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Adaptive Pinning Synchronization of Complex Networks with Negative Weights and Its Application in Traffic Road Network

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Abstract: As local traffic congestion and uncertainty factors existing on roads may lead to cascading failures or even large area traffic network congestion, a pinning control method is proposed to divert the traffic and then restore the smooth flow of traffic. To eliminate the impacts of uncertainties and negative weights for the traffic network performance, the adaptive pinning control and coupling adjustment strategies are designed to estimate controller parameters and adjust coupling strength to compensate for the impacts on the pinned nodes and unpinned nodes. Based on Lyapunov stability theory, adaptive pinning controllers and network adjusters are developed to guarantee the achievement of network synchronization even in the presence of the uncertainties and negative weights. In addition, we investigate the effects of the type of nodes on pinning synchronization performance. Numerical simulations show that if the network's degree and the single node energy index are considered, better synchronization performance can be obtained by comparing with the pervious pinning schemes.

Keywords: Adaptive pinning control, complex traffic road network, negative weights, synchronization.

1. INTRODUCTION

The studies of synchronization problem on real-world complex dynamical systems with nonlinearities have been focused over the last few years. Since a system will always be affected by some unexpect factors, such as time-delays [1], actuator faults and saturations [2–5], insensitive parameters [6, 7], many useful control techniques have been presented to deal with the factors and ensure the stability of the synchronous state of complex dynamical systems, including adaptive feedback control, backsteeping, linear matrix inequality (LMI), sliding mode, stochastic control, and so on (see e.g., [8–15]). Under normal conditions, complex networks need to be controlled to have a very large quantity of nodes, hence the complete control which constructs controllers for each node in the networks is expensive, and even can not be achieved. To reduce computational burden and economize equipment resource, the pinning control method which constructs controllers for only a small portion of nodes has been presented in the past few years [15–18].

It should be noted that the above results are obtained for the unweighted or positive weighted complex networks. However, some factors in the complex networks have a negative effect on the stability of the network [15]. For instance, the fault of resistances in the Chua's circuits may cause the damage of the whole circuitry [19]; the attenuating connection among ions of neuron networks may be unable to trigger the networks [20]; the heavy traffic flow in the traffic road networks may lead to road congestion, and even result in cascading failures.

On the other hand, the recent study of urban traffic road networks based on the complex network theory has made a lot of meaningful results [21–23]. However, these researches are mostly concerned about the static properties of the traffic network, for example, the degree distribution of traffic intersections and roads, average shortest paths, and so on [23, 24]. There are few researches on the dynamic characteristic, including the consideration of traffic flow, driving behaviors, road conditions, traffic incidents, weather conditions, especially their negative weight impacts on the urban traffic road networks.

Inspired by the above discussions, the problem of urban traffic road networks is investigated from a new perspective of complex networks with negative weights in this paper. First, we take the negative weight as the negative im-

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pect of vehicle jam on the urban traffic road networks. Based on that, a new urban traffic road networks model with negative weights is established by introducing dynamic characteristic approaches. Then, to combine the adaptive feedback control techniques with pinning control methods, the adaptive pinning control of urban road traffic networks with negative weights is studied based on the Lyapunov stability theory. Compared with the existing state-feedback control and output-feedback control, such as [25], the proposed control strategy can self-acting counterbalance the effect of nonlinearity and uncertainties of systems. At last, the efficient select pinned-nodes strategies for complex networks are proposed based on the network topology and the node energy index.

The rest of the paper is organized as follows. The model and the theory of complex traffic networks with negative weights are constructed in Section 2. The synchronization criterion is presented in Sections 3, which includes constructing adaptive pinning controllers. Numerical simulations are given to verify the effectiveness of the obtained results in Section 4. In Section 5, conclusions of this paper are given.

2. THE COMPLEX NETWORK MODEL WITH NEGATIVE WEIGHTS

2.1. A new urban traffic road network model

A new urban traffic road network model, which is not only focusing on static network topology characteristics, but also considering dynamic characteristics of the traffic road networks, is established in this subsection. In a traditional traffic road network, urban traffic intersections are considered as the network nodes and there is one edge between two nodes if there is a road between two traffic intersections. Different from traditional traffic road network model, every edge of the proposed model in this paper has positive or negative weight properties according to the dynamic information of every road. The dynamic information includes: road traffic flow, vehicle queues, traffic accidents, and etc.

In our model of traffic congestion, we introduce the negative weights as the new system parameters to discuss the effect of critical congestion for the whole traffic network besides 0 or 1. Let us suppose that a_{ij} represents the compositive weight of traffic flow density and the grade of the road between the ith and the jth traffic intersections. If there is the status of one section of the road with smooth traffic between the ith node and jth node, we set $a_{ij} = 1$. Contrarily, we set $a_{ij} = -1$, if there is the status of one section of the road with critical traffic congestion between i and j, which normally has a few features: 1) All of the cars on this section stop moving. 2) The congestion is becoming worse until "death". 3) Status of adjacent sections is changing from "+1" to "-1" as the features above appear. A bigger critical congestion is potentially going to happen.

