon > 0$, there exists $\mu^* > 0$ such that for any $0 < \mu < \mu^*$, the controller given by (7), (10), and (12) with $\|\Gamma_\zeta - \Gamma_\chi\| \le \alpha_0/\alpha$ ensures that $\left[e(t);\xi(t) - \xi^*(t);\chi(t) - \chi^*(t)\right] \in \Omega_l$ for all $t \ge 0$ and that $\limsup_{t\to\infty} \|q_r(t) - q(t)\| \le \varepsilon$.

Proof: We start from the representation (16) of the closed-loop system. Define $\psi = [\tilde{\xi};\tilde{\chi} + \Gamma_\zeta \tilde{\xi}]$ and $\psi^* = [\xi^*;\chi^* + \Gamma_\zeta \xi^*]$. Then, in (e,ψ)-coordinates, the system (16) becomes

$$\dot{e} = A_e e + B_{e\psi}\psi,$$
$$\mu\dot{\psi} = A_\psi \psi - \mu(\dot{\psi^*}), \tag{20}$$

where

$$A_\psi = \begin{bmatrix} -\Gamma_\zeta & -M^{-1}\bar{M} \\ -(\Gamma_\zeta - \Gamma_\chi)\Gamma_\zeta & -\Gamma_\zeta M^{-1}\bar{M} \end{bmatrix},$$
$$B_{e\psi} = \begin{bmatrix} 0 & 0 \\ 0 & -M^{-1}\bar{M} \end{bmatrix}.$$

Consider the Lyapunov function candidate V and level set Ω_l defined in (17) and (19), respectively. Let $P_\psi = \mathrm{diag}\{I, \alpha I\}$, i.e., the block diagonal matrix whose diagonal entries are I and αI. We show that with sufficiently small μ, the set Ω_l is forward invariant with respect to the dynamics (20). Suppose $(e,\psi) \in \Omega_l$ and compute

$$\begin{aligned}\dot{V} &= e^\top (P_e A_e + A_e^\top P_e)e + 2e^\top P_e B_{e\psi}\psi + \frac{1}{\mu}\psi^\top P_\psi A_\psi \psi + \psi^\top P_\psi(\dot{\psi^*}) \\ &\le -\|e\|^2 + \kappa_1\|e\|\|\psi\| + \frac{1}{\mu}\psi^\top P_\psi A_\psi \psi + \kappa_2\|\psi\|,\end{aligned}$$

where $\kappa_1 = \max_{q\in\mathbb{R}^n} \|2P_e B_{e\psi}\|$ and

$$\kappa_2 = \max_{(e,\psi)\in\Omega_l, \|\mathsf{q}_r\| \le \mathsf{q}_r^+, \|\mathsf{t}_d\| \le \mathsf{t}_d^+} \|P_\psi(\dot{\psi^*})\|.$$

The term $\psi^\top P_\psi A_\psi \psi$ can be bounded as follows:

$$\begin{aligned}\psi^\top P_\psi A_\psi \psi =& -\psi_1^\top \Gamma_\zeta \psi_1 \\ &-\psi_1^\top \left(M^{-1}\bar{M} + \alpha\Gamma_\zeta(\Gamma_\zeta - \Gamma_\chi)\right)\psi_2 \\ &-\alpha\psi_2^\top \Gamma_\zeta M^{-1}\bar{M}\psi_2 \\ \le& -\lambda_{\min}(\Gamma_\zeta)\|\psi_1\|^2 + \alpha_1\|\psi_1\|\|\psi_2\| \\ &+\alpha_0\|\Gamma_\zeta\|\|\psi_1\|\|\psi_2\| - \alpha\alpha_2\|\psi_2\|^2,\end{aligned}$$

where α_0, α_1, and α_2 are defined in (18) and the relation $\|\Gamma_\zeta - \Gamma_\chi\| \le \alpha_0/\alpha$ has been applied. Note that with α chosen as (18), one can easily see that there exists $\kappa_3 > 0$ such that $\psi^\top P_\psi A_\psi \psi \le -\kappa_3\|\psi\|^2$.

