International Journal of Control, Automation and Systems 16(2) (2018) 630-639
http://dx.doi.org/10.1007/s12555-017-0001-x

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Least Squares based Iterative Parameter Estimation Algorithm for Stochastic Dynamical Systems with ARMA Noise Using the Model Equivalence

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Abstract: By means of the model equivalence theory, this paper proposes a model equivalence based least squares iterative algorithm for estimating the parameters of stochastic dynamical systems with ARMA noise. The proposed algorithm reduces the number of the unknown noise terms in the information vector and can give more accurate parameter estimates compared with the generalized extended least squares algorithm. The validity of the proposed method is evaluated through a numerical example.

Keywords: Dynamical system, iterative method, least squares, model equivalence, parameter estimation.

1. INTRODUCTION

The mathematical models are the basis of controller design [1, 2] and model reduction [3]. The parameter estimation of the models from observation data is the center of system identification [4, 5], filter design [6, 7] and signal processing [8]. Modeling a practical system is important by using mathematical equations. The mathematical equations that describe the behaviors and characteristics of a natural system or a man-made system are called the mathematical models, e.g., transfer function models [9, 10]. The mathematical models are the important tools of studying the motion laws of systems in theory [11, 12]. System identification is the theory and methods of establishing the mathematical models of systems.

Typical mathematical models include time series models such as autoregressive (AR) models, moving average (MA) models and autoregressive moving average (ARMA) models. When a system is disturbed by a stochastic noise, it is called a stochastic system and the disturbances include the AR process, the MA process and the ARMA process.

Stochastic systems can be divided simply into three categories: time series models, output-error type models and equation-error type models. These models closely reflect the characteristics of systems with relatively simple structures. The identification methods can be roughly classified into the iterative methods [13–17], the maximum likelihood methods [18–20], the recursive methods [21, 22] and so on. Among various types of equation-error models, the equation-error ARMA model is quite popular. Recently, Li proposed the parameter estimation algorithm for Hammerstein equation-error ARMA systems based on the Newton iteration [23].

On the basis of the previous work in [24], this paper derives new least squares based iterative identification algorithms for an equation-error ARMA system in terms of the model equivalence [25,26]. It is assumed that the structure of the system is known in advance, while the parameters of the system are unknown. First, we multiply both sides of the system model by a polynomial to obtain an equation-error moving average model, which can be identified by the least squares iterative method. Next, using the comparative coefficient way in [27], the parameter estimates of the original system can be computed. Then, according to the acquired parameter estimates, the remaining parameter estimates can be computed.

The remainder of this paper is organized as follows. Section 2 formulates the identification problem for equation-error ARMA systems. Section 3 gives the generalized extended least squares algorithm. Sections 4 and 5 propose the model equivalence based least squares iterative algorithm and the proposed algorithm is tested through a numerical example in Section 6. Finally, some

Manuscript received January 1, 2017; revised April 25, 2017 and June 15, 2017; accepted July 7, 2017. Recommended by Associate Editor Yongping Pan under the direction of Editor Duk-Sun Shim. This work was supported by the National Natural Science Foundation of China (Nos. 60474039, 61164015) and the Flexible Distinguished Top-Level Talent Plan of Jiangxi Province Talent Project 555.

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concluding remarks are given in Section 7.

2. THE SYSTEM DESCRIPTION AND THE IDENTIFICATION MODEL

Consider the stochastic system described by the equation-error ARMA model,

$$A(z)y(t)=B(z)u(t)+\frac{D(z)}{C(z)}v(t), \tag{1}$$

where $\{u(t)\}$ and $\{y(t)\}$ are the input and output sequences of the system, $\{v(t)\}$ is a white noise sequence with zero mean and variance σ^2, and the polynomials $A(z)$, $B(z)$, $C(z)$ and $D(z)$ in the unit backward shift operator z^{-1} are defined as

$$\begin{aligned}
A(z) &:= 1+a_1z^{-1}+a_2z^{-2}+\cdots+a_{n_a}z^{-n_a},\\
B(z) &:= b_1z^{-1}+b_2z^{-2}+\cdots+b_{n_b}z^{-n_b},\\
C(z) &:= 1+c_1z^{-1}+c_2z^{-2}+\cdots+c_{n_c}z^{-n_c},\\
D(z) &:= 1+d_1z^{-1}+d_2z^{-2}+\cdots+d_{n_d}z^{-n_d}.
\end{aligned}$$

It is assumed that the initial values are set to be $y(t)=0$, $u(t)=0$, $v(t)=0$ for $t\leqslant 0$. Let $n:=n_a+n_b+n_c+n_d$. The involved parameter vectors and the information vectors are defined as

$$\begin{aligned}
\theta &:= \begin{bmatrix}\theta_{\rm s}\\ \theta_{\rm n}\end{bmatrix}\in\mathbb{R}^n,\\
\theta_{\rm s} &:= [a_1,a_2,\cdots,a_{n_a},b_1,b_2,\cdots,b_{n_b}]^{\rm T}\in\mathbb{R}^{n_a+n_b},\\
\theta_{\rm n} &:= [c_1,c_2,\cdots,c_{n_c},d_1,d_2,\cdots,d_{n_d}]^{\rm T}\in\mathbb{R}^{n_c+n_d},\\
\varphi(t) &:= \begin{bmatrix}\varphi_{\rm s}(t)\\ \varphi_{\rm n}(t)\end{bmatrix}\in\mathbb{R}^n,\\
\varphi_{\rm s}(t) &:= [-y(t-1),-y(t-2),\cdots,-y(t-n_a),\\
&\qquad u(t-1),u(t-2),\cdots,u(t-n_b)]^{\rm T}\in\mathbb{R}^{n_a+n_b},\\
\varphi_{\rm n}(t) &:= [-w(t-1),-w(t-2),\cdots,-w(t-n_c),\\
&\qquad v(t-1),v(t-2),\cdots,v(t-n_d)]^{\rm T}\in\mathbb{R}^{n_c+n_d}.
\end{aligned}$$