Now, in order to determine the importance of intersections, we define a energy index [26] by (1). The initial energy of the intersection is derived from the traffic road network itself, which can be the design traffic capacity of the intersection.

Definition 1: The energy index [26] is defined by

$$EI_i = EI_{ii} + \sum_{j=1}^{d} c_{ij} EI_{jj}, \quad 1 \le i \le N, \tag{1}$$

where EI_{ii} is normal energy of the ith node by

$$EI_{ii} = \frac{|\bar{E}I_{ii}|}{\bar{E}I_{sum}}, \tag{2}$$

where $\bar{E}I_{ii}$ is the original energy of the ith node , $\bar{E}I_{sum}$ is the overall energy of the network, d is the degree of the node i, which represents the sum of all roads through the ith intersection, c_{ij} represents the coupling strength between i and j. Note that if the intersection provides a positive role to the traffic network, $\bar{E}I_{ii} > 0$, otherwise, $\bar{E}I_{ii} < 0$.

According to the formula (1), the energy index of the intersection includes not only traffic capacity of the intersection itself, but also the importance in the subnetwork. The larger the energy index EI_i of the intersection is, the greater the contribution of the intersection to the subnetwork. If the intersection with bigger energy index exists traffic accidents or congestion, the subnetwork may appear cascading failures or even a large area of traffic network congestion.

2.2. The theory of complex traffic network with negative weights

In 1955, two British famous scholars *Lighthill* and *Whitham*, presented the kinematic model of traffic flow, and set up a continuous motion equation [27]. Based on the study of this literature, we construct a complex traffic road network consisting of N nodes, traffic flow of the ith node at time t is updated by the following law:

$$\dot{x}_i = f(x_i) + \sum_{j=1}^{N} c_{ij} a_{ij} \Gamma x_j + g_i(x_i), i = 1, 2, \ldots, N, \tag{3}$$

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T \in R^n$ is the traffic flow vector of the ith node at time t; $f(x_i) \in R^n$ is a nonlinear function, which is the index of the traffic flow and is continuously differentiable. The constant matrix Γ is a positive-definite diagonal and an inner coupling; $g_i(x_i) \in R^m$ describes the uncertainness in traffic networks that may include traffic incidents, driving behavior, external road conditions, etc; c_{ij} represents the coupling strength between the traffic intersection i and j, C is a constant parameter, and $0 < c_{ij} \le C$; a_{ij} denotes the traffic road network topology. Here, $a_{ij} = 1$ represents there is a road between the intersection i and j, while $a_{ij}=0$ represents there is no road. It is worth noting that if the traffic capacity between intersection i and j is very heavy or there is a traffic accident in this section, the topology property is denoted as $a_{ij}=-1$. Moreover, it can be transformed at intervals according to the change of traffic information, such as the handling of the accident, the change of traffic flow. Here, we call L as the graph Laplacian and define it as

$$l_{ij}=\begin{cases} c_{ij}a_{ij} & \text{if} \quad j\neq i,\\ -\sum\limits_{k=1,k\neq i}^{N} c_{ik}a_{ik} & \text{if} \quad j=i. \end{cases} \tag{4}$$

Based on the characteristic of Laplacian matrix $L:=[l_{ij}]_{n\times n}$ defined in (4), we can determine that if there are many $a_{ij}=-1$, the Laplacian matrix L will have some positive eigenvalues [15, 28]. Here, based on the practical traffic cases, we assume any one case of traffic network congestion can be considered to belong into a congestion mode. Then, we denote the following congestion mode set:

$$\Delta_{L^d}=\{L^d(t): a_{ij}^d=-1,\ i=1,2,\ldots,N,\ j\neq i\},$$

where $d=1,2,\ldots,D$ is the congestion mode and D is the total mode.

To achieve the synchronization of (3), many adaptive pinning controllers can be designed to control a small fraction of the network nodes. Then (3) can be written as

$$\dot{x}_i=f(x_i)+\sum_{j=1}^{N} c_{ij}a_{ij}\Gamma x_j+g_i(x_i)+\delta_i u_i, \quad i=1,2,\ldots,N, \tag{5}$$

where $\delta_i=1$ when the ith node is pinned, otherwise $\delta_i=0$.

If s is a solution of node equation, $\dot{x}=f(x_i(t))$, we denote $e_i=x_i-s$. Based on the characteristic of Laplacian matrix $L:=[l_{ij}]_{n\times n}$ defined in (4), we have

$$\begin{aligned}\sum_{j=1}^{N} c_{ij}a_{ij}\Gamma x_j &= \sum_{j=1,j\neq i}^{N} c_{ij}a_{ij}\Gamma(x_j-x_i)\\ &= \sum_{j=1,j\neq i}^{N} c_{ij}a_{ij}\Gamma(e_j-e_i)=\sum_{j=1}^{N} c_{ij}a_{ij}\Gamma e_j.\end{aligned}$$