Based on the discussion so far and applying Young's inequality, one has

$$\dot{V} \le -\frac{1}{2}\|e\|^2 + \left(\frac{\kappa_1^2}{2} + \frac{1}{4\bar{\varepsilon}} - \frac{\kappa_3}{\mu}\right)\|\psi\|^2 + \bar{\varepsilon}\kappa_2^2,$$

where $\bar{\varepsilon}$ is an arbitrary positive constant. Take $\mu^* > 0$ such that

$$\frac{\kappa_1^2}{2} + \frac{1}{4\bar{\varepsilon}} - \frac{\kappa_3}{\mu^*} \le -\frac{\max\{1,\alpha\}}{4\lambda_{\max}(P_e)}.$$

Then, for any $0 < \mu < \mu^*$, it holds that $\dot{V} \le -\lambda V + \bar{\varepsilon}\kappa_2^2$ where $\lambda = \frac{1}{2\lambda_{\max}(P_e)}$. Thus, when $\bar{\varepsilon} \le \frac{\lambda l}{\kappa_2^2} =: \bar{\varepsilon}_1$, it holds that $\dot{V} \le -\lambda(V - l) \le 0$ on the boundary of Ω_l, which implies that Ω_l is forward invariant.

Since the initial condition of ζ is chosen so that $\tilde{\xi}(0) = -\xi^*$, it holds that $[e(0);\tilde{\xi}(0);\tilde{\chi}(0)]$ belongs to the set Ω_l, and the forward invariance of Ω_l results in that the trajectory $[e(t);\tilde{\xi}(t);\tilde{\chi}(t)]$ remains in Ω_l for all $t \ge 0$. Moreover, applying the comparison lemma yields $V(t) \le e^{-\lambda t}V(0) + \frac{\bar{\varepsilon}\kappa_2^2}{\lambda}$, from which it holds that $\limsup_{t\to\infty} V(t) \le \frac{\bar{\varepsilon}\kappa_2^2}{\lambda}$ and that $\limsup_{t\to\infty}\|e(t)\| \le \sqrt{\frac{\bar{\varepsilon}\kappa_2^2}{\lambda\lambda_{\min}(P_e)}}$. Therefore, if we take $\bar{\varepsilon} \le \min\{\bar{\varepsilon}_1, \bar{\varepsilon}_2\}$ where $\bar{\varepsilon}_2 = \lambda\lambda_{\min}(P_e)\frac{\varepsilon^2}{\kappa_2^2}$, the trajectory of the closed-loop system remains bounded and satisfies that $\limsup_{t\to\infty}\|q_d(t) - q(t)\| \le \varepsilon$, which completes the proof. □

When Γ_ζ and Γ_χ are chosen to be identical, i.e., $\Gamma_\zeta = \Gamma_\chi = \Gamma$, and symmetric positive definite, then the proposed controller given by (7), (10), and (12) can be implemented in a simplified manner with n integrators rather than $2n$ integrators. In fact, let

$$\xi = \chi + \frac{1}{\mu}\Gamma\zeta.$$

Then, one has

$$\begin{aligned}\dot{\xi} &= -\frac{1}{\mu}\Gamma\xi + \frac{1}{\mu}\Gamma\left(\bar{M}^{-1}(q)\tau + \frac{1}{\mu}\Gamma\dot{q}\right), \\ \xi(0) &= \frac{1}{\mu}\Gamma\dot{q}(0).\end{aligned} \quad (21)$$

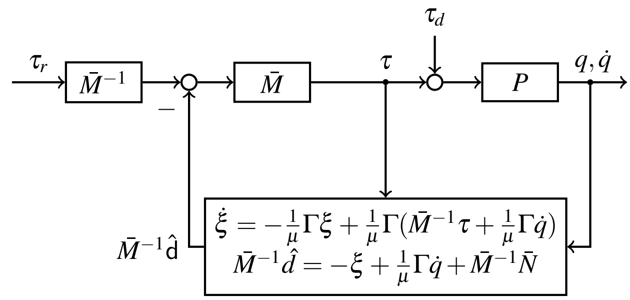

Fig. 3. Structure of proposed controller, simplified implementation.