Define the intermediate variable

$$w(t):=\frac{D(z)}{C(z)}v(t), \tag{2}$$

or

$$\begin{aligned}
w(t) =&[1-C(z)]w(t)+D(z)v(t)\\
=&(-c_1z^{-1}-c_2z^{-2}-\cdots-c_{n_c}z^{-n_c})w(t)\\
&+(1+d_1z^{-1}+d_2z^{-2}+\cdots+d_{n_d}z^{-n_d})v(t)\\
=&\varphi_{\rm n}^{\rm T}(t)\theta_{\rm n}+v(t).
\end{aligned}\tag{3}$$

Using (2) and (3), equation (1) can be written as

$$\begin{aligned}
y(t) =&[1-A(z)]y(t)+B(z)u(t)+w(t)\\
=&(-a_1z^{-1}-a_2z^{-2}-\cdots-a_{n_a}z^{-n_a})y(t)\\
&+(b_1z^{-1}+b_2z^{-2}+\cdots+b_{n_b}z^{-n_b})u(t)+w(t)\\
=&\varphi_{\rm s}^{\rm T}(t)\theta_{\rm s}+w(t) \qquad (4)\\
=&\varphi_{\rm s}^{\rm T}(t)\theta_{\rm s}+\varphi_{\rm n}^{\rm T}(t)\theta_{\rm n}+v(t)\\
=&\varphi^{\rm T}(t)\theta+v(t). \qquad (5)
\end{aligned}$$

This is the identification model for the equation-error ARMA system in (1). The objective of this paper is presenting new identification algorithms to identify the parameters a_i, b_i, c_i and d_i in the parameter vector θ from measured input-output data.

3. THE GENERALIZED EXTENDED LEAST SQUARES ALGORITHM

In order to derive the generalized extended least squares algorithm, it is necessary to define the stacked vector Y_t and the stacked matrix H_t as

$$Y_t:=\begin{bmatrix}y(1)\\ y(2)\\ \vdots\\ y(t)\end{bmatrix}\in\mathbb{R}^t,\ \ H_t:=\begin{bmatrix}\varphi^{\rm T}(1)\\ \varphi^{\rm T}(2)\\ \vdots\\ \varphi^{\rm T}(t)\end{bmatrix}\in\mathbb{R}^{t\times n}.$$

Introduce a quadratic cost function:

$$J(\theta):=\sum_{j=1}^{t}[y(j)-\varphi^{\rm T}(j)\theta]^2=(Y_t-H_t\theta)^{\rm T}(Y_t-H_t\theta).$$

Letting the partial derivative of $J(\theta)$ with regard to θ be zero, we obtain the least squares estimate of θ:

$$\hat{\theta}(t)=(H_t^{\rm T}H_t)^{-1}H_t^{\rm T}Y_t. \tag{6}$$

Define the covariance matrix $P(t)$ and the vector $\xi(t)$ as

$$P^{-1}(t):=H_t^{\rm T}H_t=P^{-1}(t-1)+\varphi^{\rm T}(t)\varphi(t), \tag{7}$$
$$\xi(t):=H_t^{\rm T}Y_t=\xi(t-1)+\varphi(t)y(t). \tag{8}$$

Then, equation (6) can be expressed as

$$\hat{\theta}(t)=P(t)\xi(t). \tag{9}$$

To compute the matrix inversion of $P(t)$, applying the matrix inversion lemma

$$(A+BC)^{-1}=A^{-1}-A^{-1}B(I+CA^{-1}B)^{-1}CA^{-1}$$

to (7) gives

$$P(t)=P(t-1)-\frac{P(t-1)\varphi(t)\varphi^{\rm T}(t)P(t-1)}{1+\varphi^{\rm T}(t)P(t-1)\varphi(t)}. \tag{10}$$

The algorithm in (8)–(10) is impossible to realize because the information vector $\varphi(t)$ on the right-hand sides of (8) and (10) contains the unknown intermediate variables $w(t-i)$ and the unmeasurable noise terms $v(t-i)$. Here, replacing the unknown $w(t-i)$ and $v(t-i)$ in $\varphi_{\rm n}(t)$ with

Table 1. The computation of the GELS algorithm.