Hence, (3) can be described by

$$\begin{aligned}\dot{e}_i=&f(x_i,t)-f(s,t)+\sum_{j=1}^{N} c_{ij}a_{ij}\Gamma e_j+g_i(x_i)+\delta_i u_i,\\ &i=1,2,\ldots,N.\end{aligned} \tag{6}$$

Here we introduce two normal assumptions in complex network researches as follows:

Assumption 1: Suppose that $f_i(x)$ always satisfies a Lipshitz condition, namely, there is a positive definite matrix $P=\text{diag}\{p_1,p_2,\ldots,p_n\}$ make the following inequality hold true:

$$(x_1-x_2)^T P(f_i(x_1,t)-f_i(x_2,t))\leq q_i\|x_1-x_2\|^2, \tag{7}$$

where $q_i>0$, $x_1,x_2\in R^n$. $\underline{p}$ is a minimum value of p_1 to p_n, that is to say, $\underline{p}=\min\{p_1,p_2,\ldots,p_n\}$.

Assumption 2: Suppose that $g_i(x)$ and $s(t)$ are piecewise continuous functions, and there exist three positive constants h_1, h_2 and σ satisfying

$$\|g_i(x_i(t),t)\|\leq h_{1i}\|x_i(t)\|+h_{2i}, \quad \|s(t)\|\leq\sigma. \tag{8}$$

The objective here is designing a pinning controller and coupling adjustor to make the synchronization error converge to a small region with the effects of uncertainties and negative weights.

3. THE SYNCHRONIZATION CRITERION OF COMPLEX NETWORK WITH NEGATIVE WEIGHTS

To achieve the synchronization of (6), we design the following pinning controllers to eliminate the effects of uncertainties and negative weights for pinned nodes:

$$\begin{aligned}&u_i=-\hat{k}_{1i}e_i(t)-k_{2i}, \quad i=1,2,\ldots,N_{pin},\\ &u_i=0,\ \ i=N_{pin}+1,N_{pin}+2,\ldots,N,\end{aligned} \tag{9}$$

where N_{pin} is the number of pinned nodes, $\hat{k}_{1i}$ is the estimate of k_{1i} which is defined by

$$k_{1i}:=\max\{(q_i+\lambda\|P\|\|\Gamma\|)\underline{p}^{-1}\}, \tag{10}$$

where λ is the largest eigenvalue of matrix L_s, $L_s=\frac{L+L^T}{2}$ for any $a_{ij}\in\Delta_{L^d}$ when choosing $c_{ij}(t)$ as $c_{ij}(0)$, and L is defined in (4).

Here, $\hat{k}_{1i}$, $i=1,2,\ldots,N_{pin}$ is is defined as:

$$\frac{d\hat{k}_{1i}(t)}{dt}=\kappa_i\underline{p}\|e_i\|^2, \tag{11}$$

where κ_i, $i=1,2,\ldots,N_{pin}$ is a positive constant; k_{2i} is a control gain function to counterbalance the impact of the nonlinearity and uncertainties of networks, which is constructed by

$$k_{2i}(t)=\frac{\beta_i P e_i\hat{k}_{3i}(t)}{\|e_i^T P\|+\alpha_i}, \quad i=1,2,\ldots,N_{pin}, \tag{12}$$

where α_i and β_i are two suitable positive constants satisfying

$$\beta_i\|e_i^T P\|\geq\|e_i^T P\|+\alpha_i, \quad i=1,2,\ldots,N_{pin}, \tag{13}$$

and $\hat{k}_{3i}(t)\in R$ is defined as:

$$\frac{d\hat{k}_{3i}(t)}{dt}=\gamma_i\|e_i^T P\|, \quad i=1,2,\ldots,N_{pin}, \tag{14}$$

where γ_i is any positive constant.

For the sake of compensating for the impacts of uncertainties and negative weights for un-pinned nodes, the following adaptive law is designed to adjust the coupling strength $c_{ij}(t)$:

$$\frac{dc_{ij}(t)}{dt} = \mathrm{Proj}_{[\underline{c}_{ij},\bar{c}_{ij}]}\{L_{c_{ij}}(t)\}$$
$$\begin{cases} 0, & \text{if} \quad c_{ij}(t) = \underline{c}_{ij} \text{ and } L_{c_{ij}}(t) \le 0, \\ L_{c_{ij}}(t), & \text{otherwise}, \end{cases} \tag{15}$$

where $\underline{c}_{ij}$ is the lower bound of c_{ij}, $\bar{c}_{ij}$ is the upper bound of c_{ij}, and

$$L_{c_{ij}}(t) = -\hat{\eta}_{ij}|a_{ij}|\|e_i^T\|^2, \tag{16}$$

where $i = N_{pin}+1, N_{pin}+2, \ldots, N$, $\hat{\eta}_{ij}$ is the estimates of η_{ij} satisfying

$$\sum_{j=1,j\ne i}^{N} \eta_{ij}|a_{ij}| > \sum_{j=1,j\ne i}^{N} (q_i + \lambda_i\|P\|\|\Gamma\|), \tag{17}$$

with the updated adaptive law

$$\frac{d\hat{\eta}_{ij}(t)}{dt}(t) = r_{ij}c_{ij}|a_{ij}|\|e_i^T\|^2,$$
$$i = N_{pin}+1, N_{pin}+2, \ldots, N, \tag{18}$$

where $r_{ij} > 0$ is the weighting of $\hat{\eta}_{ij}$.