With (21), the control input (12) becomes

$$\tau = \bar{M}(q)\left(\xi - \frac{1}{\mu}\Gamma\dot{q}\right) - \bar{N}(q,\dot{q}) + \tau_r. \quad (22)$$

Note that the new controller given by (21) and (22), whose structure is shown in Fig. 3, involves only n integrators which is the half of the controller (7), (10), and (12).

Although the new implementation has a simpler structure, it retains all benefits of the previous controller, which can be seen by the stability analysis given as follows. As before, the closed-loop system under the controller (21)-(22) is written in the coordinates $(e,\eta) := (e, \xi - \frac{1}{\mu}\Gamma\dot{q})$ as follows.

$$\begin{aligned}\dot{e}_1 &= e_2, \\ \dot{e}_2 &= -M^{-1}\bar{M}\eta + \ddot{q}_r - M^{-1}(\tau_r - \bar{N} - N + \tau_d), \\ \mu\dot{\eta} &= -\Gamma M^{-1}\bar{M}\eta + \Gamma\bar{M}^{-1}(\tau_r - \bar{N}) \\ &\quad - \Gamma M^{-1}(\tau_r - \bar{N} - N + \tau_d), \ \ \eta(0) = 0,\end{aligned} \quad (23)$$

from which the quasi-steady-state vector η^* can be computed as

$$\eta^* = (\bar{M}^{-1}M - I)\bar{M}^{-1}(\tau_r - \bar{N}) + \bar{M}^{-1}(N - \tau_d). \quad (24)$$

With $\tilde{\eta} = \eta - \eta^*$, the dynamics (23) becomes

$$\begin{aligned}\dot{e} &= A_e e + B_{e\tilde{\eta}}\tilde{\eta}, \\ \mu\dot{\tilde{\eta}} &= -\Gamma M^{-1}\bar{M}\tilde{\eta} - \mu(\dot{\eta^*}), \ \tilde{\eta}(0) = -\eta^*(0),\end{aligned}$$

where A_e is given in (16) and $B_{e\tilde{\eta}} = \left[0; -M^{-1}\bar{M}\right]$.

To proceed, we consider the set $S_e(\delta_e)$ ($\delta_e > 0$ is given) defined earlier. Noting that η^* is a function of e, q_r, $\dot{q}_r$, and τ_d, we can find $\delta_{\tilde{\eta}} > 0$ such that $\tilde{\eta}(0) = -\eta^*(0) \in S_{\tilde{\eta}}(\delta_{\tilde{\eta}}) := \{\tilde{\eta} \in \mathbb{R}^n | \|\tilde{\eta}\| \le \delta_{\tilde{\eta}}\}$. Let P_e be the solution of $P_eA_e + A_e^\top P_e = -I$. Define a Lyapunov candidate function V as

$$V(e,\tilde{\eta}) = e^\top P_e e + \frac{1}{2}\tilde{\eta}^\top\tilde{\eta}, \quad (25)$$

and a level set Ω_l for V defined similarly to (19) with $\tilde{\eta}$ instead of $(\tilde{\xi},\tilde{\chi})$ being used.

Theorem 2: Let $\delta_e > 0$ be given and suppose $e(0) \in S_e(\delta_e)$. Let $l > 0$ be such that $S_e(\delta_e) \times S_{\tilde{\eta}}(\delta_{\tilde{\eta}}) \subset \Omega_l$. Then, for any given $\varepsilon > 0$, there exists $\mu^* > 0$ such that for any $0 < \mu < \mu^*$, the controller given by (21) and (22) ensures that $\left[e(t); \xi(t) - \frac{1}{\mu}\Gamma\dot{q} - \eta^*(q,\dot{q},t)\right] \in \Omega_l$ for all $t \geq 0$ and that $\limsup_{t\to\infty} \|q_r(t) - q(t)\| \leq \varepsilon$.