Expressions	Number of multiplication	Number of addition
$\hat{\theta}(t)=P(t)\xi(t)$	n^2	$n(n-1)$
$\xi(t)=\xi(t-1)+\hat{\varphi}(t)y(t)$	n	n
$P(t)=P(t-1)-\frac{P(t-1)\hat{\varphi}(t)\hat{\varphi}^{\mathrm{T}}(t)P(t-1)}{1+\hat{\varphi}^{\mathrm{T}}(t)P(t-1)\hat{\varphi}(t)}$	$4n^2+2n$	$n^2+(n-1)(n+1)$
Sum	$5n^2+3n$	$3n^2-1$
The sum of flops	$N_1:=8n^2+3n-1$	

their corresponding estimates $\hat{w}(t-i)$ and $\hat{v}(t-i)$, the substituted information vector is denoted by

$$\begin{aligned}\hat{\varphi}_{\mathrm{n}}(t):=&[-\hat{w}(t-1),-\hat{w}(t-2),\cdots,-\hat{w}(t-n_c),\\&\hat{v}(t-1),\hat{v}(t-2),\cdots,\hat{v}(t-n_d)]^{\mathrm{T}}\in\mathbb{R}^{n_c+n_d}.\end{aligned}$$

Define

$$\hat{\varphi}(t):=\left[\begin{array}{c}\varphi_{\mathrm{s}}(t)\\ \hat{\varphi}_{\mathrm{n}}(t)\end{array}\right]\in\mathbb{R}^{n}.$$

Let $\hat{\theta}(t)=\left[\begin{array}{c}\hat{\theta}_{\mathrm{s}}(t)\\ \hat{\theta}_{\mathrm{n}}(t)\end{array}\right]$ be the estimate of $\theta=\left[\begin{array}{c}\theta_{\mathrm{s}}\\ \theta_{\mathrm{n}}\end{array}\right]$. From (4), we have $w(t)=y(t)-\varphi_{\mathrm{s}}^{\mathrm{T}}(t)\theta_{\mathrm{s}}$. Substituting $\hat{\theta}_{\mathrm{s}}(t)$ for θ_{s} obtains the estimate of $w(t)$: $\hat{w}(t)=y(t)-\varphi_{\mathrm{s}}^{\mathrm{T}}(t)\hat{\theta}_{\mathrm{s}}(t)$. From (5), we have $v(t)=y(t)-\varphi^{\mathrm{T}}(t)\theta(t)$. Replacing $\varphi(t)$ and θ with $\hat{\varphi}(t)$ and $\hat{\theta}(t)$, we acquire the estimate of $v(t)$: $\hat{v}(t)=y(t)-\hat{\varphi}^{\mathrm{T}}(t)\hat{\theta}(t)$.

Notice that $\hat{\varphi}(t)$ is known at time t, with all above preparations, replacing $\varphi(t)$ in (8)–(10) with $\hat{\varphi}(t)$, the related unknown variables are replaced with the corresponding estimates, we have the generalized extended least squares (GELS) algorithm of estimating the parameter vector θ in (5) as following:

$$\hat{\theta}(t)=P(t)\xi(t),\quad \xi(0)=\mathbf{0},\tag{11}$$
$$\xi(t)=\xi(t-1)+\hat{\varphi}(t)y(t),\quad P(0)=p_0I_n,\tag{12}$$
$$P(t)=P(t-1)-\frac{P(t-1)\hat{\varphi}(t)\hat{\varphi}^{\mathrm{T}}(t)P(t-1)}{1+\hat{\varphi}^{\mathrm{T}}(t)P(t-1)\hat{\varphi}(t)},\tag{13}$$
$$\hat{\varphi}(t)=\left[\begin{array}{c}\varphi_{\mathrm{s}}(t)\\ \hat{\varphi}_{\mathrm{n}}(t)\end{array}\right],\quad \hat{\theta}(t)=\left[\begin{array}{c}\hat{\theta}_{\mathrm{s}}(t)\\ \hat{\theta}_{\mathrm{n}}(t)\end{array}\right],\tag{14}$$
$$\begin{aligned}\varphi_{\mathrm{s}}(t)=&[-y(t-1),-y(t-2),\cdots,-y(t-n_a),\\&u(t-1),u(t-2),\cdots,u(t-n_b)]^{\mathrm{T}},\end{aligned}\tag{15}$$
$$\begin{aligned}\hat{\varphi}_{\mathrm{n}}(t)=&[-\hat{w}(t-1),-\hat{w}(t-2),\cdots,-\hat{w}(t-n_c),\\&\hat{v}(t-1),\hat{v}(t-2),\cdots,\hat{v}(t-n_d)]^{\mathrm{T}},\end{aligned}\tag{16}$$
$$\hat{w}(t)=y(t)-\varphi_{\mathrm{s}}^{\mathrm{T}}(t)\hat{\theta}_{\mathrm{s}}(t),\tag{17}$$
$$\hat{v}(t)=y(t)-\hat{\varphi}^{\mathrm{T}}(t)\hat{\theta}(t),\tag{18}$$
$$\hat{\theta}_{\mathrm{s}}=[\hat{a}_1(t),\cdots,\hat{a}_{n_a}(t),\hat{b}_1(t),\cdots,\hat{b}_{n_b}(t)]^{\mathrm{T}},\tag{19}$$
$$\hat{\theta}_{\mathrm{n}}=[\hat{c}_1(t),\cdots,\hat{c}_{n_c}(t),\hat{d}_1(t),\cdots,\hat{d}_{n_d}(t)]^{\mathrm{T}}.\tag{20}$$

The computation load of the GELS algorithm is shown in Table 1 ($n=n_a+n_b+n_c+n_d$).

4. THE MODEL EQUIVALENCE BASED LEAST SQUARES ITERATIVE ALGORITHM

The model equivalence based recursive least squares algorithms have been proposed for equation-error autoregressive systems [25] and for Box-Jenkins systems [26]. This paper derives a model equivalence based least squares iterative algorithm for equation-error ARMA systems in (1).

The basic idea is multiplying both sides of (1) by $C(z)$ to get a controlled autoregressive moving average (CARMA) model, and then we can identify the obtained CARMA model by using the least squares based iterative algorithm.