Remark 1: According to (15) and (16), we know that $\dot{c}_{ij}(t)$ is a decreasing function. Then, the initial value $c_{ij}(0)$ is the largest value of $c_{ij}(t)$ based on the adjustment law of (15).

Let

$$\tilde{k}_{1i} = \hat{k}_{1i} - k_{1i},$$
$$\tilde{k}_{3i} = \hat{k}_{3i} - k_{3i}, \tag{19}$$

where $i = 1, 2, \ldots, N_{pin}$.

Substituting (9) into (6) has the following error system:

$$\dot{e}_i = f(x_i,t) - f(s,t) + \sum_{j=1}^{N} c_{ij}a_{ij}\Gamma e_j + g_i(x_i)$$
$$- \delta_i(\hat{k}_{1i}e_i + k_{2i}), \quad i = 1, 2, \ldots, N. \tag{20}$$

Theorem 1: Under Assumptions 1 and 2, we choose controllers u_i, $i = 1, 2, \ldots, N_{pin}$ described in (9) with adaptive laws (11), (14), and if the control gain functions (10), (12) and coupling strength adjustment (15) with the condition (17) and adaptive law (18) hold, then the complex system signals are bounded and the synchronization error is bounded by ε where

$$\varepsilon \le \max(\varepsilon_1, \varepsilon_2), \tag{21}$$

where

$$\varepsilon_1 := \sum_{i=N_{pin}+1}^{N} \frac{(h_{1i}\sigma + h_{2i})\|P\|}{((N-1)\underline{c}_{ij} - 1)(q_i + \lambda\|P\|\|\Gamma\|)}, \tag{22}$$

and

$$\varepsilon_2 := \sum_{i=1}^{N_{pin}} \frac{\alpha_i}{(\beta_i - 1)\|P\|}. \tag{23}$$

Proof: Consider the following Lyapunov function:

$$V(t) = \sum_{i=1}^{N} e_i^T P e_i + \sum_{i=1}^{N_{pin}} \kappa_i^{-1}\tilde{k}_{1i}^2 + \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\ne i}^{N} c_{ij}^2$$
$$+ \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\ne i}^{N} r_{ij}^{-1}\tilde{\eta}_{ij}^2 + \sum_{i=1}^{N_{pin}} z_i^{-1}\tilde{k}_{3i}^2, \tag{24}$$

where $\tilde{\eta}_{ij} = \hat{\eta}_{ij} - \eta_{ij}$. Then, the derivative of $V(t)$ along the trajectories of Eq. (20) can be calculated as

$$\frac{dV(t)}{dt}$$
$$= \sum_{i=1}^{N} \dot{e}_i^T P e_i + \sum_{i=1}^{N} e_i^T P \dot{e}_i + \sum_{i=1}^{N_{pin}} 2\kappa_i^{-1}\tilde{k}_{1i}\dot{\hat{k}}_{1i}$$
$$+ \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\ne i}^{N} 2c_{ij}\dot{c}_{ij} + \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\ne i}^{N} 2r_{ij}^{-1}\tilde{\eta}_{ij}\dot{\hat{\eta}}_{ij}$$
$$+ \sum_{i=1}^{N_{pin}} 2z_i^{-1}\tilde{k}_{3i}\dot{\hat{k}}_{3i}$$
$$= 2\sum_{i=1}^{N} e_i^T P(f(x_i,t) - f(s,t)) + 2\sum_{i=1}^{N} e_i^T P \sum_{j=1}^{N} c_{ij}a_{ij}\Gamma e_j$$
$$+ 2\sum_{i=1}^{N} e_i^T P g_i(x_i,t) - 2\sum_{i=1}^{N_{pin}} e_i^T P \hat{k}_{1i}e_i - 2\sum_{i=1}^{N_{pin}} e_i^T P k_{2i}$$
$$+ \sum_{i=1}^{N_{pin}} 2\kappa_i^{-1}\tilde{k}_{1i}\dot{\hat{k}}_{1i} + \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\ne i}^{N} 2c_{ij}\dot{c}_{ij}$$
$$+ \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\ne i}^{N} 2r_{ij}^{-1}\tilde{\eta}_{ij}\dot{\hat{\eta}}_{ij} + \sum_{i=1}^{N_{pin}} 2z_i^{-1}\tilde{k}_{3i}\dot{\hat{k}}_{3i}. \tag{25}$$