Proof: Consider the Lyapunov function candidate V given by (25) and level set Ω_l. Following the arguments in the proof of Theorem 1, we have

$$\dot{V} \leq -\|e\|^2 + \kappa_1\|e\|\|\tilde{\eta}\| + \kappa_2\|\tilde{\eta}\| - \frac{1}{\mu}\kappa_3\|\tilde{\eta}\|^2,$$

where

$$\kappa_1 = \max_{q\in\mathbb{R}^n} \|2P_eB_{e\tilde{\eta}}\|, \quad \kappa_2 = \max_{(e,\tilde{\eta})\in\Omega_l, \|\mathrm{q}_r\|\leq \mathrm{q}_r^+, \|\mathrm{t}_d\|\leq \mathrm{t}_d^+} \|(\dot{\eta^*})\|,$$
$$\kappa_3 = \min_{q\in\mathbb{R}^n} \lambda_{\min}(\Gamma M^{-1}\bar{M}).$$

Take μ^* such that $\frac{\kappa_1^2}{2} + \frac{1}{4\bar{\varepsilon}} - \frac{\kappa_3}{\mu^*} \leq -\frac{1}{4\lambda_{\max}(P_e)}$. The rest of the proof is almost identical to that of Theorem 1, and thus omitted. □

Remark 1: The controllers, one given by (7), (10), and (12), and the other given by (21) and (22) involve some design parameters which can be chosen as follows. First of all, given the size of initial conditions, one determines the level set Ω_l by choosing l, during which one can use

$$\xi^* = -\Gamma_\zeta^{-1}(\ddot{q}_r + K_pe_1 + K_de_2),$$
$$\chi^* = \bar{M}^{-1}M(\ddot{q}_r + K_pe_1 + K_de_2) + \bar{M}^{-1}(N - \tau_d),$$
$$\eta^* = (\bar{M}^{-1}M - I)(\ddot{q}_r + K_pe_1 + K_de_2) - \bar{M}^{-1}(N - \tau_d), \tag{26}$$

which are obtained by substituting τ_r given by (4) (with $\bar{q}$ replaced by q) to (15) and (24). Given Ω_l, one can compute κ_1, κ_2, and κ_3 to choose μ^*, during which it is required to compute the bound of $(\dot{\psi^*})$ or $(\dot{\eta^*})$, which is rather involved. It is worth noting that the relations $\dot{\bar{M}} = \bar{C} + \bar{C}^\top$, $\dot{M} = C + C^\top$, and $\frac{d}{dt}(\bar{M}^{-1}) = -\bar{M}^{-1}\dot{\bar{M}}\bar{M}^{-1}$ simplify the computation; see Section 4. In practice, one can obtain the bound analytically considering the size of Ω_l and the bound of system parameters, or via repeated simulations. It is also noted that κ_1 and κ_2 can be replaced by larger constants, i.e., $\kappa_1' \geq \kappa_1$ and $\kappa_2' \geq \kappa_2$, respectively, while κ_3 by a smaller one $\kappa_3' \leq \kappa_3$, which is often useful to obtain μ^*.

4. APPLICATION TO 2-DOF MANIPULATOR

In this section, we apply the proposed trajectory tracking controller to a 2-DOF robot manipulator with revolute joints. It is noted that the matrices $M(q)$ and $C(q,\dot{q})$ and the vector $G(q)$ can be represented in the form given by (see, e.g., [20] for details)

$$M(q) = \begin{bmatrix} \alpha_1 + 2\alpha_2\cos q_2 & \alpha_3 + \alpha_2\cos q_2 \\ \alpha_3 + \alpha_2\cos q_2 & \alpha_3 \end{bmatrix},$$
$$C(q,\dot{q}) = \beta\sin q_2 \begin{bmatrix} -\dot{q}_2 & -\dot{q}_1 - \dot{q}_2 \\ \dot{q}_1 & 0 \end{bmatrix},$$
$$G(q) = \begin{bmatrix} \gamma_1\cos q_1 + \gamma_2\cos(q_1+q_2) \\ \gamma_2\cos(q_1+q_2) \end{bmatrix},$$

where q_1 and q_2 are the angular displacement of the first joint and the second joint, respectively.