Multiplying both sides of (1) by $C(z)$ makes

$$A(z)C(z)y(t)=B(z)C(z)u(t)+D(z)v(t).\tag{21}$$

Let $n_p:=n_a+n_c$ and $n_q:=n_b+n_c$. Define

$$\begin{aligned}P(z):=&C(z)A(z)\\=&1+p_1z^{-1}+p_2z^{-2}+\cdots+p_{n_p}z^{-n_p},\end{aligned}\tag{22}$$
$$\begin{aligned}Q(z):=&C(z)B(z)\\=&q_1z^{-1}+q_2z^{-2}+\cdots+q_{n_q}z^{-n_q}.\end{aligned}\tag{23}$$

Inserting (22) and (23) into (21) yields

$$P(z)y(t)=Q(z)u(t)+D(z)v(t).\tag{24}$$

Equation (24) is a controlled autoregressive moving average model, which can be identified by the least squares based iterative algorithm. Define the parameter vector ϑ and the information vector $\phi(t)$ as

$$\begin{aligned}\vartheta:=&[p_1,\cdots,p_{n_p},q_1,\cdots,q_{n_q},d_1,\cdots,d_{n_d}]^{\mathrm{T}},\\ \phi(t):=&[-y(t-1),-y(t-2),\cdots,-y(t-n_p),\\&u(t-1),u(t-2),\cdots,u(t-n_q),\\&v(t-1),v(t-2),\cdots,v(t-n_d)]^{\mathrm{T}}.\end{aligned}$$

Equation (24) can be equivalently written as

$$y(t)=\phi^{\mathrm{T}}(t)\vartheta+v(t).\tag{25}$$

Consider a group of data with length L from $t=1$ to $t=L$, define the stacked output vector $Y(L)$ and the stacked information vector $\Phi(L)$ as

$$Y(L) := \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(L) \end{bmatrix} \in \mathbb{R}^{L},\ \Phi(L) := \begin{bmatrix} \phi^{\mathrm{T}}(1) \\ \phi^{\mathrm{T}}(2) \\ \vdots \\ \phi^{\mathrm{T}}(L) \end{bmatrix} \in \mathbb{R}^{L\times n}.$$

Define the quadratic cost function:

$$J(\vartheta) := \|Y(L) - \Phi(L)\vartheta\|^2.$$

Suppose that the information vector $\varphi(t)$ is persistently exciting, which means that $\Phi^{\mathrm{T}}(L)\Phi(L)$ is invertible. Minimizing $J(\vartheta)$ gives the least squares estimate of ϑ:

$$\hat{\vartheta}(L) = [\Phi^{\mathrm{T}}(L)\Phi(L)]^{-1}\Phi^{\mathrm{T}}(L)Y(L). \tag{26}$$

It is impossible to compute the least squares estimate $\hat{\vartheta}(L)$ by the above equation, since $\Phi(L)$ contains the unmeasurable noise terms $v(t-i)$. An effective solution is adopting the hierarchical identification principle: Let $k = 1,2,3,\cdots$ be an iterative variable, and $\hat{\vartheta}_k$ be the iterative estimate of ϑ. Replacing the unknown $v(t-i)$ in $\phi(t)$ with its estimate $\hat{v}_{k-1}(t-i)$. Define the estimate of $\phi(t)$:

$$\begin{aligned}\hat{\phi}_k(t) :=&[-y(t-1), -y(t-2), \cdots, -y(t-n_p),\\ &u(t-1), u(t-2), \cdots, u(t-n_q),\\ &\hat{v}_{k-1}(t-1), \hat{v}_{k-1}(t-2), \cdots, \hat{v}_{k-1}(t-n_d)]^{\mathrm{T}}.\end{aligned}$$

From (25), we have $v(t) = y(t) - \phi^{\mathrm{T}}(t)\vartheta$. Replacing the $\phi(t)$ and ϑ with $\hat{\phi}_k(t)$ and $\hat{\vartheta}_k$, the iterative estimate $\hat{v}_k(t)$ of $v(t)$ can be computed by using $\hat{v}_k(t) = y(t) - \hat{\phi}_k^{\mathrm{T}}(t)\hat{\vartheta}_k(t)$. Use $\hat{\phi}_k(t)$ to define

$$\hat{\Phi}_k(L) := \begin{bmatrix} \hat{\phi}_k^{\mathrm{T}}(1) \\ \hat{\phi}_k^{\mathrm{T}}(2) \\ \vdots \\ \hat{\phi}_k^{\mathrm{T}}(L) \end{bmatrix} \in \mathbb{R}^{p\times n}.$$

Replacing $\Phi(L)$ in (26) with $\hat{\Phi}_k(L)$, we can obtain the least squares based iterative algorithm for ϑ:

$$\hat{\vartheta}_k = [\hat{\Phi}_k^{\mathrm{T}}(L)\hat{\Phi}_k(L)]^{-1}\hat{\Phi}_k^{\mathrm{T}}(L)Y(L), \tag{27}$$

$$\hat{\Phi}_k(L) = [\hat{\phi}_k(1), \hat{\phi}_k(2), \cdots, \hat{\phi}_k(L)]^{\mathrm{T}}, \tag{28}$$

$$Y(L) = [y(1), y(2), \cdots, y(L)]^{\mathrm{T}}, \tag{29}$$

$$\begin{aligned}\hat{\phi}_k(t) = &[-y(t-1), -y(t-2), \cdots, -y(t-n_p),\\ &u(t-1), u(t-2), \cdots, u(t-n_q),\\ &\hat{v}_{k-1}(t-1), \cdots, \hat{v}_{k-1}(t-n_d)]^{\mathrm{T}},\end{aligned} \tag{30}$$

$$\hat{v}_k(t) = y(t) - \hat{\phi}_k^{\mathrm{T}}(t)\hat{\vartheta}_k,\ t = 1,2,\cdots,L. \tag{31}$$

From (27)–(31), we can compute the parameter estimate $\hat{\vartheta}_k$, i.e., the estimates of the parameters p_i, q_i and d_i.