Under the condition of Assumptions 1, 2 and $\sum_{j=1,j\ne i}^{N} c_{ij}a_{ij} > 0$, we can rewrite (25) as

$$\frac{dV(t)}{dt}$$
$$\le 2\sum_{i=1}^{N} q_i\|e_i^T\|^2 - 2\sum_{i=1}^{N_{pin}} e_i^T P \hat{k}_{1i}e_i$$
$$+ 2\sum_{i=1}^{N} \|e_i^T P\| \sum_{j=1}^{N} c_{ij}a_{ij}\|\Gamma\|\|e_j\| + \sum_{i=1}^{N_{pin}} 2\kappa_i^{-1}\tilde{k}_{1i}\dot{\hat{k}}_{1i}$$
$$- 2\sum_{i=1}^{N_{pin}} e_i^T P k_{2i} + 2\sum_{i=1}^{N} (h_{1i}\sigma + h_{2i})\|e_i^T P\|$$
$$+ \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\ne i}^{N} 2c_{ij}\dot{c}_{ij} + \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\ne i}^{N} 2r_{ij}^{-1}\tilde{\eta}_{ij}\dot{\hat{\eta}}_{ij}$$
$$+ \sum_{i=1}^{N_{pin}} 2z_i^{-1}\tilde{k}_{3i}\dot{\hat{k}}_{3i}$$
$$\le 2\sum_{i=1}^{N} q_i\|e_i^T\|^2 + 2\|P\|\|\Gamma\|\zeta^T L_s \zeta - 2\sum_{i=1}^{N_{pin}} e_i^T P \hat{k}_{1i}e_i$$
$$+ \sum_{i=1}^{N_{pin}} 2\kappa_i^{-1}\tilde{k}_{1i}\dot{\hat{k}}_{1i} - 2\sum_{i=1}^{N_{pin}} e_i^T P k_{2i} + 2\sum_{i=1}^{N} (h_{1i}\sigma + h_{2i})\|e_i^T P\|$$

$$+\sum_{i=1}^{N_{pin}} 2z_i^{-1}\tilde{k}_{3i}\dot{\tilde{k}}_{3i} + \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\neq i}^{N} 2c_{ij}\dot{c}_{ij} + \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\neq i}^{N} 2r_{ij}^{-1}\tilde{\eta}_{ij}\dot{\tilde{\eta}}_{ij}, \quad (26)$$

where $\zeta = (\|e_1\|,\ldots,\|e_N\|)^T$, $L_s = \frac{L+L^T}{2}$, $L=[l_{ij}]_{(N)\times(N)}$ defined in (4). Since L_s is a real symmetric matrix, then there exists an orthogonal matrix Q satisfying $L_s = Q^T\mathrm{diag}\{\lambda_1,\ldots,\lambda_N\}Q$, where $\lambda_1 \geq \cdots \geq \lambda_N$ are the eigenvalues of the matrix L_s. Thus, $\zeta^T L_s \zeta = \zeta^T Q^T \mathrm{diag}\{\lambda_1,\ldots,\lambda_N\}Q\zeta \leq \sum_{i=1}^{N}\lambda\|e_i\|^2$, where λ is the largest value of λ_i for any $a_{ij}\in\Delta_{L^d}$ when choosing $c_{ij}(t)$ as $c_{ij}(0)$.

From the definitions (10) and (19), we have

$$\begin{aligned}\frac{dV(t)}{dt} &\leq 2\sum_{i=1}^{N}(q_i+\lambda_i\|P\|\|\Gamma\|)\|e_i^T\|^2 - 2(\sum_{i=1}^{N_{pin}}\underline{p}\|e_i^T\|^2\tilde{k}_{1i}\\ &\quad+\sum_{i=1}^{N_{pin}}\underline{p}\|e_i^T\|^2k_{1i}) + \sum_{i=1}^{N_{pin}}2\kappa_i^{-1}\tilde{k}_{1i}\dot{\tilde{k}}_{1i} + \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\neq i}^{N}2c_{ij}\dot{c}_{ij}\\ &\quad+\sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\neq i}^{N}2r_{ij}^{-1}\tilde{\eta}_{ij}\dot{\tilde{\eta}}_{ij} + 2\sum_{i=1}^{N}(h_{1i}\sigma+h_{2i})\|e_i^TP\|\\ &\quad-2\sum_{i=1}^{N_{pin}}e_i^TPk_{2i} + 2\sum_{i=1}^{N_{pin}}\gamma_i^{-1}\tilde{k}_{3i}\dot{\tilde{k}}_{3i}. \end{aligned} \quad (27)$$

Then, by using the constructed control gain function k_{2i} in (12) with adaptive law (13), we have

$$\begin{aligned}\frac{dV(t)}{dt} &\leq 2\sum_{i=N_{pin}+1}^{N}(q_i+\lambda_i\|P\|\|\Gamma\|)\|e_i^T\|^2\\ &\quad-2\sum_{i=1}^{N_{pin}}\underline{p}\|e_i^T\|^2\tilde{k}_{1i} + \sum_{i=1}^{N_{pin}}2\kappa_i^{-1}\tilde{k}_{1i}\dot{\tilde{k}}_{1i}\\ &\quad+\sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\neq i}^{N}2c_{ij}\dot{c}_{ij} + \sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\neq i}^{N}2r_{ij}^{-1}\tilde{\eta}_{ij}\dot{\tilde{\eta}}_{ij}\\ &\quad+2\sum_{i=1}^{N}(h_{1i}\sigma+h_{2i})\|e_i^TP\| - 2\sum_{i=1}^{N_{pin}}\|e_i^TP\|\hat{k}_{3i}\\ &\quad+2\sum_{i=1}^{N_{pin}}\gamma_i^{-1}\tilde{k}_{3i}\dot{\tilde{k}}_{3i}. \end{aligned} \quad (28)$$