Table 1. Simulation parameters.

parameter	min.	nominal	max.	actual
α_1	12.75	15	17.25	17
α_2	2.55	3	3.45	2.65
α_3	4.25	5	5.75	4.25
β	2.975	3.5	4.025	4
γ_1	5.95	7	8.05	6
γ_2	2.125	2.5	2.875	2.875

The numerical values for system parameters are summarized in Table 1. We assume that uncertainties in the system parameters lie within about 15% of the nominal values, e.g., $0.85\bar{\alpha}_1 \leq \alpha_1 \leq 1.15\bar{\alpha}_1$ where $\bar{\alpha}_1$ is the nominal value of α_1. The computed torque controller, designed for nominal system, are tuned so that the natural frequency and the damping ratio of the error dynamics are $\sqrt{2}$ rad/s and $1/\sqrt{2}$, respectively, i.e., $K_p = 2I_2$ and $K_d = 2I_2$. The reference trajectory and the disturbance are given by $q_r(t) = [a_{r1}\sin\omega_1 t; a_{r2} + a_{r3}\cos\omega_1 t]$, $\tau_d(t) = [a_{d1}\sin\omega_2 t + a_{d2}\sin\omega_3 t; a_{d3}\cos\omega_4 t]$ with $a_{r1} = \pi/6$ rad, $a_{r2} = \pi/12$ rad, $a_{r3} = \pi/6$ rad, $a_{d1} = 1.5$ Nm, $a_{d2} = 2$ Nm, $a_{d3} = 1.5$ Nm, $\omega_1 = \pi/5$ rad/s, $\omega_2 = \pi/20$ rad/s, $\omega_3 = \pi/2$ rad/s, and $\omega_4 = \pi$ rad/s. The initial condition of the manipulator is given by $q(0) = [\pi/36; \pi/4.5]$ and $\dot{q}(0) = [0;0]$.

The controller employed in this simulation is the one given by (21) and (22). Let $\varepsilon = 0.1$ and $\Gamma = 170$. In order to determine μ^*, we need to compute κ_1, κ_2, and κ_3. As mentioned in Remark 1, conservative bounds κ_i's replacing κ_i's can be used, especially for κ_2. Precisely, using the properties for $\dot{\bar{M}}$ and $\frac{d}{dt}(\bar{M}^{-1})$ from Remark 1, we have

$$\begin{aligned}(\dot{\eta^*}) =& \bar{M}^{-1}(\bar{C} + \bar{C}^\top)\bar{M}^{-1}M(\ddot{q}_r + K_pe_1 + K_de_2) \\ &+ \bar{M}^{-1}(C + C^\top)(\ddot{q}_r + K_pe_1 + K_de_2) \\ &+ (\bar{M}^{-1}M - I)(q_r^{(3)} + K_p\dot{e}_1 + K_d\dot{e}_2) \\ &+ \bar{M}^{-1}(\bar{C} + \bar{C}^\top)\bar{M}^{-1}(N - \tau_d) - \bar{M}^{-1}(\dot{N} - \dot{\tau}_d),\end{aligned}$$

whose bound can be obtained by taking an upper bound for each term separately. The bounds used in the design are $\kappa_1' = 3.1$, $\kappa_2' = 49.7$, $\kappa_3' = 232.8$. Taking $\bar{\varepsilon} = 5.0 \times 10^{-6}$, we have $\Gamma = \mathrm{diag}\{170, 170\}$ and $\mu^* = 0.001$.