Next, we compute the estimates $\hat{a}_i(t)$ and $\hat{b}_i(t)$ of the parameters a_i and b_i of the original system according to the obtained estimates $\hat{p}_i(t)$ and $\hat{q}_i(t)$ of the parameters p_i and q_i after model transformation.

5. THE COMPUTATION OF THE PARAMETER ESTIMATES

Once the estimates $\hat{p}_i(t)$ and $\hat{q}_i(t)$ of p_i and q_i are obtained, the parameter estimates $\hat{a}_i(t)$, $\hat{b}_i(t)$ and $\hat{c}_i(t)$ of the original system can be computed. Here uses the comparative coefficient approach in [25,27] to compute the parameter estimates of the original system.

Define the estimates of $A(z)$, $B(z)$ and $C(z)$ as

$$\begin{aligned}\hat{A}(t,z) &:= 1 + \hat{a}_1(t)z^{-1} + \hat{a}_2(t)z^{-2} + \cdots + \hat{a}_{n_a}(t)z^{-n_a},\\ \hat{B}(t,z) &:= \hat{b}_1(t)z^{-1} + \hat{b}_2(t)z^{-2} + \cdots + \hat{b}_{n_b}(t)z^{-n_b},\\ \hat{C}(t,z) &:= 1 + \hat{c}_1(t)z^{-1} + \hat{c}_2(t)z^{-2} + \cdots + \hat{c}_{n_c}(t)z^{-n_c}.\end{aligned}$$

From (22) and (23), we approximately use the relations:

$$\begin{aligned}\hat{P}(t,z) &= \hat{C}(t,z)\hat{A}(t,z)\\ &= 1 + \hat{p}_1(t)z^{-1} + \cdots + \hat{p}_{n_p}(t)z^{-n_p},\end{aligned} \tag{32}$$

$$\begin{aligned}\hat{Q}(t,z) &= \hat{C}(t,z)\hat{B}(t,z)\\ &= \hat{q}_1(t)z^{-1} + \cdots + \hat{q}_{n_q}(t)z^{-n_q}.\end{aligned} \tag{33}$$

Using the above assumptions, it follows that $\hat{B}(t,z)\hat{P}(t,z) = \hat{A}(t,z)\hat{Q}(t,z)$. Substituting the polynomials $\hat{B}(t,z)$, $\hat{P}(t,z)$, $\hat{A}(t,z)$ and $\hat{Q}(t,z)$ into it, we have

$$\begin{aligned}&[\hat{b}_1(t)z^{-1} + \cdots + \hat{b}_{n_b}(t)z^{-n_b}]\\ &\quad\times[1 + \hat{p}_1(t)z^{-1} + \cdots + \hat{p}_{n_p}(t)z^{-n_p}]\\ &= [1 + \hat{a}_1(t)z^{-1} + \cdots + \hat{a}_{n_a}(t)z^{-n_a}]\\ &\quad\times[\hat{q}_1(t)z^{-1} + \cdots + \hat{q}_{n_q}(t)z^{-n_q}].\end{aligned}$$

Expanding the above equation and comparing the coefficients of the same power of z^{-1} on both sides, we can set up $(n_b + n_p)$ equations:

$$\begin{aligned}z^{-1}:&\quad \hat{b}_1(t) = \hat{q}_1(t),\\ z^{-2}:&\quad \hat{b}_1(t)\hat{p}_1(t) + \hat{b}_2(t) = \hat{q}_1(t)\hat{a}_1(t) + \hat{q}_2(t),\\ z^{-3}:&\quad \hat{b}_1(t)\hat{p}_2(t) + \hat{b}_2(t)\hat{p}_1(t) + \hat{b}_3(t)\\ &\quad\quad = \hat{q}_1(t)\hat{a}_2(t) + \hat{q}_2(t)\hat{a}_1(t) + \hat{q}_3(t),\\ &\quad\vdots\\ z^{-(n_b+n_p)+1}:&\quad \hat{b}_{n_b-1}(t)\hat{p}_{n_p}(t) + \hat{b}_{n_b}(t)\hat{p}_{n_p-1}(t)\\ &\quad\quad = \hat{q}_{n_q-1}(t)\hat{a}_{n_a}(t) + \hat{q}_{n_q}(t)\hat{a}_{n_a-1}(t),\\ z^{-(n_b+n_p)}:&\quad \hat{b}_{n_b}(t)\hat{p}_{n_p}(t) = \hat{q}_{n_q}(t)\hat{a}_{n_a}(t),\end{aligned}$$

which can be written in a matrix form,

$$\begin{aligned}&S(t)\hat{\vartheta}_1(t) = B(t),\\ &S(t) := [S_p(t), -S_q(t)] \in \mathbb{R}^{(n_b+n_p)\times(n_a+n_b)},\\ &\hat{\vartheta}_1(t) := [\hat{b}_1(t), \cdots, \hat{b}_{n_b}(t), \hat{a}_1(t), \cdots, \hat{a}_{n_a}(t)]^{\mathrm{T}},\end{aligned}$$

Table 2. The computation of the ME-LSI algorithm.