According to (11), (15), and (16), (28) can be rewritten as

$$\begin{aligned}\frac{dV(t)}{dt} &\leq 2\sum_{i=N_{pin}+1}^{N}(q_i+\lambda_i\|P\|\|\Gamma\|)\|e_i^T\|^2\\ &\quad-2\sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\neq i}^{N}\eta_{ij}c_{ij}|a_{ij}|\|e_i^T\|^2\\ &\quad+2\sum_{i=N_{pin}+1}^{N}(h_{1i}\sigma+h_{2i})\|e_i^TP\|\\ &\quad+2\sum_{i=1}^{N_{pin}}(h_{1i}\sigma+h_{2i})\|e_i^TP\|\\ &\quad-2\sum_{i=1}^{N_{pin}}\|e_i^TP\|\hat{k}_{3i} + 2\sum_{i=1}^{N_{pin}}\gamma_i^{-1}\tilde{k}_{3i}\dot{\tilde{k}}_{3i}. \end{aligned} \quad (29)$$

Since h_{1i}, h_{2i}, σ are unknown bounded constants, there always exist constants k_{3i} such that

$$k_{3i}\parallel e_i^TP\parallel \geq (h_{1i}\sigma+h_{2i})\parallel e_i^TP\parallel,\ \ i=1,2,\ldots,N_{pin}. \quad (30)$$

Next, based on the adaptive law (14), and by using (17) and (19), (29) can be rewritten as

$$\begin{aligned}\frac{dV(t)}{dt} &\leq 2\sum_{i=N_{pin}+1}^{N}(q_i+\lambda_i\|P\|\|\Gamma\|)\|e_i^T\|^2\\ &\quad-2\sum_{i=N_{pin}+1}^{N}\sum_{j=1,j\neq i}^{N}c_{ij}(q_i+\lambda_i\|P\|\|\Gamma\|)\|e_i^T\|^2\\ &\quad+2\sum_{i=N_{pin}+1}^{N}(h_{1i}\sigma+h_{2i})\|e_i^TP\|\\ &< 2\zeta^{\frac{T}{2}}\sum_{i=N_{pin}+1}^{N}((q_i+\lambda\|P\|\|\Gamma\|\\ &\quad-(N-1)\underline{c}_{ij}(q_i+\lambda\|P\|\|\Gamma\|))\|e_i^T\|\\ &\quad+(h_{1i}\sigma+h_{2i})\|P\|)\zeta^{\frac{1}{2}}. \end{aligned} \quad (31)$$

If

$$\|e_i^T\| > \frac{(h_{1i}\sigma+h_{2i})\|P\|}{((N-1)\underline{c}_{ij}-1)(q_i+\lambda\|P\|\|\Gamma\|)}, \quad (32)$$

then $\frac{dV(t)}{dt} < 0$.

On the other hand, according to (13), we can get

$$\|e_i^T\| \geq \frac{\alpha_i}{(\beta_i-1)\|P\|}. \quad (33)$$

By using (21)-(23), the synchronization error is bounded by ε. This ends the proof. □

Remark 2: Theorem 1 is indicated that the adjustment of coupling strength can bring positive effects of synchronization of networks. It is well known that the network topological structure and coupling strength afford the essential information of the synchronization of networks. Some studies (see e.g., [10, 11]) have demonstrated that synchronization of un-pinned notes can be achieved by adjusting coupling strength, which means it can indeed engender control effects for un-pinned notes.

Remark 3: There are two parts of control efficiency to deal with the congestion problem in this paper. The first one is the control inputs from adaptive controllers which are designed for the pinned nodes. The second one is the adjustments of coupling strength from adaptive adjustors which are designed for un-pinned nodes. According to the results of Theorem 1, the adjustments of coupling strength can also afford the control efficiency as controller provided, and the congestion problem of un-pinned nodes can be dealt with.