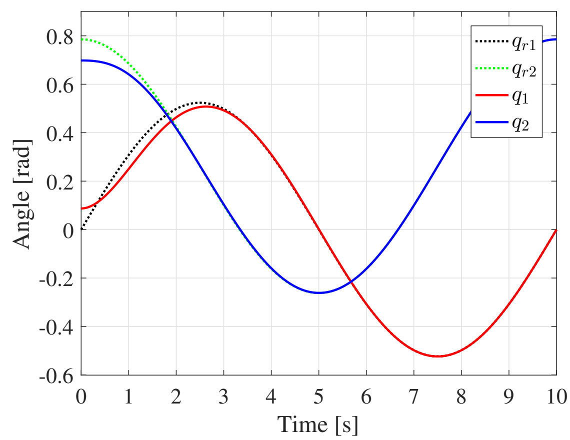

Fig. 4. Response of nominal closed-loop system.

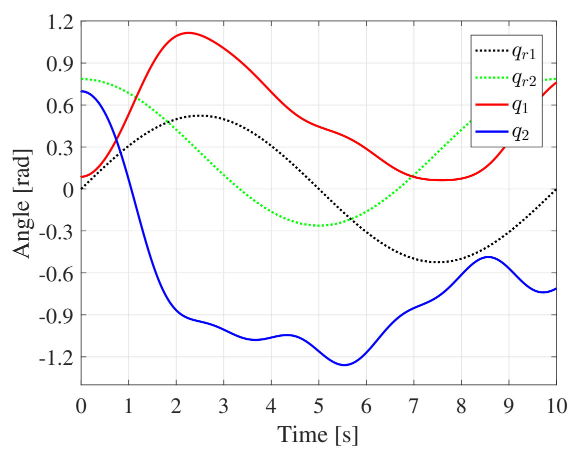

Fig. 5. Performance degradation under the computed torque controller (4) in the presence of disturbance and plant uncertainty.

In Fig. 4, the trajectory of $\bar{q}(t)$ of the nominal closed-loop system composed of (3) and (4) without disturbance are shown. Since there is no plant uncertainty and external disturbance, the computed torque control (4) works well and it is seen that $\bar{q}(t)$ tracks $q_r(t)$ with no steady-state error. Since the computed torque control is based on the feedback linearization theory, it is not robust against plant uncertainties and disturbances. As can be seen in Fig. 5, the tracking performance becomes poor when the computed torque control is applied to the real system which is subject to external disturbances and system uncertainties.

The performance of the closed-loop system under proposed controller is shown in Fig. 6 with $\mu = 0.001$ and one can see that the proposed controller almost recovers the performance of nominal closed-loop system shown in Fig. 4 despite the presence of plant uncertainties and external disturbances.

In addition, our controller is compared with that of [15].

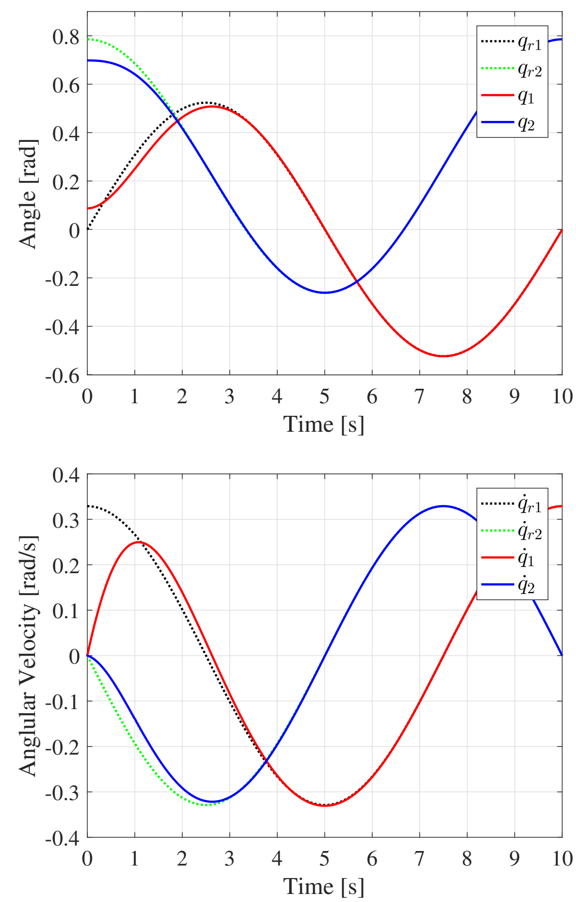

Fig. 6. Performance recovery under proposed controller.