Expressions	Number of multiplication	Number of addition
$\hat{\vartheta}_k=[\hat{\Phi}_k^{\mathrm{T}}(L)\hat{\Phi}_k(L)]^{-1}\hat{\Phi}_k^{\mathrm{T}}(L)Y(L)$	$(2L-1)n_1^2-n_1$	$2n_1^2L+n_1L$
$\hat{\vartheta}_1(t)=[S^{\mathrm{T}}(t)S(t)]^{-1}S^{\mathrm{T}}(t)B(t)$	$(2n_3-1)n_2^2-n_2$	$2n_2^2n_3+n_2n_3$
$\hat{c}(t)=[S_1^{\mathrm{T}}(t)S_1(t)]^{-1}S_1^{\mathrm{T}}(t)B_1(t)$	$(2n_p-1)n_c^2-n_c$	$2n_c^2n_p+n_cn_p$
Sum	$(2L-1)n_1^2+(2n_3-1)n_2^2+(2n_p-1)n_c^2$ $-(n_1+n_2+n_c)$	$2n_1^2L+2n_2^2n_3+2n_c^2n_p$ $+n_1L+n_2n_3+n_cn_p$
The sum of flops	$N_2:=(4L-1)n_1^2+(4n_3-1)n_2^2+(4n_p-1)n_c^2+n_1(L-1)+n_2(n_3-1)+n_c(n_p-1)$	

$$S_p(t):=\begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ \hat{p}_1(t) & 1 & \ddots & & \vdots \\ \hat{p}_2(t) & \hat{p}_1(t) & 1 & \ddots & \vdots \\ \vdots & \hat{p}_2(t) & \hat{p}_1(t) & \ddots & 0 \\ \hat{p}_{n_p-1}(t) & & \ddots & \ddots & 1 \\ \hat{p}_{n_p}(t) & \hat{p}_{n_p-1}(t) & & \ddots & \hat{p}_1(t) \\ 0 & \hat{p}_{n_p}(t) & \ddots & & \hat{p}_2(t) \\ \vdots & \ddots & \ddots & \hat{p}_{n_p-1}(t) & \vdots \\ \vdots & & \ddots & \hat{p}_{n_p}(t) & \hat{p}_{n_p-1}(t) \\ 0 & \cdots & \cdots & 0 & \hat{p}_{n_p}(t) \end{bmatrix},$$

$$B(t):=\begin{bmatrix} \hat{q}_1(t) \\ \hat{q}_2(t) \\ \vdots \\ \hat{q}_{n_q}(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\in\mathbb{R}^{n_b+n_p},$$

$$S_q(t):=\begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ \hat{q}_1(t) & 0 & \ddots & & \vdots \\ \hat{q}_2(t) & \hat{q}_1(t) & 0 & \ddots & \vdots \\ \vdots & \hat{q}_2(t) & \hat{q}_1(t) & \ddots & 0 \\ \hat{q}_{n_q-1}(t) & & \ddots & \ddots & 0 \\ \hat{q}_{n_q}(t) & \hat{q}_{n_q-1}(t) & & \ddots & \hat{q}_1(t) \\ 0 & \hat{q}_{n_q}(t) & \ddots & & \hat{q}_2(t) \\ \vdots & \ddots & \ddots & \hat{q}_{n_q-1}(t) & \vdots \\ \vdots & & \ddots & \hat{q}_{n_q}(t) & \hat{q}_{n_q-1}(t) \\ 0 & \cdots & \cdots & 0 & \hat{q}_{n_q}(t) \end{bmatrix},$$

where the dimensions of $S_p(t)$ and $S_q(t)$ are $(n_b+n_p)\times n_b$ and $(n_b+n_p)\times n_a$. It is easy to know that

$$\hat{\vartheta}_1(t)=[S^{\mathrm{T}}(t)S(t)]^{-1}S^{\mathrm{T}}(t)B(t). \tag{34}$$

From (34), we can get the estimates $\hat{a}_i(t)$ and $\hat{b}_i(t)$ of a_i and b_i. According to the definition of $\hat{P}(t,z)$ in (32), we have

$$\begin{aligned}&[1+\hat{c}_1(t)z^{-1}+\cdots+\hat{c}_{n_c}(t)z^{-n_c}]\\ &\quad\times[1+\hat{a}_1(t)z^{-1}+\cdots+\hat{a}_{n_a}(t)z^{-n_a}]\\ &=[1+\hat{p}_1(t)z^{-1}+\cdots+\hat{p}_{n_p}(t)z^{-n_p}].\end{aligned}$$

Similarly, expanding this equation and comparing the coefficients on both sides of it give the matrix equation,

$$S_1(t)\hat{c}(t)=B_1(t), \tag{35}$$

$$\hat{c}(t):=[\hat{c}_1(t),\hat{c}_2(t),\cdots,\hat{c}_{n_c}(t)]^{\mathrm{T}}\in\mathbb{R}^{n_c},$$