4. NUMERICAL SIMULATIONS

Previous research results show that urban traffic road networks have the characteristics of small-world networks [29] and the traffic flow fulfills the nonlinear behavior [21, 27]. Suppose that traffic flow fulfills Lorenz chaotic system, thus, the node dynamics can be described by

$$\begin{cases} \dot{x}_1(t) = 10[x_2(t) - x_1(t)], \\ \dot{x}_2(t) = 28x_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - \dfrac{8}{3}x_3(t). \end{cases} \tag{34}$$

Consider a WS small-world network with $N = 16, p = 0.2$. The topology matrix $L = (a_{ij})_{16\times 16}$ is composed by the elements with $a_{ij} = 0$, $a_{ij} = 1$, or $a_{ij} = -1$ which means the traffic flow is very heavy between i and j. The negative connection is assumed to appear randomly but satisfies the following condition $\sum_{j=l+1}^{N} a_{ij} > 0$. Then we given the following parameters and initial conditions for the simulation:

$$\begin{aligned} &\hat{k}_{1i}(0) = 5,\ \ \hat{k}_{3i}(0) = 0,\ \ \kappa_i = 1,\ \ \alpha_i = 0.05, \\ &\beta_i = 10,\ \ \gamma_i = 20, \\ &\qquad i = 1, 2, \ldots, 12, \\ &\hat{\eta}_{ij}(0) = 10,\ \ c_{ij}(0) = 20,\ \ \underline{c}_{ij} = 2, \\ &\qquad i = 13, 14, \ldots, 16, \\ &P = I_3,\ \ g(x_i) = \log_2(|x_i| + 2) + \sin(t), \\ &\qquad i = 1, 2, \ldots, 16, \end{aligned} \tag{35}$$

where I_3 denotes a 3-dimensional identity matrix.

Figs. 1 and 2 denote the orbits of $x_i(t)$ and $e_i(t)$ respectively, which exemplify the designed distribute adaptive controllers can guarantee network synchronization. Fig. 3 denotes the response curves of the estimate of $\hat{k}_{3i}(t)$. Figs. 4 and 5 show the orbits of network adjustment c_{ij} and the estimates of weighings η_{ij}. It is clear that all signals of the system can be ensured to converge into a small bounded region.

Finally, the average network synchronization time is investigated according to three kinds of pinning schemes: (a) the random pinning scheme (labeled RPS in Fig. 6), (b) the specifical pinning scheme according to the higher de-

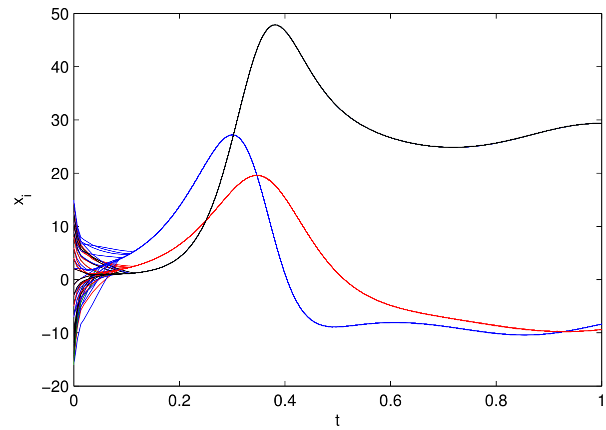

Fig. 1. The orbits of $x_i(t)$ (red, blue and black curves represent x_{i1}, x_{i2} and x_{i3}, $i = 1, 2, \ldots, N$, respectively).

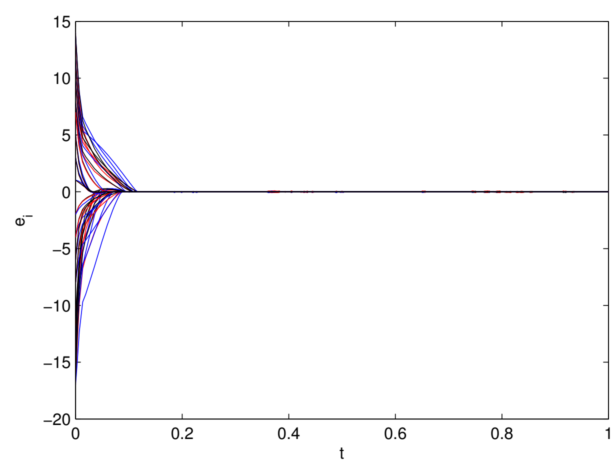

Fig. 2. The orbits of $e_i(t)$ (red, blue and black curves represent e_{i1}, e_{i2} and e_{i3}, $i = 1, 2, \ldots, N$, respectively).

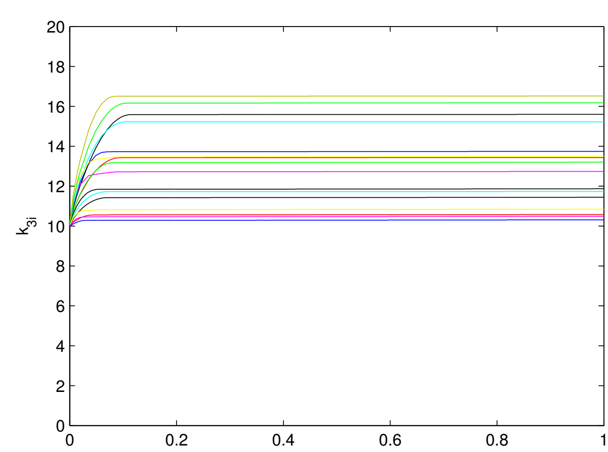

Fig. 3. The orbits of $k_{3i}(t)$, $i = 1, 2, \ldots, N_{pin}$.