We follow the design described in [15] and the gain of the disturbance observer is chosen as $c = 500$.

Fig. 7 shows the disturbance estimation error when there is no uncertainties. When constant disturbance $\tau_d = [5;3]$Nm is applied to the system, both controllers successfully estimate the disturbance, while proposed estimator yields smaller estimation error when sinusoidal disturbance is present. Fig. 8 shows the tracking error of two controllers when external disturbances and uncertainties are present, and one can see that the proposed controller results in much smaller steady-state error.

5. CONCLUSION

In this paper, we have presented a disturbance observer-based robust tracking controller for uncertain robot manipulators assuming that angular displacements as well as angular velocities are available for feedback. Through rigorous stability analysis, it is shown that proposed controller can successfully compensate the disturbance and the effect of plant uncertainties, and guarantee a desired bound of tracking error in the steady-state. The performance has been validated via numerical simulations on a

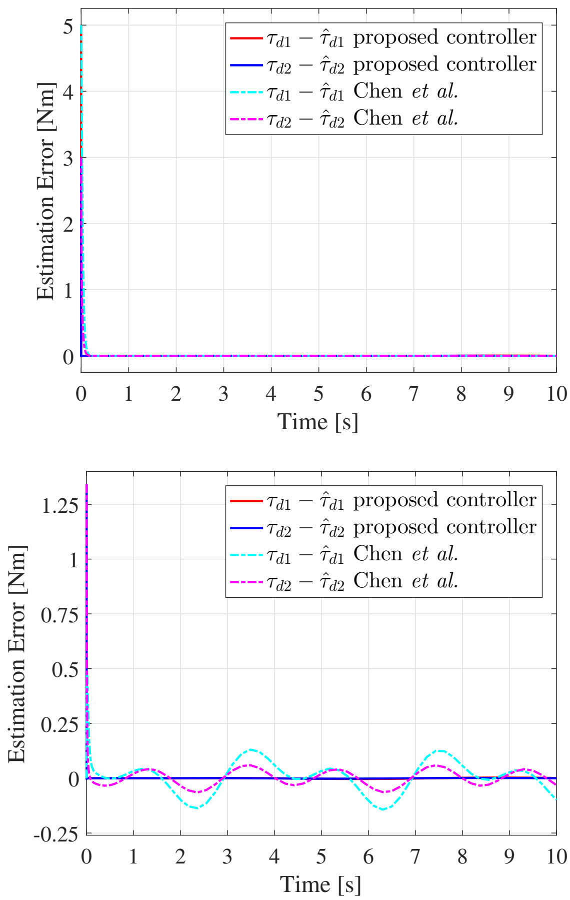

Fig. 7. Comparison of disturbance estimation performance: constant (top), sinusoid (bottom).

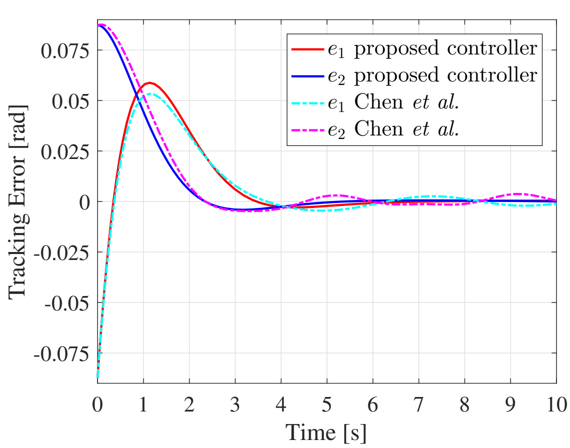

Fig. 8. Tracking error of closed-loop system with disturbance and uncertainties.

2-DOF manipulator. Extensions to more general nonlinear systems, and to the case with known disturbance models are future research topics.

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