$$S_1(t):=\begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ \hat{a}_1(t) & 1 & \ddots & & \vdots \\ \hat{a}_2(t) & \hat{a}_1(t) & 1 & \ddots & \vdots \\ \vdots & \hat{a}_2(t) & \hat{a}_1(t) & \ddots & 0 \\ \hat{a}_{n_a-1}(t) & & \ddots & \ddots & 1 \\ \hat{a}_{n_a}(t) & \hat{a}_{n_a-1}(t) & & \ddots & \hat{a}_1(t) \\ 0 & \hat{a}_{n_a}(t) & \ddots & & \hat{a}_2(t) \\ \vdots & \ddots & \ddots & \hat{a}_{n_a-1}(t) & \vdots \\ \vdots & & \ddots & \hat{a}_{n_a}(t) & \hat{a}_{n_a-1}(t) \\ 0 & \cdots & \cdots & 0 & \hat{a}_{n_a}(t) \end{bmatrix},$$

$$B_1(t):=\begin{bmatrix} \hat{p}_1(t)-\hat{a}_1(t) \\ \hat{p}_2(t)-\hat{a}_2(t) \\ \vdots \\ \hat{p}_{n_a}(t)-\hat{a}_{n_a}(t) \\ \hat{p}_{n_a+1}(t) \\ \hat{p}_{n_a+2}(t) \\ \vdots \\ \hat{p}_{n_p}(t) \end{bmatrix}\in\mathbb{R}^{n_p}.$$

where the dimension of $S_1(t)$ is $n_p\times n_c$. Then we obtain

$$\hat{c}(t)=[S_1^{\mathrm{T}}(t)S_1(t)]^{-1}S_1^{\mathrm{T}}(t)B_1(t). \tag{36}$$

According to (36), we can compute the estimate $\hat{c}_i(t)$ of c_i from the parameter estimates $\hat{a}_i(t)$ and $\hat{p}_i(t)$.

The computation load of the model equivalence based least squares iterative algorithm is shown in Table 2 where $n_1:=n_p+n_q+n_d$, $n_2:=n_a+n_b$ and $n_3:=n_b+n_p$.

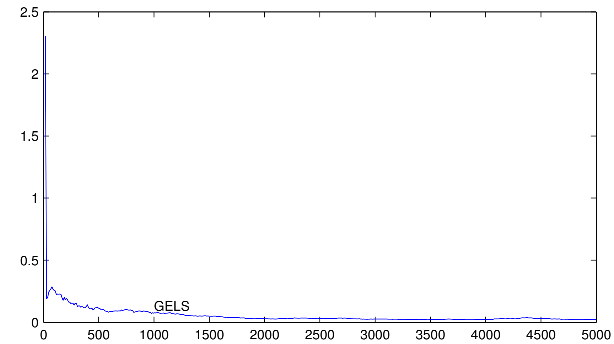

Fig. 1. The GELS estimation errors ($\sigma^2 = 0.50^2$).

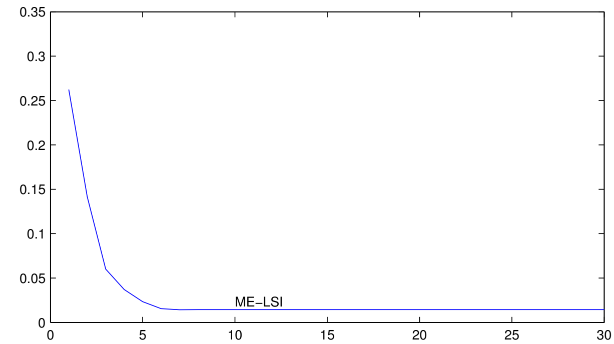

Fig. 2. The ME-LSI estimation errors ($\sigma^2 = 0.50^2$).

The model equivalence based least squares iterative (ME-LSI) algorithm increases computational complexity compared with the GELS algorithm. But the information vector of the ME-LSI contains fewer noise terms to be estimated, the estimation accuracy becomes higher.

6. EXAMPLE

Consider the following equation-error ARMA system,

$$\begin{aligned}
&A(z)y(t) = B(z)u(t) + \frac{D(z)}{C(z)}v(t),\\
&A(z) = 1 + a_1z^{-1} + a_2z^{-2} = 1 - 1.40z^{-1} + 0.68z^{-2},\\
&B(z) = b_1z^{-1} + b_2z^{-2} = 0.38z^{-1} + 0.42z^{-2},\\
&C(z) = 1 + c_1z^{-1} + c_2z^{-2} = 1 - 0.50z^{-1} + 0.34z^{-2},\\
&D(z) = 1 + d_1z^{-1} + d_2z^{-2} = 1 + 0.48z^{-1} - 0.36z^{-2},\\
&\boldsymbol{\theta} = [a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2]^{\mathrm{T}}.
\end{aligned}$$

In simulation, the input $\{u(t)\}$ is taken as a persistent excitation signal sequence with zero mean and unit variance, $\{v(t)\}$ as an uncorrelated noise sequence with zero mean. Apply the GELS algorithm and the ME-LSI algorithm to identify this equation-error ARMA system, the parameter estimates and errors are shown in Tables 3–4 and Tables 5–6 with $\sigma^2 = 1.00^2$ and $\sigma^2 = 0.50^2$. The estimation errors of the GELS algorithm versus t is shown in Fig. 1. The estimation errors of the ME-LSI algorithm versus the iteration k is shown in Fig. 2.

From Tables 3–6 and Figs. 1–2, we can see that for the parameter estimates given by using the GELS algorithm approach the true values as the data length t increases. The larger iteration k, the smaller the estimation errors of the ME-LSI algorithm. The algorithm has the higher estimation accuracy when the noise variance is small. Compared with the GELS algorithm, the parameter estimates of the ME-LSI algorithm can converge faster to their true values.