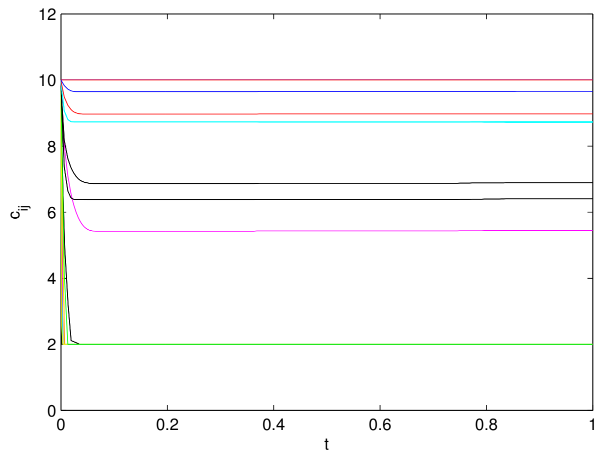

Fig. 4. The orbits of $c_{ij}(t)$, $i = N_{pin}+1,\ldots,N$.

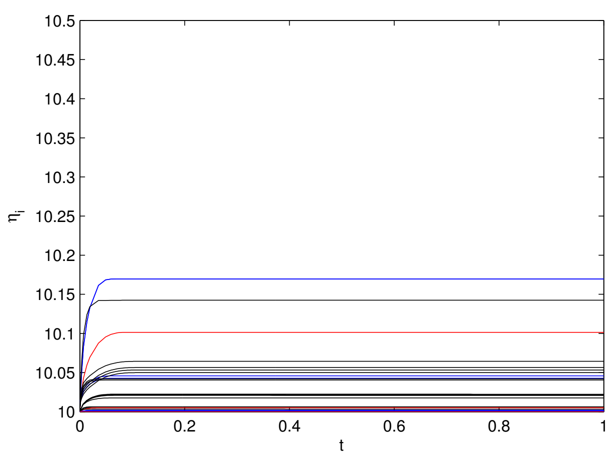

Fig. 5. The orbits of $\eta_{ij}(t)$, $i = N_{pin}+1,\ldots,N$.

gree (labeled SPS HD in Fig. 6), (c) the specifical pinning scheme according to the the higher energy index (labeled SPS HEI in Fig. 6). The parameters and initial conditions are the same ones previously defined, and there is only one pinned node, $N_{pin} = 1$. In Fig. 6, the abscissa indicates different types of networks, the coordinate "1" represents a random network, the coordinate "2" represents a small-world network, and the coordinate "3" represents a scale-free network. Fig. 6 shows that the pinning nodes are chosen based on the bigger energy index of the network, which can better realize the synchronization of the network. Hence the type of the pinned nodes plays a crucial role in constructing the controllers.

The simulation result of Fig. 6 reveals that the appropriate readjustment of the traffic capacity of the pivotal traffic intersection is beneficial for maintaining the complex traffic road network. For example, city planners can try to improve the traffic capacity of the pivotal traffic intersection to increase the the energy index of the node, and to reduce the average synchronization time during the rush

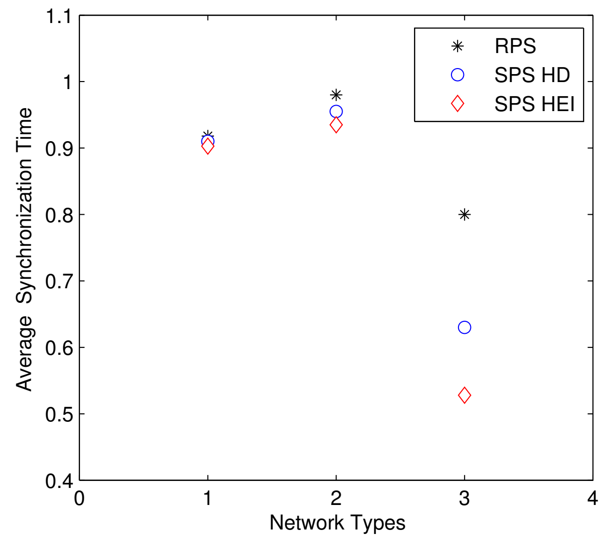

Fig. 6. The average network synchronization time for three kinds pinning schemes in three kinds of network, respectively, with $N = 10$, $N_{pin} = 1$.

hours by controlling several traffic intersections.

5. CONCLUSIONS

In this paper, an adaptive pinning control strategy has been investigated to handle the synchronization problem for complex traffic road networks with negative weights. The energy index has also been introduced to indicate the importance of the node. Moreover the relationship between pinning strategies and network synchronization performance has been studied. The results indicated that higher energy index nodes have greater impact on the network synchronization.

There are many issues which are worth our further studying. First of all, how many nodes should be pinned to obtain the network synchronization under scale-free networks with negative weights? In addition, how many negative links can result in whole network instability? Furthermore, the importance of the node's energy index in traffic road networks as well as the locations of the negative links are also worthy to be further investigated.

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