7. CONCLUSIONS

By means of the model equivalence transformation, this paper proposes an ME-LSI algorithm for identifying the parameters of equation-error ARMA systems. The proposed algorithm can enhance the parameter estimation accuracy and the simulation results verify the performance of the proposed identification algorithms. Although the method in this paper is presented for single-input single-output systems, the methods can be extended to study parameter identification problems of state space systems [28], multivariable systems and nonlinear systems with colored noises [29–31] and applied to other fields [32–45].

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Table 3. The GELS estimates and errors ($\sigma^2 = 1.00^2$).

t	a_1	a_2	b_1	b_2	c_1	c_2	d_1	d_2	$\delta(\%)$
100	-1.48748	0.84567	0.56323	-0.63220	-0.38582	0.44794	0.84312	0.27068	45.75716
200	-1.51203	0.84226	0.61038	-0.53723	-0.37925	0.40937	0.59241	0.08357	34.28058
500	-1.49277	0.80704	0.67098	-0.43876	-0.42203	0.36778	0.63058	0.05704	30.40011
1000	-1.47259	0.77374	0.68758	-0.44801	-0.47994	0.33777	0.60291	-0.06800	23.84688
2000	-1.45829	0.75915	0.67073	-0.40721	-0.54479	0.29897	0.57813	-0.14600	18.44572
3000	-1.45333	0.75202	0.68126	-0.39460	-0.55080	0.28931	0.60307	-0.15063	18.28997
4000	-1.44823	0.74645	0.67968	-0.37443	-0.58296	0.28029	0.59231	-0.19049	15.74600
5000	-1.43871	0.73742	0.67411	-0.37099	-0.59875	0.25830	0.59722	-0.21935	14.65639
True values	-1.40000	0.68000	0.38000	0.42000	-0.50000	0.34000	0.48000	-0.36000	

Table 4. The ME-LSI estimates and errors ($\sigma^2 = 1.00^2$).

k	a_1	a_2	b_1	b_2	c_1	c_2	d_1	d_2	$\delta(\%)$
1	-1.44599	0.72180	1.59922	-0.32968	-0.93702	0.63330	0.06437	0.03604	29.47235
2	-1.41210	0.67247	1.62125	-0.27618	-1.00682	0.45767	0.16686	-0.42576	19.07080
3	-1.38188	0.63326	1.61197	-0.22490	-0.98847	0.40495	0.32636	-0.53585	12.48922
4	-1.37925	0.63250	1.59279	-0.22664	-0.92859	0.41865	0.41615	-0.44800	8.28276
5	-1.41024	0.66595	1.59297	-0.28356	-0.87123	0.41324	0.43825	-0.38618	7.33075
6	-1.39120	0.64270	1.59753	-0.25551	-0.88884	0.41742	0.44514	-0.39430	6.93917
7	-1.37547	0.62625	1.59569	-0.22908	-0.89640	0.42400	0.45426	-0.38441	6.27337
True values	-1.40000	0.68000	0.38000	0.42000	-0.50000	0.34000	0.48000	-0.36000	

Table 5. The GELS estimates and errors ($\sigma^2 = 0.50^2$).

t	a_1	a_2	b_1	b_2	c_1	c_2	d_1	d_2	$\delta(\%)$
100	-1.29556	0.60106	0.78993	-0.05738	-1.00566	0.50000	0.21772	-0.39921	24.00496
200	-1.35170	0.62284	0.72791	-0.17117	-0.97636	0.48378	0.23471	-0.39313	18.67484
500	-1.39463	0.66964	0.68576	-0.24975	-0.81088	0.43350	0.28486	-0.32609	11.38673
1000	-1.39717	0.67697	0.68111	-0.29286	-0.80003	0.40089	0.35841	-0.31430	7.21108
2000	-1.39794	0.67927	0.66811	-0.30932	-0.75580	0.33439	0.44948	-0.34978	2.82803
3000	-1.39776	0.68060	0.67740	-0.31218	-0.76192	0.34050	0.44754	-0.35127	2.54321
4000	-1.39885	0.68081	0.67963	-0.31715	-0.76382	0.32895	0.46156	-0.36209	2.08623
5000	-1.39720	0.67832	0.67975	-0.31051	-0.76436	0.32523	0.46717	-0.35593	2.07695
True values	-1.40000	0.68000	0.38000	0.42000	-0.50000	0.34000	0.48000	-0.36000	

Table 6. The ME-LSI estimates and errors ($\sigma^2 = 0.50^2$).

k	a_1	a_2	b_1	b_2	c_1	c_2	d_1	d_2	$\delta(\%)$
1	-1.47912	0.78172	1.61154	-0.38537	-0.86029	0.56132	0.01262	0.01275	26.24172
2	-1.45866	0.76402	1.61271	-0.36071	-0.89708	0.35863	0.16498	-0.42877	14.15537
3	-1.44847	0.74803	1.61122	-0.34205	-0.82058	0.30052	0.36704	-0.49176	6.00016
4	-1.44704	0.74236	1.60803	-0.33738	-0.70132	0.31712	0.51709	-0.32436	3.71254
5	-1.45638	0.75275	1.60891	-0.35410	-0.71047	0.33232	0.49803	-0.31756	2.33543
6	-1.44979	0.74606	1.60947	-0.34416	-0.74454	0.33851	0.46783	-0.34513	1.55322
7	-1.44925	0.74564	1.60938	-0.34287	-0.74297	0.33624	0.47073	-0.34572	1.43162
True values	-1.40000	0.68000	0.38000	0.42000	-0.50000	0.34000	0.48000	-0.36000	

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